

EXAMINATIONS

13 September 2001 (pm)

Subject 109 — Financial Economics

Time allowed: Three hours

INSTRUCTIONS TO THE CANDIDATE

1. *Write your surname in full, the initials of your other names and your Candidate's Number on the front of the answer booklet.*
2. *Mark allocations are shown in brackets.*
3. *Attempt all 8 questions, beginning your answer to each question on a separate sheet.*

Graph paper is not required for this paper.

AT THE END OF THE EXAMINATION

Hand in BOTH your answer booklet and this question paper.

<p><i>In addition to this paper you should have available Actuarial Tables and an electronic calculator.</i></p>
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- 1** (i) Explain, using formulae where necessary, the main features and assumptions of Arbitrage Pricing Theory (APT). [5]
- (ii) Show how the CAPM can be considered as a simplified version of APT, stating any assumptions required. [4]
- [Total 9]

2 The price of a zero coupon bond can be written as

$$P(\tau) = \exp[-D(\tau)r - (\tau - D(\tau))L - \frac{\beta}{2}D(\tau)^2]$$

where τ is the bond term

and $D(\tau) = \frac{1 - e^{-\alpha\tau}}{\alpha} \quad \alpha > 0$

- (i) Show that the spot rate is

$$= \frac{r}{\tau} D(\tau) + \frac{(\tau - D(\tau))L}{\tau} - \frac{\beta}{2} \frac{D(\tau)^2}{\tau} \quad [1]$$

- (ii) Derive an expression for the limiting short spot yield and the limiting long spot yield. [4]

(Hint: $e^x \approx 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots$)

- (iii) Show that the limiting forward yields are the same as those for the spot yield. [3]
- [Total 8]

- 3** (i) Define a replicating portfolio. [2]
- (ii) Explain what is meant by a “risk neutral probability measure” and state briefly what it implies about the pricing of derivatives relative to the prices of the underlying asset. [2]
- (iii) Consider a one period problem with a non-dividend paying stock at S_0 which may move up or down to give $S_1 = S_0u$ or $S_1 = S_0d$. Consider a derivative which pays c_u or c_d on the up or down event. The risk free rate of return (continuously compounded) is r .
- (a) Derive an equation for the price of the derivative at $t = 0$.

- (b) Hence find the price for a derivative where the payoff is defined as $|S_1 - S_0|$, assuming $d < 1$ and $u > 1$.
- (c) Explain how to synthesise the derivative in (iii) (b) from simpler options. [8]
[Total 12]

- 4 (i) (a) State the stochastic differential equation that implies that the price of an asset at time t , $S(t)$, follows a geometric Brownian motion.
- (b) Describe and discuss the plausibility of the assumptions behind this equation. [5]
- (ii) (a) For the standard Black Scholes call price formula, calculate the price of a European call on a non-dividend paying stock with the following features:
- risk free rate 5% p.a.
 - volatility 20% p.a.
 - time to expiry 1 year
 - current price of underlying 100p
 - strike price 100p

The Black-Scholes price for a call option on a non-dividend paying share is

$$c = S_t \Phi(d_1) - Ke^{-r(T-t)} \Phi(d_2)$$

$$\text{where } d_1 = \frac{\log S_t / K + (r + \frac{1}{2}\sigma^2)(T-t)}{\sigma\sqrt{T-t}}$$

$$d_2 = d_1 - \sigma\sqrt{T-t}$$

and S_t = current price σ = volatility $T-t$ = time to expiry
 K = strike price r = risk free rate

- (b) Explain in general terms why the prices actually quoted in the market may differ from the theoretical Black-Scholes price.
- (c) Calculate the volatility to within 1% p.a. implied by a price of 8p. [8]
[Total 13]

- 5** (i) Distinguish between the weak, semi-strong and strong forms of the Efficient Market Hypothesis (EMH). [3]
- (ii) Consider the following three investment techniques where:
- Fundamental analysis* includes the analysis of balance sheets, consideration of company strategy, the environment in which the company operates, etc.
- Insider trading* is illegal in the UK stock market and involves trading on the basis of information that has not been published or made known to the public.
- Technical analysis* is the study of chart patterns. A variety of trends, triangles and peak formations are used as bullish or bearish indicators.
- Explain how these investment techniques relate to different forms of the EMH. [6]
[Total 9]
- 6** (i) (a) State and briefly describe the four axioms of utility theory.
- (b) Define what is meant by non-satiation and risk seeking in relation to a utility function, U .
- (c) Explain what the existence of an insurance market and a national lottery that partially pays away revenue to charity implies about U for some individuals. [9]
- (ii) A project has the following possible outcomes for each unit, costing 1, of initial investment:
- | | <i>Return</i> | <i>Probability</i> |
|------------------|---------------|--------------------|
| High success | 3 | 0.2 |
| Moderate success | 1 | 0.6 |
| Failure | 0 | 0.2 |
- The risk-free rate of return is zero.
- (a) Evaluate whether or not this project is worth undertaking on an expected return basis.
- (b) Let $U = \ln(W)$ where W is the wealth at the end of the project, be the utility function of the investor.
- Calculate the proportion of initial wealth that should be invested in the project assuming that any wealth not invested has to be held in the risk-free asset. [7]
[Total 16]

7 In one period, the returns on two assets A and B are r_A and r_B . A portfolio is constructed of the two assets in the proportions X_A and X_B , such that $X_A + X_B = 1$.

(i) State an expression for the return of the portfolio, r_p , in one period. [1]

(ii) (a) State an expression for the expected return per period on the portfolio over the long run, $E[r_p]$.

(b) State and discuss the plausibility of the assumptions made in moving from the single period to the long run.

[4]

(iii) State an expression for the standard deviation, σ_p , of the one period return on the portfolio. [2]

(iv) (a) Derive the proportions X_A and X_B to give the minimum risk of the portfolio.

(b) Explain the conditions under which the risk will be zero.

[9]

(v) In a simple example of pairs trading on an arbitrage desk, consider two companies in the same industry, A and B. They both have the same number of shares issued in the market, and they are both debt free. A is priced at 100p per share, B at 110p. A merger is announced where A and B come together to form C. The terms are 1 share of A or 1 share of B becomes 1 share of C.

Describe how a risk free profit can be made stating all assumptions made.

[5]

[Total 21]

- 8 (i) Consider a class of investor for whom risk is defined as “the probability that the value of the portfolio of risky assets is less than if it had been invested in cash”.

Show how the above definition of risk can be related mathematically to the variance of the asset returns assuming that these are normally distributed.

State all assumptions made. [2]

- (ii) Consider a second class of investors for whom the measure of success is the return on the fund less the return on a “benchmark” portfolio (the relative return).

(a) Discuss how the existence of this class of investors can be consistent (or not) with the CAPM by referring to how different types of risk are priced.

(b) A measure of risk for this class of investors is called the tracking error, which is defined as the standard deviation of the relative return.

Show that the tracking error of a fund with a cash benchmark is the standard deviation of the portfolio return.

[4]

- (iii) A bond fund has a benchmark of gilts. The fund actually holds a proportion X of initial wealth in corporate bonds (and $1 - X$ in gilts) in order to try to outperform the gilt benchmark.

(a) Prove that the tracking error introduced by the corporate bond component is linear in X , assuming that the gilt component of the fund is invested exactly in line with the gilt benchmark.

(b) Suppose the investment characteristics of the bond components are as follows:

	<i>Gilts</i>	<i>Corporate Bonds</i>
Expected return	5%	6%
Standard deviation	8%	12%
Correlation	0.9	

Calculate the upper limit on X so that the probability that the relative return will be less than -1% is less than 0.05.

[6]

[Total 12]