Quebec Pension Plan (QPP) multi-population data analysis

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Content

QPP data overview

- 11 sub-populations ordered by increasing cohort pension amount in 10% bands.
- Only contains Quebec pensioners.
- Age over 65-89, and year over 1991-2015. $(11 \times 25 \times 25)$

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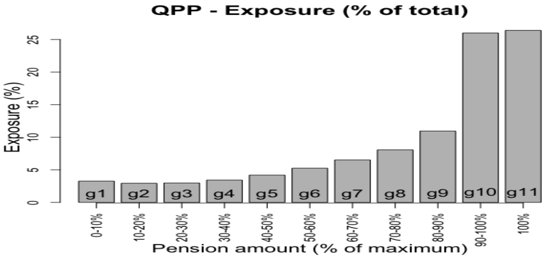
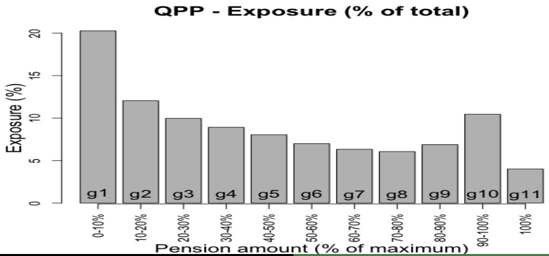
QPP data overview (cont.)

- Males

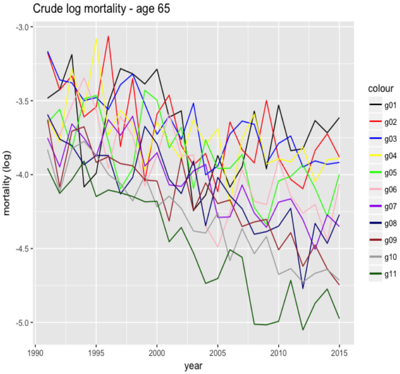


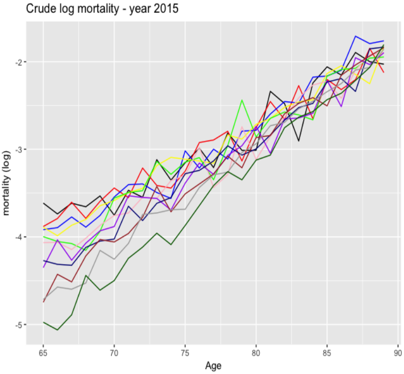
- Females



QPP data overview (cont.)

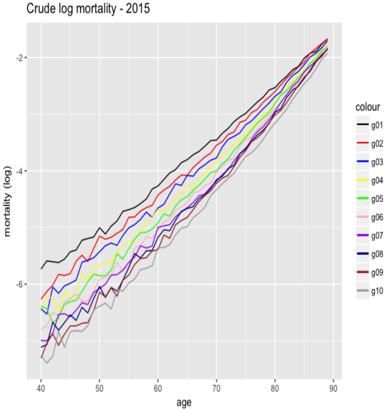
- Group-wise crude death rates (log-scale):





QPP data overview (cont.)

- Comparison with **England IMD** (larger sample size, with groups evenly splited)

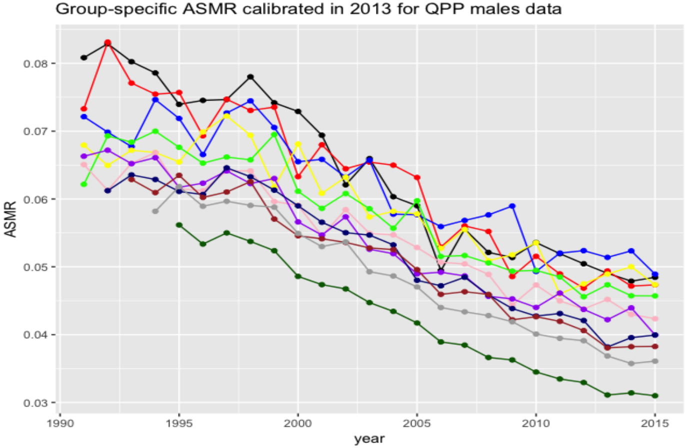


- **Age-Standardized Mortality Rate (ASMR)**

- ASMR is a weighted average of the crude death rates over a defined age range, for certain specific calender year t.
- E_x^s is the 'standard population' at age x (from European Standard Population, calibrated in 2013).
- m_{tx} is crude death rate.
- $$ASMR(t) = \frac{\sum_x m_{tx} E_x^s}{\sum_x E_x^s}$$
- Use of ASMR:
  - comparison of mortality over different populations;
  - assessment of mortality term structure;
  - assessment of singal-to-noise ratio.

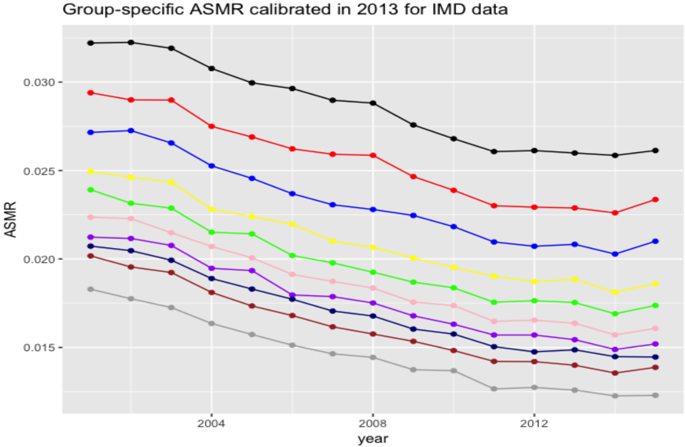
QPP data overview (cont.)

- ASMR of QPP males over age 65-89:



QPP data overview (cont.)

- (For comparison) ASMR of England IMD:



QPP data overview (cont.)

- ASMR is smoothier than the crude death rates, but still quite volatile for QPP males.
- Group 10 and 11 (larger size) are smoothier than others.
- Groups with higher pension tends to have lower mortality.
- QPP applies different grouping methodology (pension level) from England IMD (deprivation index) - less powerful predictor.

Model specification

m1 $\log m_{xti} = \alpha_{xi} + \beta^1_{xi}\kappa^1_{ti} + \beta^2_{xi}\kappa^2_{ti}$ (Renshaw and Haberman, 2003)

m2 $\log m_{xti} = \alpha_{xi} + \beta^1_{xi}\kappa^1_{ti} + \beta^2_{x}\kappa^2_{ti}$

m3 $\log m_{xti} = \alpha_{xi} + \beta^1_{x}\kappa^1_{t} + \beta^2_{xi}\kappa^2_{ti}$ (Li and Lee, 2005)

m4 $\log m_{xti} = \alpha_{xi} + \beta^1_{xi}\kappa^1_{ti}$ (Lee-Carter, 1992)

m5 $\log m_{xti} = \alpha_{xi} + \beta^1_{x}\kappa^1_{ti} + \beta^2_{x}\kappa^2_{ti}$ (CAE model by Kleinow, T, 2014)

m6 $\log m_{xti} = \alpha_{x} + \beta^1_{x}\kappa^1_{ti} + \beta^2_{x}\kappa^2_{ti}$ (CAE model with common α_x)

Model specification

m1	$\log m_{xti} = \alpha_{xi} + \beta^1_{xi}\kappa^1_{ti} + \beta^2_{xi}\kappa^2_{ti}$	(Renshaw and Haberman, 2003)
m2	$\log m_{xti} = \alpha_{xi} + \beta^1_{xi}\kappa^1_{ti} + \beta^2_{x}\kappa^2_{ti}$	
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m6	$\log m_{xti} = \alpha_{x} + \beta^1_{x}\kappa^1_{ti} + \beta^2_{x}\kappa^2_{ti}$	(CAE model with common α_x)

- α, β and κ are stochastic parameters capturing age/period effect.
- α provides a form of base mortality table (while κ is zero).
- β determines the relative rates of mortality improvement at different ages.

Model specification

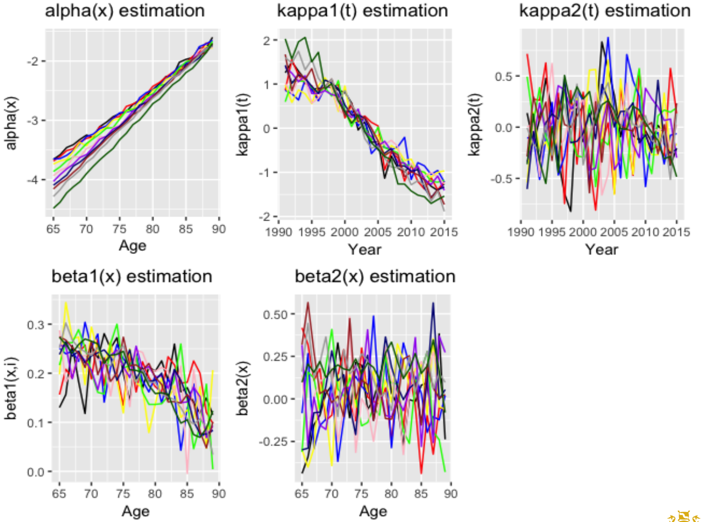
m7	$\log m_{xti} = \alpha_{xi} + \kappa^1_{ti} + (x - \bar{x})\kappa^2_{ti}$	(Plat, 2009)
m8	$\log m_{xti} = \alpha_{x} + \kappa^1_{ti} + (x - \bar{x})\kappa^2_{ti}$	(Plat model with common α_x)
m9	$\log m_{xti} = \alpha_{xi} + \kappa^1_{t} + (x - \bar{x})\kappa^2_{ti}$	(Plat model with common κ^1_t)
m10	$\log m_{xti} = \alpha_{xi} + \kappa^1_{ti} + (x - \bar{x})\kappa^2_{t}$	(Plat model with common κ^2_t)
m11	$\log m_{xti} = \alpha_{xi} + \kappa^1_{t} + (x - \bar{x})\kappa^2_{t}$	(Plat model with common κ^1_t and κ^2_t)

Model specification (cont.)

- m1 is the 'basis' with most specified structure among all others.
- All other models are simplifications of m1.
- Parameters are estimated by Poisson assuption on number of deaths with Maximum Log-likelihood Estimation (MLE).

Parameter estimation and Model selection (cont.)

- Model m1 - estimated parameters (males)



Parameter estimation and Model selection (cont.)

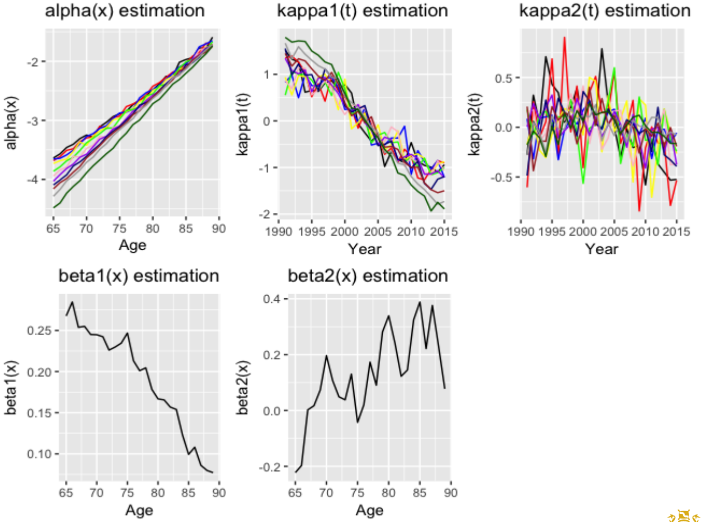
- Model m1:

$$\log m_{xti} = \alpha_{xi} + \beta^1_{xi}\kappa^1_{ti} + \beta^2_{xi}\kappa^2_{ti}$$

- Group specific α_{xi} gives observable group rankings.
- κ^1_{ti} and β^1_{xi} have decreasing pattern for all groups.
- κ^2_{ti} and β^2_{xi} are quite volatile.

Parameter estimation and Model selection (cont.)

- Model m5 - estimated parameters (males)



- Model m5:

$$\log m_{xti} = \alpha_{xi} + \beta_x^1 \kappa_{ti}^1 + \beta_x^2 \kappa_{ti}^2$$

- Group specific α_{xi} gives observable group rankings.
- κ_{ti}^1 has similar decreasing pattern for all groups.
- κ_{ti}^2 is quite volatile.
- β_x^1 decreases over age and is less volatile than β_x^2.

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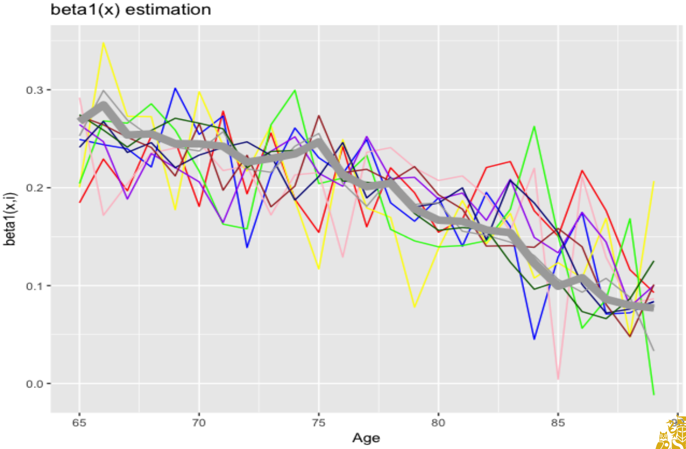
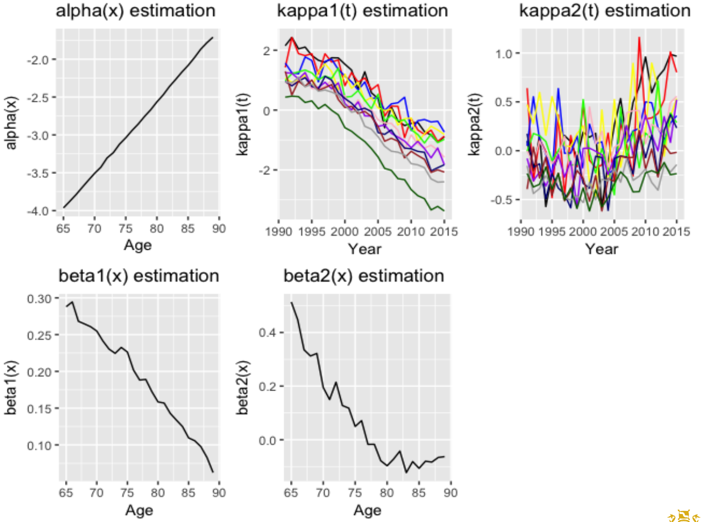
Parameter estimation and Model selection (cont.)

- Pattern of β^1 - model m5 (common - the grey fat solid line) and m1 (group-specific)



- Model m6 - estimated parameters (males)



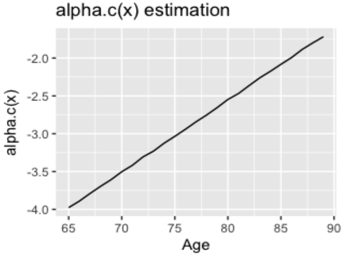
- Model m6:

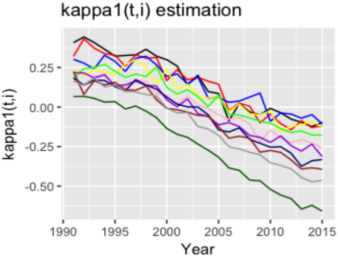
$$\log m_{xti} = \alpha_x + \beta_x^1 \kappa_{ti}^1 + \beta_x^2 \kappa_{ti}^2$$

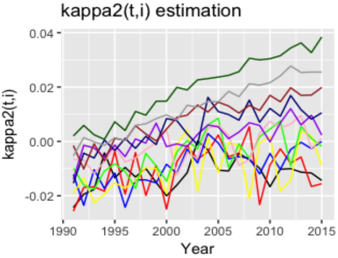
- As α_x is common, variations between subgroups are captured by κ_{ti}^1 and κ_{ti}^2.
- Group 11 stands clear of others in terms of κ_{ti}^1.
- β_x^1 and β_x^2 decreases over age, β_x^2 is smoothier than under m5.

Parameter estimation and Model selection (cont.)

- Model m8 - estimated parameters (males)







Parameter estimation and Model selection (cont.)

- Model m8:

$$\log m_{xti} = \alpha_x + \kappa_{ti}^1 + \kappa_{ti}^2 (x - \bar{x})$$

- As α_x is common, variations between subgroups are captured by κ_{ti}^1 and κ_{ti}^2.
- Group 11 stands well below and above others for κ_{ti}^1 and κ_{ti}^2 respectively.

- Model selection criteria: log-likelihood and BIC: males

Bayes Information Criterion (BIC) is a statistic based on log-likelihood that penalises over-parameterized models and is used as a purely numerical criterion for selecting out the best model (m8).

Model	log-likelihood	# parameters	df	BIC
m1	-22,252.44	1375	1331	56,265.12
m5	-22,628.04	875	851	52,775.22
m6	-22,771.52	625	621	51,029.98
m8	-22,867.36	575	573	50,797.55

Parameter estimation and Model selection (cont.)

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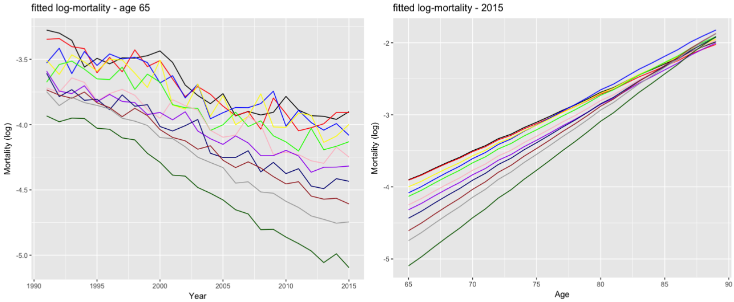
Diagnostic on fitting results

- m8 has fewest parameters and better BIC than other three.
- More parameters improves log-likelihood but is also penalized for over-parameterization.
- Greater complexity does not necessarily improve fitting significantly.
- Additional diagnostic is also required for selection.

Diagnostic on fitting results

- Fitted mortalities (log-scale) from model m8 (males)



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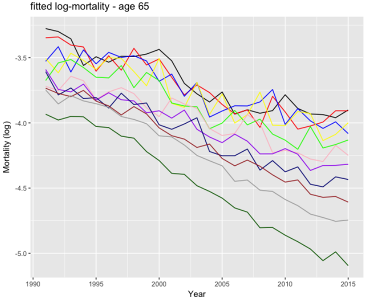
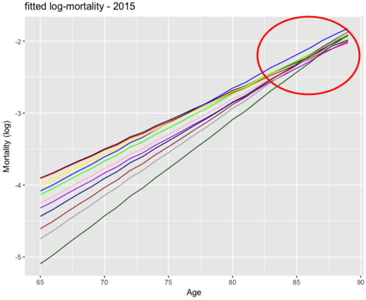
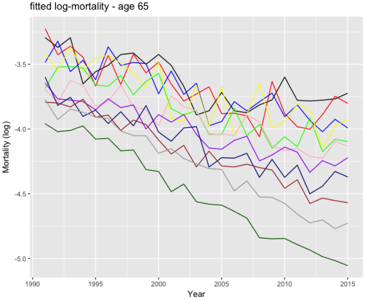
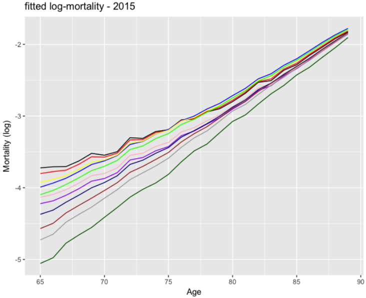




Diagnostic on fitting results

- Fitted mortalities (log-scale) from model m6 (males)





Diagnostic on fitting results

- Standardized Residuals

$$Z_{txi} = \frac{D_{txi} - E_{txi}\hat{m}_{txi}}{\sqrt{E_{txi}\hat{m}_{txi}}}$$

- Measures standardized difference between crude and estimated figures.
- Not affected by absolute scale of observations.
- Well-fitted model is expected to have random standardized residuals.

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Diagnostic on fitting results

- Standardized residuals from m6: QPP males

Diagnostic on fitting results

- Standardized residuals from m8: QPP males

Diagnostic on fitting results

- Both m6 and m8 have quite random standardized residuals.
- There is no significant non-random cluster along x-axis (year), y-axis (age) or diagonal (cohort).
- m6 doesn't have significant crossover in fitted mortality curves. m8 has crossovers at high ages.
- m6 is selected as the most suitable model for QPP males.

Cluster analysis on QPP data

- QPP has relatively small population size.
- Subpopulations are not evenly grouped.
- Crude mortalities are quite volatile.
- Some adjacent groups typically have quite similar levels of mortality.
- We consider to re-cluster the QPP dataset.

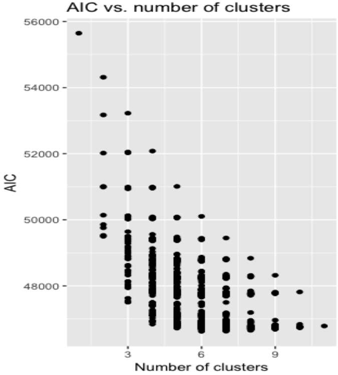
Cluster analysis on QPP data

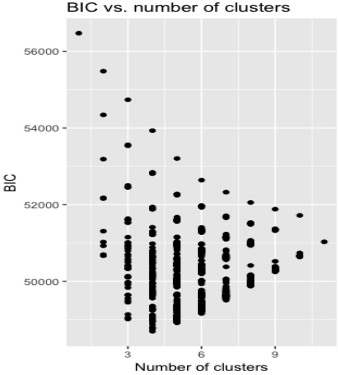
- Algorithm:

1. Restructure the data by combining neighbouring groups into clusters. Each cluster could contain $1, 2, \ldots, 11$ groups.
2. We obtain new restructured datasets with ≤ 11 groups.
3. There are $1,024$ different combinations in total. $(\sum_{i=0}^{11-1} C_{11-1}^{i})$
4. Fit underlying models to each reclustered dataset.

Cluster analysis on QPP data

- AIC and BIC for all $1,024$ cluster combinations fitted for model m6:

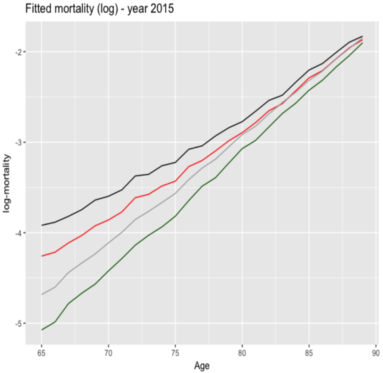




BIC is $48,694.69$ under the optimized scenario (used to be $51,029.98$).

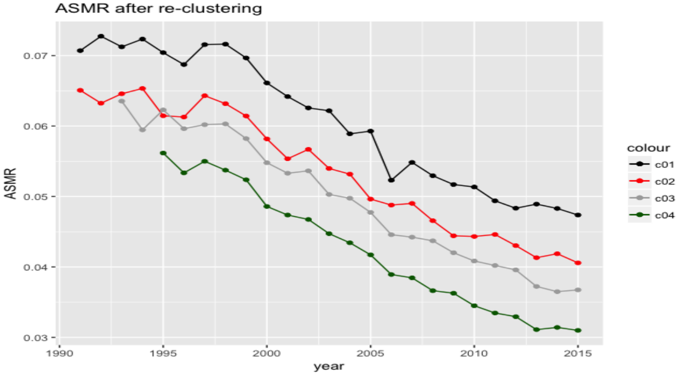
Cluster analysis on QPP data

- Fitted mortalities (log-scale) from m6 after re-clustered into 4 groups.



Cluster analysis on QPP data

- ASMR of QPP males after re-clustering:



Cluster analysis on QPP data

- Conclusion from cluster analysis:
  - All models suggest the same optimal clustering by BIC - with 4 clusters:
    - Cluster 1: group 1-5;
    - Cluster 2: group 6-8;
    - Cluster 3: group 9 and 10;
    - Cluster 4: group 11.
  - Volatilities are reduced significantly.
  - It enables us to see more clearly the different trends of clusters.

Summary

- For volatile population, models with simpler structure fits better, i.e. model m6 and m8 over m1.
- Besides quantitative criteria, qualitative criteria like graphical diagnostics are the same important.
- Clustering improves fitting quality and signal-to-noise ratio.
- Future researches: Smoothing of modelling results; More detailed cluster analysis; Long-term mortality projection.

ANY QUESTIONS ?