

# EXAMINATIONS

10 September 2002 (am)

## Subject 105 — Actuarial Mathematics 1

*Time allowed: Three hours*

### ***INSTRUCTIONS TO THE CANDIDATE***

1. *Enter all the candidate and examination details as requested on the front of your answer booklet.*
2. *You must not start writing your answers in the booklet until instructed to do so by the supervisor.*
3. *Mark allocations are shown in brackets.*
4. *Attempt all 14 questions, beginning your answer to each question on a separate sheet.*

***Graph paper is not required for this paper.***

### ***AT THE END OF THE EXAMINATION***

*Hand in BOTH your answer booklet, with any additional sheets firmly attached, and this question paper.*

*In addition to this paper you should have available Actuarial Tables and your own electronic calculator.*

- 1** Explain in words what is meant by the logistic model for projecting the size of a population.
- Write down a differential equation whose solution gives a formula for the size of a population based on the logistic model.
- Define carefully all symbols that you use. [3]
- 2** Describe how option pricing techniques may be used to determine the value of the guarantee under a deferred annuity policy with a guaranteed minimum annuity. [3]
- 3** Define each of the following terms and give an example of each in life assurance business:
- (a) class selection
  - (b) spurious selection
  - (c) adverse selection
- [3]
- 4** Explain what is meant by the following, in the context of with profit life insurance contracts:
- (a) earned asset share
  - (b) retrospective valuation reserve
- [4]
- 5** A life insurance company issues a whole life assurance policy to a life aged exactly 60, paying a sum assured  $S$ , together with attaching bonuses, immediately on death. Compound bonuses are added annually in advance. Premiums under the policy are payable annually in advance, ceasing at exact age 85 (the last premium is payable at age 84 exact) or on earlier death.
- Write down an expression for the net future loss random variable at outset for this policy. Define carefully all the symbols that you use. [4]
- 6** A life insurance company issues a temporary annuity policy to two independent lives, each aged exactly 60. The annuity of £10,000 per annum is payable quarterly in arrears, while at least one of the lives is alive. The annuity is payable for a maximum of 10 years.
- Calculate the single premium.
- Basis: mortality: A1967–70 Ultimate  
interest: 4% per annum  
expenses: ignore
- [4]

- 7 Two lives,  $(x)$  and  $(y)$ , are assumed to be independent with respect to mortality and are each assumed to be subject to a constant force of mortality of 0.01. Calculate the probability that  $(x)$  dies more than 10 years after  $(y)$ . [4]

- 8 Calculate

$$A_{\overline{20}|20}^2$$

using A1967–70 mortality and interest of 4% per annum. [4]

- 9 Members of a pension scheme are subject to two modes of decrement namely death ( $d$ ) and withdrawal ( $w$ ). The following assumptions are made in respect of the two decrements:

Independent rate  $q_x^d$  is A1967–70 Ultimate;

Independent rate  $q_x^w$  is 0.05 per annum at age 20 last birthday and increases by 5% at each successive age attained. (For example, the annual rate of withdrawal at age  $20 + t$  last birthday is  $(0.05) \times (1.05)^t$ );

the decrements are statistically independent;

each decrement is uniformly distributed in its single decrement table.

Calculate the probability that a new entrant aged exactly 20 will withdraw from the scheme at age 22 last birthday. [5]

- 10 (i) Define, giving a formula, the term “Standardised Mortality Ratio”. Define all the symbols that you use. [2]
- (ii) Show how the Standardised Mortality Ratio may be expressed as a weighted average, setting out clearly what function is averaged and what the weights are. [3]

[Total 5]

**11** A pension scheme provides the following deferred benefits for a member aged 55 exact, who leaves service before Normal Pension Age (NPA), which is age 65 exact.

- (a) A deferred pension of £10,000 per annum, payable monthly in advance from NPA. The pension is payable for a minimum of 60 monthly payments. The pension increases monthly in deferment and payment at the effective rate of 3.846% per annum compound.
- (b) On the death of the member after NPA, a dependant's pension of 50% of the member's pension entitlement at the date of death. The pension is payable monthly in advance beginning on the first day of the month following the date of the member's death, or the fifth anniversary of the member's NPA, if later and increases monthly in payment at the effective rate of 3.846% per annum compound.

Calculate the expected present value of the deferred benefits.

Basis: Mortality:	A1967–70 Ultimate	
Interest:	8% per annum	
Proportion with dependants:	90% of members have dependants at the date of retirement	
Age difference:	members are the same age as their dependants (assume that females are treated exactly the same as males)	
Expenses:	none	[11]

**12** A life insurance company issues a disability insurance contract to a healthy life aged exactly 30. Under the contract, a benefit of £20,000 per annum is payable weekly in the event of disability. The benefit continues to be payable during disability, until the policyholder recovers or reaches age 65. The benefit increases continuously in payment at the rate of 3% per annum compound.

There is no waiting period or deferred period. Premiums continue to be payable during periods of disability.

Disability benefit payments are valued using rates of claim inception and termination.

- (i) Describe the method of valuing disability benefit payments under this contract, setting out the data required. [6]
- (ii) Derive commutation functions for valuing the benefits payable under the contract, stating clearly any assumptions that you make and defining carefully all the symbols that you use. [7]

[Total 13]

- 13** A life insurance company issues a long-term care contract to a healthy life aged 50 exact. Under the contract, the life insurance company will pay the costs of long-term care while the policyholder satisfies the conditions for payment.

The conditions for payment are assessed each year on the policy anniversary, just before payment of the premium then due. If the policyholder satisfies the conditions, the full annual amount of the benefit payable is paid immediately.

Regular premiums are payable annually in advance under the policy until death and are waived during periods of benefit payment.

For those lives needing care at 100% of maximum, the current payment on the policy anniversary is £50,000. The company uses the following data in respect of the expected proportions of lives at each age needing care at different expected cost levels, for pricing the long-term care contract.

<i>Exact age</i>	<i>Proportion needing care at 50% of maximum</i>	<i>Proportion needing care at 100% of maximum</i>
51–70	0.01	0.01
71–85	0.04	0.06
86+	0.08	0.10

Basis: Mortality: A1967–70 Ultimate  
 Interest: 6% per annum  
 Benefit inflation: Maximum payment at 100% care level at policy anniversary  $t(t = 1, 2, \dots) = £50,000 \times (1.019231)^t$   
 Expenses: 10% of each premium

- (i) Write down an expression for the expected present value of benefits (including the waiver of premium benefit) at outset for the contract. Define carefully all the symbols that you use. [4]
- (ii) Calculate the annual premium payable under the contract. [10]
- [Total 14]

- 14** A life insurance company issues a 2-year unit-linked endowment assurance contract to a male life aged exactly 63, under which level annual premiums of £6,000 are payable in advance throughout the term of the policy, or until earlier death.

102% of each year's premium is invested in units at the offer price.

The premium in the first year is used to buy capital units, with the second year's premium being used to buy accumulation units. There is a bid-offer spread in unit values, with the bid price being 95% of the offer price.

The annual management charges are 5% of the bid value of capital units and 1% of the bid value of accumulation units. Management charges are deducted at the end of each year, before death, surrender or maturity benefits are paid.

On the death of the policyholder during the term of the policy, there is a benefit payable at the end of the year of death of £10,000, or the full bid value of the units allocated to the policy, if greater. On maturity, the full bid value of the units is payable.

The policyholder may surrender the policy only at the end of the first policy year. The surrender value is equal to 87% of the bid value of the capital units.

The life insurance company uses the following assumptions in carrying out profit tests of this contract:

Mortality:	A1967–70 Ultimate
Surrender:	10% of policies then in force, occurring at the end of the first policy year
Expenses: Initial:	£500
Renewal:	£100 at the start of the second policy year
Unit fund growth rate:	8% per annum
Non unit fund interest rate:	4% per annum
Risk discount rate:	15% per annum

- (i) Calculate the net present value on this contract, assuming that the company holds unit reserves equal to the full bid value of the accumulation units and capital units. [12]
- (ii) Assume that the company holds unit reserves equal to the full bid value of the accumulation units and a proportion,  $A_{63+t:2-t|}$  (calculated at 4% and A1967–70 Ultimate mortality), of the full bid value of the capital units ( $t = 0, 1$ ).  
Calculate the net present value on the contract. [9]
- (iii) Explain what the effect would be on the answers in parts (i) and (ii) if the mortality assumption were changed to mortality of A1967–70 Select. [2]  
[Total 23]