EXAMINATIONS

21 April 2004 (pm)

Subject 109 — Financial Economics

Time allowed: Three hours

INSTRUCTIONS TO THE CANDIDATE

1. *Enter all the candidate and examination details as requested on the front of your answer booklet.*

2. *You must not start writing your answers in the booklet until instructed to do so by the supervisor.*

3. *Mark allocations are shown in brackets.*

4. *Attempt all 11 questions, beginning your answer to each question on a separate sheet.*

Graph paper is not required for this paper.

AT THE END OF THE EXAMINATION

Hand in BOTH your answer booklet, with any additional sheets firmly attached, and this question paper.

In addition to this paper you should have available Actuarial Tables and your own electronic calculator.

1 State, defining all symbols used, the third order stochastic dominance theorem and describe the conditions for it to apply. [6]

2 Outline the three forms of the Efficient Market Hypothesis and explain their implications for investment techniques. [6]

3 (i) Define each of the following:

(a) variance of return
(b) downside semi variance of return
(c) expected shortfall

[3]

(ii) Describe the form of the utility function for an investor who is able to compare investment opportunities using expected return and the measure of risk in (i)(b) above. [1]

[Total 4]

4 Let: $R(t)$ = gross real yield on an irredeemable index linked gilt at the end of year t

$K(t)$ = force of dividend growth during year t

$Y(t)$ = equity dividend yield at end of year t

$D(t)$ = cumulative dividend index at end of year t, given by $\ln D(t) = \ln D(t-1) + K(t)$

$I(t)$ = force of inflation growth during the year

$Q(t)$ = cumulative inflation index at end of year t, given by $\ln Q(t) = \ln Q(t-1) + I(t)$

(i) Assuming dividends are paid at year end and the yield at year end is an ex-dividend yield, write down an expression for the total return on an equity from t to $t + 1$ and an index-linked bond from t to $t + 1$. [2]

(ii) Show that the following expression gives the equity risk premium

$$\log\left(\frac{Y(t)}{R(t)}\right)+\mathbf{E}\left(\log\left(\frac{1}{Y(t+1)}+1\right)-\log\left(\frac{1}{R(t+1)}+1\right)+K(t+1)-I(t+1)\,\middle|\,U(t)\right)$$

where the equity risk premium is defined as the expectation of the log relative return on equities and irredeemable index linked gilts, conditional on state vector $U(t)$. [2]

(iii) Explain, by referring to the expression in (ii), under what conditions the model above would be consistent with an efficient market. [2]

[Total 6]

5 A building society issues a three year bond which entitles the holder to the return on the FTSE 100 Share Index up to a maximum level of 50% growth over the three year period. The bond has a guaranteed minimum level of return so that investors will receive at least x% of their initial investment back. Investors cannot redeem their bonds prior to three years.

(i) Determine the value of x at which the building society makes neither profit nor loss.

[Hint: you may ignore dividends, profit loading and expenses, and you may use linear interpolation where necessary.]

You may assume the following parameters:

volatility of FTSE 100 index = 30% p.a.;
continuously compounded risk free rate of return = 4% p.a.

[7]

(ii) Explain why the Building Society does not need to make an assumption about the growth of the FTSE 100 index in determining x. [3]

[Total 10]

6 An investment market consists of the following four risky assets:

Asset i	*Expected rate of return* E_i	*Market capitalisation*
1	5%	250,000
2	7%	250,000
3	9%	750,000
4	10%	50,000

In addition, investors can invest in a risk free asset with a return of 3% p.a.

You may assume that the returns on the assets are uncorrelated and that the CAPM holds and in particular that investors have homogeneous expectations.

(i) Determine the composition of the portfolio that a rational investor will select if they desire an expected return of 6% p.a. [3]

(ii) By referring to the mathematical form of the CAPM, state four reasons why the actual returns on the portfolio in (i) will vary over successive years. [4]

[Total 7]

7 (i) Define the First Vasicek model for bond prices, defining any terms used. [2]

(ii) List four useful features of the model. [2]

(iii) List four deficiencies of the model. [2]

(iv) Derive an expression for the limiting short spot yield by using (i) in the formula for the spot yield and letting the term to maturity tend to zero. [Hint: you may use Taylor's expansion where necessary.] [4]

[Total 10]

8 Three assets have the following characteristics:

Asset i	*Expected rate of return (E_i)*	*Volatility (σ_i)*
1	6%	5%
2	7%	15%
3	8%	20%

The correlation between any two assets is 0.5.

(i) State the Lagrangian function that can be minimised to find the minimum variance portfolio associated with a given expected return. Define any notation used. [2]

(ii) By deriving the partial derivatives of the function in (i) state the five equations that could be solved to determine the minimum variance portfolio associated with an expected return of 7%. [5]

[You do not have to find the minimum variance portfolio.]

(iii) Determine the composition of the corner portfolio where asset 1 is not present. [6]

[Total 13]

9 (i) State and briefly describe the four axioms of utility theory. [4]

(ii) Define what is meant by non-satiation and risk seeking in relation to a utility function U. [2]

(iii) Let an investor's utility function be described as $U(w) = a + bw + cw^2$.

(a) Determine the condition on the parameters for the function a, b and c for this utility function to represent a risk averse investor.

(b) Determine the risk averse investor's range of wealth for which the function satisfies the condition of non-satiation.

[2]

(iv) An investor has the following utility function

$$U(w) = 2\,(w^{½} - 1)$$

Determine the absolute risk aversion of the investor and comment on the implications for the amount and type of assets this investor might choose to hold. [3]

[Total 11]

10 A special European-style call option provides for the following payoff function at time $t = T$

$$\text{Payoff} = \begin{cases} 0 & \text{where} \quad S_T < 100 \\ S_T - 100 & \quad 100 \le S_T < 200 \\ 1.5S_T - 200 & \quad 200 \le S_T \end{cases}$$

where S_t = the underlying stock price at time t.

You may assume that: the continuously compounded risk free interest rate, r, is 5% p.a.; volatility σ = 50% p.a.; no dividends are payable on the stock and that the Black-Scholes formula holds for pricing vanilla European call options.

(i) Explain how the special call option above can be constructed as a portfolio of vanilla European call options. [2]

(ii) Using the portfolios in (i) or otherwise calculate the value of the special call option, with 6 months to expiry and a current share price of 120. [4]

(iii) Evaluate the delta of the special option at $S_0 = 150$ and $S_0 = 250$. [3]

(iv) By considering the changes in delta from $S_0 = 150$ to $S_0 = 250$ for the special call option and for a simple call option with exercise price 100, comment on the relative value of gamma for these two options and the implication for the corresponding hedging strategies. [3]

[Total 12]

11 Suppose that a stock price is modelled by a two period recombining binomial model, where each period is six months, with the following parameters:

risk free interest rate r = 3% p.a.;
volatility σ = 10% p.a.;
initial share price S_0 = 100.

The up step is given by $u = \exp\left(\frac{0.1}{\sqrt{2}}\right)$.

Consider a European call option, expiring in one year's time with exercise price 90.

(i) Construct a two period recombining tree, showing the share price at each node, and derive the risk neutral probabilities assuming that no dividends are payable. [4]

(ii) Using the binomial tree in (i), determine the value of the option. [3]

Assume now that a dividend of 20 is payable immediately before the expiry of the option.

(iii) (a) Write down the share prices at the final three nodes of the binomial tree in (ii) assuming the same total return as before.

(b) Write down the payoff at each of these three nodes of the option . [2]

(iv) Using the risk neutral probabilities from (i) determine the value of the payoff under the option at the end of the first time period, assuming the share price has jumped up in the first time period . [1]

(v) Suppose now that the option is American, not European.

(a) Determine the payoff at the node described in (iv) (i.e. at the end of the first time period, assuming the share price has moved up in the first time period).

(b) Using (a), determine whether the holder of the American option should exercise early at the node described in (iv).

(c) State with reasons whether the value at time zero of the American option is either:

(1) strictly less than
(2) greater than or equal to
(3) strictly greater than

the value of the European option. [3]

(vi) Describe briefly Black's approximation to take account of early exercise. [2]
[Total 15]

END OF PAPER