Mortality and Deprivation

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Outline

- Index of Multiple Deprivation (IMD)
- Observed mortality by age and year
- Simultaneously modelling mortality in all deciles:
    - choosing common parameters
    - comparing the goodness of fit

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Index of Multiple Deprivation

The IMD is a weighted combination of seven indices of deprivation:

- Income (22.5%)
- Employment (22.5%)
- Education (13.5%)
- Health (13.5%)
- Crime (9.3%)
- Barriers to Housing and Services (9.3%)
- Living environment (9.3%)

source: GOV.UK

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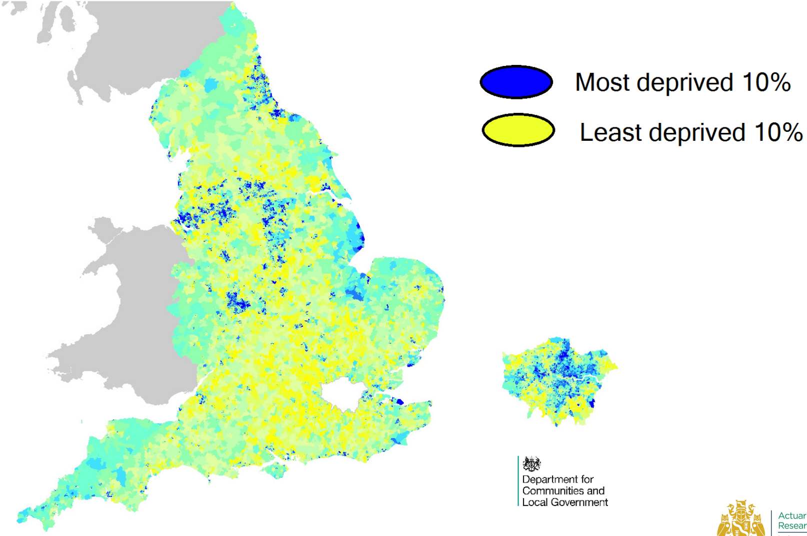
source: GOV.UK

- just over 30,000 LSOAs (Lower Layer Super Output Area) in England
- ordered and split into ten deciles:
  10% most deprived, ..., 10% least deprived

Index of Multiple Deprivation (IMD) areas



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Outline

- Index of Multiple Deprivation (IMD)
- **Observed mortality by age and year**
- Simultaneously modelling mortality in all deciles

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Data

- We consider mortality data for males in England for the ten IMD deciles (2015).
- ages: 40-89, years: 2001-2015
- source: Office for National Statistics

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There are more deaths per 1,000 lives in the most deprived areas compared to the least deprived areas.

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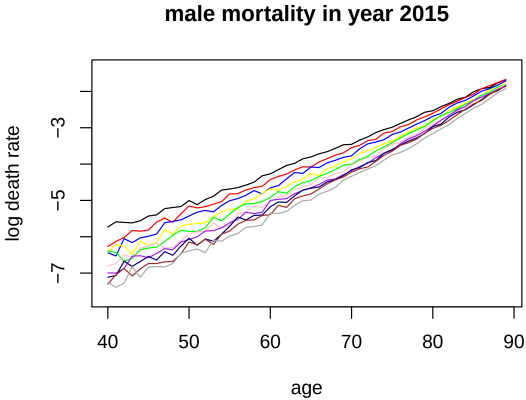
Mortality rates by IMD decile in 2001

- roughly linear in age (Gompertz line)
- mortality differentials are decreasing with age

Mortality rates by IMD decile in 2001

- similar shape as male log mortality, but lower level, slightly smaller differences
- again, mortality differentials are decreasing with age

Mortality rates by IMD decile in 2001



- similar shape as in 2001
- differences at high ages are larger

Mortality rates by IMD decile

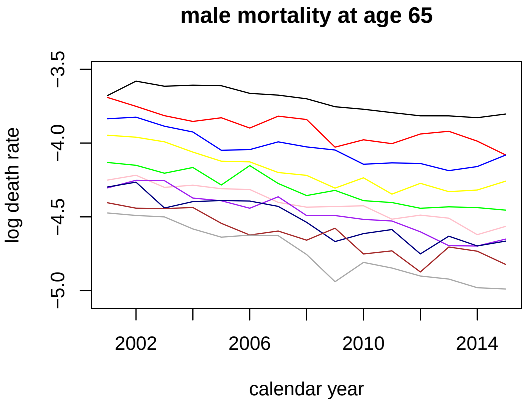
- downward shift from 2001 to 2015
- differences between most deprived and least deprived have increased since 2001
- higher differences at high ages

Number of deaths per 1,000 lives by IMD decile - Change over time

	Deaths per 1,000 lives		
	most deprived	least deprived	ratio
2001	25.3	11.4	2.219
2005	27.0	9.7	2.784
2010	23.0	8.2	2.805
2015	22.3	6.8	3.279

Males aged 65

Mortality rates by IMD decile for males aged 65



- downward trend strongest for least deprived

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Stochastic Models for the Number of Deaths by Decile

Moving away from the number of deaths per 1,000 lives, we consider a stochastic mortality model:
For each period (calendar year) t, age x and IMD decile i we define

D_{xti}: Number of deaths,

E_{xti}: Central exposure-to-risk

m_{xti}: force of mortality

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$$D_{xti} \quad \sim \quad \text{Poisson}\,(m_{xti} E_{xti})$$

So, expected number of deaths, $\mathrm{E}[D_{xti}] = m_{xti} E_{xti}$

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So, expected number of deaths, $\mathrm{E}[D_{xti}] = m_{xti} E_{xti}$

Aim: to compare different models for the force of mortality m_{xti}.

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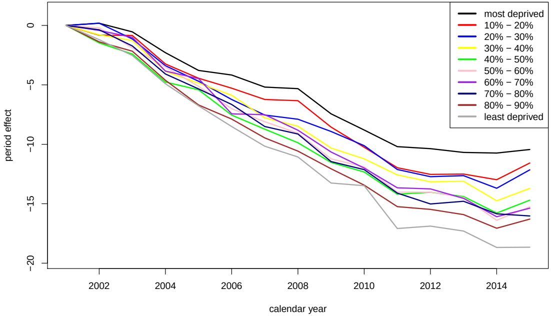
Lee-Carter model fitted to individual IMD decile

$$\log m_{xti} = \alpha_{xi} + \beta_{xi}\kappa_{ti}$$



- downward trend strongest for least deprived
- no improvements for most deprived since 2011
- slowdown of improvements for least deprived since 2011

Focus of ARC research - Models

Focus of our research:

- Stochastic models that describe mortality experiences in all socio-economic groups simultaneously.
- Model uncertainty addressed by comparing a wide variety of models (Goodness of fit, robustness, ...)
- Leading to projections, and more importantly, mortality scenario generation allowing us
  - to put probabilities on certain scenarios and ...
  - then use those for Value at Risk calculations, annuity pricing, etc.

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Models

All considered models are variants of group specific Lee-Carter type models with the extension to a second age-period effect by Renshaw & Haberman (2003):

$$\log m_{xti} \quad = \quad \alpha_{xi} + \beta^1_{xi}\kappa^1_{ti} + \beta^2_{xi}\kappa^2_{ti} + \gamma_{ci}$$

where $c = t - x$ is the cohort (year of birth).

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where $c = t - x$ is the cohort (year of birth).
Specific versions include models with:

common age effect : $\alpha_{xi} = \alpha_x$

non-parametric common age effects : $\beta^k_{xi} = \beta^k_x$ (Kleinow, 2015)

fixed age effects : constant $\beta^1_{xi} = 1$ and linear $\beta^2_{xi} = x - \bar{x}$, where $\bar{x}$ is the mean age in the data set. (Plat, 2009)

common period effects : $\kappa^k_{ti} = \kappa^k_t$ (Li and Lee, 2005)

and variations with and without cohort effects.

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Some research questions

$$\log m_{xti} \quad = \quad \alpha_{xi} + \beta^1_{xi}\kappa^1_{ti} + \beta^2_{xi}\kappa^2_{ti} + \gamma_{ci}$$

- What parameters should be chosen to be group specific and which parameters are common?
- Should age-effects be estimated?
- Should we include cohort effects (common or group specific)?
- What parameters show the greatest differences between IMD groups?

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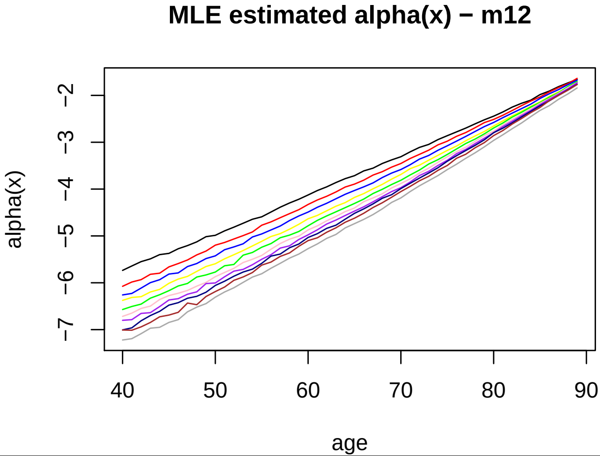
Parameter estimates - m12 - the most general model

$$\log m_{xti} = \alpha_{xi} + \beta^1_{xi}\kappa^1_{ti} + \beta^2_{xi}\kappa^2_{ti} \text{ (Renshaw\&Haberman, 2003)}$$



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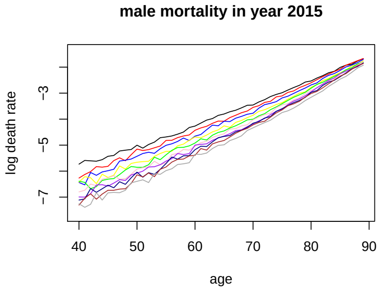

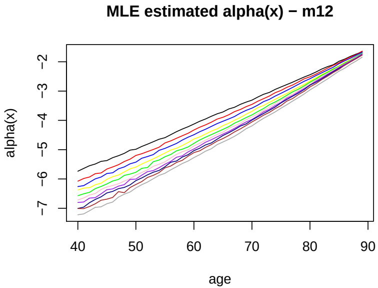

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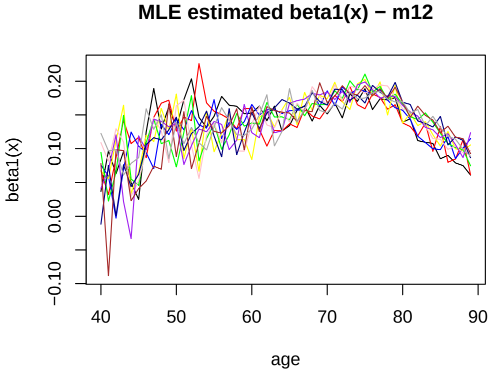

- general shape seems to be the same for all deciles
- differences might not be important

Parameter estimates - m12 - the most general model

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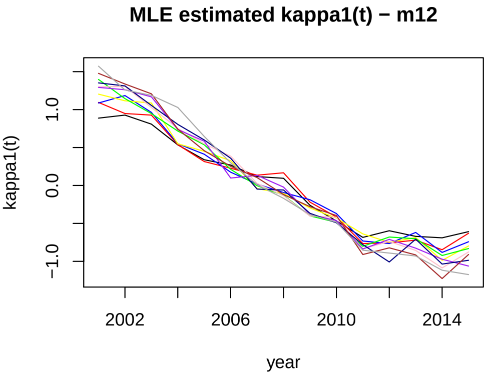


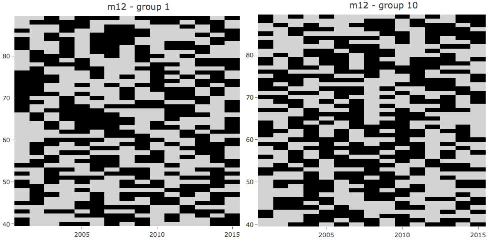
strongest slope for least deprived decile (grey)

weakest improvements for most deprived decile (black)

Residuals - m12 - the most general model

$$\log m_{xti} = \alpha_{xi} + \beta^1_{xi}\kappa^1_{ti} + \beta^2_{xi}\kappa^2_{ti} \text{ (Renshaw\&Haberman, 2003)}$$

Residuals = Fitted log rates - observed log rates



black shows positive residuals (fitted rates are too high)
grey shows negative residuals (fitted rates are too low)
no obvious pattern - good

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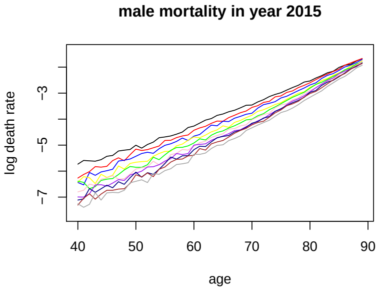

Parameter estimates - m12 - the most general model

Bayesian Information Criterion: $k \log n - 2 \log(L)$

	Log-likelihood	parameters	constraints	d.o.f.	BIC
m12	-30,131.47	1800	40	1760	75,966.82

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- good fit
- very large number of parameters

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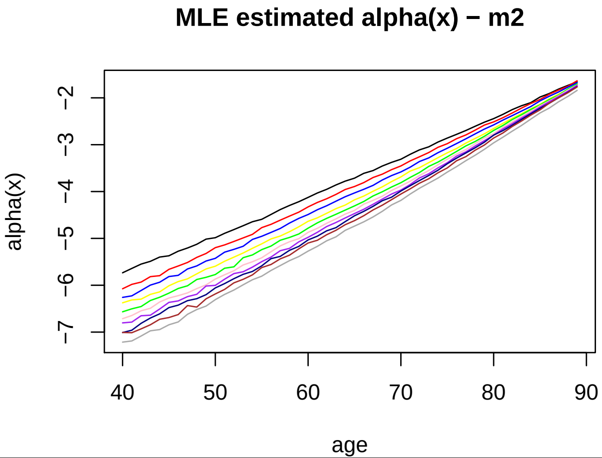
Parameter estimates - m2 - common beta

$$\log m_{xti} = \alpha_{xi} + \beta_x^1 \kappa_{ti}^1 + \beta_x^2 \kappa_{ti}^2 \text{ (Kleinow, 2015)}$$



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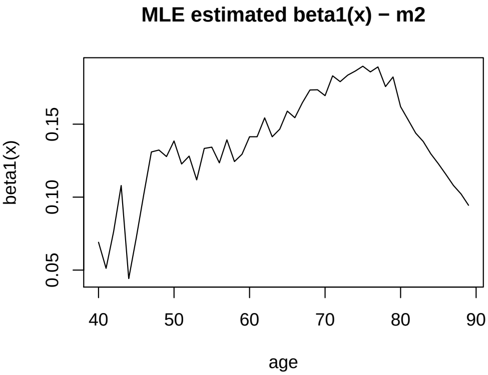

general shape is similar to shape of individual β^1 in previous model

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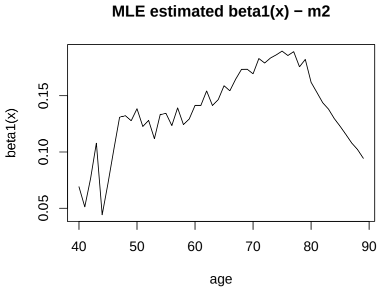

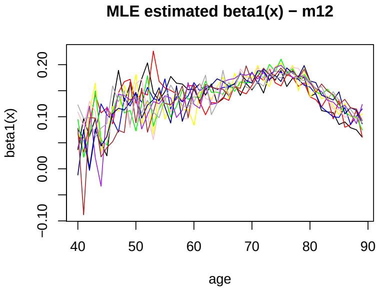

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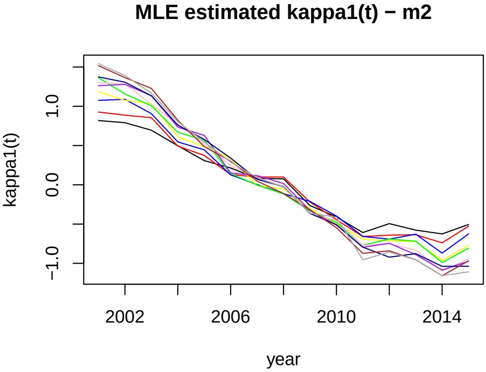

similar to period effects in previous (more general) model

clearly different improvements for different IMD deciles

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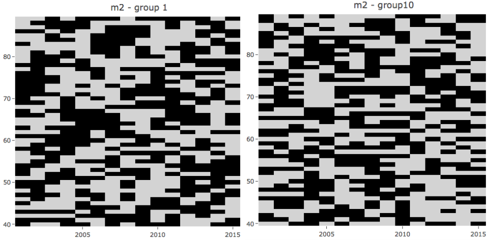

black shows positive residuals (fitted rates are too high)
grey shows negative residuals (fitted rates are too low)
no obvious pattern - age effects seem to be common to all deciles

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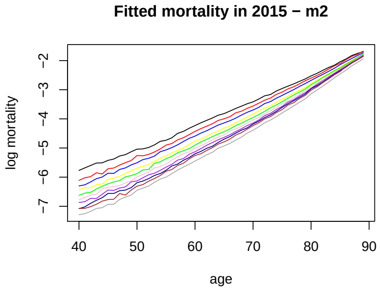

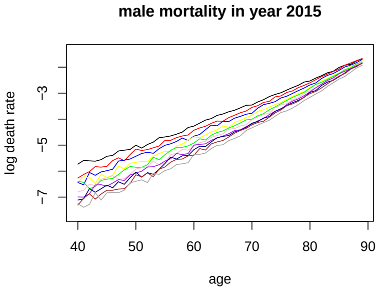

Parameter estimates - m2 - common beta

Bayesian Information Criterion: $k \log n - 2\log(L)$

	Log-likelihood	parameters	constraints	d.o.f.	BIC
m2	-30,591.20	900	22	878	69,016.49
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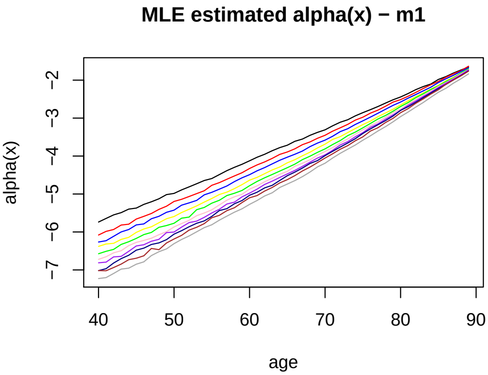
- reasonable fit
- smaller number of parameters than m12
- BIC improved

Parameter estimates - m1 - constant & linear beta

$$\log m_{xti} = \alpha_{xi} + \kappa^1_{ti} + (x - \bar{x})\kappa^2_{ti} \text{ (Plat, 2009)}$$



The first age effect is almost identical to α in the other two models

Parameter estimates - m1

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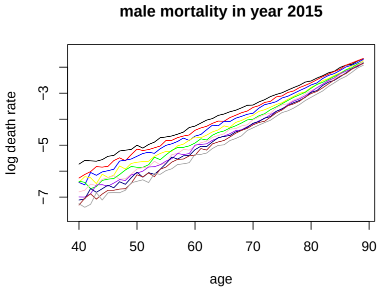

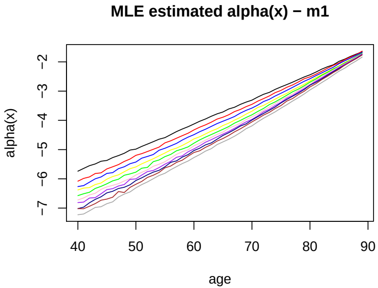

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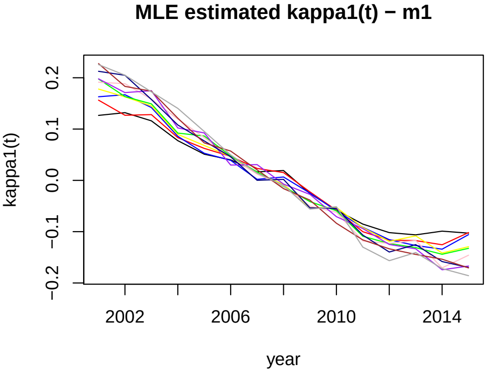

The first period effect is also similar to the period effects in the other two models

Parameter estimates - m1

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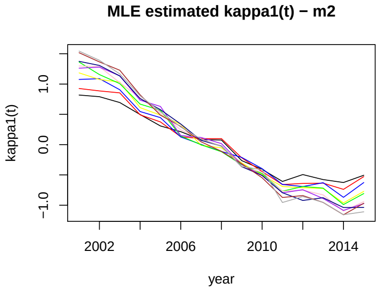

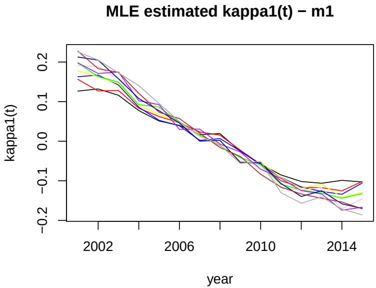

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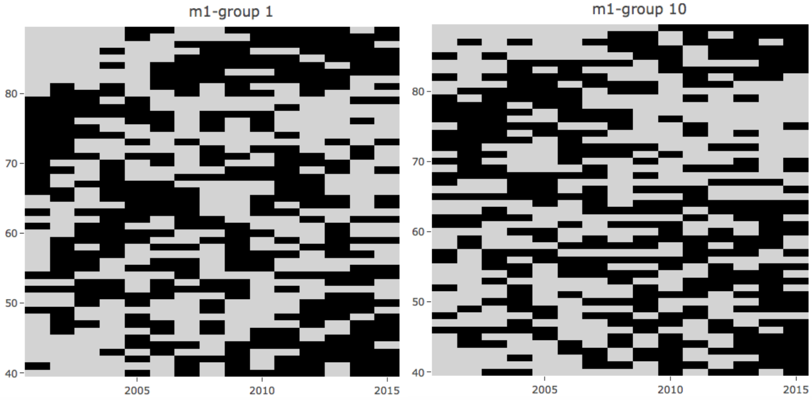

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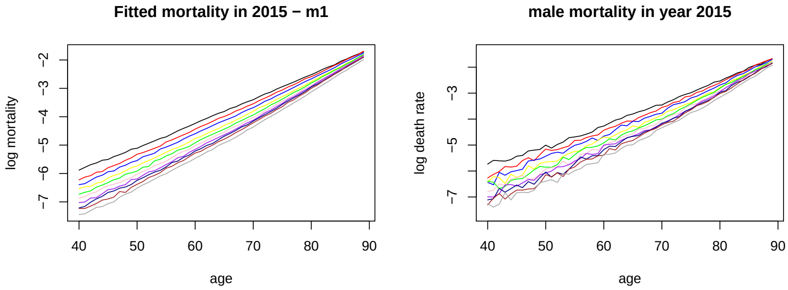

Fitted mortality rates are now very smooth

Parameter estimates - m1

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- model does not fit as well as m2 and m12 (lower likelihood)
- number of parameters reduced further
- but BIC does not improve

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Next step

$$
\begin{array}{llcll}
\text{m1:} & \log m_{xti} & = & \alpha_{xi} + \kappa^1_{ti} + (x - \bar{x})\kappa^2_{ti} & \text{(Plat, 2009)} \\
\text{m2:} & \log m_{xti} & = & \alpha_{xi} + \beta^1_x \kappa^1_{ti} + \beta^2_x \kappa^2_{ti} & \text{(Kleinow, 2015)} \\
\text{m12:} & \log m_{xti} & = & \alpha_{xi} + \beta^1_{xi} \kappa^1_{ti} + \beta^2_{xi} \kappa^2_{ti} & \text{(Renshaw\&Haberman, 2003)}
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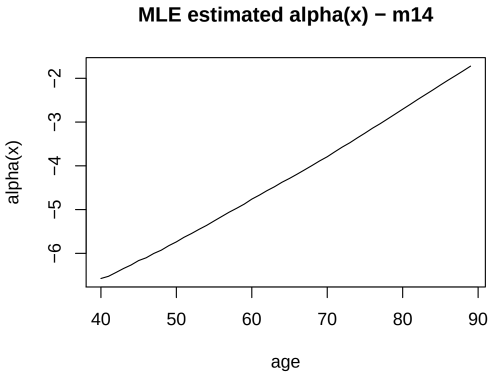
\end{array}
$$

Try a common alpha for m1 and m2.

Parameter estimates - m14 - common alpha and beta

$$\log m_{xti} = \alpha_x + \beta_x^1 \kappa_{ti}^1 + \beta_x^2 \kappa_{ti}^2$$



The main age effect is now very smooth

Decile specific noise is averaged out

Parameter estimates - m14 - common alpha and beta

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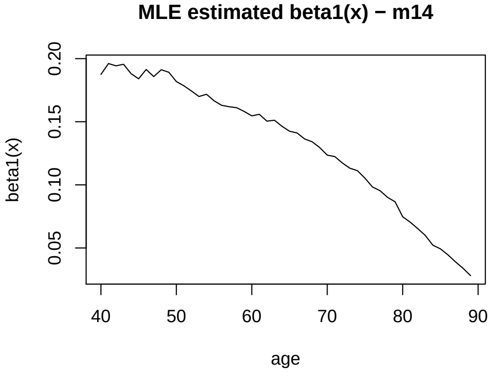

Different shape compared to common β in previous model

Parameter estimates - m14 - common alpha and beta

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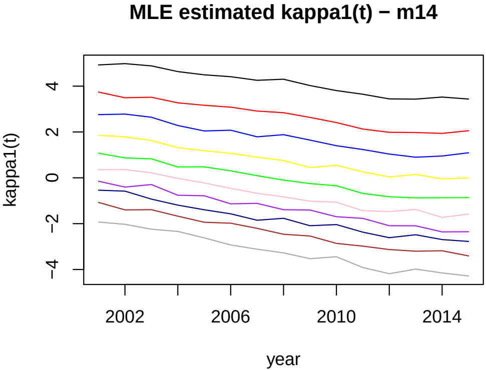
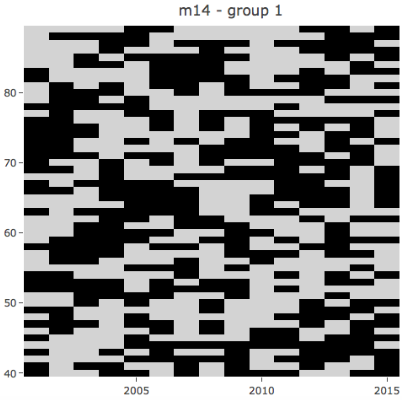
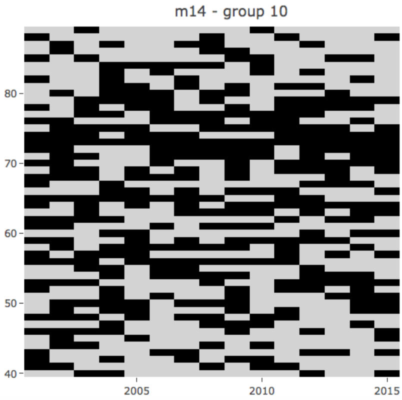


The period effect κ is now showing differences in the level of mortality (since α are common)

Parameter estimates - m14 - common alpha and beta

$$\log m_{xti} = \alpha_x + \beta_x^1 \kappa_{ti}^1 + \beta_x^2 \kappa_{ti}^2$$





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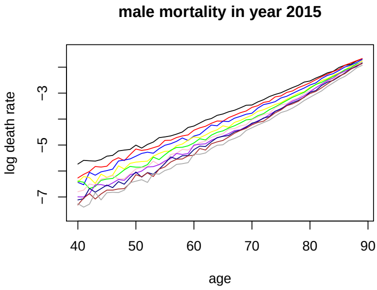

Fitted rates are now very smooth

Parameter estimates - m14 - common alpha and beta

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Bayesian Information Criterion: $k \log n - 2 \log(L)$

	Log-likelihood	parameters	constraints	d.o.f.	BIC
m1	-31,403.22	800	20	780	69,766.10
m2	-30,591.20	900	22	878	69,016.49
m12	-30,131.47	1800	40	1760	75,966.82
m14	-30,852.96	450	4	446	65,685.42

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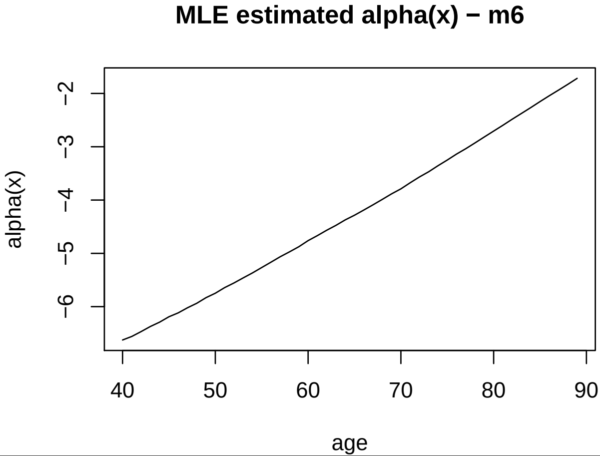
- higher likelihood than m1 with fewer parameters
- best BIC

Parameter estimates - m6 - M1 with common alpha

$$\log m_{xti} = \alpha_x + \kappa^1_{ti} + (x - \bar{x})\kappa^2_{ti}$$



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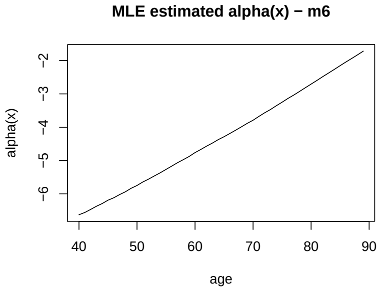

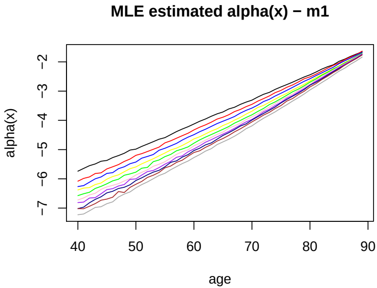

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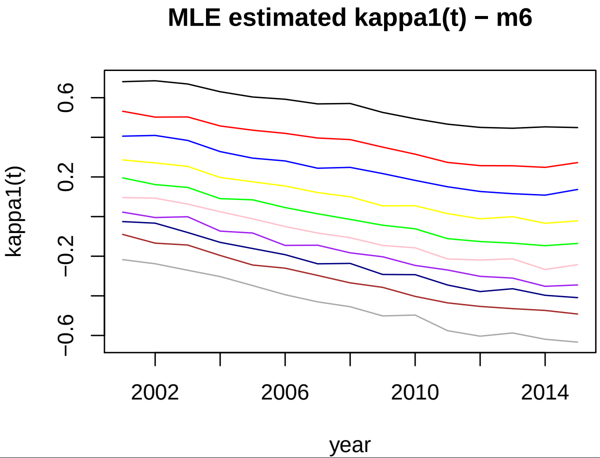

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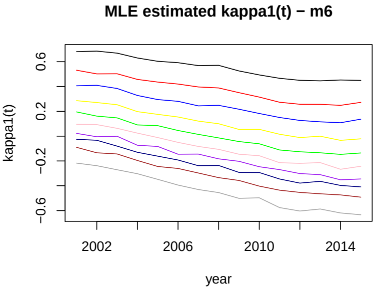

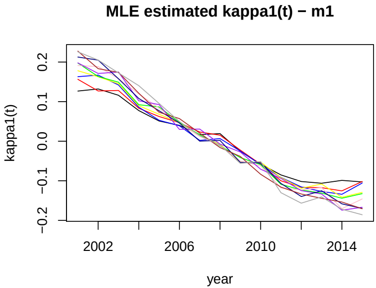

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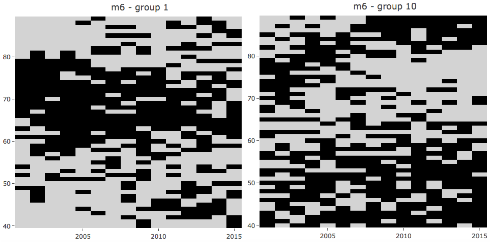
Residuals = Fitted log rates - observed log rates



black shows positive residuals (fitted rates are too high)
grey shows negative residuals (fitted rates are too low)
seems to be a pattern (age direction)

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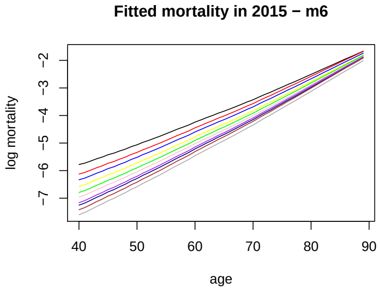

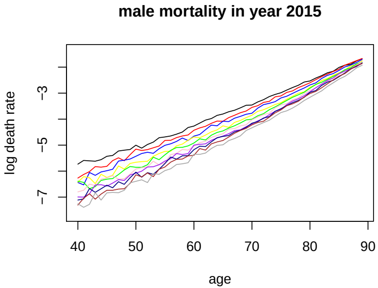

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- smallest number of paramters
- second best model in terms of BIC
- better BIC than m1

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- Cohort effects do not improve the fit of those models.
- If a cohort effect is included it should be a common cohort effect.

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Thank You!

Questions and Comments