EXAMINATIONS

19 September 2000 (pm)

Subject 109 — Financial Economics

Time allowed: Three hours

INSTRUCTIONS TO THE CANDIDATE

1. *Write your surname in full, the initials of your other names and your Candidate's Number on the front of the answer booklet.*

2. *Mark allocations are shown in brackets.*

3. *Attempt all 11 questions, beginning your answer to each question on a separate sheet.*

Graph paper is not required for this paper

AT THE END OF THE EXAMINATION

Hand in BOTH your answer booklet and this question paper.

In addition to this paper you should have available Actuarial Tables and an electronic calculator.

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1 (i) State what is meant by put-call parity. [2]

(ii) Derive an expression for the put-call parity of a European option that has a dividend payable prior to the exercise date. [6]

(iii) If the equality in (ii) does not hold, explain how an arbitrageur can make a riskless profit. [3]
[Total 11]

2 Discuss the following statement:

The existence of fund managers who sell their services based on their alleged ability to select over-performing sectors and stocks and so add value to portfolios demonstrates that capital markets are not efficient. [6]

3 In the Wilkie stochastic investment model, the force of inflation from $t-1$ to t, $I(t)$, is modelled as

$$I(t) = QMU + QA[I(t-1) - QMU] + QSD \,.\, QZ(t)$$

where

$$QZ(t) \sim N(0, 1)$$

and QMU, QA and QSD are fixed parameters.

The parameter values recommended by Wilkie in his 1995 paper based on historic data are:

$QMU = 0.047$
$QA = 0.58$
$QSD = 0.0425$

(i) Calculate, given that the annual rate of inflation is 2.5%, the 95% confidence interval for the rate of inflation over the following year. [4]

(ii) Comment on the result in (i). [2]
[Total 6]

4 (i) Explain the following terms (stating any assumptions needed) in the context of mean-variance portfolio theory:

(a) opportunity set
(b) efficient frontier for a portfolio of risky assets
(c) indifference curves
(d) optimal portfolio [7]

(ii) Describe, using a sketch, the effect on the efficient frontier of introducing a risk-free asset that can be bought or sold in unlimited quantities. [2]

(iii) Explain what the Separation Theorem implies about optimal investment strategies. [3]

[Total 12]

5 Explain the four axioms that are required to derive the expected utility theorem. [6]

6 An investor has the following utility function:

$$U(w) = 1 - e^{-aw}$$

where $w > 0$ is her wealth.

(i) Show what constraints exist on the value of a if she prefers more wealth to less wealth (is non-satiated) and is risk averse.

(ii) Derive her absolute and relative risk aversion functions.

(iii) Explain the implications of the results in (i) and (ii) for the proportion of wealth she will invest in risky assets as her level of investible wealth changes. [6]

7 (i) Describe systematic and specific risks and returns. [3]

(ii) Explain how the risks of a portfolio can be reduced by diversification, without impacting on expected return. [3]

[Total 6]

8 A non-dividend paying stock has a current price of £100. In any unit of time the price of the stock is expected to increase by 10% or decrease by 5%. The continuously compounded risk-free interest rate is 4% per unit of time.

A European call option is written with a strike price of £103 and is exercisable after two units of time, at $t = 2$.

Establish, using a binomial tree, the replicating portfolio for the option at the start and end of the first unit of time, i.e. at $t = 0, 1$. Hence, calculate the value of the option at $t = 0$. [14]

9 (i) Define first and second order stochastic dominance. Illustrate the definitions by sketching cumulative distribution functions of two random variables which represent the returns on two investments, one of which dominates the other. [5]

(ii) Explain how an investor's economic characteristics will affect his choice of an investment that:

(a) first order stochastically dominates another
(b) second order stochastically dominates another [3]

[Total 8]

10 (i) The following unconventional model has been suggested for short-term interest rates, r:

$$dr = \mu r dt + \sigma r dZ$$

where Z is a standard Brownian motion and μ and σ are fixed parameters.

Outline the properties of the model and comment on its appropriateness. [5]

(ii) Outline the properties of the following two models for interest rates:

(a) the one-factor Vasicek model
(b) the Cox-Ingersoll-Ross model [3]

[Total 8]

11 (i) (a) A conventional single-index model is defined by:

$$R_i = \alpha_i + \beta_i R_m + \varepsilon_i$$

where R_i = return on security i

α_i, β_i = security-specific parameters

R_m = return on market index

ε_i = zero-mean random variate

$$\text{Cov}(\varepsilon_i, \varepsilon_j) = 0 \qquad i \neq j$$

$$= \sigma^2_{\varepsilon_i} \qquad i = j$$

Explain in detail the reduction in data required for mean-variance portfolio selection achieved by using the model. [4]

(b) If the number of securities being analysed is 50, what is the difference in the number of data items required between the single-index model and the full covariance (Markowitz) model? [2]

(ii) Explain the major conceptual difference between the single-index model and the basic Sharpe-Lintner CAPM. [3]

(iii) Suppose that the CAPM beta of a security has been estimated from 60 months of historical data to be 1.5.

Over the next 12 months, a stock market index returns 10%. The return on a risk-free asset over the same period is 5%.

(a) Under the CAPM, what is the expected return on the security? [2]

(b) Suppose the return on the security is 6%. State and explain four reasons why the realised return is not the same as your answer to (a). [6]

[Total 17]