EXAMINATIONS

September 2002

Subject 104 — Survival Models

EXAMINERS' REPORT

Introduction

The attached subject report has been written by the Principal Examiner with the aim of helping candidates. The questions and comments are based around Core Reading as the interpretation of the syllabus to which the examiners are working. They have however given credit for any alternative approach or interpretation which they consider to be reasonable.

K G Forman
Chairman of the Board of Examiners
12 November 2002

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EXAMINERS' COMMENTS

By its nature Survival Models is a statistical subject and the examination may contain questions that require the use of the skills taught in Subject 101. As in previous papers, the Examiners aim to strike a balance between questions requiring numerical solutions and those requiring verbal and algebraic answers, as well as between those with and without a statistical theme. The questions set will always aim to cover Survival Modelling in all its aspects, Life Contingencies including their stochastic treatment, Graduation including its statistical aspects and the determination of exposures.

The solutions presented to questions 1, 2, 5 and 9 were generally good. The general standard on the remaining questions was below that expected. Comments on the individual questions follow:

Question 3. Few candidates applied their knowledge to the circumstances of the question, with the majority of candidates stating general reasons for graduation.

Question 4. Many candidates failed to state appropriate circumstances for the different types of censoring, in particular failing to relate their answer to life insurance policyholders as stated in the question.

Question 6. Few candidates showed a clear derivation of the formula in part (i).

Question 7. Some candidates failed to describe the method using the exact central exposed to risk. Of those that described the correct method, a large number of failed to provide sufficient clear detail.

Question 8. Many candidates struggled to express themselves clearly in this question and answers generally lacked sufficient detail.

Question 10. A large number of candidates derived the maximum likelihood estimator under the two-state model rather than the Poisson model. Many candidates also failed to distinguish between μ *and* $\hat{\mu}$.

Question 11. Parts (i) and (ii) were generally well answered. The remaining parts were not so well answered and few candidates showed any understanding of Thiele's equation.

1 (a)

K_x	0	1	2	3
Value	$a_{\overline{0}\vert}$	$a_{\overline{1}\vert}$	$a_{\overline{2}\vert}$	$a_{\overline{3}\vert}$ …
	$= 0$	$= v$	$= v + v^2$	$= v + v^2 + v^3$ …

Whole of Life immediate annuity of 1 p.a. payable in arrear until the death of life now aged x.

(b)

K_x	0	1	2 …	$n + 1$
Value	$\ddot{a}_{\overline{1}\vert} - 1$	$\ddot{a}_{\overline{2}\vert} - 1$	$\ddot{a}_{\overline{3}\vert} - 1$	$\ddot{a}_{\overline{n}\vert} - 1$
	$= 1 - 1$	$= 1 + v - 1$	$= 1 + v + v^2 - 1$	$= 1 + v + v^n - 1$
	$= 0$	$= v$	$= v + v^2$	$= v + v^2 \ldots + v^n$

K_x	$n + 2$	$n + 3$
Value	$\ddot{a}_{\overline{n+1}\vert} - 1$	$\ddot{a}_{\overline{n+1}\vert} - 1$
	$= 1 + v + v^2 \ldots + v^n - 1$	$= 1 + v + v^2 \ldots + v^n - 1$
	$= v + v^2 + \ldots + v^n$	$= v + v^2 \ldots + v^n$

Temporary life annuity of 1 p.a. with term of n years payable in arrear until the death of a life now aged x or until the life attains $x + n$ whichever event occurs first.

Only a correct statement was required in each case for full marks.

2 Value of premiums $= P\ddot{a}_{30:\overline{30}|} = 14.349P$ (from tables)

Value of survival benefit
$$= 50{,}000 \times \frac{l_{65}}{l_{30}} \times 1.06^{-35}$$
$$= 50{,}000 \times \frac{27{,}442.681}{33{,}839.370} \times 1.06^{-35}$$
$$= 50{,}000 \times 0.1055113$$
$$= 5{,}275.5651$$

Value of death benefit
$$= 10{,}000 \times \bar{A}^{1}_{30:\overline{35}|}$$
$$= 10{,}000 \times (1.06^{1/2}) \times (A_{30:\overline{35}|} - A^{1}_{30:\overline{35}|})$$
$$= 10{,}000 \times (1.06^{1/2}) \times (\underset{\text{tables}}{0.14989} - \underset{\text{above}}{0.10551})$$
$$= 456.9201$$

Equation of value:

$$14.349P = 5{,}275.5651 + 456.9201$$

$$P = £399.50 \text{ per annum}$$

There is very little difference using alternative calculations for $\bar{A}^{1}_{30:\overline{35}|}$:

Using 1.03 instead of $1.06^{½}$ leads to value of death benefit = 457.114 and P = 399.52.

Alternative calculation of $A^{1}_{30:\overline{35}|}$

$$= A_{30} - \frac{l_{65}}{l_{30}} \times 1.06^{-35} \times A_{65}$$

$$= 0.09372 - \frac{27{,}442.681}{33{,}839.370} \times 1.06^{-35} \times 0.46764$$

$$= 0.09372 - 0.04934$$

$$= 0.04438$$

3 Points to be made:

Yes, there are lots of deaths and many years of exposed to risk but these will be spread across about 110 years of age, so numbers at any particular age will be much smaller.

The numbers will not be divided evenly across ages. At high ages (say > 80) there will be little exposed to risk and at younger ages (say < 30) there will be few deaths.

So there will be ages at which the standard errors are substantial, particularly when compared to the estimated value of the rates at these ages.

The rate for each age is estimated independently of the rates at all other ages. There is nothing in the estimation process that ensures that rates increase smoothly with age as we would expect a priori.

If there is any irregularity in the published table that would cause difficulties when the table is used for financial calculations, e.g. state pension liabilities.

4 (i) Random censoring occurs when the time at which the *i*th life is censored is a random variable. The observation will be censored if the censoring time is less than the (random) lifetime of the life. In an investigation into the mortality of life insurance policyholders, a life will be censored if either the period of investigation ends before the life dies, or the life withdraws from the investigation while still alive (perhaps because the policy lapses). Both these mechanisms will generate random censoring.

Type I censoring occurs if the censoring times are known in advance. In most investigations withdrawals do occur, and if it is not known in advance whether (let alone when) a life will withdraw, then the censoring is not Type I.

If the period of investigation is known in advance and if there are no withdrawals from the investigation while still alive, the censoring will be Type I.

(ii) Censoring is non-informative if it gives no information about lifetimes. In the case of random censoring, the independence for all lives of the random variables measuring the future lifetime and the time until censoring is sufficient to ensure that the censoring is non-informative. Censoring would be informative if, for example, censored lives were subject to lighter mortality than other lives. Since persons who allow their life insurance policies to lapse tend to have lighter mortality than those who keep up their payments, it is likely that in most investigations, censoring due to withdrawals while still alive is informative. Censored observations are likely to have longer lives than lives which are not censored.

5 (i) (a) UDD: ${}_tq_x = t.q_x$

$${}_{0.5}p_x = 1. - 0.5.q_x = 1. -0.5 \times (1 - 0.9) = 0.95$$

$$p_x = {}_{0.5}p_{x+0.5} \cdot {}_{0.5}p_x$$

$$\Rightarrow {}_{0.5}p_{x+0.5} = 0.9/0.95 = 0.9474$$

(b) Balducci: ${}_{1-t}q_{x+t} = (1 - t).q_x$

$${}_{0.5}p_{x+0.5} = 1 - 0.5.\ q_x = 1 -0.5 \times (1 - 0.9) = 0.95$$

$$p_x = {}_{0.5}p_{x+0.5} \cdot {}_{0.5}p_x$$

$$\Rightarrow {}_{0.5}p_x = 0.9 / 0.95 = 0.9474$$

(ii) Under UDD assumption, ${}_{0.5}p_{x+0.5} < {}_{0.5}p_x$, so the force of mortality is increasing between x and $x + 1$. Conversely, under the Balducci assumption, the force of mortality is decreasing. So UDD assumption seems the more appropriate for most ages. The Balducci assumption would be appropriate for either very young ages or the back of the accident hump.

6 (i) Since deaths are classified on an "age nearest birthday" basis, we need to estimate the exposed to risk on the same basis.

The number of persons aged x last birthday on 1 January in calendar year t is $P_{x,t}$. Let the number of persons aged x nearest birthday on 1 January in calendar year t be $P^*_{x,t}$. Then, assuming that dates of birth are evenly distributed across each calendar year, $P^*_{x,t} = 0.5(P_{x-1,t} + P_{x,t})$.

The central exposed to risk at age x over the two year period between 1 January 1999 and 1 January 2001 may be estimated using the formula

$$E_x^c = 0.5P^*_{x,1999} + P^*_{x,2000} + 0.5P^*_{x,2001}$$

assuming $P^*_{x,t}$ is linear in t over the calendar years 1999 and 2000.

Thus, substituting for $P^*_{x,t}$ this becomes

$$E_x^c = 0.5[0.5(P_{x-1,1999} + P_{x,1999})] + 0.5(P_{x-1,2000} + P_{x,2000})$$
$$+ 0.5[0.5(P_{x-1,2001} + P_{x,2001})].$$

Then $\hat{\mu}_x = \dfrac{\theta_x(1999) + \theta_x(2000)}{E_x^c}$ estimates μ_x.

assuming that the force of mortality is constant between age $x - ½$ and age $x + ½$.

(ii) So, using the data given, we have

$$E_{41}^c = 0.25(473 + 450) + 0.5(512 + 470) + 0.25(491 + 482) = 965$$
$$E_{42}^c = 0.25(450 + 490) + 0.5(470 + 460) + 0.25(482 + 480) = 940.5$$

The total deaths at ages 41 and 42 nearest birthday are 38 and 40 respectively, so

$$\hat{\mu}_{41} = \frac{38}{965} = 0.03938 \text{ and } \hat{\mu}_{42} = \frac{40}{940.5} = 0.04253.$$

7 (i) Essential data is:

- date of birth (or date of x^{th} birthday or exact age)
- multiple policy indicator

Either

- date of purchase of term assurance policy (*)
- date of policy lapse, date of policy expiry or date of death (*)
- if occurred between 1.1.95 and 31.12.98

Or

- Date of entry into investigation
- Date of exit from investigation
- Reason for exit

(ii) Data for all lives that had died during the Period of Investigation (1.1.95 to 31.12.98) would be tabulated by age last birthday, x at date of death. Counts of the number of deaths at each age x, θ_x would be recorded for all x.
For each age x and for each life two dates would be calculated.

START DATE

The latest date of	1 January 1995
	date of purchase of policy
	date of xth birthday

END DATE

The earliest date of	31 December 1998
	date of death or date of leaving (if any)
	date of x + 1th birthday

Then calculate END DATE − START DATE (if this is > 0) and total these values for all lives. Record this answer in years. This is the Central Exposed to Risk at age, x, E_x^c.

Tabulate these values for all x.

Then: $\hat{\mu}_x = \dfrac{\theta_x}{E_x^c}$ is an estimate of the force of mortality at age $x + ½$, assuming that the force is constant over the year of age x to $x + 1$.

ALTERNATIVELY

Tabulations could be produced for age nearest birthday, in which START DATE must be amended to use "date of attaining x nearest birthday", and END DATE to use "date of attaining x + 1 nearest birthday".

Then: $\hat{\mu}_x = \dfrac{\theta_x}{E_x^c}$ is an estimate of the force of mortality at age x, assuming that the force is constant over the year of age $(x - \frac{1}{2}, x + \frac{1}{2})$.

8 (i) (a) Mathematical smoothness is usually defined in terms of differentiality — a continuous function which is differentiable everywhere is smooth.

Empirical smoothness is about the curvature and rate of change of curvature of a fitted function.

Smoothness implies no sharp curves.

This is usually checked by using finite differences.

Small first differences, and smaller second differences with a regular progression with age imply low curvature and no rapid change of curvature with age.

(b) Observed rates are smoothed (graduated) and replaced by the smoothed or graduated rates.

While we want the rates we use to be smooth they should not deviate too far from the observed rates.

If the observed rates look like a set of estimates that might have been obtained from a population in which the graduated rates are the true rates they are said to "adhere to the data".

(ii) (a) Maximum smoothness would be achieved by ignoring the plotted estimated rates and drawing a graduation curve which is very smooth, e.g. straight line. The deviations between the rates read from such a curve and the observed rates are likely to be very large. The graduation curve will be very smooth but have poor "adherence to data".

On the other hand joining up a plot of the estimated values will give perfect adherence to data but is likely to produce a "curve" with rapidly changing curvature which would not satisfy the smoothness criteria.

Suitably explained graphs demonstrating the above points were given credit.

(b) Graduation aims to resolve these conflicts by choosing a half way house.

Graduated rates can be obtained by many methods, some ensure smoothness, e.g. graduation by a mathematical form (the chosen functional form will ensure smoothness), reference to a standard table (a simple relationship with an already smooth set of standard table rates will ensure smoothness). In this case the graduated rates just need to satisfy tests of adherence to data.

Graphical graduation does not ensure smoothness, so graduated rates must be checked for smoothness and adherence to data. The graduation process must be repeated until both criteria are satisfied.

9 (i)

Time t_j	c_j	d_j	n_j	d_j/n_j	$\Lambda_t = \Sigma d_j/n_j$
$0 \le t < 6$	3	0	50	0	0
$6 \le t < 12$	2	2	47	2/47	0.04255
$12 \le t < 15$	0	1	43	1/43	0.06581
$15 \le t < 20$	1	1	42	1/42	0.08962
$20 \le t < 23$	0	2	40	2/40	0.13962
$t \ge 23$	0	1	38	1/38	0.16593

(ii) $F(t) = 1 - \exp(-\Lambda_t)$

$S(18) = 1 - F(18) = \exp(-\Lambda_{18}) = \exp(-0.08962) = 0.91428$

10 (i) The Poisson distribution is used to model the number of "rare" events occurring during some period of time.

Since death is a rare event, the Poisson distribution can thus be used to model the number of deaths among a group of lives, given the time spent exposed to risk and assuming that the force of mortality for lives aged x is constant over $(x, x + 1)$ and over time during the period of observation/investigation.

The Poisson model is not always an "exact" model because, under some observational plans, it allows a non-zero probability of more than N deaths among N lives. However, observational plans can, at least in theory, be adjusted to overcome this problem.

(ii) Let d be the total number of deaths we observe among the N individuals. This value d is a sample value of a random variable $\mathbf{D}$.

The maximum likelihood estimate of the force of mortality, $\hat{\mu}$, is the value which maximises the probability that $\mathbf{D} = d$.

The total waiting time is equal to $\sum_{i=1}^{N} v_i$. The Poisson model assumes that **D** has a Poisson distribution with parameter $\mu\sum_{i=1}^{N} v_i$. The Poisson likelihood is

$$L = \Pr[\mathbf{D} = d] = \frac{e^{-\mu\sum_{i=1}^{N} v_i}\left(\mu\sum_{i=1}^{N} v_i\right)^d}{d!}.$$

The maximum likelihood estimate of μ maximises this. To find it, we differentiate $\ln L$ with respect to μ and set the derivative equal to zero. Thus

$$\ln L = -\mu\sum_{i=1}^{N} v_i + d\ln\left(\mu\sum_{i=1}^{N} v_i\right) - \ln d!$$

and

$$\frac{d\ln L}{d\mu} = -\sum_{i=1}^{N} v_i + \frac{d\sum_{i=1}^{N} v_1}{\mu\sum_{i=1}^{N} v_i} = -\sum_{i=1}^{N} v_i + \frac{d}{\mu}.$$

Setting this equal to zero produces the maximum likelihood estimate of μ,

$$\hat{\mu} = \frac{d}{\sum_{i=1}^{N} v_i}.$$

Since $\frac{d^2\ln L}{d\mu^2} = -\frac{d}{\mu^2}$ is negative, we have a maximum.

(iii) $E[\hat{\mu}] = \mu$.

$$\text{Var}[\hat{\mu}] = \frac{\mu}{\sum_{i=1}^{N} v_i}.$$

(iv) 95% confidence interval for $\hat{\mu}$, estimate of constant force at age x

$$\hat{\mu} \pm 1.96\sqrt{\frac{\hat{\mu}}{\sum_{i=1}^{N} v_i}}$$

The forces would be estimated separately for each age classification x.

The estimates $\hat{\mu}_x$ would be plotted against x.

A confidence band would be plotted around each estimate.

The end points of the confidence bands would be joined to form a "tunnel".

This "tunnel" would be used as a guide in drawing the smooth curve to produce the graphically graduated rates. The "tunnel" would be wider at ages where there were few deaths (say in 50s) or little exposed to risk (say in the 90s).

We would expect the graduation curve to stay within the "tunnel" for 19 out of 20 ages on average, i.e. all but 2 to 4 of the ages 50 to 98 say.

11 (i) (a)

$$\bar{A}^{1}_{50:\overline{20|}} = \bar{A}_{50} - \frac{D_{70}}{D_{50}}\bar{A}_{70}$$

$$= 0.31675 - (1.06)^{-20} \times \frac{54,806}{90,085} \times 0.61701$$

$$= 0.31675 - 0.18970 \times 0.61701$$

$$= 0.19970$$

$$\bar{A}_{50:\overline{20|}} = 0.19970 + 0.18970 = 0.38940$$

$$\bar{a}_{50:\overline{20|}} = \bar{a}_{50} - \frac{D_{70}}{D_{50}}\bar{a}_{70}$$

$$= 11.726 - 0.18970 \times 6.573$$

$$= 10.47910$$

Then: $1{,}000\bar{P}(\bar{A}_{50:\overline{20}|}) = 1{,}000 \times \dfrac{0.38940}{10.47910}$

$= £37.16$

(b) $1{,}000\bar{P}(\bar{A}^{1}_{50:\overline{20}|}) = 1{,}000 \times \dfrac{0.19970}{10.47910}$

$= £19.06$

ALTERNATIVE 1

(i) (a) $\bar{A}^{1}_{50:\overline{20}|} = 0.19970$ as previously

$\bar{A}_{50:\overline{20}|} = 0.38940$ as previously

Then using the premium conversion relationship

$$1{,}000\bar{P}(\bar{A}_{50:\overline{20}|}) = 1{,}000\left\{\frac{1}{\bar{a}_{50:\overline{20}|}} - \delta\right\}$$

$$= 1{,}000\left\{\frac{\delta}{1-\bar{A}_{50:\overline{20}|}} - \delta\right\}$$

$$= 1{,}000\left\{\frac{0.058269}{1-0.38940} - 0.058269\right\}$$

$$= £37.16$$

(b) $\bar{a}_{50:\overline{20}|} = \dfrac{1-\bar{A}_{50:\overline{20}|}}{\delta}$

$= \dfrac{1-0.38940}{0.058269}$

$= 10.47899$

$1{,}000\bar{P}(\bar{A}^{1}_{50:\overline{20}|}) = \dfrac{1{,}000 \times 0.19970}{10.47899}$

$= £19.06$

ALTERNATIVE 2

(i) (a) $\bar{A}^{1}_{50:\overline{20}|} = 0.19970$ as previously

$\bar{A}_{50:\overline{20}|} = 0.38940$ as previously

$$\bar{a}_{50:\overline{20}|} = \frac{1-\bar{A}_{50:\overline{20}|}}{\delta}$$

$= 10.47899$ as previously

$$1{,}000\bar{P}(\bar{A}_{50:\overline{20}|}) = 1{,}000 \times \frac{0.38940}{10.47899}$$

$$= £37.16$$

(b) $$1{,}000\bar{P}(\bar{A}^{1}_{50:\overline{20}|}) = 1{,}000 \times \frac{0.19970}{10.47899}$$

$$= £19.06$$

ALTERNATIVE 3

(i) (a) $\bar{a}_{50:\overline{20}|} = 10.47910$ as previously

Then using the premium conversion relationship

$$1{,}000\bar{P}(\bar{A}_{50:\overline{20}|}) = 1{,}000\left\{\frac{1}{\bar{a}_{50:\overline{20}|}} - \delta\right\}$$

$$= 1{,}000\left\{\frac{1}{10.47910} - 0.058269\right\}$$

$$= £37.16$$

(b) $\bar{A}_{50:\overline{20}|} = 1 - \delta\bar{a}_{50:\overline{20}|}$

$= 1 - 0.058269 \times 10.47910$

$= 0.38939$

$$\bar{A}^{1}_{50:\overline{20}|} = \bar{A}_{50:\overline{20}|} - \frac{D_{70}}{D_{50}}$$

$$= 0.38939 - \frac{54,806}{90,085} \times (1.06)^{-20}$$

$$= 0.19969$$

$$1,000\bar{P}(\bar{A}^{1}_{50:\overline{20}|}) = \frac{1,000 \times 0.19969}{10.47910}$$

$$= £19.06$$

(ii) (a) $${}_{15}\bar{V}(\bar{A}_{50:\overline{20}|}) = \bar{A}_{65:\overline{5}|} - \bar{P}(\bar{A}_{50:\overline{20}|})\bar{a}_{65:\overline{5}|}$$

$$\bar{A}^{1}_{65:\overline{5}|} = \bar{A}_{65} - \frac{D_{70}}{D_{65}}\bar{A}_{70}$$

$$= 0.54139 - (1.06)^{-5} \times \frac{54,806}{68,490} \times 0.61701$$

$$= 0.54139 - 0.59796 \times 0.61701$$

$$= 0.17244$$

$$\bar{A}_{65:\overline{5}|} = 0.17244 + 0.59796 = 0.77040$$

$$\bar{a}_{65:\overline{5}|} = \bar{a}_{65} - \frac{D_{70}}{D_{65}}\bar{a}_{70}$$

$$= 7.871 - 0.59796 \times 6.573$$

$$= 3.94061$$

Then $1,000\,{}_{15}\bar{V}(\bar{A}_{50:\overline{20}|}) = 1,000 \times 0.77040 - 37.16 \times 3.94061$

$$= £623.97$$

(b) $$1,000\,{}_{15}\bar{V}(\bar{A}^{1}_{50:\overline{20}|}) = 1,000\bar{A}^{1}_{65:\overline{5}|} - 19.06\bar{a}_{65:\overline{5}|}$$

$$= 1,000 \times 0.17244 - 19.06 \times 3.94061$$

$$= £97.34$$

ALTERNATIVE 1

(ii) (a) $${}_{15}\bar{V}(\bar{A}_{50:\overline{20}|}) = \bar{A}_{65:\overline{5}|} - \bar{P}(\bar{A}_{50:\overline{20}|})\bar{a}_{65:\overline{5}|}$$

$\bar{A}^{1}_{65:\overline{5}|}$ = 0.17244 as previously

$\bar{A}_{65:\overline{5}|}$ = 0.77040 as previously

$$\bar{a}_{65:\overline{5}|} = \frac{1-\bar{A}_{65:\overline{5}|}}{\delta}$$

$$= \frac{1-0.77040}{0.058269}$$

$$= 3.94035$$

Then $1{,}000\,{}_{15}\bar{V}(\bar{A}_{50:\overline{20}|}) = 1{,}000 \times 0.77040 - 37.16 \times 3.94035$

$= £623.98$

(b) $$1{,}000\,{}_{15}\bar{V}(\bar{A}^{1}_{50:\overline{20}|}) = 1{,}000\bar{A}^{1}_{65:\overline{5}|} - 19.06 \times \bar{a}_{65:\overline{5}|}$$

$= 1{,}000 \times 0.17244 - 19.06 \times 3.94035$
$= £97.34$

ALTERNATIVE 2

(ii) (a) $${}_{15}\bar{V}(\bar{A}_{50:\overline{20}|}) = \bar{A}_{65:\overline{5}|} - \bar{P}(\bar{A}_{50:\overline{20}|})\bar{a}_{65:\overline{5}|}$$

$\bar{a}_{65:\overline{5}|}$ = 3.94061 as previously

$$\bar{A}_{65:\overline{5}|} = 1 - \delta\bar{a}_{65:\overline{5}|}$$

$$= 1 - 0.058269 \times 3.94061$$

$$= 0.77038$$

$$\bar{A}^{1}_{65:\overline{5}|} = \bar{A}_{65:\overline{5}|} - \frac{D_{70}}{D_{65}}$$

$$= 0.77038 - \frac{54{,}806}{68{,}490} \times (1.06)^{-5}$$

$= 0.17242$

Then $1{,}000\,{}_{15}\overline{V}(\overline{A}_{50:\overline{20|}}) = 1{,}000 \times 0.77038 - 37.16 \times 3.94061$

$= £623.95$

(b) $1{,}000\,{}_{15}\overline{V}(\overline{A}^{1}_{50:\overline{20|}}) = 1{,}000 \times 0.17242 - 19.06 \times 3.94061$

$= £97.31$

ALTERNATIVE 3

(ii) (a) $${}_{15}\overline{V}(\overline{A}_{50:\overline{20|}}) = 1 - \frac{\overline{a}_{65:\overline{5|}}}{\overline{a}_{50:\overline{20|}}}$$

$$\overline{a}_{65:\overline{5|}} = \overline{a}_{65} - \frac{D_{70}}{D_{65}}\overline{a}_{70}$$

$= 7.871 - 0.59796 \times 6.573$

$= 3.94061$

$$1{,}000\,{}_{15}\overline{V}(\overline{A}_{50:\overline{20|}}) = 1{,}000\left\{1 - \frac{3.94061}{10.47910}\right\}$$

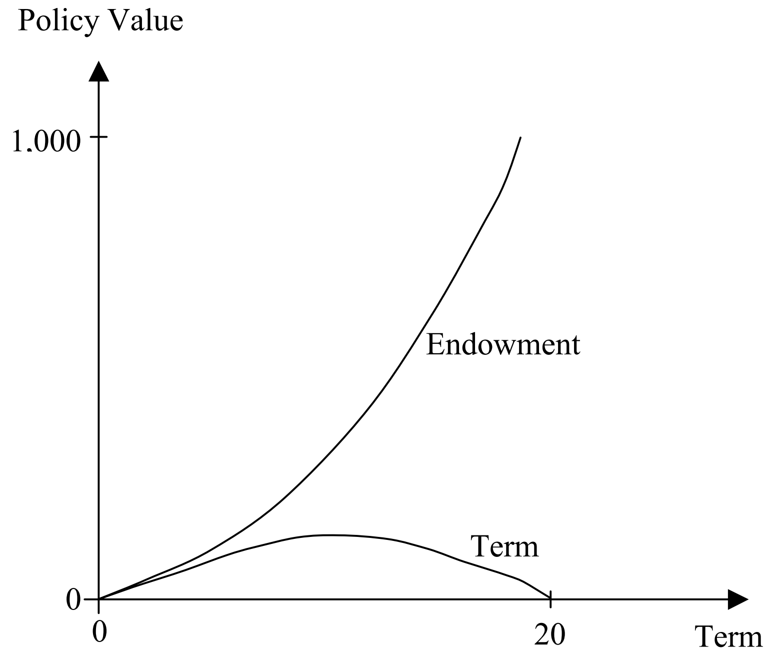
$= £623.96$

(b) $\overline{A}^{1}_{65:\overline{5|}} = 0.17244$ as previously

$1{,}000\,{}_{15}\overline{V}(\overline{A}^{1}_{50:\overline{20|}}) = 1{,}000 \times 0.17244 - 19.06 \times 3.94061$

$= £97.34$

(iii)



(iv) $\frac{\partial}{\partial t}{}_t\bar{V}(\bar{A}^1_{50:\overline{20}|}) = \delta\, {}_t\bar{V}(\bar{A}^1_{50:\overline{20}|}) + \bar{P}(\bar{A}^1_{50:\overline{20}|}) - (1 - {}_t\bar{V}(\bar{A}^1_{50:\overline{20}|}))\mu_{x+t} \quad 0 < t < 20$

(v) At time 15 the components of change in policy value are

+ Investment Return $\delta\, {}_t\bar{V}(\quad)$
+ Premium Income $\bar{P}(\quad)$
– Death Strain $(1 - {}_tV(\quad))\mu_{x+t}$

These are expressed as equivalent annual rates of change measured at policy duration 15.

Here $\delta = \log_e 1.06 = 0.058269$

So components are

	Endowment	*Term*
+ Investment Return	0.058269 × 623.97 = £36.35	0.058269 × 97.34 = £5.67
+ Premium Income	£37.16	£19.06
– Death Strain	(1,000–623.97)0.03553 = £13.36	(1,000–97.34)0.03553 = £32.07
Net Annual Rate of Change	+£60.15	–£7.34

So at policy duration 15 the endowment assurance is increasing in value and the term assurance is decreasing in value.