Faculty of Actuaries Institute of Actuaries

EXAMINATIONS

April 2000

Subject 104 — Survival Models

EXAMINERS' REPORT

1 (a) **Parametric Formula**

The chosen formula, e.g. GM(r,s) can be approximated by a polynominal of low order/a curve with few parameters.

The smoothness criterion of small second or third order differences of the graduated rates will be ensured by the approximate low order of the parametric formula chosen as the best fitting curve.

(b) **Reference to a Standard Table**

The standard table will already have fulfilled the smoothness criterion.

The formula chosen to be fitted to the crude rates will usually represent an approximately linear transformation of the rates in the standard table, e.g. $q_x = aq_x^s + b$ so that the second and third order differences of the graduated rates will be the same as those of the standard table.

2 (i) The underlying model of mortality is that rates change smoothly with age.

Two aspects of estimation are in conflict with this model

- rates are estimated <u>separately</u> for each age and there is no constraint in the estimation procedures which ensures that rates at adjacent ages conform to the model
- superimposed on this is the sampling error of each estimate. This will be an important effect at ages where there is a small exposed to risk.

If these features of estimation are not corrected before rates are used then, for example, premium rates may not progress smoothly by age. This could present opportunities for "lapse and re-entry" options by healthy policyholders resulting in increased expenses, and an irregular progression would be hard to justify in practice.

Creit given for other relevant examples.

3 $\bar{A}_{50:\overline{15}|} = \bar{A}^{1}_{50:\overline{15}|} + A_{50:\overset{1}{\overline{15}|}}$

$$\simeq (1 + ½i)\ A^{1}_{50:\overline{15}|} + A_{50:\overset{1}{\overline{15}|}}$$

$$= (1 + ½i)\ \left\{A_{50:\overline{15}|} - A_{50:\overset{1}{\overline{15}|}}\right\} + A_{50:\overset{1}{\overline{15}|}}$$

$$= (1.03)\ \left\{0.44395 - \overset{0.35050}{A_{50:\overset{1}{\overline{15}|}}}\right\} + \overset{0.35050}{A_{50:\overset{1}{\overline{15}|}}}$$

as

$$A_{50:\overset{1}{\overline{15}|}} = \frac{\overset{27442.681}{l_{65}}}{\underset{32669.855}{l_{50}}}.(1.06)^{-15} = 0.35050$$

$$\therefore\ \bar{A}_{50:\overline{15}|} = 1.03 \times 0.09345 + 0.35050$$

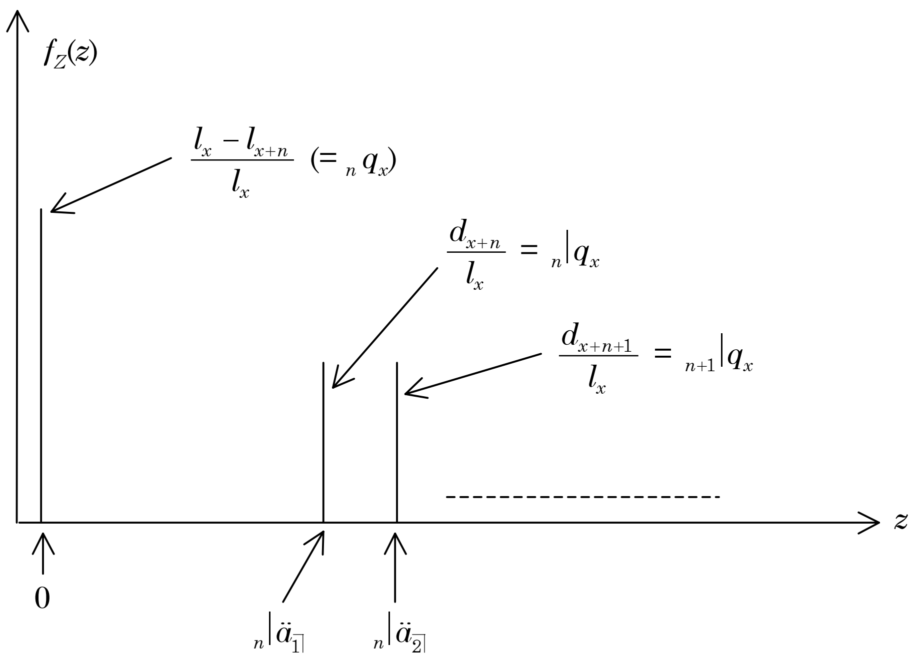
$$= 0.09625 + 0.35050$$

$$= 0.44675$$

(not! $1.03 \times 0.44395 = 0.45727$)

Other "acceleration" factors acceptable. Solution also possible using premium conversation relationship.

4 (i) $Z = \begin{cases} 0 & 0 \le k < n \\ {}_{n|}\ddot{a}_{\overline{k+1-n}|} & k \ge n \end{cases}$



(ii) Equation of value:

$$P \,.\, \ddot{a}_{38:\overline{25|}} = 10000 \,.\, \frac{D_{63}}{D_{38}} \,.\, \ddot{a}_{63}$$

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$$\ddot{a}_{38:\overline{25|}} = \frac{N_{38} - N_{63}}{D_{38}} = 15.7271$$

7575.028

$$\therefore\ P = \left(10000 \times \frac{2418.2107}{7575.028} \times 11.463\right) \div 15.7271$$

$$= \underline{\underline{2326.81}}$$

5 (i) The life annuity will be secured by a single payment at age x and so the policy value at duration t will be

$$ {}_t\overline{V} = \overline{a}_{x+t}$$

$$ {}_t\overline{V} = \overline{a}_{x+t} = \int_{s=0}^{s=\infty} e^{-\delta s} \,.\, {}_sp_{x+t} \,.\, ds$$

So: $$\frac{\partial}{\partial t}\, {}_t\overline{V} = \frac{\partial}{\partial t} \int_{s=0}^{s=\infty} e^{-\delta s}\, {}_sp_{x+t} \,.\, ds$$

$$= \int_{s=0}^{s=\infty} e^{-\delta s} \frac{\partial}{\partial t}\, {}_sp_{x+t} \,.\, ds$$

Now $$\frac{\partial}{\partial t}\, {}_sp_{x+t} = \frac{\partial}{\partial t}\left(\frac{l_{x+t+s}}{l_{x+t}}\right)$$

$$= \frac{l_{x+t}(-\mu_{x+t+s} \,.\, l_{x+t+s}) - l_{x+t+s}(-\mu_{x+t}\; l_{x+t})}{l_{x+t}^2}$$

$$= {}_sp_{x+t}\,(\mu_{x+t} - \mu_{x+t+s})$$

$$\therefore \quad \frac{\partial}{\partial t}\, {}_t\overline{V} = \int_{s=0}^{s=\infty} e^{-\delta s} \,.\, {}_sp_{x+t}\,(\mu_{x+t} - \mu_{x+t+s})\; ds$$

$$= \mu_{x+t}\; \overline{a}_{x+t} - \int_{s=0}^{s=\infty} e^{-\delta s}\, {}_sp_{x+t}\; \mu_{x+t+s} \,.\, ds$$

$$= \mu_{x+t}\ \bar{a}_{x+t} - \left\{ -e^{-\delta s}\ .\ {}_s p_{x+t} \Big|_0^{\infty} - (-\delta)\int_{s=0}^{s=\infty} e^{-\delta s}\,{}_s p_{x+t}\ .\ ds \right\}$$

$$= \mu_{x+t}\ \bar{a}_{x+t} - 1 + \delta \bar{a}_{x+t}$$

$$= \mu_{x+t}\ .\ {}_t\bar{V} - 1 + \delta\ .\ {}_t\bar{V}$$

A derivation is required for (i). Alternative "steps" are possible.

(ii) If we consider the short time interval $(t,\ t + dt)$ then equation implies

$${}_{t+dt}\bar{V} - {}_t\bar{V} = {}_t\bar{V}\ .\ \delta dt - 1\ .\ dt + {}_t\bar{V}\ \mu_{x+t}\ dt + o(dt)$$

where

${}_t\bar{V}\delta dt$	Interest earned on the reserve over $(t,\ t + dt)$
$-\ 1\ .\ dt$	Annuity payments made in $(t,\ t + dt)$
$+\ {}_t\bar{V}\ \mu_{x+t}\ .\ dt$	Reserves "released" as a result of deaths in $(t,\ t + dt)$

and these are all the "changes" that can happen over $(t,\ t + dt)$.

[Note: the candidates may write the expression as:

$${}_t\bar{V}(1 + \delta\ .\ dt) = {}_{t+dt}\bar{V} + 1\ .\ dt - dt\ .\ \mu_{x+t}\ .\ {}_t\bar{V} + o(dt)$$

which might be easier to interpret as [income] = [outgo]]

6 (a) H_0: the observed transition rates $\hat{\mu}_{x+½}$ come from a population in which the standard table rates are the true rates.

χ^2 with m degrees of freedom

(b) H_0: the observed transition rates $\hat{\mu}_{x+½}$ come from a population in which the graduated transition rates are the true rates

In the graduation process four parameters have been estimated so sampling distribution is χ^2 with $m - 4$ degrees of freedom.

(c) H_0: same as in (b).

1 degree of freedom is lost for each parameter estimated, plus an unknown additional number for each constraint imposed by the standard table; say p degrees of freedom, so:

χ^2 with $m - 1 - p$ degrees of freedom.

(d) H_0: same as in (b)

The best fitting curve will usually be drawn as a series of curved segments joined smoothly.

Each segment imposes a constraint of height, slope and curvature; so lose 2 or 3 degrees of freedom for each section of about 10 ages drawn; so in 2 sections for example we have

χ^2 with say $m - 5$ degrees of freedom.

7 (i) $X - Y$ is the present value of a deferred whole of life assurance with a sum assured of 1 payable at the end of the year of death of a life now aged x provided the life dies after age $x + n$.

(ii) $X = v^{k+1}$ all k $\qquad Y = \begin{cases} v^{k+1} & 0 \le k < n \\ 0 & k \ge n \end{cases}$

$\mathrm{Cov}(X, Y) = E[XY] - E[X]\,E[Y]$

Now $E[XY] = \sum_{k=0}^{k=n-1} (v^{k+1})^2 \; P[K_x = k] + \sum_{k=n}^{k=\infty} v^{k+1} \times 0 \times P[K_x = k]$

$= \sum_{k=0}^{k=n-1} (v^2)^{k+1} \; P[K_x = k]$

$= {}^2A^{1}_{x:\overline{n}|}$

Where ${}^2A$ is determined using a discount function v^2, i.e. using an interest rate

$i^* = (1 + i)^2 - 1 = 2i + i^2$

Then: $\mathrm{Cov}(X, Y) = {}^2A^{1}_{x:\overline{n}|} - A_x \,.\, A^{1}_{x:\overline{n}|}$

Now: $\text{Var}(X - Y) = \text{Var}(X) + \text{Var}(Y) - 2\,\text{Cov}(X, Y)$

$$= ({}^2A_x - (A_x)^2) + ({}^2A^1_{x:\overline{n|}} - (A^1_{x:\overline{n|}})^2) - 2({}^2A^1_{x:\overline{n|}} - A_x \,.\, A^1_{x:\overline{n|}})$$

$$= ({}^2A_x + {}^2A^1_{x:\overline{n|}} - 2\,{}^2A^1_{x:\overline{n|}}) - ((A_x)^2 + (A^1_{x:\overline{n|}})^2 - 2A^1_{x:\overline{n|}}A_x)$$

$$= {}^2A_x - {}^2A^1_{x:\overline{n|}} - (A_x - A^1_{x:\overline{n|}})^2$$

$$= {}^2A_x - {}^2A^1_{x:\overline{n|}} - ({}_{n|}A_x)^2$$

Alternative approaches are possible.

8 (i) Hazard Rate, $h(x) = \lim\limits_{h \to 0+} \dfrac{P[x < X \le x + h \mid X > x]}{h}$

Alternative expressions are also correct.

Integrated Hazard $H(x) = \displaystyle\int_{u=0}^{u=x} h(u)\,.\,du = -\ln(S(x))$

$$= -\ln\,.\,P[X > x]$$

where $S(x) = P[X > x]$

(ii) From (i) $\quad H(x) = \displaystyle\int_{u=0}^{u=x} \alpha\lambda u^{\alpha-1}\,.\,du$

$$= \frac{\alpha}{\alpha}\lambda u^{\alpha}\Big|_{u=0}^{u=x}$$

$$= \lambda x^{\alpha} \qquad\qquad x > 0$$

(iii) $h(x) = (\alpha_0 + \alpha_1 z_1)(\lambda_1 z_1 + \lambda_2 z_2)\ x^{\alpha_0 + \alpha_1 z_1 - 1}$

Then $\dfrac{h(x \mid z)}{h(x \mid z^*)} = \dfrac{(\alpha_0 + \alpha_1 z_1)(\lambda_1 z_1 + \lambda_2 z_2)\ x^{\alpha_0 + \alpha_1 z - 1}}{(\alpha_0 + \alpha_1 z_1^*)(\lambda_1 z_1^* + \lambda_2 z_2^*)\ x^{\alpha_0 + \alpha_1 z^* - 1}}$

which is not in general independent of x, so hazards are not proportional.

If $\alpha_1 = 0$, then $\dfrac{h(x|z)}{h(x|z^*)} = \dfrac{\alpha_0(\lambda_1 z_1 + \lambda_2 z_2)\, x^{\alpha_0 - 1}}{\alpha_0(\lambda_1 z_1^* + \lambda_2 z_2^*)\, x^{\alpha_0 - 1}}$

$$= \frac{\lambda_1 z_1 + \lambda_2 z_2}{\lambda_1 z_1^* + \lambda_2 z_2^*}$$

which is independent of x, so the hazards are proportional.

Alternatively if $\alpha_1 = 0$, then

$$h(x\,|\,z) = \alpha_0(\lambda_1 z_1 + \lambda_2 z_2)\, x^{\alpha_0 - 1}$$

which is of the form $h_0(x) = \alpha_0 x^{\alpha_0 - 1} \times c(z)$; where $c(z) = \lambda_1 z_1 + \lambda_2 z_2$

9 (i) Expected Present Value of death benefit $= 10000\bar{A}^{\,1}_{40:\overline{25}|}$

Expected Present Value of survival benefit $= 15000A_{40:\overset{1}{\overline{25}|}}$

Expected Present Value of Premiums to be paid

$$= P\ddot{a}_{40:\overline{25}|} + P\,{}_{5|}\ddot{a}_{40:\overline{20}|} + P\,{}_{10|}\ddot{a}_{40:\overline{15}|}$$

Evaluation

$$\bar{A}^{\,1}_{40:\overline{25}|} \simeq (1 + \tfrac{1}{2}i)\, A^{\,1}_{40:\overline{25}|}$$

$$= 1.02 \times \left(\overset{0.40005}{A_{40:\overline{25}|}} - \overset{0.30690}{A_{40:\overset{1}{\overline{25}|}}} \right)$$

$$= 0.09501$$

as $A_{40:\overset{1}{\overline{25}|}} = \dfrac{\overset{2144.1713}{D_{65}}}{\underset{6986.4959}{D_{40}}} = 0.30690$

Also $\ddot{a}_{40:\overline{25}|} = 15.599$

$$_{5|}\ddot{a}_{40:\overline{20}|} = \frac{\overset{5689.1776}{D_{45}}}{\underset{6986.4959}{D_{65}}} \overset{13.488}{\ddot{a}_{45:\overline{20}|}} = 10.9834$$

and $$_{10|}\ddot{a}_{40:\overline{15}|} = \frac{\overset{4597.0607}{D_{50}}}{\underset{6986.4959}{D_{40}}} \overset{10.995}{\ddot{a}_{50:\overline{15}|}} = 7.2346$$

Equation of Value

$$\underset{950.10}{10000\bar{A}^{\,1}_{40:\overline{25}|}} + \underset{4603.50}{15000A_{40:\overset{\,1}{\overline{25}|}}} = P(\underbrace{\ddot{a}_{40:\overline{25}|} + {}_{5|}\ddot{a}_{40:\overline{20}|} + {}_{10|}\ddot{a}_{40:\overline{15}|}}_{33.817})$$

$$P = \frac{5553.60}{33.817} = £164.2251 \qquad \underline{£164.23}$$

(ii) Prospective Policy Value after 12 years

$$\underset{0.11272}{10000\bar{A}^{\,1}_{52:\overline{13}|}} + \underset{0.50965}{15000A_{52:\overset{\,1}{13}}} - 3 \times 164.23\underset{9.876}{\ddot{a}_{52:\overline{13}|}}$$

Evaluation

$$A_{52:\overset{\,1}{\overline{13}|}} = \frac{\overset{2144.1713}{D_{65}}}{\underset{4207.1417}{D_{52}}} = 0.50965$$

$$\bar{A}^{1}_{52:\overline{13}|} \simeq 1.02(\overset{0.62016}{A_{52:\overline{13}|}} - \overset{0.50965}{A_{52:\overset{\,1}{\overline{13}|}}}) = 0.11272$$

Policy Value 1127.2 + 7644.75 − 4865.806 = £3906.14

10 (i) Those lives who are released from prison during the period of investigation or who are still alive and resident in the prison at the end of the period of investigation are *right censored*.

Duration is recorded to the nearest month so there is *interval censoring*.

Being released from prison is a form of *random censoring*.

(ii) We can summarise the data to obtain the statistics necessary to complete the estimation.

j	t_j months	R_j Risk set $= R_{j-1} - d_{j-1} - C_{j-1}$	C_j # censored in $[t_j, t_{j+1}]$	d_j # of deaths
1	6	14	1	3
2	7	10	1	1
3	10	8	2	1
4	13	5	0	1
5	16	4	1	1
6	20	2	1	1

Then estimates of survival probabilities are

j	t_j	$s_j = s_{j-1} \times \dfrac{(R_{j-1} - d_{j-1})}{R_{j-1}}$	
1	6	1	
2	7	1 × (14 – 3) / 14	= .7857
3	10	.7857 × (10 – 1) / 10	= .7071
4	13	.7071 × (8 – 1) / 8	= .6188
5	16	.6188 × (5 – 1) / 5	= .4950
6	20	.4950 × (4 – 1) / 4	= .3712
7	23	.3712 × (2 – 1) / 2	= .1856

So estimates of the survival function is:

$$S(t) = \begin{cases} 1 & 0 \le t \le 6 \\ .79 & 6 < t \le 7 \\ .71 & 7 < t \le 10 \\ .62 & 10 < t \le 13 \\ .50 & 13 < t \le 16 \\ .37 & 16 < t \le 20 \\ .19 & 20 < t \le 23 \end{cases}$$

(iii) Non-informative censoring: Time to censoring i.e. leaving for reasons other than death is independent of Time to death.

If we believe that more healthy lives will tend to leave and thus have lighter mortality, this assumption is violated. [OR could say less healthy lives may tend to leave — same conclusion.]

However if we believe that being released is unrelated to state of health, then assumption is OK.

Lives are independent i.e. time to censoring, i.e. being released, or time to death determined independently for each life.

Other relevant comments received credit.

11 (i) $${}_{t+h}p_x^{11} = {}_tp_x^{11} \cdot {}_hp_{x+t}^{11}$$

assuming probabilities of transition are dependent upon age and the current state only (Markov assumption).

Using $1 = {}_hp_{x+t}^{11} + {}_hp_{x+t}^{12} + {}_hp_{x+t}^{13} + {}_hp_{x+t}^{14}$

i.e. law of total probability

$${}_{t+h}p_x^{11} = {}_tp_x^{11}(1 - [{}_hp_{x+t}^{12} + {}_hp_{x+t}^{13} + {}_hp_{x+t}^{14}])$$

Now ${}_hp_{x+t}^{1j} = h \,.\, \mu_{x+t}^{1j} + o(h)$ for $j \neq 1$

where $o(h)$ is such that $\lim\limits_{h\to 0+} \dfrac{o(h)}{h} = 0$

$$\therefore \; {}_{t+h}p_x^{11} - {}_tp_x^{11} = -\,{}_tp_x^{11} \,.\, h \,.\, (\mu_{x+t}^{12} + \mu_{x+t}^{13} + \mu_{x+t}^{14}) + o(h)$$

$$\therefore \; \frac{{}_{t+h}p_x^{11} - {}_tp_x^{11}}{h} = -\,{}_tp_x^{11} \cdot \sum_{j=2}^{4} \mu_{x+t}^{1j} + \frac{o(h)}{h}$$

$$\therefore \; \lim_{h\to 0+} \left(\frac{{}_{t+h}p_x^{11} - {}_tp_x^{11}}{h} \right) = \frac{\partial\, {}_tp_x^{11}}{\partial t} = -\,{}_tp_x^{11} \cdot \sum_{2}^{4} \mu_{x+t}^{1j}$$

Hence $\dfrac{\partial \log_e {}_tp_x^{11}}{\partial t} = -\sum\limits_{2}^{4} \mu_{x+t}^{1j}$

$$\therefore \; \log_e {}_tp_x^{11} - \log_e {}_op_x^{11} = \int_0^t -\sum_{2}^{4} \mu_{x+s}^{1j}\, ds$$

As ${}_0p_x^{11} = 1$

then $\log_e {}_tp_x^{11} = \int_0^t -\sum_2^4 \mu_{x+s}^{1j} ds$

and ${}_tp_x^{11} = \exp\left(-\int_0^t \sum_2^4 \mu_{x+s}^{1j}\, ds\right)$

(ii) $\frac{\partial}{\partial t}\, {}_tp_x^{1j} = {}_tp_x^{11} \,.\, \mu_{x+t}^{1j} \qquad j \neq 1$

Hence ${}_tp_x^{1j} - {}_op_x^{1j} = \int_0^t {}_rp_x^{11} \,.\, \mu_{x+r}^{1j}\, dr$

As ${}_op_x^{1j} = 0$

then ${}_tp_x^{1j} = \int_0^t {}_rp_x^{11} \,.\, \mu_{x+r}^{1j}\, dr$

(iii) Substituting 1 for t, assuming constant forces, and putting (i) into (ii) we get:

$${}_1p_x^{1j} = \int_0^1 e^{-\int_0^r \sum_2^4 \mu_{x+½}^{1k} ds} \,.\, \mu_{x+½}^{1j}\, dr$$

$$= \mu_{x+½}^{1j} \,.\, \int_0^1 e^{-r.\sum_2^4 \mu_{x+½}^{1k}}\, dr$$

$$= \frac{\mu_{x+½}^{1j}}{-\sum_2^4 \mu_{x+½}^{1k}} \left[e^{-r.\sum_2^4 \mu_{x+½}^{1k}} \right]_0^1$$

$$= \frac{\mu_{x+½}^{1j}}{\sum_2^4 \mu_{x+½}^{1k}} \left[1 - e^{-\sum_2^4 \mu_{x+½}^{1k}} \right]$$

(iv) $\quad {}_2p_{60}^{12} = {}_1p_{60}^{12} + {}_1p_{60}^{11} \cdot {}_1p_{61}^{12}$

Now $\quad {}_1p_{60}^{12} = \dfrac{.25}{.342}(1 - e^{-.342}) = .2117$

$${}_1p_{60}^{11} = e^{-.342} = .7103$$

$${}_1p_{61}^{12} = \dfrac{.10}{.154}(1 - e^{-.154}) = .0927$$

$\therefore \quad {}_2p_{60}^{12} = .2117 + .7103 \times .0927 = \underline{\underline{.2775}}$

12 (i) (a) There is considerable evidence from other studies that mortality rates vary with smoking status (smokers have higher rates at all ages),
sex (males have higher mortality at all ages)
and policy class (permanent assurances i.e. whole life and endowment have different mortality than temporary assurances).
To ensure homogeneity policies should be divided so that those of the same smoking status, sex and policy class fall in one group.
All of the statistical models of mortality which might be parameterised from the data require the data to be homogeneous.

(b) Too many subdivisions of data can make sample sizes very small.

- so that the estimates will have large standard errors
- and hence little meaningful interpretation can be made
- may lead to identification of spurious selection

Other relevant comments given credit.

(ii) Choose a period of investigation from time 0 to time T, where T is a whole number of calendar years (say about 4) and 0 corresponds to the start of a calendar year.

The x = age next birthday on policy anniversary before death.

Let $\Sigma\theta_x$ be total deaths labelled x in all calendar years during period of investigation.

Let $P_x(t)$ be a census at time t after start of period of investigation of those lives having age label x at time t. Then

$$E_x^c = \int_{t=0}^{t=T} P_x(t)\,.\,dt$$

$$\simeq \tfrac{1}{2}P_x(o) + \sum_{t=1}^{t=T-1} P_x(t) + \tfrac{1}{2}P_x(T)$$

assuming $P_x(t)$ varies linearly with t over each calendar year.

Then: $\hat{\mu}_x = \dfrac{\Sigma\theta_x}{E_x^c}$ estimates μ_x

Policy year rate interval, average x – ½ at start assuming birthdays are uniformly distributed over the policy year, and that the force of mortality is constant over each year of age.

(iii) The investigations are for widely sold classes of business, so graduation with reference to a standard mortality table for similar classes of business would be appropriate,
e.g. using a published table for male permanent (whole life and endowment) assurances.

Separate graduations would probably be necessary by type of policy, sex, and smoking status.

The standard table should be chosen so that the characteristics of the company's business are similar to those of the standard table, e.g. geographical regions in which the business is sold, policy terms and conditions.

Plots of the standard table and the observed experience should be compared for "shape". It is important that the shapes are similar.

Graphical investigations should find a suitable relationship $f(q_x^s), f(\mu_x^s)$, between the standard table rates and the current experience, e.g. $\mathring{q}_x = a + bq_x^s$ reflects and approximate straight line relationship when $\hat{q}_x$ is plotted against q_x^s. A perfect fit is not expected particularly at ages where the data are scanty. [$\mathring{q}_x$ is graduated rate, $\hat{q}_x$ is crude estimate].

A best fitting line should be fitted. Weighted least squares provides a satisfactory fitting method, e.g. choose a and b to minimise

$$\sum_x w_x(\mathring{q}_x - \hat{q}_x)^2 = \sum w_x(a + bq_x^s - \hat{q}_x)^2$$

where $w_x = [\mathrm{Var}(\tilde{q}_x)]^{-1}$ $\tilde{q}_x$ is estimator of $\hat{q}_x$

The resulting graduated rates are tested for adherence to the crude estimates.
Smoothness of $\mathring{q}_n$ will be ensured by the method of graduation, since smoothness is "borrowed" from the standard table.

Tests for "adherence to data" should include a test for overall fit (χ^2 test)

and tests for the pattern of residuals $(\mathring{q}_x - \hat{q}_x)$
by size (sign test, standardised deviations test)
and by pattern with age (cumulative deviations test, serial correlation test.

If satisfactory adherence to data cannot be obtained then the chosen relationship should be reassessed, i.e. choose new $f(q_x^s)$ or $f(\mu_x^s)$ and repeat fitting and testing.
If a satisfactory relationship cannot be found, then the choice of standard table should be reassessed and the fitting procedure repeated.

Similar description of fitting parametric formula also acceptable.
Graphical methods not acceptable.

The graduated rates, e.g. $\mathring{q}_x = \hat{a} + \hat{b}q_x^s$ can be tabulated with sufficient significant figures to facilitate their use in premium rate calculations.