EXAMINATIONS

9 September 2002 (pm)

Subject 104 — Survival Models

Time allowed: Three hours

INSTRUCTIONS TO THE CANDIDATE

1. *Enter all the candidate and examination details as requested on the front of your answer booklet.*

2. *You must not start writing your answers in the booklet until instructed to do so by the supervisor.*

3 *Mark allocations are shown in brackets.*

4 *Attempt all 11 questions, beginning your answer to each question on a separate sheet.*

Graph paper is required for this paper.

AT THE END OF THE EXAMINATION

Hand in BOTH your answer booklet, with any additional sheets firmly attached, and this question paper.

In addition to this paper you should have available Actuarial Tables and your own electronic calculator.

1 If K_x is a random variable which measures the curtate future lifetime of a life aged x, describe the benefits payable under policies that have the present value of their benefits expressed as

(a) $a_{\overline{K_x}|}$ $\qquad K_x = 0, 1, 2, 3, \ldots$

(b) $\ddot{a}_{\overline{\min(K_x+1,n+1)}|} - 1$ $\qquad K_x = 0, 1, 2, 3, \ldots$ [3]

2 An endowment assurance issued to a man aged exactly 30 provides the following benefits:

- a sum assured of £10,000 payable immediately on death before age 65
- a sum assured of £50,000 on survival to age 65

Premiums are payable annually in advance for 30 years, or until death if earlier.

Calculate the level annual premium payable. [5]

Basis: Mortality: A1967–70 Ultimate Males
Interest: 6% per annum effective throughout
Expenses are ignored

3 The Australian National Life Table for males 1980–1982 was estimated using the death registrations for males during the three calendar years 1980, 1981 and 1982 (184,197 deaths) and the male population enumerated at the census held on 30 June 1981 (7,416,090 lives).

A student has argued that with such a large number of lives and deaths the standard errors of the estimated mortality rates will be very small and that graduation of the rates is not necessary.

Explain why this conclusion is untrue. [5]

4 (i) Explain the differences between random censoring and Type I censoring in the context of an investigation into the mortality of life insurance policyholders. Include in your explanation a statement of the circumstances in which the censoring will be random, and the circumstances in which it will be Type I. [4]

(ii) Explain what is meant by non-informative censoring in the investigation in (i). Describe a situation in which censoring might be informative in this investigation. [3]
[Total 7]

5 (i) Given that $p_x = 0.9$, calculate ${}_{0.5}p_x$ and ${}_{0.5}p_{x+0.5}$ using the following assumptions about mortality between ages x and $x + 1$:

(a) uniform distribution of deaths
(b) Balducci assumption [4]

(ii) Comment on how appropriate you think each of these assumptions is. [2]
[Total 6]

6 In an investigation of the mortality of whole of life insurance policyholders, information was available about the number of policyholders aged x last birthday on 1 January 1999, 1 January 2000 and 1 January 2001. Counts of deaths for the calendar years 1999 and 2000 were available classified by age nearest birthday on the date of death. The table below shows an extract from the data.

Age	*Number of persons aged x last birthday on*			*Deaths aged x nearest birthday*	
	1 Jan 1999	*1 Jan 2000*	*1 Jan 2001*	*1999*	*2000*
x	$P_{x,1999}$	$P_{x,2000}$	$P_{x,2001}$	$\theta_x(1999)$	$\theta_x(2000)$
40	473	512	491	17	18
41	450	470	482	20	18
42	490	460	480	21	19

Assuming that the forces of mortality at each age are constant over the whole period from 1 January 1999 to 1 January 2001:

(i) Derive a formula to estimate the constant force of mortality for lives aged x nearest birthday. State any assumptions you make in deriving your formula.
[5]

(ii) Use your formula in (i) to calculate numerical estimates of μ_{41} and μ_{42}. [2]
[Total 7]

7 (i) You have been asked to undertake an investigation of the mortality of male term assurance policyholders for the period 1 January 1995 to 31 December 1998. List the data that should be recorded for each life so that the investigation can be undertaken using a life year rate interval with an exact calculation of the Central Exposed to Risk. The list should include only data items that are essential to complete the calculations. [3]

(ii) Using the data items from (i) describe how the force of mortality at particular ages would be estimated. [4]
[Total 7]

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8 (i) In the context of the graduation of a set of estimated mortality rates explain what is meant by:

(a) smoothness
(b) adherence to data [4]

(ii) Graduation is said to "resolve the conflicting requirements of smoothness and adherence to data".

Explain:

(a) how this conflict arises, and
(b) how the process of graduation resolves the conflict [4]
[Total 8]

9 In a clinical trial, 50 patients are observed for two years following treatment with a new drug. The following data show the period in complete months from the initial treatment to the end of observation for those patients who died or withdrew from the trial before the end of the two year period.

Deaths	6, 6, 12, 15, 20, 20, 23
Withdrawals	1, 3, 5, 8, 10, 18

(i) Calculate the Nelson-Aalen estimate of the integrated hazard function, Λ_t. [6]

(ii) Hence, or otherwise, estimate the probability of a patient surviving for at least 18 months after the initial treatment. [2]
[Total 8]

10 (i) Explain the rationale behind the use of the Poisson distribution to model the number of deaths among a group of lives. Include in your explanation a discussion of why the Poisson Model is not always an exact model. [4]

(ii) A group of N lives is observed for some finite period between the ages of x and $x + 1$. Let v_i be the observed waiting time for life i. Assuming a constant force of mortality μ between ages x and $x + 1$, derive the maximum likelihood estimator $\hat{\mu}$ of this constant force under the Poisson model. [6]

(iii) Write down the expected value and the variance of $\hat{\mu}$. [2]

(iv) (a) Write down an approximate 95% confidence interval for the constant force, μ.

(b) Explain how this confidence interval could be used to help in the graphical graduation of a set of estimated mortality rates, $\hat{\mu}_x$ (x = 50, 51, 52, …, 98). [6]
[Total 18]

11 (i) (a) An endowment assurance with a term of 20 years is sold to a life aged exactly 50. The assurance has a sum assured of £1,000 payable immediately on death or on survival to the end of the term. Premiums are paid continuously until the end of the term of the policy or until earlier death.

Show that the net level annual premium is £37.16.

(b) Show that the net level annual premium for a term assurance with the same sum assured and the same term is £19.06. [7]

Basis: Mortality: English Life Table No. 12 Males
Interest: 6% per annum effective
Expenses are ignored

(ii) Using the same basis as in (i), calculate the net premium policy value at duration 15 for:

(a) the endowment assurance described in (i)(a)
(b) the term assurance described in (i)(b) [7]

(iii) On the same scale and axes sketch graphs of the net premium policy values of each of the endowment assurance and the term assurance against the term of the policy. [4]

(iv) Thiele's equation for the endowment assurance policy is

$$\frac{\partial}{\partial t}\,{}_t\bar{V}(\bar{A}_{50:\overline{20|}}) = \delta\,{}_t\bar{V}(\bar{A}_{50:\overline{20|}}) + \bar{P}(\bar{A}_{50:\overline{20|}}) - (1 - {}_t\bar{V}(\bar{A}_{50:\overline{20|}}))\ \mu_{x+t} \qquad 0 < t < 20$$

Write down Thiele's equation for the term assurance policy. [2]

(v) Use the equations from (iv) and the calculations from (i) and (ii) to explain the shape at policy duration 15 of the graphs you have drawn in (iii). You should provide calculations to support your explanations. [6]
[Total 26]