# EXAMINATIONS

9 April 2002 (am)

## Subject 105 — Actuarial Mathematics 1

*Time allowed: Three hours*

***INSTRUCTIONS TO THE CANDIDATE***

1. *Enter all the candidate and examination details as requested on the front of your answer booklet.*

2. *You must not start writing your answers in the booklet until instructed to do so by the supervisor.*

3. *Mark allocations are shown in brackets.*

4. *Attempt all 14 questions, beginning your answer to each question on a separate sheet.*

***Graph paper is not required for this paper.***

*AT THE END OF THE EXAMINATION*

*Hand in BOTH your answer booklet, with any additional sheets firmly attached, and this question paper.*

*In addition to this paper you should have available Actuarial Tables and your own electronic calculator.*

**1** (i) Explain what is meant by the following expression:

$$ {}_{3|4}q_{[40]+1} \cdot $$

(ii) Calculate its value using A1967–70 mortality. [3]

**2** Two lives, each aged exactly 35, are independent with respect to mortality and are each subject to a constant force of mortality of 0.02.

Calculate the value of the following expression for these lives:

$$ {}_{10|20}q^{1}_{35:35} $$ [4]

**3** (i) Define the term Total Fertility Rate and explain the difference between rates calculated on a cohort basis and a period basis. [3]

(ii) In the context of population projection, state, with a reason, which basis is preferable. [2]

[Total 5]

**4** Due to a downturn in the economy, the numbers unemployed in a certain country are expected to increase. The current number unemployed is 100,000, and this is expected to rise towards but not exceed 300,000 following the logistic growth model. The initial rate of increase in unemployment will be 25% per annum. Calculate how long it will take for the unemployed population to reach 200,000. [5]

**5** A life insurance company sold a number of 4-year single premium policies with a guaranteed amount payable at maturity.

The closest matching investment available was a 5-year zero-coupon bond. Interest rates at the time of the insurance company selling the policies and investing the money in the bonds were 5.25% effective per annum.

The office invested all the premiums received in these assets. The insurance company guaranteed a return of 5.0% per annum at maturity. On death, the return was not guaranteed but the company promised to pay out the full market value of the related asset immediately at the date of death.

If the distribution of $1 + i$ is log-normal with parameters $\mu = 0.05$ and $\sigma = 0.01$, and mortality follows English Life Table No. 12 – Males, calculate the probability that the office makes a loss on a policy sold to someone aged exactly 56. You should assume that the company sells the matching asset at the time of any claim.

(Remember that if $X$~lognormal with parameters $\mu$ and $\sigma$, then $\log_e(X)$ is normally distributed with mean $\mu$ and standard deviation $\sigma$.) [5]

**6** A retiring employee aged exactly 60 is given a choice between the following two pensions:

Pension A is payable annually in arrear throughout the pensioner's lifetime, with at least 4 payments guaranteed to be made. The first payment is £20,000 and payments increase by 0.9709% per annum compound thereafter.

Pension B is payable annually in arrear, with an initial payment of £13,000. Each subsequent payment increases by £1,000 and payments cease immediately on death.

Calculate the expected present value of each pension using the following basis:

Mortality: A1967–70 Select
Interest: 4% per annum [6]

**7** Describe how nutrition and education affect mortality. [6]

**8** A mortality investigation of pensioners who retired due to ill health is being undertaken to investigate if there are any initial temporary select effects. Data in respect of deceased pensioners are categorised as follows:

$x$: age last birthday at death
$r$: curtate number of years between retirement and death

(i) Estimates of $\mu_{[y]+t}$ are made by dividing the death data by its corresponding central exposed to risk. Derive the values of $y$ and $t$ in terms of $x$ and $r$, stating clearly any assumptions you need to make. [3]

(ii) The following data are also available in respect of one pensioner:

| | |
|---|---|
| Date of birth | 1 August 1936 |
| Date of retirement | 1 November 1998 |
| Date of death | 1 July 2001 |

Calculate the contribution of this individual to each of the appropriate central exposed to risk figures corresponding to the available summary data. [3]

[Total 6]

**9** A life insurance company sells 4-year unit-linked endowment assurance contracts to males aged 61 exact. Premiums of £1,000 are payable annually in advance.

Capital units are bought by the premium in the first year, and accumulation units are bought thereafter. 102% of each year's premium is invested in units at the offer price. There is a bid-offer spread in unit values, with the bid price being 95% of the offer price.

Capital units bear a management charge of 5% per annum of the bid value of the fund, and this charge is deducted at the end of every year.

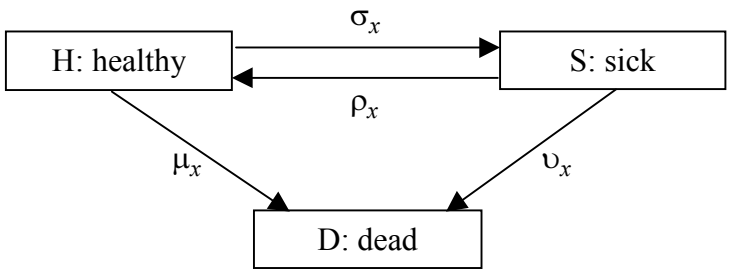
The death benefit under the policy is paid at the end of the year of death, and is the full bid value of units under the policy, after deduction of relevant management charges.

The pricing actuary assumes that fund growth will be 7.5% per annum and that mortality experience will follow A1967–70 Select. He is contemplating using part of the management charge for actuarial funding of capital units. The actuarial factor at duration t would be $A_{[61]+t:\overline{4-t}|}$ at a rate of interest of 4% per annum, with mortality as above.

Assuming non-unit fund growth is 5% per annum, and ignoring expenses, calculate the non-unit fund cash flow for the first year of the policy if:

(a) the full value of capital units is held
(b) only the actuarially funded value of capital units is held [6]

**10** The following 3 state model is used to price various sickness policies. The forces of transition $\sigma$, $\rho$, $\mu$ and $\upsilon$ depend only on age.




The following probabilities are defined:

$p_{x,t}^{ij}$ is the probability that a life aged $x$ in state $i$ will be in state $j$ at age $x + t$;

$p_{x,t}^{\overline{ii}}$ is the probability that a life aged $x$ in state $i$ will remain in state $i$ until age $x + t$;

${}_t p_{x,z}^{ij}$ is the probability that a life aged $x$ in state $i$ will be in state $j$ at age $x + t$, having been in state $j$ for period $z$.

Using these probabilities and / or forces of transition, write down an expression for the expected present value of each of the following sickness benefits for a life currently aged 35 and healthy. The constant force of interest is $\delta$.

(a) £1,000 per annum payable continuously while sick, but all benefits cease at age 65

(b) £1,000 per annum payable continuously while in the sick state for any continuous period in excess of a year. However, any benefit period is limited to 5 years payments, but the number of possible benefit periods is unlimited

(c) £1,000 per annum payable continuously throughout the first period of sickness only [6]

**11** (i) A life insurance company prices endowment assurance policies allowing for mortality, expenses and interest. For surrenders, it wants to base the values it is prepared to pay on gross premium reserves, using the premium basis unchanged except for the interest rate. If it is to make a profit on surrenders, state in what direction it should change the interest rate element of the basis, if the reserves are:

(a) retrospective
(b) prospective

Give reasons for your answers. You should assume that experience is the same as the premium basis. [4]

(ii) A policyholder aged exactly 60 has 5 years remaining on his endowment policy which has a sum assured of £100,000 payable immediately on death, or maturity, whichever occurs first. He can no longer afford to pay any further premiums. He is offered a choice of:

(a) a surrender value of £41,000

(b) a paid up sum assured of £54,000

(c) a whole life policy, without future premiums, with a death benefit, payable immediately on death, of £100,000

Show which he should choose, assuming he wants the one with the highest expected present value of benefits.

Basis: Mortality: A1967–70 Ultimate
Interest: 6% per annum [4]
[Total 8]

**12** A small employer decides to set up a pension scheme for his 2 employees, who are described by the following details:

| *Age (exact)* | *Past service (years)* | *Expected salary in next year (£)* |
|---|---|---|
| 30 | 5 | 25,000 |
| 35 | 6 | 20,000 |

The scheme will provide a pension of $1/60^{th}$ of pensionable salary for each year of service (fractions of a year counting proportionally) on retirement for any reason. Pensionable salary is the average annual salary earned in the final 36 months of employment.

The employer meets the full cost of the scheme. The contribution rate is determined by equating the expected present value of the total scheme liabilities to the expected present value of contributions. Contributions are calculated to be a constant percentage of the total salaries of the members at any time.

(i) Using the symbols defined in, and assumptions underlying, the Formulae and Tables for Actuarial Examinations, calculate the contribution rate required for the scheme. Ignore the possibility of new members joining the scheme. [8]

(ii) Immediately after the scheme is set up, a new employee joins the company and pension scheme. She is aged exactly 40, and will earn £30,000 in the next year. The employer decides to maintain the contribution rate determined in part (i) and to apply it to the new total salaries. Determine whether the funding rate is sufficient to meet the liabilities of the extended membership. [3]

[Total 11]

**13** 100 people aged exactly 50 are each sold a 15-year endowment assurance policy with sum assured £100,000. The premiums are paid annually in advance, and the sum assured is paid on maturity or at the end of the year of earlier death.

The life insurance company's assumptions are:

Mortality: A1967–70 Ultimate, and the lives are independent with respect to mortality

Interest: 6% per annum

Expenses: Initial: £300
Renewal: 2.5% of each premium, including the first

Let $P$ be the gross annual premium.

(i) State the gross future loss random variable for one policy at the outset. [3]

(ii) Using your answer to part (i) or otherwise, evaluate, in terms of $P$,

(a) the mean and variance of the loss (in present value terms) for a single policy at outset

(b) the mean and variance of the loss (in present value terms) for the entire portfolio at outset. [7]

Note: $A_{50:\overline{15}|}$ at 12.36% per annum = 0.20426

(iii) Show what values the gross annual premium $P$ can take if the company requires that the probability it incurs a loss (in present value terms) on the entire portfolio has to be less than 2.5%. Use the Normal approximation. [4]

[Total 14]

**14** A life insurance company issues a number of 3-year term assurance contracts to lives aged exactly 60. The sum assured under each contract is £200,000, payable at the end of the year of death. Premiums are payable annually in advance for the term of the policy, ceasing on earlier death.

The company carries out profit tests for these contracts using the following assumptions:

| | |
|---|---|
| Initial expenses: | £200 plus 35% of the first year's premium |
| Renewal expenses: | £25 plus 3% of the annual premium, incurred at the beginning of the second and subsequent years |
| Mortality: | A1967–70 Ultimate |
| Investment return: | 7% per annum |
| Risk discount rate: | 15% per annum |
| Reserves: | One year's office premium |

(i) Show that the office premium, to the nearest pound, is £4,396, if the net present value of the profit is 25% of the office premium. [10]

(ii) Calculate the cash flows if the company held zero reserves throughout the contract, using the premium calculated in part (i). [2]

(iii) Explain why the company might not hold reserves for this contract and the impact on profit if they did not hold any reserves. [3]

[Total 15]