Practical Challenges in Reserve Risk

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Abstract

This document describes the challenges experienced in quantifying reserve risk with practical solutions suggested. It provides a pragmatic development towards advancing the assessment of reserve risk. It is aimed at general insurance actuaries who want an understanding of the challenges encountered in reserve risk without becoming too engaged in the technical details which can often be a distraction to the underlying issues. However, owing to the nature of the subject, technical content is unavoidable and can be found in the appendices.

The first part of this document is a review of the fundamental concepts of reserve risk while the second part provides readers with a deeper understanding of the challenges faced within practical implementation of reserve risk. The authors hope that this publication will generate discussions in the general insurance actuarial community prompting the development of a common best practice approach to reserve risk   As such it is recommended that this document to be circulated to all practitioners involved in reserving and reserve risk.

Keywords: *reserving, reserve risk, stochastic reserving, bootstrap, Mack's model*

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Notes on Reproduction of this Document

The motivation of this document is to educate and encourage improvement of practices across the general insurance industry in the assessment of reserve risk. As such the authors encourage this document to be distributed freely but in its full original form. All constructive comments are welcomed via email as provided above.

About the Authors

This document shares the personal views of both authors, Kevin Chan and Michael Ramyar, reflecting their knowledge and research gathered in the field of reserve risk.

Kevin Chan has been a Fellow of the Casualty Actuarial Society since 2004. He has been with XL Group for over 10 years, leading the reserve risk work stream since 2012. He has prior experience in Casualty reserving and pricing, as well as consulting from PriceWaterhouseCoopers Toronto.

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Both Kevin and Michael have extensive knowledge of reserve risk, not only have they participated in the Institute and Faculty of Actuaries (IFoA) Working Party on Pragmatics Stochastic Reserving, they have presented at various conferences including plenary session titles “Alleviating Reserving Stress” at the 2015 General Insurance Research Organising Committee (GIRO) conference.

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SECTION 1 – INTRODUCTION

The move towards a risk-based capital environment and the ever changing regulatory environment has meant that the quantification of reserve risk has become vital for the wider general insurance sector. In response to this, over the recent past a considerable amount of research and development has already taken place to better quantify reserve risk.

The authors firmly believe that all existing practices and knowledge should be consolidated so that common generally accepted market practice guidance can be created for the present and future reserving and reserve risk practitioners. This paper attempts to address this by providing an overview of the common practical challenges faced and proposing possible solutions. Ideally, if stochastic reserving techniques were more embedded by the wider market into the reserving process, the derivation of the reserve uncertainty will have less challenges. However, the highly technical nature of stochastic reserving has in many instances created a barrier for reserving actuaries and practitioners. As a consequence, this has led to one of the biggest practical challenges of evaluating reserve risk which will be explored in this document.

The remainder of this document is structured as follows:

- Section 2 provides a brief tutorial of what reserve risk is and how it can be estimated. Not only does this section cover basic terminologies and fundamental concepts used throughout the document, it also serves as an educational piece to the topic of reserve risk.

- Section 3 is the core of this document detailing practical challenges experienced in reserve risk. Building on the basic concepts of reserve risk provided in the previous section, the reader will be able to understand why these challenges exist. A clear understanding of the cause of these challenges will allow practitioners to develop high-level solutions.

  The challenges are categorised into the following three sub-sections with cross-references provided throughout the document since most of the challenges are interlinked:
  - *Inconsistency between reserving and reserve risk* allowing the reader to fully understand the source of the most problematic issue; namely "scaling" the reserve distribution mean to the best estimates of reserve.
  - *Common misconceptions* are another major challenge creating obstacles in the advancement of reserve risk. Providing clarity on these concepts and offering potential solutions is a step to eliminating these issues. This sub-section also leads to a theoretical study on *"How wrong is our estimate of reserve risk"* which is outlined in detail in Appendix A.
  - *Other practical challenges* include other issues relating to reserve risk that are worth highlighting. Some of these are covered extensively in other literature including the authors' previous work.
- Section 4 provides the conclusion, which summarises the salient points of discussion in this document.

Please note that all suggested approaches should not be interpreted as the only approach but are intended to initiate discussion for further development of a common industry wide generally accepted market practice. A list of references including suggested further readings are indicated in square brackets throughout the document which corresponds to the list in the *Reference* section.

SECTION 2 – A BRIEF TUTORIAL OF RESERVE RISK

2.1 Basic Concepts of Reserve Risk

Across the general insurance actuarial community there are large variations in the fundamental understanding of what reserve risk is and what it is estimating. This issue is the most fundamental challenges in embracing and advancing reserve risk market practices. The remainder of this section aims to address this challenge.

2.1.1 What is Reserve Risk?

It is important to establish conventional terminology in order to describe the basic concept of reserve risk. The **required reserve** is the payment ultimately be paid to settle all incurred claims. This value can take on a range of possible outcome and can only be known with certainty when all claims have been finally settle in the future. The **estimated reserve** is a point estimate of the average required reserve resulted from reserving methodologies using parameters derived from historical data.

Reserve risk is the uncertainty in the estimation of the required reserve. It is related to the difference between the required reserve and the estimated reserve. In an ideal world, if the true mean of the required reserve is known, the estimated reserve would be set as the true mean and the reserve risk would simply be quantified by the *variance* of the distribution of required reserve *R*:

$$Var(R) = E[(R - E(R))^2]$$

The *variance* measures the volatility of a random process by taking the average of the squared error between all *possible outcome* and the *true mean outcome*. The *standard deviation* is the square root of this quantity.

The term ***prediction error*** arises because within the well-known variance definition, the *true mean outcome*, *E(R)* is actually unknown. In practice, *E(R)* is replaced by an estimate $\hat{R}$. Thus the result is a *mean square error of prediction* (MSEP), not a *variance*, and the square root is not a *standard deviation* but a ***prediction error***. The definition is as follows:

$$MSEP(\hat{R}) \;=\; E\left[(R - \hat{R})^2 \,|\, D\right] \qquad (2.1.1)$$

The MSEP is conditioned on data *D* because data is needed to derive parameters of a selected model that in turn are used to calculate $\hat{R}$. The reader is encouraged to compare and contrast the similarities and differences, term by term, of MSEP versus variance. Take careful note of the “hats”, these signify a deterministic point estimate, rather than the random variable.

With some arithmetic work, the MSEP can be broken down into two components:

$$MSEP(\hat{R}) = \;\; Var[R|D] + \;(\hat{R} - E(R|\, D))^2 \qquad (2.1.2)$$

The first term is related to the *process risk*, the uncertainty arises from the difference between all *possible required reserve* and the *true mean required reserve*, and the second term is related to the *parameter error*, the difference between the *estimated reserve* and the *true mean required reserve*. These correspond to the two sources of reserve risk. Note that once an estimate $\hat{R}$ is made, the second term is a fixed constant hence termed “parameter error” instead of “parameter risk” for the rest of this paper.

Now, the difference between the *standard deviation* and *prediction error* is clear. The prediction error not only has captured the intrinsic uncertainty of a random process around the *true mean*, but also the error in estimating the true mean. These sources of error are further discussed below.

2.1.2 Models

In order to quantify the *prediction error* of reserve, a model is specified. It is impossible for any mathematical model to capture the full multitude of factors of the real underlying process. A model is a simplification of reality whilst reflecting the significant dynamics. In reserving context, regardless of how complex the reserving methodology is, it will not fully reflect the reality. In terms of reserve risk, since the prediction error of reserve relates to both the required reserve (R) and the estimated reserve ($\hat{R}$), the error in model specification is not only related to how good the model reflects reality, but also how consistent the model is with the reserving process. This point will be further examined in Section 3.1.4.

2.1.3 Sources of Reserve Risk

In summary, uncertainty in reserve can arise through the following three sources though the focus of the prediction error is only on the latter two:

Model specification error – A model is a simplification of the real world. The difference between the selected model and how reality behaves leads to model specification error. For reserve risk, this includes the inconsistency between reserving and reserve risk. It is a key source of risk which will be discussed in great detail in Section 3.1.

Process risk – This is the risk that the actual required reserve eventually ended up higher or lower than the mean required reserve. Even if the true model and true parameters are known, this uncertainty would still exist. Using the chain-ladder terminology, process error is the uncertainty around the projected bottom claim triangle.

Parameter error – Once a model is selected, historical data can be used to estimate the parameters, which are in turn used to project the future. Since historical data contains historical process error, the parameters are only an estimate of the true parameters. Hence, even if there is no model specification error, parameter error would still exist due to the noise embedded in past data. Using the chain-ladder terminology, the parameters are the selected link ratios and since they are estimated using top claim triangle, it leads to parameter error.

2.2 How to Estimate Prediction Error

It is very important to remember the actual prediction error, $MSEP(\hat{R})$, is unknown, but, given historical data, it can be estimated and is denoted by $\widehat{MSEP}(\hat{R})$. In practice, this is often expressed as a percentage of the estimate and is referred to as the coefficient of variation, *CoV*.

It is worth noting that in addition to the three sources of uncertainty mentioned in the previous section, there is a further uncertainty often got overlooked. Since any estimate contains uncertainty, the estimation of uncertainty in reserve must also contain uncertainty itself. This uncertainty in estimation of reserve risk will be discussed in Section 3.2.7 with further details provided in Appendix A.

For now, two general approaches to derive prediction error are illustrated below:

1. Analytical approach
2. Empirical approach

2.2.1 Analytical approach

Perhaps the most well-known formula for reserve risk is from the Mack 1993 paper [9] where a prediction error is derived for chain-ladder reserve estimates under Mack's model. The formulae below are for reference, and the detail is not important for this document.

$$\widehat{\text{msep}(\hat{R}_i)} = \hat{C}_{iI}^2 \sum_{k=I+1-i}^{I-1} \frac{\hat{\sigma}_k^2}{\hat{f}_k^2} \left(\frac{1}{\hat{C}_{ik}} + \frac{1}{\sum_{j=1}^{I-k} C_{jk}} \right)$$

$$\widehat{\text{msep}(\hat{R})} = \sum_{i=2}^{I} \left\{ (\text{s.e.}(\hat{R}_i))^2 + \hat{C}_{iI} \left(\sum_{j=i+1}^{I} \hat{C}_{jI} \right) \sum_{k=I+1-i}^{I-1} \frac{2\hat{\sigma}_k^2 / \hat{f}_k^2}{\sum_{n=1}^{I-k} C_{nk}} \right\}$$

$\hat{f}_j$ – Volume all-year weighted link ratio of development period j

$C_{i,j}$ – Cumulative claims at accident period i, development period j

$\hat{\sigma}_j$ – Estimate of volatility parameter of development period j

Equation 2.2.1 - Mack's formulae for prediction error

The reader must note the following: firstly, the MSEP has a hat demonstrating that it is an estimate and secondly, it gives only the prediction error and not the full predictive distribution.

The above formulae are occasionally incorrectly referred to as the "Mack's Model"; in fact, Mack's model was assumed in order to derive the above formulae. For explicitness Mack's model is:

$$\boldsymbol{C_{i,j+1} = f_j C_{i,j} + \sigma_j \sqrt{C_{i,j}} \varepsilon_{i,j+1}}$$

$\varepsilon_{i,j}$ – Zero mean, unit variance, independent and identically distributed noise of origin period i, development period j

Equation 2.2.2 - Mack's model

Equation 2.2.1 is described as "distribution free" meaning that in the calculation of prediction error, no assumption is made about the noise variable $\varepsilon_{i,j}$ beyond its first two moments. This is not to be confused with "model free" since it assumes the cumulative claim amounts follow Mack's model above. Note that $\varepsilon_{i,j}$ is not completely without other constraints since $C_{i,j+1}$ must remain positive.

Another commonly used model is the Over-dispersed Poisson known as the "ODP model" [13]. The ODP and Mack's models are commonly used since the mean reserve estimates produced from these models are exactly the same as the chain-ladder reserve estimates and hence they are implemented in popular reserving software. Note that no analytical formula exists for the ODP model.

There are non-chain-ladder based models explored in existing literature on reserve risk, for example, both Alai, et al. [1] [2] and Mack [10] [11] provide a Bornhuetter-Ferguson-based model and England and Verrall [7] consider a wide range of stochastic reserving models for general insurance including the log-normal model. The important point is that a model is required when estimating prediction error, either by using an analytical formula if it exists or by using an empirical approach, the latter of which is covered next.

2.2.2 Empirical approach

Bootstrapping is a statistical technique for estimation; in particular, it can be used for estimating the prediction error in reserving. It is a powerful technique because no analytical formula is needed to obtain the prediction error. Thus, given a model, it avoids the challenge of deriving the formulae as in the case for the Mack's model, and that is a closed-form formula needs to exist in the first place.

In England & Verrall [5] and England [6], bootstrapping was applied to chain-ladder based models. The mechanics of exactly how bootstrapping can be implemented is well documented and is a common functionality found in many reserving software. A brief summary of the bootstrap technique is:

- Assume a model for the data
- Fit the data to the model to obtain a set of parameters
- Derive the fitted data using these parameters
- Subtract the fitted from actual to get a set of residuals
- Create many sets of pseudo data using the residuals
- Project every set of pseudo data to derive a range of possible outcomes

Not only does this method avoid the challenges in deriving a formula, it produces a full predictive distribution. It is important to note that a bootstrap technique can be applied to any model with appropriate data. It is common to hear references to the "bootstrap model", however, there is no such thing as a "bootstrap model"! A model is required before one can apply the bootstrap technique.

A common approach in estimating prediction error for reserve is often referred to as the "Mack bootstrap". This assumes the same Mack's model in Equation 2.2.2 as described in the previous section but is estimated using the bootstrap technique. An important link must be made, given that Mack's model formulae approach and Mack bootstrap approach are estimating the same quantity, if consistent assumptions are used then the estimated prediction errors would be the same from both approaches aside from simulation error. This fact allows one to validate the implementation of a bootstrap when the prediction error formula is available. Recall that bootstrapping is simply a numerical technique and it is the appropriateness of the model that is important when considering the validity of the results.

It is also important to note that the bootstrap technique is not the only possible empirical approach. In principle one could take any model and directly parameterise it using the data but the difficulty is ensuring both parameter and process error are incorporated when estimating the prediction error. For example, if a Generalized Linear Model (GLM) is used to fit the data, the full prediction distribution can be derived using the Markov Chain Monte Carlo (MCMC) simulation. Readers who are interested in exploring this further can refer to Scollnik [14] and England and Verrall [8]. However, reserve risk practitioners are advised that such sophisticated models may actually create further disconnect between the reserving and reserve risk process as described the next section.

SECTION 3 – PRACTICAL CHALLENGES

From Section 2, it is clear that reserve risk should capture both the error in estimated reserve (parameter risk) and the uncertainty in required reserve (process risk). In addition, a model is required in quantifying reserve risk. Given these concepts and terminologies, the challenges and limitations faced in quantifying reserve risk become clearer and are discussed below.

Note that for the rest of this document, the word "method" refers to deterministic reserving point estimate approach, while "model" refers to stochastic approach which provides a distribution of reserve estimates. Also, as this section focuses on the fundamental concepts in order to illustrate practical challenges in reserve risk, readers are advised not to be distracted by the of time horizon of reserve risk at this point, i.e. the ultimate-view versus the 1-year view. Section 3.3.4 contains a briefly discussion on this topic with reference to other literature.

3.1 Inconsistency between Reserving and Reserve Risk

Since all established reserving methods are deterministic without considering the stochastic aspect of reserve, the reserving process does not often have a view of the full reserve distribution before setting the mean. It is only in the recent decades that the general insurance industry has developed an increasing interest in the concept of reserving beyond a point estimate.

Consequently, instead of deriving the reserve distribution first and then using its mean to set the reserve, practitioners are now encountering the task of deriving a reserve distribution and hoping that the resulting mean would not deviate too much from the predetermined reserve amount set independently by the reserving process. This leads to the common practice of scaling the reserve distribution so that the mean is exactly equal to the predetermined reserve and it's often described as the "scaling issue".

Given that the actuarial profession now faces the challenge of deriving reserve distributions around a well-embedded deterministic reserving process, to have meaningful reserve distributions and to reduce the scaling issue, consistency between these two processes is crucial. The rest of this section addresses why inconsistency exists in the first place. With an understanding these sources of inconsistency a more sensible scaling approach can be selected. In this section, a typical reserving process is assumed whereby reserve estimates are derived using various common chain-ladder based triangulation methodologies and the reserve risk is based on Mack or OPD bootstrap.

3.1.1 Inconsistencies in Data

One would expect at the very least that the two processes would start off with the same data, but in practice, one could face the following challenges:

Granularity of Data – As the reserve estimates are derived at a given granularity, it follows intuitively that the reserve risk should be analysed at the same granularity in order to truly quantify the reserve uncertainty. Unfortunately, the data volume at the reserving granularity level is often not sufficient for estimating reserve risk. It is sometimes unavoidable that the reserve risk has to be performed at a less granular level by grouping multiple reserving lines of business. This immediately creates an inconsistency; specifically, the mean reserve from the reserve risk analysis will differ to the reserve set by the reserving process. The reason is obvious since totalling the reserve estimates from chain-ladder on many claim triangles will be significantly different than the reserve estimates from chain-ladder on the aggregated claim triangle, differ both by accident year and in total. If the results were very similar, then reserving would not need to be performed at a more granular level in the first place.

Claim Triangle Period – Similar to the granularity reason above, quarter-quarter triangles may be more suitable to reserve for short-tail business but it could be too sparse for reserve risk analysis and hence a year-year triangle is used. Again, chain-ladder analysis based on different triangle periods could lead to significantly different reserve estimates between the reserving and reserve risk analysis.

Incurred or Paid Data – Even though incurred triangles may be used for reserving, some practitioners still prefer using paid triangles for reserve risk in order to satisfy particular assumptions for certain models; for instance, the restriction of positive incremental claims under the ODP or log-normal models. Using different data leads to a complete disconnection between reserving and reserve risk. It should be the data driving the choice of model rather than the model driving the choice of data.

Cut-Off Date – If the valuation date is at year-end, the obvious choice of data is a year-end triangle. However, owing to reporting timelines, some companies perform reserving based on an earlier cut-off date, e.g. Q3 triangle (9, 21, 33 months…), and roll-forward the results for year-end reserving. To be consistent with reserving, a bootstrap on Q3 triangles seems appropriate. However, how to eliminate the potential extra uncertainty arising from one additional quarter of unearned exposure is not obvious. On the other hand, reflecting the reduction in reserve uncertainty from the roll-forward process is also not intuitive.

Gross Versus Net Data – If reserving is done based on gross (of reinsurance recoveries) data before "netting down", then reserve risk should be done on a consistent manner by modelling gross data before netting it down. Conversely, if the reserving is done by modelling the net data directly, then the reserve risk should also be done on net data. Any other way leads to inconsistency between the two processes and hence less meaningful results. More details of this can be found in Section 3.3.1.

3.1.2 Inconsistencies in Model

As mentioned in Section 2, a model is needed in order to derive reserve distributions and ideally a model that is consistent with the deterministic reserving method. Perhaps the greatest practical issue that practitioners currently face is that the reserving analysis often does not build around a model.

Method versus Model – The most popular stochastic reserving models are Mack's model and the ODP model since both are consistent with the chain-ladder method. As mentioned in Section 2.2.1, there are also Bornhuetter-Ferguson method based models. However, these models require more assumptions and are yet to be widely tested using actual claims experience in practice. As a result, the industry is limited to using chain-ladder based models for reserve risk even for lines of business where reserving is not based on the chain-ladder method.

Mix of Methods – It is common practice for reserving processes to use multiple methods, for example, Bornhuetter-Ferguson to set the reserve for the most recent accident years while using chain-ladder for more developed accident years.

Mixing multiple methods on the same triangle might be straightforward for deterministic reserving, but when it comes to stochastic reserving, how can models be mixed such that some years it is Bornhuetter-Ferguson based while some years are chain-ladder based? It is no longer just the mean that is in question, but the noise component or in fact the distribution of each cell in a claim triangle that needs to be considered.

For recent accident years where loss ratio reserving method is used, one common practice is to use underwriting risk CoV, but this raises many technical questions such as how to aggregate the volatility with other accident years or how suitable is this approach when underwriting risk and reserve risk are fundamentally different quantities (see Section 3.2.9).

It is evident that the use of a single model on a stochastic basis for the reserve risk estimation while multiple methods are used deterministically to estimate the underlying reserve are likely leads to an inconsistent output. Therefore, ultimately the choice of model one should use is as much a reserving question as it is a reserve risk question.

Non-Triangle Information – Since not all information is necessarily captured in a claim triangle, other adjustments or non-triangulation methods are often incorporated in setting reserve. A simple example is a line of business where separate reserve estimates are set for specific large claims or extra provisions are included to the indicated reserve from the standard reserving methods. In such instances where the reserve amounts do not come directly from triangulation methods, any triangle based model will not be able to fully capture the reserve uncertainty.

Since there is not an existing model that can fully reflect the reserving process, a single model is often assumed in reserve risk for the entire triangle, and most commonly it is the chain-ladder method that is assumed. Although it may be seen as a simplistic approach, this in itself should not prompt practitioners to seek a more complex model. Some practitioners have made minor tweaks to the bootstrap to include calendar year correlations while others have moved towards a GLM framework. However, fundamentally if the reserve estimates are not set using the same model, using a more sophisticated model for reserve risk will create further inconsistencies which leads to less meaningful reserve risk results.

3.1.3 Inconsistencies in Parameters

Even if the ideal case of both the data used and model selected are being consistent with reserving, the underlying parameter selections could still differ. For example, the chain-ladder in reserving might select a number of years in calculating loss development factors and the number of years selected can differ for each development period while Mack's model in reserve risk assumes all-year volume-weighted averages. In addition, curve fitted or judgementally selected loss development factors used within the reserving process would further complicate the issue. Even though some stochastic reserving software can overcome some of these issues by allowing users to adjust loss development factor methods, many inconsistencies still remain.

The inconsistency in parameters is further amplified by the inconsistency in data and model. For example, if reserving is performed at a more granular level whereby one line of business uses an all-year volume-weighted average while another line of business uses a 5-year volume-weighted average, it is impossible to have a loss development factor method for the combined triangle that replicates the reserve amount by accident year and in total.

3.1.4 The Scaling Issue

In an ideal world if reserving is done at the same level as the reserve risk using chain-ladder for all accident years without any of the inconsistencies mentioned above, then the mean of the reserve distribution from the Mack or ODP bootstrap will match the reserve estimate for every accident year aside from simulation error. Unfortunately, this is often not the case in practice and hence scaling the mean of the resulting reserve distributions to the estimated reserve is required.

The "scaling issue" should be seen as a form of model specification error. As mentioned in Section 2.1.2, since prediction error of reserve relates to both the required reserve and the estimated reserve, the model specification error actually consists of two components – first component relates to how good does the model reflects reality (i.e. the required reserve) and the second components related to how consistent the model is in replicating the reserving process (i.e. estimate reserve). It should be clear by now that the amount of scaling reflects the amount of the second component of the model specification error.

Despite this scaling issue being an obvious indication that reserve risk is not modelling what it should be modelled, the general approach in practice is to continue the process by scaling the distribution. There are three common scaling approaches to calibrating the reserve distributions in order for the mean to match the reserve estimate. These are as follows:

- **Multiplicative Scaling** – For each accident year, multiply the reserve distribution by a constant so that the mean ties with the estimated reserve of that accident year. This preserves the CoV by accident year but not necessarily the total CoV. Another option would be to maintain the CoV in total by multiplying the reserve distributions by the same constant for all accident years.

- **Additive Scaling** – For each accident year, add a constant to the reserve distribution so that the mean ties with the estimated reserve. This preserves the prediction error both by accident year and also at the total level.

- **Mixed Scaling –** A combination of additive and multiplicative scaling is applied to the reserve distribution. This allows the user to target a CoV, for example, if underwriting risk CoV is used for the recent accident years.

The common choice in practice is to use multiplicative scaling by keeping the same CoV. However, this implies the prediction error of the reserve estimate varies in proportion to the amount of model specification error which there is no reason to believe so. (Also, is CoV always an absolute risk measure? Refer to Section 3.2.3) Therefore, before selecting any of the scaling methods, it is important to understand the sources of inconsistency between reserving and the reserve risk process in order to select a more suitable scaling method. For example, if in addition to the chain-ladder reserve, there are additional reserve set for specific claims (e.g. a certain full limit loss pending to be settled), then it may make sense to fix the prediction error by selecting additive scaling instead of increasing the prediction error by using multiplicative scaling.

3.1.5 Summary

The main reason for a scaling issue to arise is the inconsistency between the reserving and reserve risk process which highlight the existence of model specification error. Given this, there is often regular discussion over which scaling method is more appropriate or more prudent. However, in the authors' opinion, the "additive-versus-multiplicative arguments" are a distraction to the main problem of why large scaling issues occur in the first place. Fundamentally all scaling methods are just a way to force a consistent output from both processes retrospectively and are therefore far from the ideal.

Knowing that it would be an almost impossible task for most companies to drastically modify their reserving process in the short run, the industry has to accept the scaling issue and practitioners have to settle with one of the scaling approaches in the meantime. A good understanding of the components of inconsistency between the reserving and reserve risk estimation processes should enable a more sensible scaling approach to be selected. As usual, validation of the selected scaling approach is imperative to ensure results are not unreasonable.

Ideally, to eliminate the scaling issue, there needs to be a complete consistency between reserving and reserve risk. The authors' suggestion for the industry is to incorporate stochastic reserving methodologies into the reserving process, either on an aggregated level or at an individual claim level, and eventually reserving and reserve risk would be a result of stochastic reserving. This is the only way to eliminate the scaling issue and to reduce the model specification error in reserve risk in the long run.

3.2 Common Misconceptions

Even if one were to achieve consistency between the reserving and reserve risk process, there still remains numerous challenges when it comes to the design and implementation of a reserve risk framework. This is driven primarily by possible gaps in knowledge that leads to a disproportionate amount of time and resource being spent tackling the wrong issues or seeking solutions to the wrong questions.

It is the authors' belief that a lack of commonality of understanding has led to many misconceptions surrounding the implementation of a reserve risk estimation process. The remainder of this sub-section seeks to address the common misconceptions.

3.2.1 "The bootstrap model doesn't work. We need a better model."

This misconception is already touched upon in the discussion of the "scaling issue", but given that it leads to one of the most common misconceptions in the industry, it is being discussed further below.

Firstly, as mentioned in Section 2.2.2, bootstrap is not a model but rather an estimation technique which requires an underlying model. Secondly, if the method used for reserving are consistent with the model used for reserve risk, then the mean of the reserve distribution will match the reserve aside from simulation error. Hence, there is nothing wrong with bootstrapping, but instead, inconsistencies between reserving and reserve risk exist in practice as discussed in Section 3.1 leads to the "scaling issue".

Some practitioners in the market are spending a great deal of resources into researching and implementing sophisticated models for reserve risk which will not resolve any of the fundamental issues unless reserving is also applying the same model to set the reserve. The problem is not finding a better models, but finding a model that is consistent with the reserving methods.

3.2.2 "Paid triangle should be used to estimates the reserve volatility"

Since incurred losses includes case reserves, a chain-ladder on incurred triangle will includes a provision for incurred but not enough reported (IBNER) and a provision for incurred but not reported ("pure IBNR"). This led some practitioners to believe estimating reserve volatility using incurred triangle will capture the volatility from IBNER and "pure IBNR" but not volatility from case reserves, and then expressing the CoV as a percentage of total reserve (where the denominator includes the case reserves) would further understate the volatility. Moreover, many practitioners prefer paid triangle due to the restriction of positive incremental claims under for ODP or log-normal models.

These four points should clarify this misconception:

- Incurred triangle already contains volatility from case reserves (which could offset some volatility of the paid movement) hence the resulting prediction error would reflect the volatility embedded in the triangle. Instead, paid triangle does not contain case reserves; any projection using paid will not contain any volatility from case reserves.

- Reserve risk is fundamentally about the misestimation of reserve estimates. The resulting distribution is only meaningful if it is derived using the same data used for reserving.
- It should be the data driving the choice of model rather than the model driving the choice of data. Practitioners should not use paid triangle for reserve risk solely because incurred triangle is not suitable for a particular model.
- How the volatility is being expressed, CoV as a percentage of total reserve in this case, is not important as along as it will be consistently applied to get back the same prediction error (i.e. the CoV will be multiply by total reserve that includes the case reserves)

In summary, if reserving is using paid triangle, the reserve distribution should be derived using paid triangle; if reserving is using incurred triangle, the reserve distribution should also be derived using incurred triangle.

3.2.3 "We had a good year so why has the CoV increased?"

It is not uncommon to hear a variation of the following reaction, *"We had a good year so why has the CoV increased?"*

Practitioners often spend a significant amount of time analysing the details of the reserve risk estimation mechanics to explain why the latest claims experience has not impacted the estimated reserve volatility (measured by CoV) in the same way as they expected. Since there are many moving components, to fully understand the change in volatility, it is advisable to focus on the movement of prediction error on an accident year by accident year basis rather than a total CoV across all accident years.

Using Mack's model as an example, the change in volatility from an addition diagonal of loss development factors can be explained by considering the impact of each of the following factors separately on the overall movement in prediction error:

Maturity – If the latest diagonal is exactly as expected, the prediction error would reduce since there is one less future development period contributing to the prediction error. This is purely due to the claims are being more mature.

Volume – Now replacing the expected diagonal by the actual diagonal but leaving the parameters (e.g. f's and σ's) unchanged. If the actual diagonal is higher/lower than expected for the accident year, the prediction error for all future development would increase/decrease for that accident year.

Parameter – Finally, update the parameters but including the actual diagonal in the volume-weighted average. The impact to f's are straightforward from the actual experience, but the impact to σ's is less obvious since too good an experience (much lower development than expected) can also increase the σ's. The movement of both parameters impact the prediction error in the same direction for all future development for all accident years that relies on those development periods.

It should be clear by now that the combination of above impact is very complex and the direction of movement will vary case by case let alone the additional accident year will also contribute to the overall prediction error. Moreover, the change in reserve estimates from last year would have a direct impact to the CoV as it impacts the denominator. Hence, practitioners should not infer the direction of CoV movement purely from the experience. A full analysis of change should be conducted by breaking down the prediction error movement into above components in order to fully understand the driver of reserve volatility.

Refer to [3] for more detail of how the latest diagonal impacts the reserve volatility.

3.2.4 Is the CoV an absolute risk measure?

Often in academic literature, reserve risk is quantified as a prediction error, square root of MSEP, while in practice the prediction error is expressed as a percentage of the reserve, the CoV. A misconception is that the CoV is always the correct risk measure to use. As mentioned in the previous two misconceptions and in the discussion on multiplicative scaling in Section 3.1.4, since CoV is a percentage of reserve, sometimes it is difficult to get a sense of the absolute amount of risk without discussing the prediction error. As another example; it is not uncommon to hear the question "Why is net of reinsurance CoV greater than gross of reinsurance CoV?" since one would expect a reduction in reserve volatility from its reinsurance program. Indeed, the net prediction error (the numerator) is lower. However, since the net reserve (the denominator) is also lower; this can lead to a higher net CoV. Basically, the reinsurance protection in place can lead to a proportionally higher reduction in reserve than the prediction error.

In many situations, the absolute prediction error is a better risk measure, rather than being potentially obscured as percentage. Other risk measures could include percentiles, or more specifically, the 99.5th percentile minus the reserve as a proxy capital measure.

In conclusion as highlighted above, careful consideration must be given to determine whether the CoV is the most appropriate risk measure for each situation. This highlights the point that CoV should not be viewed in isolation in all situations as a risk measure. This is especially important when using a CoV as a benchmark or making comparisons across lines of business and this leads to the next two topics.

3.2.5 Using the CoV directly from bootstraping an industry triangle

When performing reserving analysis on a sparse claim triangle where the volume of claims is insufficient, it is a standard practice to source loss development factors from an industry triangle of a comparable line of business. This is a sensible approach due to the "law of large numbers", where the noise around the mean reduces as the volume increases hence an industry triangle will produce more credible loss development factors than the ones from a sparse triangle.

However, some practitioners not only use the loss development factor from the industry triangle but also the CoV as well. Again, also due to the law of large number, the industry triangle is more diversified and hence less volatile than a sparse triangle, using the industry CoV directly will underestimate the true volatility unless an adjustment is applied to remove the diversification effect.

Here are a few possible ways to adjust the CoV for diversification effect:

- Use a generic diversification adjustment as outlined in Appendix B.

- Based on industry data, use historical ultimate loss information from a selected set of companies. Given a particular line of business, for each company, calculate the volatility using historical ultimate movements and plot it against reserve amounts such that each point on the graph corresponds to a company. A power curve, for example, can be fitted to derive the relationship between size of reserve and its volatility. Once the parameter has been established, it would be a simple calculation to adjust the CoV to any other size of reserve.

- Adjust the diversification impact from the industry CoV by modifying the bootstrap mechanics as outlined below:

  i) Bootstrap the industry triangle without incorporating process risk. Hence, the set of pseudo project bottom triangles reflects only the parameter error of the industry triangle.
  ii) For each of the pseudo project bottom triangle, load in the process risk using the residuals of the underlying sparse triangle. In other words, the process risk will be based on the sparse triangle itself.

  This approach is consistent with the reserving process where the parameters of a sparse triangle come from another source. The benefit is in reducing the uncertainty of the parameter estimation while the volatility of the actual development should not be reduced because an industry loss development factors have been used.

On the whole, regardless of what method is used to adjust an external CV, the authors strongly encourage practitioners not to rely on CoV in isolation as it should always be considered in conjunction with the volume.

3.2.6 Every triangle is unique!

Is using the CoV from a different source appropriate in the first place? Consideration must be given when comparing CoVs because every triangle is unique. This point is one of the key observations from the study outlined in Appendix A. Even when two companies write the same line of business with the same volume, they will have different loss experiences resulting in different claim triangles which will lead to different loss development factor selections. Hence they will have different reserve estimates and it should not be a surprise that they will have different levels of reserve risk even though they wrote the same line of business with the same volume.

In simple terms, a company that is unfortunate to have more extreme noises in its history will have more uncertainty in the reserve estimate and therefore a higher CoV even though it may write the same exposures as other companies. This illustrates the point that every triangle is unique! Hence, by using a CoV from another source requires a leap of faith, whereby not only are you assuming they will experience the same volatility in future claim payments (process risk), but also they have experienced the same volatility in the past (parameter error).

3.2.7 Uncertainty in the estimation of reserve risk

Section 2.2 covered the general approach to estimate prediction error, and in particular the desirable outcome that both Mack's model and the ODP model produce the same reserve estimate as the traditional deterministic chain-ladder method. Therefore, bootstraps based on these two models are implemented in most popular reserving software and have become the standard approach to quantify reserve risk.

These popular approaches may be robust in calculating an estimate of the MSEP or CoV, however some practitioners may have placed an overreliance on these results and overlooked that since data is only one realisation of all possible outcome, any quantity derived from it is only an estimate. Just like the estimated reserve is only an estimate of the true mean required reserve (which exists although is unobservable), the MSEP from any estimation method, is also only an estimate of the true MSEP (also unobservable in practice).

While the reserving process estimates the mean and reserve risk estimates "how wrong" is the reserve estimate, the authors decided to examine "how wrong" is the estimate of reserve risk. This is a study on the uncertainty in the estimation of reserve risk based on Mack's model and it is briefly outlined here:

- The study generates 1,000 triangles based on the Mack's model with the same underlying parameters.
- Since the parameters are known, the *True CoV* for each generated triangle can be calculated which in reality would be unknown. Then for each of these generated triangles, the *Estimated CoV* is obtained by Mack bootstrap. (Mack bootstrap is chosen as the triangles were generated using Mack's model. This is to remove the impact of model specification risk focusing solely on the parameter error and process risk.)
- The objective is to compare the *True CoV* to the *Estimated CoV* for all 1,000 triangles. This will demonstrate the potential magnitude of the uncertainty in reserve risk estimate.

One of the key observations from this analysis was that even in a short-tail stable line of business where the model is known, the *Estimated CoV* can be significantly different to the *True CoV*. Note that the study can be repeated for other parameters to reflect characteristics for different business (e.g. long-tail volatile line of business). It can also be extended to other models.

Overall the aim of the study carried out by the authors was not to criticise any of the common models used for reserve risk in the industry, but rather to draw attention to the uncertainty surrounding the estimation of reserve risk itself. There may also be instances where some practitioners and regulators react strongly to relatively small changes in CoV when in truth the movements might not be statistically significant. In such cases, the focus may detract from more important issues underlying reserving and reserve risk estimates. The key observations from this study, which can be found in Appendix A, provide clarity to many misconceptions discussed in the rest of this section highlighting the importance of validation and warning against spurious accuracy especially as model specification error could be as material as the prediction error.

3.2.8 Is using an underwriting year triangle appropriate?

One of the assumptions for both Mack's model and ODP model when applied to a claim triangle is that the rows are independent. This assumption generally holds for an accident year triangle since each row represents a completely different set of accidents. However, for an underwriting year triangle, a major event can impact policies from successive underwriting years since one accident can trigger claims from polices written in more than one underwriting year. This therefore results in the violation of the independence assumption which has led some practitioners to strongly against using Mack's or ODP model on an underwriting year triangle.

Despite this, it is important to take a moment to think if violating an assumption within a model should always result in the negation of the use of the model. In practice, if an assumption is slightly violated, it may not necessarily mean that the result has no value at all. Also, it is possible to make appropriate and reasonable adjustments to mitigate any know violations of assumption. For instance, claims in multiple underwriting years related to the same major event could be removed and analysed separately. This would reduce the dependency between rows in the triangle.

Even though appropriate allowance can be made to an underwriting year triangle, a more fundamental obstacle still prevails. In particular, the CoV for recent underwriting years represent full underwriting year reserve, not just earned reserve, therefore it includes the volatility of the unearned reserve. One suggested approach to overcome this is to infer the volatility of the earned reserve. Since the volatility of the full underwriting year reserve consists of the earned reserve and the unearned reserve, by assuming a CoV for the unearned reserve (e.g. underwriting risk CoV with diversification adjustment on the volume) and a correlation between earned and unearned, the CoV for the earned reserve can be calculated.

Again, one should consider the fundamental which is to maintain consistency with the reserving process. If the reserve is set using an underwriting year triangle, the prediction error of reserve should also be derived using the same triangle though it might require pragmatic adjustments.

3.2.9 Comparisons between underwriting risk and reserve risk CoV

Some practitioners compare the underwriting risk CoV to the reserve risk CoV as a reasonability checks. This practice raises more questions than answers.

Reserve should be viewed as a conditional expectation while reserve risk is a conditional variance (plus parameter error) conditioning on a set of incurred claims. Underwriting risk is the variance of future claim in respect of business earned over multiple future accident years. The point is that they are fundamentally different quantities related to a different block of claims with different maturity.

Furthermore, comparing total reserve risk CoV across all-accident year basis is not meaningful since underwriting risk CoV relates to only one underwriting year. However, it is reasonable to expect similarity between an underwriting risk CoV and reserve risk CoV of the most recent accident years for lines of business with a long reporting lags. Again, it is important to adjust the CoVs to a more comparable volume, perhaps using the diversification adjustment described in Section 3.2.5.

3.2.10 Dependency between lines of business

Most practitioners will be aware of the challenges in quantifying the dependency between lines of business. These challenges are well discussed elsewhere in existing market wide literature.

The important point on this topic is that the dependency of process risk and parameter error should be considered separately. A typical approach in evaluating the dependency of process risk is to consider the common drivers of future claims that could impact both lines. Since reserve is a conditional mean given a set of already incurred claims, the common drivers that could impact the reserve outcome for both lines must be a subset of the common drivers that impact the underwriting outcome for both lines since the claims have already incurred. For example, a future catastrophic event could have a potential of impacting multiple lines from an underwriting risk perspective, but it would be a known event for reserve risk.

For dependency of parameter error, which often ignored in practice, it is important to consider what are the common drivers in mis-estimating the parameters for both lines? Basically, if the loss development factors were underestimated for a particular line of business, how likely would the loss development factors be also underestimated for the other line of business? For example, if the reserve estimates are derived by the same actuary, perhaps the same level of conservatism is embedded in the estimates. Or if the case reserves for both lines were set by the same claims department, it might create the same bias these lines of business.

As with many topics in reserve risk, it is vital to consider the parameter and process components separately. With this in mind, it is not difficult to envisage two lines that have low underwriting risk correlation but high reserve risk correlation, and vice versa. Moreover, if bringing model risk into the picture, it would likely increase the dependency if the reserve estimates for the lines were derived using the same methodologies.

3.2.11 Summary

The authors believe that these misconceptions created a major barrier in the advancement of reserve risk. Although it is not a comprehensive list, each item has been discussed and expanded to cover various related topics with potential remedial solutions. In addition, appendices and external references are provided where applicable so that reserve risk practitioners can explore further when tackling these issues.

3.3 Other Practical Challenges

The previous two sub-sections described in detail the main practical challenges in quantifying reserve risk. In addition to these, there are other further challenges worth noting and are discussed briefly in the remainder of this section.

3.3.1 Netting down an aggregate gross distribution

Given that the net (of reinsurance recoveries) results are the main focus for capital calculations, the question is how to derive the net reserve distribution? For instance, should the modelling be done on the net triangle or on the gross triangle before "netting down" the results? And how to net down a gross aggregate distribution since many reinsurance arrangements are non-proportional and on a per claim basis?

In order to address this challenge, the authors presented a "conditional ceded" approach which is a practical approach to calculate the ceded distribution conditional on the gross distribution. Appendix C provides a summary of this approach and refer to [4] for the presentation.

3.3.2 How much data is required

One of the most common questions asked is how many years of history is required for a claim triangle to be credible for estimating the reserve volatility. Intuitively, the longer the development pattern the more history is required to produce a credibility estimate of CoV. However, the underlying true volatility of the triangle should have no impact on how credible the reserve volatility estimate is. A more volatility triangle would have a higher CoV, but it does not mean it requires more history to produce a credible estimate of the CoV.

Rather than judgementally deciding on a minimum number of years for a credible estimate, the study described in Section 3.2.7 (which is further expanded in Appendix A) can infer a guideline of the number of years required based on a selected confidence level and an acceptance tolerance. The study uses Mack's model as an example, but it can be easily applied to other models. As outlined before, given a development pattern, 1,000 triangles are generated for a fixed number of years. For each triangle, the *True CoV* and the *Estimate CoV* are then computed. Finally, determine the percentage of triangles with an *Estimated CoV* within a selected acceptance tolerance from the *True CoV*. By repeating this exercise for a number of different number of years and different development patterns categorised into short, medium and long tail durations, the minimum number of years in order for the *Estimated CoV* be within an acceptance tolerance level of the *True CoV* can be determined given a selected confidence level.

Below table summarises the percentage of triangles with an *Estimate CoV* within +/- 10% of *True CoV* based on three development patterns from the above study. For example, the medium tail line of business used in the study has a duration of 3 years with a development length of 7 years. If the *True*

CoV of this triangle is 20%, then given 20 years of history, 61% of the time the *Estimate CoV* will be between 18% to 20%.

% of triangles	Development Pattern Examples (duration in year; length in year)		
Number of Years	**Short (2; 5)**	**Medium (3; 8)**	**Long (5; 11)**
10	43%	41%	34%
11	44%	44%	45%
12	45%	47%	46%
13	50%	48%	49%
14	51%	50%	51%
15	52%	52%	55%
16	55%	56%	52%
17	55%	56%	57%
18	58%	58%	58%
19	60%	60%	60%
20	60%	60%	60%

Surprisingly, regardless of the develop payment pattern, it seems a triangle of more than 14 years would be sufficient to produce an *Estimate CoV* within 10% of the *True CoV* more than half of the time. The results provided a "rule of thumb" of the minimum number of historical years that is required.to estimating reserve volatility.

3.3.3 High sensitivity to changes in assumptions

All reserve risk practitioners who have experience in estimating reserve risk know how sometimes a small change in assumptions can lead to an unreasonable change in the results. For example, in the case of Mack bootstrap, a change in the development pattern will change the "fitted triangle" and therefore completely change the set of residuals. Sometimes the exclusion of just one development factor or a different choice of tail fit can materially alter the results.

Similarly, with GLMs, often the parameters are derived based on the maximum likelihood, hence altering one data point or changing the number of free parameters and can have a large impact on parameters in all three dimensions. Does this mean a better model is needed to better explain the data? As mentioned earlier, this is as much a reserving question as it is a reserve risk question.

3.3.4 One-year time horizon

The authors highlight this issue for completeness as it is a large topic in its own right. Most literatures for quantifying prediction error focused on volatility on ultimate basis. However, for Solvency II, Article 101 of the Solvency II Directive states:

"The Solvency Capital Requirement (SCR) shall correspond to the Value-at-Risk (VaR) of the basic own funds of an insurance or reinsurance undertaking subject to a confidence level of 99.5% over a one-year period."

This leads to development of "one-year" approaches such as the "Actuary-in-a-box" or Merz and Wüthrich [12] along with other "time-scaling" method to convert the ultimate CoV to a one-year CoV using emergence patterns. For an overview of this topic and various models, refer to the presentation by White and Margetts [15].

SECTION 4 – CONCLUSION

This document has provided a detailed discussion on the key practical challenges encountered in implementing a robust reserve risk estimation process. Specifically, the following key messages are noteworthy:

Scaling Issue - Inconsistencies between reserving and reserve risk process which lead to the "scaling issue" is a fundamental challenge. Fundamentally all scaling methods are just a way to force a consistent output which is far from the ideal. This paper provided an insight to the scaling issue with a detail description of each component of inconsistency between reserving and reserve risk process. The inconsistencies can be viewed as a form of model specification error. Ultimately the author's suggestion for the industry is to incorporate stochastic reserving methodologies into the reserving process and eventually reserving and reserve risk would be a result of stochastic reserving. This is the only way to eliminate the scaling issue and to reduce the model specification error in reserve risk in the long run.

Other Practical Issues - The other practical challenge is the existence of many misconceptions among practitioners. Misconceptions are a burden creating obstacles in advancing the topic of reserve risk. A disproportionate amount of time and resource can be spent on tackling issues without addressing the fundamental problems. Below are the key points clarifying the common misconceptions:

- There is nothing wrong with the bootstrap technique; the challenge is due to inconsistency between reserving and reserve risk.
- Practitioner should use data (paid or incurred) that is consistent with the reserving.
- Claims experience and reserve volatility does not need to move in the same direction.
- The CoV is not an appropriate risk measure in all situations.
- Using the industry CoV as a benchmark will underestimate the true CoV unless adjusted to remove the diversification effect.
- Even when two companies write the same business with the same volume, they should have different volatilities in reserve since every triangle is unique!
- Practitioners and regulators should not react strongly to relatively small changes in CoV when in truth the movements may not be statistically significant.
- Applying a Mack or ODP model on an underwriting year triangle can still be appropriate.
- Underwriting risk and reserve risk are fundamentally different quantities.
- Related to dependency assumption between lines of business, practitioners are encouraged to consider the common drivers for both process risk and parameter error.

The aim of the document is to bring clarity, or maybe a new perspective, to each of the above challenges with suggested solutions where applicable. It is acknowledged that some of the issues highlighted will remain difficult to solve for some companies and some may never be entirely solved. As challenges and limitations will always exist, the validation of results will become especially important.

The authors invite the readers to consider the contents of this paper and to contribute to the subject. Hopefully this document will prompt the industry to develop a set of generally accepted principles and approaches in tackling challenges in reserve risk. This is the only way the industry can strive towards advancing the assessment of reserve risk.

APPENDIX A – How Wrong is the Estimation of Reserve Risk?

Background

Due to the heavy technical nature in many stochastic reserving literatures, one might easily overlook the fact that both the analytical or empirical approach in calculating the Mean Squared Error of Prediction (MSEP) is after all only an estimate even without error in model specification. This is an analogy to the fact that reserve is only an estimate of the true mean required reserve, the MSEP from any estimation method is also only an estimate of the true MSEP. So while reserving estimates the mean and reserve risk estimates "how wrong" is the reserve, the authors decided to examine "how wrong" is the estimation of reserve risk.

The most common measure used to assess the uncertainty in reserve is the Coefficient of Variation; the square root of MSEP expressed as a percentage of the mean reserve and will be referred to as the CoV in the rest of this appendix.

In the study described below, Mack's model was used for illustration purposes, however it could also be replicated for other models. The study began with a set of the selected parameters, namely, the development factors (f) and the noise parameters (σ) for each period (consistent with the notations in Mack's paper [9]). Using Equation 2.2.2 with normally distributed error $\varepsilon_{i,j}$, 1,000 sample claim triangles were generated. The values for the parameters were selected to represent a short-tail line of business for the purpose of the study. Note that in practice there is only one observed claim triangle, but in this study, 1,000 triangles were generated in order to address the topic in question.

Since the true parameters were known in this study, using Equation 2.1.1, the true MSEP was calculated and hence the *True CoV* for each triangle was obtained by dividing by the estimated reserve of each triangle. Moreover, the *CoV* for each triangle was estimated by bootstrapping using the Mack's model (or again by using Equation 2.1.1 but using estimated parameters instead). The *Estimated CoV* versus the *True CoV* for each of the 1,000 sample triangles can now be compared and is displayed in Figure A1.

Readers are advised to have a clear distinction between the *True CoV* and *Estimated CoV* before proceeding. Given a triangle, the *True CoV* is the reserve uncertainty which is unknown in reality while the *Estimated CoV* is the reserve uncertainty calculated from the triangle (using Mack formulae or Mack bootstrap in this case).

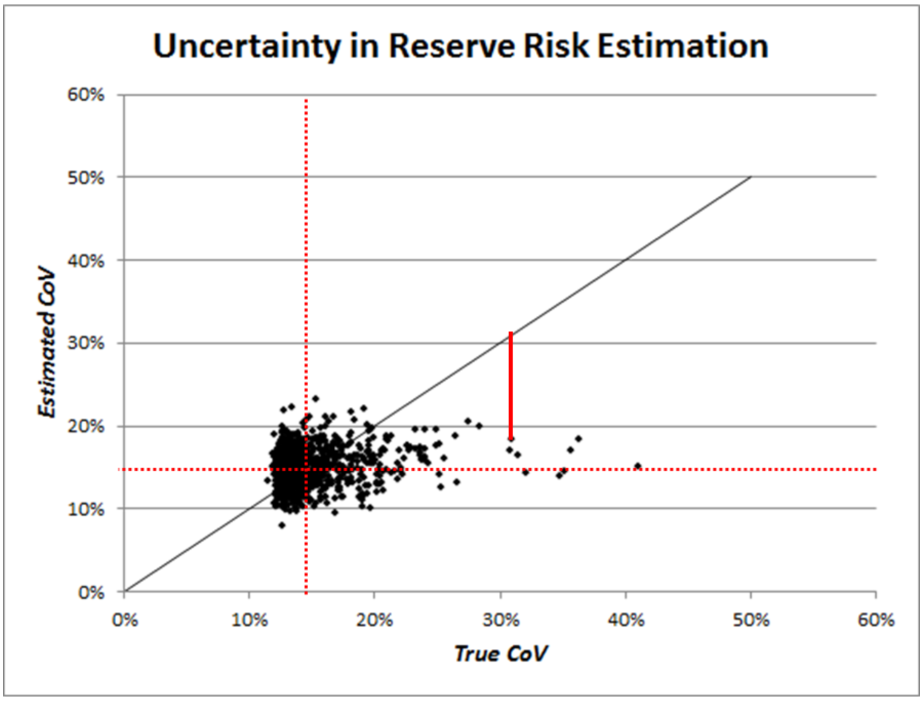


Figure A1 Estimated CoV versus True CoV

Each point on the above graph corresponds to a sample triangle with the axis indicating the *Estimated CoV* and the *True CoV*. The points above the line $y=x$ are from triangles where the CoV were overestimated whereas the opposite is true for ones below the line. The vertical distance indicated by the solid red line (or the horizontal distance from the line y=x) represents the error of the CoV estimation, difference between the True and Estimated CoV.

Observation 1

Despite the fact that all 1,000 triangles came from the same model with the same set of underlying parameters, each one has a different True CoV.

By taking three different sample triangles shown in Figure A2 below, it illustrates the point that even with the same model and parameters, each sample triangle is in fact unique and has a different *True CoV*. The selected parameters for the model, f and σ, are shown in the first two rows which are the same for each of all sample triangles. The loss developments are colour-coded with green to indicate better than true development (i.e. factors that are lower than the true loss development factor for that column) and red to indicate worse than true development.

Figure A2a shows an example of an “average” sample – average in the sense that roughly half of the factors in each column are higher and half are lower than the true loss development factors. The final row highlights the differences between the true f and the estimated $\hat{f}$. As expected, the differences are small for an “average” triangle due to that fact that the parameter error in the reserve estimates for this particular triangle would also be relatively small and hence a relatively low *True CoV*.

	2/1	3/2	4/3	5/4	6/5	7/6	8/7	9/8	10/9
f	2.20	1.40	1.20	1.10	1.01	1.01	1.00	1.00	1.00
σ	300	100	100	100	50	20	5	0	0

Development

Year i	1	2	3	4	5	6	7	8	9	10
1	1,000,000	2,391,911	3,506,223	4,239,223	4,566,354	4,634,674	4,701,945	4,797,599	4,797,599	4,797,599
2	1,000,000	2,107,052	3,087,878	3,678,489	4,265,311	4,205,696	4,398,225	4,374,044	4,374,044	
3	1,000,000	2,341,249	3,129,773	3,700,816	4,267,585	4,285,892	4,245,529	4,340,855		
4	1,000,000	2,010,549	2,840,260	3,522,843	3,803,993	3,832,101	4,037,680			
5	1,000,000	1,888,299	2,468,546	2,866,506	3,545,037	4,003,298				
6	1,000,000	1,750,916	2,553,273	2,965,042	3,042,827					
7	1,000,000	2,411,699	3,327,589	4,077,663						
8	1,000,000	2,559,981	3,713,097							
9	1,000,000	1,970,140								
10	1,000,000									

Development

Year i	2/1	3/2	4/3	5/4	6/5	7/6	8/7	9/8	10/9
1	2.39	1.47	1.21	1.08	1.01	1.01	1.02	1.00	1.00
2	2.11	1.47	1.19	1.16	0.99	1.05	0.99	1.00	
3	2.34	1.34	1.18	1.15	1.00	0.99	1.02		
4	2.01	1.41	1.24	1.08	1.01	1.05			
5	1.89	1.31	1.16	1.24	1.13				
6	1.75	1.46	1.16	1.03					
7	2.41	1.38	1.23						
8	2.56	1.45							
9	1.97								
$\hat{f}$	2.16	1.41	1.20	1.12	1.03	1.03	1.01	1.00	1.00
$\hat{f}-f$	-0.04	0.01	0.00	0.02	0.02	0.02	0.01	0.00	0.00

Figure A2a “Average” triangle

To further elaborate on this concept, Figure A2b and Figure A2c show two “extreme” sample triangles but in opposite directions, i.e. an “extreme unfavourable” and an “extreme favourable”. By “extreme”, it means most of the development in the triangle happens to lie on the same side from the true f’s. In both cases, the volume-weighted average $\hat{f}$’s provide a relatively poor estimate of the true f’s. Hence the true parameter error in reserve estimates for these particular triangles would be relatively high and so would the *True CoV*. Note that reserve risk is about mis-estimation in either direction, so regardless of whether the triangle shows extremely favourable or unfavourable experience, both exhibit higher parameter error relative to the “average” triangle in Figures A2a.

It should be very clear now that despite the triangles being generated from the same model with the same set of underlying parameters, not only would each triangle has a different reserve estimate, but would also have a different true uncertainty in the reserve estimates. Refer to Section 3.2.6 for further discussion on the practical implications of this observation.

	2/1	3/2	4/3	5/4	6/5	7/6	8/7	9/8	10/9
f	2.20	1.40	1.20	1.10	1.01	1.01	1.00	1.00	1.00
σ	300	100	100	100	50	20	5	0	0

Year	Development								
i	2/1	3/2	4/3	5/4	6/5	7/6	8/7	9/8	10/9
1	2.36	1.45	1.24	1.05	1.04	1.02	1.00	1.00	1.00
2	2.75	1.33	1.25	1.21	0.81	1.01	1.00	1.00	
3	2.51	1.45	1.19	1.13	1.02	1.00	1.00		
4	2.46	1.34	1.20	1.06	1.00	1.01			
5	1.90	1.40	1.26	1.14	0.97				
6	2.28	1.57	1.28	1.03					
7	2.31	1.51	1.11						
8	2.35	1.42							
9	2.05								
$\hat{f}$	2.33	1.43	1.22	1.10	0.96	1.01	1.00	1.00	1.00
$\hat{f}-f$	0.13	0.03	0.02	0.00	-0.05	0.00	0.00	0.00	0.00

Figure A2b “Extreme unfavourable” triangle

	2/1	3/2	4/3	5/4	6/5	7/6	8/7	9/8	10/9
f	2.20	1.40	1.20	1.10	1.01	1.01	1.00	1.00	1.00
σ	300	100	100	100	50	20	5	0	0

Year	Development								
i	2/1	3/2	4/3	5/4	6/5	7/6	8/7	9/8	10/9
1	2.09	1.42	1.12	1.13	0.98	1.00	1.00	1.00	1.00
2	1.96	1.43	1.14	1.05	0.97	1.08	1.01	1.00	
3	1.63	1.38	1.18	1.11	0.99	1.01	0.99		
4	2.15	1.19	1.28	1.01	1.07	0.95			
5	1.95	1.37	1.18	1.16	1.01				
6	2.19	1.36	1.16	1.06					
7	2.08	1.44	1.12						
8	2.18	1.35							
9	2.21								
$\hat{f}$	2.05	1.37	1.17	1.08	1.00	1.01	1.00	1.00	1.00
$\hat{f}-f$	-0.15	-0.03	-0.03	-0.02	-0.01	0.00	0.00	0.00	0.00

Figure A2c “Extreme favourable” triangle

Observation 2
The True CoV has a wider range than the Estimated CoV.

As may be expected, “average” triangles like the one *Figure A2a* are typical triangles observed and the *Estimated CoV* would be relatively close to the *True CoV* corresponding to the cluster of points around $y=x$ in Figure A1 whereby the *Estimated CoV* is closer to the *True CoV*. However why are there points skewed to the right where the *True CoV* is much higher than the *Estimated CoV*?

The answer to this question will be more obvious by referring back to the “extremely unfavourable” triangle in *Figure A2b*. As mentioned before, this triangle has a relatively high *True CoV* due to relatively higher parameter error as indicated in the final row. Now imagine that the true f’s are unknown as in reality without any indication of green or red, the CoV will have to be estimated based on the triangle. Though this is an “extremely unfavourable” triangle, but in fact, it appears to be very stable since most of the developments happen to be clustered on the same side from the true f’s (higher in this case)! So in practice the *Estimated CoV* would be relativity lower than the *True CoV*.

What about the “extreme favourable” triangle in *Figure A2c*? This triangle also has a relatively high *True CoV* for the same reasons above, but because most of the developments also happen to be clustered on one side (lower in this case), the observed triangle looks relatively stable and hence would lead to a relatively low *Estimated CoV* as well. It should be clear now why both “extreme unfavourable” and “extreme favourable” triangles would lead to a significant underestimation of reserve uncertainty while the “typical” triangles would cluster near $y=x$ as indicated in Figure A1.

This observation clearly illustrates a limitation of the estimation process since the range of *Estimated CoV* is narrower than the *True CoV* range indicating that in the extreme cases, the estimated reserve uncertainty could be significantly underestimated.

Observation 3
The average Estimated CoV is approximately the same as the average True CoV across the 1,000 triangles. Moreover, majority of the time the CoV will be overestimated but in the extreme case, it could be significantly underestimated.

It might sound concerning that triangles coming out from the same model with same set of parameters could have a wide range of *Estimated CoV* and even a wider range of *True CoV*, but the good news is that on average they are correct! The average *True CoV* across the 1,000 triangles matched closely to the average *Estimated CoV* across the 1,000 triangles. The estimation process works, on average! In other words, the difference between the true and estimated MSEP varies on different sample triangles but on average it is close to zero[1]. The histogram below shows the skewness of the error distribution of the 1,000 triangles. Note that for the majority of the time the CoVs were overestimated but in the extreme case, it could be substantially underestimated.

[1]This difference can also be viewed as the parameter error in estimating MSEP since the *Estimated CoV* is calculated based on data, one triangle in this case, and any estimate based on data contains parameter error.

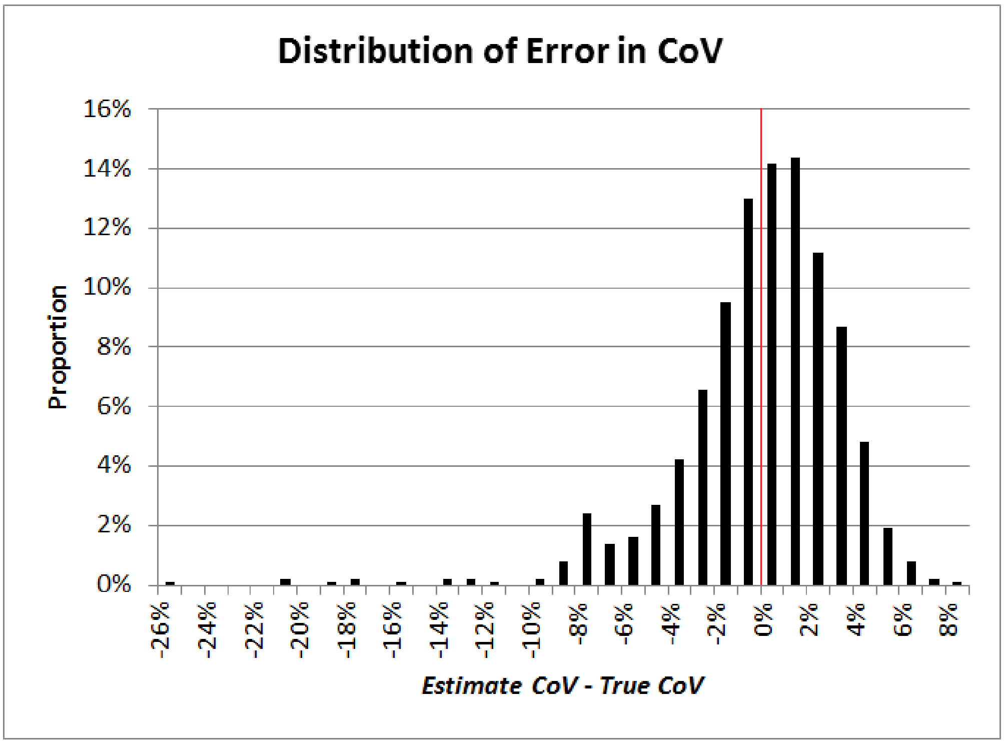


Figure A3 Distribution of Error in CoV

Not only did this observation reassure that the implementation of this study was done correctly, but it also demonstrated the remarkable fact that based on just one triangle, without knowing the true parameters, the estimated reserve uncertainty is, in fact, correct on average[2].

Observation 4

Given a triangle with an Estimated CoV, the range and distribution of the True CoV can vary significantly.

Since the *True CoV* is not observable in reality, what most practitioners should be interested in is the horizontal cross-section distribution of Figure A1. In other words, given the value of *Estimated CoV* what could be the range and distribution of the *True CoV*? This directly addresses the original question of this study – while reserving estimates the mean and reserve risk estimates "how wrong" is the reserve, this study examines "how wrong" is the estimation of reserve risk.

Below are two cuts of the cross-section of 1% of the width of the *Estimated CoV* centred at 13.5% and at 18.5%. The thin black line indicates the average of *True CoV* while the red line indicates the *Estimated CoV*. For the first case, Figure A4a, 75% of the time the *True CoV* is between 12% to 15% and 5% of the time it is above 20%. So most of the time the *Estimated CoV* will be higher or within a close range, but in the unfortunate case where the observed is an "extreme triangle" like ones in Figure A2b and A2c, the CoV would be significantly underestimated, which leads to an underestimation on average.

[2]A subtle point should be made is that the average *Estimated CoV* and the average *True CoV* will not match exactly. This is due to the fact that, the Mack formulae in Equation 2.2.1 for the *Estimated CoV* is not an unbiased estimator of the *True CoV*.

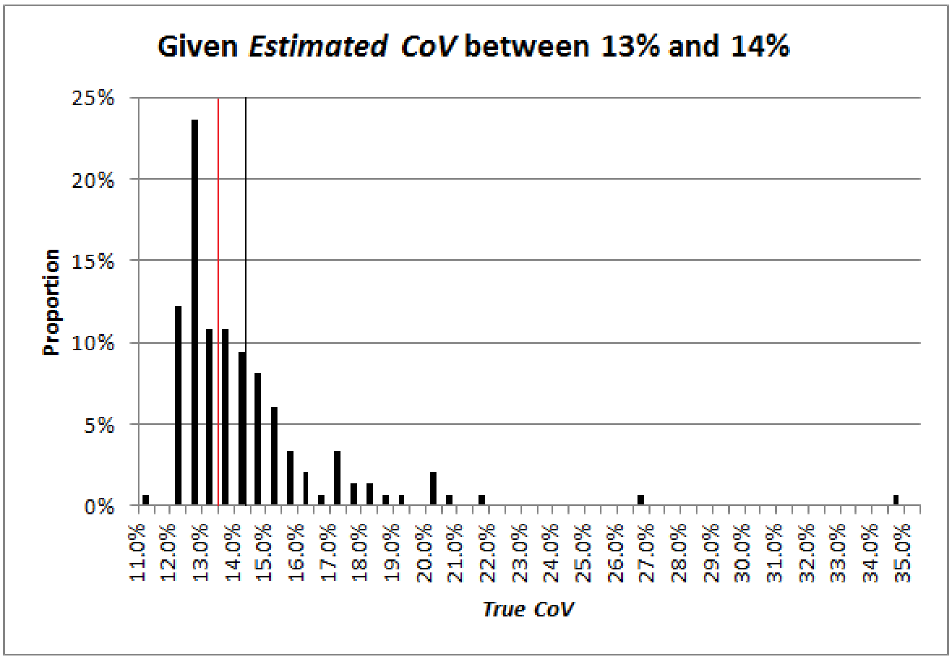


Figure A4a Conditional distribution of True CoV given Estimate CoV is between 13% and 14%

For the second case, Figure A4b, the distribution is quite different. Again, in extreme cases the CoV would be significantly underestimated but most of the time the *True CoV* is significantly below the *Estimated CoV* and on average it overestimates the CoV.

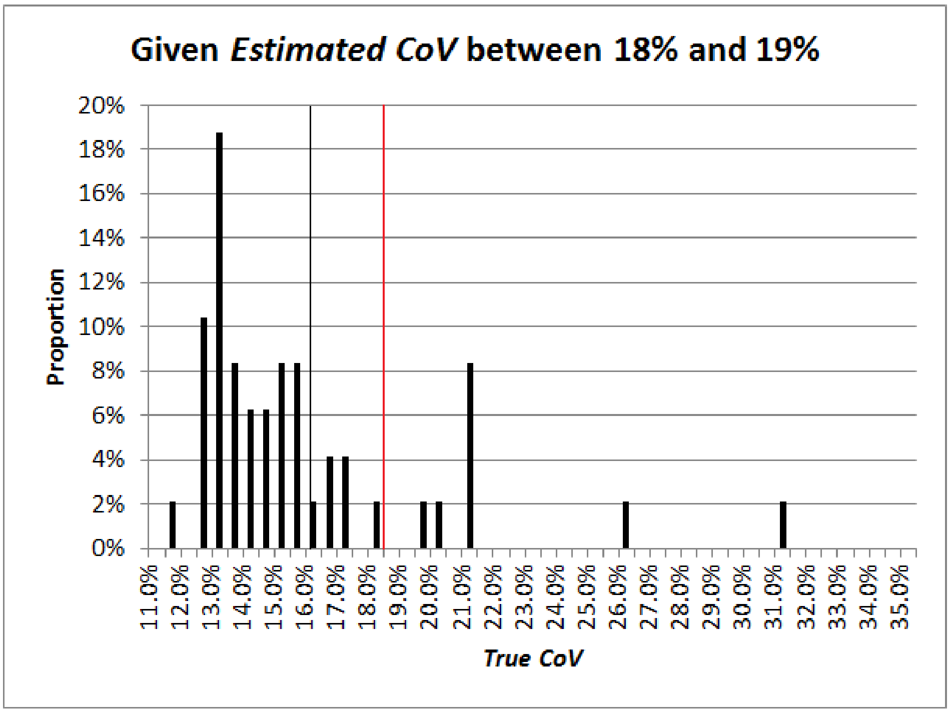


Figure A4b Conditional distribution of True CoV given Estimate CoV is between 18% and 19%

Observation 5

The parameter error has completely different behaviour than process risk and it is the parameter error that leads to material reserve risk mis-estimation for "extreme triangles".

As outlined in Section 2.1.1, the MSEP can be split into process risk and parameter error. In the context of Mack's model, the parameters are the loss development factors which are estimated using the claim triangle. Since the loss development factors are then used to calculate the estimated reserve, it contains parameter error. Moreover, even if the parameters are known, the estimated reserve is only the mean of the required reserve, not the required reserve which is still yet to be known. The difference between the estimated and required reserve is described as the process risk.

Recall the fundamental fact that an unbiased estimate of the variance of a population can be derived even if the true population mean is unknown. This is comparable to estimating the process risk of required reserve. It should not be a surprise that the estimated volatility would produce a reasonable estimate of the process risk. However, for parameter error, it would be a much more challenging task since the true f's are never known in practice.

To confirm the point that the key driver of the mis-estimation of reserve risk is in fact from the estimation of parameter error, Figure A1 in the study can be dissected into process risk and parameter error. To do this, using Equation 2.1.2, the *True CoV* can be split into the two components, and from Mack's derivation of Equation 2.2.1, the *Estimated CoV* can also be separated into the two components. This allows a comparison of the True versus Estimated CoV separately for both process risk and parameter error.

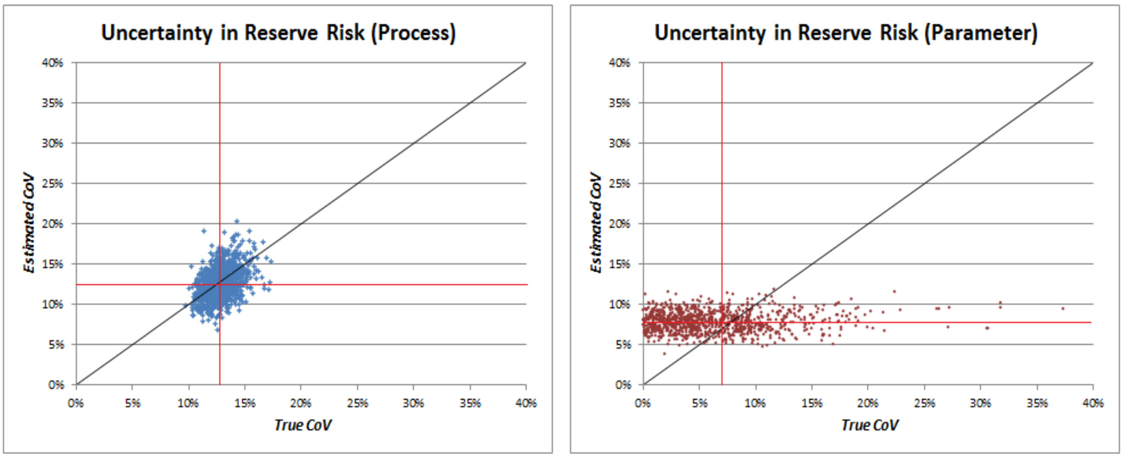


Figure A5 Process Risk vs Parameter Error

As shown above, in terms of process risk, the *Estimated CoV* will be within a closer range of the *True CoV*; at least this would be the case based on Mack's model with a normally distributed error as assumed in this study. In terms of parameter error, the figure above clearly demonstrates the limitation of the estimation of parameter error. Regardless of whether the true parameter error is small or large, the estimated parameter error is always within a much narrower range. This limitation is due to the fact that the true underlying parameters cannot be known in practice when estimating the CoV which led to the wide range of CoVs in Figure A4. Nevertheless, it is pleasing that even though the estimation will always be wrong given a single triangle, but the estimation works on average. The average *True CoV* across the 1,000 triangles matched closely to the average *Estimated CoV* across the 1,000 triangles for both process risk and parameter error as denoted by the red line.

Observation 6

The uncertainty in the estimation of reserve risk behaves differently by accident year; both the parameter error and process risk.

So far, the focus has been on the reserve risk for the total reserve of all accident years. It should not be a surprise that the same graphs of *Estimated CoV* versus *True CoV* could vary significantly by accident year. The reserve for less developed accident years are based on more loss development factors, hence larger parameter error, and since those are the accident years where it takes more development periods to reach ultimate, hence more process risk as well. The proportion of parameter error and process risk could vary significantly between accident years.

To illustrate this point, Figure A6 display both the parameter error and process risk for the most recent three accident years of the study: Year 8, 9, and 10. It is recommended to focus on the shape of the distribution rather than the magnitude since the reserve amount, the denominator of CoV, differ by accident years. For example, Year 8 is more developed and hence it consists of less process risk and parameter error even though the graph might indicate otherwise due to a smaller reserve in the denominator.

Note that as indicated by the red lines, the *Estimated CoV* is the very close to the *True CoV* for each component and for each accident year on average across the 1,000 triangles. Again, it is evident that the estimation process works, on average.

Conclusion

This appendix illustrated many interesting observations based on a simple question. It provided a deeper understanding of what reserve risk is trying to quantify. From examining some sample triangles, the concept of *True CoV* versus *Estimate CoV* becomes more apparent and the distinction between the two sources of reserve risk becomes much more clear.

This study demonstrated the potential magnitude of uncertainty in the reserve risk estimate. Even in a short-tail stable case assuming the model is known (i.e. no model specification error), the *True CoV* could be significantly different from the *Estimated CoV*. It is not uncommon for some practitioners and regulators to react strongly to relatively small changes in CoV when in truth the movement may not be statistically significant as the estimate itself could contain large uncertainty. Such concerns may detract from more important issues in the process.

As mentioned earlier, the objective is to draw attention to the uncertainty surrounding the estimation of reserve risk and not to criticise one of the most common estimation methods of reserve risk in the industry. The results highlighted the importance of validation and warn against a false sense of accuracy especially the error in model specification could be as material, if not significantly more. This study is an educational piece for practitioners as it provides a valuable insight in the topic of estimation of reserve risk.

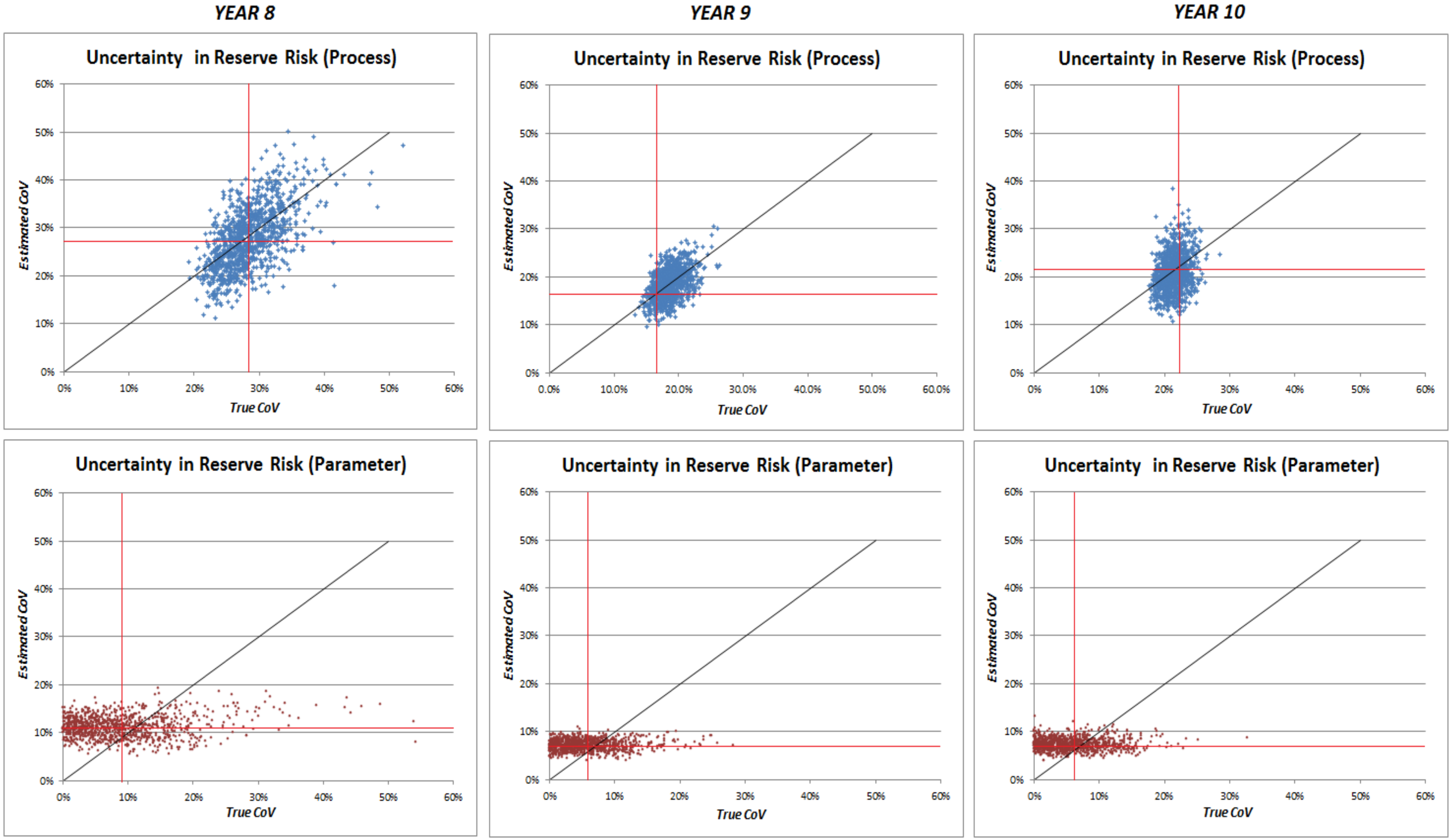


Figure A6 Uncertainty of process risk vs parameter error by year

APPENDIX B – Diversification Adjustment

As illustrated in section 3.2.4 to 3.2.6, using CoV as a risk measure is not ideal in all situations. The CoV is the prediction error expressed as a percentage of reserve. Using CoV as a comparative measure of reserve risk basically assumes prediction error would move by the same proportion as the reserve moves. This appendix outlines a general approach to adjust for the non-linear relationship between prediction error and reserve.

The rationale behind this is if the amount of business written has been doubled, with everything else being equal, it would have doubled the number of claims resulting in a more stable triangle due to diversification and hence the triangle would contain less process risk. Since the parameters are derived from the triangle, it would also lead to less parameter error. This means before drawing comparison between CoVs of two lines of business, they must be adjusted to the same volume. Here is a general diversification adjustment approach.

Let X_i be a random variable with mean μ, standard deviation σ, and correlation coefficient between any two X_i's is ρ. Now, let *Y* be the sum of y X_i's. The mean and variance of *Y* can be easily calculated:

$$E[Y] = y\mu \qquad \text{and} \qquad Var(Y) = y\sigma^2 + y(y-1)\rho\sigma^2$$

So, $$CoV(Y) = \sigma\sqrt{1+\rho(y-1)} \, / \, \sqrt{y}\mu$$

Similar, let *Z* be the sum of *z* X_i's, so $CoV(Z) = \sigma\sqrt{1+\rho(z-1)} \, / \, \sqrt{z}\mu$

The ratio of the two CoVs would be:

$$CoV(Z)/\, CoV(Y) = \sqrt{y}\sqrt{1+\rho(z-1)} \, / \, \sqrt{z}\sqrt{1+\rho(y-1)}$$

The above formula provides a practical and simple way to adjust for the diversification effect due to volume for lines of business applicable for both underwriting and reserve risk. In practice, the correlation between two incurred claims would be very low. For example, assuming 5% correlation between any two claims (i.e. $\rho = 0.05$), if a line of business with a 100 claims (i.e. $y = 100$) has a CoV of 10% doubled in volume (i.e. $z = 200$), the CoV would be 9.6%. In practice, if the number of claims is not readily available, though not ideal, but the reserve amount could be used as a proxy for the claim volume.

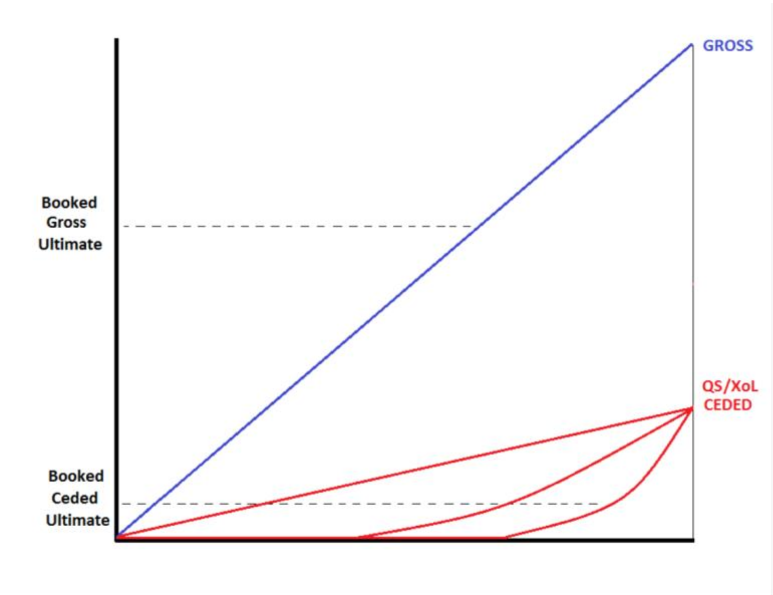
With this in mind and as concluded in Section 3.2.4, the CoV should not be used in isolation and must be considered in conjunction with the volume. Either for the purpose of comparing CoV across different lines of business or using CoV as a benchmark, the CoV must be adjusted with respect to the volume accordingly.

APPENDIX C – "Netting Down" A Gross Aggregate Distribution

To generate a net (of reinsurance recoveries) reserve distribution, one approach is to directly model net of reinsurance data without explicit consideration of the reinsurance program in place. While this approach is practical to implement, it raises questions about the appropriateness. For example, any method applied on a net triangle will not be able to distinguish if a low development is due to good experience or due to bad experience but good reinsurance program. And, if a class had material recovery with changing reinsurance program throughout its history, by applying bootstrapped where residuals are resampled across different accident years will hardly give any meaningful result at all. Hence, a more intuitive approach is to first analyse gross of reinsurance data to obtain the gross reserve distribution before capturing the reinsurance impact. This is described as "netting down" in this document.

Since gross reserve distribution is on an aggregated claim level, netting down is trivial for an aggregate reinsurance program but the task becomes challenging for individual claim/occurrence based reinsurance structure. Refer to [4] for a pragmatic netting down method, the "Conditional Ceded" method, by incorporating individual excess of loss reinsurance programs which can then be extended to other reinsurance programs. Below is a high level overview to briefly illustrate the framework.

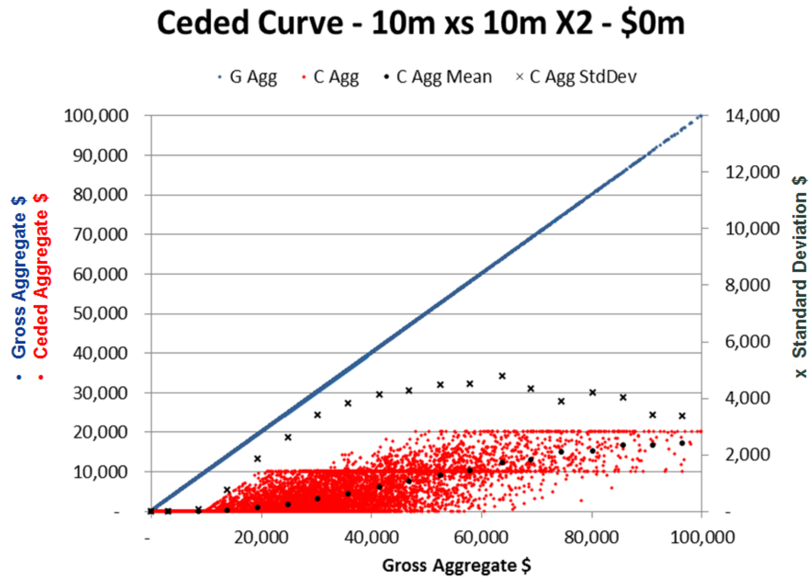
Given an empirical gross distribution from say a gross bootstrap, this netting down method relies on deriving the "ceded curve", the expected ceded conditional on each gross amount (hence the name "Conditional Ceded" method). This concept is easy to visualise in the following diagram and using ultimate instead of reserve amount:



Where:

- *x-axis is gross ultimate loss*
- *y-axis is gross ultimate loss but gross basis for the blue curve and ceded for the red curve*
- *Blue curve represents gross distribution (line y=x)*
- *Red curves represent three ceded distributions: Straight line represents quota share while the other two represent two examples of excess of loss programs.*
- *For the excess of loss ceded curve, it is initially flat and recoveries begins after a certain threshold of gross amount*
- *The average height across the ceded curve represents the mean and should tie back to ceded booked ultimate.*

With the above framework in mind, the objective now becomes finding an approach to approximate the ceded curve for a given reinsurance program. To understand the true shape of the ceded curve, frequency-severity based simulation was implemented in order to apply excess of loss reinsurance on a claim-by-claim basis with different excess of loss parameters. For example, in the next figure, the black points indicate a ceded curve for a $10m excess of $10m program with one reinstatement and no annual aggregate deductible.



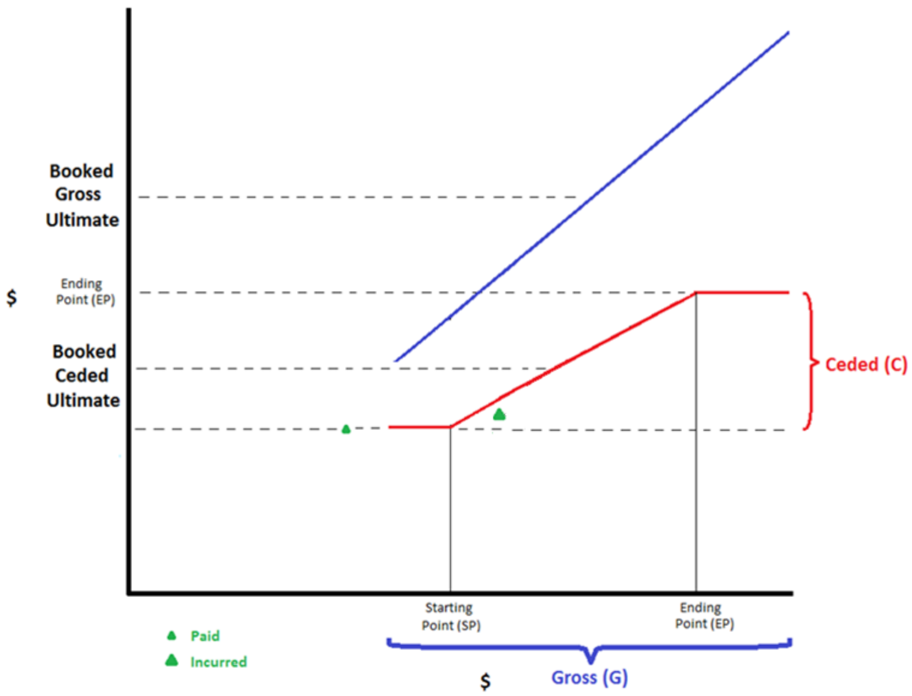
From the simulation study, it illustrated all excess of loss programs have ceded curve of similar characteristic. It begins flat and then "takes off" as gross ultimate amount becomes larger before it gets flatten due to exhaustion. Hence, the pragmatic approach to approximate the ceded curve using simple piecewise linear relationship – horizontal line until a ***Starting Point***, then a constant ***Slope***, before another horizontal line after an ***Ending Point*** parameters, a total of three parameters.

Note that since the average height of the ceded curve has to balance back to the ceded booked ultimate, there are only two free parameters. For ease of implementation, the two parameters *Starting Point* and *Ending Point* can be set by reserving actuaries while the *Slope* can be *goal seek* using MS Excel feature. Moreover, the selection of the parameters would be more intuitive using percentile instead of monetary amount, e.g. The reinsurance program would be exhausted every 20 accident years or 100 accident years (which correspond to *Ending Point* of 95th or 99th percentile)?

Once the ceded curve is parameterised, it can be applied to the empirical gross distribution to derive the empirical ceded distribution and the net distribution can then be obtained.

The above illustration is more appropriate for underwriting risk rather than Reserve Risk since there was no consideration of actual paid and incurred loss. Readers are suggested to apply this ceded curve concept to reserve distribution. The diagram below provides an illustration:

Readers are recommended to further explore this concept in order to reflect reinsurance program when generating the net of reinsurance reserve distribution. Note that expert judgements is required since the original challenge still exist - the gross reserve distribution is on an aggregated claim basis without full knowledge of individual claims.



Readers are recommended to further explore this concept in order to reflect underlying reinsurance programs when deriving the net of reinsurance reserve distribution. Note that expert judgement is required since the original challenge still exist - the gross reserve distribution is on an aggregated claim basis without full knowledge of individual claims.

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