

INSTITUTE AND FACULTY OF ACTUARIES

EXAMINATION

1 October 2014 (am)

Subject CT5 – Contingencies Core Technical

Time allowed: Three hours

INSTRUCTIONS TO THE CANDIDATE

1. *Enter all the candidate and examination details as requested on the front of your answer booklet.*
2. *You must not start writing your answers in the booklet until instructed to do so by the supervisor.*
3. *Mark allocations are shown in brackets.*
4. *Attempt all 14 questions, beginning your answer to each question on a new page.*
5. *Candidates should show calculations where this is appropriate.*

Graph paper is NOT required for this paper.

AT THE END OF THE EXAMINATION

Hand in BOTH your answer booklet, with any additional sheets firmly attached, and this question paper.

<i>In addition to this paper you should have available the 2002 edition of the Formulae and Tables and your own electronic calculator from the approved list.</i>

- 1** Define Time Selection giving a distinct example. [2]
- 2** With respect to life assurance contracts, discuss the possible effects of policy lapses on subsequent mortality experience. [3]
- 3** Calculate ${}_{2.5}q_{75.75}$ using the method of Uniform Distribution of Deaths. [4]
Basis: Mortality PMA92C20
- 4** Calculate ${}_{5|3}q_{40:40}^1$. [4]
Basis: Mortality AM92
- 5** (i) Define in words and give a formula for the area comparability factor F. [2]
(ii) Mortality levels for a certain country have been studied at national and regional level.
Explain the circumstances under which a particular region may have an Area Comparability Factor of 1.5. [2]
[Total 4]
- 6** A pension scheme provides an ill-health retirement pension of $1/80$ of Final Pensionable Salary for each year of company service limited to a maximum of 50% of Final Pensionable Salary, with fractions of a year to count proportionately. Retirement due to ill-health may take place at any time before age 65 the normal retirement age.
Final Pensionable Salary is defined as the average annual salary over the three-year period preceding retirement.
Calculate the present expected value of past and future service ill-health benefits for a male now aged 30 exact who has a current salary of £30,000 per annum and has 10 years past service. His salary will increase in 1 year's time.
Basis:
PEN Tables in the Formulae and Tables for Actuarial Examinations – Interest 4% per annum [4]

- 7** A life aged 40 exact purchases an endowment assurance policy whereby the sum assured on survival at age 60 exact is £20,000 and the benefit payable on death during the term is £10,000. Death benefits are payable at the end of the year of death.

Calculate the expected present value and variance of the benefits under this policy.

Basis: Mortality AM92 Select
 Interest 4% per annum
 Expenses Ignore

[6]

- 8** (i) In the context of random variables define T_x and K_x . [2]

(ii) State the random variable for the following expected values:

(a) \bar{A}_x

(b) a_x

(c) $A_{[x]:\overline{n}|}$

(d) ${}_5|\ddot{a}_x$

[5]

[Total 7]

- 9** A life aged 60 exact purchases a special deferred term assurance policy for an overall term of 20 years.

Under this policy a sum assured of £100,000 is paid on death but only on death from age 65 exact up to the end of the term. On death between age 60 and 65 the benefit is equal to the total premiums paid without interest.

All payments on death are made at the end of the year of death. An annual premium paid in advance is payable for the full 20 year term.

Calculate the annual premium payable. [7]

Basis:

Mortality AM92 Ultimate
 Interest 4% per annum
 Expenses Ignore

- 10** A life insurance company issues a non-profit assurance policy for a term of n years to a life aged x exact.

For $t = 1, 2, \dots, n$:

- The level annual premium payable at the start of year t is P .
- The expense at the start of policy year t is E_t .
- The benefits payable at the end of the t^{th} policy year on death, surrender and survival are D_t , B_t and S_t respectively.
- The rate of interest earned on net cash flows during the t^{th} policy year is i_t .
- The dependent rates of mortality and surrender at age $x+t$ are $(aq)_{x+t}^d$ and $(aq)_{x+t}^w$ respectively.

Assume that the insurance company **does not** set up a reserve for the policy.

- (i) Write down an expression for $(CF)_t$, the accumulation to the end of the t^{th} policy year of the expected net cash flow arising during the t^{th} policy year per policy in force at the start of that year. [2]
- (ii) Derive an expression which could be used to calculate the level annual premium that the company should charge if the company requires the expected net present value of profit on the policy to be zero assuming a risk discount rate of $j\%$ per annum defining any notation used. [3]

Assume that the insurance company **does** set up a reserve ${}_{t-1}V$ for the policy at the start of the t^{th} policy year.

- (iii) Write down an expression for the expected profit at the end of the t^{th} policy year for each policy in force at the start of that year. [2]
- [Total 7]

- 11** A life assurance company has issued whole of life assurance policies over a number of years. Premiums on these policies are payable annually in advance and the sums assured are payable at the end of the year of death.

You are given the following information relating to a group of policies within the portfolio of whole of life assurance policies:

<i>Age exact on 1 January 2013</i>	<i>Sums assured in force on 1 January 2013</i>	<i>Reserves held on 31 December 2013 for policies in force at that date</i>
69	£740,000	£371,000

During 2013, there was 1 death claim (on a policy which was issued on 1 January 2000 for a sum assured of £15,000) arising from this group of policies.

- (i) Calculate the mortality profit or loss for 2013 to the company in respect of this group of policies assuming net premiums are held on the following basis:

Mortality AM92 Ultimate
Interest 4% per annum

[5]

- (ii) Calculate the amount of expected death claims in 2013 for this group of policies. [1]

- (iii) Compare your answer in part (ii) with the amount of actual claims and comment on your answer with reference to your answer in part (i) above.

[2]

[Total 8]

- 12** (i) Calculate the probability that a life now aged 30 exact will die between the ages of 55 and 65 both exact.

Basis: Mortality ELT15 (Males)

[2]

- (ii) Calculate the above probability again assuming the basis below

Basis: Mortality $\mu_x = 0.005e^{0.09(x-20)}$ for $20 \leq x \leq 70$

[7]

[Total 9]

- 13** A life insurance company issues 3-year policies to lives aged 55 exact who are employees of a manufacturer. These policies offer the following benefits during the term of the policy:

- On death whilst still an employee, £200,000 paid at the end of year of death.
- On ill-health retirement, £100,000 paid at the end of the year of retirement.
- On leaving their employer other than on death or ill-health retirement, a return of all premiums paid accumulated with interest at a rate of 2% per annum payable at the end of the year of leaving.
- On survival as an employee at the end of 3 years, £10,000 is payable.

The company uses the following basis to calculate annual premiums for this policy:

Independent rate of mortality	110% of AM92 Ultimate
Interest earned on cash flows	5% per annum
Initial expenses	£150
Renewal expenses	£25 at the start of the second and third policy year
Reserves	None held

In addition, you are given the following independent rates of ill-health retirement and withdrawal. You can assume that the decrements operate uniformly over each year of age in each single decrement table.

<i>Age</i>	<i>Ill-health retirement</i>	<i>Withdrawal</i>
55	0.04	0.10
56	0.05	0.08
57	0.06	0.06

- (i) Calculate the dependent rates of mortality, ill-health retirement and withdrawal for each policy year. [3]

The company sets premiums so that the net present value of the profit for the policy is 5% of the annual premium, using a risk discount rate of 5% per annum.

- (ii) Calculate the level premium payable annually in advance for this policy. [9]
- (iii) Discuss briefly whether the life insurance company needs to hold reserves at the beginning and end of each policy year for this policy. [2]

Assume that the company does hold reserves at the beginning and end of each policy year for this policy and that reserves earn interest at 5% per annum.

- (iv) Explain, without doing further calculations, whether the premium would be higher, the same or lower than that calculated in part (ii) above. [2]
- [Total 16]

- 14** A life insurance company, is proposing to launch a “Low Start” unit-linked endowment policy for a term of 3 years under which premiums increase by a fixed monetary amount each year and are payable yearly in advance throughout the term of the policy or until earlier death. The premium payable and the amount of premium allocated to units in each policy year are as follows:

<i>Policy Year</i>	<i>Premium Payable</i> £	<i>Allocation Rate</i> %
1	1500	50
2	2250	105
3	3000	115

If the policyholder dies during the term of the policy, the death benefit of £6,750 (i.e. the total amount of premiums due to be paid on the policy if held to maturity) or the bid value of the units, whichever is higher, is payable at the end of the policy year of death. The policyholder may surrender the policy only at the end of each policy year. On surrender or on survival to the end of the term, the bid value of the units is payable at the end of the policy year of exit.

The units are subject to a bid-offer spread of 6% and an annual management charge of 1% of the bid value of units is deducted at the end of each policy year. Management charges are deducted from the unit fund before death, surrender and maturity benefits are paid.

You should use the following assumptions in carrying out profit tests of this policy:

Rate of growth on assets in the unit fund	4.5% per annum
Rate of interest on non-unit fund cash flows	2.5% per annum
Mortality	90% AM92 Ultimate
Surrender	7.5% of policies in force at the end of year 1 and 2.5% of policies in force at the end of year 2 then surrender
Initial expenses	£200
Renewal expenses	£55 per annum on the second and third premium dates
Initial commission	5% of first premium
Renewal commission	2.5% of the second and third years' premiums
Claim expense	£75 (payable only on death and surrender)
Risk discount rate	6.5% per annum

- (i) Calculate the profit margin for the policy issued to a life aged 61 exact on the assumption that the company does not set up sterling reserves for this policy. [13]
- (ii) Explain why a life insurance company might need to set up non-unit reserves in respect of a unit-linked life assurance policy. [2]
- (iii) Calculate the profit margin for the policy on the assumption that the company does set up reserves for this policy. [4]

[Total 19]

END OF PAPER