

EXAMINATIONS

21 April 2004 (am)

Subject 105 — Actuarial Mathematics 1

Time allowed: Three hours

INSTRUCTIONS TO THE CANDIDATE

1. *Enter all the candidate and examination details as requested on the front of your answer booklet.*
2. *You must not start writing your answers in the booklet until instructed to do so by the supervisor.*
3. *Mark allocations are shown in brackets.*
4. *Attempt all 14 questions, beginning your answer to each question on a separate sheet.*

Graph paper is not required for this paper.

AT THE END OF THE EXAMINATION

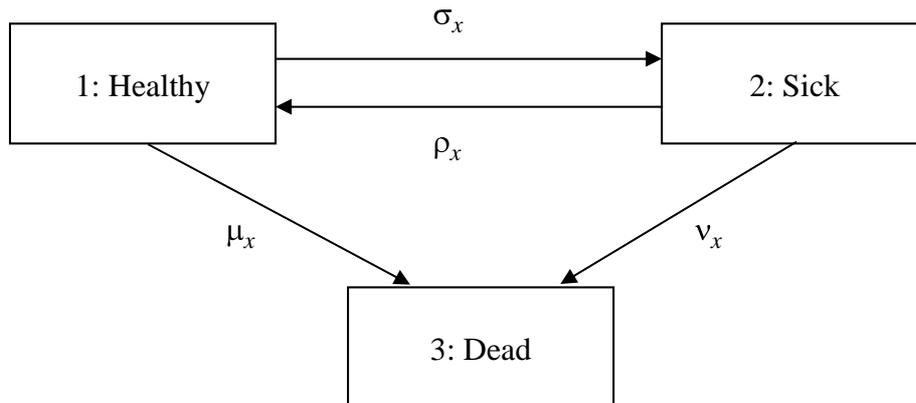
Hand in BOTH your answer booklet, with any additional sheets firmly attached, and this question paper.

In addition to this paper you should have available Actuarial Tables and your own electronic calculator.

- 1 (a) Give a formula for the Area Comparability Factor, defining all terms you use.
- (b) Explain the role of this Factor in calculating standardised mortality rates, indicating any advantages it has over other available methods.

[3]

- 2 A life insurance company uses the following model for pricing and valuing sickness and other contracts.



$p_{x,t}^{ab}$ is the probability that a life now aged x and in state a will be in state b when aged $x + t$

$\overline{p_{x,t}^{aa}}$ is the probability that a life now aged x and in state a will remain continuously in state a until age $x + t$

Explain what is represented by each of the following integrals:

(a)
$$\int_0^{65-x} 12,000e^{-\delta t} p_{x,t}^{12} dt$$

(b)
$$\int_0^{30} \int_0^{30-t} 10,000e^{-\delta(t+r)} \overline{p_{35,t}^{11}} \sigma_{x+t} \overline{p_{35+t,t}^{22}} dr dt$$

[3]

- 3 Explain the main differences in approach between the conventional and North American methods for pricing mortality options in life assurance contracts.

[4]

- 4** In a certain country, the population has reached a stationary size, and there is no immigration or emigration. Women between the ages of 20–39 inclusive are regarded as being of childbearing age and mortality in this age range is zero. In the past every woman had a new baby on each of her 21st, 26th, 31st and 36th birthdays. From 1 January 2004, a change in birth patterns means that every woman is expected to have a new baby on each of her 23rd, 28th, 33rd and 38th birthdays. During the transition from one pattern to the other, it is expected that every woman will still have 4 babies, with a gap of at least 5 years between consecutive births.

Calculate the Total Fertility Rate for:

- (a) the calendar year 2003
- (b) the calendar year 2004
- (c) women born in 1962
- (d) women born in 1982

[4]

- 5** (a) Explain what is meant by

$${}_nq_{[x][y]}^2$$

- (b) Evaluate ${}_{25}q_{[40][40]}^2$ assuming both lives are subject to AM92 mortality.

[4]

- 6** In a select mortality investigation, $\theta_{x,r}$ corresponds to the number of deaths aged x nearest birthday at death with duration r at the policy anniversary preceding death. $\theta_{x,r}$ divided by the appropriate central exposed to risk gives an estimate of $\mu_{[y]+t}$.

Derive the values of y and t to which this estimate applies, stating any assumptions used.

[5]

- 7** The future lifetimes of two individuals aged x and y are independent, and subject to constant forces of mortality of 0.02 and 0.03 respectively.

- (i) Calculate the probability that their first death occurs after 3 years and before 8 years from now. [3]

- (ii) Calculate the probability that their second death occurs after 3 years and before 8 years from now. [3]

[Total 6]

- 8** A company issues a block of 5-year single premium investment policies to lives each aged 60 exact at commencement of a policy. It guarantees simple annual reversionary bonuses of 8% per annum of the single premium, with the possibility of a terminal bonus at maturity. The death benefit is 5 times the single premium.

All premiums received under this policy are invested in an asset class where 5-year returns have a normal distribution with a mean of 50% and standard deviation of 25%. The company intends to declare terminal bonuses on maturity such that the proceeds of the policy are the greater of the guaranteed amount and 90% of the underlying asset value.

Calculate the probability that:

- (a) the insurer makes a loss on a particular policy.
- (b) a policyholder receives a terminal bonus.

Basis: Mortality: ELT15 (Females)
Expenses: Ignore

[6]

- 9** A retirement benefits scheme provides a lump sum retirement benefit equal to $\frac{3}{80}$ ths of the salary rate at retirement for each completed year of service in the scheme. Fractions of a year do not get credit. Retirement can occur at any age after attaining age 60 but not later than a member's 65th birthday.

Calculate the total service liability for the lump sum benefit in respect of a member aged 63 exact on the valuation date who has exactly 30 years of past service and is earning £40,000 per annum.

Basis: Interest: 6% per annum
Salary increases: Nil
Independent mortality rates: PMA92Base
Independent retirement rates: Age 63 last birthday 10%
Age 64 last birthday 6%

State any other assumptions you rely on. [6]

- 10** List the main categories of costs incurred by life insurance companies, giving an example of each, and indicating the manner in which they are usually allowed for in calculating premiums. [8]

11 (i) In the context of Manchester Unity Sickness Tables, state the meaning of:

- (a) the force of sickness \bar{z}_x
- (b) the annual rate of sickness s_x

[2]

(ii) An insurance sickness policy provides combined endowment and sickness benefits. The sickness benefit is £200 per week for the first 26 weeks of sickness, £150 per week for the next 26 weeks and £100 per week thereafter while sickness lasts. All sickness payments cease on a policyholder's 65th birthday. There are no waiting or deferred periods.

The endowment part of the policy pays £10,000 immediately on the death of the policyholder or on survival to age 65.

Premiums are waived during periods of sickness.

Calculate the level premium per annum payable continuously by a new policyholder aged 35. Premiums are payable to age 65 but cease on earlier death.

Basis: Sickness: S(MU)
Mortality: ELT 15 (Males)
Interest: 4% per annum
Expenses: Nil

[7]

[Total 9]

- 12** On 1 January 1993, a life insurance company issued a number of 25-year without profit endowment assurance policies to lives then aged 35 exact. Level premiums were payable annually in advance throughout the term of the policy, ceasing on the earlier death of the life assured. The sum assured was payable on survival to the end of the term, or at the end of the year of death, if earlier.

Premiums and reserves were calculated on the following basis:

Mortality: AM92 Select
Interest: 6% per annum
Expenses: 60% of the first premium
 5% of each premium excluding the first

Calculate, as at 31 December 2003, the profit or loss for the calendar year 2003 in respect of these policies, given the following information:

- The total sums assured in force on 1 January 2003 were £50,000,000.
- The total death claims occurring during 2003 and paid on 31 December 2003 were £200,000.
- During 2003, policies with sums assured of £2,500,000 were surrendered. Surrender values, paid on 31 December 2003, were calculated as the retrospective reserve using the above basis, but with interest at 4% per annum.
- During 2003, policies with sums assured of £1,000,000 (before alteration) were made paid up with effect from 31 December 2003. Paid-up sums assured were calculated on a proportionate basis, namely the original sum assured * $t/25$ where t is the number of premiums actually paid.
- The company incurred expenses of £100,000 on 1 January 2003.
- The company earned a total return of 7% on its assets during 2003.

Ignore tax, and assume that reserves for paid-up policies ignore future expenses.

[10]

13 A life insurance company issues a policy to male lives aged 45 exact, providing the following benefits:

- A decreasing term assurance with a death benefit, which is payable immediately on death, of £200,000 in the first year, £190,000 in the second year thereafter reducing by £10,000 each year until the benefit is £10,000 in the 20th year, with cover ceasing at age 65.
- An annuity of £25,000 per annum, increasing by £2,000 each year, where the first payment is made on the policyholder's 65th birthday, and continues annually for life thereafter.

The policy is paid for by level quarterly premiums payable in advance for 20 years, ceasing on earlier death.

Calculate the premium, using the equivalence principle.

Basis:

Mortality: AM92 Select

Interest: 4% per annum

Expenses: Initial: £200 plus 35% of the premiums paid in the first year
Renewal: 5% of all subsequent premiums and £40 per annum, increasing by 4% per annum compound, on each policy anniversary
Claim: Death: £250*(1.04)^t where *t* is the exact duration of the policy at death, measured in years with fractions counting
Annuity: 2% of annuity payments

[14]

- 14 (i) Under a 4-year unit-linked policy issued to a male aged 60 exact, the following non-unit cash flows, NUCF_t , ($t = 1, 2, 3, 4$) are obtained at the end of year t per policy in force at the start of year t .

Year t	1	2	3	4
NUCF_t	-400	210	-190	450

Mortality follows AM92 Select.

- (a) Show that the annual internal rate of return lies between 5% and 6%.
- (b) If the rate of interest earned on non-unit reserves is 7.5% per annum, calculate the reserves required at times $t = 1, 2$ and 3 in order to zeroise future negative cash flows.
- (c) Without doing any further calculations, explain what effect the zeroisation of future negative cash flows in part (b) above will have on the internal rate of return relative to that in (a) above. [7]
- (ii) A unit-linked endowment policy with an annual premium of £5,000 and a term of 2 years is to be issued to a male life aged 60 exact. 97.5% of each premium will be allocated to units at the offer price. The units will be subject to a bid-offer spread of 4%.

At the end of each year a management charge of 1% of the bid value of the units will be deducted from the unit fund.

If the policyholder dies during the term of the contract the office will pay out the greater of £40,000 and the bid value of the units at the end of the year of death (after the deduction of the management charge).

The company carries out all profit test calculations on the contract using the following basis:

Mortality: AM92 Select

Rate of growth on assets in the unit fund: 9% per annum

Rate of interest on non-unit fund cash flows: 6% per annum

Expenses: £250 at time 0; £50 at time 1

Risk discount rate: 12% per annum

- (a) If the policyholder dies in the second year of the contract, calculate the amounts of the non-unit fund cash flows in both of the years of the contract.
- (b) Hence calculate the net present value of the profit assuming that the policyholder dies during the second year of the contract.
- (c) The policyholder could also die in the first year, or survive to the end of the term of the contract. Calculate the net present value of the profit for each of these two events.
- (d) Hence or otherwise, calculate the expected net present value of the profit under this contract.

[11]

[Total 18]

END OF PAPER