

INSTITUTE AND FACULTY OF ACTUARIES

EXAMINATION

15 April 2013 (pm)

Subject CT1 – Financial Mathematics Core Technical

Time allowed: Three hours

INSTRUCTIONS TO THE CANDIDATE

1. *Enter all the candidate and examination details as requested on the front of your answer booklet.*
2. *You must not start writing your answers in the booklet until instructed to do so by the supervisor.*
3. *Mark allocations are shown in brackets.*
4. *Attempt all 10 questions, beginning your answer to each question on a separate sheet.*
5. *Candidates should show calculations where this is appropriate.*

Graph paper is NOT required for this paper.

AT THE END OF THE EXAMINATION

Hand in BOTH your answer booklet, with any additional sheets firmly attached, and this question paper.

<p><i>In addition to this paper you should have available the 2002 edition of the Formulae and Tables and your own electronic calculator from the approved list.</i></p>
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1 The value of the assets held by an investment fund on 1 January 2012 was £1.3 million.

On 30 September 2012, the value of the assets was £1.9 million.

On 1 October 2012, there was a net cash outflow from the fund of £0.9 million.

On 31 December 2012, the value of the assets was £0.8 million.

- (i) Calculate the annual effective time-weighted rate of return (TWRR) for 2012. [2]
 - (ii) Calculate the annual effective money-weighted rate of return (MWRR) for 2012 to the nearest 1%. [3]
 - (iii) Explain why the MWRR is significantly higher than the TWRR. [2]
- [Total 7]

2 (i) Explain the main difference:

- (a) between options and futures.
- (b) between call options and put options.

[4]

- (ii) A one-year forward contract is issued on 1 April 2013 on a share with a price at that date of £10.50. Dividends of £1.10 per share are expected on 30 September 2013 and 31 March 2014. On 1 April 2013, the 6-month risk-free spot rate of interest is 4.5% per annum convertible half-yearly and the 12-month risk-free spot rate of interest is 5% per annum convertible half-yearly.

Calculate the forward price at issue, stating any further assumptions made. [4]
[Total 8]

3 Three bonds each paying annual coupons in arrear of 6% and redeemable at £103 per £100 nominal reach their redemption dates in exactly one, two and three years' time, respectively. The price of each bond is £97 per £100 nominal.

- (i) Calculate the gross redemption yield of the 3-year bond. [3]
 - (ii) Calculate the one-year and two-year spot rates implied by the information given. [3]
- [Total 6]

4 An investor is interested in purchasing shares in a particular company.

The company pays annual dividends, and a dividend payment of 30 pence per share has just been made.

Future dividends are expected to grow at the rate of 5% per annum compound.

- (i) Calculate the maximum price per share that the investor should pay to give an effective return of 9% per annum. [4]
- (ii) Without doing any further calculations, explain whether the maximum price paid will be higher, lower or the same if:
 - (a) after consulting the managers of the company, the investor increases his estimate of the rate of growth of future dividends to 6% per annum.
 - (b) as a result of a government announcement, the general level of future price inflation in the economy is now expected to be 2% per annum higher than previously assumed.
 - (c) general economic uncertainty means that, whilst the investor still estimates future dividends will grow at 5% per annum, he is now much less sure about the accuracy of this assumption.

You should consider the effect of each change separately. [6]
[Total 10]

5 The force of interest per unit time at time t , $\delta(t)$, is given by:

$$\delta(t) = \begin{cases} 0.1 - 0.005t & \text{for } t < 6 \\ 0.07 & \text{for } t \geq 6 \end{cases}$$

- (i) Calculate the total accumulation at time 10 of an investment of £100 made at time 0 and a further investment of £50 made at time 7. [4]
- (ii) Calculate the present value at time 0 of a continuous payment stream at the rate $£50e^{0.05t}$ per unit time received between time 12 and time 15. [5]
[Total 9]

- 6** A cash sum of £10,000 is invested in a fund and held for 15 years. The yield on the investment in any year will be 5% with probability 0.2, 7% with probability 0.6 and 9% with probability 0.2, and is independent of the yield in any other year.
- (i) Calculate the mean accumulation at the end of 15 years. [2]
 - (ii) Calculate the standard deviation of the accumulation at the end of 15 years. [5]
 - (iii) Without carrying out any further calculations, explain how your answers to parts (i) and (ii) would change (if at all) if:
 - (a) the yields had been 6%, 7% and 8% instead of 5%, 7%, and 9% per annum, respectively.
 - (b) the investment had been held for 13 years instead of 15 years.
- [4]
[Total 11]

- 7** An insurance company has liabilities of £6 million due in 8 years' time and £11 million due in 15 years' time. The assets consist of two zero-coupon bonds, one paying £X in 5 years' time and the other paying £Y in 20 years' time. The current interest rate is 8% per annum effective. The insurance company wishes to ensure that it is immunised against small changes in the rate of interest.
- (i) Determine the values of £X and £Y such that the first two conditions for Redington's immunisation are satisfied. [8]
 - (ii) Demonstrate that the third condition for Redington's immunisation is also satisfied. [2]
- [Total 10]

- 8** A car manufacturer is to develop a new model to be produced from 1 January 2016 for six years until 31 December 2021. The development costs will be £19 million on 1 January 2014, £9 million on 1 July 2014 and £5 million on 1 January 2015.

It is assumed that 6,000 cars will be produced each year from 2016 onwards and that all will be sold.

The production cost per car will be £9,500 during 2016 and will increase by 4% each year with the first increase occurring in 2017. All production costs are assumed to be incurred at the beginning of each calendar year.

The sale price of each car will be £12,600 during 2016 and will also increase by 4% each year with the first increase occurring in 2017. All revenue from sales is assumed to be received at the end of each calendar year.

- (i) Calculate the discounted payback period at an effective rate of interest of 9% per annum. [9]
 - (ii) Without doing any further calculations, explain whether the discounted payback period would be greater than, equal to, or less than the period calculated in part (i) if the effective rate of interest were substantially less than 9% per annum. [2]
- [Total 11]

- 9** A fixed-interest security pays coupons of 8% per annum half yearly on 1 January and 1 July. The security will be redeemed at par on any 1 January from 1 January 2017 to 1 January 2022 inclusive, at the option of the borrower.

An investor purchased a holding of the security on 1 May 2011, at a price which gave him a net yield of at least 6% per annum effective. The investor pays tax at 30% on interest income and 25% on capital gains.

On 1 April 2013 the investor sold the holding to a fund which pays no tax at a price to give the fund a gross yield of at least 7% per annum effective.

- (i) Calculate the price per £100 nominal at which the investor bought the security. [5]
 - (ii) Calculate the price per £100 nominal at which the investor sold the security. [3]
 - (iii) Show that the effective net yield that the investor obtained on the investment was between 8% and 9% per annum. [6]
- [Total 14]

- 10** A loan is repayable by annual instalments in arrear for 20 years. The initial instalment is £5,000, with each subsequent instalment decreasing by £200.

The effective rate of interest over the period of the loan is 4% per annum.

(i) Calculate the amount of the original loan. [3]

(ii) Calculate the capital repayment in the 12th instalment. [3]

After the 12th instalment is paid, the borrower and lender agree to a restructuring of the debt.

The £200 reduction per year will no longer continue. Instead, future instalments will remain at the level of the 12th instalment and the remaining term of the debt will be shortened. The final payment will then be a reduced amount which will clear the debt.

- (iii) (a) Calculate the remaining term of the revised loan.
(b) Calculate the amount of the final reduced payment.
(c) Calculate the total interest paid during the term of the loan.

[8]

[Total 14]

END OF PAPER