

EXAMINATIONS

11 April 2001 (am)

Subject 105 — Actuarial Mathematics 1

Time allowed: Three hours

INSTRUCTIONS TO THE CANDIDATE

1. *Write your surname in full, the initials of your other names and your Candidate's Number on the front of the answer booklet.*
2. *Mark allocations are shown in brackets.*
3. *Attempt all 14 questions, beginning your answer to each question on a separate sheet.*

Graph paper is not required for this paper.

AT THE END OF THE EXAMINATION

Hand in BOTH your answer booklet and this question paper.

*In addition to this paper you should have available
Actuarial Tables and an electronic calculator.*

- 1 In the context of a unit-linked contract, state a key reason for the use of actuarial funding of capital units. [2]
- 2 In the context of a pension fund, state what is meant by a transfer value. [2]
- 3 Under the Manchester Unity model of sickness, you are given the following values:

$${}_tP_x = 1 - .05t^2 \quad (0 \leq t \leq 1)$$

$$\bar{z}_{x+t} = 0.1 \quad (0 \leq t \leq 1)$$

Calculate s_x . [3]

- 4 Some time ago, a life office issued an assurance policy to a life now aged exactly 55. Premiums are payable annually in advance, and death benefits are paid at the end of the year of death. The office calculates reserves using gross premium policy values. The following information gives the reserve assumptions for the policy year just completed. Expenses are assumed to be incurred at the start of the policy year.

Reserve brought forward at the start of the policy year	£12,500
Annual premium	£1,150
Annual expenses	£75
Death benefit	£50,000
Mortality	A1967–70 ultimate
Interest	5.5% per annum

Calculate the reserve at the end of the policy year. [3]

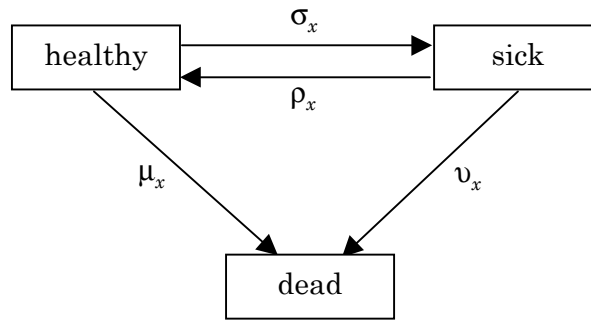
- 5 Life insurance company A calculates paid-up policy values for endowment assurance policies by applying the net premium reserve as a single premium at the time of the alteration. It holds net premium reserves based on A1967–70 ultimate mortality and 3% per annum interest.

Life insurance company B calculates its corresponding paid-up values by reducing the sum assured to (t/n) times the original sum assured, where n is the original term of the policy and t is the number of premiums which have been paid at the time of the calculation of the paid-up sum assured.

The sum assured is paid at the end of the term or the end of the year of death, if earlier. Premiums are payable annually in advance.

Identify, showing your calculations, which company pays the higher paid-up sum assured after 15 years, immediately before payment of the 16th premium, for a 25-year endowment assurance policy originally taken out by a life then aged exactly 40. [4]

- 6** A life office prices sickness insurance contracts using the following three state model in which the forces of transition depend on age:



Level premiums are payable continuously. Benefits are payable continuously during periods of sickness. There is no death benefit, and the contracts have a deferred period of three months and include a waiver of premiums during periods of benefit payment. Reserves are always positive under the normal premium basis.

State briefly, with reasons, what effect the following changes will have on the premium (certain increase, certain decrease, not certain), if the same net present value of profit is to be achieved:

- (a) an increase in the death rate from the sick state together with an increase in the rate of transition from the healthy state to the sick state
- (b) a fall in the death rate from the sick state together with a fall in the rate of transition from the sick state to the healthy state. [4]

- 7** A population is subject to two modes of decrement, α and β , between ages x and $x + 1$. In the single decrement tables

$${}_tP_x^\alpha = \left(\frac{x}{x+t} \right)^2$$

and

$${}_tP_x^\beta = \left(\frac{x}{x+t} \right)^3$$

where $0 \leq t \leq 1$.

Write down an integral expression for $(aq)_x^\alpha$. Hence or otherwise obtain an expression for this probability in terms of x only. [6]

- 8** The following data relate to a population projection being carried out using the component method, and specifically give information about the female population of the country.

$P(x, t)$ = population at 1 January 2001 + t aged x last birthday, $x, t = 0, 1, 2, \dots$

$M(x, t)$ = estimated net number of emigrants from the population during the year 2001 + t , aged x last birthday at 1 January 2001 + t

$q(x, t)$ = independent probability that a life who attains exact age x during the year 2001 + t , dies during that year

The following is a selection from the available data.

x	$P(x, 0)$	$q(x, 0)$	$q(x, 1)$	$M(x, 0)$	$M(x, 1)$
54	728,610	.0121	.0115	37,013	31,461
55	700,369	.0136	.0129	35,868	30,126
56	678,123	.0152	.0144	34,312	28,994
57	620,975	.0170	.0161	31,179	24,943

Calculate, from the information given, the projected number of females aged 57 last birthday at 1 January 2003. State any assumptions you make. [6]

- 9** Define each of the following terms and give one example of each:

- (a) class selection
- (b) selective decrement
- (c) spurious selection [6]

- 10** A life office has just sold a single premium deferred pension policy to a lady aged exactly 45. This policy guarantees to pay a cash sum of £200,000 on her 60th birthday, which must be used to buy a whole of life annuity at that time. The policy also carries an annuity option whereby the policyholder can elect to receive a pension of £15,000 per annum payable monthly in advance from the same date, until her death.

The office invests the single premium such that the value of related assets on the policyholder's 60th birthday will be normally distributed with a mean value of £250,000 and a standard deviation of £50,000. It also believes that the annual interest rate, i , which will be available on the policyholder's 60th birthday is a random variable where $i =$

.04	with probability .25
.06	with probability .50
.08	with probability .25

The distribution of the value of assets on the policyholder's 60th birthday is independent of this annual interest rate, i .

Calculate the probability that the value of assets on the policyholder's 60th birthday is less than the cost of providing the annuity benefit, assuming the policyholder is alive at age 60.

Basis for annuity rates: Mortality: $a(55)$ female ultimate
Interest: i per annum
Expenses: Nil [8]

11 An annuity of £40,000 per annum is payable annually in arrear in respect of two lives both aged 40.

- The first payment is deferred until the end of the year in which the first of the two lives dies, and
- Payments continue until 5 years after the death of the survivor.

Assume the two lives are independent with respect to mortality.

Calculate the expected present value of this annuity.

Basis: A1967–70 ultimate mortality
4% per annum interest [10]

12 Two life offices operating in the same economy have maintained the following records in respect of their male assured lives data:

deaths, subdivided by age last birthday at the preceding policy anniversary, and duration at the preceding policy anniversary

in force, each 1 January, subdivided by age next birthday at issue, and calendar year of policy issue

- (i) Describe how you would calculate select forces of mortality, defining carefully all symbols you use and stating necessary assumptions. State clearly the ages and durations to which the resultant rates would apply. [8]
- (ii) Discuss briefly the advantages and disadvantages of pooling the data of the two companies to form one mortality rate estimate for each combination of age and duration. [3]

[Total 11]

- 13** A life insurance company sells with-profit whole life policies, with the sum assured payable immediately on the death of the life assured and with premiums payable annually in advance ceasing with the policyholder's death or on reaching age 65 if earlier.

The company markets two versions of this policy, one with simple reversionary bonuses and the other with compound reversionary bonuses. In both cases the bonuses are added at the end of the policy year.

The company prices the products using the following basis:

Mortality		A1967–70 select
Interest		4% per annum
Expenses	initial	£250
	renewal	2% of second and subsequent premiums
	claim	£150 at termination of contract
Bonuses	simple	6% of basic sum assured per annum
	compound	4% of accumulated sum assured and bonuses per annum

- (i) Write down an expression for the gross future loss at the point of sale for each of these policies, assuming they are sold to a life aged x exact ($x < 65$) at outset. Write the expression in terms of functions of the random variables T_x and K_x , which represent the exact future lifetime and the curtate future lifetime of (x) , respectively. [5]
- (ii) Calculate the gross premium required for each of the two policies for a sum assured of £200,000 and a life aged 40 exact at outset, using the equivalence principle. [8]
- (iii) After 10 years, bonuses totalling £90,000 have been declared for the compound reversionary bonus contract. Calculate the net premium reserve for that policy at that time, using A1967–70 ultimate mortality and interest of 4% per annum. [4]

[Total 17]

- 14** A life office issues an endowment assurance with a term of five years to a life aged exactly 55. The sum assured is £100,000, payable at the end of the five years or at the end of the year of death if earlier. Premiums are payable annually in advance throughout the term of the policy.

The office assumes that initial expenses will be £300, and renewal expenses, which are incurred at the beginning of the second and subsequent years of the policy, will be £30 plus 2.5% of the premium. The funds invested for the policy are expected to earn 7.5% per annum, and mortality is expected to follow the A1967–70 select life table. The office holds net premium reserves, using A1967–70 ultimate mortality and interest of 4% per annum.

The office sets premiums so that the net present value of the profit on the contract is 15% of the annual premium, using a risk discount rate of 12% per annum.

- (i) Calculate the premium. [12]
- (ii) Without carrying out any further calculations, state with brief reasons what the effect on the premium would be in each of the following cases:
 - (a) The reserves are calculated using a lower rate of interest.
 - (b) The office uses a risk discount rate of 15%.
 - (c) Mortality is assumed to be A1967–70 ultimate.

[6]

[Total 18]