EXAMINATION

27 April 2010 (am)

Subject CT1 — Financial Mathematics
Core Technical

Time allowed: Three hours

INSTRUCTIONS TO THE CANDIDATE

1. *Enter all the candidate and examination details as requested on the front of your answer booklet.*

2. *You must not start writing your answers in the booklet until instructed to do so by the supervisor.*

3. *Mark allocations are shown in brackets.*

4. *Attempt all 11 questions, beginning your answer to each question on a separate sheet.*

5. *Candidates should show calculations where this is appropriate.*

Graph paper is NOT required for this paper.

AT THE END OF THE EXAMINATION

Hand in BOTH your answer booklet, with any additional sheets firmly attached, and this question paper.

In addition to this paper you should have available the 2002 edition of the Formulae and Tables and your own electronic calculator from the approved list.

1 (i) Explain the difference

(a) between options and futures
(b) between call options and put options

[4]

A security is priced at £60. Coupons are paid half-yearly. The next coupon is due in two months' time and will be £2.80. The risk-free force of interest is 6% per annum.

(ii) Calculate the forward price an investor should agree to pay for the security in three months' time assuming no arbitrage. [3]

[Total 7]

2 In January 2008, the government of a country issued an index-linked bond with a term of two years. Coupons were payable half-yearly in arrear, and the annual nominal coupon rate was 4%. Interest and capital payments were indexed by reference to the value of an inflation index with a time lag of six months.

A tax-exempt investor purchased £100,000 nominal at issue and held it to redemption. The issue price was £98 per £100 nominal.

The inflation index was as follows:

Date	*Inflation Index*
July 2007	110.5
January 2008	112.1
July 2008	115.7
January 2009	119.1
July 2009	123.2

(i) Calculate the investor's cashflows from this investment and state the month when each cashflow occurs. [3]

(ii) Calculate the annual effective money yield obtained by the investor to the nearest 0.1% per annum. [3]

[Total 6]

3 A company issues ordinary shares to an investor who is subject to income tax at 20%.

Under the terms of the ordinary share issue, the investor is to purchase 1,000,000 shares at a purchase price of 45p each on 1 January 2011.

No dividend is expected to be paid for 2 years. The first dividend payable on 1 January 2013 is expected to be 5p per share. Dividends will then be paid every 6 months in perpetuity. The two dividend payments in any calendar year are expected to be the same, but the dividend payment is expected to increase at the end of each year at a rate of 3% per annum compound.

Calculate the net present value of the investment on 1 January 2011 at an effective rate of interest of 8% per annum. [5]

4 An investor is considering purchasing a fixed interest bond at issue which pays half-yearly coupons at a rate of 6% per annum. The bond will be redeemed at £105 per £100 nominal in 10 years' time. The investor is subject to income tax at 20% and capital gains tax at 25%.

The inflation rate is assumed to be constant at 2.8571% per annum.

Calculate the price per £100 nominal if the investor is to obtain a net real yield of 5% per annum. [7]

5 Let f_t denote the one-year forward rate of interest over the year from time t to time $(t+1)$.

The current forward rates in the market are:

time, t	0	1	2	3
one-year forward rate, f_t	4.4% p.a.	4.7% p.a.	4.9% p.a.	5.0% p.a.

A fixed-interest security pays coupons annually in arrear at the rate of 7% per annum and is redeemable at par in exactly four years.

(i) Calculate the price per £100 nominal of the security assuming no arbitrage. [3]

(ii) Calculate the gross redemption yield of the security. [3]

(iii) Explain, without doing any further calculations, how your answer to part (ii) would change if the annual coupon rate on the security were 9% per annum (rather than 7% per annum). [2]

[Total 8]

6 The annual returns, i, on a fund are independent and identically distributed. Each year, the distribution of $1 + i$ is lognormal with parameters $\mu = 0.05$ and $\sigma^2 = 0.004$, where i denotes the annual return on the fund.

(i) Calculate the expected accumulation in 25 years' time if £3,000 is invested in the fund at the beginning of each of the next 25 years. [5]

(ii) Calculate the probability that the accumulation of a single investment of £1 will be greater than its expected value 20 years later. [5]

[Total 10]

7 A pension fund has to pay out benefits at the end of each of the next 40 years. The benefits payable at the end of the first year total £1 million. Thereafter, the benefits are expected to increase at a fixed rate of 3.8835% per annum compound.

(i) Calculate the discounted mean term of the liabilities using a rate of interest of 7% per annum effective. [5]

The pension fund can invest in both coupon-paying and zero-coupon bonds with a range of terms to redemption. The longest-dated bond currently available in the market is a zero-coupon bond redeemed in exactly 15 years.

(ii) Explain why it will not be possible to immunise this pension fund against small changes in the rate of interest. [2]

(iii) Describe the other practical problems for an institutional investor who is attempting to implement an immunisation strategy. [3]

[Total 10]

8 A loan is repayable by annual instalments paid in arrear for 20 years. The first instalment is £4,650 and each subsequent instalment is £150 greater than the previous instalment.

Calculate the following, using an interest rate of 9% per annum effective:

(i) the amount of the original loan [3]
(ii) the capital repayment in the tenth instalment [4]
(iii) the interest element in the last instalment [2]
(iv) the total interest paid over the whole 20 years [2]

[Total 11]

9 A company is undertaking a new project. The project requires an investment of £5m at the outset, followed by £3m three months later.

It is expected that the investment will provide income over a 15 year period starting from the beginning of the third year. Net income from the project will be received continuously at a rate of £1.7m per annum. At the end of this 15 year period there will be no further income from the investment.

Calculate at an effective rate of interest of 10% per annum:

(i) the net present value of the project [3]
(ii) the discounted payback period [4]

A bank has offered to loan the funds required to the company at an effective rate of interest of 10% per annum. Funds will be drawn from the bank when required and the loan can be repaid at any time. Once the loan is paid off, the company can earn interest on funds from the venture at an effective rate of interest of 7% per annum.

(iii) Calculate the accumulated profit at the end of the 17 years. [4]

[Total 11]

10 A pension fund's assets were invested with two fund managers.

On 1 January 2007 Manager A was given £120,000 and Manager B was given £100,000. A further £10,000 was invested with each manager on 1 January 2008 and again on 1 January 2009.

The values of the funds were:

	31 December 2007	*31 December 2008*	*31 December 2009*
Manager A	£130,000	£135,000	£180,000
Manager B	£140,000	£145,000	£150,000

(i) Calculate the time-weighted rates of return earned by Manager A and Manager B over the period 1 January 2007 to 31 December 2009. [4]

(ii) Show that the money-weighted rate of return earned by Manager A over the period 1 January 2007 to 31 December 2009 is approximately 9.4% per annum. [2]

(iii) Explain, without performing further calculations, whether the money-weighted rate of return earned by Manager B over the period 1 January 2007 to 31 December 2009 was higher than, lower than or equal to that earned by Manager A. [3]

(iv) Discuss the relative performance of the two fund managers. [3]

[Total 12]

11 The force of interest $\delta(t)$ is a function of time and at any time t, measured in years, is given by the formula

$$\delta(t) = \begin{cases} 0.04 + 0.02t & 0 \le t < 5 \\ 0.05 & 5 \le t \end{cases}.$$

(i) Derive and simplify as far as possible expressions for $v(t)$, where for $v(t)$ is the present value of a unit sum of money due at time t. [5]

(ii) (a) Calculate the present value of £1000 due at the end of 17 years.

(b) Calculate the rate of interest per annum convertible monthly implied by the transaction in part (ii)(a). [4]

A continuous payment stream is received at a rate of $10e^{0.01t}$ units per annum between $t = 6$ and $t = 10$.

(iii) Calculate the present value of the payment stream. [4]

[Total 13]

END OF PAPER