

INSTITUTE AND FACULTY OF ACTUARIES

EXAMINATION

24 April 2012 (am)

Subject CT1 – Financial Mathematics Core Technical

Time allowed: Three hours

INSTRUCTIONS TO THE CANDIDATE

1. *Enter all the candidate and examination details as requested on the front of your answer booklet.*
2. *You must not start writing your answers in the booklet until instructed to do so by the supervisor.*
3. *Mark allocations are shown in brackets.*
4. *Attempt all 10 questions, beginning your answer to each question on a separate sheet.*
5. *Candidates should show calculations where this is appropriate.*

Graph paper is NOT required for this paper.

AT THE END OF THE EXAMINATION

Hand in BOTH your answer booklet, with any additional sheets firmly attached, and this question paper.

<p><i>In addition to this paper you should have available the 2002 edition of the Formulae and Tables and your own electronic calculator from the approved list.</i></p>
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- 1** In a particular bond market, n -year spot rates can be approximated by the function $0.06 - 0.02e^{-0.1n}$.
- (i) Calculate the gross redemption yield for a 3-year bond which pays coupons of 3% annually in arrear, and is redeemed at par. Show all workings. [6]
 - (ii) Calculate the 4-year par yield. [3]
- [Total 9]
- 2** The value of the assets held by an investment fund on 1 January 2011 was £2.3 million.
- On 30 April 2011, the value of the assets had risen to £2.9 million and, on 1 May 2011, there was a net cash inflow to the fund of £1.5 million. On 31 December 2011, the value of the assets was £4.2 million.
- (i) Calculate the annual effective time-weighted rate of return (TWRR) for 2011. [2]
 - (ii) Calculate, to the nearer 0.1%, the annual effective money-weighted rate of return (MWRR) for 2011. [4]
 - (iii) Explain why the TWRR is significantly higher than the MWRR for 2011. [2]
- [Total 8]
- 3** A company has borrowed £500,000 from a bank. The loan is to be repaid by level instalments, payable annually in arrear for ten years from the date the loan is made. The annual instalments are calculated at an effective rate of interest of 9% per annum.
- (i) Calculate:
 - (a) the amount of the level annual instalments.
 - (b) the total amount of interest which will be paid over the ten-year term. [3]
- At the beginning of the eighth year, immediately after the seventh instalment has been made, the company asks for the loan to be rescheduled over a further four years from that date. The bank agrees to do this on condition that the rate of interest is increased to an effective rate of 12% per annum for the term of the rescheduled instalments and that repayments are made quarterly in arrear.
- (ii)
 - (a) Calculate the amount of the new quarterly instalment.
 - (b) Calculate the interest content of the second quarterly instalment of the rescheduled loan repayments. [5]
- [Total 8]

- 4** (i) Explain what is meant by the “no arbitrage” assumption in financial mathematics. [2]

An investor entered into a long forward contract for a security four years ago and the contract is due to mature in five years' time. The price of the security was £7.20 four years ago and is now £10.45. The risk-free rate of interest can be assumed to be 2.5% per annum effective throughout the nine-year period.

- (ii) Calculate, assuming no arbitrage, the value of the contract now if the security will pay dividends of £1.20 annually in arrear until maturity of the contract. [3]
- (iii) Calculate, assuming no arbitrage, the value of the contract now if the security has paid and will continue to pay annually in arrear a dividend equal to 3% of the market price of the security at the time of payment. [3]
- [Total 8]

- 5** An investor is considering two projects, Project A and Project B. Project A involves the investment of £1,309,500 in a retail outlet. Rent is received quarterly in arrear for 25 years, at an initial rate of £100,000 per annum. It is assumed that the rent will increase at a rate of 5% per annum compound, but with increases taking place every five years. Maintenance and other expenses are incurred quarterly in arrear, at a rate of £12,000 per annum. The retail outlet reverts to its original owner after 25 years for no payment.

Project B involves the purchase of an office building for £1,000,000. The rent is to be received quarterly in advance at an initial rate of £85,000 per annum. It is assumed that the rent will increase to £90,000 per annum after 20 years. There are no maintenance or other expenses. After 25 years the property reverts to its original owner for no payment.

- (i) Show that the internal rate of return for project A is 9% per annum effective. [5]
- (ii) Calculate the annual effective internal rate of return for Project B. Show your working. [4]
- (iii) Discuss the extent to which the answers to parts (i) and (ii) above will influence the investor's decision over which project to choose. [3]
- [Total 12]

- 6** A fixed-interest bond pays annual coupons of 5% per annum in arrear on 1 March each year and is redeemed at par on 1 March 2025.

On 1 March 2007, immediately after the payment of the coupon then due, the gross redemption yield was 3.158% per annum effective.

- (i) Calculate the price of the bond per £100 nominal on 1 March 2007. [3]

On 1 March 2012, immediately after the payment of the coupon then due, the gross redemption yield on the bond was 5% per annum.

- (ii) State the new price of the bond per £100 nominal on 1 March 2012. [1]

A tax-free investor purchased the bond on 1 March 2007, immediately after payment of the coupon then due, and sold the bond on 1 March 2012, immediately after payment of the coupon then due.

- (iii) Calculate the gross annual rate of return achieved by the investor over this period. [2]

- (iv) Explain, without doing any further calculations, how your answer to part (iii) would change if the bond were due to be redeemed on 1 March 2035 (rather than 1 March 2025). You may assume that the gross redemption yield at both the date of purchase and the date of sale remains the same as in parts (i) and (ii) above. [3]

[Total 9]

- 7** The annual yields from a fund are independent and identically distributed. Each year, the distribution of $1 + i$ is log-normal with parameters $\mu = 0.05$ and $\sigma^2 = 0.004$, where i denotes the annual yield on the fund.

- (i) Calculate the expected accumulation in 20 years' time of an annual investment in the fund of £5,000 at the beginning of each of the next 20 years. [5]

- (ii) Calculate the probability that the accumulation of a single investment of £1 made now will be greater than its expected value in 20 years' time. [5]

[Total 10]

8 The force of interest, $\delta(t)$, at time t is given by:

$$\delta(t) = \begin{cases} 0.04 + 0.003t^2 & \text{for } 0 < t \leq 5 \\ 0.01 + 0.03t & \text{for } 5 < t \leq 8 \\ 0.02 & \text{for } t > 8 \end{cases}$$

- (i) Calculate the present value (at time $t = 0$) of an investment of £1,000 due at time $t = 10$. [4]
 - (ii) Calculate the constant rate of discount per annum convertible quarterly, which would lead to the same present value as that in part (i) being obtained. [2]
 - (iii) Calculate the present value (at time $t = 0$) of a continuous payment stream payable at the rate of $100e^{0.01t}$ from time $t = 10$ to $t = 18$. [4]
- [Total 10]

9 An ordinary share pays dividends on each 31 December. A dividend of 35p per share was paid on 31 December 2011. The dividend growth is expected to be 3% in 2012, and a further 5% in 2013. Thereafter, dividends are expected to grow at 6% per annum compound in perpetuity.

- (i) Calculate the present value of the dividend stream described above at a rate of interest of 8% per annum effective for an investor holding 100 shares on 1 January 2012. [4]

An investor buys 100 shares for £17.20 each on 1 January 2012. He expects to sell the shares for £18 on 1 January 2015.

- (ii) Calculate the investor's expected real rate of return.

You should assume that dividends grow as expected and use the following values of the inflation index:

Year:	2012	2013	2014	2015
Inflation index at start of year:	110.0	112.3	113.2	113.8

[5]
[Total 9]

10 A company has the following liabilities:

- annuity payments of £200,000 per annum to be paid annually in arrear for the next 20 years
- a lump sum of £300,000 to be paid in 15 years

The company wishes to invest in two fixed-interest securities in order to immunise its liabilities.

Security A has a coupon rate of 9% per annum and a term to redemption of 12 years. Security B has a coupon rate of 4% per annum and a term to redemption of 30 years.

Both securities are redeemable at par and pay coupons annually in arrear. The rate of interest is 8% per annum effective.

- (i) Calculate the present value of the liabilities. [3]
- (ii) Calculate the discounted mean term of the liabilities. [4]
- (iii) Calculate the nominal amount of each security that should be purchased so that Redington's first two conditions for immunisation against small changes in the rate of interest are satisfied for this company. [8]
- (iv) Describe the further calculations that will be necessary to determine whether the company is immunised against small changes in the rate of interest. [2]

[Total 17]

END OF PAPER