

INSTITUTE AND FACULTY OF ACTUARIES

EXAMINATION

3 October 2012 (am)

Subject CT1 – Financial Mathematics Core Technical

Time allowed: Three hours

INSTRUCTIONS TO THE CANDIDATE

1. *Enter all the candidate and examination details as requested on the front of your answer booklet.*
2. *You must not start writing your answers in the booklet until instructed to do so by the supervisor.*
3. *Mark allocations are shown in brackets.*
4. *Attempt all 10 questions, beginning your answer to each question on a separate sheet.*
5. *Candidates should show calculations where this is appropriate.*

Graph paper is NOT required for this paper.

AT THE END OF THE EXAMINATION

Hand in BOTH your answer booklet, with any additional sheets firmly attached, and this question paper.

<p><i>In addition to this paper you should have available the 2002 edition of the Formulae and Tables and your own electronic calculator from the approved list.</i></p>
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- 1** An investor is considering two investments. One is a 91-day deposit which pays a rate of interest of 4% per annum effective. The second is a treasury bill.
- Calculate the annual simple rate of discount from the treasury bill if both investments are to provide the same effective rate of return. [3]
- 2** The nominal rate of discount per annum convertible quarterly is 8%.
- (i) Calculate the equivalent force of interest. [1]
 - (ii) Calculate the equivalent effective rate of interest per annum. [1]
 - (iii) Calculate the equivalent nominal rate of discount per annum convertible monthly. [2]
- [Total 4]
- 3** An investment fund is valued at £120m on 1 January 2010 and at £140m on 1 January 2011. Immediately after the valuation on 1 January 2011, £200m is paid into the fund. On 1 July 2012, the value of the fund is £600m.
- (i) Calculate the annual effective time-weighted rate of return over the two-and-a-half year period. [3]
 - (ii) Explain why the money-weighted rate of return would be higher than the time-weighted rate of return. [2]
- [Total 5]
- 4** A ten-month forward contract was issued on 1 September 2012 for a share with a price of £10 at that date. Dividends of £1 per share are expected on 1 December 2012, 1 March 2013 and 1 June 2013.
- (i) Calculate the forward price assuming a risk-free rate of interest of 8% per annum convertible half-yearly and no arbitrage. [4]
 - (ii) Explain why it is not necessary to use the expected price of the share at the time the forward matures in the calculation of the forward price. [2]
- [Total 6]

- 5** (i) State the characteristics of a Eurobond [4]
- (ii) (a) State the characteristics of a certificate of deposit.
- (b) Two certificates of deposit issued by a given bank are being traded. A one-month certificate of deposit provides a rate of return of 12 per cent per annum convertible monthly. A two-month certificate of deposit provides a rate of return of 24 per cent per annum convertible monthly.
- Calculate the forward rate of interest per annum convertible monthly in the second month, assuming no arbitrage. [4]
- [Total 8]
- 6** A loan is to be repaid by an increasing annuity. The first repayment will be £200 and the repayments will increase by £100 per annum. Repayments will be made annually in arrear for ten years. The repayments are calculated using a rate of interest of 6% per annum effective.
- (i) Calculate the amount of the loan [2]
- (ii) (a) Calculate the interest component of the seventh repayment.
- (b) Calculate the capital component of the seventh repayment. [4]
- (iii) Immediately after the seventh repayment, the borrower asks to have the original term of the loan extended to fifteen years and wishes to repay the outstanding loan using level annual repayments. The lender agrees but changes the interest rate at the time of the alteration to 8% per annum effective.
- Calculate the revised annual repayment. [3]
- [Total 9]

7 An individual wishes to make an investment that will pay out £200,000 in twenty years' time. The interest rate he will earn on the invested funds in the first ten years will be either 4% per annum with probability of 0.3 or 6% per annum with probability 0.7. The interest rate he will earn on the invested funds in the second ten years will also be either 4% per annum with probability of 0.3 or 6% per annum with probability 0.7. However, the interest rate in the second ten year period will be independent of that in the first ten year period.

- (i) Calculate the amount the individual should invest if he calculates the investment using the expected annual interest rate in each ten year period. [2]
 - (ii) Calculate the expected value of the investment in excess of £200,000 if the amount calculated in part (i) is invested. [3]
 - (iii) Calculate the range of the accumulated amount of the investment assuming the amount calculated in part (i) is invested. [2]
- [Total 7]

8 The force of interest, $\delta(t)$, is a function of time and at any time t , measured in years, is given by the formula

$$\delta(t) = \begin{cases} 0.03 + 0.01t & \text{for } 0 \leq t \leq 9 \\ 0.06 & \text{for } 9 < t \end{cases}$$

- (i) Derive, and simplify as far as possible, expressions for $v(t)$ where $v(t)$ is the present value of a unit sum of money due at time t . [5]
- (ii) (a) Calculate the present value of £5,000 due at the end of 15 years.
- (b) Calculate the constant force of interest implied by the transaction in part (a). [4]

A continuous payment stream is received at rate $100e^{-0.02t}$ units per annum between $t = 11$ and $t = 15$.

- (iii) Calculate the present value of the payment stream. [4]
- [Total 13]

- 9 (i) Describe three theories that have been put forward to explain the shape of the yield curve. [7]

The government of a particular country has just issued five bonds with terms to redemption of one, two, three, four and five years respectively. The bonds are redeemed at par and have coupon rates of 4% per annum payable annually in arrear.

- (ii) Calculate the duration of the one-year, three-year and five-year bonds at a gross redemption yield of 5% per annum effective. [6]
- (iii) Explain why a five-year bond with a coupon rate of 8% per annum would have a lower duration than a five-year bond with a coupon rate of 4% per annum. [2]

Four years after issue, immediately after the coupon payment then due the government is anticipating problems servicing its remaining debt. The government offers two options to the holders of the bond with an original term of five years:

Option 1: the bond is repaid at 79% of its nominal value at the scheduled time with no final coupon payment being paid.

Option 2: the redemption of the bond is deferred for seven years from the original redemption date and the coupon rate reduced to 1% per annum for the remainder of the existing term and the whole of the extended term.

Assume the bonds were issued at a price of £95 per £100 nominal.

- (iv) Calculate the effective rate of return per annum from Options 1 and 2 over the total life of the bond and determine which would provide the higher rate of return. [6]
- (v) Suggest two other considerations that bond holders may wish to take into account when deciding which options to accept. [2]

[Total 23]

10 Two investment projects are being considered.

- (i) Explain why comparing the two discounted payback periods or comparing the two payback periods are not generally appropriate ways to choose between two investment projects. [3]

The two projects each involve an initial investment of £3m. The incoming cash flows from the two projects are as follows:

Project A

In the first year, Project A generates cash flows of £0.5m. In the second year it will generate cash flows of £0.55m. The cash flows generated by the project will continue to increase by 10% per annum until the end of the sixth year and will then cease. Assume that all cash flows are received in the middle of the year.

Project B

Project B generates cash flows of £0.64m per annum for six years. Assume that all cash flows are received continuously throughout the year.

- (ii) (a) Calculate the payback period from Project B. [5]
- (b) Calculate the discounted payback period from Project B at a rate of interest of 4% per annum effective. [5]
- (iii) Show that there is at least one “cross-over point” for Projects A and B between 0% per annum effective and 4% per annum effective where the cross-over point is defined as the rate of interest at which the net present value of the two projects is equal. [6]
- (iv) Calculate the duration of the incoming cash flows from Projects A and B at a rate of interest of 4% per annum effective. [6]
- (v) Explain why the net present value of Project A appears to fall more rapidly than the net present value of Project B as the rate of interest increases. [2]
- [Total 22]

END OF PAPER