EXAMINATIONS

9 April 2002 (pm)

Subject 109 — Financial Economics

Time allowed: Three hours

INSTRUCTIONS TO THE CANDIDATE

1. *Enter all the candidate and examination details as requested on the front of your answer booklet.*
2. *You must not start writing your answers in the booklet until instructed to do so by the supervisor.*
3. *Mark allocations are shown in brackets.*
4. *Attempt all 8 questions, beginning your answer to each question on a separate sheet.*

Graph paper is required for this paper.

AT THE END OF THE EXAMINATION

Hand in BOTH your answer booklet, with any additional sheets firmly attached, and this question paper.

In addition to this paper you should have available Actuarial Tables and your own electronic calculator.

1 (i) State expressions for the following measures of investment risk:

(a) variance of return
(b) downside semi-variance
(c) expected shortfall conditional upon a shortfall occurring [3]

(ii) Returns, R, for an investment are distributed $R \sim \exp(\lambda)$.

Find expressions for:

(a) the variance

(b) the downside semi-variance

(c) the expected shortfall below a return of k conditional on a shortfall occurring

and evaluate for $\lambda = ½$, for values of $k = 2$. [11]
[Total 14]

2 (i) Explain what is meant by an "excessively volatile" market. [2]

(ii) Describe how you would test if a market is "excessively volatile". [7]

(iii) Explain the practical and conceptual difficulties in using a test of an excessively volatile market to establish whether or not a market is efficient. [4]
[Total 13]

3 Total returns for the market are assumed to be lognormally distributed with parameters μ, σ^2.

(i) Calculate the parameters μ, σ^2 given an expected rate of return for the market of 8% per annum and a standard deviation of 16% per annum. [3]

(ii) (a) Show that the probability of a rate of return exceeding 50% in a year is 0.161%. [3]

(b) In any given year there are 200 trading days. Calculate the probability of the rate of return on the market exceeding 0.25% in any day. [3]

(c) Find the rate of return required on the market in a day such that this event has a probability of 0.161%. [2]

(iii) Comment on the results in (ii). [3]
[Total 14]

4 Explain in broad terms how a model that assumes a flat yield curve will give rise to arbitrage opportunities and hence why a flat yield curve would exist for only short periods of time. [5]

5 (i) Explain what is meant by:

(a) risk-neutral models
(b) equilibrium models [6]

(ii) Outline the advantages and disadvantages of the two different approaches for modelling asset prices and returns. [3]
[Total 9]

6 Suppose that the price, S_t, of a share that pays no dividends is described in continuous time by

$$S_t = S_0 \exp[(\mu - ½\sigma^2)\, t + \sigma Z_t]$$

where μ, σ are constant parameters

S_0 is the initial price at time 0

and Z_t is a standard brownian motion.

This process is to be approximated by a discrete time re-combining binomial model such that

$$S_a = S_0 u^{N_a} d^{n-N_a}\ ; \text{ where,}$$

$$N_a = \sum_{k=1}^{n} I_k$$

and I_k = 1 if the price increases from day $k - 1$ to day k
= 0 if the price decreases from day $k - 1$ to day k

u = return factor (S_k/S_{k-1}) for a price increase
d = return factor (S_k/S_{k-1}) for a price decrease.

Let Q be an equivalent martingale measure under which the probability of a price increase over any day is q.

Let r be the continuously compounded risk free rate per annum.

(i) Write down, and explain the rationale behind, two different expressions relating the discrete time parameters u, d and q and the continuous time parameters r and σ, by considering:

(a) the martingale property of the stock price
(b) the volatility of S_t [5]

(ii) Assume $r = 6\%$ p.a.
$\sigma = 15\%$ p.a.
$S_0 = 100$
$u = \exp(\sigma.200^{-0.5})$

and that there are 200 trading days in a year.

Using the binomial approximation:

(a) Find the price of a call option that expires in 2 days with a strike price of 101.

(b) Calculate the percentage error that the binomial model produces compared with the continuous time model of Black-Scholes. [7]

[Total 12]

7 Let S_t be the price of a non-dividend paying stock. Suppose that S_t follows the process.

$$dS_t = S_t(\mu dt + \sigma dZ_t)$$

where Z_t is a standard brownian motion.

Let $f(t, S)$ be the price of a derivative security on the stock S_t.

(i) Write down an expression for $df(t, S)$ involving S_t, dZ_t, dt and the partial derivatives of $f(t, S)$. [2]

(ii) Show how a portfolio containing stock and a short position in the derivative can be constructed to be risk free. [4]

(iii) Define the "Greeks":

Δ (delta);
Γ (gamma);
θ (theta);
K (kappa). [4]

(iv) By considering the portfolio in (ii) derive an expression for the value of the derivative f in terms of the Greeks. [3]

[Total 13]

8 Three assets have the following characteristics:

Asset i	*Expected return (μ_i)*	*Volatility (σ_i)*
1	6%	7%
2	8%	15%
3	9%	20%

The correlation between each pair of assets is 0.2.

(i) In (μ, σ) space sketch the following frontiers of minimum variance portfolios in a single diagram:

(a) The minimum variance portfolios obtained by positive holdings in each pair of assets.

(b) The minimum variance portfolios obtained by unconstrained combinations of holdings in all three assets.

(c) The minimum variance portfolios obtained by non-negative holdings in all three assets. [4]

(ii) (a) You are given that the following two portfolios are on the unconstrained efficient frontier:

Asset	*Portfolio 1*	*Portfolio 2*
1	.986	−.220
2	.043	.661
3	−.029	.559

Calculate the composition of the two corner portfolios between the portfolio containing 100% asset 1 and 100% asset 3 that lie on the non-negative holdings constrained minimum variance frontier.

Calculate the expected return on these two corner portfolios. [8]

(b) Determine the non-negative holdings minimum variance portfolio given that the expected return on this portfolio is less than 6.146%.

[8]

[Total 20]