EXAMINATIONS

23 September 2004 (pm)

Subject 104 — Survival Models

Time allowed: Three hours

INSTRUCTIONS TO THE CANDIDATE

1. *Enter all the candidate and examination details as requested on the front of your answer booklet.*

2. *You must not start writing your answers in the booklet until instructed to do so by the supervisor.*

3. *Mark allocations are shown in brackets.*

4. *Attempt all 11 questions, beginning your answer to each question on a separate sheet.*

Graph paper is NOT required for this paper.

AT THE END OF THE EXAMINATION

Hand in BOTH your answer booklet, with any additional sheets firmly attached, and this question paper.

In addition to this paper you should have available Actuarial Tables and your own electronic calculator.

1 Describe the benefit which has present value random variable Z below; here T denotes the future lifetime of a life aged x.

$$Z = \begin{cases} v^T \bar{a}_{\overline{n-T}|} & \text{if } T < n \\ 0 & \text{if } T \geq n \end{cases}$$

[2]

2 A life office issued a term assurance policy to a life aged exactly 45 years. The term of the policy is 20 years and the benefit, payable at the end of the year of death, is £20,000. The policy is funded by a level annual premium P, payable in advance.

(i) Using standard actuarial notation, write down the equation of value for this policy. [1]

(ii) Write down expressions for the prospective and retrospective policy values of this policy when the life is aged exactly 45 + t years. Show that the policy values are equal if calculated on the same basis as the premiums. [5]

[Total 6]

3 A study has been undertaken into the effect of a new treatment on the survival times of patients suffering from a tropical disease. The following model has been fitted:

$$h_i(t) = h_0(t)\ \exp(\underline{\beta}^T \underline{z})$$

where $h_i(t)$ is the hazard at time t, where t is the time since treatment
$h_0(t)$ is the baseline hazard at time t

$\underline{z}$ is a vector of covariates, where
z_1 = period from diagnosis to treatment in years
z_2 = 0 if existing treatment given, 1 if new treatment given
z_3 = 0 if female, 1 if male

$\underline{\beta}$ is a vector of parameters, where
$\beta_1 = 0.5$
$\beta_2 = 0.01$
$\beta_3 = -0.05$

(i) State the group of lives to which the baseline hazard applies. [1]

(ii) For a male who was given the new treatment 6 months after diagnosis:

(a) Write down the hazard function, in terms of $h_0(t)$ only.

(b) Express the survival function, in terms of $h_0(t)$ only.

[3]

(iii) For a female given the new treatment at the time of diagnosis, the probability of survival for 5 years is 0.75. Calculate the probability that the male in (ii) will survive 5 years. [3]

[Total 7]

4 (i) Z is a random variable representing the present value of the benefits payable under an immediate life annuity which pays 1 per year, issued to a life aged x.

Show that $Var(z) = \dfrac{1}{d^2}\left({}^2A_x - (A_x)^2\right)$, where ${}^2A_x$ is an assurance calculated at a rate of interest which you should specify. [5]

(ii) A life office issues such a policy to a life aged exactly 60. The benefit is £100 per annum. Calculate the standard deviation of the annuity.

Basis: Mortality: AM92 Ultimate
Interest: 4% per annum throughout
Expenses are ignored

[3]

[Total 8]

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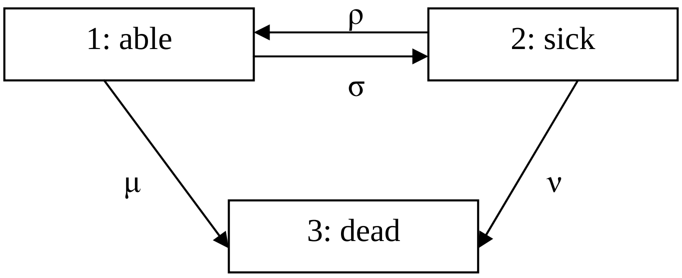
5 (i) Derive an expression, in terms of ${}_t p_x$ only, for the complete expectation of life at age x. [3]

(ii) In a certain population the force of mortality is constant at 0.02 for all ages. For a life aged exactly 25 calculate the complete expectation of life. [3]

[Total 6]

6 A life insurance company uses the following 3-state Markov model in the pricing of its long term sickness policies. The forces of transition are assumed to be constant.



For a group of policyholders, over a one year period the following data were recorded:

Transition from	*Number*
1 to 2	15
1 to 3	6
2 to 1	5
2 to 3	1

The total waiting times recorded were:

State 1	625 years
State 2	35 years

(i) Write down the likelihood function for this model and show that this is maximized when $\sigma = 0.024$. [5]

(ii) (a) State the asymptotic distribution of $\tilde{\sigma}$, the maximum likelihood estimator of σ.

(b) Calculate an estimate of the standard deviation of $\tilde{\sigma}$. [3]

[Total 8]

7 A life insurance company is investigating the mortality of a group of policyholders. The possible definitions of age by which deaths may be grouped are as follows:

(a) age at birthday in calendar year of death
(b) age last birthday at entry plus curtate duration
(c) age next birthday at policy anniversary in calendar year of death

The force of mortality for lives labelled aged x is to be estimated as

$$\hat{\mu}_x = \frac{\text{number of deaths aged } x}{\text{Central exposed to risk for age } x}$$

$\hat{\mu}_x$ estimates the force of mortality μ_{x+f}.

For each of the possible age definitions, state the type of rate interval and determine the value of f. State any assumptions that you make. [10]

8 On 1 January 2002 a pension scheme has 100 members aged exactly 75, each in receipt of an annual pension of £10,000 payable in arrears. In addition, the scheme pays a lump sum of £2,000 on the death of a member, the lump sum being payable at the end of the year of death. No further premiums are payable in respect of these benefits.

Of these members, 7 die during 2002. Calculate the mortality profit or loss to the scheme for 2002.

Basis: Mortality: PMA92C20
Interest: 4% per annum throughout
Expenses are ignored [10]

9 In a clinical trial, 100 patients were treated with a new vaccine for a serious but treatable disease and then observed over a 2 year period. The results were as follows:

Withdrew from study: 13 patients
Contracted disease during study: 8 patients
Still clear of disease at end of study: 79 patients

The times (in months) at which patients withdrew or contracted the disease were as follows:

Withdrew	1	3	3	4	4	5	7	7	9	10	11	13	15
Contracted disease	2	2	2	6	6	8	8	17					

(i) Calculate the Kaplan-Meier estimate of the survival function based on this data. [5]

(ii) A control group of patients who were not treated with the vaccine, was also observed over 2 years. The Kaplan-Meier estimate of the survival function and the implied hazard function for this control group are:

	Survival
$0 \leq t < 5$	1.0
$5 \leq t < 8$	0.92
$8 \leq t < 12$	0.895
$t \geq 12$	0.87

t	*Hazard function*
5	0.08
8	0.02717
12	0.02793

The cost of treating a patient who contracts the disease is £1,000, paid as a lump sum at the time of diagnosis. The vaccine costs £20 per patient.

Compare the expected present value of the cost over 2 years of treating a vaccinated patient with that for a patient not given the vaccine. Hence calculate the expected saving made by vaccinating a patient.

Basis: Interest: 4% per annum throughout

[6]
[Total 11]

10 A medium-sized UK pension scheme has carried out an investigation of the mortality of its pensioners over the period 2000–2002.

(i) Explain why the crude rates will require graduation, and suggest with reasons an appropriate method of graduation in this case. [4]

(ii) The data used to produce the crude rates, and the proposed graduated rates, are shown below.

Age x	*Central Exposed to Risk* E_x^c	*Number of Deaths* d_x	*Crude Force of Mortality* $\mu_{x+½}$	*Graduated Force of Mortality* $\mathring{\mu}_{x+½}$	*Standardised Deviation* z_x
60–64	1,388.9	10	0.0072	0.0061	0.5249
65–69	1,188.8	17	0.0143	0.0131	0.3615
70–74	880.5	28	0.0318	0.0262	1.0266
75–79	841.6	34	0.0404	0.0487	–1.0912
80–84	402.8	41	0.1018	0.0839	1.2394
85–89	123.9	19	0.1533	0.1338	0.5949
90–94	27.9	7	0.2509	0.1975	0.6346
95–99	10.0	3	0.3000	0.2706	0.1787
100+	7.5	2	0.2666	0.3455	–0.3673

Test the proposed graduation for:

(a) overall goodness of fit; and
(b) bias

[8]

(iii) Comment on the use of the graduated rates to value the benefits payable from the scheme. [2]

[Total 14]

11 In a certain country, people finance their retirement by purchasing deferred annuities and pure endowment policies. According to the law in this country, these policies must both be purchased on the birthday nearest age *a* years, where *a* is the age at which a person's complete expectation of life is equal to his or her current age: $\overset{\circ}{e}_a = a$. The value of *a* is calculated, for each person, on the basis of a set of life tables published for different classes of life by the Government Actuary.

Both types of policy are funded by a single premium paid by the life on the birthday nearest age *a*. The deferred annuity pays a level weekly amount from the policyholder's 65th birthday until the birthday nearest the age 2*a*, or until death (if earlier). If the policyholder survives to the birthday nearest age *2a* the pure endowment provides a sum assured at that time.

(i) The Government Actuary publishes life tables for different classes of lives. List some relevant factors the Government Actuary might wish to use to classify lives for this purpose. [2]

(ii) The mortality experience of one class of lives in this country is the same as that described in AM92 Ultimate. Show that at age 39.778 the complete expectation of life is 39.778 (to 3 decimal places) and that, therefore, the birthday nearest age *a* is the 40^{th} birthday. You should assume a uniform distribution of deaths within each year of age and may use the approximation $\overset{\circ}{e}_x = e_x + 0.5.$ [7]

(iii) A member of this class of lives who is aged exactly 40 wishes to purchase one of each of these policies. The weekly annuity payable will be a sum equivalent to £10,000 per year from the 65^{th} birthday to the birthday nearest exact age 2*a*. The pure endowment lump sum will be £100,000 on survival to the birthday nearest exact age 2*a*. Calculate the total premium.

Basis: Mortality: AM92 Ultimate
Interest: 4% per annum throughout
Expenses are ignored

[6]

(iv) Discuss the factors that companies selling these annuities and pure endowments might wish to take into account when pricing them. [3]

[Total 18]

END OF PAPER