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DEFINING AMBITION

# Population Structure: Impact on Asset Values

Stephen Bonnar, Lori Curtis, Kate Rybzynski, Mark Zhou - uWaterloo

Miguel Leon-Ledesma, Jaideep Oberoi - University of Kent

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# Motivation

- Baby boomers entering retirement
  - concerns about diminished returns, compromised pensions

- Higher old-age dependency ratio may lead to
  - less saving (dissaving) and investment
  - shift of individuals’ asset allocations toward low risk / low return assets
  - reduced labour force growth

- All with implications for asset returns and retirement outcomes

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# Model Framework

- Overlapping Generations Model (OLG) with
  - aggregate uncertainty
  - two asset classes (risky and risk-free)
  - multi-pillar pension systems (savings, pay-go, earnings based)
  - endogenous labour supply
- Generates standard age-specific labour, consumption, asset holdings, and portfolio allocation qualitatively consistent with the data
- Older population $\longrightarrow$ moderately lower asset returns

# Demographics

- Overlapping generations, $j \in \{1,2,\ldots 20\}$, ages 18 - 97
- Five life stages: YW, MW, W, SR, R

- Intra-cohort heterogeneity, $i \in \{1,2\}$, baseline $i = 1$
- Fertility rate: n
- Survival probability: $\phi_j^i \in [0,1), \phi_J^i = 0$

$$N_{j,t}{}^{i} = (1+n)\mathrm{X}^{i} N_{0,t-1}, \qquad \text{if } \; j = 1$$

$$N_{j,t}{}^{i} = \phi_{j-1}^{i}\mathrm{X}^{i} N_{j-1,t-1}, \qquad \text{if } \; 1 < j \leq J$$

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# Household Time Endowment

- $$H_j = H(1 - FC_j - FE_j), \qquad j \in \{YW, MW\},$$
  $$H_j = H, \qquad j \in \{W, SR, R\}.$$

- Fixed constant H units of time
- Education (FE) and child rearing (FC)
- SR can work maximum of $\iota_p H$

# Household Preferences

- Periodic utility from Consumption and Leisure

$$u^i(c,h) = \frac{c^{1-\gamma_c}}{1-\gamma_c} + \Psi \frac{(H_j - h)^{1-\gamma_h}}{1-\gamma_h}$$

- Coefficient of relative risk aversion: $\gamma_c$
- Parameter that regulates the Frisch elasticity of labour supply: $\gamma_h$
- Utility weight of leisure relative to consumption: $\Psi$

# Assets

- Total asset holdings: $\theta^i_{j,t}$

*Risk Free Bonds*

- Return in period t+1: $\overline{r}_t$
- Share of total assets in risk free: $\eta^i_{j,t}$
- Zero net supply: $\Sigma_j \Sigma_i \eta^i_{j,t} \theta^i_{j,t} N^i_{j,t} = 0$

*Risky Capital*

- Return in period t+1: $r_{t+1}$
- Share of total assets: $1 - \eta^i_{j,t}$
- Total capital: $K_t = \Sigma_j \Sigma_i (1 - \eta^i_{j,t}) \theta^i_{j,t-1} N^i_{j,t-1}$

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# Production

- $Y_t = z_t K_t^{\alpha} H_t^{1-\alpha}$ and $K_{t+1} = (1-\delta)K_t + q_t I_t$

$$\ln(z_t) = \rho \ln(z_{t-1}) + v_t \text{ where } v_t \sim N(0,\sigma_z^2)$$

$$\ln(q_t) = \rho_q \ln(q_{t-1}) + v_{q,t} \text{ where } v_{q,t} \sim N(0,\sigma_q^2)$$

$$corr(\sigma_q^2, \sigma_z^2) = 0$$

- Aggregate efficient labour is: $H_t = \Sigma_j \Sigma_i \epsilon_j^i h_{j,t}^i N_{j,t}^i$
- Baseline: $\epsilon_j^i = 1$ $\longrightarrow$ no age or type-specific labour productivity

# Pay-as-you-go Pension

- Pay-as-you-go proportional pension scheme

$$p_{j,t} = 0, \qquad \text{if} \qquad j \in \{YW, MW, W\},$$

$$p_{j,t} = \frac{\tau_s w_t H_t}{\sum_{j \in \{SR,R\}} \sum_i N_{j,t}^i} \qquad \text{if} \qquad j \in \{SR, R\}.$$

- Fixed tax, $\tau_s$, on labour income uniformly distributed to retirees

# Partially Funded Pension

- Partially funded, employment earnings based pension

$$p^G_{j,t} = 0, \qquad \text{if} \qquad j \in \{YW, MW, W\},$$

$$p^G_{j,t} = \kappa_j \left( \frac{w_{ss} \Sigma_i \epsilon^i_{SR-1} h^i_{SR-1,SS} N^i_{SR-1,SS}}{\Sigma_i N^i_{SR-1,SS}} \right) \qquad \text{if} \qquad j \in \{SR, R\}.$$

- Government taxes working cohorts at rate, $\tau^G_s$ , and pays out a fraction, $\kappa_j$ , of pre-retirement income

# Government Budget

- *In the two pillar model:*

$$\Sigma_{j=SR}^{W} p_j^G N_{j,t}^i = [\eta_G(1+(1-\tau_r)\overline{r}_{t-1}) + (1-\eta_G)(1+(1-\tau_r)r_t)]\theta_G$$

$$+\tau_s^G w_t H_t + B_t^G$$

*In the three pillar model:*

$$\Sigma_j \Sigma_i \eta_{j,t}^i \theta_{j,t}^i N_{j,t}^i + \eta_G \theta_G = B_t^G$$

$$K_t = \Sigma_j \Sigma_i (1-\eta_{j,t}^i)\theta_{j,t-1}^i N_{j,t-1}^i + (1-\eta_G)\theta_G$$

- Government holds a pool of assets, $\theta_G$, with proportion, $\eta_G$, in risk-free bonds, and issues bonds, $B_t^G$, to balance the budget.

# Taxes and Bequests

- *Taxes*
  - Consumption tax: $\tau_c$
  - Labour income tax: $\tau_h$
  - Investment income tax: $\tau_r$
  - Tax on pension income: $\tau_p$
  - Tax for pay-go pension and social security: $\tau_s$ and $\tau_s^G$

- *Bequests*
  - Base model has accidental bequests only
  - Bequest motive – utility from leaving bequest $v(X) = \Gamma \dfrac{X^{1-\gamma_b}}{1-\gamma_b}$

# Household Decision

- $V_j^i(s_t;z_t) = \max\left[c_{j,t}^i, h_{j,t}^i, \theta_{j,t}^i, \eta_{j,t}^i\right]\left\{u^i(c_{j,t}^i, h_{j,t}^i) + \beta\phi_j^i E_t\left[V_{j+1}^i(s_{t+1};z_{t+1})\right]\right\}$

  subject to

$$(1+\tau_c)c_{j,t}^i + \theta_{j,t}^i \leq$$
$$\left\{(1-\tau_s-\tau_s^G-\tau_h)w_t\epsilon_j^i h_{j,t}^i + x_{j,t}^i + (1-\tau_p)(p_{j,t}+p_j^G) + \xi_t - HC\right\}$$

  where

$$\begin{aligned} h_{j,t}^i \leq H_j^c &= H_j && \text{if} \quad j \in \{YW, MW, W\} \\ &= \iota_p H_j && \text{if} \quad j \in \{SR\} \\ &= 0 && \text{if} \quad j \in \{R\} \end{aligned}$$

$$HC_j = 0 \qquad \text{if} \quad j \in \{YW, MW, W\}$$

$$HC_j = 0.2\exp\left(\frac{4(j-12)}{J-12} - 4\right) \qquad j \in \{SR, R\}$$

# Household Decision - Oldest Generation

- $V_J^i(s_t;z_t) = \max\left[c_{j,t}^i, \theta_{j,t}^i, \eta_{j,t}^i\right]\left\{u^i(c_{J,t}^i, 0) + \beta E_t\left[v^i(X_{J+1,t+1}^i)\right]\right\}$

  where

$$X_{J+1,t+1}^i = \left[\eta_{J,t}^i(1+(1-\tau_r)\overline{r}_t) + (1-\eta_{J,t}^i)(1+(1-\tau_r)r_{t+1})\right]\theta_{J,t}^i$$

  and

$$v(X) = \Gamma\frac{X^{1-\gamma_b}}{1-\gamma_b}$$

# Firm Decision

- Firm maximizes profits, resulting in:

$$r_t = \alpha z_t K_t^{\alpha-1} H_t^{1-\alpha} - \delta$$

$$w_t = (1-\alpha) z_t K_t^{\alpha} H_t^{-\alpha}$$

where $\delta \in [0,1]$

# Recursive Competitive Equilibrium

- Value functions
- Household policy functions for consumption, labour supply, total saving, and share of savings invested in risk-free bonds
- Inputs (capital and hours of labour) for the representative firm
- Government policy for pensions and bond issuance
- Rates of return for risk-free bonds and risk capital, and wages

*Such that in each period, the:*

- household problems are solved,
- the competitive firm maximizes profits, and
- all markets clear.

# Lifecycle Consumption, Labour, Assets

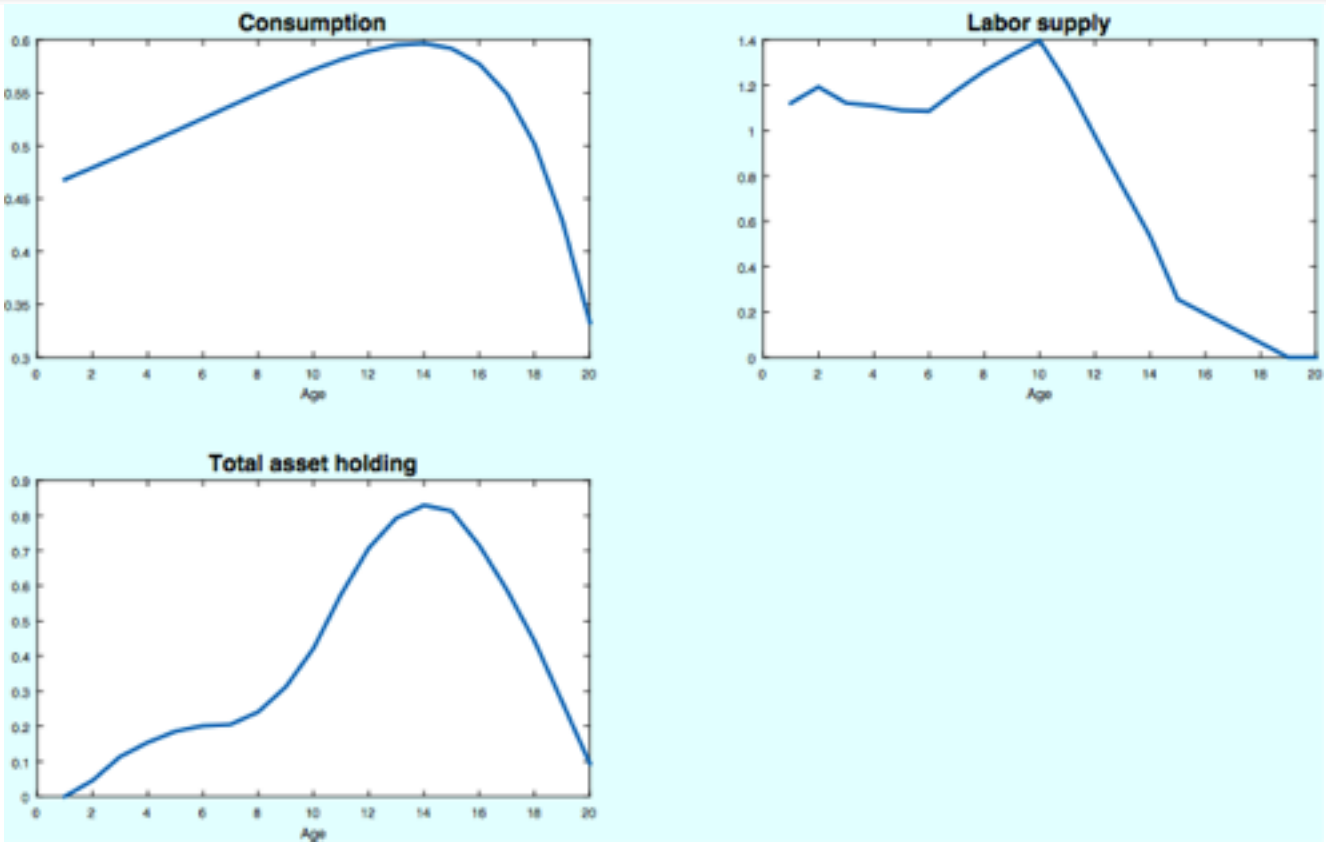


*Figure 1 - Lifecycle consumption, labour and asset profiles*

# Observed Age-Specific Portfolio Allocation

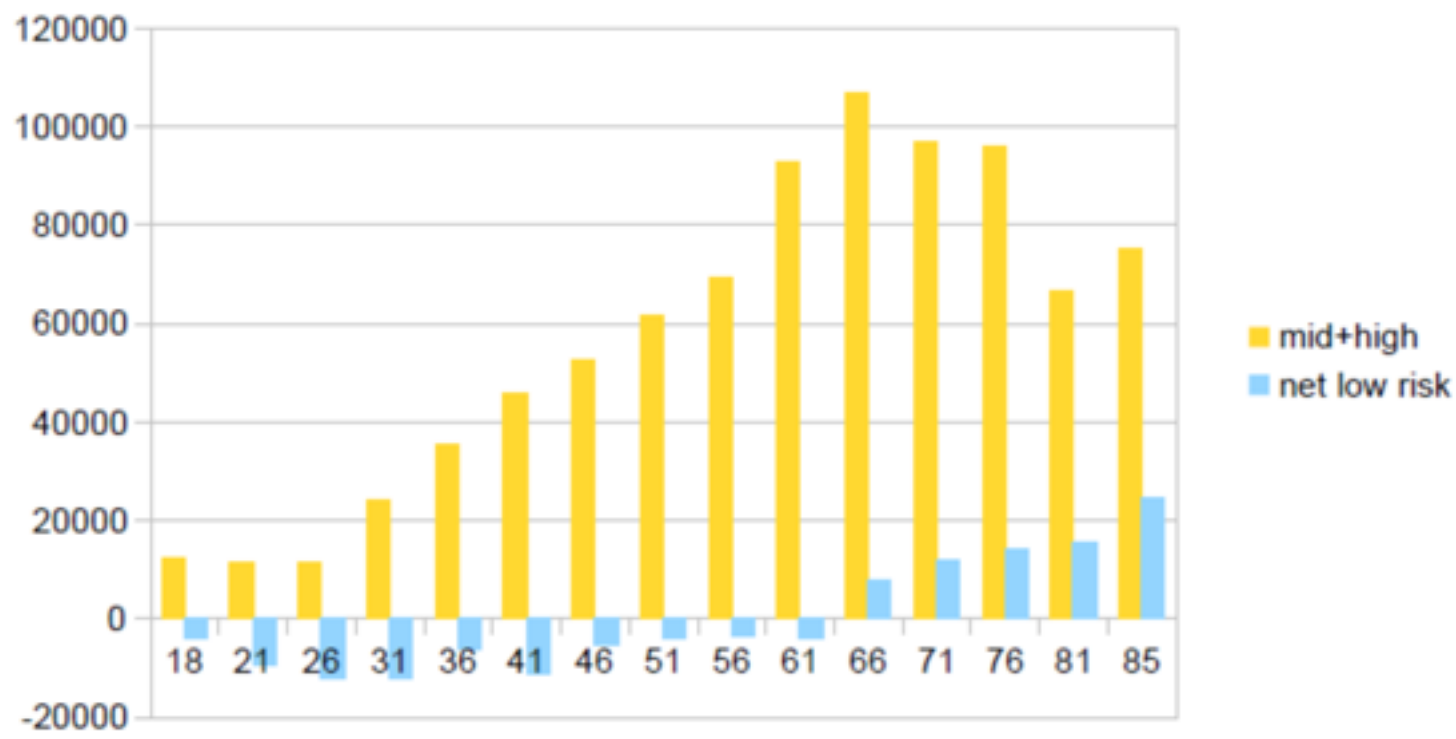


*Figure 2 - Portfolio allocation by age: risky vs net low-risk financial assets*

# Portfolio Allocation — 2 Pillar Pension Model

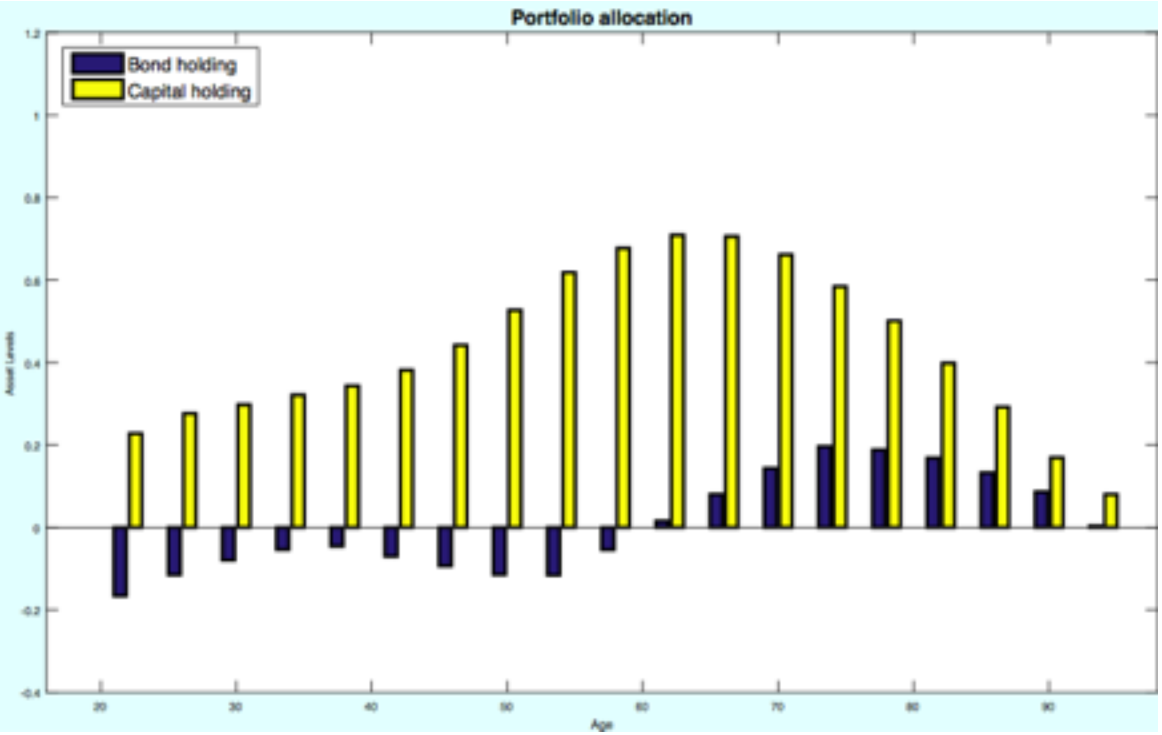


*Figure 3 - Portfolio allocation in 2 pillar model*

# Portfolio Allocation
# 3 Pillar Pension Model — Baseline

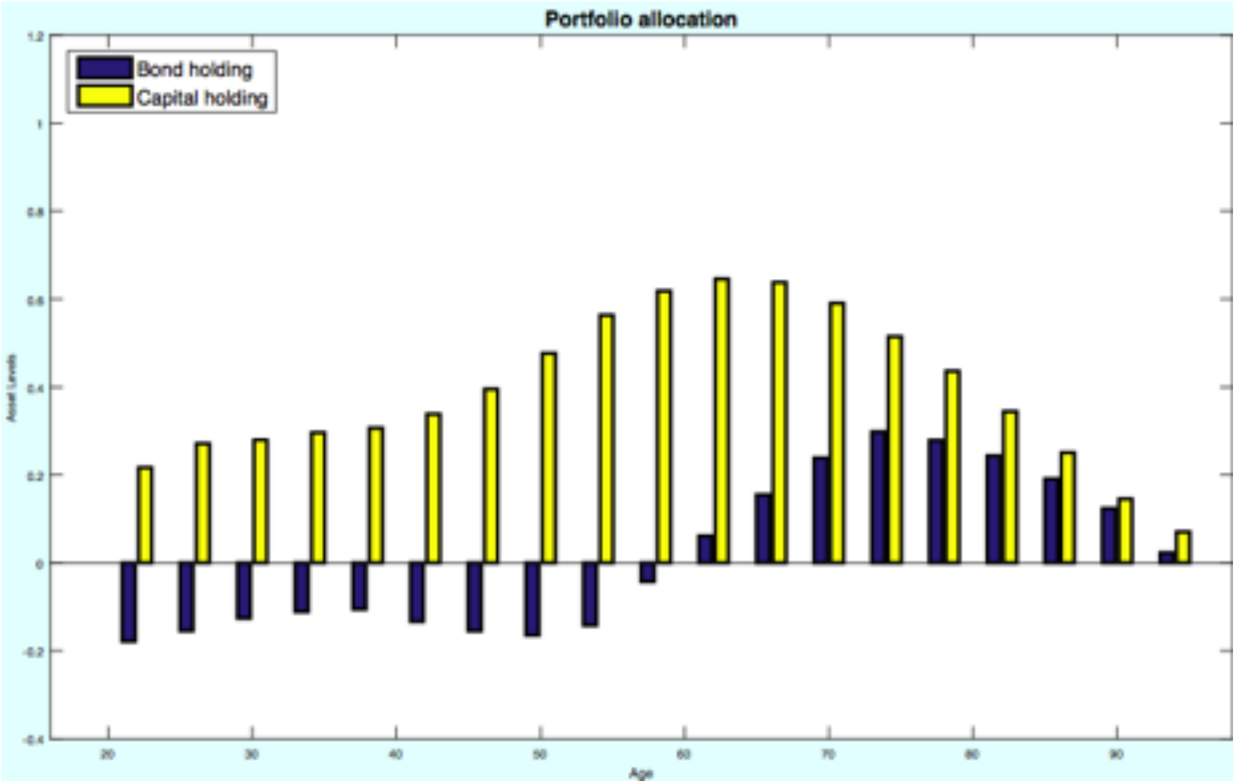


*Figure 4 - Portfolio allocation in 3 pillar model*

# 3-Pillar Model Results under Alternative Demographic Structures

| Variable | Base-3 Pillar | +10% | +20% | -10% | -20% |
|---|---|---|---|---|---|
| $E_t(r_{t+1})$ | 0.2855 | 0.2788 | 0.2735 | 0.2919 | 0.2965 |
| $\overline{r}_t$ | 0.2851 | 0.2784 | 0.2730 | 0.2915 | 0.2961 |
| Priv. risky assets / GDP | 0.5223 | 0.5233 | 0.5362 | 0.5214 | 0.5206 |
| $c_{20,t}$ | 0.3327 | 0.3771 | 0.4183 | 0.2984 | 0.2512 |

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# Portfolio Allocation
# 3 Pillar + Health Costs + Bequests

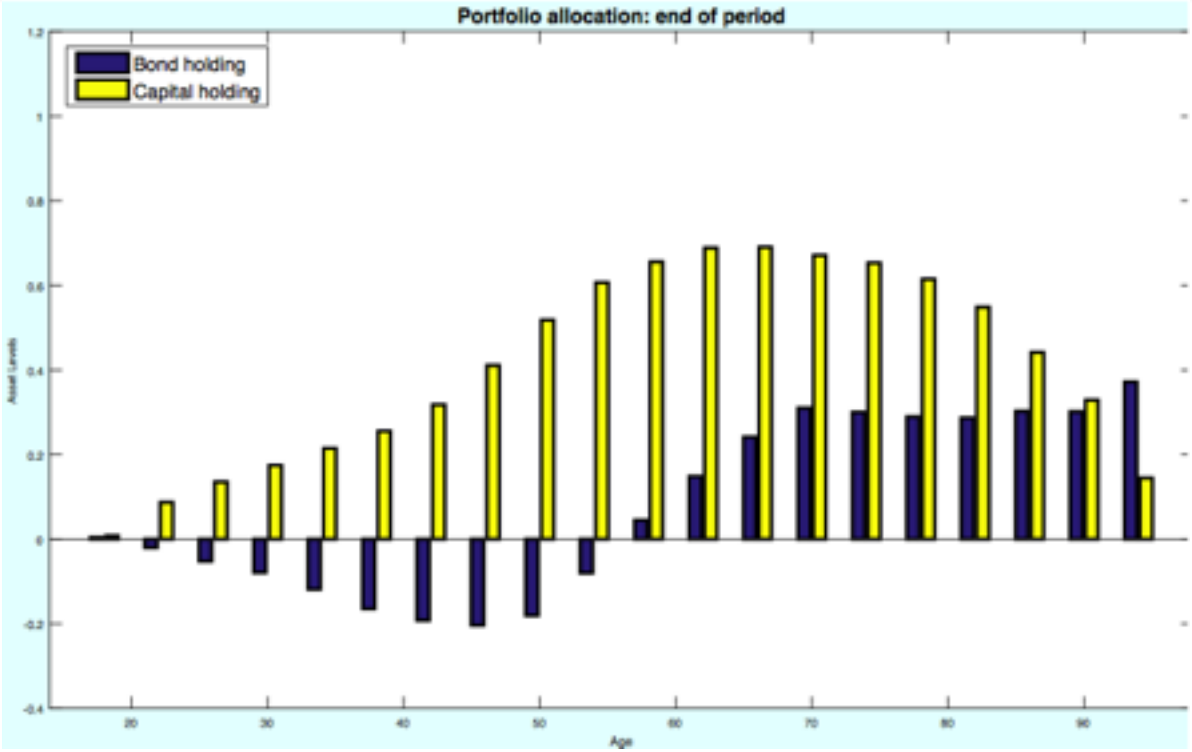



*Figure 5 - Portfolio allocation 3 pillar + health costs + bequests*

# Discussion and Next Steps

- Asset prices are moderately lower with older population: Higher survival probability for age 65+ (max 20% at j=J) → approximately 4% lower returns on capital and on bonds

- Higher replacement ratio → lower asset accumulation

**Next Steps:**

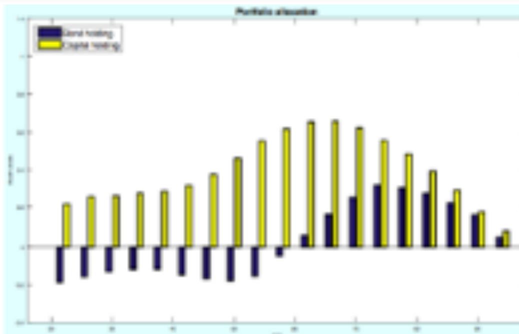
- Improve portfolio allocation match
  - consumption saturation
  - intra-cohort heterogeneity
- Explore further intra-cohort heterogeneity models

# Appendix

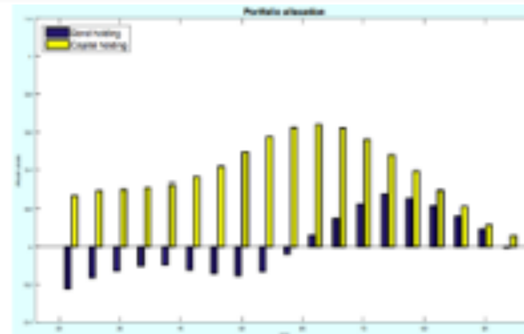
# Parameterization

| Parameter | Value | Description |
|---|---|---|
| H | 4 | Time available to household (one period represents 4 yrs) |
| $\beta$ | 0.8515 | Discount factor (0.95 annual) |
| $\alpha$ | 0.3 | Capital's share of production |
| $\rho_z$ | 0.4401 | Autocorrelation coefficient for TFP |
| $\sigma_z$ | 0.0305 | Std. Deviation of error for TFP process |
| $\rho_q$ | 0.4401 | Autocorrelation coefficient for IST |
| $\sigma_q$ | 0.1221 | Std. Deviation of error for IST process |
| $\delta$ | 0.192 | Depreciation Rate |
| n | 0.0489 | Population Growth rate |
| $\gamma_c$ | 2.0 | Relative risk aversion – consumption |
| $\gamma_b$ | 2.0 | Relative risk aversion - bequest |
| $\gamma_l$ | 3.0 | Inverse of intertemporal elasticity of substitution of non-market time |
| $\Psi$ | 21.833 | Utility weight of non-market time relative to consumption |
| $\tau_c, \tau_r, \tau_p$ | 0.123, 0.167, 0.167 | Tax rates on consumption, investment income, pension, |
| $\tau_h + \tau_s + \tau_s^G$ | 0.167 | Tax on labour income |
| $ratio_s$ | 1.0 | Proportion of labour tax to social security |
| $\iota_p$ | 0.08 | Labour constraint for SR |

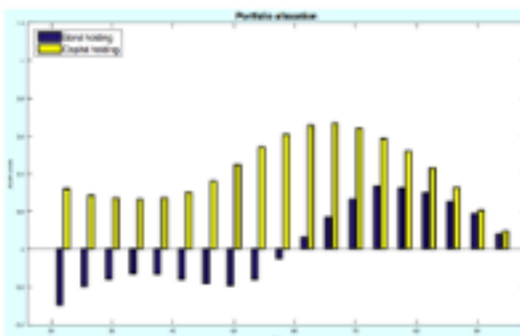
# Portfolio Allocation under Alternative Demographic Structures



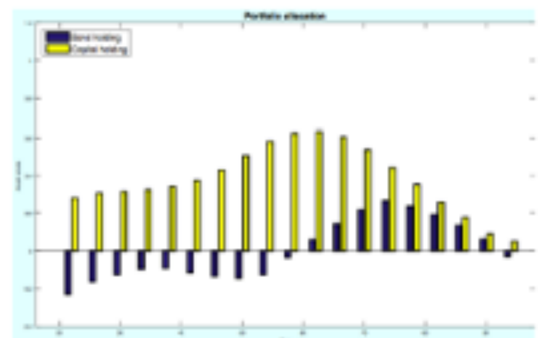
(a) Baseline + 10% maximal higher survival probability



(b) Baseline - 10% maximal higher survival probability



(c) Baseline + 20% maximal higher survival probability



(d) Baseline - 20% maximal higher survival probability

*Figure 6 - Portfolio allocation alternative demographics*

# THANK YOU FOR YOUR ATTENTION!