EXAMINATIONS

18 April 2000 (pm)

Subject 109 — Financial Economics

Time allowed: Three hours

INSTRUCTIONS TO THE CANDIDATE

1. *Write your surname in full, the initials of your other names and your Candidate's Number on the front of the answer booklet.*

2. *Mark allocations are shown in brackets.*

3. *Attempt all 12 questions, beginning your answer to each question on a separate sheet.*

Graph paper is not required for this paper.

AT THE END OF THE EXAMINATION

Hand in BOTH your answer booklet and this question paper.

In addition to this paper you should have available Actuarial Tables and an electronic calculator.

1 (i) Set down the equations for the expected returns based on the Capital Asset Pricing Model (CAPM) and the Arbitrage Pricing Theory (APT). Define all symbols used. [3]

(ii) Briefly explain the major differences between these models. [5]
[Total 8]

2 (i) Briefly discuss how liabilities can be incorporated into portfolio selection models. [3]

(ii) Outline how Monte Carlo simulation can be used in asset liability modelling. [3]
[Total 6]

3 Define the following measures of investment risk:

(i) variance of return

(ii) downside semi-variance of return

(iii) shortfall probability [6]

4 Describe how the Black-Scholes pricing formula for a European call option can be used to determine implied volatility.
[3]

5 (i) Let $U(W)$ be the utility function of an investor's wealth, W. The first and second derivatives of U with respect to W are positive.

Explain what these conditions imply about the investor's economic characteristics. [3]

(ii) You are given the following information on projects A and B:

State of nature	*Probability*	*Return on project A*	*Return on project B*
1	10%	–3%	–2%
2	20%	4%	4.5%
3	50%	6%	7%
4	20%	12%	14%

Explain with reasons which of the two projects the investor in (i) above should choose in order to maximise his expected utility of wealth. [3]
[Total 6]

6 The Black-Scholes formula for the value of a European call option on a non-dividend paying stock at time t can be written as:

$$c = S\Phi(d_1) - Ke^{-r(T-t)}\,\Phi(d_2)$$

where

$$d_1 = \frac{\ln(S/K) + \left(r + \sigma^2/2\right)(T-t)}{\sigma\sqrt{T-t}}$$

$$d_2 = \frac{\ln(S/K) + \left(r - \sigma^2/2\right)(T-t)}{\sigma\sqrt{T-t}}$$

K = strike price
T = time of maturity
S = price of stock at time t
r = risk-free rate
σ = volatility

and $\Phi(\cdot)$ = cumulative distribution function of the standard normal distribution.

Using the Black-Scholes formula show that the call price, c, is the maximum of $S - Ke^{-r(T-t)}$ or zero, depending on the strike price, when σ tends to zero. [9]

7 An investor wishes to construct a portfolio consisting of a risk-free and a risky asset. His expected utility is given by

$$E[U] = \overline{r_p} - \tfrac{1}{2}\sigma_p^2$$

where $\overline{r_p}$ and σ_p are the mean and standard deviation of the portfolio rates of return. The risk-free asset has an expected rate of return of 5% p.a. The risky asset has an expected rate of return of 8% p.a. and variance of 4%% p.a.

Determine the portfolio that will maximise the investor's expected utility. [9]

8 (i) Define the delta, gamma and theta of an option. [3]

(ii) Describe, using a numerical example, the concept of delta hedging. [6]
[Total 9]

9 (i) Describe the forms of the Efficient Markets Hypothesis (EMH). [3]

(ii) Outline the rôle that portfolio managers have even if the market is perfect and fully efficient. [3]
[Total 6]

10 You are given the following information for returns on two stocks I and Z.

	I	Z
α	0.04	0.09
β	1.20	1.50
σ_e	0.25	0.40

$\bar{R}_m = 0.16$ and $\sigma_m = 0.20$

The returns generating process is assumed to be as follows:

$$R = \alpha + \beta R_m + e$$

$$\sigma^2 = \beta^2 \ \sigma_m^2 + \sigma_e^2$$

where R is a random variable representing the return on the stock; R_m is a random variable representing the return on a market index; e is the residual term; σ^2 , σ_m^2 and σ_e^2 are the variances of the stock, index and residual term, respectively. The residual terms of the returns generating processes for stocks I and Z are assumed uncorrelated with each other and have a mean of zero.

Calculate the following:

(i) The mean and variance of the returns of each stock. [3]

(ii) The covariance of returns between the stocks. [1]

(iii) The beta of an equally-weighted portfolio of the two stocks. [1]

(iv) The expected return and variance of an equally-weighted portfolio of the two stocks. [3]

[Total 8]

11 The price of a non-dividend paying stock at time 1, S_1 , is related to the price at time 0, S_0 , as follows:

$S_1 = uS_0$ with probability p and $S_1 = dS_0$ with probability $(1 - p)$. The continuously compounded rate of return on a risk-free asset is r.

(i) Derive an expression for the replicating portfolio for a European call option written on the stock that expires at time 1 and has a strike price of k, where $dS_0 < k < uS_0$. [5]

(ii) Show that the price of the option in (i) can be written as the discounted expected payoff under a probability measure Q. Hence find an expression for the probability, q, of an upward move in the stock price under Q. [7]

(iii) Explain the relationship between the Q probability measure in (ii) and the real world probability measure. Explain what relationship you would expect q and p to have if all investors are (a) risk averse, (b) risk-seeking, or (c) risk-neutral. [5]

[Total 17]

12 Let

$R(t)$ = gross real yield on an irredeemable index linked bond at the end of year t

$Q(t)$ = cumulative inflation index at the end of year t

$C(t)$ = gross nominal yield on an irredeemable conventional bond

$I(t) = \ln \dfrac{Q(t)}{Q(t-1)}$

$EI(t) = ED\, I(t) + (1 - ED)\, EI(t-1)$, where ED is a constant.

(i) Assuming that coupons are paid annually in arrear and that there is no lag in the indexing, write down an expression for the total return earned on the index linked bond over the year t to $t + 1$ in terms of Q and R. [4]

(ii) Explain why $EI(t)$ might be regarded as representing expected inflation. [3]

(iii) Let $CZ(t)$ and $RZ(t)$ be independent standard normal variates.

CMU, QMU, RMU, RSD, CSD, CA, $RA1$, $RA2$ are all constants.

State in words the key features and properties of the following two models for long term interest rates.

Comment on their plausibility for stochastic asset modelling.

(a) $\ln R(t) = \ln RMU + RA\,[\ln R(t-1) - \ln RMU] + RSD\,.\,RZ(t)$

$\ln(1 + C(t)) = EI(t) + \ln(1 + R(t)) + CMU + CSD\,.\,CZ(t)$

(b) $\ln(1 + C(t)) = CMU + CA1\,[EI(t) - QMU] + CSD\,.\,CZ(t)$

$\ln(1 + R(t)) = RMU + RA1\,[(\ln(1 + C(t)) - CMU) - (EI(t) - QMU)]$

$+ RA2\,[\ln(1 + R(t-1)) - RMU] + RSD\,.\,RZ(t)$ [6]

[Total 13]