EXAMINATIONS

13 September 2000 (pm)

Subject 104 — Survival Models

Time allowed: Three hours

INSTRUCTIONS TO THE CANDIDATE

1. *Write your surname in full, the initials of your other names and your Candidate's Number on the front of the answer booklet.*

2. *Mark allocations are shown in brackets.*

3. *Attempt all 13 questions, beginning your answer to each question on a separate sheet.*

Graph paper is not required for this paper.

AT THE END OF THE EXAMINATION

Hand in BOTH your answer booklet and this question paper.

In addition to this paper you should have available Actuarial Tables and an electronic calculator.

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1 (i) Explain precisely what is meant by ${}_{3|5}q_{50}$. [1]

(ii) Write down an integral expression for ${}_{3|5}q_{50}$ in terms of the hazard rates over the appropriate age range only. [2]

(iii) Evaluate ${}_{3|5}q_{50}$ using the A1967-70 Ultimate mortality table. [1]

[Total 4]

2 A life insurance company sells whole-life assurance policies with a sum assured of £20,000, payable at the end of the year of death. The premium is £520 payable annually in advance until the death of the policyholder.

A life now aged 50 purchased a policy exactly one year ago, and is now due to pay the second annual premium.

(i) Find the expected present value of the future loss to the company arising from this policy. [2]

(ii) Show that the variance of the present value of the future loss from this policy can be expressed as:

$$b \ . \ A'_{50} + c$$

Determine the numerical values of b and c, and the rate of interest used to evaluate A'_{50}. [3]

Basis: mortality A1967-70 (Ultimate), interest 4% per annum. Ignore expenses.

[Total 5]

3 Is it possible for ${}_tV_x$ to exceed ${}_tV_{x:\overline{n}|}$? Justify your answer, either by providing proof or by showing a suitable example, as appropriate. [5]

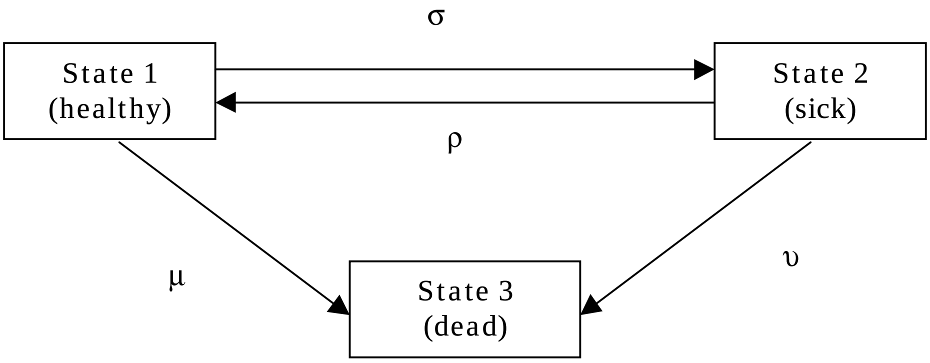
4 You are given that $q_{75} = 0.06229$.

(i) Calculate ${}_{¼}p_{75}$ and ${}_{¼}p_{75¾}$, assuming a uniform distribution of deaths between integer ages. [4]

(ii) Repeat part (i), using the alternative assumption that a constant hazard rate applies between integer ages. [2]

[Total 6]

5 A life insurance company prices its long-term sickness policies using the following 3-state continuous-time Markov model, in which the forces of transition σ, ρ, μ and υ are assumed to be constant:



For a group of policyholders observed over a 1-year period, there are:

10 transitions from state 1 to state 2;
7 transitions from state 2 to state 1;
2 deaths from state 1;
3 deaths from state 2.

The total time spent in state 1 is 512 years, and in state 2 is 20 years.

(i) Write down the likelihood function for these data. [2]

(ii) Hence derive the maximum likelihood estimate of σ. [2]

(iii) Calculate an estimate of the standard error of $\hat{\sigma}$, where $\hat{\sigma}$ is the maximum likelihood estimator of σ. [2]

[Total 6]

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6 Suppose you have fitted the following proportional hazards regression model to the mortality data for a sample of life-assurance policyholders:

$$h_i(t) = h_0(t)\exp\{0.01(x_i - 30) + 0.2y_i - 0.05z_i\},$$

where:

$h_i(t)$ denotes the hazard function for life i at duration t;
$h_0(t)$ denotes the baseline hazard function at duration t;
x_i denotes the age at entry of life i;
$y_i = 1$ if life i is a smoker, otherwise zero; and
$z_i = 1$ if life i is female, zero if male.

(i) Describe the class of lives to which the baseline hazard function applies. [2]

(ii) What does the model (if correct) tell you about the survival function of a male smoker aged 30 at entry, relative to that of a female smoker aged 40 at entry? [3]

(iii) What does the model (if correct) tell you about the survival function of a female smoker aged 30 at entry, relative to that of a male non-smoker aged 40 at entry? [2]
[Total 7]

7 (i) Consider an annuity payable continuously during the lifetime of (x), but for at most 20 years. The rate of payment at time t is £2,000 per annum for $0 \le t \le 5$, and £10,000 per annum for $5 < t \le 20$. The force of mortality to which (x) is subject is assumed to be constant at 0.01.

Calculate the expected present value of this annuity at a force of interest of 0.06. [5]

(ii) Calculate the expected present value of the benefits of an endowment assurance issued to the same life as in part (i), paying a sum assured of £20,000 at the end of five years or immediately on earlier death. The mortality and interest assumptions are the same as in part (i). [2]
[Total 7]

8 In a mortality investigation the total number of deaths at age x during the period of investigation is θ_x. Age x is defined as:

x = [age next birthday at the start of the policy]
+ [curtate duration at the date of death]

(i) State, precisely, the rate interval implied by this definition. [2]

(ii) Describe, precisely, the central exposed to risk E_x^c that would correspond to this definition. [1]

(iii) Give the age, $x + f$, for which the estimate $\left(\mathring{\mu}_{(x)} = \dfrac{\theta_x}{E_x^c}\right)$ estimates μ_{x+f}.

State and explain any assumptions you make in determining f. [2]

(iv) Explain how your answer to (iii) above would change if an investigation showed that, on average, lives purchased their policies two months before a birthday. [3]

[Total 8]

9 Life insurance companies, investigating the mortality experience of their policyholders, usually count the number of policies in force and the number of policies giving rise to death claims, rather than the number of lives assured in force and the number of lives who die during the period of investigation.

(i) Explain why this is done. Describe the statistical problems that might arise from an investigation based on policies rather than on the lives assured. [4]

(ii) If N identical and independent lives aged x exact are observed until age $x + 1$ (or until earlier death), and π_i is the proportion of those lives having exactly i policies ($i = 1, 2, 3, \ldots$), then show algebraically how the statistical problems you described in part (i) arise. [4]

[Total 8]

10 The following data relate to 12 patients who had an operation which was intended to correct a life-threatening condition, where time 0 is the start of the period of the investigation:

Patient number	*Time of operation (in weeks)*	*Time observation ended (in weeks)*	*Reason observation ended*
1	0	120	Censored
2	0	68	Death
3	0	40	Death
4	4	120	Censored
5	5	35	Censored
6	10	40	Death
7	20	120	Censored
8	44	115	Death
9	50	90	Death
10	63	98	Death
11	70	120	Death
12	80	110	Death

You can assume that censoring was non-informative with regard to the survival of any individual patient.

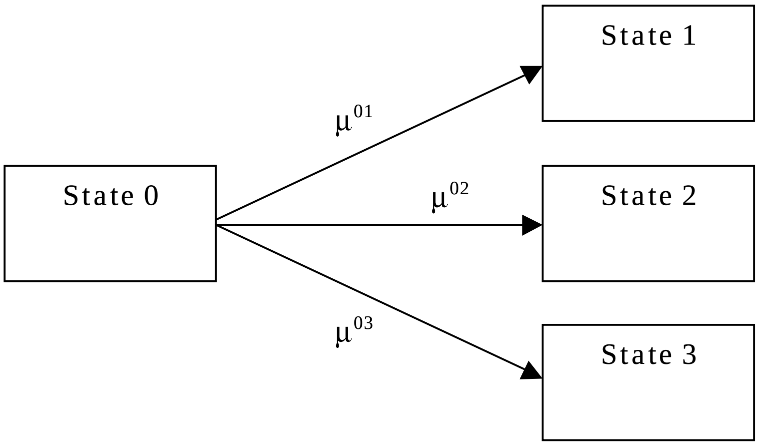
(i) Compute the Nelson-Aalen estimate of the cumulative hazard function, $\Lambda(t)$, where t is the time since having the operation. [6]

(ii) Using the results of part (i), deduce an estimate of the survival function for patients who have had this operation. [2]

(iii) Estimate the probability of a patient surviving for at least 70 weeks after undergoing the operation. [1]

[Total 9]

11 Consider the following multiple-decrement model, in which $S(t)$, the state occupied at time t of a life initially aged x, is assumed to follow a continuous-time Markov process.



The usual notation ${}_tp_x^{ab} = P(S(t) = b \mid S(0) = a)$ is used, and the forces of transition (μ^{0k}, $k = 1, 2, 3$) are assumed to be constant.

(i) Derive differential equations for ${}_tp_x^{00}$ and ${}_tp_x^{01}$, and state the relevant boundary condition for each equation. [5]

(ii) Hence find simple explicit expressions for ${}_tp_x^{00}$ and ${}_tp_x^{01}$.
(Your final answers should be expressed in terms of μ^{0k}, $k = 1, 2, 3$ only.) [5]
[Total 10]

12 The following data have been taken from a mortality investigation of a life insurance company's male whole of life policyholders aged between 25 and 65. The graduated rates were obtained by fitting a mathematical formula to the data. The formula has four parameters, which were estimated using the method of maximum likelihood.

Age (x)	*Initial exposed to risk* E_x	*Observed deaths* θ_x	*Graduated rate* Q_x	*Expected deaths* $E_x q_x$	$E_x q_x p_x$	*Standardised deviations* $\dfrac{\theta_x - E_x q_x}{\sqrt{E_x q_x p_x}}$
35	14 211	17	0.001998	28.39	28.33	–2.14
36	12 381	21	0.002061	25.52	25.47	–0.89
37	11 704	27	0.002124	24.86	24.81	–0.43
38	11 038	24	0.002187	24.14	24.09	–0.03
39	10 947	29	0.002250	24.63	24.57	+0.88
40	13 885	21	0.002314	32.13	32.06	–1.97
41	11 507	30	0.002378	27.36	27.29	+0.51
	85 673	169		187.03	186.62	

(i) Perform the Chi-squared test, and two other appropriate tests for the adherence of this graduation to the data. Comment on the results of the tests. [8]

(ii) Describe the statistical rationale underlying the cumulative deviations test, a test which is often used to assess adherence to data in a graduation. State an important limitation of the test when used for this purpose, and explain how you could overcome this limitation. [4]

[Total 12]

13 A life insurance company issues a 20-year temporary assurance to lives aged 45. The sum assured, which is payable immediately on death, is £300,000 for the first 10 years, and £100,000 thereafter. Level annual premiums are payable in advance for 20 years, or until earlier death.

The premium basis is:

Mortality: A1967-70 Ultimate
Interest: 4% per annum.
Expenses: nil.

(i) Show that the premium payable is approximately £1,339.57 per annum. [4]

(ii) Find the net premium policy value ten years after the commencement of the policy, immediately before the payment of the eleventh premium, assuming the reserving basis is the same as the premium basis. [4]

(iii) Give an explanation of your numerical answer to part (ii). Describe the disadvantages to the insurance company of issuing this policy. [3]

(iv) How could the terms of the policy be altered, so as to remove the disadvantages described in part (iii)? [2]
[Total 13]