

# EXAMINATION

5 April 2006 (am)

## Subject CT8 — Financial Economics Core Technical

*Time allowed: Three hours*

### ***INSTRUCTIONS TO THE CANDIDATE***

1. *Enter all the candidate and examination details as requested on the front of your answer booklet.*
2. *You must not start writing your answers in the booklet until instructed to do so by the supervisor.*
3. *Mark allocations are shown in brackets.*
4. *Attempt all 9 questions, beginning your answer to each question on a separate sheet.*
5. *Candidates should show calculations where this is appropriate.*

***Graph paper is not required for this paper.***

### ***AT THE END OF THE EXAMINATION***

*Hand in BOTH your answer booklet, with any additional sheets firmly attached, and this question paper.*

*In addition to this paper you should have available the 2002 edition of the Formulae and Tables and your own electronic calculator.*

- 1** An investor can construct a portfolio using only two assets A and B with the following properties:

	<i>A</i>	<i>B</i>
Variance of return	24% %	12% %
Correlation coefficient between assets	0.25	

- (i) Derive a formula for and determine the composition of the investor's minimum variance portfolio. [3]
- (ii) Explain in general terms the benefits of diversification. [3]
- [Total 6]

**2**

<i>State</i>	<i>Asset</i>				<i>Probability of state</i>
	<i>A</i>	<i>B</i>	<i>C</i>	<i>D</i>	
1	5%	5%	5%	2%	0.3
2	4%	7%	5%	6%	0.2
3	7%	3%	5%	9%	0.5
Value of asset	10,000	20,000	n/a	10,000	

The table above gives the returns on all four assets in an investment market under the three possible states of the world.

- (i) Calculate the market price of risk under CAPM. [4]
- (ii) Outline the limitations of CAPM. [3]
- [Total 7]

- 3**
- (i) State five key defining properties of a standard Brownian motion. [5]
- (ii) Outline the advantages and disadvantages of using the continuous time Lognormal model for stock prices by considering both the theoretical features of the model and its consistency with empirical evidence. [6]
- [Total 11]

- 4** (i) List the desirable characteristics of a term structure model. [4]

Under the real world probability measure,  $P$ , the price of a zero coupon bond with maturity  $T$  is given by:

$$B(t, T) = \exp(T(T-t)r_t + \sigma^2(T-t)^3/6)$$

where  $r_t$  is the risk free rate of interest at time  $t$ .

$r_t$  satisfies the following SDE under the real world measure  $P$ :

$$dr_t = \mu r_t dt + \sigma dz_t$$

where  $\mu > 0$  and  $dz_t$  is a standard Brownian motion under  $P$ .

- (ii) Derive:

- (a) the market price of risk and
- (b) the SDE for  $r_t$  under the risk neutral measure  $Q$

[7]

[Total 11]

- 5** (i) Comment on the implications for an assessment of stock market efficiency of the necessity for a ban on the management of a company trading in their company stock particularly during takeover talks. [2]
- (ii) Explain why it is not straight forward to identify when the semi-strong form of stock market efficiency holds. [3]
- (iii) Comment on the implications of stock market efficiency for passive and active fund managers. [2]

[Total 7]

- 6** In the Wilkie model, the force of inflation from time  $t - 1$  to  $t$ ,  $I(t)$ , is modelled as:

$$I(t) = Qmu + QA[I(t - 1) - Qmu] + QSD \cdot QZ(t)$$

where  $QZ(t) \sim N(0, 1)$

and  $Qmu$ ,  $QA$  and  $QSD$  are fixed parameters as follows:

$$Qmu = 0.03$$

$$QA = 0.55$$

$$QSD = 0.45$$

- (i) Calculate the 95% confidence interval for the force of inflation over the following year given inflation over the past year was 2.75%. [4]
  - (ii) Explain an economic justification for using an  $AR(1)$  process for inflation. [1]
  - (iii) Explain whether a model of the form that is used for inflation is also suitable for share prices. [3]
- [Total 8]

- 7**
- (i) Describe the advantages of the martingale approach to derivative valuation compared with an approach based on deriving an appropriate partial differential equation. [3]
  - (ii) State and compare the risk-neutral and state price deflator approaches to valuing derivatives. [4]
- [Total 7]

- 8** An employer contracts with his staff to give each of them 1,000 shares in one year's time provided the share price increased from its current level of £1 to at least £1.50 at the end of the year.

You may assume the following parameters:

- risk free interest rate: 4% p.a. continuously compounded
- stock price volatility: 30% p.a.
- dividend yield: nil

- (i) Calculate the value of the contract with each employee by considering the terms of the Black-Scholes formula, [6]

The employer now wishes to limit the gain to each employee to £2,000.

- (ii) Calculate the value of this revised contract. [6]

- (iii) An employee has said that he believes the original uncapped contract is worth £300. He has determined this by saying that he believes there is a 30% chance of the share price being at least £1.50 therefore  $30\% \times £1 = 30p$ .

- (a) Compare the approach taken by the employee and the approach used in (i).
- (b) Comment on the implications of the differences in (iii) (a) if there were a market in such contracts.

[4]

[Total 16]

**9** Consider a recombining binomial model for the price of a share where:

risk free interest rate  $r = 4\%$  p.a. (equivalent to 0.016% per trading day)  
continuously compounded

volatility  $\sigma = 20\%$  p.a.

initial share price  $S_0 = 100$

the ratio of the share price after an “up jump” compared with the share price before the jump is given by  $u = \exp(\sigma \cdot 250^{-1/2})$

There are 250 trading days per year and ignore dividends except where specifically mentioned.

- (i) Calculate the price at time 0 of a European-style put option with a strike price of 101p that expires in 2 days time. [6]
- (ii) (a) Sketch a graph of the delta of a put option against share price with exercise price 100.  
(b) Explain the key features of the graph. [6]
- (iii) Derive, using the binomial lattice, the price of a European-style call option with exercise price 101p expiring in 2 days time. [1]
- (iv) (a) State the relationship known as “put-call parity”.  
(b) Prove it from first principles. [4]
- (v) Compare the result of (iii) to the result of (i) using put-call parity. [2]

Assume now that an investor holds a call option and a put option, both with exercise prices of 101p, that can be exercised at the investor’s option at the end of day 1 or the end of day 2. At the end of day 1, just before the investor is allowed to exercise the options, the company announces unexpectedly that a dividend of 3p per share will be paid at the end of day 2 immediately prior to the expiry of the options.

- (vi) (a) Construct the binomial lattice of share prices allowing for the dividend payment.  
(b) Explain the conditions under which the holders of the put and call options will exercise at the end of day 1 after the announcement of the dividend. [8]

[Total 27]

**END OF PAPER**