EXAMINATIONS

22 September 2004 (am)

Subject 105 — Actuarial Mathematics 1

Time allowed: Three hours

INSTRUCTIONS TO THE CANDIDATE

1. *Enter all the candidate and examination details as requested on the front of your answer booklet.*

2. *You must not start writing your answers in the booklet until instructed to do so by the supervisor.*

3. *Mark allocations are shown in brackets.*

4. *Attempt all 13 questions, beginning your answer to each question on a separate sheet.*

Graph paper is not required for this paper.

AT THE END OF THE EXAMINATION

Hand in BOTH your answer booklet, with any additional sheets firmly attached, and this question paper.

In addition to this paper you should have available Actuarial Tables and your own electronic calculator.

1 A life insurance company issues an annuity policy to two lives aged 65 and 62 exact in return for a single premium. Under the policy an annuity of £10,000 per annum is payable monthly in advance while at least one of the lives is alive.

Calculate the single premium.

Basis:
Mortality: PMA92C20 in respect of the life aged 65 exact
PFA92C20 in respect of the life aged 62 exact
Interest: 4% per annum
Expenses: none

[3]

2 A member of a pension scheme is aged 50 exact, having joined the scheme at age 30 exact. His current salary is £50,000 per annum. Final pensionable salary is defined as the annual average earnings over the three years immediately prior to retirement. Normal Retirement Age is a member's 65^{th} birthday. Salary increases take place six months before the member's birthday.

Using the functions and symbols defined in, and the assumptions underlying, the Example Pension Scheme Table in the Actuarial Tables, calculate the expected present value of the following:

A pension on retirement at any stage on grounds of ill health of one-sixtieth of final pensionable salary for each year of service, with fractions of a year counting proportionately. [4]

3 A life insurance company issues a policy to a life aged 50 exact. The policy provides the following sickness benefit:

£100 per week for the first two years of sickness, reducing to £50 per week thereafter during sickness. Sickness benefit ceases at age 65, or on earlier recovery or death. There is no waiting or deferred period.

Level premiums under the policy are payable weekly in advance until age 65 or until earlier death. Any premiums falling due during periods of sickness are waived.

Calculate the weekly premium.

Basis:
Mortality: ELT 15 (Males)
Sickness Table: S(ID) in the Actuarial Tables
Interest: 6% per annum
Expenses: 5% of each premium
(Expenses continue even when premiums are waived)

[5]

4 (i) Describe the use of risk classification by life insurance companies in underwriting life assurance policies. [2]

(ii) State two limitations to the use of risk classification and explain how life insurance companies deal with these limitations. [3]

[Total 5]

5 A life insurance company issued a non-profit term assurance policy to a life aged x exact at the outset, with a term of 20 years. Under the policy, the sum assured of £100,000 is payable at the end of the year of death. Premiums under the policy are level and payable monthly in advance for 20 years, or until earlier death.

The company values the policy at duration t years using a gross premium prospective policy value, ${}_tV'$.

Derive algebraically the relationship between ${}_tV'$ and ${}_{t+1}V'$. Define all the symbols that you use, where necessary. [6]

6 On 1 January 2001, a life insurance company issued a 10-year joint life non-profit term assurance policy to two lives aged 50 exact. Under the policy, the sum assured of £500,000 is payable immediately on the death of the first of the lives to die. Premiums of £1,000 per annum are payable annually in advance for 10 years, or until the first death of the lives assured.

On 31 December 2003 the policy is still in force. Calculate the gross premium prospective policy value at this date, using the following valuation assumptions:

Mortality:		PMA92C20 for the first life and PFA92C20 for the second life
Interest:		4% per annum
Expenses:	Renewal:	3% of each premium
	Claim:	£200 on payment of a claim

[6]

7 A double decrement table is to be constructed from two single decrement tables. The modes of decrement are α and β. For each of the single decrement tables you are given

$$l_{x+t}^{\alpha} = l_x^{\alpha} - t^2.d_x^{\alpha} \text{ and } l_{x+t}^{\beta} = l_x^{\beta} - t^3.d_x^{\beta} \text{ for } 0 \le t \le 1$$

where l_x^i = the number of lives in the single decrement table i at age x exact ($i = \alpha, \beta$)

d_x^i = the number of decrements over $[x, x + 1]$ in the single decrement table i ($i = \alpha, \beta$)

(i) Show that

$${}_t p_x^{\alpha}.\mu_{x+t}^{\alpha} = 2tq_x^{\alpha} \text{ for } 0 \le t \le 1$$

where

${}_t p_x^i$ = the probability that a life aged x exact survives t years

μ_{x+t}^i = the force of decrement by cause i at age $x + t$

q_x^i = the probability that a life aged x exact becomes a decrement by cause i over $[x, x + 1]$

in the single decrement table for cause i ($i = \alpha, \beta$).

[3]

(ii) Hence or otherwise show that the dependent initial rate of decrement at age x exact due to cause α is:

$$(aq)_x^{\alpha} = q_x^{\alpha}\left(1 - \frac{2}{5}q_x^{\beta}\right)$$

[3]
[Total 6]

8 A life insurance company issues 10-year non-profit term assurance policies, for a sum assured S, to lives aged x exact. It offers an option on the policies to effect, either on the fifth policy anniversary or at the expiry of the 10-year term, a whole life non-profit policy for the same sum assured, without evidence of health. Premiums under the term assurance policies are payable annually in advance for 10 years, or until earlier death, or until the fifth policy anniversary, if the option is then exercised. Premiums under the whole life policy are payable annually in advance for the whole of life. The sums assured under the term assurance and whole life policies are payable at the end of the year of death.

An additional single premium is charged at the outset under the term assurance policy for the mortality option. The company uses the North American method for pricing options.

Give formulae for calculating the additional single premium charged at outset for the mortality option. You may ignore expenses. Define all the symbols that you use, where necessary.

[8]

9 A life insurance company sells with profit whole life policies, with the sum assured and attaching bonuses payable immediately on the death of the life assured and with level premiums payable annually in advance ceasing with the policyholder's death or on reaching age 65 if earlier.

Simple reversionary bonuses vest under the policies at the end of each year.

The company prices the product using the following basis:

Mortality:		AM92 Select
Interest:		4% per annum
Expenses:	Initial:	£250
	Renewal:	2% of second and subsequent years' premiums
	Claim:	£150 at termination of contract
Bonuses:	Simple:	6% of basic sum assured per annum

(i) Write down an expression for the gross future loss at the point of sale for one of these policies, assuming it is sold to a life aged x exact ($x < 65$) at the outset. Write the expression in terms of functions of the random variables $T_{[x]}$ and $K_{[x]}$, which represent the exact future lifetime and the curtate future lifetime of (x) respectively. [3]

(ii) Calculate the gross premium required for one of these policies for a sum assured of £200,000 and issued to a life aged 40 exact at the outset, using the equivalence principle. State any assumptions you make. [6]

[Total 9]

10 The following data are available from a life insurance company, relating to the mortality experience of its term assurance policyholders.

$\theta_{x,d}$ The number of deaths over the period 1 January 2000 to 30 September 2003, aged x nearest birthday at entry and having exact duration d at the next policy anniversary following the date of death.

$P_{y,e}(n)$ The number of policyholders with policies in force at time n, aged y nearest birthday at entry and having curtate duration e at time n, where n = 1.1.2000, 30.9.2000, 30.9.2002 and 30.9.2003.

(a) Develop formulae for the calculation of the crude select forces of mortality corresponding to the $\theta_{x,d}$ deaths.

(b) Derive the age and duration to which these estimates apply.

Assume that all months are of equal length. State all other assumptions that you make.

[11]

11 A special 3-year endowment assurance policy provides that the death benefit payable at the end of year of death is £10,000 plus the endowment assurance net premium reserve for that year that would have been held had death not occurred. £10,000 is payable on survival to the end of the 3 years.

On the basis set out below, use a discounted cash flow method to calculate the level annual premium payable in advance for a life aged 57 exact. The requirement is that at the discount rate defined below the value of the annual emerging surpluses should sum to zero.

Basis:		
	Mortality:	AM92 Select for experience and reserves
	Expenses:	20% of the first annual premium 5% of subsequent premiums
	Reserves:	Value as a normal endowment assurance for a 3-year term on a net premium basis using a valuation rate of interest of 4% per annum. Ignore the effect on reserving of the extra death benefit defined above.
	Interest earnings:	7% per annum on cash flow
	Discount rate:	10% per annum

Ignore tax and any other items.

[12]

12 A pension scheme provides the following benefits in respect of a former male member of the scheme who has just left service:

(a) A pension to him for life of £10,000 per annum if he survives to age 65: the pension commences on his 65th birthday and is guaranteed payable for five years in any event.

(b) A spouse's pension of £5,000 per annum, commencing immediately on the death of the former member before his 65th birthday and payable for life to the spouse.

A spouse's pension is payable on death in deferment if the former member is married at the date of death.

Pensions are payable monthly in advance.

Pensions in payment and deferment are increased monthly in arrears at the effective rate of 2.8846% per annum.

The former member is now aged 62 exact. You are not given any information as to whether he has a spouse.

Calculate the expected present value of these benefits using the following basis:

Basis:		
	Valuation rate of interest:	7% per annum
	Mortality in deferment and in retirement:	PMA92C20 for the former member and PFA92C20 for his spouse
	Proportion of former members with a spouse at each age up to age 65:	90%
	Age difference of spouses:	Females are exactly 3 years younger than their husbands

Assume that death before retirement occurs at the mid-point of the year of age in respect of each year of age.

[12]

13 You are a member of a committee responsible for monitoring the trend in assured lives mortality rates. You have been presented with the following ratios of actual to expected mortality rates on the basis of a standard table constructed twenty years ago ("Standard Table A") and the total expected deaths over the period 2000–2003 based on this table.

Age	*Ratio of Actual to Expected Mortality Rates*		*Total Expected Deaths (000's)*	
	2000–2001	*2002–2003*	*2000–2001*	*2002–2003*
15–44	1.80	2.00	10	10
45+	0.90	0.80	20	20

You have also been given details of the exposed to risk data in the two age groups 15–44 and 45+ corresponding to Standard Table A. The exposed to risk data are described as "Standard Population A".

(i) Define, giving a formula, the term "Standardised Mortality Ratio". Define all the symbols that you use. [2]

(ii) Show how the Standardised Mortality Ratio may be expressed as a weighted average. Describe the function averaged and the weights. [3]

(iii) Calculate the Standardised Mortality Ratios for the periods 2000–2001 and 2002–2003 with reference to Standard Table A, using the data presented. [2]

(iv) The committee measured the change in mortality between the periods 2000–2001 and 2002–2003 by calculating a "Comparative Mortality Factor" (CMF) for each period. This factor was calculated as

r_1 / r_2, where

r_1 was the expected number of deaths for the period obtained by applying the observed mortality rates to Standard Population A

r_2 is the expected number of deaths in Standard Population A over a two-year period based on Standard Table A

The CMF was 0.95 for the period 2001–2001 and 0.99 for the period 2002–2003, which led the committee to conclude that mortality was deteriorating.

(a) Explain the difference between the results of your calculation of the Standardised Mortality Ratios in part (iii) and these CMF figures. (Hint: Express the CMF figures as weighted averages.)

(b) State, giving a reason, which set of figures you think provides the better results.

(c) Comment on the conclusion of the committee that mortality was deteriorating. [6]

[Total 13]

END OF PAPER