

INSTITUTE AND FACULTY OF ACTUARIES

EXAMINATION

27 September 2011 (am)

Subject CT1 — Financial Mathematics Core Technical

Time allowed: Three hours

INSTRUCTIONS TO THE CANDIDATE

1. *Enter all the candidate and examination details as requested on the front of your answer booklet.*
2. *You must not start writing your answers in the booklet until instructed to do so by the supervisor.*
3. *Mark allocations are shown in brackets.*
4. *Attempt all 10 questions, beginning your answer to each question on a separate sheet.*
5. *Candidates should show calculations where this is appropriate.*

Graph paper is NOT required for this paper.

AT THE END OF THE EXAMINATION

Hand in BOTH your answer booklet, with any additional sheets firmly attached, and this question paper.

<p><i>In addition to this paper you should have available the 2002 edition of the Formulae and Tables and your own electronic calculator from the approved list.</i></p>
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- 1** A 91-day treasury bill is issued by the government at a simple rate of discount of 8% per annum.

Calculate the annual effective rate of return obtained by an investor who purchases the bill at issue. [3]

- 2** State the characteristics of index-linked government bonds. [3]

- 3** An individual intends to retire on his 65th birthday in exactly four years' time. The government will pay a pension to the individual from age 68 of £5,000 per annum monthly in advance. The individual would like to purchase an annuity certain so that his income, including the government pension, is £8,000 per annum paid monthly in advance from age 65 until his 78th birthday. He is to purchase the annuity by a series of payments made over four years quarterly in advance starting immediately.

Calculate the quarterly payments the individual has to make if the present value of these payments is equal to the present value of the annuity he wishes to purchase at a rate of interest of 5% per annum effective. Mortality should be ignored. [6]

- 4** A pension fund makes the following investments (£m):

<i>1 January 2009</i>	<i>1 July 2009</i>	<i>1 January 2010</i>
1.5	6.0	4.0

The rates of return earned on money invested in the fund were as follows:

<i>1 January 2009 to 30 June 2009</i>	<i>1 July 2009 to 31 December 2009</i>	<i>1 January 2010 to 31 December 2010</i>
1%	2%	5%

Assume that 1 January to 30 June and 1 July to 31 December are precise half-year periods.

- (i) Calculate the time-weighted rate of return per annum effective over the two years from 1 January 2009 to 31 December 2010. [3]
- (ii) Calculate the money-weighted rate of return per annum effective over the two years from 1 January 2009 to 31 December 2010. [3]
- [Total 6]

- 5** A nine-month forward contract is issued on 1 March 2011 on a stock with a price of £9.56 per share at that date. Dividends of 20 pence per share are expected on both 1 April 2011 and 1 October 2011.
- (i) Calculate the forward price, assuming a risk-free rate of interest of 3% per annum effective and no arbitrage. [4]
 - (ii) (a) Explain why the expected price of the share in nine months' time is not needed to calculate the forward price.
 - (b) Explain why the price of an option would be explicitly dependent on the variance of the share price but the price of a forward would not be. [4]
- [Total 8]
- 6** The force of interest, $\delta(t)$, is a function of time and at any time t , measured in years, is $a + bt$ where a and b are constants. An amount of £45 invested at time $t = 0$ accumulates to £55 at time $t = 5$ and £120 at time $t = 10$.
- (i) Calculate the values of a and b . [5]
 - (ii) Calculate the constant force of interest per annum that would give rise to the same accumulation from time $t = 0$ to time $t = 10$. [2]
- [Total 7]
- 7** An investment manager is considering investing in the ordinary shares of a particular company.
- The current price of the shares is 12 pence per share. It is highly unlikely that the share will pay any dividends in the next five years. However, the investment manager expects the company to pay a dividend of 2 pence per share in exactly six years' time, 2.5 pence per share in exactly seven years' time, with annual dividends increasing thereafter by 1% per annum in perpetuity.
- In five years' time, the investment manager expects to sell the shares. The sale price is expected to be equal to the present value of the expected dividends from the share at that time at a rate of interest of 8% per annum effective.
- (i) Calculate the effective gross rate of return per annum the investment manager will obtain if he buys the share and then sells it at the expected price in five years' time. [6]
 - (ii) Calculate the net effective rate of return per annum the investment manager will obtain if he buys the share today and then sells it at the expected price in five years' time if capital gains tax is payable at 25% on any capital gains. [3]
 - (iii) Calculate the net effective real rate of return per annum the investment manager will obtain if he buys the share and then sells it at the expected price in five years' time if capital gains tax is payable at 25% on any capital gains and inflation is 4% per annum effective. There is no indexation allowance. [3]
- [Total 12]

- 8** (i) State the conditions that are necessary for an insurance company to be immunised from small, uniform changes in the rate of interest. [2]

An insurance company has liabilities to pay £100m annually in arrear for the next 40 years. In order to meet these liabilities, the insurance company can invest in zero coupon bonds with terms to redemption of five years and 40 years.

- (ii) (a) Calculate the present value of the liabilities at a rate of interest of 4% per annum effective. [5]
- (b) Calculate the duration of the liabilities at a rate of interest of 4% per annum effective. [5]
- (iii) Calculate the nominal amount of each bond that the fund needs to hold so that the first two conditions for immunisation are met at a rate of interest of 4% per annum effective. [5]
- (iv) (a) Estimate, using your calculations in (ii) (b), the revised present value of the liabilities if there were a reduction in interest rates by 1.5% per annum effective.
- (b) Calculate the present value of the liabilities at a rate of interest of 2.5% per annum effective.
- (c) Comment on your results to (iv) (a) and (iv) (b). [6]
- [Total 18]

- 9** (i) Describe the information that an investor can obtain from the following yield curves for government bonds:
- (a) A forward rate yield curve.
- (b) A spot rate yield curve.
- (c) A gross redemption yield curve. [6]

An investor is using the information from a government bond spot yield curve to calculate the present value of a corporate eurobond with a term to redemption of exactly five years. The investor will value each payment that is due from the bond at a rate of interest equal to $j = i + 0.01 + 0.001t$ where:

- t is the time in years at which the payment is due
- i is the annual t -year effective spot rate of interest from the government bond spot yield curve and $i = 0.02t$ for $t \leq 5$

The eurobond pays annual coupons of 10% of the nominal amount of the bond and is redeemed at par.

- (ii) Calculate the present value of the eurobond. [6]
- (iii) Calculate the gross redemption yield from the eurobond. [3]

- (iv) Explain why the investor might use such a formula for j to determine the interest rates at which to value the payments from the corporate eurobond. [3]
[Total 18]

10 A country's football association is considering whether to bid to host the World Cup in 2026. Several countries aspiring to host the World Cup will be making bids. Regardless of whether the bid is successful, the association will incur various costs. For two years, starting on 1 January 2012, the association will incur costs at a rate of £2m per annum, assumed to be paid continuously, to prepare the bid.

If the football association is successful, the following costs will be incurred from 1 January 2016 until 31 December 2025:

- One stadium will be built each year for ten years. The first stadium will be built in 2016 and is expected to cost £200m; the stadium built in 2017 is expected to cost £210m; and so on, with the cost of each stadium rising by 5% each year. The costs of building each stadium are assumed to be incurred halfway through the relevant year.
 - Administration costs at a rate of £100m per annum will be incurred, payable monthly in advance from 1 January 2025 until 31 December 2026.
 - Revenues from television, ticket receipts, advertising and so on are expected to be £3,300m and are assumed to be received continuously throughout 2026.
- (i) Explain why the payback period is not a good indicator of whether this project is worthwhile. [3]

The football association decides to judge whether to go ahead with the bid by calculating the net present value of the costs and revenues from a successful bid on 1 January 2012 at a rate of interest of 4% per annum effective.

- (ii) Determine whether the association should make the bid. [13]

The football association is discussing how it might factor into its calculations the fact that it is not certain to win the right to host the World Cup because other countries are also bidding.

- (iii) Explain how you might adjust the above calculations if the probability of winning the right to host the World Cup is 0.1 and whether this adjustment would make it more likely or less likely that the bid will go ahead. [3]
[Total 19]

END OF PAPER