

# EXAMINATIONS

9 April 2003 (am)

## Subject 105 — Actuarial Mathematics 1

*Time allowed: Three hours*

### ***INSTRUCTIONS TO THE CANDIDATE***

1. *Enter all the candidate and examination details as requested on the front of your answer booklet.*
2. *You must not start writing your answers in the booklet until instructed to do so by the supervisor.*
3. *Mark allocations are shown in brackets.*
4. *Attempt all 14 questions, beginning your answer to each question on a separate sheet.*

***Graph paper is not required for this paper.***

### ***AT THE END OF THE EXAMINATION***

*Hand in BOTH your answer booklet, with any additional sheets firmly attached, and this question paper.*

*In addition to this paper you should have available Actuarial Tables and your own electronic calculator.*

**1** In the context of Manchester Unity sickness functions, state the relationship between  $s_{26}^{26|26}$  and  $z_{26}^{26|26}$ . [2]

**2** (i) In the context of with profit policies, describe the super compound method of adding bonuses. [2]

(ii) Give a reason why a life insurance company might use the super compound method of adding bonuses as opposed to the compound method. [1]

[Total 3]

**3** Under a policy issued by a life insurance company, the benefit payable on death, at the end of the year of death, is a return of premiums paid without interest. A level premium of £1,500 is payable annually in advance, throughout the term of the policy.

For a policy in force at the start of the tenth year, you are given the following information:

Reserve at the start of the year, ${}_9V$ :	£11,300
Reserve at the end of the year per survivor, ${}_{10}V$ :	£13,200
Probability of death during the year:	0.04
Rate of interest earned:	5% p.a.

Calculate the profit expected to emerge at the end of the tenth year per policy in force at the start of that year. Ignore expenses and all decrements other than death. [3]

**4** Compare the use of the component method and the logistic mathematical modelling method for projecting the size of the population in a certain country. [4]

**5** A researcher into international mortality experience is interested in comparing death rates in different countries by cause of death (cancer, heart disease, accidents etc.). An initial study compares crude death rates by cause of death for each country, and indicates a wide range of experience among the different countries.

(i) Comment on the approach of using crude rates for this comparison, indicating any advantages and disadvantages of this method. [2]

(ii) Suggest an alternative approach which addresses any shortcomings identified in (i). You should assume any data required are available. [2]

[Total 4]

- 6** A life insurance company sells disability insurance contracts, under which the benefit is £100 per week, payable while a life insured is alive, disabled and aged not more than 65. It calculates premiums and reserves using the inception rate / disability annuity methodology.

Calculate the expected present value of future benefit payments for the following two policyholders:

- (a) A 45 year old who is healthy at the valuation date, and whose policy has a deferred period of one year. The value should take into account all possible future periods of sickness claims.
- (b) A 55 year old who has been receiving benefit payments for the last two years. The value should allow only for the remaining payments under the current sickness claim.

Basis: Interest: 6% per annum  
Morbidity & Mortality: S(ID) in the Actuarial Tables [4]

- 7** A life insurance company issues a reversionary annuity policy to a male and a female, both of whom are aged exactly 60.

The annuity commences immediately on the death of the first of the lives to die and is payable subsequently while the second life is alive, for a maximum period of 20 years after the commencement date of the policy.

The annual amount of the annuity is £10,000 and is payable continuously.

Calculate the single premium for the policy.

Basis: Mortality: PMA92C20 for the male life and PFA92C20 for the female life.  
The lives are independent with respect to mortality.

Interest: 4% per annum

Expenses: Initial: £300 incurred at the outset

Annuity: 2% per annum of the annuity payment, incurred continuously while the annuity is being paid

[7]

- 8** An insurer sells a special 3-year single premium non-profit term assurance policy for an initial sum assured of £250,000. This policy includes an option such that the policyholder can double the sum assured at the end of the second year of the policy by paying an additional premium at that time, based on normal mortality rates, without evidence of health.

All death benefits are payable immediately on death.

The company uses the North American method for pricing this policy.

Calculate the premiums payable by a female life aged exactly 55 at the outset who does take up the option.

Basis:	Normal mortality:	ELT15 (Females)	
	Mortality of those who exercise the option:	300% of ELT15 (Females)	
	Interest:	5% per annum	
	Expenses:	None	
	Proportion of policyholders who exercise the option:	40% of those alive on the second policy anniversary	[8]

- 9**
- (i) Explain why a life insurance company might need to set up non-unit reserves in relation to a unit-linked assurance contract. [3]
- (ii) A ten-year contract has the following profit signature before non-unit reserves are set up:
- (-1, 0, +1, -2, +1, +1, 0, -1, 0, +1)
- If positive non-unit reserves are set up to zeroise negative cash flows, write down the revised profit signature. You should ignore interest. [2]
- (iii) State the advantages of cash flow techniques for product pricing compared with traditional commutation functions. [3]
- [Total 8]

- 10** A member of a pension scheme is aged exactly 40, having joined the scheme at age exactly 22. He earned £30,000 in the immediately preceding 12 months. Final pensionable salary is defined as the annual average earnings over the three years immediately prior to retirement. Normal Retirement Age is a member's 65th birthday.

Using the functions and symbols defined in, and assumptions underlying, the Example Pension Scheme Table in the Actuarial Tables, calculate the expected present value of each of the following:

- (i) A pension on ill-health retirement of two-thirds of final pensionable salary. [3]
- (ii) A pension on retirement at any stage on grounds other than ill-health of one-eightieth of final pensionable salary for each year of service (fractions of a year counting proportionately), subject to a maximum of 40 years. [3]
- (iii) A lump sum on retirement at any age for any reason of £50,000. [3]
- [Total 9]

- 11** In a select mortality investigation, tabulations of in force populations are available for a certain class of business, in the following 2 ways:

On each of 1 January 2000, 2001 and 2002,  $P_{x,t}$  is available where  $x$  and  $t$  are defined as:

<i>Method</i>	<i>x</i>	<i>t</i>
A	Age last birthday	Curtate duration
B	Age next birthday at issue plus calendar year of census minus calendar year of issue	Duration at policy anniversary during year of census

Two different tabulations of deaths in each of the years 2000–2002 are also available,  $\theta_{y,r}$  where  $y$  and  $r$  are defined as:

<i>Method</i>	<i>y</i>	<i>r</i>
1	Age last birthday at policy anniversary prior to death	Duration at policy anniversary following death
2	Age last birthday at death	Curtate duration at death

These data are to be used to estimate select forces of mortality. For each tabulation of deaths:

- (i) Determine the ages and durations to which these estimates apply, stating all assumptions you make. [6]
- (ii) Indicate which of the tabulations of census data gives the best match to each of the tabulations of deaths and write down an appropriate approximation to the required exposed to risk. State all assumptions you make. [4]
- [Total 10]

**12** You are the actuary of a life insurance company which issued 5,000 with-profit endowment assurance policies to lives then aged exactly 40 on 1 January 2002. Each policy had an original sum assured of £100,000 and a term of 20 years, with annual premiums of £4,300 payable in advance throughout the term, ceasing on earlier death or discontinuance.

You are given the following information, most but not all of which is needed to calculate asset shares:

- The office holds net premium prospective reserves for in force policies based on AM92 Ultimate mortality and 4% per annum interest.
- On death, policies receive the original sum assured plus previously declared reversionary bonuses and any applicable terminal bonuses. The claim payment is made at the end of the calendar year of death.
- On discontinuance within the first two years, policies receive a surrender value equal to 25% of premiums paid. The surrender value is payable at the end of the calendar year of discontinuance.
- On 31 December 2002, the office declared a reversionary bonus of 2% of the original sum assured for all policies fully in force on that date (i.e. not including any policies terminating during 2002 for reason of death or surrender).
- On 31 December 2002, the office also declared a terminal bonus for death claims which arose in the previous 12 months whereby the total death benefit payable is 125% of the original sum assured plus 125% of any attaching reversionary bonuses.
- Expenses incurred were £15.0 million on 1 January 2002.
- During 2002, 4 policyholders died and 200 discontinued.
- The office earned interest of 6.5% on its assets during 2002.

The company uses actual death claims when calculating asset shares and ignores all other factors affecting profit or expenses not given above.

- (i) Calculate the asset share per in force policy on 31 December 2002. [7]
- (ii) State with reasons which information given is not required for your calculation in (i). [3]
- [Total 10]

- 13 (i) Describe the benefit whose present value is shown below.  $T_x$  and  $T_y$  are the complete future lifetimes of two lives aged  $x$  and  $y$  respectively:

$$\begin{cases} g(T) = \text{£}100,000v^{T_y} & \text{if } T_x < T_y \\ g(T) = 0 & \text{otherwise} \end{cases} \quad [2]$$

- (ii) The policy in (i) was originally paid for by a single premium at outset. The policyholders, who are both still alive, now request that the benefit be modified immediately to be paid on the earlier death of either life.

Calculate the level premium payable annually in advance from now until the first death of either life if the policy is amended in the manner requested.

Basis: Mortality: (x) subject to force of mortality of 0.02  
(y) subject to force of mortality of 0.03  
(x), (y) independent with respect to mortality

Interest: force of interest of 0.04

Renewal expenses: 2.5% of all premiums payable from the alteration date

Alteration expenses: £100 [8]

- (iii) State, with reasons, any actions the life insurance company should undertake before proceeding with the alteration described in (ii). [2]  
[Total 12]

- 14** A life insurance company sells 4-year decreasing term assurance policies, with level premiums payable annually in advance for the term of the policy, but ceasing on earlier death. The initial sum assured is £200,000 decreasing by £50,000 at each policy anniversary and the death benefit is payable at the end of the year of death.

The company allows for the following when calculating premiums:

Initial expenses: £300 plus 25% of the annual premium

Renewal expenses: £30 per annum plus 2.5% of annual premium, incurred at the time of payment of the second and subsequent premiums

Mortality: AM92 Select

Interest: 4% per annum (for all rates needed)

For a male aged exactly 60 at outset:

- (i) Write down the gross future loss random variable at the outset of the policy. [3]
- (ii) Calculate the office premium using commutation functions, setting the expected value of the gross future loss random variable to zero. [4]
- (iii) Calculate the office premium using a discounted cash flow projection, assuming no withdrawals, ignoring reserves and using the same profit criterion as in (ii). [6]
- (iv) Without further calculation explain the effect of:
  - (a) allowing for the setting up of reserves in the calculation in part (iii)
  - (b) having set up the reserves in (iv)(a), increasing the discount rate to 10% per annum

[3]

[Total 16]