INSTITUTE AND FACULTY OF ACTUARIES

EXAMINERS' REPORT

September 2011 examinations

Subject CT1 — Financial Mathematics
Core Technical

Purpose of Examiners' Reports

The Examiners' Report is written by the Principal Examiner with the aim of helping candidates, both those who are sitting the examination for the first time and who are using past papers as a revision aid, and also those who have previously failed the subject. The Examiners are charged by Council with examining the published syllabus. Although Examiners have access to the Core Reading, which is designed to interpret the syllabus, the Examiners are not required to examine the content of Core Reading. Notwithstanding that, the questions set, and the following comments, will generally be based on Core Reading.

For numerical questions the Examiners' preferred approach to the solution is reproduced in this report. Other valid approaches are always given appropriate credit; where there is a commonly used alternative approach, this is also noted in the report. For essay-style questions, and particularly the open-ended questions in the later subjects, this report contains all the points for which the Examiners awarded marks. This is much more than a model solution – it would be impossible to write down all the points in the report in the time allowed for the question.

T J Birse
Chairman of the Board of Examiners

December 2011

© Institute and Faculty of Actuaries

General comments on Subject CT1

CT1 provides a grounding in financial mathematics and its simple applications. It introduces compound interest, the time value of money and discounted cashflow techniques which are fundamental building blocks for most actuarial work.

Please note that different answers may be obtained to those shown in these solutions depending on whether figures obtained from tables or from calculators are used in the calculations but candidates are not penalised for this. However, candidates may be penalised where excessive rounding has been used or where insufficient working is shown.

Comments on the September 2011 paper

The general performance was considerably better than in September 2010 and also slightly better than in April 2011. Well-prepared candidates scored well across the whole paper.
As in previous diets, questions that required an element of explanation or analysis, such as Q5(ii) and Q9(iv) were less well answered than those that just involved calculation. Marginal candidates should note that it is important to explain and show understanding of the concepts and not just mechanically go through calculations. The comments that follow the questions concentrate on areas where candidates could have improved their performance. Where no comment is made the question was generally answered well by most candidates.

1 $\left(1-\frac{91}{365}\times 0.08\right)=(1+i)^{-91/365}$

$0.980055=(1+i)^{-91/365}$

$1+i=1.08416 \Rightarrow i=8.416\%$

2 Issued by the government
Pay regular interest
Redeemable at a given redemption date
Normally liquid/marketable
More or less risk-free relative to inflation
Low expected return
Low default risk
Coupon and capital payments linked to an index of prices…
… with a time lag.

This type of bookwork question is common in CT1 exam papers. As such, it was disappointing that only about one-sixth of candidates obtained full marks here (which could be achieved by listing six distinct features).

3 Let the annual rate of payment = X

Present value of the payments = $X\ddot{a}_{\overline{4}|}^{(4)}$

Present value of the payments needed from the annuity is:

$8,000\ddot{a}_{\overline{3}|}^{(12)}v^4+3,000\ddot{a}_{\overline{10}|}^{(12)}v^7$

$X\ddot{a}_{\overline{4}|}^{(4)}=8,000\ddot{a}_{\overline{3}|}^{(12)}v^4+3,000\ddot{a}_{\overline{10}|}^{(12)}v^7$

$a_{\overline{3}|}=2.7232 \qquad i/d^{(4)}=1.031059$

$a_{\overline{4}|}=3.5460 \quad a_{\overline{10}|}=7.7217 \quad i/d^{(12)}=1.026881 \quad v^4=0.82270 \quad v^7=0.71068$

$X\frac{i}{d^{(4)}}a_{\overline{4}|}=8,000\frac{i}{d^{(12)}}a_{\overline{3}|}v^4+3,000\frac{i}{d^{(12)}}a_{\overline{10}|}v^7$

$$X\times 1.031059\times 3.5460=8,000\times 1.026881\times 2.7232\times 0.82270$$
$$+3,000\times 1.026881\times 7.7217\times 0.71068$$

$3.65614X=18,404.80+16.905.51$

$X = £9,657.81$

$\therefore$ Quarterly payment is: £2,414.45.

Many candidates struggled to allow correctly for the Government pension. In some cases, candidates would have scored more marks if they had explained their methodology and their workings more clearly.

4 (i) The fund value on 30 June 2009 will be:

$$1.5 \times 1.01 = 1.515$$

The fund value on 31 December 2009 will be:

$$(1.5 \times 1.01 + 6) \times 1.02 = 7.6653$$

The fund value on 31 December 2010 will be:

$$\left[(1.5 \times 1.01 + 6) \times 1.02 + 4\right] \times 1.05 = 12.2486$$

TWRR is i such that:

$$\frac{1.515}{1.5} \times \frac{7.6653}{7.515} \times \frac{12.2486}{11.6653} = (1+i)^2 = 1.0817$$

$$\therefore i = 4.005\%$$

(This can also be calculated directly from the rates of return for which no marks would be lost).

(ii) The equation of value is:

$$1.5(1+i)^2 + 6.0(1+i)^{1\frac{1}{2}} + 4(1+i) = 12.2486$$

Try i = 4% LHS = 12.146

Try i = 4.5% LHS = 12.22754

Try i = 5% LHS = 12.3094

Interpolate:

$$i = 0.045 + \frac{12.2486 - 12.22754}{12.3094 - 12.22754} \times 0.005$$

= 0.04629 or 4.63%

A common error was to assume that the 1% and 2% rates of return were annualised figures rather than returns over a six-month period.

5 (i) Forward price is accumulated value of the share less the accumulated value of the expected dividends:

$$F = 9.56(1.03)^{9/12} - 0.2(1.03)^{8/12} - 0.2(1.03)^{2/12}$$

= 9.7743 – 0.20398 – 0.20099

= £9.3693

(ii) (a) Although the share will be bought in nine months, it is not necessary to take into account the expected share price. The current share price already makes an allowance for expected movements in the price and the investor is simply buying an instrument that is (more or less) identical to the underlying share but with deferred payment. As such, under given assumptions, the forward can be priced from the underlying share.

(b) An option does not have to be exercised. As such, movements in the share price in one direction will benefit the holder whereas movements in the other direction will not harm him. The more volatile is the underlying share price, the more potential there is for gain for the holder of the option (with limited risk of loss), compared with holding the underlying share. This is not the case for a forward which has to be exercised.

Part (i) was well-answered but part (ii) was very poorly answered. The examiners anticipated that many candidates would find part (ii)(b) challenging but it was pleasing to see some of the strongest candidates give some well-reasoned explanations for this part.

6 (i) $45e^{\int_0^5 (a+bt)dt} = 55 \quad (1)$

$45e^{\int_0^{10} (a+bt)dt} = 120 \quad (2)$

From (1)

$$45\exp\left[at + \frac{bt^2}{2}\right]_0^5 = 55$$

$$\ln\left(\frac{55}{45}\right) = 5a + 12.5b = 0.2007 \qquad (1a)$$

From (2)

$$45\exp\left[at + \frac{bt^2}{2}\right]_0^{10} = 120$$

$$\ln\left(\frac{120}{45}\right) = 10a + 50b = 0.98083$$

From (1a)

$$10a = 0.4014 - 25b \qquad (2a)$$

Substituting into (2a)

$$0.4014 + 25b = 0.98083$$

$$\therefore b = \frac{0.98083 - 0.4014}{25} = 0.02318$$

Substituting into (1a)

$$5a + 12.5 \times 0.02318 = 0.2007$$

$$\therefore a = \frac{0.2007 - 12.5 \times 0.0231772}{5} = -0.01781$$

(ii) $45e^{10\delta} = 120$

$$e^{10\delta} = \frac{120}{45}; 10\delta = \ln\left(\frac{120}{45}\right) = 0.98083$$

$\therefore \delta = 0.09808$ or 9.808 %

7 (i) Expected price of the shares in five years is:

$$X = 2v + 2.5v^2 + 2.5\times 1.01\times v^3 + 2.5\times 1.01^2 v^4 + \ldots$$

$$= 2v + 2.5v^2 + 2.5v^2\left(1.01v + 1.01^2 v^2 + \ldots.\right)$$

$1.01v + 1.01^2 v^2 + \ldots$ at 8% $= \dfrac{1}{i'}$

where $i' = \dfrac{1.08}{1.01} - 1 = 0.069307$

$$X = 2\times 0.92593 + 2.5\times 0.85734 + \frac{2.5\times 0.85734}{0.069307}$$

= 3.9952 + 30.9254 = 34.9206

Equation of value for the investor is:

$$12(1+i)^5 = 34.9206$$

$i = 0.23817$ or 23.817%

(ii) $12(1+i)^5 = 34.9206 - (34.9206 - 12)\times 0.25$

where i is the net rate of return.

$$12(1+i)^5 = 29.1905$$

$i = 0.1946$ or 19.46%

(iii) The cash flow received in nominal terms is still the same: 29.190495

The equation of value expressed in real terms is:

$12 = \dfrac{29.1905}{(1+f)^5} v^5$ where $f = 0.04$

$$v^5 = \frac{12\times(1.04)^5}{29.1905} = 0.50016$$

$$\therefore v = 0.50016^{1/5} = 0.87061$$

i = 14.86%

8 (i) The present value of the assets is equal to the present value of the liabilities at the starting rate of interest.

The duration /discounted mean term/volatility of the assets is equal to that of the liabilities.

The convexity of the assets (or the spread of the timings of the asset cashflows) around the discounted mean term is greater than that of the liabilities.

(ii) (a) PV of liabilities is: $£100m\, a_{\overline{40|}}$ at 4%

$= £100\text{m} \times 19.7928$

$= £1{,}979.28\text{m}$

(b) The duration of the liabilities is:

$$\sum_{t=1}^{t=40} 100t\, v^t \Big/ \sum_{t=1}^{t=40} 100v^t \quad \text{(working in £m)}$$

$$= \frac{100 \sum_{t=1}^{t=40} t\, v^t}{1{,}979.28} = \frac{100(Ia)_{\overline{40|}}}{1{,}979.28} \text{ at 4\%}$$

$$= \frac{100 \times 306.3231}{1{,}979.28} = 15.4765 \text{ years}$$

(iii) Let x = nominal amount of five-year bond
y = nominal amount of 40-year bond.

working in £m

$$1{,}979.28 = xv^5 + yv^{40} \qquad \text{— (1)}$$
$$30{,}632.31 = 5xv^5 + 40yv^{40} \qquad \text{— (2)}$$

multiply equation (1) by 5.

$$9{,}896.4 = 5xv^5 + 5yv^{40} \qquad \text{— (1a)}$$

subtract (1a) from (2) to give

$$20735.91 = 35yv^{40}$$

$$\frac{20{,}735.91}{35 \times v^{40}} = y$$

with $v^{40} = 0.20829$

$y = 2,844.38$

Substitute into (1) to give:

$1,979.28 = Xv^5 + 2,844.38 \times 0.20829$

$v^5 = 0.82193$

$$\frac{1,979.28 - 2,844.38 \times 0.20829}{0.82193} = x = 1,687.28$$

Therefore £1,687.28m nominal of the five-year bond and £2,844.38*m* nominal of the 40-year bond should be purchased.

(iv) (a) The duration of the liabilities is 15.4765

Therefore the volatility of the liabilities is $\frac{15.4765}{1.04}$ =14.88125%

The value of the liabilities would therefore change by:

$1.5 \times 0.1488125 \times 1,979.28m = £441.81m$

and the revised present value of the liabilities will be £2,421.09*m*.

(b) PV of liabilities is: $£100m\ a_{\overline{40|}}$ at 2.5%

$= £100\text{m} \times \dfrac{1 - 1.025^{-40}}{0.025}$

= £2,510.28*m*.

(c) The PV of liabilities has increased by £531*m*. This is significantly greater than that estimated in (iv) (a). This estimation will be less valid for large changes in interest rates as in this case.

The first three parts were generally well-answered but, in part (iv), the examiners were surprised that so few candidates were able to use the duration to estimate the change in the value of the liability.

9 (i) (a) The theoretical rate of return that could be achieved over a given time period in the future from investment in government bonds today.

(b) The theoretical rate of return that could be achieved between the current time and a given future time from investment in government bonds.

(c) The gross redemption yield that could be theoretically achieved by investing in government bonds of different terms to redemption. The yield curve represents a statistical average gross redemption yield.

(ii)

Time	*Government bond yield*	*Valuation rate of interest*	*P.V factor*
1	0.02	0.031	0.96993
2	0.04	0.052	0.90358
3	0.06	0.073	0.80947
4	0.08	0.094	0.69812
5	0.1	0.115	0.58026

$$PV = 10(0.96993 + 0.90358 + 0.80947 + 0.69812 + 0.58026) + 100 \times 0.58026$$

$= 97.6396$.

(iii) GRY is such that: $97.6396 = 10a_{\overline{5}|} + 100v^5$

Try 11% $a_{\overline{5}|} = 3.69590$ $v^5 = 0.59345$ RHS = 96.30397

Try 10% $a_{\overline{5}|} = 3.7908$ $v^5 = 0.62092$ RHS = 100 [calculation not necessary]

Interpolate to find i:

$$i = -\frac{97.6396 - 96.30397}{100 - 96.30397} \times 0.01 + 0.11$$

$\Rightarrow i = 0.10639$ or 10.64%

(iv) It is reasonable for the investor to price a corporate bond with reference to the rates of return from government bonds which may be (more or less) risk free.

A risk premium will then need to be added.

It is also not unreasonable that this risk premium rises with term as the uncertainty regarding credit risk rises.

This question proved to be the most difficult on the paper. The examiners had anticipated that some candidates would have difficulty with part (i) but it was disappointing to see the number of candidates who were unable to give even a basic description of a spot rate and a forward rate. Part (iv) was also very poorly answered and whilst it had been anticipated that only the *strongest candidates would make all the relevant points, the examiners were surprised at how many candidates failed to score any marks on this part.*

10 (i) The payback period measures the earliest time at which the project breaks even but takes no account either of interest on borrowings or on cash flows received after the payback period. It is therefore a poor measure of ultimate profitability.

(ii) The present value of preparation costs is (in £m):

$2\bar{a}_{\overline{2|}}$ @ 4% per annum effective.

$$= 2.\frac{i}{\delta}.a_{\overline{2|}} \qquad \frac{i}{\delta} = 1.019869 \quad a_{\overline{2|}} = 1.8861$$

$$= 2 \times 1.019869 \times 1.8861 = 3.847$$

The present value the stadium building costs is (in £m):

$$200v^{4\frac{1}{2}} + 200 \times 1.05v^{5\frac{1}{2}} + 200 \times 1.05^2 v^{6\frac{1}{2}} + \ldots + 200 \times 1.05^9 v^{13\frac{1}{2}}$$

$$200v^{4\frac{1}{2}}\left(1 + 1.05v + 1.05^2 v^2 + \ldots + 1.05^9 v^9\right)$$

$$= 200v^{4\frac{1}{2}}\left[\frac{1 - 1.05^{10} v^{10}}{1 - 1.05v}\right]$$

with $v = 0.96154 \quad v^{10} = 0.67556 \quad 1.05^{10} = 1.62889 \quad v^{4\frac{1}{2}} = 0.83820$

$$= 200 \times 0.83820 \times \left(\frac{1 - 1.62889 \times 0.67556}{1 - 1.05 \times 0.96154}\right)$$

$$= £1,750.837$$

Present value of admin. costs is (£m):

$$100\ddot{a}_{\overline{2|}}^{(12)} v^{13} \qquad @\ 4\%$$

with $\dfrac{i}{d^{(12)}} = 1.021537 \; v^{13} = 0.60057 \; a_{\overline{2|}} = 1.8861$

$$= 100 \times 1.021537 \times 1.8861 \times 0.60057$$

$$= 115.714$$

Present value of revenue (£m):

$3,300\bar{a}_{\overline{1}|}v^{14}$ with $\frac{i}{\delta} = 1.019869$ $a_{\overline{1}|} = 0.9615$ $v^{14} = 0.57748$

$= 3,300 \times 1.019869 \times 0.9615 \times 0.57748$

= 1,868.781

NPV = 1,868.781 – 115.714 – 1,750.837 – 3.847 = –£1.617m.

Therefore should not make a bid.

(iii) One way of dealing with this would be to multiply the NPV of all the revenues and costs that are only received if the bid is won by 0.1.

The costs of preparing the bid would be incurred for certain and therefore not multiplied by 0.1. This adjustment would make it less likely the bid will go ahead because the only certain item is a cost.

This question contained a potential ambiguity regarding the timing of the administration costs. Although the examiners felt that the approach given in the model solution was the most logical, candidates who assumed that the administration costs were only payable during 2025 were given full credit. This question was answered well and it was very pleasing to see that (a) candidates managed their time efficiently and so left enough time to make a good attempt at the question with the most marks and (b) candidates who made calculation errors still clearly explained their method and so were able to pick up significant marks for their working.

END OF EXAMINER'S REPORT