# REPORT OF THE BOARD OF EXAMINERS

September 2003

## Subject 109 — Financial Economics

EXAMINERS' REPORT

**Introduction**

The attached subject report has been written by the Principal Examiner with the aim of helping candidates. The questions and comments are based around Core Reading as the interpretation of the syllabus to which the examiners are working. They have however given credit for any alternative approach or interpretation which they consider to be reasonable.

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Chairman of the Board of Examiners

11 November 2003

# EXAMINATIONS

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EXAMINERS' REPORT

**1** (i) The expected utility theorem states that a function, $U(w)$, can be constructed to represent an investor's preferences for wealth, $W$, at some future date, and that the investor will act so as to maximize their expected utility

The characteristic of non-satiation means that investors always prefer more wealth to less wealth and is represented by $U'(w) > 0$.

(ii) (a) If the consultant prefers more to less wealth, then $U'(w) > 0$. Since $U(w) = w - 0.4w^2$, we have $U'(w) = 1 - 0.8w > 0$ or $w < 1.25$.

He is also risk-averse for all $w > 0$ which follows from $U''(w) = -0.8 < 0$. So no extra constraints on the range of $w$ follow, and the salary range for which the utility function is appropriate will be up to 1.25.

(b) His expected salary will be $0.65(1.0) + 0.35 \times 0.8 = 0.93$ or £93,000.

His expected utility
$$\begin{aligned} &= 0.65(1 - 0.4 \times 1^2) + 0.35(0.8 - 0.4 \times 0.8^2) \\ &= 0.65 \times 0.6 + 0.35 \times 0.544 \\ &= 0.39 + 0.1904 \\ &= 0.5804 \end{aligned}$$

(c) The minimum level of fixed salary will be $x$ (in £'00,000s), the certainty equivalent, where

$$x - 0.4x^2 = 0.58 \Rightarrow 0.4x^2 - x + 0.58 = 0 \Rightarrow x = 1.585 \text{ or } 0.915$$

1.585 is outside the appropriate range of $w$, so we can ignore this value. Hence, the minimum will be 0.915.

**2** (i) A portfolio is efficient if the investor cannot find a better one with both higher expected return and lower variance of the return. An efficient portfolio is one that has the lowest variance for the given expected return (except where a lower variance can be attained with a higher expected return than that given) or the highest expected return for the given variance.

The efficient frontier is the representation in expected return-variance space or expected return-standard deviation space of the set of efficient portfolios. Or, more abstractly, it is the set of efficient portfolios.

(ii) The existence of a risk-free asset has the effect of making the frontier curve a straight line that is tangent to the original frontier for risky assets (in mean-standard-deviation space) and passes through the point (0, *r*) where *r* is the risk-free rate of return.

At the point of tangency, the portfolio is a diversified one without risk-free assets. To the left of the point of tangency, the portfolios will have a mix of diversified assets and risk-free assets. To the right of the point of tangency, the portfolios will consist of more than 100% diversified assets, as the investor would have borrowed at risk-free rate and invested in diversified assets.

**3** (i) Delta: the rate of change in option price with respect to change in the price of underlying asset.

Gamma: the rate of change of delta with respect to change in the price of underlying asset.

Theta: the rate of change in the value of the option with respect to time to expiration.

(ii) Delta hedging involves establishing:

- a risk-less portfolio
- consisting of a position in a derivative on a stock and a position in the stock

Assume that the delta ($\Delta$) of a call option is 0.4. This means for a small change in the stock price, the option price changes by about 40% of the change. Imagine the investor sold 20 option contract, that is, options to buy 2,000 shares. The investor's position could be hedged by buying $0.4 \times 2{,}000 = 800$ shares.

If the stock price goes up by 1p, the investor will make a gain of 800p. However, he will also make a loss of $2{,}000 \times 0.4 \times 1\text{p} = 800\text{p}$ on the options written.

In general, the gain (loss) on the option position would offset the loss (gain) on the stock position.

[*Other valid examples were, of course, acceptable.*]

**4** The put–call parity relationship is given by:

$$c_t + Ke^{-r(T-t)} = p_t + S_t$$

where $c_t$ = price of call option at time $t$; $p_t$ is the price of put option; $K$ = strike price; $S_t$ is the price of the underlying and $T - t$ is the time to expiry.

Given the values of parameters,

$42 + 620e^{-0.07\times0.5} = P_t + 600$
$42 + 620 \times 0.9656 = P_t + 600$
$42 + 598.675 = P_t + 600$
$P_t = 40.68$
$\therefore$ Price of the put option = 40.68p

We assume here that the markets in which the share and options are traded, there is no arbitrage.

**5** (i) The intrinsic value is 120 – 110 = 10p.

(ii) The time value is premium minus the intrinsic value, 14p – 10p = 4p.

(iii) An increase in market expectations of volatility of the share price.
Decrease in interest rates
Decrease in dividend payments

(iv) The option holder could exercise to realise the intrinsic value of 120p – 50p = 70p. When the premium payment of 14p is considered, the net profit is 70p – 14p = 56p. (The option writer would pay 120p for shares worth 50p. This loss of 70p is partially offset by the premium receipt of 14p. The net loss of the writer is therefore 70p – 14p = 56p.)

*The solution above assumes that the information given in the question relates to the purchase date of the option. Candidates who made a different assumption were given full credit if their subsequent methodology was correct.*

(v) The maximum loss for the writer would occur in the event of the share price falling to zero. It would equal the strike price minus the premium receive, 120p – 14p = 106p.

**6** (i)

- Earnings are sampled from a geometric Brownian motion with drift, which is not an implausible economic model.
- Price earnings ratio mean reverts around $\mu_{PE}$, which is not inconsistent with historical evidence.
- The model allows $P/E$ to be negative, which is implausible.
- PE and ln E are independent AR(1) processes

(ii) (a) $P(t) = E(t)\, PE(t)$

$E[P(t) \mid E(t-1), PE(t-1)]$

$= E[E(t)PE(t) \mid E(t-1), PE(t-1)]$

$= E[E(t) \mid E(t-1)]\; E[PE(t) \mid PE(t-1)]$ from independence

$= E(t-1)\; e^{\mu_E + ½\sigma_E^2} * [(1-\alpha)\,\mu_{PE} + \alpha PE(t-1)]$

(b) (1) $E[P(t) \mid E(t-1), PE(t-1)] = 1 \times e^{0.02+½0.05^2} \times [0.5 \times 20 + 0.5 \times 30]$

$= 25e^{0.02+½0.05^2} = 25.5369$

(2) $E[P(E) \mid E(t-1)\; PE(t-1)]$

$= 15e^{0.02+½0.05^2} = 15.3222$

(c) The model suggests a very inefficient market as expected prices depend significantly on the current *PE* value. That implies some predictability in the market that can be exploited.

For example, in (1) the expected price at time $t$ is less than the price at $t-1$ ($30 \times 1 = 30$), whereas in (2) the expected price is higher than the price at $t-1$ ($10 \times 1 = 10$).

High/low PE values result in negative/positive expected changes in price.

**7** (i) The EMH (in its various forms) states that asset prices reflect information. However it does not explicitly tell us how new information affects prices. It is also empirically difficult to establish precisely when information arrives — for example, many events are widely rumoured prior to official announcements.

Many studies show that the market over-reacts to certain events and under-reacts to other events. The over/under-reaction is corrected over a long time period. If this is true then traders could take advantage of the slow correction of the market, and efficiency would not hold.

(ii)

- Past performance predictability:
    - past winners subsequently underperform
- Accounting ratios have predictive power:
    - e.g. *P/E* predicts low future returns
- Firms coming to market have poor subsequent performance.
- Stock prices take some time to react to earnings announcements.
- Abnormally poor performance following mergers.
- Abnormally good performance following demergers.

(iii)

- Type I errors in hypothesis test: testing for many anomalies will inevitably generate some fake positives by chance.
- Some "information" may actually be a proxy for risk, which should be associated with differential return.
- Terminology: is the market efficient only if transaction costs are taken into account, because these can stop anomalies being exploited?
- Invalid statistical tests: assumptions of normality of the return distributions may lead to the rejection of EMH only because the returns are not normally distributed.
- Timescale: clearly arbitrage possibilities do arise, but nevertheless the market may be efficient on timescales which ignore fleeting arbitrage opportunities.
- Rare events (shocks): the market may reflect small probabilities of large "shocks" which nevertheless do not occur during long periods covered by a given data set.

**8** The Black Scholes formula values an option relative to other assets.

It is based on constructing a portfolio of assets that replicates the payoff from the option (a hedging or replicating portfolio).

By constructing such a portfolio, we avoid taking a view as to the likelihood of a stock price movement.

Therefore no view is taken about stock price growth.

**9** (i) *SIM*: $R_i = \alpha_i + \beta_i R_M + \varepsilon_i$

$R_i$ = return on stock $i$

$\alpha_i$, $\beta_i$ = parameters that are specific to stock $i$

$R_M$ = return on market index

$\varepsilon_i$ = random stock-specific element with zero mean and standard deviation $\sigma_i$

$\varepsilon_i$ and $\varepsilon_j$ are uncorrelated for $i \neq j$
$\varepsilon_i$ and $R_M$ are uncorrelated for all $i$

For an individual stock

$$\mathrm{Var}(R_i) = \beta_i^2 \,\mathrm{Var}(R_M) + \mathrm{Var}(\varepsilon_i)$$

For a portfolio with proportion of wealth $w_i$ invested in stock $i$, with $\Sigma w_i = 1$

$$\mathrm{Var}(\Sigma w_i R_i) = (\Sigma w_i \beta_i)^2 \,\mathrm{Var}(R_M) + \Sigma w_i^2 \,\mathrm{Var}(\varepsilon_i)$$

Provided $w_i$ is small for all $i$, i.e. well-diversified portfolio

$$\mathrm{Var}(\Sigma w_i R_i) \approx (\Sigma w_i \beta_i)^2 \,\mathrm{Var}(R_M)$$

Hence $\beta_i^2 \text{Var}(R_M)$ is referred to as systematic risk
$\text{Var}(\varepsilon_i)$ is diversifiable risk since it becomes unimportant in portfolios

(ii) (a) $RR_i = R_i - R_m$

$$\begin{aligned}
\text{Var}(RR_i) &= \text{Var}(R_i - R_m) \\
&= \text{Var}(\alpha_i + \beta_i R_m + \varepsilon_i - R_m) \\
&= \text{Var}((\beta_i - 1) R_m + \varepsilon_i) \\
&= (\beta_i - 1)^2 \text{Var}(R_m) + \text{Var}(\varepsilon_i) \\
&= \text{“systematic”} + \text{“diversifiable” risks}
\end{aligned}$$

(b)
$$\begin{aligned}
RR_p &= \Sigma w_i R_i - R_m \\
&= \Sigma w_i (R_i - R_m) \text{ provided } \Sigma w_i = 1 \\
&= \Sigma[w_i \alpha_i + w_i(\beta_i - 1) R_m + w_i \varepsilon_i] \\
&= \alpha_p + (\beta_p - 1) R_m + \Sigma w_i \varepsilon_i
\end{aligned}$$

where $\alpha_p = \Sigma w_i \alpha_i$, $\beta_p = \Sigma w_i \beta_i$

$$\text{Var}(RR_p) = (\beta_p - 1)^2 \text{Var}(R_m) + \Sigma w_i^2 \text{Var}(\varepsilon_i)$$

(iii) (a) $\Sigma w_i \beta_i = 0$, $\Sigma w_i = 1$

If $\beta_i > 0 \ \forall\ i$,

then $w_i < 0$ for some $i$

(b)
$$\begin{aligned}
\text{Var}(RR_{MN}) &= (\beta_{MN} - 1)^2 \text{Var}(R_m) + \Sigma w_i^2 \text{Var}(\varepsilon_i) \\
&= \text{Var}(R_m) + \Sigma w_i^2 \text{Var}(\varepsilon_i)
\end{aligned}$$

**10** (i) State prices

$$u = \exp\left(\frac{0.4}{\sqrt{365}}\right) = 1.0211597$$

$\Rightarrow d = 0.9792807$

The risk neutral probability is $p = \dfrac{e^{\frac{0.05}{365}} - 0.97928}{1.02116 - 0.97928}$

$= 0.498018$

State price

$s_1$ $\quad 1 \times (0.498018)^2 \times e^{-\frac{0.05\times 2}{365}} = 0.24795$

$s_2$ $\quad 1 \times 2 \times (0.498018) \times (0.501982) \times e^{-\frac{0.05\times 2}{365}} = 0.499855$

$s_3$ $\quad 1 \times (0.501982)^2 \times e^{-\frac{0.05\times 2}{365}} = 0.251917$

(ii) Three possible share prices at time 2

$s_1 = 104.2763$
$s_2 = 100.0$
$s_3 = 95.899$

Real world probabilities

$s_1$ $\quad 0.24795 \div 0.81977 = 0.30246$

$s_2$ $\quad 0.499855 \div (1.00981) = 0.495$

$s_3$ $\quad 0.251917 \div 1.24390 = 0.20252$

$\Rightarrow$ Expected share price $104.2763 \times 0.30246$
$+ 100 \times 0.495 + 95.899 \times 0.20252$

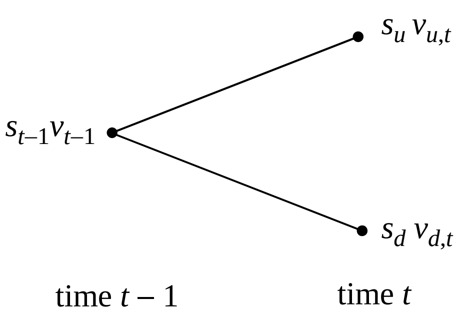
$= 100.46288$

$\Rightarrow$ Return is 0.23% per day

(iii)

| *(1)* | *(2)* | *(3)* | *(4)* | *(2) × (3) × (4)* |
|---|---|---|---|---|
| *Share price* | *Option payoff* | *Probability* | *Deflator* | |
| 104.2763 | 87.8289 | .30246 | .81977 | 21.77 |
| 100.0 | 50 | .495 | 1.00981 | 24.99 |
| 95.899 | 20.899 | .20252 | 1.2439 | 5.26 |
| | | | | 52.03 |

(iv)




where $s_u$ and $s_d$ are share prices at upper and lower nodes respectively

where $v_u$ and $v_d$ are the values of the option at the upper and lower nodes respectively at time $t$

$v_{t-1}$ is the value of the option/hedging portfolio at time $t-1$ and $s_{t-1}$ is the share price at $t-1$.

Let $\phi s_{t-1} + \psi = v_{t-1}$ where $\psi$ represents the cash component and $\phi s_{t-1}$ represents the share component.

We know

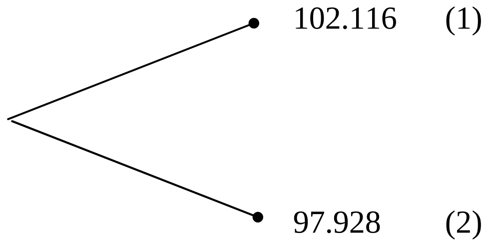
$$\phi s_u + \psi e^r = v_{u,t} \text{ and}$$

$$\phi s_d + \psi e^r = v_{d,t}$$

$$\Rightarrow \qquad \frac{v_{u,t} - v_{d,t}}{s_u - s_d} = \phi \text{ and}$$

$$\psi = e^{-r}(v_{u,t} - s_u\phi)$$

At time 1 we have two nodes




Value of option at node (1)

$$\frac{3\times(104.2763-75)-2\times(100-75)}{(104.2763-100)} = \phi = 8.8462$$

$$\psi = [3 \times (104.2763 - 75) - (104.2763) \times 8.8462]\ e^{-\frac{0.05}{365}}$$

$$= -834.51$$

$\Rightarrow$ Value at (1) $8.8462 \times 102.116 - 834.51 = 68.83$

Value at node (2)

$$\frac{2\times(100-75)-(95.899-75)}{(100-95.899)} = \phi = 7.09607$$

$$\psi = e^{-\frac{0.05}{365}} [2 \times (100 - 75) - 7.09607 \times 100]$$

$$= -659.52$$

$\Rightarrow$ Value at (2) $7.09607 \times 97.928 - 659.52 = 35.384$

At time zero

$$\phi = \frac{68.83-35.38}{102.116-97.928} = 7.9871$$

$$\psi = e^{-\frac{0.05}{365}} (68.83 - 102.116 \times 7.9871)$$

$$= -746.68$$

$\Rightarrow$ Value at time $t = 0$ $7.9871 \times 100 - 746.68 = 52.03$