EXAMINATIONS

19 September 2000 (am)

Subject 105 — Actuarial Mathematics 1

Time allowed: Three hours

INSTRUCTIONS TO THE CANDIDATE

1. *Write your surname in full, the initials of your other names and your Candidate's Number on the front of the answer booklet.*

2. *Mark allocations are shown in brackets.*

3. *Attempt all 15 questions, beginning your answer to each question on a separate sheet.*

Graph paper is not required for this paper

AT THE END OF THE EXAMINATION

Hand in BOTH your answer booklet and this question paper.

In addition to this paper you should have available, Actuarial Tables and an electronic calculator.

1   Two lives, each aged x, are subject to the same mortality table. According to the mortality table and a certain rate of interest, $A_x = 0.5$ and $A_{xx} = 0.8$.

Calculate A_{xx}^2, using the same mortality table and interest rate.   [2]

2   The following data are available in relation to a particular country and one of its regions:

Age group	Region A Population at 30 June 2000 (000s)	Deaths in 2000	Country Population at 30 June 2000 (000s)	Deaths in 2000
0–39	645	350	13,580	8,347
40–59	450	2,295	8,100	45,360
60+	385	27,500	6,290	489,860

Calculate the standardised mortality ratio for region A by reference to the country as a whole.   [2]

3   (i)   A life insurance policy provides a benefit of £10,000 payable immediately on the death of a life (x), if (x) dies after a life (y). Express in integral form the expected present value of the benefit under this policy.   [1]

(ii)   Set out, giving a reason, the most appropriate annuity factor to value annual premiums payable under the policy.   [1]
[Total 2]

4   A healthy life aged exactly 35 has a policy providing an income benefit of £50 per week payable during sickness. The benefit is not payable beyond age 60. There is no deferred or waiting period.

Calculate the present value of this benefit.

Basis: Mortality: English Life Table No. 12-Males
       Sickness: Manchester Unity Sickness Experience 1893/97
                   Occupation Group AHJ
       Interest: 4% per annum   [3]

5   An annuity of 1 is payable annually in arrears while at least one of two lives, (x) and (y), is alive.

Derive an expression in terms of joint-life and single life functions for the variance of the present value of the annuity.   [3]

6   Describe three types of bonus that may be given to a with profits contract.   [3]

7   In the context of a life insurance contract, explain how an asset share may be built up using a recursive formula.   [3]

8   (i)   On 1 January 1990 a life insurance company issued a 20-year annual premium without profits endowment assurance policy to a life then aged exactly 40, which is still in force. The sum assured of £100,000 is payable at the end of the year of death within the term of the policy, or on survival. The company values the policy using a modified net premium method, with a Zillmer adjustment.

Calculate the reserve for the policy on 31 December 1999.

Basis:	Mortality:	A1967–70 Select
	Interest:	4% per annum
	Zillmer adjustment:	2% of the sum assured [3]

(ii)   Without carrying out any further calculations, explain how the value of the policy would differ if the company used a Zillmer adjustment of 1% of the sum assured, with the same mortality and interest assumptions.   [2]

[Total 5]

9   A life insurance company issues a special reversionary annuity contract. Under the contract an annuity of £10,000 per annum is payable monthly for life, to a female life now aged exactly 60, on the death of a male life now aged exactly 65, provided the male life dies within 10 years of the start date of the policy. Payments commence on the first monthly policy anniversary after the date of death.

Calculate the single premium required for the contract.

Basis:	Mortality:	a(55) Ultimate mortality, male or female as appropriate
	Interest:	6% per annum
	Expenses:	none [5]

10   A pension scheme provides an ill-health retirement pension of 1/60 of Final Pensionable Salary for each year of company service, with fractions of a year to count proportionately, subject to a maximum pension of 40/60 of Final Pensionable Salary. Retirement due to ill-health may take place at any age before age 65. Final Pensionable Salary is defined as the average annual salary over the three-year period preceding retirement.

Derive commutation functions to value the ill-health retirement pension for a member aged exactly 25, who has completed exactly 5 years company service to date. Define carefully all the symbols that you use.   [7]

11   Describe the component method of population projection used for British Official Projections, stating carefully any assumptions that you make and defining all the symbols that you use.   [7]

12 A life insurance company issues only single premium without profit term assurance policies.

The premium is to be calculated for a special 3-year term assurance for lives aged exactly 60 where the basic sum assured is £100,000, payable at the end of the year of death.

This special policy carries a "guaranteed insurability" option that may be selected at the outset of the 3-year policy in return for the payment of an additional single premium.

This option provides a guarantee to the policyholder that a further £100,000 of sum assured may be purchased, at a subsequent policy anniversary, on normal premium rates and without evidence of health.

The further sum assured purchased will not itself carry any further options, and will expire at the end of the 3-year term of the original policy.

A policyholder who has paid the additional single premium can subsequently decide whether or not to effect the increase in sum assured and then at which policy anniversary — the first or second, but not both.

The company uses the "North American experience" method for pricing the option.

Calculate the additional single premium payable at outset for a policyholder choosing the option.

Basis: Mortality: A1967–70 Select, except in the case of policyholders who decide to exercise their option to increase the sum assured. For these policyholders, the mortality basis assumed to apply, from the point of increase in sum assured, is 150% of A1967–70 Ultimate.

Interest: 5.5% per annum

Proportion of policyholders at the first anniversary who decide to increase their sum assured at that point: 20%

Proportion of policyholders at the second anniversary who decide to increase their sum assured at that point: 20%

Expenses: none   [7]

13 A life insurance company uses the following 3-state model, to estimate the profit in respect of a 2-year combined death benefit and sickness policy issued to a healthy policyholder aged exactly 55 at inception.
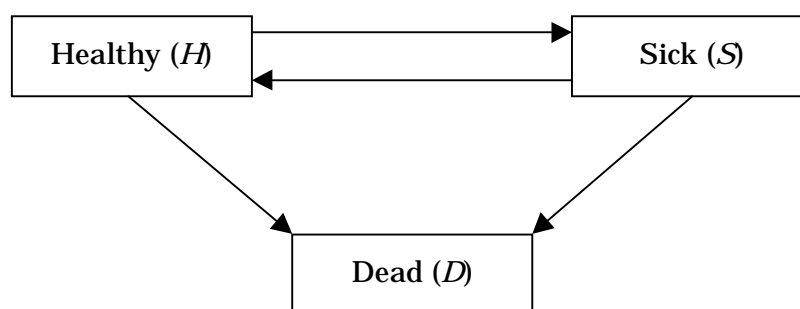
In return for a single premium of £6,000 payable at the outset the company will pay the following benefits:

£16,000 if the policyholder dies within 2 years, payable at the end of the year of death;

£8,000 at the end of each of the 2 years if the policyholder is sick at those times.

Let S_t represent the state of the policyholder at age $55 + t$, so that $S_0 = H$ and for $t = 1$ and 2, $S_t = H$, S or D.

The company uses transition probabilities defined as follows:

$$p_{55+t}^{ij} = P(S_{t+1} = j \mid S_t = i)$$

For $t = 0$ and 1 the transition probabilities are:

$$p_{55+t}^{HD} = 0.08 \qquad p_{55+t}^{SD} = 0.15 \qquad p_{55+t}^{SH} = 0.75 \qquad p_{55+t}^{HS} = 0.12$$

The transitions in the multiple state model are the only sources of randomness.

(i) One possible outcome for this policy is that the policyholder is healthy at times 0, 1 and 2. List all the possible outcomes and the associated cash flows. [3]

(ii) Calculate the probability that each outcome occurs. [5]

(iii) Assuming a rate of interest of 8% per annum, calculate the net present value at time 0 of the profit for each outcome. [2]

(iv) Calculate the mean and standard deviation of the net present value of the profit at time 0 for the policy. [5]

[Total 15]

PLEASE TURN OVER

14   On 1 September 1992, a life insurance company issued a whole life with profits policy to a life then aged exactly 45. The basic sum assured was £100,000. The sum assured and attaching bonuses are payable immediately on death. Level monthly premiums are payable in advance to age 85 or until earlier death. The company calculated the premium on the following basis:

Mortality:	A1967–70 Select
Interest:	4% per annum
Bonus loading:	0.97087% per annum compound, vesting at the beginning of each policy year
Expenses: initial:	50% of the first year's premiums, incurred at the outset
renewal:	5% of the second and each subsequent year's premiums, incurred at the beginning of the respective policy years.

(i)   Show that the monthly premium is £229, to the nearest £.   [7]

(ii)   Immediately before payment of the premium due on 1 September 2000, at the request of the policyholder, the insurance company alters the policy to a paid-up policy, with no future premiums payable. The sum assured under the policy is reduced, with no further bonuses payable.

The insurance company calculates the reduced sum assured after alteration by equating prospective gross premium policy reserves immediately before and after alteration, allowing for an expense of alteration of £100.

Bonuses have vested at the rate of 4% per annum compound at the beginning of each policy year from the date of issue of the policy. The company calculates prospective gross premium policy reserves for the purpose of the alteration using the following assumptions:

Mortality:	A1967–70 Ultimate
Interest:	4% per annum
Expenses:	none
Allowance for future bonuses:	none

Calculate the sum assured after alteration.   [6]

[Total 13]

15   A life insurance company issues a 3-year unit-linked endowment assurance contract to a male life aged exactly 62 under which level annual premiums of £4,000 are payable in advance throughout the term of the policy or until earlier death. 101% of each year's premium is invested in units at the offer price.

The premium in the first year is used to buy capital units, with subsequent years' premiums being used to buy accumulation units. There is a bid-offer spread in unit values, with the bid price being 95% of the offer price.

The annual management charges are 5.25% on capital units and 1.25% on accumulation units. Management charges are deducted at the end of each year, before death, surrender or maturity benefits are paid.

On the death of the policyholder during the term of the policy, there is a benefit payable at the end of the year of death of £10,000 or the bid value of the units allocated to the policy, if greater. On maturity, the full bid value of the units is payable.

A policyholder may surrender the policy only at the end of each year. On surrender, the bid value of the accumulation units plus a proportion of the capital units is payable. The proportion of the capital units payable on surrender is determined by the year of surrender, as follows:

Year of surrender	Proportion of capital units paid out
1	0.85
2	0.90
3	1

The life insurance company uses the following assumptions in carrying out profit tests of this contract:

Mortality:	A1967–70 Ultimate
Expenses: initial:	£300
renewal:	£60 at the start of each of the second and third policy years
Unit fund growth rate:	9% per annum
Sterling fund interest rate:	4.5% per annum
Risk discount rate:	15% per annum
Surrender rates:	15% of all policies still in force at the end of each of the first and second years

(i) The company holds unit reserves equal to the full bid value of the accumulation units and a proportion $A_{62+t:\overline{3-t}|}$ (calculated at 4%), of the full bid value of the capital units, calculated just after the payment of the premium due at time t ($t = 0$, 1 and 2). The company holds no sterling reserves.

Calculate the profit margin on the contract. [17]

(ii) Assume instead that the company holds unit reserves equal to the full bid value of both the accumulation and capital units and that the company also holds sterling reserves, at the start of each policy year, equal to 10% of the annual premium. Calculate the revised profit margin on the contract. [6]

[Total 23]