EXAMINATIONS

16 September 2003 (pm)

Subject 109 — Financial Economics

Time allowed: Three hours

INSTRUCTIONS TO THE CANDIDATE

1. *Enter all the candidate and examination details as requested on the front of your answer booklet.*

2. *You must not start writing your answers in the booklet until instructed to do so by the supervisor.*

3. *Mark allocations are shown in brackets.*

4. *Attempt all 10 questions, beginning your answer to each question on a separate sheet.*

Graph paper is not required for this paper.

AT THE END OF THE EXAMINATION

Hand in BOTH your answer booklet, with any additional sheets firmly attached, and this question paper.

In addition to this paper you should have available Actuarial Tables and your own electronic calculator.

1 (i) State the expected utility theorem and the characteristic of non-satiation. [3]

(ii) A consultant, who is risk-averse and non-satiated, has been offered a one year contract with profit-sharing arrangements. His basic fee will be £80,000. In addition, he will be paid an extra £20,000 if the company profits exceed a certain target. The probability of the company making enough profits is 0.65.

(a) His utility function is of the form $U(w) = w - 0.4w^2$ (w is expressed as a proportion of £100,000). Derive the remuneration range over which $U(w)$ gives an appropriate representation of his individual preferences . [3]

(b) Calculate the expected total remuneration and the expected utility offered by the job. [2]

(c) If he is to be offered the alternative of a fixed fee, calculate the minimum he should accept. [4]

[Total 12]

2 (i) Define "efficient portfolio" and explain "efficient frontier" in the context of Mean-Variance Portfolio Theory [2]

(ii) Explain how an efficient frontier changes its shape with the introduction of risk-free lending and borrowing. [4]

[Total 6]

3 (i) Explain the terms delta, gamma and theta of an option. [3]

(ii) Describe, including a numerical example, the concept of delta hedging of options. [4]

[Total 7]

4 A stock is currently priced at 600p. The price of a six month European call option with an exercise price of 620p is 42p. Calculate the price of a six month European put option with the same exercise price if the risk free interest rate (continuously compounded) is 7% p.a. and no dividends are payable during the life of the option. State the assumptions you make in this calculation. [5]

5 The price per share of a quoted security is £1.10. A European put option on the share with a strike price of £1.20 is priced at 14p.

(i) Calculate the intrinsic value of the option. [1]

(ii) Calculate the time value of the option. [1]

(iii) State the factors that might cause the time value to increase with no change in intrinsic value. [3]

(iv) If the share price falls to £0.50 at the expiry date, calculate the profit/loss for the holder and writer of the options. [2]

(v) Calculate the maximum possible loss for the writer of the options. [2]
[Total 9]

6 A modeller has proposed the following model for earnings and the price-earnings ratio for the stockmarket

$$E(t) = E(t-1)\ e^{\mu_E + \varepsilon_E(t)}$$
$$PE(t) = \mu_{PE} + \alpha(PE(t-1) - \mu_{PE}) + \varepsilon_{PE}(t)$$

where

$E(t)$ = earnings index at time t
$PE(t)$ = price/earnings ratio at time t

α, μ_E, μ_{PE} are constant parameters (Assume $0 < \alpha < 1$.)

$\varepsilon_E(t)$ and $\varepsilon_{PE}(t)$ are independent normal random variables with mean 0 and variance σ_E^2 and σ_{PE}^2, respectively.

(i) Briefly describe the economic plausibility of the model and describe its statistical properties. [4]

(ii) Denote the price index for the stock market at time t by $P(t)$.

(a) Derive an expression for the expected value of $P(t)$ conditional on the values of $PE(t-1)$ and $E(t-1)$.

(b) Evaluate the conditional expectation in (a), assuming $\alpha = 0.5$, $\mu_E = 0.02$, $\mu_{PE} = 20$, $\sigma_E = 0.05$, $E(t-1) = 1$ in the cases where

(1) $PE(t-1) = 30$, and
(2) $PE(t-1) = 10$.

(c) Comment on the efficiency of the market implied by the model, referring to your calculations in (ii)(b). [8]
[Total 12]

7 (i) Describe informational efficiency in the context of the Efficient Markets Hypothesis. [4]

(ii) Briefly outline five examples of effects that have been claimed to exist in stock markets that might be considered examples of informational inefficiency. [5]

(iii) List the reasons, with brief explanations, why it is difficult to assess empirically whether or not the market is efficient. [4]
[Total 13]

8 Explain why the Black Scholes formula does not take account of the (real world) expected growth in share price. [4]

9 (i) Derive expressions for diversifiable and systematic risk in the context of a single index model for stock returns. Define all notation used and state all assumptions made. You should consider first an individual stock, then a portfolio, explaining why diversifiable risk is so-called. [7]

(ii) Let the "relative return" of a stock be defined as the difference between its return and that on the market index.

(a) Derive a decomposition of the variance of the relative return ("the relative risk") that enables systematic and diversifiable relative risks to be identified for one stock.

(b) Derive a similar decomposition for the relative risk of a portfolio of stocks. [4]

(iii) Assume that all stocks in a market have positive betas and that a hedge fund manager wishes to construct a portfolio from these stocks that is "market neutral" (i.e. has zero systematic risk).

(a) State the conditions concerning the stock allocations and betas that are necessary for this to be achieved and state what this implies about the portfolio weights.

(b) Derive an expression for the relative risk (as defined in (ii)) of the hedge fund. [3]
[Total 14]

10 Suppose that a stock price is modelled by a two-period recombining binomial model with the following parameters

r = 5% p.a. = risk-free rate, continuously compounded
σ = 40% p.a. = volatility of stock price process
S_0 = 100 = price at time 0
$u = \exp(\sigma/365^{1/2})$ = rate of return per period of an up-step

(Assume 365 days in the year for all calculations and that each period represents one day.)

Let s_1, s_2, s_3 denote the three possible share prices at the end of day 2. s_1 denotes the share price resulting from two up-steps, s_2, the result of an up then down (or down then up) step and s_3 the result of two down-steps.

Let the state price deflator be as follows:

State	*Price*	*State price deflator*
1	s_1	0.81977
2	s_2	1.00981
3	s_3	1.24390

(i) Calculate the state prices for the three states. [4]

(ii) Evaluate the expected rate of return on the share over the two days (expressed as a daily rate). [4]

(iii) A special type of call option with a 2 day term has the following payoff function

$\max(s - 75, 0)$ where $s < 100$
$2 \times (s - 75)$ where $100 \le s < 102$
$3 \times (s - 75)$ otherwise

By using the state price deflators given above, determine the value of this special type of call option at time zero. [2]

(iv) By determining the hedging portfolio at each node at $t = 0$ and $t = 1$, show that the value in (iii) is identical to the value of the hedging portfolio at time zero. [8]

[Total 18]