

EXAMINATIONS

13 September 2001 (am)

Subject 105 — Actuarial Mathematics 1

Time allowed: Three hours

INSTRUCTIONS TO THE CANDIDATE

1. *Write your surname in full, the initials of your other names and your Candidate's Number on the front of the answer booklet.*
2. *Mark allocations are shown in brackets.*
3. *Attempt all 14 questions, beginning your answer to each question on a separate sheet.*

Graph paper is not required for this paper.

AT THE END OF THE EXAMINATION

Hand in BOTH your answer booklet and this question paper.

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| <p><i>In addition to this paper you should have available Actuarial Tables and an electronic calculator.</i></p> |
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- 1** Under the Manchester Unity model of sickness, you are given the following values:

$$s_x = 5$$

$$\int_0^1 {}_t p_x dt = 0.9$$

Calculate the value of z_x . [2]

- 2** Give a formula for $P_{21}(2003)$ in terms of $P_{20}(2002)$, based on the component method of population projection. $P_x(n)$ denotes the population aged x last birthday at mid-year n .

State all the assumptions that you make and define carefully all the symbols that you use. [3]

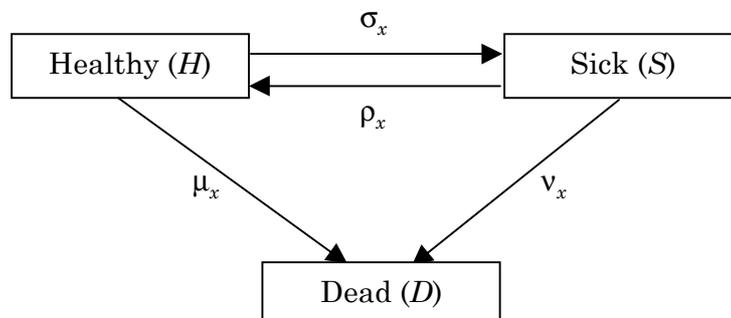
- 3** A life insurance company issues a policy under which sickness benefit of £100 per week is payable during all periods of sickness. There is a waiting period of 1 year under the policy.

You have been asked to calculate the premium for a life aged exactly 30, who is in good health, using the Manchester Unity model of sickness.

Describe how you would allow for the waiting period in your calculation, giving a reason for your choice of method. [3]

- 4 An employer recruits lives aged exactly 20, all of whom are healthy when recruited. On entry, the lives join a scheme that pays a lump sum of £50,000 immediately on death, with an additional £25,000 if the deceased was sick at the time of death.

The mortality and sickness of the scheme members are described by the following multiple-state model, in which the forces of transition depend on age only.



All surviving members retire at age 65 and leave the scheme regardless of their state of health.

$p_{x,t}^{ab}$ is defined as the probability that a life who is in state a at age x ($a = H, S, D$) is in state b at age $x + t$ ($t \geq 0$ and $b = H, S, D$).

Write down an integral expression for the expected present value, at force of interest δ , of the death benefit in respect of a single new recruit. [3]

- 5 A pension scheme provides a pension of 1/60 of career average salary in respect of each full year of service, on age retirement between the ages of 60 and 65. A proportionate amount is provided in respect of an incomplete year of service.

At the valuation date of the scheme, a new member aged exactly 40 has an annual rate of salary of £40,000.

Calculate the expected present value of the future service pension on age retirement in respect of this member, using the Pension Fund Tables in the Formulae and Tables for Actuarial Examinations. [3]

- 6** A life insurance company issues a special annuity contract to a male life aged exactly 70 and a female life aged exactly 60.

Under the contract, an annuity of £10,000 per annum is payable monthly to the female life, provided that she survives at least 10 years longer than the male life. The annuity commences on the monthly policy anniversary next following the tenth anniversary of the death of the male life and is payable for the balance of the female's lifetime.

Calculate the single premium required for the contract.

Basis: Mortality: a(55) Ultimate, males or females as appropriate
Interest: 8% per annum
Expenses: none [4]

- 7** The staff of a company are subject to two modes of decrement, death and withdrawal from employment.

Decrements due to death take place uniformly over the year of age in the associated single-decrement table: 50% of the decrements due to withdrawal occur uniformly over the year of age and the balance occurs at the end of the year of age, in the associated single-decrement table.

You are given that the independent rate of mortality is 0.001 per year of age and the independent rate of withdrawal is 0.1 per year of age.

Calculate the probability that a new employee aged exactly 20 will die as an employee at age 21 last birthday. [4]

- 8** The following data are available from a life insurance company relating to the mortality experience of its temporary assurance policyholders.

$\theta_{x,d}$ The number of deaths over the period 1 January 1998 to 30 June 2001, aged x nearest birthday at entry and having duration d at the policy anniversary next following the date of death.

$P_{y,e}(n)$ The number of policyholders with policies in force at time n , aged y nearest birthday at entry and having curtate duration e at time n , where $n = 1.1.1998, 30.6.1998, 30.6.2000$ and $30.6.2001$.

Develop formulae for the calculation of the crude central select rates of mortality corresponding to the $\theta_{x,d}$ deaths and derive the age and duration to which these rates apply. State all the assumptions that you make. [6]

- 9** (i) State the conditions necessary for gross premium retrospective and prospective reserves to be equal. [3]
- (ii) Demonstrate the equality of gross premium retrospective and prospective reserves for a whole life policy, given the conditions necessary for equality. [4]
- [Total 7]

10 A life insurance company issues a special term assurance policy to two lives aged exactly 50 at the issue date, in return for the payment of a single premium. The following benefits are payable under the contract:

- (i) In the event of either of the lives dying within 10 years, a sum assured of £100,000 is payable immediately on this death.
- (ii) In the event of the second death within 10 years, a further sum assured of £200,000 is payable immediately on the second death.

Calculate the single premium.

Basis: Mortality: A1967–70 Ultimate
Interest: 4% per annum
Expenses: None

[8]

- 11** A life insurance company sells term assurance policies with terms of either 10 or 20 years.

As an actuary in the life office, you have been asked to carry out the first review of the mortality experience of these policies. The following table shows the statistical summary of the mortality investigation. In all cases, the central rates of mortality are expressed as rates per 1,000 lives.

| <i>Age</i> | <i>All policies</i> | | <i>10-year policies</i> | | <i>20-year policies</i> | |
|------------|------------------------|-------------------------------|-------------------------|-------------------------------|-------------------------|-------------------------------|
| | <i>Number in force</i> | <i>Central mortality rate</i> | <i>Number in force</i> | <i>Central mortality rate</i> | <i>Number in force</i> | <i>Central mortality rate</i> |
| –24 | 6,991 | 1.08 | 6,013 | 0.86 | 978 | 2.12 |
| 25–44 | 6,462 | 2.05 | 5,438 | 1.74 | 1,024 | 3.68 |
| 45–64 | 5,815 | 13.26 | 4,942 | 11.55 | 873 | 22.94 |
| 65– | 3,051 | 75.70 | 2,570 | 71.53 | 481 | 97.70 |
| Total | 22,319 | | 18,963 | | 3,356 | |

- (i) Calculate the directly standardised mortality rate and the standardised mortality ratio separately in respect of the 10-year and 20-year policies. In each case, use the “all policies” population as the standard population. [6]
- (ii) You have been asked to recommend which of these two summary mortality measures should be monitored on a regular basis.

Give your recommendation, explaining the reasons for your choice. [3]

[Total 9]

12 A life insurance company offers an option on its 10-year without profit term assurance policies to effect a whole life without profits policy, at the expiry of the 10-year term, for the then existing sum assured, without evidence of health. Premiums under the whole life policy are payable annually in advance for the whole of life, or until earlier death.

(i) Describe the conventional method of pricing the mortality option, stating clearly the data and assumptions required. Formulae are not required. [3]

(ii) A policyholder aged exactly 30 wishes to effect a 10-year without profits term assurance policy, for a sum assured of £100,000.

Calculate the additional single premium, payable at the outset, for the option, using the conventional method.

The following basis is used to calculate the basic premiums for the term assurance policies.

Basis: Mortality: A1967–70 Select
Interest: 6% per annum
Expenses: none [4]

(iii) Describe how you would calculate the option single premium for the policy described in part (ii) above using the North American method, stating clearly what additional data you would require and what assumptions you would make. [4]

(iv) State, with reasons, whether it would be preferable to use the conventional method or the North American method for pricing the mortality option under the policy described in part (ii) above. [3]

[Total 14]

- 13** (i) On 1 September 1996, a life aged exactly 50 purchased a deferred annuity policy, under which yearly benefit payments are to be made. The first payment, being £10,000, is to be made at age 60 exact if he is then alive. The payments will continue yearly during his lifetime, increasing by 1.923% per annum compound.

Premiums under the policy are payable annually in advance for 10 years or until earlier death.

If death occurs before age 60, the total premiums paid under the policy, accumulated to the end of the year of death at a rate of interest of 1.923% per annum compound, are payable at the end of the year of death. Calculate the annual premium.

| | | |
|--------------------|----------------|--|
| Basis: Mortality: | before age 60: | A1967–70 Ultimate |
| | after age 60: | a(55) Males Ultimate |
| Interest: | | 6% per annum |
| Expenses: initial: | | 10% of the initial premium, incurred at the outset |
| | renewal: | 5% of each of the second and subsequent premiums, payable at the time of premium payment |
| | claim: | £100, incurred at the time of payment of the death benefit [9] |

- (ii) On 1 September 2001, immediately before payment of the premium then due, the policyholder requests that the policy be altered so that there is no benefit payable on death and the rate of increase of the annuity in payment is to be altered. The premium under the policy is to remain unaltered as is the amount of the initial annuity payment.

The life insurance company calculates the revised terms of the policy by equating gross premium prospective reserves immediately before and after the alteration, calculated on the original pricing basis, allowing for an expense of alteration of £100.

Calculate the revised rate of increase in payment of the annuity. [7]
[Total 16]

- 14 A life insurance company issues a 3-year unit-linked endowment assurance contract to a male life aged exactly 60 under which level annual premiums of £5,000 are payable in advance throughout the term of the policy or until earlier death. 102% of each year's premium is invested in units at the offer price.

The premium in the first year is used to buy capital units, with subsequent years' premiums being used to buy accumulation units. There is a bid-offer spread in unit values, with the bid price being 95% of the offer price.

The annual management charges are 5% on capital units and 1% on accumulation units. Management charges are deducted at the end of each year, before death, surrender or maturity benefits are paid.

On the death of the policyholder during the term of the policy, there is a benefit payable at the end of the year of death of £12,000 or the bid value of the units allocated to the policy, if greater. On maturity, the full bid value of the units is payable.

The policy may be surrendered only at the end of the first or the second policy year. On surrender, the life insurance company pays the full bid value of the accumulation units and 80% of the nominal bid value of the capital units, calculated at the time of surrender.

The company holds unit reserves equal to the full bid value of the accumulation units and a proportion, $A_{60+t:3-t}$ (calculated at 4% interest and A1967-70 Ultimate mortality), of the full bid value of the capital units, calculated just after the payment of the premium due at time t ($t = 0, 1$ and 2). The company holds no sterling reserves.

The life insurance company uses the following assumptions in carrying out profit tests of this contract:

Mortality: A1967–70 Ultimate

Expenses: initial: £400
renewal: £80 at the start of each of the second and third policy years

Unit fund growth rate: 8% per annum

Sterling fund interest rate: 5% per annum

Risk discount rate: 15% per annum

Surrender rates: 20% of all policies still in force at the end of each of the first and second years

Calculate the profit margin on the contract.

[18]