EXAMINATIONS

9 April 2003 (pm)

Subject 109 — Financial Economics

Time allowed: Three hours

INSTRUCTIONS TO THE CANDIDATE

1. *Enter all the candidate and examination details as requested on the front of your answer booklet.*

2. *You must not start writing your answers in the booklet until instructed to do so by the supervisor.*

3. *Mark allocations are shown in brackets.*

4. *Attempt all 9 questions, beginning your answer to each question on a separate sheet.*

Graph paper is not required for this paper.

AT THE END OF THE EXAMINATION

Hand in BOTH your answer booklet, with any additional sheets firmly attached, and this question paper.

In addition to this paper you should have available Actuarial Tables and your own electronic calculator.

1 Assume that the Capital Asset Pricing Model holds and returns are generated in accordance with a consistent form of the Single Index Model.

(i) Given that the annual risk free rate of return is 5%, that the expected rate of return on the market portfolio is 6% and the expected rate of return on a chosen portfolio is 7%, calculate the beta of the chosen portfolio. [1]

(ii) Briefly explain, in words, what is meant by systematic and specific risk. [2]

(iii) Calculate the systematic and specific risk (as measured by variance) of an asset with expected rate of return of 5.5%, standard deviation of return of 30%. Assume that the expected rate of return on the market portfolio and risk free rate of return are as in part (i) and the standard deviation of the return on the market portfolio is 20%. [2]

[Total 5]

2 (i) List the steps involved in the process of conventional asset liability modelling. [4]

(ii) Outline the inputs to the model and the considerations to be taken into account when choosing the inputs. [5]

(iii) List the practical constraints to the asset liability modelling process. [4]

[Total 13]

3 (i) State what is meant by "excessive volatility" in an investment market. [2]

(ii) Describe the work of Schiller in testing whether equity markets are "excessively volatile" stating the conclusions drawn and any criticisms of this investigation.

[8]

[Total 10]

4 (i) Demonstrate using a simple example relating to long and short positions in zero-coupon bonds of terms one year, two years and three years that a model based on a flat yield curve permits arbitrage opportunities. [4]

(ii) Consider a model of yield curves where the forward yield at term τ is:

$$\text{forward yield}(\tau) = e^{-\alpha\tau}R + (1 - e^{-\alpha\tau})L$$

where R is the limiting yield for short bonds
L is the limiting long forward yield
α is a positive parameter

(a) Derive an expression for the price of a zero coupon bond of term T.

(b) Derive an expression for the spot yield at term T and hence also an expression for the limiting spot yield.

[5]

[Total 9]

5 Let $U(w)$ denote an investor's utility function, where w denotes the investor's level of wealth.

(i) In terms of the derivatives of $U(w)$, state the condition for:

(a) absolute risk aversion
(b) relative risk aversion [1]

(ii) Show, for each of the utility functions listed below, whether they display increasing, decreasing, or constant relative and absolute risk aversion.

(a) $U(w) = 1 - e^{-aw}$ where $a > 0$

(b) $U(w) = \dfrac{w^{\gamma} - 1}{\gamma}$ where $\gamma \in (0, 1)$ [4]

(iii) State with reasons the most suitable characteristic (in terms of absolute risk aversion) of a utility function for a typical investor. [2]
[Total 7]

6 (i) State the assumptions underlying Mean Variance Portfolio Theory. [2]

(ii) Let A and B be two assets with volatility of returns σ_A and σ_B respectively. The coefficient of correlation between the returns of A and B is denoted by ρ.

Derive an expression for the proportion of the asset A held in the minimum variance portfolio comprising only the assets A and B. [2]

(iii) Show that the condition for non negative holdings of A and B is
$\rho < \min\left(\dfrac{\sigma_A}{\sigma_B}, \dfrac{\sigma_B}{\sigma_A}\right)$. [3]

(iv) State what is meant by:

(a) indifference curves
(b) optimal portfolio
(c) opportunity set
(d) efficient frontier

[4]

(v) Sketch, in return/standard deviation space, a set of indifference curves and the opportunity set and label clearly the efficient frontier. [3]
[Total 14]

7 An investor can invest in two assets A and B.

	A	B
expected return	6%	8%
variance	4%%	25%%

The correlation coefficient of the rate of return of the two assets is denoted by ρ and is assumed to take the value 0.5.

The investor is assumed to have expected utility functions of the form $E_\alpha(U) = E(r_p) - \alpha\text{Var}(r_p)$ where α is a positive constant and r_p is the rate of return on the assets held by the investor.

(i) Determine, as a function of α, the portfolio that maximises the investor's expected utility. [8]

(ii) Show that, as α increases, the investor selects an increasing proportion of asset A. [1]

[Total 9]

8 A stock price is modelled by a two period recombining binomial model with the following parameters where each period represents a day. Assume that there are 365 days in a year.

r = 5% p.a. (risk-free rate, continuously compounded)
σ = 30% p.a. = volatility of share price process
$s_0 = 100$ = share price at time 0
$u = \exp(\sigma.365^{-½})$ = return per unit investment of an up jump
probability of an up jump = 60%

(i) By considering the risk neutral probability or otherwise, evaluate the state price $\psi(s)$ (at time zero) for each of the three possible states the share price is in after two steps (assume that one step is one day). [3]

(ii) Calculate the state price deflator (at time 0) for each of the three possible states. [2]

(iii) By considering the cashflows arising from a call option on the stock with exercise price $k = 99$ at time two, use the above to determine the value of this option at time zero. [2]

(iv) Estimate the delta of the option in (iii) at time zero. [3]

(v) Determine the appropriate minimum value delta hedging investment portfolio at time zero for the option described in (iii). [1]

(vi) Explain what is meant by risk neutral valuation. [2]

[Total 13]

9 Consider two non-dividend paying stocks s_1 and s_2. The price per share of stock s_i at time t, $s_i(t)$, has volatility σ_i for $i = 1, 2$. The coefficient of correlation between s_1 and s_2 is denoted by ρ.

An exchange option is the right (but not the obligation) to exchange one share in stock s_2 for one share in stock s_1 at a specified time T. The payoff function is therefore

$$\max(0, s_1(T) - s_2(T)).$$

(i) By considering stock s_1 priced, not in currency units, but in units of stock s_2, and noting that

$$\max(0, s_1(T) - s_2(T)) = s_2(T) \times \max\left(\frac{s_1(T)}{s_2(T)} - 1, 0\right)$$

prove that the value of an exchange option at time 0 in currency units is given

by $s_1(0)\ \Phi\dfrac{\left(\log\left(\dfrac{s_1(0)}{s_2(0)}\right) + ½v^2T\right)}{v\sqrt{T}} - s_2(0)\ \Phi\dfrac{\left(\log\left(\dfrac{s_1(0)}{s_2(0)}\right) - ½v^2T\right)}{v\sqrt{T}}$

where $v^2 = \sigma_1{}^2 + \sigma_2{}^2 - 2\rho\sigma_1\sigma_2$.

You may assume that the Black-Scholes formula holds in pricing any European options. [7]

(ii) Derive expressions for the delta of the exchange option with respect to s_1 and s_2. [6]

(iii) State the constituent components of the replicating portfolio. [2]

(iv) The current price of s_1 = £2 and s_2 = £1.75. A 6 month European call option over share s_1 with exercise price of £1.50 has value of 53p. The volatility of s_2 is 45% p.a. and ρ = ½. The risk-free rate is 4.5% p.a. continuously compounded. Determine the value of an exchange option, with remaining time to expiry of 6 months.

[Hint: you will need to calculate the implied volatility to within 1% p.a.] [5]
[Total 20]