EXAMINATIONS

10 September 2002 (pm)

Subject 109 — Financial Economics

Time allowed: Three hours

INSTRUCTIONS TO THE CANDIDATE

1. *Enter all the candidate and examination details as requested on the front of your answer booklet.*

2. *You must not start writing your answers in the booklet until instructed to do so by the supervisor.*

3. *Mark allocations are shown in brackets.*

4. *Attempt all 8 questions, beginning your answer to each question on a separate sheet.*

Graph paper is not required for this paper.

AT THE END OF THE EXAMINATION

Hand in BOTH your answer booklet, with any additional sheets firmly attached, and this question paper.

In addition to this paper you should have available Actuarial Tables and your own electronic calculator.

1 Consider the following investment opportunities, A and B, and their rates of return in each of the two states, Failure and Success.

	Failure	*Success*
A	−100%	300%
B	50%	50%

The probability of each state occurring is 0.5.

Explain how:

(a) a risk-neutral investor will allocate money between these opportunities [3]

(b) a log utility investor, i.e. $U(w) = \ln(w)$, will allocate money [4]

[Total 7]

2 Consider an investment in a risky asset class, with returns in period t of r_t which are independent and identically distributed with a normal distribution with standard deviation, σ.

Consider the process $R_t = 0.5 \times (r_t + r_{t-1})$.

(a) Derive expressions for the unconditional mean return and volatility of this process. [4]

(b) State with detailed reasons whether or not this process stochastically dominates (to second order) its underlying asset class. [5]

[Total 9]

3 (i) Compare the properties of the Vasicek and the Cox-Ingersoll-Ross models for interest rates. [6]

(ii) Under the Vasicek model the price model (given term t, short rate R, long rate L and parameters α and β) is

$$P(R, t) = \exp[-D(t)R - (t - D(t))L - (\beta/2)D(t)^2]$$

where $D(t) = [1 - \exp(-\alpha t)]/\alpha$.

Derive expressions for:

(a) the spot yield
(b) the forward yield
(c) the limit of the spot and forward yields as t tends to zero [6]

[Total 12]

4 (i) (a) State what is meant collectively by "the Greeks" of an option.

(b) Define delta for a derivative. [2]

(ii) Assume that the assumptions underlying the Black-Scholes model hold.

Derive a formula for the delta of a European call option for a non-dividend paying stock.

Evaluate the delta of an at-the-money call, when the risk free rate is zero, time to expiry is one year, and volatility is 20% p.a. [7]

[Hint: *The Black-Scholes formula for the price of a European call option is given by* $f = s\Phi(d_1) - ke^{-ru}\Phi(d_2)$ *where* $d_1 = [\ln(s/k) + (r + \sigma^2/2)u] / \sigma\sqrt{u}$; $d_2 = d_1 - \sigma\sqrt{u}$, *and* f, s, k, r, σ *and* u *are the price of the option, the price of the stock, the strike price, the risk-free rate, the volatility and time to expiry respectively.*]

(iii) One investor, A, holds £1 million of equity. Another, B, holds £1 million in cash, and the call option evaluated in (ii), over £1 million of the same equity. The equity market rises instantly by 20%.

Explain which of these investors now owns greater value. [3]
[Total 12]

5 (i) Explain the concept of a replicating portfolio and its use in derivative pricing. [4]

(ii) Consider a one-period binomial tree, with a non-dividend paying stock at price S_0 which may move up or down to $S_1 = S_0u$ or S_0d respectively at the end of the period. The continuously compounded risk free rate of return is r.

The random payoff at the end of one period of a derivative based on this stock is defined as $S_0 - S_1$.

(a) Derive an expression for the price of the derivative at $t = 0$.

(b) Explain whether dividends would raise or lower the price of derivative. [7]
[Total 11]

6 (i) Describe the three forms of the Efficient Markets hypothesis. [3]

(ii) Describe, in broad terms, for each of the three forms, how you might go about quantitatively testing whether each form held true or not. [6]

(iii) Discuss the theoretical and practical problems associated with your suggested tests in (ii) and any other tests based on empirical research. [6]
[Total 15]

7 Assume that an investor can invest (long or short) in only two assets, the characteristics of which are summarised in the following table:

Asset	*Expected rate of return (p.a.)*	*Standard deviation (p.a.)*
A	10%	20%
B	5%	4%

The correlation between the returns on A and B is 0.5.

Assume that the random returns on these assets have normal distributions. Assume that the investment horizon is one year and that no adjustment is possible to any portfolio during the year.

(i) Determine the overall minimum variance portfolio. [3]

(ii) Assume now that the investor has to pay a random sum of money (the liability) to a creditor at the end of the year which can be written as

£1,000(1 + X)

where X is a normal random variable with mean 8% and standard deviation 10%. X has a correlation of 0.8 with the return on A and 0.2 with the return on B. Surplus is defined as the value of the portfolio less the liability at the end of the year.

(a) Assume the investor has £1,000 at the start of the year. Determine the portfolio that will result in the lowest standard deviation of surplus.

(b) Assume that the investor has £1,050 at the start of the year and that she wishes to construct a portfolio (the "least risk portfolio") that minimises the probability that the surplus is negative.

Determine the expected value of the surplus, $f(w)$, and the variance of surplus, $g(w)$, as functions of w, the proportion of initial wealth invested in asset A.

(c) Demonstrate that

$$f'(w) - \tfrac{1}{2}\frac{f(w)}{g(w)}g'(w) = 0$$

in the case of the "least risk portfolio".

(You do not have to calculate w.) [11]

[Total 14]

8 (i) Derive, using the Single Index Model, expressions for systematic and specific risks of a stock's return. Define all notation used. [4]

(ii) Demonstrate why only the systematic risks of stocks are important to investors constructing well diversified portfolios. State any assumptions made. [4]

(iii) Consider a hypothetical market that comprises 20 stocks with equal market capitalisation. Each stock has a systematic risk of 0.04 and a specific risk also of 0.04.

(a) Calculate the systematic and specific risks of the return on an index constructed as a market capitalisation weighted average of the stocks' returns.

(b) Calculate and comment briefly on how the risk properties of the index in (iii)(a) are changed by the introduction into the market of a 21st stock which has three times the market capitalisation of each of the other stocks, but the same systematic risk and specific risk. [6]

(iv) Explain in detail the theoretical and mathematical similarities and differences between the Single Index Model and the basic Capital Asset Pricing Model (CAPM). [6]

[Total 20]