

EXAMINATIONS

16 September 2003 (am)

Subject 105 — Actuarial Mathematics 1

Time allowed: Three hours

INSTRUCTIONS TO THE CANDIDATE

1. *Enter all the candidate and examination details as requested on the front of your answer booklet.*
2. *You must not start writing your answers in the booklet until instructed to do so by the supervisor.*
3. *Mark allocations are shown in brackets.*
4. *Attempt all 14 questions, beginning your answer to each question on a separate sheet.*

Graph paper is not required for this paper.

AT THE END OF THE EXAMINATION

Hand in BOTH your answer booklet, with any additional sheets firmly attached, and this question paper.

In addition to this paper you should have available Actuarial Tables and your own electronic calculator.

- 1** A life insurance company issues a number of 3-year unit-linked policies to lives each aged 40 exact. The year-end non-unit fund cash flows $(NUCF)_t$, per policy in force at the start of policy year t , are as follows (in £'s):

<i>Year (t)</i>	<i>1</i>	<i>2</i>	<i>3</i>
$(NUCF)_t$	100	100	−150

Non-unit fund reserves are to be set up at each year end for each policy then in force to zeroise future negative cash flows. Calculate the adjusted value of $(NUCF)_t$ at the end of year 1, assuming that interest is earned on reserves at the rate of 5% per annum and that the mortality basis is AM92 Select. [3]

- 2** Describe four benefit options that may be available to an individual member of a pension scheme who leaves the scheme before normal pension age. [4]

- 3** A life insurance company issues a disability insurance policy to a healthy life aged exactly 45.

The benefits under the policy are as follows. There is no waiting period, but there is a deferred period of one year. A benefit of £10,000 per annum is payable continuously while the policyholder is sick, after the completion of the deferred period. The benefit is payable until the policyholder reaches age 65, dies or recovers. Premiums are waived while the policyholder is in receipt of benefit payment.

Level annual premiums are payable continuously under the policy until age 65 or the policyholder's earlier death.

Calculate the annual premium.

Basis:	Sickness:	S(ID) Tables
	Mortality:	ELT(15) Males
	Interest:	6% per annum
Expenses:	Initial:	60% of the annual rate of premium
	Regular:	£50 per annum, assumed incurred continuously in all years of the policy, including periods of sickness
	Claim:	1.5% of sickness benefit payments made to the policyholder

[4]

- 4 Describe the calculation of a surrender value for a without-profit endowment assurance policy, under which level annual premiums are payable monthly in advance and cease on earlier death or surrender and the sum assured is payable immediately on death. Give formulae, defining carefully all the symbols that you use. [5]

- 5 A life insurance company issues a term assurance policy to a life aged 55 exact for a term of 10 years. The sum assured is payable immediately on death. The sum assured is given by

$$£100,000 \times (1 + 0.05t) \quad t = 0, 1, 2, \dots, 9.$$

where t denotes the curtate duration in years since the inception of the policy.

Level premiums are payable monthly in advance for a period of 10 years or until earlier death. The life insurance company calculates the premium using the equivalence principle.

Calculate the annual premium.

Basis: Mortality: AM92 Select
Interest: 4% per annum
Expenses: None

[5]

- 6 A pension scheme provides a pension of $1/60$ of Final Pensionable Salary for each year of scheme service upon retirement for any reason. Fractional years of service count proportionately. Final Pensionable Salary is defined as the average annual salary in the three years immediately prior to retirement. Members are required to contribute continuously at the rate of 5% of salary.

You are given the following data in respect of Member A as at 1 January 2003:

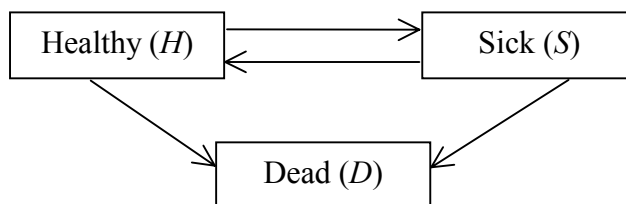
Age: 50 exact
Annual rate of salary: £50,000

Using the data in the *Actuarial Tables*, calculate, in respect of Member A:

- (i) The expected present value of future contributions payable. [3]
(ii) The expected present value of the pension benefits on retirement for any reason based on future service. [2]

[Total 5]

- 7 A life insurance company uses the following 3-state model, to calculate premiums for a 3-year sickness policy issued to healthy policyholders age 50 exact at inception.



In return for a single premium of P payable at the outset the company will pay a benefit of £10,000 at the end of each of the 3 years if the policyholder is sick at that time.

Let S_t represent the state of the policyholder at age $50+t$, so that $S_0 = H$ and for $t = 1, 2$ and 3 , $S_t = H, S$ or D .

The life insurance company uses transition probabilities defined as follows:

$$p_{50+t}^{ij} = P(S_{t+1} = j | S_t = i)$$

For $t = 0, 1$ and 2 the transition probabilities are:

$$p_{50+t}^{HD} = 0.05 \quad p_{50+t}^{SD} = 0.15 \quad p_{50+t}^{SH} = 0.80 \quad p_{50+t}^{HS} = 0.1$$

The life insurance company calculates P as the expected present value of the benefit payments, assuming interest at 6% per annum and expenses of 5% of P .

Calculate P . [5]

- 8 A life insurance company issues 10-year unit linked policies to lives aged exactly 50. Premiums paid in the first two years of the policies are applied to purchase capital units, with premiums in subsequent years being applied to purchase accumulation units.

The management charge on the capital unit fund is 5% of the bid value of the units, deducted at the end of each policy year. The management charge on the accumulation unit fund is 1% of the bid value of the units, deducted at the end of each policy year.

The life insurance company wishes to use actuarial funding assuming a rate of interest of 3% per annum. In calculating the actuarial funding factors, the life insurance company assumes that mortality is constant, with

$$q_x = 0.001 \text{ for } 50 \leq x \leq 60.$$

The life insurance company ignores surrenders.

- (i) Calculate the actuarial funding factor to be applied at the end of the third year of a policy. [4]
- (ii) The life insurance company is considering using a higher rate of interest for actuarial funding factors. It wishes to assume the same mortality basis and to ignore surrenders in calculating the revised actuarial funding factors.

Describe how you would determine the maximum rate of interest it would be prudent to use in calculating the actuarial funding factor to be applied at the end of the third year of the policy. Set out the considerations you would take into account. [5]
[Total 9]

- 9 You are a consulting actuary to a client who wishes to invest £1m now to provide an immediate income for his partner and himself in retirement. Both the client and his partner are aged 60 exact.

The client wishes to provide a payment annually in advance each year while either he or his partner is alive. He wishes the amount of the payment to be

$$£I \times (1.05)^t \quad t = 0, 1, 2, \dots$$

where I denotes the amount of the initial payment and t denotes the curtate duration in years since the inception of the policy.

The client further requests that he wishes the amount of the initial payment I to be such that the capital of £1m is at least 95% likely to be sufficient to provide the required payments and he asks you to advise what the maximum value of the initial payment I should be.

In carrying out the calculations, you assume that the only source of random variation is the future mortality of the client and his partner.

Calculate the required value of I based on the following assumptions.

Mortality: The client and his partner are independent with respect to mortality and are each subject to the mortality of PMA92C20.

Rate of future investment returns: 6% per annum

Expenses: none

[9]

- 10** A life insurance company offers an option on its 10-year level term assurance policies to effect a whole life without profits policy, for the sum assured, without evidence of health. The option may be exercised once only, either on the fifth anniversary of the policy or at the expiry of the 10-year term. If the option is exercised on the fifth policy anniversary, the term assurance policy ceases immediately.

The sums assured under the 10-year term assurance policy and under the whole life policy are both payable immediately on death. A single premium, inclusive of the option premium, is payable at the outset under the term assurance policy and level premiums under the whole life policy are payable annually in advance until death. The premiums under the whole life policy are calculated using the company's normal annual premium basis.

- (i) Describe the conventional method of pricing the mortality option, stating clearly the data and assumptions required. [4]
- (ii) A policyholder aged exactly 45 wishes to effect a 10-year without profits term assurance policy, for a sum assured of £200,000.

Calculate the total single premium payable under the term assurance policy, using the conventional method to calculate the option premium.

The following basis is used to calculate the basic term assurance premium:

Basis: Mortality: AM92 Select
Interest: 4% per annum
Expenses: none [5]
[Total 9]

- 11** On 1 January 2000, a life insurance company issued an endowment assurance policy to a life aged exactly 50 for a term of 10 years.

Under the policy, a sum assured of £100,000 is payable on survival to age 60 exact or at the end of the year of death on earlier death. Level premiums are payable annually in advance for 10 years or until earlier death.

On 1 January 2003, the policy is still in force and the life insurance company calculates on a prospective basis both the gross premium reserve and the net premium reserve for the policy at this date, using the assumptions shown below. The same assumptions were used to calculate the gross premium at inception as follows:

Mortality: AM92 Ultimate
Interest: 4% per annum
Expenses: Initial: £300 incurred at the outset
Renewal: 5% of each premium

- (i) Calculate the gross premium reserve as at 1 January 2003. [3]
- (ii) Calculate the net premium reserve, with Zillmer adjustment, as at 1 January 2003. Identify clearly the Zillmer adjustment. [2]

- (iii) Explain why the net premium reserve with Zillmer adjustment calculated in part (ii) might be used in preference to the net premium reserve with no Zillmer adjustment, calculated as at 1 January 2003, using the same assumptions. [2]

- (iv) Assume instead that the life insurance company calculated the gross premium reserve as at 1 January 2003 using a rate of interest of 3.5% per annum following a general fall in market interest rates, with all other assumptions unchanged. Assume also that the net premium reserve with a Zillmer adjustment, calculated in part (ii), is unchanged.

State, giving a reason, whether you consider it appropriate to use this unchanged net premium reserve with a Zillmer adjustment for reserving purposes. [2]
[Total 9]

12 A life insurance company issues a two-year without-profit policy to a member, aged exactly 50, of a certain club. The policy provides the following benefits:

- (a) on death as a member during two years, a sum of £10,000
- (b) on withdrawal from the club within two years, a return of 75% of premiums paid without interest
- (c) on survival as a member to the end of two years, the sum of £5,000

Death and withdrawal benefits are payable at the end of the year of death or withdrawal respectively and the survival benefit is payable on the maturity date of the policy. There are no decrements from membership of the club other than death or withdrawal.

A premium of £3,000 is payable annually in advance under the policy for 2 years or until earlier death or withdrawal.

Calculate the net present value of the profit under the policy to the life insurance company.

Basis:	Mortality:	the independent rate of mortality is that of AM92 Select
	Withdrawal:	the independent rate of withdrawal is 5% per annum
	Rate of decrements:	Mortality and withdrawal occur uniformly throughout each policy year in the respective associated single decrement tables.
	Expenses:	£150 incurred at outset
	Rate of interest:	5% per annum
	Reserves:	Ignore
	Risk discount rate:	15% per annum

[9]

- 13** A life insurance company issues a special annuity policy to a male and a female life, both aged exactly 60.

Under the policy, an annuity is payable annually in arrear for a maximum of 4 years, ceasing on the first death of the two lives. The first payment under the policy is £10,000 and subsequent payments increase by 1.9231% per annum compound.

- (i) Calculate the standard deviation of the present value of benefits under the annuity policy.

Basis: Mortality: The male and the female lives are independent with respect to mortality and are subject to the mortality of PMA92C20 and PFA92C20 respectively.

Interest: 6% per annum

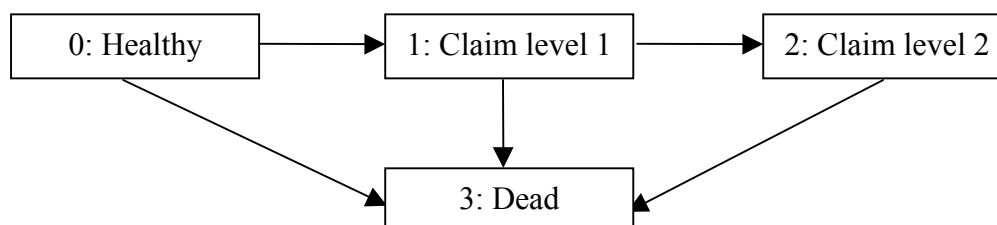
[8]

- (ii) State, with reasons, whether the standard deviation would be higher, lower or the same if the annuity were to cease on the second death of the two lives, other conditions remaining unchanged.

[2]

[Total 10]

- 14** A life insurance company uses the following multiple-state model for pricing and valuing annual premium long-term care contracts, which are sold to lives that are healthy at outset.



Under each contract, the life company will pay the costs of long-term care while the policyholder satisfies the conditions for payment. These conditions are assessed every year on the policy anniversary, just before payment of the premium then due. If the policyholder satisfies the conditions, the annual amount of the benefit payable is paid immediately. A maximum of four benefit payments may be made under the policy, after which time the policy expires. The policy also expires on earlier death.

Premiums are payable annually in advance under the policy until expiry, and are waived if a benefit is being paid at a policy anniversary.

For lives at claim level 1, benefits of 60% of the maximum level are paid, while lives at claim level 2 receive 100% of the maximum level. The current maximum level is £50,000 per annum and is expected to increase by 6% per annum compound in the future.

p_x^{ij} is the probability that a life aged x in state i will be in state j at age $x+1$ and the insurer uses the following probabilities for all values of x :

$$\begin{array}{lll} p_x^{00} = 0.87 & p_x^{01} = 0.1 & p_x^{02} = 0.0 \\ p_x^{11} = 0.6 & p_x^{12} = 0.3 & p_x^{22} = 0.6 \end{array}$$

- (i) Calculate the annual premium under the contract.

Basis: Interest: 6% per annum
Expenses: 7.5% of each premium

[9]

- (ii) A policyholder has already received two benefit payments at level 1, and is about to receive a third benefit instalment. Calculate the reserves the office should hold for this policy immediately after the benefit payment is made, if the policyholder is assessed as entitled to either:

- (a) benefit at level 1 = £42,000 per annum
(b) benefit at level 2 = £70,000 per annum

Reserve basis: Transition probabilities: as given

Interest: 5% per annum

Benefit inflation: Inflation of the maximum benefit level of 7% per annum.

[5]

[Total 14]