EXAMINATIONS

22 September 2004 (pm)

Subject 109 — Financial Economics

Time allowed: Three hours

INSTRUCTIONS TO THE CANDIDATE

1. *Enter all the candidate and examination details as requested on the front of your answer booklet.*

2. *You must not start writing your answers in the booklet until instructed to do so by the supervisor.*

3. *Mark allocations are shown in brackets.*

4. *Attempt all 10 questions, beginning your answer to each question on a separate sheet.*

Graph paper is not required for this paper.

AT THE END OF THE EXAMINATION

Hand in BOTH your answer booklet, with any additional sheets firmly attached, and this question paper.

In addition to this paper you should have available Actuarial Tables and your own electronic calculator.

1 (i) State and describe briefly the three forms of the Efficient Markets Hypothesis. [3]

(ii) Give two reasons why it is hard to test whether any of them hold. [2]

[Total 5]

2 (i) Define an Arrow-Debreu security. Define any terms used. [1]

(ii) Consider an arbitrage-free one-period model with three states.

There are (only) three assets in the market and their current prices and payoffs in each of the three states at time 1 are as follows:

Asset	*Price at time 0*	*Payoff at time 1*		
		State 1	*2*	*3*
1	100	200	110	50
2	90	100	100	60
3	75	75	75	75

The (real world) probability of state 2 occurring is 50% and the state price deflator for state 1 is 0.5. Calculate the (real world) probability of state 3 occurring.

[5]

[Total 6]

3 Defining carefully any terms you use:

(i) Explain in general terms how a model for bond prices which assumes a flat yield curve will give rise to arbitrage opportunities and hence why a flat yield curve would only exist for short periods of time. [4]

(ii) Compare the properties of the Vasicek and Cox-Ingersoll-Ross (CIR) models for interest rates. [4]

[Total 8]

4 (i) Define three Greeks other than Delta and explain briefly what they measure about an option. Define any symbols used. [3]

(ii) A non-dividend paying stock has volatility $\sigma = 25\%$ p.a. and the continuously compounded risk-free rate of interest is $r = 12\%$ p.a. An option on the stock has a theoretical price of $S^3 e^{2rt+3\sigma^2 t}$, when the current price is S and the time to expiry is t.

(a) Use the Cox-Ross-Rubinstein binomial lattice approximation with $q = (e^{r\Delta t} - d)/(u - d)$, $u = e^{\sigma\Delta t}$ and $d = e^{-\sigma\Delta t}$ (with $\Delta t = 1$ day) to estimate Γ when $S = 1$ and $t = 1$ year. Assume that there are 365 days in a year.

(b) Compare the estimate in (a) to the value obtained by differentiating (as appropriate) the formula for the option price and comment. [5]

[Total 8]

5 A non-dividend paying stock has a current price of 1000p. In any unit of time the price of the stock either increases by 25% or decreases by 20%. The risk-free rate of interest is 5% per unit of time.

(i) Find the risk-neutral probability measure for the model. [3]

(ii) Denoting the stock price after t time units by S_t:

(a) Find the price of a path-dependent option on the stock with expiry date $t = 2$ which pays $S_2 - M_2$, where $M_2 = \min_{0 \le t \le 2} S_t$.

(b) Find the hedging portfolio for the option at $t = 1$ if the stock price S_1 is 800p. [7]

[Total 10]

6 The model for the price of a non-dividend paying asset at time t, S_t is given by

$$dS_t = S_t\,(\mu dt + \sigma dZ_t)$$

where Z is a standard Brownian Motion and μ and σ are fixed parameters.

(i) Describe the relationship between μ and the risk free rate of interest if the model is an equivalent martingale measure. [1]

(ii) Use Ito's formula to prove that the solution is

$$S_t = S_0 \exp((\mu - \sigma^2/2)\,t + \sigma Z_t)$$ [3]

(iii) Discuss the properties of the solution and its plausibility as a model of a non-dividend paying share price. [3]

(iv) Describe, with justification, how to simulate monthly returns from this model. [3]

[Total 10]

7 (i) Explain the validity of the criticism that the Wilkie model has more parameters to estimate than are evident in the ARIMA equations. Hint: refer to the VARMA formulation of the model and construct a detailed example involving both autoregressive and moving average terms to illustrate the issue. Define any notation used. [7]

(ii) Consider the following model of dividends:

$$D(t) = D(t-1)\exp\{DW\,DM(t) + DX\,I(t) + DMU + DSD\,DE(t)\}$$

where

$D(t)$ = dividend index at time t
$I(t)$ = force of inflation over $t-1$ to t
$DE(t)$ = independent standard normal random variate
$DM(t) = DD\,I(t) + (1 - DD)\,DM(t-1)$

DX, DMU, DW, DD, DSD are constant parameters.

(a) Explain what is meant when it is said that the dividend process has unit gain with respect to changes in inflation.

(b) State any constraints needed on the parameters in the model above to ensure that the dividend process has unit gain with respect to changes in inflation. [3]

[Total 10]

8 A stock is currently priced at 400p. The price of a six month European call option with an exercise price of 420p is 41p. The annual risk-free interest rate (continuously compounded) is 7% and no dividends are payable during the life of the option.

Assume the Black-Scholes pricing formula applies.

(i) Calculate the current price of a six month European put option with the same exercise price. State the assumptions you make in the calculation. [2]

(ii) Estimate the implied volatility of the stock to within 1%. [3]

(iii) Derive an expression for the Delta of the option and calculate its value. [4]

(iv) Find the hedging portfolio of stock and cash (earning the risk-free rate) that a writer of 10,000 units of the call-option should hold. [3]

[Total 12]

9 (i) State and describe briefly the expected utility theorem and the axioms from which it is derived. [5]

(ii) Define what is meant by "risk averse" and "non-satiated". [2]

(iii) A software contractor, who is risk-averse and non-satiated, has been offered a contract to write a piece of software for a company. Her basic fee will be £1,000,000. In addition, she may choose a contract deadline. If she chooses a deadline of T she will receive a bonus of $\pounds\dfrac{1{,}000{,}000}{T^6}$ **provided** that she completes the contract within the deadline. If not then she receives no bonus. Her utility function u is given by

$$u(w) = (w - 1)^{½},$$

where w is expressed in £1,000,000s, and she knows that the time to complete the contract is X, a Uniform[1, 2] random variable.

(a) Calculate her expected total remuneration and the expected utility offered by the job, if she opts for deadline T, with $1 \leq T \leq 2$.

(b) Determine the deadline that she should choose if she is a rational agent.

[6]

[Total 13]

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10 (i) Consider the following returns-generating model

$$R_i = r_f + \beta_i(R_M - r_f) + \sigma_i\, \varepsilon_i \text{ for } i = 1 \ldots n$$

where

R_i = random variable representing the rate of return on security i

r_f = (known) rate of return on a risk-free asset

R_M = random variable representing the contemporaneous rate of return on the market index

ε_i = standard normal random variable (that is independent of R_M and of ε_j for each j distinct from i)

β_i, σ_i are unknown constants specific to security i.

Explain, in detail, how the above model considerably simplifies the estimation problems in mean-variance portfolio modelling when n is large. [3]

(ii) (a) State the CAPM and APT formulae for the expected return on security i. Define all notation used.

(b) State the main assumptions needed in each case to ensure consistency between the returns-generating model in (i) and the CAPM and the APT.

[5]

(iii) An investor can invest (long or short) in only two assets with annual returns R_A and R_B with the following (annualised) characteristics:

Asset	*Expected rate of return*	*Standard deviation*
A	0.15	0.2
B	0.1	0.1

The correlation between the returns on the assets is 0.5.

No adjustments to the investor's portfolio are possible within the year.

(a) Determine the global minimum variance portfolio proportions.

(b) Derive expressions for both the expected return and standard deviation of a general portfolio on the efficient frontier in terms of a common parameter and hence explain how the efficient frontier can be derived.

[6]

The market is now expanded by the addition of a risk free asset with annualised rate of return of 7%.

(iv) Derive the equation for the new efficient frontier. [4]

[Total 18]

END OF PAPER