EXAMINATIONS

11 April 2001 (pm)

Subject 109 — Financial Economics

Time allowed: Three hours

INSTRUCTIONS TO THE CANDIDATE

1. *Write your surname in full, the initials of your other names and your Candidate's Number on the front of the answer booklet.*

2. *Mark allocations are shown in brackets.*

3. *Attempt all 8 questions, beginning your answer to each question on a separate sheet.*

Graph paper is not required for this paper.

AT THE END OF THE EXAMINATION

Hand in BOTH your answer booklet and this question paper.

In addition to this paper you should have available Actuarial Tables and an electronic calculator.

1 (i) State expressions for relative and absolute risk aversion with reference to a utility function, U. [2]

(ii) Show, for the utility functions below, whether they display increasing, decreasing or constant, relative and absolute risk aversion.

(a) $U(w) = a + bw + cw^2$, where $b > 0$ and $c < 0$ and $w \leq -\dfrac{b}{2c}$

(b) $U(w) = \ln w$ [7]

(iii) An individual who prefers more to less has a quadratic utility function and initial wealth of 100. She faces a random loss that is normally distributed with mean 5 and standard deviation 10. She is indifferent between facing this loss and paying 5.5 to fully protect herself from the loss.

(a) Find the form of her utility function.

(b) The individual is considering entering a lottery in which the first prize is 1000. Explain whether the utility function from (iii)(a) can be used to calculate how much should be paid for the ticket.

(c) Comment on the appropriateness of the quadratic utility function. [7]

[Total 16]

2 (i) Define delta, gamma, kappa, rho and theta and explain briefly what they measure about an option. Define any notation used. [8]

(ii) Consider a call option on an underlying which is a non-dividend paying stock.

- Expiration date 30 days
- Underlying Price 1000p
- Strike Price 1000p
- Risk free rate 5%
- Volatility 20%

The outputs of a computer model of the valuation of the option are:

Theoretical Value	25.04p
Delta	0.53981
Gamma	0.00689p^{-1}
Theta	$-0.045030\text{pday}^{-1}$
Kappa	$1.15379\text{p}\%^{-1}$
Rho	$0.42949\text{p}\%^{-1}$

After one day the underlying price moves to 1010p, volatility goes to 22% and the risk free rate moves to 4.5%.

(a) Estimate the price of the option.

(b) Theoretically, the new price is 32.31p. Explain why your estimate differs from the theoretical price. [4]

[Total 12]

3 (i) Distinguish between systematic risk and specific risk in the single index model. [3]

(ii) Show that the volatility of a portfolio of n equally weighted assets increases proportionately with $\sqrt{n}$. The assets are uncorrelated and have equal variance. [2]

(iii) Find the limit of the volatility of an equally-weighted portfolio as n becomes large, assuming that the pairwise correlations between the stocks are all equal. [4]

(iv) Describe the effect on portfolio risk of an investment manager shifting from a broad spectrum of UK stocks to a themed portfolio of internet stocks. [2]

[Total 11]

4 (i) Outline the central rôle that the inflation model plays within the Wilkie model. [3]

(ii) The Wilkie model proposes an AR(1) process for the continuously compounded rate of inflation, $I(t)$, that can be written as:

$$I(t) = a + bI(t-1) + \varepsilon(t)$$

where $\varepsilon(t) \sim N(0, \sigma^2)$ and a and b are constants, with $-1 < b < 1$.

(a) Derive an expression for the long-term average inflation rate in terms of a and b.

(b) Explain an economic justification for using an AR(1) process to model inflation.

(c) Explain why a model of the form above would not be suitable for share prices. [8]

[Total 11]

5 An investor buys, for a premium of 187.06, a call option on a non-dividend paying stock whose current price is 5,000. The strike price of the call is 5,250 and the time to expiry is 6 months. The risk free rate of return over this period is 5% p.a.

The Black-Scholes price for a call option on a non-dividend paying share is

$$c = S_t \Phi(d_1) - Ke^{-r(T-t)} \Phi(d_2)$$

where $$d_1 = \frac{\log S_t / K + \left(r + \tfrac{1}{2}\sigma^2\right)(T-t)}{\sigma\sqrt{T-t}}$$

$$d_2 = d_1 - \sigma\sqrt{T-t}$$

and S_t = current price, σ = volatility, $T - t$ = time to expiry, K = strike price, r = risk free rate

(i) Calculate the price of a put option with the same time to maturity and strike price as the call. [2]

(ii) The investor buys a put option with strike price 4,750 with the same time to maturity. Calculate the price of the put option if the implied volatility were the same as that in (i).

[You need to estimate the implied volatility to within 1% p.a. of the correct value.]

Draw a diagram of the investor's payoff profile if the investor purchases both the put and the call options. [10]

(iii) Explain why the market price for the put might differ from that calculated in (ii). [2]

[Total 14]

6 The price of a one year zero coupon bond is currently £0.94. On maturity it will pay £1. An equity also has a current price of £0.94. In one year the price of the equity will depend on the state of the economy. If the economy is in a good state the equity will have a value of £1.15 and if the economy is in a bad state the equity will have a value of £0.90.

It is estimated that the economy will be in a good state with probability 0.6 and in a bad state with probability 0.4.

A call option is sold on the equity which will expire in 1 year and has a strike price of £0.95.

(i) Show that the option price can be derived using the binomial model. [5]

(ii) A pricing kernel K_t , is a stochastic process such that the total return index for any asset P_t obeys the relationship.

$$P_t K_t = E_t[P_s K_s] \text{ for all } s > t$$

The value of the pricing kernel at time t depends on the state of the economy at that time.

(a) Using the binomial model for equities and bonds, derive the values of the pricing kernel at the end of one year in the good and bad states of the economy.

(b) Show that the option price calculated in (i) can also be derived using the pricing kernels from (ii)(a). [7]

[Total 12]

7 Assets A and B have the following distribution of returns in various states:

State	*Asset A*	*Asset B*	*Probability*
1	10%	–2%	0.2
2	8%	15%	0.2
3	25%	0%	0.3
4	–14%	6%	0.3
			1.0

(i) Calculate the correlation between the returns on asset A and asset B. [3]

(ii) Calculate the proportion of assets that should be invested in asset A to obtain the minimum risk portfolio. [4]

(iii) An investor decides to hold 25% of his wealth in asset A and 75% in asset B. He is concerned that the correlation between the assets may not remain constant over time.

Calculate the lowest value of the correlation between assets A and B for which the investor still gets diversification benefits from holding 25% in asset A. Assume that the variances are unchanged. [4]

[Total 11]

8 (i) Consider a fund where the surplus at time t, $S(t)$, is measured as the excess of the value of a portfolio of assets at time t, $A(t)$, over the value of a portfolio of liabilities at time t, $L(t)$.

(a) Write down a differential equation for $S(t)$ in terms of $A(t)$, $L(t)$, the deterministic rate of return earned on the assets, r_p, and the deterministic rate of increase in the liabilities, r_L.

(b) State the circumstances under which the surplus diminishes.

[3]

(ii) Consider a charitable fund that is invested in a portfolio of assets and disburses continuously a constant cash flow at a rate of L per annum.

(a) Write down and solve a differential equation for $F(t)$, the fund size at time t, in terms of the deterministic annual rate of return earned on the assets, r_p, and L.

(b) State the circumstances under which the fund is constant through time.

(c) Describe the alternatives to ensure that the fund grows.

[7]

(iii) Outline how asset liability modelling may be done in practice to analyse the problems in (i) and (ii) in the cases where r_p is stochastic. [3]

[Total 13]