INSTITUTE AND FACULTY OF ACTUARIES

EXAMINERS' REPORT

September 2014 examinations

Subject CT1 – Financial Mathematics
Core Technical

Introduction

The Examiners' Report is written by the Principal Examiner with the aim of helping candidates, both those who are sitting the examination for the first time and using past papers as a revision aid and also those who have previously failed the subject.

The Examiners are charged by Council with examining the published syllabus. The Examiners have access to the Core Reading, which is designed to interpret the syllabus, and will generally base questions around it but are not required to examine the content of Core Reading specifically or exclusively.

For numerical questions the Examiners' preferred approach to the solution is reproduced in this report; other valid approaches are given appropriate credit. For essay-style questions, particularly the open-ended questions in the later subjects, the report may contain more points than the Examiners will expect from a solution that scores full marks.

The report is written based on the legislative and regulatory context at the date the examination was set. Candidates should take into account the possibility that circumstances may have changed if using these reports for revision.

F Layton
Chairman of the Board of Examiners

November 2014

General comments on Subject CT1

CT1 provides a grounding in financial mathematics and its simple applications. It introduces compound interest, the time value of money and discounted cashflow techniques which are fundamental building blocks for most actuarial work.

Please note that different answers may be obtained to those shown in these solutions depending on whether figures obtained from tables or from calculators are used in the calculations but candidates are not penalised for this. However, candidates may be penalised where excessive rounding has been used or where insufficient working is shown.

Comments on the September 2014 paper

The comments that follow the questions concentrate on areas where candidates could have improved their performance. Where no comment is made the question was generally answered well by most candidates. In general the non-numerical questions were answered poorly by marginal candidates. This applied to bookwork questions such as Q1 and Q8(i) as well as questions requiring interpretation of answers such as Q4(iii), Q8(iv) and (v) and Q9(iv) and (v).

1 One party agrees to pay to the other a regular series of fixed amounts for a certain term. In exchange, the second party agrees to pay a series of variable amounts in the same currency based on the level of a short term interest rate. [2]

2 (i) Expected annual interest rate

$= 0.25 \times 4 + 0.75 \times 7 = 6.25\%$

Let premium = P

$P\,(1.0625)^{20} = 200{,}000$

$\therefore\ P = £59{,}490.99$ [2]

(ii) Expected accumulation is:

$59{,}490.99\,(0.25 \times 1.04^{20} + 0.75 \times 1.07^{20})$

$= 205{,}246.55$

$\therefore$ Expected profit = £5,246.55 [2]

[Total 4]

Many candidates struggled to distinguish between the use of an expected annual interest rate and the expected accumulation after 20 years.

3 (i) $98.83 = 100\,(1 + i)^{-91/365}$

$$\ln(1+i) = \left(-\frac{365}{91}\right) \times \ln(98.83/100) = 0.047205$$

Therefore $i = 0.04834$. [3]

(ii) The rate of interest over 91 days is

$(100 - 98.83) / 98.83 = 0.011839$

The simple rate per annum is:

$$0.011839 \times \frac{365}{91} = 0.04748$$ [2]

[Total 5]

4 (i) Let i be the TWRR per annum effective, then:

$$1+i=\frac{2.1}{2.0}\times\frac{4.2}{2.1+2.5}=0.95870$$

$$\Rightarrow \text{TWRR}=-4.130\%\text{p.a.}$$ [2]

(ii) Let i be the MWRR per annum effective, then:

$$2.0(1+i)+2.5(1+i)^{2/3}=4.2$$

Try: -8% LHS = 4.20482
-9% LHS = 4.16765

Then $$-i=0.08+0.01\times\left(\frac{4.20482-4.2}{4.20482-4.16765}\right)$$

$i\approx-8.130\%=-8.1\%$p.a. (Exact answer is -8.12985%) [3]

(iii) The MWRR is affected by the timing and amount of cashflows. The fund performs relatively worse when the size of the fund is largest and this will have a greater effect on the MWRR which is consequently lower than the TWRR. [2]

[Total 7]

Parts (i) and (ii) were well-answered. In part (iii), examiners were looking for specific comments regarding this scenario and not just a statement of the bookwork.

5 (i) $\frac{i}{i^{(12)}}a_{\overline{5}|}$ = 1.022715 × 4.3295 = 4.4278 [1]

(ii) $a_{\overline{19}|}-a_{\overline{4}|}$ = 12.0853 – 3.5460 [1]

= 8.5394

(iii) $$\frac{\ddot{a}_{\overline{10}|}-10v^{10}}{\delta}$$

$$=\frac{1.05\times7.7217-10\times0.61391}{0.04879}$$

$=40.3501$ [1]

(iv) $$\frac{\bar{a}_{\overline{10|}} - 10v^{10}}{\delta} = \frac{1.024797 \times 7.7217 - 10 \times 0.61391}{0.04879}$$

$$= 36.3613.$$ [1]

(v) Present value is:

$$13\ddot{a}_{\overline{10|}} - (I\ddot{a})_{\overline{10|}}$$

$$= 1.05 \times (13 \times 7.7217 - 39.3738)$$

$$= 64.0592$$ [2]

[Total 6]

Generally well-answered although some candidates were unable to distinguish between the increasing annuities in parts (iii) and (iv).

6 (i) $P = 0.8 \times 5\, a_{\overline{10|}} + 100v^{10}$ @ 4% per annum.

$a_{\overline{10|}} = 8.1109 \qquad v^{10} = 0.67556$

$P = 0.8 \times 5 \times 8.1109 + 100 \times 0.67556 = 100$

No loss of marks for general reasoning. [2]

(ii) $$\text{DMT} = \frac{\sum tC_t\, v^t}{\sum C_t v^t}$$

$$= \frac{5(Ia)_{\overline{10|}} + 10 \times 100v^{10}}{5a_{\overline{10|}} + 100v^{10}}$$

$$= \frac{5 \times 41.9922 + 10 \times 100 \times 0.67556}{5 \times 8.1109 + 100 \times 0.67556}$$

$$= \frac{885.525}{108.1108} = 8.19 \text{ years}$$ [3]

(iii) (a) On average the gross cash flows are earlier because of the higher coupon payments. Therefore the discounted mean term would be lower.

(b) Term of the bond
The gross redemption yield/interest rate at which payments are discounted.

[3]

(iv) All payments are 3 months closer. Therefore, purchase price would be

$$100(1.04)^{1/4} = 100.9853$$

[1]
[Total 9]

Parts (i) and (ii) were answered well. Many candidates' arguments in part (iii)(a) were unclear.

7 (i)

$$A(0,10) = e^{\int_0^{10} 0.03\,dt} = e^{[0.03t]_0^{10}} = e^{0.3}$$

$$A(10,20) = e^{\int_{10}^{20} 0.003t\,dt}$$

$$= e^{\left[\frac{0.003t^2}{2}\right]_{10}^{20}}$$

$$= e^{(0.6-0.15)} = e^{0.45}$$

$$A(20,28) = e^{\int_{20}^{28} 0.0001t^2\,dt}$$

$$= e^{\left[\frac{0.0001t^3}{3}\right]_{20}^{28}} = e^{0.73173-0.26666} = e^{0.46507}$$

Required PV

$$= \frac{1}{A(0,10)\,A(10,20)\,A(20,28)} = e^{-0.3-0.45-0.46507} = e^{-1.21507}$$

$$= 0.29669$$

[7]

(ii) (a) $0.29669 = e^{-28\delta}$

$$\frac{\ln 0.29669}{-28} = \delta = 0.04340 = 4.340\% \text{ per annum}$$

(b) $(1-d)^{28} = 0.29669$

$1 - d = 0.95753$

$d = 0.04247 = 4.247\%$ per annum [3]

(iii) $v(t) = e^{-\int_0^t 0.03ds} = e^{-0.03t}$

$\rho(t) = e^{-0.04t}$

We require:

$$\int_3^7 e^{-0.03t}e^{-0.04t}dt = \int_3^7 e^{-0.07t}dt$$

$$= \left[\frac{-e^{-0.07t}}{0.07}\right]_3^7$$

$= -8.75181 + 11.57977$

$= 2.82797$ [4]

[Total 14]

Answered well. The common mistake was to calculate the effective rate of interest rather than the effective rate of discount in part (ii)(b).

8 (i) (a) Bonds of different terms are attractive to different investors, who will choose assets that are appropriate for their liabilities. The shape of the yield curve is determined by supply and demand at different terms to redemption.

(b) Longer dated bonds are more sensitive to interest rate movements than short dated bonds. It is assumed that risk averse investors will require compensation (in the form of higher yields) for the greater risk of loss on longer bonds.

[4]

(ii) Let i_t be the spot yield over t years.

One year: yield is 6% therefore $i_1 = 0.06$
Two years: $(1 + i_2)^2 = 1.06 \times 1.05$ therefore $i_2 = 0.054988$
Three years: $(1 + i_3)^3 = 1.06 \times 1.05 \times 1.04$ therefore $i_3 = 0.049968$.
Four years: $(1 + i_4)^4 = 1.06 \times 1.05 \times 1.04 \times 1.03$ therefore $i_4 = 0.04494$.

[4]

(iii) Price of bond is:

$$4[(1.06)^{-1} + (1.054988)^{-2} + (1.049968)^{-3} + (1.04494)^{-4}]$$
$$+ 110 \times 1.04494^{-4}$$
$$= 4 \times 3.54454 + 92.26294$$
$$= 106.4411$$

Find the gross redemption yield from:

$$106.4411 = 4a_{\overline{4}|} + 110v^4$$

Try 4%

$$a_{\overline{4}|} = 3.6299 \quad v^4 = 0.85480$$

RHS = 108.5476

Try 5%

$$a_{\overline{4}|} = 3.5460 \quad v^4 = 0.82270$$

RHS = 104.681

Interpolate between 4% and 5%:

$$i = 0.04 + 0.01 \times \frac{108.5476 - 106.4411}{108.5476 - 104.681}$$

$$= 0.0454$$

$$= 4.54\%$$

[5]

(iv) On average, the payments would be received earlier and discounted at higher spot rates. This means that the gross redemption yield (which is a weighted average of the interest rates used to discount the payments) would be higher.
[2]

(v) The earlier spot rates are likely to fall as a result of greater demand for the bonds with shorter terms to redemption. [2]
[Total 17]

Parts (i) and (iii) were generally answered well with correct approaches in part (iii) given full credit even if the calculations in part (ii) had been incorrect. In common with other similar questions on this paper, the reasoning questions in parts (iv) and (v) were poorly answered.

9 (i)

Date	*Nominal Cash Flow £m*	*Indexed Cash Flow £m*
1/12/2012	0.0075×3.5 = 0.02625	(116/112)×0.02625 = 0.0271875
1/6/2013	0.0075×3.5 = 0.02625	(117/112)×0.02625 = 0.0274219
1/12/2013	0.0075×3.5 = 0.02625	(120/112)×0.02625 = 0.028125
1/6/2014	(1 + 0.0075)×3.5 = 3.52625	(121/112)×3.52625 = 3.8096094

[5]

(ii)

Date	*Indexed Cash Flow £m*	*Index Ratio*	*Real Value of Cash flow £m*
1/12/2012	0.0271875	113/117	0.0262580
1/6/2013	0.0274219	113/118	0.0262599
1/12/2013	0.028125	113/121	0.0262655
1/6/2014	3.8096094	113/122	3.5285726

[4]

(iii) Value of £3.5m nominal is:

$$0.0262580v^{½} + 0.0262599v + 0.0262655\text{v}^{1½} + 3.5285726v^2$$

$$= 0.0262580 \times 0.992583 + 0.0262599 \times 0.98522 + 0.0262655 \times 0.97791$$

$$+ 3.5285726 \times 0.97066$$

$= £3.502657\text{m}$

Per £100 nominal $= \dfrac{3.502671}{3.5} \times 100$

$= £100.0763$ [3]

(iv) The expected rate of return at issue is likely to have been higher. Although the investor is compensated for the higher-than-expected inflation, the time lag used for indexation is likely to mean that he is not fully compensated. Therefore the actual real value of the cash flows is less than the expected real value of the cash flows at issue. [2]

(v) It is likely that the price will fall. The expected real value of the cash flows measured will be lower because the cash flows will be linked to an index expected to rise at a lower rate. [2]

[Total 16]

The most poorly answered question on the paper. Better candidates took advantage of the relatively large number of marks available in parts (i) and (ii) for straightforward calculation work. The important point in part (iii) is to note that the real redemption yield *equation uses inflation adjusted cashflows (in terms of 1 June 2012 prices in this case). In part (iv), the important point is that the time lag causes the investor not to be fully protected against inflation. If there had been no time lag, the actual increase in the retail price index would have no effect on the investor's real rate of return.*

10 (i) Present value of earning if university is not attended:

$$15,000\ a_{\overline{1|}}^{(12)} + 18,000\ a_{\overline{1|}}^{(12)} v + 20,000\ a_{\overline{1|}}^{(12)} v^2 + 20,000 a_{\overline{1|}}^{(12)}$$

$$\times 1.01 v^3 (1 + 1.01v + \ldots + 1.01^{36} v^{36})$$

$$= \frac{i}{i^{(12)}}\ a_{\overline{1|}} (15,000 + 18,000v + 20,000v^2)$$

$$+ \left(20,000 \frac{i}{i^{(12)}}\ a_{\overline{1|}} * 1.01 v^3 \right) \left(\frac{1 - 1.01^{37} v^{37}}{1 - 1.01v} \right)$$

$$\frac{i}{i^{(12)}} = 1.031691; \quad a_{\overline{1|}} = v = 0.93458$$

$v^2 = 0.87344$; $v^3 = 0.81630$; $1.01^{37} = 1.445076$

$v^{37} = 0.08181.$

$$1.031691 \times 0.93458\ (15,000 + 18,000 \times 0.93458 + 20,000 \times 0.8734)$$

$$+ (20,000 \times 1.031691 \times 0.93458$$

$$\times 1.01 \times 0.81630) \left(\frac{1 - 1.445076 \times 0.081809}{1 - 1.01 \times 0.9346} \right)$$

$= 47,527.46 + 15,898.86 \times 15.7252$

$= £297,537.30$ [7]

(ii) The cost of going to university is:

$$15,000 \times \ddot{a}_{\overline{3|}} @ 7\% = 42,120.27$$

$$\ddot{a}_{\overline{3|}} = 2.6243 \times 1.07 = 2.8080$$

The PV of the salary from attending university at the time of leaving university is:

$$22,000\, a_{\overline{1|}}^{(12)} + 25,000\, {}_{\overline{1|}}^{(12)} v + 28,000\, a_{\overline{1|}}^{(12)} v^2$$

$$+28,000 \quad a_{\overline{1|}}^{(12)} \times 1.015 v^3 (1 + 1.015 v + \ldots + 1.015^{33} v^{33})$$

$$= \frac{i}{i^{(12)}} a_{\overline{1|}} (22,000 + 25,000 v + 28,000 v^2)$$

$$+ \left(28,000 \frac{i}{i^{(12)}} a_{\overline{1|}} 1.015 v^3 \right) \left(\frac{1 - 1.015^{34} v^{34}}{1 - 1.015 v} \right)$$

$1.015^{34} = 1.658996$

$v^{34} = 0.10022$

= 1.031691 × 0.93458(22,000 + 25,000 × 0.93458

+ 28,000 × 0.87344) + 28,000 × 1.031691 × 0.93458 × 1.015

$$\times\ 0.81630 \left(\frac{1 - 1.658996 \times 0.10022}{1 - 1.015 \times 0.93458} \right)$$

= 67,321.02 + 22,368.60 × 16.21996

= 430,138.80

PV at time of decision = 430,138.80 × v^3

= 430,138.80 × 0.81630 = £351,121.46

There are various ways in which the answer can be rationalised.

NPV of benefit of going to university (net of earnings lost through the alternative course of action)

= 351,121.46 – 42,120.27 – 297,537.30

= £11,463.89 [9]

(iii) The costs of going to university are incurred earlier and the benefits received later. If the rate of interest is lower, then any loans taken out to finance attendance at university will be repaid more easily at a lower interest cost (answer could say that value of payments received later will rise by more when the interest rate falls). [2]

(iv) Tax is paid on income only at rate t.

Therefore, equation of value is:

$$351{,}121.46\,(1-t) = 42{,}120.27 + 297{,}537.30 \times (1-t)$$

$$351{,}121.46 - 42{,}120.27 - 297{,}537.30$$

$$= t\,(351{,}121.46 - 297{,}537.30)$$

$$\therefore\ 11{,}463.89 = 53{,}584.16t$$

$$\therefore\ t = 0.2139 \text{ or } 21.39\%$$

[2]
[Total 20]

Parts (i) and (ii) were often well-answered although marginal candidates would have benefited from setting out their working more clearly and some candidates failed to' determine whether attending university would be a more attractive option' despite having completed the requisite calculations.

Part (iii) was poorly answered by marginal candidates with few such candidates correctly considering the relative timing of the costs and benefits. Few such candidates attempted part (iv) perhaps because of time pressure. Stronger candidates, however, often obtained close to full marks on the question.

END OF EXAMINERS' REPORT