EXAMINATIONS

22 April 2004 (pm)

Subject 104 — Survival Models

Time allowed: Three hours

INSTRUCTIONS TO THE CANDIDATE

1. *Enter all the candidate and examination details as requested on the front of your answer booklet.*

2. *You must not start writing your answers in the booklet until instructed to do so by the supervisor.*

3. *Mark allocations are shown in brackets.*

4. *Attempt all 9 questions, beginning your answer to each question on a separate sheet.*

Graph paper is NOT required for this paper.

AT THE END OF THE EXAMINATION

Hand in BOTH your answer booklet, with any additional sheets firmly attached, and this question paper.

In addition to this paper you should have available Actuarial Tables and your own electronic calculator.

1 A life office has a portfolio of whole of life assurance policies, issued to lives aged exactly 60. The benefit of £3,000 under each policy is payable at the end of year of death. No further premiums are payable in respect of these policies.

The reserve held per policy after 5 years, V_5, is £1,200.

Assuming a rate of interest of 6% and mortality rates given below, calculate V_6, the reserve held per policy after 6 years.

Age	q_x
65	0.020
66	0.025
67	0.030

[3]

2 A trial for the life of a new battery was carried out by placing 1,000 batteries inside 1,000 fluffy electrical toys. The toys were turned on and left to run for 24 hours. A researcher returned every hour to count how many toys had stopped operating. On close analysis it was discovered that some of the toys had ceased to operate due to mechanical problems and that some of the toys were not switched on in the first place. Some of the toys were still running after 24 hours.

For each of the following types of censoring, state with reasons whether it is present in the investigation:

(a) left-censoring
(b) interval censoring
(c) type I censoring
(d) non-informative censoring

[4]

3 A particularly morbid actuarial student is considering the mortality of the population of the village in which she lives. The village graveyard contains tombstones which bear inscriptions of the form "A N Other beloved mother of …. , Born 1903, Died 1976".

(i) If the student were to use the inscriptions as a source of death data, state the exact age to which the calculated mortality rate would apply assuming a Binomial model was used. [2]

(ii) Comment on the difficulties the student is likely to face in attempting to calculate mortality rates in this way. [3]

[Total 5]

4 The following data come from the national life table of a developed country.

Age x	*Survivors to age x,* l_x
80	22,933
81	20,010

(i) Estimate ${}_{0.5}p_{80}$ assuming:

(a) that deaths between exact ages 80 and 81 are uniformly distributed
(b) that the force of mortality is constant between exact ages 80 and 81

[3]

(ii) Explain why the two estimates in (i) are different. State with reasons which estimate you would prefer to use. [2]

[Total 5]

5 (i) A random variable X measures the duration until some event occurs. Write down a definition of the hazard, $h(t)$ of the event occurring at duration t, in terms of probabilities relating to the random variable X, and state what the definition means in words. [2]

(ii) You wish to investigate the effect of two factors, Z_1 and Z_2, on the duration until the event occurs. Someone suggests that you use a proportional hazards model. Explain what is meant by the term proportional hazards model. [2]

(iii) The Weibull distribution has a survival function given by the formula

$$S(t) = \exp[-(\lambda t)^{\alpha}]$$

where λ and α are parameters.

Show that, by choosing λ so that it depends on Z_1 and Z_2 only, the Weibull distribution can be used as a proportional hazards model to investigate the effects of Z_1 and Z_2 on the duration. [4]

[Total 8]

6 A life office issues a special 3 year term assurance policy to a man aged exactly 65. The benefit payable at the end of the year of death is £10,000 if death occurs in the first year, £15,000 if death occurs in the second year and £30,000 if death occurs in the third year. Premiums of £400 per year are payable annually in advance for 3 years or until earlier death. The random variable L is defined as the present value of the loss under this policy calculated at the date of issue, so that:

L = present value of benefit outgo – present value of premium income

(i) Calculate all the possible values that L can take, and hence find the mean and standard deviation of L. [7]

The office issues an endowment assurance contract to an identical life; death benefits are the same as under the term assurance policy described above. In addition an endowment benefit of £30,000 is payable on survival to the end of the 3 year term. Premiums of £9,500 are payable annually in advance for three years or until earlier death. The loss under this policy is denoted L^*.

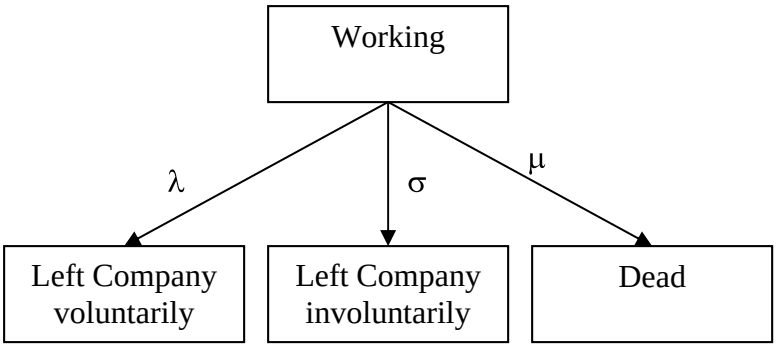
(ii) Calculate the mean and standard deviation of L^*. [4]

(iii) Comment on your results. [3]

Basis: Mortality: AM92 Ultimate
Interest: 3% per annum throughout
Expenses are ignored

[Total 14]

7 A Company is modelling its workforce using the multiple state model shown below.



The Company is interested in how many of its employees leave employment voluntarily each year and so wishes to estimate the transition intensity λ from the "working" state to the "left company voluntarily" state. Consider employees aged between ages 30 and 31 exact and suppose that the ith individual spends time W_i employed by the Company between those ages.

(i) Derive the maximum likelihood estimate $\hat{\lambda}$ of λ and write down the variance of $\hat{\lambda}$. [6]

(ii) Suppose the Company has available data on the number of employees who left service voluntarily in 2002, classified by age last birthday on the previous anniversary of the date they started employment.

The Company also has census data $P(x, t)$ for $t = 0$ and $t = 1$ where $P(x, t)$ represents the number of employees aged x nearest birthday on 1 January in year $2002 + t$.

Assuming that the dates on which employment began are distributed uniformly across both calendar years and life years:

(a) Derive a formula to approximate the exposed to risk in the year 2002 for lives classified as age 30.

(b) If you use the exposed to risk in (a) to estimate λ, state the exact age to which your formula applies.

[6]

(iii) You subsequently discover that, actually, all workers begin employment with the Company on their 16th birthdays. Write down the revised estimate of the exposed to risk based on the data available. [2]

[Total 14]

8 The main activity of a certain charity is training and providing guide dogs for blind people. The charity wishes to estimate the lifetime of the dogs it provides, and has therefore attempted to keep track of 100 dogs that were all born in 1990. Of these 100 dogs, 81 remained in service on reaching age 10 years. Of the remaining dogs, some died and some left active guide service for other reasons.

The ages of the deaths and other exits were recorded as follows:

Age (years)	*Number of deaths*	*Number of other exits*
0.25		2
0.5	1	
1		3
2.25		3
3.5		2
4	2	
4.5		1
6	1	
7.5		2
9		2

(i) Calculate the Nelson-Aalen estimate of the integrated hazard for these guide dogs. [6]

Guide dogs undergo an initial period of one year's training, which costs £3,000 payable continuously over the year. They then work for up to 9 years. The charity undertakes to provide food, medical expenses, equipment and so on throughout the ten years of the dog's working life. This is estimated to cost £2,000 per year, payable continuously over the year. If the dog reaches age 10, the charity will find it a new home for its retirement, the expense of this resettlement being £300.

(ii) Using the integrated hazard calculated in (i), calculate the capital cost to the charity of supporting a single guide dog. [10]

The charity has two principal sources of finance. The first is a new scheme whereby members of the public commit to make regular payments for ten years (but ceasing on death) to the charity. Payments are to be made annually in advance. The total annual income from this scheme at the outset is £125,000. The average age of the contributors is now exactly 50.

The second source of income is a commitment to leave the charity a legacy when the benefactor dies. The charity has commitments to legacies worth a total of £900,000 from benefactors, the average age of whom is now exactly 70. Due to delays in settling estates, the charity generally receives these legacies one year after the death of the benefactor.

(iii) Calculate the expected present value of the current commitments to the charity. Hence estimate the number of guide dogs that the charity can undertake to provide. [7]

Basis: Mortality of humans: PFA92C20
Interest: 4% per annum throughout

[Total 23]

9 An investigation into the mortality of young adult males in a developed country has been undertaken. The table below shows an extract from the results.

Age	*Exposed-to-risk*	*Observed deaths*	*Standardised deviation* z_x
18	34,000	40	1.9159
19	33,000	35	1.4541
20	29,500	27	0.4460
21	30,000	26	0.0394
22	25,500	22	–0.1469
23	24,000	19	–0.5106
24	17,000	13	–0.5067
25	23,500	20	–0.0467
26	18,000	12	–0.8437
27	14,000	11	–0.2609

(i) Someone suggests to you that the underlying mortality of these young men is the same as that in English Life Table 15 (Males) and that you should test this using the chi-squared test.

(a) Define the standardised deviation.
(b) State the null hypothesis to be tested.
(c) State the test statistic and its distribution under the null hypothesis.
(d) Carry out the test.

[6]

(ii) (a) Describe two possible differences between the underlying mortality of the men in the sample and English Life Table 15 (Males) which the chi-squared test might fail to detect.

(b) For each of the differences you describe, carry out a different test to see if the observed experience is significantly different from that of English Life Table 15 (Males). For each test you carry out, state explicitly what the test is designed to detect, and state your conclusions.

[9]

(iii) Comment on your results in (i) and (ii) above. [4]

(iv) Explain how you would graduate the observed experience by reference to a standard mortality table. [5]

[Total 24]

END OF PAPER