EXAMINATIONS

September 2001

Subject 104 — Survival Models

EXAMINERS' REPORT

1 Smoothness can be checked by calculating second or third order finite differences of the graduated rates.
These values should be small and vary little with age.

2 (i) If the ith life dies the contribution to the likelihood is

$$_{t_i-a_i}p_{x+a_i}\ \mu_{x+t_i}$$

$$= e^{-\mu(t_i-a_i)}\mu \quad \text{with assumption of constant force, } \mu$$

If the ith life survives to $x + t_i$ the contribution to the likelihood is

$$_{t_i-a_i}p_{x+a_i} = e^{-\mu(t_i-a_i)} \quad \text{with assumption of constant force, } \mu.$$

So the overall contribution for the ith life can be written

$$e^{-\mu(t_i-a_i)}\ \mu^{d_i}$$

where $d_i = 1$ for a death
$d_i = 0$ for a survivor

So for N independent lives we have:

$$L = \prod_{i=1}^{i=N} e^{-\mu(t_i-a_i)}\ \mu^{d_i}$$

$$= e^{-\mu\sum_{i=1}^{i=N}(t_i-a_i)}\ \mu^{\sum_{i=1}^{i=N}d_i}$$

Then $\text{Log}_e L = -\mu\sum_{i=1}^{i=N}(t_i-a_i) + \sum_{i=1}^{i=N}d_i \log_e\mu$

So $$\frac{\partial \log_e L}{\partial\mu} = -\sum_{i=1}^{i=N}(t_i-a_i) + \frac{\sum_{i=1}^{i=N}d_i}{\mu}$$

And the maximum likelihood estimator is

$$\hat{\mu} = \frac{\sum_{i=1}^{i=N}d_i}{\sum_{i=1}^{i=N}(t_i-a_i)}$$

Now $\quad \dfrac{\partial^2 \log_e L}{\partial \mu^2} = -\dfrac{\sum_{i=1}^{i=N} d_i}{\mu^2} < 0$ so solution is a maximum

(ii) Asymptotic Sampling Variance is given by

$$-\frac{1}{E\left[\dfrac{\partial^2 \log_e L}{\partial \mu^2}\right]} = \frac{\mu^2}{E[D]}$$

which can be estimated by

$$= \frac{\mu^2}{\mu \sum_{i=1}^{i=N} (t_i - a_i)}$$

$$= \frac{\mu}{\sum_{i=1}^{i=N} (t_i - a_i)}$$

So $\hat{\mu} \sim N\left(\mu, \dfrac{\mu}{\sum (t_i - a_i)}\right)$

3 (i) $\bar{a}_{x:\overline{n}|} = E(\bar{a}_{\overline{S}|})$ where $S = \min(T_x, n)$

$\ddot{a}_{x:\overline{n}|} = E(\ddot{a}_{\overline{U}|})$ where $U = \min(K_x + 1, n)$

(ii) $E(\bar{a}_{\overline{S}|}) = \int_0^n \bar{a}_{\overline{t}|}\, f_x(t)\, dt + \int_n^\infty \bar{a}_{\overline{n}|}\, f_x(t)\, dt$

where $f_x(t) = {}_t p_x\, \mu_{x+t}$

The second term = $\bar{a}_{\overline{n}|}\; {}_n p_x$, and the first term is:

$$\int_0^n \int_0^t v^s\, ds\, f_x(t)\, dt = \int_0^n v^s \int_s^n f_x(t)\, dt ds$$

$$= \int_0^n v^s ({}_s p_x - {}_n p_x)\, ds = \int_0^n v^s\, {}_s p_x\, ds - \bar{a}_{\overline{n}|}\, {}_n p_x$$

Hence result.

Also $\ddot{a}_{x:\overline{n}|} = \sum_{k=0}^{n-1} v^k \, {}_k p_x$

(iii) $\bar{a}_{x:\overline{n}|} = \sum_{k=0}^{n-1} \int_k^{k+1} v^s \, {}_s p_x \, ds \approx \sum_{k=0}^{n-1} \tfrac{1}{2}(v^k \, {}_k p_x + v^{k+1} \, {}_{k+1} p_x)$

$= \tfrac{1}{2} + \sum_{k=1}^{n-1} v^k \, {}_k p_x + \tfrac{1}{2} v^n \, {}_n p_x$

$= -\tfrac{1}{2} + \sum_{k=0}^{n-1} v^k \, {}_k p_x + \tfrac{1}{2} v^n \, {}_n p_x$

$= \ddot{a}_{x:\overline{n}|} - \tfrac{1}{2}(1 - v^n \, {}_n p_x)$

(iv) $\bar{a}_x = \lim_{n\to\infty} \bar{a}_{x:\overline{n}|} \simeq \lim_{n\to\infty} \ddot{a}_{x:\overline{n}|} - \tfrac{1}{2}(1-0) = \ddot{a}_x - \tfrac{1}{2}$

4 (i) The present value $= \int_0^{T_x} \rho(s) e^{-\delta s} \, ds$

(ii) The expected present value

$= E\left[\int_0^{T_x} \rho(s) e^{-\delta s} \, ds\right] = \int_0^{\infty}\left(\int_0^t \rho(s) e^{-\delta s} ds\right) f(t) dt$

where f denotes the probability density function of T_x.

Hence EPV $= \int_0^{\infty} \rho(s) e^{-\delta s}\left(\int_s^{\infty} f(t)\, dt\right) ds = \int_0^{\infty} \rho(s) e^{-\delta s} \, {}_s p_x \, ds.$

(iii) EPV $= \int_0^5 e^{-0.05s} \, 5{,}000 e^{-0.01s} \, ds + e^{-5\times 0.05} . e^{-5\times 0.01} \int_0^5 e^{-0.05s} \, 10{,}000 \, e^{-0.02s} \, ds$

$= 5{,}000 \bar{a}_{\overline{5}|}$ @ force of 0.06 + $10{,}000 e^{-5(0.06)} \, \bar{a}_{\overline{5}|}$ @ 0.07 = 52,851.69

(iv) $$40{,}000\left(\int_0^5 e^{-0.05t}\,0.01e^{-0.01t}\,dt + \int_5^{10} e^{-0.05t}\,0.02e^{-0.01\times 5}\,e^{-0.02(t-5)}\,dt\right)$$

$$= \left(0.01\bar{a}_{\overline{5|}\delta=0.06} + 0.02e^{-5(0.06)}\,\bar{a}_{\overline{5|}\delta=0.07}\right)\times 40{,}000$$

$$= 4{,}228.14$$

Other approaches are possible, in particular the use of premium conversion relationships in (iv).

5 (i) $${}_tp_x^{11} = \exp\left(-\int_0^t (\sigma_{x+s} + \mu_{x+s})\,ds\right);$$

$${}_tp_x^{22} = \exp\left(-\int_0^t \upsilon_{x+s}\,ds\right);$$

$${}_tp_x^{33} = 1.$$

(ii) $${}_tp_x^{23} = 1 - {}_tp_x^{22} = 1 - \exp\left(-\int_0^t \upsilon_{x+s}\,.\,ds\right)$$

Other expressions are possible.

(iii) $${}_{t+h}p_x^{12} = \sum_{k=1}^{3} {}_tp_x^{1k}\;{}_hp_{x+t}^{k2} \text{ with Markov assumption}$$

$$= {}_tp_x^{11}\;{}_hp_{x+t}^{12} + {}_tp_x^{12}\;{}_hp_{x+t}^{22} \text{ model constraints}$$

We know that

$${}_hp_{x+t}^{12} = h\sigma_{x+t} + o(h) \qquad \text{definition}$$

$${}_hp_{x+t}^{23} = h\upsilon_{x+t} + o(h) \qquad \text{definition}$$

$${}_hp_{x+t}^{21} + {}_hp_{x+t}^{22} + {}_hp_{x+t}^{23} = 1 \qquad \text{law of total probability}$$

Substituting and using model constraints we obtain

$${}_{t+h}p_x^{12} = {}_tp_x^{11}(h\sigma_{x+t} + o(h)) + {}_tp_x^{12}(1 - h\upsilon_{x+t} + o(h))$$

Then

$$\lim_{h\to 0} \frac{{}_{t+h}p_x^{12} - {}_tp_x^{12}}{h} = \frac{\partial}{\partial t}\left({}_tp_x^{12}\right) = \sigma_{x+t}\ {}_tp_x^{11} - \upsilon_{x+t}\ {}_tp_x^{12}$$

$${}_0p_x^{12} = 0$$

(iv) Differential equation is $\left({}_tp_x^{12}\right)' + \upsilon\ {}_tp_x^{12} = \sigma \exp(-(\sigma + \mu)t)$

Hence $\left({}_tp_x^{12}e^{\upsilon t}\right)' = \sigma \cdot \exp((\upsilon - \sigma - \mu)t)$

$$\Rightarrow {}_tp_x^{12} = e^{-\upsilon t}\left[\frac{\sigma}{\upsilon - \sigma - \mu}\exp\{(\upsilon - \sigma - \mu)t\} + c\right],$$

and ${}_0p_x^{12} = 0 \Rightarrow c = \dfrac{-\sigma}{\upsilon - \sigma - \mu}$. Hence result.

[Direct verification is a satisfactory alternative, but the solution must verify also that ${}_0p_x^{12} = 0$.]

6 (i) The probability of dying the first month

$$= \frac{D}{E}$$

where D is the deaths in the first month of life $= d_1 = 396$

and E is the initial exposed to risk of dying in the first month

$= E_1^c \times 12 + d_1 \times 0.5$ (since the unit we are dealing with is a month).

$= 1{,}536.2 \times 12 + 396 \times 0.5 = 18{,}632.4$.

Thus the probability of dying in the first month = 0.021253.

(ii) $q_0 = \dfrac{D}{E}$

where D is the deaths in the first year of life $= d_1 + d_2 + d_3 + d_4 = 898$

and E is the initial exposed to risk of dying in the first year

$= E_1^c + E_2^c + E_3^c + E_4^c + d_1\left(\frac{11½}{12}\right) + d_2\left(\frac{10}{12}\right) + d_3\left(\frac{7½}{12}\right) + d_4\left(\frac{3}{12}\right)$

= 17,869 + 640.083 = 18,509.083

Thus the probability of dying in the first year = 0.048517

(iii) $\quad {}_3q_0 = \dfrac{D}{E}$

where D is the deaths in the first three years of life

$= d_1 + d_2 + d_3 + d_4 + d_5 + d_6 = 1{,}153$

and E is the initial exposed to risk of dying in the first three years

$$= \tfrac{1}{3}\left(\sum_{t=1}^{6} E_t^c + d_1\left(2\tfrac{11½}{12}\right) + d_2\left(2\tfrac{10}{12}\right) + d_3\left(2\tfrac{7½}{12}\right) + d_4\left(2\tfrac{3}{12}\right) + 1.5d_5 + 0.5d_6\right)$$

$= \frac{1}{3}$ (51,308.4 + 2,739.6) = 18,016.0

Thus the probability of dying in the first three years = 0.063999

Alternative is to estimate each of q_1 q_2 separately and then use
${}_3q_0 = 1 - (1 - q_0)\,(1 - q_1)\,(1 - q_2)$

$$q_1 = \frac{176}{16{,}999.2 + 0.5 \times 176} = \frac{176}{17{,}087.2} = 0.010300$$

$$q_2 = \frac{79}{16{,}440.2 + 0.5 \times 79} = \frac{79}{16{,}479.7} = 0.004794$$

$$\begin{aligned} {}_3q_0 &= 1 - (1 - 0.048517)\,(1 - 0.010300)\,(1 - 0.004794) \\ &= 1 - 0.937168 \\ &= 0.062832 \end{aligned}$$

These calculations assume that deaths in each age interval occur on average at the mid point of the interval i.e. at times $\frac{½}{12}, \frac{2}{12}, \frac{4½}{12}, \frac{9}{12}, 1½$ and 2½ years.

ALTERNATIVELY

If we assume that the force of mortality is constant over each age interval, then

(i) The assumed constant force over the first month of life can be estimated by

$$\frac{396}{1{,}536.2} = 0.257779$$

and the probability of surviving the first month of life is

$$\exp\left\{-0.257779 \times \frac{1}{12}\right\} = 0.978748$$

So the probability of dying in the first month of life is 0.021252.

(ii) The assumed constant forces over the remaining age intervals in the first year of life can be estimated by

$$\text{1 to 3 months} \quad \frac{139}{3{,}041.9} = 0.045695$$

$$\text{3 to 6 months} \quad \frac{144}{4{,}498.1} = 0.032014$$

$$\text{6 to 12 months} \quad \frac{219}{8{,}792.8} = 0.024907$$

The survival probabilities for these intervals are estimated by

$$\text{1 to 3 months} \quad \exp\left\{-0.045695 \times \frac{2}{12}\right\} = 0.992413$$

$$\text{3 to 6 months} \quad \exp\left\{-0.032014 \times \frac{3}{12}\right\} = 0.992028$$

$$\text{6 to 12 months} \quad \exp\left\{-0.024907 \times \frac{6}{12}\right\} = 0.987624$$

Then $$\begin{aligned} {}_1\hat{p}_0 &= 0.978748 \times 0.992413 \times 0.992028 \times 0.987624 \\ &= 0.951654 \end{aligned}$$

and $${}_1\hat{q}_0 = 0.048346$$

(iii) The assumed constant force over (1, 2) (2, 3) can be estimated by

$$(1, 2) \quad \frac{176}{16{,}999.2} = 0.010353$$

$$(2, 3) \quad \frac{79}{16{,}440.2} = 0.004805$$

and the survival probabilities can be estimated by

$${}_1\hat{p}_1 = \exp\{-0.010353\} = 0.989700$$

$${}_1\hat{p}_2 = \exp\{-0.004805\} = 0.995207$$

So $\quad {}_3\hat{p}_0 = 0.951654 \times 0.989700 \times 0.995207$
$= 0.937338$

and $\quad {}_3\hat{q}_0 = 0.062662$

7 (i) Whole-life policies: $10{,}000(1 / \ddot{a}_{40} - d) = 144.653166$

Pure endowments:

$$\frac{10{,}000D_{60}}{N_{40} - N_{60}} = \frac{10{,}000 \times 2{,}855.5942}{132{,}001.93 - 35{,}841.261} = 296.960725$$

(ii) Policy values, per policy:

Whole-life, time 5: $10{,}000(1 - \ddot{a}_{45} / \ddot{a}_{40}) = 719.805229$

6: $10{,}000\,(1 - \ddot{a}_{46} / \ddot{a}_{40}) = 874.880915$

Pure endowment, time 5: $10{,}000(D_{60} / D_{45}) - 296.96\ddot{a}_{45:\overline{15|}}$

$= 10{,}000 \times 0.501934 - 296.96 \times 11.235 = 1{,}682.998848$

time 6: $10{,}000(D_{60} / D_{46}) - 296.96a_{46:\overline{14|}}$

$= 10{,}000 \times (0.523392) - 296.96 \times 10.672 = 2{,}064.76288$

Hence totals:

	Whole-life	*PE*	*All*
time 5 (beginning of CY 2000)	431,883	673,200	1,105,083
time 6 (end of CY 2000)	523,179	823,840	1,347,019

(iii) Alternatively ${}_6V$ can be calculated from ${}_5V$.

Policy values, time 5	431,883	673,200	1,105,083
+ premiums	86,790	118,784	205,574
	518,673	791,984	1,310,657
+4% interest	20,747	31,679	52,426
– death claims	20,000	0	20,000
– policy values reqd at time 6	523,179	823,840	1,347,019
	–3,759	–177	–3,936

Mortality loss of approximately £3,936

Alternatively Expected and Actual Death Strain can be calculated to determine mortality profit. Numercial answers will not usually agree exactly because of rounding errors.

(iv) (1,105,083 + 205,574) (i + i) = 1,347,019 + 2,000

1 + i = 1.043002 4.3% p.a. effective.

8 (i) Mortality rates are estimated separately for each year of age.

So there is no reason why the estimated rates should progress smoothly from age to age as would be expected, a priori.

This fluctuation is the result of sampling error attaching to each estimate.

The sampling error can be smoothed out by using information from adjacent ages to adjust the rate at any age. The adjusted rate is called the graduated rate.

Graduated rates will be smooth (from age to age) and will not deviate significantly from the observed rates.

So they will reflect the observed mortality experience and conform to a priori assumptions about rates. In calculations e.g. premium rates the resulting calculated values will change smoothly from age to age.

Examiners' Comment: Not all points were necessary for full marks.

(ii)

x	θ_x	$\mathring{q}_x \times 10^5$	E_x	$E_x\mathring{q}_x$	$\theta_x - E_x\mathring{q}_x$	$(\theta_x - E_x\mathring{q}_x)/\sqrt{E_x\mathring{q}_x}$
14	3	38	12,800	4.86	−1.86	−0.84
15	8	43	15,300	6.58	+1.42	0.55
16	5	48	12,500	6.00	−1.00	−0.41
17	14	53	15,000	7.95	+6.05	2.15
18	17	59	16,500	9.74	+7.26	2.33
19	9	66	10,100	6.67	+2.33	0.90
20	15	74	12,800	9.47	+5.53	1.80
21	10	83	13,700	11.37	−1.37	−0.41
22	10	93	11,900	11.07	−1.07	−0.32
Total	91			73.71	17.29	

All the $\mathring{q}_x$ are small so $E_x\mathring{q}_x$ can be used to approximate the variance.

Chi-Squared Test

H_0: The observed rates look as if they come from a population in which the graduated rates are the true rates.

Observed Value of test Statistic:

Sum of squared standardised deviations 15.55
which will be $\chi^2_{9\text{-deduction}}$ if null hypothesis is true.

Say χ^2_6 or χ^2_7
deduct one degree of freedom from each of height, slope and curvature.
$\chi^2_6(0.95) = 12.59 \quad \chi^2_7(0.95) = 14.07$

So result leads to rejection of null hypothesis. The graduated rates do not appear to fit the observed rates.

There seems to be "under-estimation" at ages 17, 18, 19 and 20.

(iii) Any **two** of the following (suitably chosen so as not to overlap in what they test).

Individual Standardised Deviations:

Table of Values

Values	$-\infty, -2$	$-2, -1$	$-1, 0$	$0, +1$	$1, 2$	$2, \infty$
Expected	0.2	1.22	3.07	3.07	1.22	0.2
Tally			1111	11	1	11
Observed	0	0	4	2	1	2

The standarised deviations seem to be skewed towards +ve values compared to the distribution expected under the null hypothesis.

There are 4 negative and 5 positive deviations compared to the expected number of each of 4.5. So no reason to worry here.

Five deviations are $>|0.67|$ and four deviations are less than $|0.67|$, compared to the expected number of each of 4.5. No reason to reject null hypothesis.

There are 2 outliers $> |2|$ in a sample of size 9. This is much greater than the expected number of 0.4, so reason to reject the null hypothesis.

Here we are examining the general level of the graduated rates, are they too high or too low?
We must conclude that they appear to be too low compared to the observed rates.

Grouping of Signs Test:

There are 9 deviations with 3 groups of negative signs and 2 groups of positive signs.

The probability of observing 2 or fewer groups of positive signs when there are 5 positive and 4 negative deviations is

$$\frac{\binom{4}{0}\binom{5}{1}}{\binom{9}{5}} + \frac{\binom{4}{1}\binom{5}{2}}{\binom{9}{5}}$$

$$= \frac{5}{126} + \frac{4 \times 10}{126}$$

$$= \frac{45}{126} = 0.357$$

So there is no reason to reject the null hypothesis.

This test looks for "clumping" of deviations of the same sign, i.e. that the graduated curve "cuts through" the curve of observed rates.

Change of Sign Test:

There are 9 deviations and 4 observed changes of signs.

Under the null hypothesis the observed number of changes of sign should follow a Binomial (8, 0.5) sampling distribution.

If the null hypothesis is true the probability of observing 4 or fewer sign changes is:

$$\left(\frac{1}{2}\right)^8 + 9\left(\frac{1}{2}\right)^8 + \frac{8.7}{2}\left(\frac{1}{2}\right)^8 + \frac{8.7.6}{6}\left(\frac{1}{2}\right)^8 + \frac{8.7.6.5}{24}\left(\frac{1}{2}\right)^8$$

$$= 163\left(\frac{1}{2}\right)^8 = 0.637$$

So there is no reason to reject the null hypothesis.

This test detects if the graduated rates are consistently higher or lower than the observed rates.

Cumulative Deviations Test:

This text tests for overall bias or long runs of deviations of the same sign.

The observed value of the cumulative deviations statistic is 17.29.

If the null hypothesis is true we expect this statistic to follow a

$N(0, 73.71)$ distribution.

Observed Standard Normal Value: $\dfrac{17.29}{\sqrt{73.71}} = 2.01$

which is significant at the 5% level. There is reason to reject the null hypothesis.

This test looks for significant under or over estimation by the graduated rates compared to the observed rates.

Serial Correlations Test:

Mean standardised deviation = $\dfrac{5.75}{9}$ = 0.64

i	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*
$Z_i - \bar{Z}$	–1.48	–0.09	–1.05	1.51	1.69	0.26	1.16	–1.05	–0.96
$Z_{i+1} - \bar{Z}$	–0.09	–1.05	1.51	1.69	0.26	1.16	–1.05	–0.96	

$$\sum_{i=1}^{i=8}(Z_i - \bar{Z})(Z_{i+1} - \bar{Z}) = 1.7251$$

$$\sum_{i=1}^{i=9}(Z_i - \bar{Z})^2 = 11.8745$$

Then: $r_1 = \dfrac{1.7251}{\frac{8}{9}\times 11.8745} = 0.1634$

Standardised $r_1 = 0.1634\sqrt{9} = 0.4903$

Examiners' Comment: using "separate" residual means gives $r_1 = 0.4598$.

which is standardised normal if H_0 is true.

So no reason to reject null hypothesis.

This test looks for "runs" of deviations of the same sign, i.e. looks to see if graduated rates ignore "key" features of the observed rates.

9 (i) There will be **right censoring** of all the lives that survive to age 45 or who withdraw before age 45.

There will be **random censoring** of all the lives that withdraw before death or attaining age 45.

(ii)

Person	*Duration (Months)*	
1	6	where + = censored
2	6+	
3	12	
4	12	
5	18+	
6	27	
7	27+	
8	27	
9	30+	
10	36+	
11	39	
12	39+	
13	51+	
14	54+	
15	57	
16	60+	
17	60+	
18	60+	
19	60+	
20	60+	

So the times of death are 6 12 27 39 57 and initial risk set is 20.

(1)	*(2)*	*(3)*	*(4)*	*(5)*	*(6)*
j	t_j	d_j	C_j	$n_j = n_{j-1} - d_{j-1} - C_{j-1}$	$S_j = \frac{(n_j - d_j)}{n_j}$
				j > 0	
deaths	*times*	*deaths*	*censored*	*risk set*	
0	0	0	0	20	$\frac{20-0}{20} = 1$
1	6	1	1	20	$\frac{(20-1)}{20} = 0.950$
2	12	2	1	18	$\frac{(18-2)}{18} = 0.889$
3	27	2	3	15	$\frac{15-2}{15} = 0.867$
4	39	1	3	10	$\frac{(10-1)}{10} = 0.900$
5	57	1	5	6	$\frac{(6-1)}{6} = 0.833$

Then the estimate of the survival function is:

time	$0 \le t < 6$	$6 \le t < 12$	$12 \le t < 27$	$27 \le t < 39$	$39 \le t < 57$	$57 \le t$
probability	1	1×0.950 $= 0.950$	0.950×0.889 $= 0.844$	0.844×0.867 $= 0.732$	0.732×0.900 $= 0.659$	0.659×0.833 $= 0.549$

Standard error of these estimates will be given by Greenwood's Formula:

$$\text{Var}(\hat{S}(t)) \simeq (\hat{S}(t))^2 \sum_{t_j \le t} \frac{d_j}{n_j\ (n_j - d_j)}$$

(1)	*(2)*	*(7)*	*(8)*
j	t_j	$\frac{d_j}{n_j(n_j - d_j)}$	$\sum_{t_j \le t} \frac{d_j}{n_j(n_j - d_j)}$
0	0	$\frac{0}{20(20-0)} = 0$	0
1	6	$\frac{1}{20(20-1)} = 0.00263$	0.00263
2	12	$\frac{2}{18(18-2)} = 0.00694$	0.00957
3	27	$\frac{2}{15(15-2)} = 0.01026$	0.01983
4	39	$\frac{1}{10(10-1)} = 0.01111$	0.03094
5	57	$\frac{1}{6(6-1)} = 0.03333$	0.06427

Then the standard errors are

time	*0 ≤ t < 6*	*6 ≤ t < 12*	*12 ≤ t < 27*	*27 ≤ t < 39*	*39 ≤ t < 57*	*57 ≤ t*
probability	1	0.950	0.844	0.732	0.659	0.549
standard	$1 \times \sqrt{0}$	$0.950 \times \sqrt{0.00263}$	$0.844 \times \sqrt{0.00957}$	$0.732 \times \sqrt{0.01983}$	$0.659 \times \sqrt{0.03094}$	$0.549 \times \sqrt{0.06427}$
error	= 0	= 0.0487	= 0.0826	= 0.1031	= 0.1159	0.1392

Then maximum likelihood estimate of Survival Function with approximate 95% confidence intervals:

	$\hat{S}(t)$	*Approx. 95% Confidence Interval*		
$0 \le t < 6$	1.000	1 ± 0	=	1
$6 \le t < 12$	0.950	0.950 ± 0.096	=	0.854, 1*
$12 \le t < 27$	0.844	0.844 ± 0.162	=	0.682, 1*
$27 \le t < 39$	0.732	0.732 ± 0.202	=	0.530, 0.934
$39 \le 4 < 57$	0.659	0.659 ± 0.227	=	0.432, 0.886
$57 \le t$	0.549	0.549 ± 0.273	=	0.276, 0.822

* Since $\hat{S}(t)$ cannot exceed 1.

Examiners' Comment: Confidence intervals calculated on log or log-log of survival probabilities also gained full credit.

(iii)