EXAMINATIONS

10 April 2003 (pm)

Subject 104 — Survival Models

Time allowed: Three hours

INSTRUCTIONS TO THE CANDIDATE

1. *Enter all the candidate and examination details as requested on the front of your answer booklet.*

2. *You must not start writing your answers in the booklet until instructed to do so by the supervisor.*

3. *Mark allocations are shown in brackets.*

4. *Attempt all 13 questions, beginning your answer to each question on a separate sheet.*

Graph paper is required for this paper.

AT THE END OF THE EXAMINATION

Hand in BOTH your answer booklet, with any additional sheets firmly attached, and this question paper.

In addition to this paper you should have available Actuarial Tables and your own electronic calculator.

1 List three advantages of the two-state model over the Binomial model for the estimation of transition intensities in a case where exact dates of entry into and exit from observation are known. [3]

2 (i) State what is meant by the assumption of a uniform distribution of deaths between integer ages x and $x + 1$. [1]

(ii) Using the assumption of a uniform distribution of deaths and ELT15(Males) mortality table calculate ${}_{0.25}p_{75}$ and ${}_{0.25}p_{75.5}$. [4]

[Total 5]

3 (i) Describe what is meant by homogeneity in a mortality investigation and explain what problems lack of homogeneity may cause for an insurer in setting premium rates. [3]

(ii) Suggest six factors in respect of which life office mortality statistics are often subdivided. [3]

[Total 6]

4 (i) Explain why crude mortality rates will be graduated before being used for premium calculations. [2]

(ii) (a) Describe how you would graduate crude rates by reference to a standard mortality table.

(b) Comment on any further considerations that an insurance company would take into account before using these graduated rates for premium calculations for annuity policies. [4]

[Total 6]

5 The Cox proportional hazards model is to be used to model the rate at which students leave a certain profession before qualification. Assuming they stay in the profession, students will qualify three years after joining the profession. In the fitted model, the hazard depends on the time, t, since joining the profession and three covariates. The covariates, their categories and the fitted parameters for each category are shown in the table below:

Covariate	*Possibility*	*Parameter*
Size of employer	large	0
	small	0.4
Degree studied	none	0.3
	Science	–0.1
	Arts	0.2
	other	0
Location	London	0
	other UK	–0.3
	overseas	0.4

(i) Defining clearly all the terms you use, write down an expression for the hazard function in this model. [3]

(ii) State the class of students which is most likely to proceed to qualification under this model, and that which is least likely. [2]

(iii) A student who has been in the profession for one year moves from a "small" employer to a "large" employer. Express the probability that he will qualify with the "large" employer P_L in terms of the probability that he would have qualified if he had stayed with the "small" employer P_S, all other factors being equal. [2]

[Total 7]

6 (i) State the age ranges over which Gompertz' Law is an appropriate model for human mortality. [1]

(ii) Show that, under Gompertz' Law, the probability of survival from age x to age $x + t$ is equal to:

$$\left[\exp\left(-\frac{B}{\log c}\right)\right]^{c^x(c^t-1)}. \qquad [3]$$

(iii) Describe a method of estimating the parameters, B and c, when graduating a set of crude mortality rates using Gompertz' Law. [3]

[Total 7]

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7 In a mortality investigation spanning $N + 1$ years, information is available on the calendar year of birth and the calendar year of death of each life dying in each year K, $K + 1, ..., K + N$. In addition, information is available on the number of lives, classified by age x last birthday, on 1 January in each of years $K, K + 1, ..., K + N + 1$.

(i) In terms of the available data, derive an approximation for the central exposed to risk which corresponds to the deaths data. [5]

(ii) The initial rates of mortality for deaths having age label x in this investigation estimate q_{x+f}. Determine f, stating any assumptions you make. [2]

[Total 7]

8 A life office wishes to compare the mortality of its male term assurance policyholders with the standard table AM92. A Chi-squared test has been carried out to test the null hypothesis that the true underlying mortality rates of the policyholders are those of the standard table. Based on the results of this test, the null hypothesis has been accepted.

(i) State three possible defects that the Chi-squared test may have failed to detect and explain why these are not detected. [3]

(ii) Describe how you would carry out a suitable test to check for **one** of the above defects. You should state which defect you are checking for. [4]

[Total 7]

9 Let X be a random variable representing the present value of the benefits of a 20 year term assurance issued to a life aged exactly 45, the benefit being payable at the end of the year of death.

Let Y be a random variable representing the present value of the benefits of a 20 year pure endowment policy issued to a (different, independent) life also aged exactly 45.

Finally, let Z denote the present value of the benefits of a 20 year endowment assurance issued to a third independent life aged exactly 45, the death benefit being payable at the end of the year of death.

The benefit payable under each policy is £1.

(i) Calculate the variances of X and Y. [5]

(ii) Without carrying out any further calculations explain why $\text{Var}(Z) < \text{Var}(X) + \text{Var}(Y)$. [2]

[Total 7]

Basis: Mortality: AM92 ultimate
Interest: 4% per annum throughout
Expenses are ignored

10 An illness-death model has three states:

1 = healthy
2 = sick
3 = dead

(i) Draw and label a diagram showing the three states and the transition intensities between them. [2]

(ii) Show, from first principles, that in this illness-death model

$$\frac{\partial}{\partial t}\,{}_t p_x^{12} = {}_t p_x^{11}\mu_{x+t}^{12} - {}_t p_x^{12}\mu_{x+t}^{21} - {}_t p_x^{12}\mu_{x+t}^{23}\,.$$ [6]

[Total 8]

11 A man aged exactly 45 has decided to invest some money to purchase a deferred annuity from an insurance company. The man plans to invest £1,500 per annum for 20 years; the first payment being made now, with subsequent payments made annually if he is alive. In addition he will have lump sums of £3,000 to invest in 5 and 10 years' time, if he is then alive.

The man has to choose between a standard whole life annuity and an annuity where the benefits are guaranteed to be paid for at least 10 years and for life thereafter. Both annuities are level, commence at age 65 and are payable annually in arrears.

(i) Write down the equations of value for the standard whole life annuity £X and the guaranteed annuity £X* that can be bought. [2]

(ii) Show that by choosing the guaranteed annuity option, the annuity payable is £155 per annum lower than under the standard option. [5]

(iii) The policyholder chooses to take the standard annuity option. He pays the premiums and reaches age 65. Calculate the probability that the policyholder survives long enough that he receives annuity payments whose nominal amount exceeds the total premiums paid. [2]

[Total 9]

Basis: Mortality before age 65: AM92 ultimate
Mortality from age 65: PMA92C20
Interest: 4% per annum throughout
Expenses are ignored

12 A manufacturer of computer chips is attempting to estimate the useful working lifetime of its products. To do so, it has been tracking 20 chips sold on 1 January 1997. The purchaser of each chip was contacted at three-monthly intervals up to 1 January 2002 to check whether the chip was still functioning correctly. The results are summarised below (where B means observation ceased because the chip stopped functioning and O means observation ceased for some other reason).

Date observation ceased	*Reason*
1 April 1997	B
1 July 1997	B
1 October 1997	O
1 October 1997	O
1 October 1997	O
1 January 1998	B
1 January 1998	B
1 July 1998	O
1 July 1998	O
1 October 1998	O
1 July 1999	O
1 July 1999	O
1 July 1999	O
1 October 1999	B
1 April 2000	O
1 July 2000	O
1 October 2000	B
1 January 2001	B
1 January 2002	O
1 January 2002	O

(i) Explain how the manufacturer's tracking method introduces censoring into the study. [2]

(ii) Calculate the Nelson-Aalen estimate of the integrated hazard for these computer chips. State any assumptions that you make about the exact time each chip stopped functioning. [6]

(iii) Use this to approximate the Kaplan-Meier estimate of the survival function and sketch the survival function. [4]

(iv) Suggest two ways in which the breakdown rate for computer chips is similar to typical patterns for human mortality. [2]

[Total 14]

13 A 20 year endowment assurance policy was issued to a man aged exactly 40 on 1 January 1991. The sum assured of £10,000 is payable immediately on death or on survival to the end of the term.

Premiums of £300 per annum are payable annually in advance for the term of the policy, ceasing on earlier death.

(i) Explain why the life insurance company will hold reserves for this policy. [2]

(ii) (a) Define the prospective and retrospective policy values for the policy.
(b) Explain why the two policy values may be different.
[5]

(iii) Calculate the prospective policy value at the end of 2003. [5]

(iv) Calculate the mortality profit for this policy for 2003 if the policyholder dies during 2003. [2]
[Total 14]

Basis: Mortality: AM92 ultimate
Interest: 6% per annum throughout
Expenses are ignored