

INSTITUTE AND FACULTY OF ACTUARIES

EXAMINATION

19 April 2011 (am)

Subject CT1 — Financial Mathematics Core Technical

Time allowed: Three hours

INSTRUCTIONS TO THE CANDIDATE

1. *Enter all the candidate and examination details as requested on the front of your answer booklet.*
2. *You must not start writing your answers in the booklet until instructed to do so by the supervisor.*
3. *Mark allocations are shown in brackets.*
4. *Attempt all 10 questions, beginning your answer to each question on a separate sheet.*
5. *Candidates should show calculations where this is appropriate.*

Graph paper is NOT required for this paper.

AT THE END OF THE EXAMINATION

Hand in BOTH your answer booklet, with any additional sheets firmly attached, and this question paper.

<i>In addition to this paper you should have available the 2002 edition of the Formulae and Tables and your own electronic calculator from the approved list.</i>

- 1** The force of interest, $\delta(t)$, is a function of time and at any time t , measured in years, is given by the formula

$$\delta(t) = \begin{cases} 0.04 + 0.003t^2 & \text{for } 0 < t \leq 5 \\ 0.01 + 0.03t & \text{for } 5 < t \end{cases}$$

- (i) Calculate the amount to which £1,000 will have accumulated at $t = 7$ if it is invested at $t = 3$. [4]
- (ii) Calculate the constant rate of discount per annum, convertible monthly, which would lead to the same accumulation as that in (i) being obtained. [3]
- [Total 7]

- 2** A one-year forward contract on a stock is entered into on 1 January 2011 when the stock price is £68 and the risk-free force of interest is 14% per annum. The stock is expected to pay an annual dividend of £2.50 with the next dividend due in eight months' time.

On 1 April 2011, the price of the stock is £71 and the risk-free force of interest is 12% per annum. The dividend expectation is unchanged.

Calculate the value of the contract to the holder of the long forward position on 1 April 2011. [6]

- 3** An investment trust bought 1,000 shares at £135 each on 1 July 2005. The trust received dividends on its holding on 30 June each year that it held the shares.

The rate of dividend per share was as given in the table below:

<i>30 June in year</i>	<i>Rate of dividend per share (£)</i>	<i>Retail price index</i>
2005	...	121.4
2006	7.9	125.6
2007	8.4	131.8
2008	8.8	138.7
2009	9.4	145.3
2010	10.1	155.2

On 1 July 2010, the investment trust sold its entire holding of the shares at a price of £151 per share.

- (i) Using the retail price index values shown in the table, calculate the real rate of return per annum effective achieved by the trust on its investment. [6]
- (ii) Explain, without doing any further calculations, how your answer to (i) would alter (if at all) if the retail price index for 30 June 2008 had been greater than 138.7 (with all other index values unchanged). [2]

[Total 8]

4 The n -year spot rate of interest y_n , is given by:

$$y_n = 0.03 + \frac{n}{1000} \quad \text{for } n = 1, 2, 3 \text{ and } 4$$

- (i) Calculate the implied one-year and two-year forward rates applicable at time $t = 2$. [3]
- (ii) Calculate, assuming no arbitrage:
 - (a) The price at time $t = 0$ per £100 nominal of a bond which pays annual coupons of 4% in arrear and is redeemed at 115% after 3 years.
 - (b) The 3-year par yield.

[6]

[Total 9]

5 A loan of nominal amount £100,000 was issued on 1 April 2011 bearing interest payable half-yearly in arrear at a rate of 6% per annum. The loan is to be redeemed with a capital payment of £105 per £100 nominal on any coupon date between 20 and 25 years after the date of issue, inclusive, with the date of redemption being at the option of the borrower.

An investor who is liable to income tax at 20% and capital gains tax of 35% wishes to purchase the entire loan on 1 June 2011 at a price which ensures that the investor achieves a net effective yield of at least 5% per annum.

- (i) Determine whether the investor would make a capital gain if the investment is held until redemption. [3]
- (ii) Explain how your answer to (i) influences the assumptions made in calculating the price the investor should pay. [2]
- (iii) Calculate the maximum price the investor should pay. [5]

[Total 10]

6 The value of the assets held by a pension fund on 1 January 2010 was £10 million. On 30 April 2010, the value of the assets had fallen to £8.5 million. On 1 May 2010, the fund received a contribution payment of £7.5 million and paid out £2 million in benefits. On 31 December 2010, the value of the fund was £17.1 million.

- (i) Calculate the annual effective money-weighted rate of return (MWRR) for 2010. [3]
- (ii) Calculate the annual effective time-weighted rate of return (TWRR) for 2010. [3]
- (iii) Explain why the MWRR is higher than the TWRR for 2010. [2]

The fund manager's bonus for 2010 is based on the return achieved by the fund over the year.

- (iv) State, with reasons, which of the two rates of return calculated above would be more appropriate for this purpose. [2]
- [Total 10]

7 A loan of £60,000 was granted on 1 July 1998.

The loan is repayable by an annuity payable quarterly in arrear for 20 years. The amount of the quarterly repayment increases by £100 after every four years. The repayments were calculated using a rate of interest of 8% per annum convertible quarterly.

- (i) Show that the initial quarterly repayment is £1,370.41. [5]
 - (ii) Calculate the amount of capital repaid that was included in the payment made on 1 January 1999. [3]
 - (iii) Calculate the amount of capital outstanding after the quarterly repayment due on 1 July 2011 has been made. [4]
- [Total 12]

8 A company has liabilities of £10 million due in three years' time and £20 million due in six years' time. The investment manager for the company is able to buy zero-coupon bonds for whatever term he requires and has adequate monies at his disposal.

- (i) Explain whether it is possible for the investment manager to immunise the fund against small changes in the rate of interest by purchasing a single zero-coupon bond. [2]

The investment manager decides to purchase two zero-coupon bonds, one for a term of four years and the other for a term of 20 years. The current interest rate is 4% per annum effective.

- (ii) Calculate the amount that must be invested in each bond in order that the company is immunised against small changes in the rate of interest. You should demonstrate that all three Redington conditions are met. [10]
- [Total 12]

- 9** A company is considering investing in a project. The project requires an initial investment of three payments, each of £105,000. The first is due at the start of the project, the second six months later, and the third payment is due one year after the start of the project.

After 15 years, it is assumed that a major refurbishment of the infrastructure will be required, costing £200,000.

The project is expected to provide a continuous income stream as follows:

- £20,000 in the second year
- £23,000 in the third year
- £26,000 in the fourth year
- £29,000 in the fifth year

Thereafter the continuous income stream is expected to increase by 3% per annum (compound) at the start of each year. The income stream is expected to cease at the end of the 30th year from the start of the project.

- (i) Show that the net present value of the project at a rate of interest of 8% per annum effective is £4,000 (to the nearest £1,000). [7]
- (ii) Calculate the discounted payback period for the project, assuming a rate of interest of 8% per annum effective. [5]
- [Total 12]

- 10** The annual rates of return from a particular investment, Investment A, are independently and identically distributed. Each year, the distribution of $(1 + i_t)$, where i_t is the rate of interest earned in year t , is log-normal with parameters μ and σ^2 .

The mean and standard deviation of i_t are 0.06 and 0.03 respectively.

- (i) Calculate μ and σ^2 . [5]

An insurance company has liabilities of £15m to meet in one year's time. It currently has assets of £14m. Assets can either be invested in Investment A, described above, or in Investment B which has a guaranteed return of 4% per annum effective.

- (ii) Calculate, to two decimal places, the probability that the insurance company will be unable to meet its liabilities if:
- (a) All assets are invested in Investment B.
- (b) 75% of assets are invested in Investment A and 25% of assets are invested in Investment B. [6]
- (iii) Calculate the variance of return from each of the portfolios in (ii)(a) and (ii)(b). [3]
- [Total 14]

END OF PAPER