EXAMINATIONS

18 April 2000 (am)

Subject 105 — Actuarial Mathematics 1

Time allowed: Three hours

INSTRUCTIONS TO THE CANDIDATE

1. *Write your surname in full, the initials of your other names and your Candidate's Number on the front of the answer booklet.*

2. *Mark allocations are shown in brackets.*

3. *Attempt all 17 questions, beginning your answer to each question on a separate sheet.*

Graph paper is not required for this paper.

AT THE END OF THE EXAMINATION

Hand in BOTH your answer booklet and this question paper.

In addition to this paper you should have available Actuarial Tables and an electronic calculator.

1 In the context of a pension scheme, explain the term "prospective service benefit" and state one example. [2]

2 In a select mortality investigation, $\theta_{x,r}$ corresponds to the number of deaths aged x next birthday at entry with duration r at the policy anniversary following death. $\theta_{x,r}$ divided by the appropriate central exposed to risk gives an estimate of $\mu_{[y]+t}$.

Derive the values of y and t to which this estimate applies. State clearly any assumptions used. [2]

3 Mortality levels for a certain country have been studied at national and regional level. Explain the circumstances under which a particular region may have an Area Comparability Factor of 0.5. [2]

4 A 25 year annual premium endowment assurance policy was sold to a life aged 40 exact at outset. Death benefits are payable at the end of the year of death. Calculate the Zillmerised net premium reserve at the end of the tenth year per unit sum assured.

Basis: Mortality: A1967–70 Select
Interest: 3% per annum
Initial expense: 2.5% of the sum assured [3]

5 A life insurance company sells an annual premium whole life assurance policy with benefits payable at the end of the year of death. Expenses are incurred at the start of each year, and claim expenses are nil.

(a) Write down a recursive relationship between the gross premium reserves at successive durations, calculated on the premium basis. Define all symbols used.

(b) Explain the meaning of this formula. [3]

6 Calculate $A^{1}_{30:\overline{30}|}$ using A1967–70 mortality and interest of 4% per annum. [3]

7 A pension scheme provides a pension of $\frac{1}{45}$ of final pensionable salary for each year of service, with a maximum of $\frac{2}{3}$ of final pensionable salary, upon retirement at age 65.

Final pensionable salary is defined as average annual salary over the 3 years immediately preceding retirement.

A member is now aged exactly 47 and has 14 years of past service. He earned £40,000 in the previous 12 months.

Calculate the expected present value now of this member's total pension on retirement, using the symbols defined in, and assumptions underlying, the Formulae and Tables for Actuarial Examinations. [3]

8 The random variables T_x and T_y represent the exact future lifetimes of two lives aged x and y respectively.

Let the random variable $g(T)$ take the following values:

$$g(T) = \begin{cases} \overline{a}_{\overline{n}|} & \text{if } \max\{T_x, T_y\} \le n \\ \overline{a}_{\overline{\max\{T_x, T_y\}}|} & \text{if } \max\{T_x, T_y\} > n \end{cases}$$

(i) Describe the benefit which has present value equal to $g(T)$. [2]

(ii) Express $E[g(T)]$ as concisely as possible in the form of an annuity function. [1]

[Total 3]

9 Define the term "asset share" in the context of a with-profit policy. [3]

10 The number of people sick with a new disease is expected to increase according to the logistic model. The initial number sick is 100,000 and it is believed that the number sick with the disease will never exceed 250,000. At the outset, sickness is assumed to grow at 5% per annum.

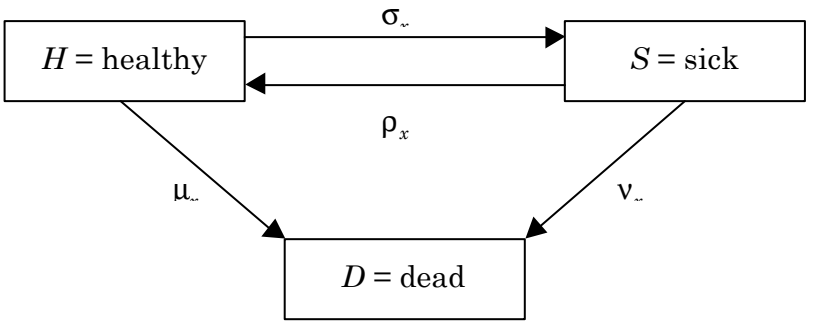
Calculate the number of people who are sick after exactly 10 years. [3]

11 A multiple decrement table is subject to two forces of decrement α and β. Under the assumption of a uniform distribution of the independent decrements over each year of age, $(aq)_x^{\alpha} = 0.2$ and $(aq)_x^{\beta} = 0.05$.

Calculate q_x^{α} and q_x^{β}. [3]

12 An insurer sells combined death and sickness policies to healthy lives aged 35. The policies, which are for a term of 30 years, pay a lump sum of £20,000 immediately on death, with an additional £10,000 if the deceased is sick at the time of death. There is also a benefit of £3,000 per annum payable continuously to sick policyholders. There is no waiting period before benefits are payable. Annual premiums of £500 are payable continuously by healthy policyholders.

The mortality and sickness of the policyholders are described by the following multiple state model, in which the forces of transition depend on age.



$p_{x,t}^{gh}$ is defined as the probability that a life aged x who is in state g($g = H$, S or D) is in state h at age $x + t$ ($t \geq 0$ and $h = H$, S or D). The force of interest is δ.

Express in integral form, using the probabilities and the various forces of transition, the expected present value of one such policy at its commencement. [4]

13 A pension scheme provides the following benefit to the spouse of a member, following the death of the member in retirement:

A pension of £10,000 per annum payable during the lifetime of the spouse, but ceasing 30 years after the death of the member if that is earlier. All payments are made on the anniversary of the member's retirement.

Calculate the expected present value of the spouse's benefit in the case of a female member retiring now on her 60th birthday, who has a husband aged exactly 64.

Basis: a(55) Ultimate mortality at 8% per annum interest [8]

14 (i) Discuss the suitability of the crude death rate, the standardised mortality rate and the standardised mortality ratio for comparing

(a) the mortality, at different times, of the population of a given country

(b) the mortality, at a certain time, of two different occupational groups in the same population [6]

(ii) The following table gives a summary of mortality for one of the occupational groups and for the country as a whole.

	Occupation A		**Whole Country**	
Age group	*Exposed to risk*	*Deaths*	*Exposed to risk*	*Deaths*
20–34	15,000	52	960,000	3,100
35–49	12,000	74	1,400,000	7,500
50–64	10,000	109	740,000	7,100
	37,000	235	3,100,000	17,700

Calculate the crude death rate, the standardised mortality rate and the standardised mortality ratio for Occupation A. [4]

[Total 10]

15 An insurer issues 15 year term assurance policies to lives aged exactly 50 who have provided satisfactory answers on a basic medical questionnaire. The sum assured of £100,000 is payable at the end of the year of death during the policy term. The policy includes an option at the end of the term which allows policyholders to convert their policy to a whole life policy for the same sum assured (payable at the end of the year of death). The premiums payable for this whole life policy are the office's standard premium rates, irrespective of the health of the policyholder effecting the option.

The insurer calculates annual premiums for all products using A1967–70 Select mortality and 4% per annum interest, with an expense allowance of 5% of all premiums.

(i) Describe:

(a) the North American method and
(b) the conventional method

for pricing mortality options. [5]

(ii) Using the conventional method calculate the extra annual premium the insurer should charge above that for a term assurance policy with no option. [5]

(iii) Without performing any further calculations, describe what other considerations would arise if the option were such that the policy could be converted on the 10th or the 15th anniversary. [3]

[Total 13]

16 A life insurance company issues a 4 year unit-linked policy with a level premium of £1,000 payable annually in advance to a life aged exactly 61. The death benefit at the end of the year of death is £4,000, or the bid value of the units if greater. The maturity value is the bid value of the units.

95% of each premium is invested in units at the offer price. The bid price is 95% of the offer price. Premiums payable in the first two years are invested in capital units which are subject to a management charge of 6% per annum. Subsequent premiums are invested in accumulation units for which the management charge is 1% per annum. Management charges are deducted at the end of each year from the bid value of units before benefits are paid.

Capital units are actuarially funded using factors of $A_{61+t:\overline{4-t}|}$ calculated using A1967–70 Ultimate with 5% per annum interest for t = 0, 1, 2 and 3.

The company uses the following assumptions to profit test this contract:

Rate of interest on unit investments:	8% per annum
Rate of interest on sterling fund:	4% per annum
Mortality:	A1967–70 Ultimate
Initial expenses:	£100 plus 20% of the first premium
Renewal expenses:	£20 on the first policy anniversary, and increasing with inflation at 5% per annum on each subsequent anniversary

(i) Using a risk discount rate of 12% per annum calculate the expected net present value of the profit on this contract. [12]

(ii) Without performing any further calculations, state with reasons whether your answer in (i) would be higher or lower for each of the following, if

(a) the risk discount rate were 10% per annum
(b) the policyholder were aged 50 exactly
(c) capital units were actuarially funded at 4% per annum [5]

[Total 17]

17 A man aged exactly 30 effected a 35 year with profit endowment assurance for a sum assured of £50,000. Level annual premiums are payable throughout the policy term, ceasing on earlier death. The sum assured, with attaching bonuses, is payable at the end of the year of death, or on maturity. Compound reversionary bonuses vest at the end of each policy year.

(i) Show that the premium (to the nearest £1) is £990 per annum using the following basis:

Mortality:	A1967–70 Ultimate
Interest:	6% per annum
Expenses:	Initial: £250 plus 60% of the annual premium
	Renewal: 2.5% of second and subsequent premiums
Bonuses:	1.923% per annum

[7]

(ii) The random variables T_x and K_x represent the exact future lifetime and the curtate future lifetime of a life aged x, respectively. Using T_x , K_x or both, express, in stochastic form, the gross future loss random variable for this policy at duration t, where t is an integer and $0 < t < 35$. Use those elements of the basis set out in part (i) as needed. Assume bonus declarations have been in line with the original bonus loadings. [3]

(iii) Immediately before the 11th premium is due, and just after the 10th bonus has brought the sum assured plus accumulated bonuses to £60,000, the policyholder wishes to convert the policy to a non-profit whole life policy, with premiums of an unchanged amount payable until death.

Using the mortality and interest elements of the premium basis set out in part (i), and allowing for renewal expenses of 2.5% of all future premiums as well as an alteration expense of £100, calculate the revised sum assured. [6]

(iv) State one other consideration, if any, that the office should take into account before completing the alteration in (iii), and explain why they should do so. [2]

[Total 18]