EXAMINATION

17 September 2008 (pm)

Subject ST6 — Finance and Investment Specialist Technical B

Time allowed: Three hours

INSTRUCTIONS TO THE CANDIDATE

1. *Enter all the candidate and examination details as requested on the front of your answer booklet.*

2. *You have 15 minutes at the start of the examination in which to read the questions. You are strongly encouraged to use this time for reading only, but notes may be made. You then have three hours to complete the paper.*

3. *You must not start writing your answers in the booklet until instructed to do so by the supervisor.*

4. *Mark allocations are shown in brackets.*

5. *Attempt all 8 questions, beginning your answer to each question on a separate sheet.*

Graph paper is required for this paper.

AT THE END OF THE EXAMINATION

Hand in BOTH your answer booklet, with any additional sheets firmly attached, and this question paper.

In addition to this paper you should have available the 2002 edition of the Formulae and Tables and your own electronic calculator from the approved list.

NOTE: In this examination, you are never required to prove the use of an arbitrage-free methodology unless clearly stated in the question.

1 A money market is based on annually compounding interest rates.

(i) (a) Derive the arbitrage neutral value of a forward interest rate f_{12} that applies between two future dates t_1 and t_2 (each less than a year from now).

(b) Derive the current value of a forward rate agreement, previously dealt at a rate of k, covering the same period.

[3]

Assume that the price S of a stock paying no dividends follows a geometric Brownian motion process with drift μ and volatility σ, i.e.

$$dS = \mu S\,dt + \sigma S\,dW(t)$$

where $W(t)$ is a standard Brownian motion. Let r be the continuously compounded (positive) risk-free rate.

(ii) Show that the forward price of S (settling at some future time T) also follows a geometric Brownian motion process, but with a drift less than μ. [4]

Consider two currencies, X and Y. The price of currency X expressed in terms of currency Y follows a similar geometric Brownian motion process to the stock in (ii) above, with drift $\mu = r_Y - r_X$, where r_X and r_Y are the continuously compounded risk-free interest rates in currencies X and Y respectively.

(iii) Show that the process followed by the price of currency Y expressed in terms of currency X also follows a geometric Brownian motion process, but with a drift greater than $-\mu$. [5]

[Total 12]

2 Consider an asset that follows a random process S and whose current value (at time 0) is s. Over the next small time step Δt, the price of the asset can change from s to one of two possible values, s_1 or s_2.

Consider also a cash bond B that accumulates at the continuously compounded risk-free rate r over time step Δt from its current value b.

At the end of the time step, a derivative claim on S pays an amount:

$$f(s) = \begin{cases} f(s_1) \text{ if } s \mapsto s_1 \\ f(s_2) \text{ if } s \mapsto s_2 \end{cases}$$

(i) (a) Derive the quantities ϕ of asset S and ψ of bond B which ensure that the portfolio (ϕ, ψ) of asset and bond accumulates to exactly the same amount as the derivative payoff, no matter which value the asset takes on at time Δt.

(b) Hence write down an expression for the current value of the derivative. [4]

(ii) Demonstrate why the value in (i)(b) is the unique arbitrage-free value. [3]
[Total 7]

3 You are managing a portfolio of interest rate options and their underlying instruments, together with short-term money market instruments. The portfolio is Delta neutral in respect of each underlying instrument. Short-term interest rates are constant.

(i) Derive an expression for changes in portfolio value in terms of the "Greeks" (i.e. Delta, Gamma, Vega and Theta) at time t, ignoring terms of higher order than Δt. [2]

A trader holds the following option portfolio based on the December 2008 Eurex German Bund future (contract size €100,000):

Option	*Price now*	*Option Delta*
Sold 500 Calls strike 114	0.76	42%
Sold 500 Puts strike 112	0.41	–27%

The options all expire on 21st November 2008, and the current price of the future is 113.50.

(ii) (a) Sketch a diagram of the profit and loss on the portfolio at expiry of the options against a suitable range of futures prices (assuming no futures hedge).

(b) Add to this graph a line indicating approximately the current profit and loss over the same range. [4]

(iii) (a) Give reasons why the trader might be hoping to gain by taking on this option position.

(b) Determine the futures position that the trader would require to Delta hedge his portfolio at the current market level.

(c) Describe the effect on the trader's profit and loss if the market falls sharply in the next few days, and describe the likely action he would take. You may assume that the portfolio was Delta hedged beforehand.

(d) Indicate whether your answer to (c) would be any different if the fall took place a week before expiry. [6]

[Total 12]

4 For a particular LIBOR-based interest rate curve, the semi-annually compounded forward rates $f(t, t + ½)$, for $t = 0, ½, 1, 1½$ ... etc., are modelled according to the formula:

$$f(t,\ t + \tfrac{1}{2}) = 4 + \frac{\sqrt{t}}{2}$$

where t is in years. (The formula gives f in percent. A semi-annual interest rate is expressed in annual terms but applies for six months.)

For example $f(0, ½) = 4\%$, and since this is semi-annual it would apply at the rate of 2% over the half-year $t = 0$ to ½.

(i) Obtain the first six discount factors (zero coupon bond prices), i.e. from $t = ½$ to $t = 3$. [3]

(ii) By writing down the cashflows, or otherwise, calculate the present value of each of the following:

(a) a 3-year semi-annually resetting Floating Rate Note with a margin of 1% above LIBOR per annum, payable semi-annually

(b) a 3-year fixed rate bond paying an <u>annual</u> coupon of 5.75%

(c) a 3-year interest rate receiver swap, in which the fixed leg has an <u>annual</u> coupon of 5.75% and the floating leg has semi-annual coupons of LIBOR + 1%. [5]

[<u>*Note*</u>*: You may ignore transaction costs, taxes and payment conventions.*]

Two years later, the 3-year receiver swap from (ii)(c) is still in force (with one year to run). Interest rates, as measured by the semi-annual forward rates, have fallen to levels exactly 1% below the path expected in the model, and the regular (net) payment on the swap has just been made.

(iii) Determine whether the market value of the swap is higher or lower than at outset. [2]

[Total 10]

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5 The option desk in a sophisticated investment bank wishes to launch a new product – the Stock Protection Logarithm, or SPLurge.

A SPLurge is similar to a European style option except that the payoff depends on the logarithm (to base e) of the stock price S with a multiplier M. Hence a Call SPLurge with strike K has a payoff equal to:

$$M \max\{ \log_e S - \log_e K, 0 \}$$

You may assume you are in a Black Scholes economic world, where the stock price process follows a lognormal Wiener process with drift μ and volatility σ.

(i) Show how to construct a valuation formula for a Call SPLurge using the probability distribution of the stock price. You do not need to evaluate your formula in detail. [4]

(ii) Assess any new risk factors (e.g. the "Greeks") or modelling features that would be encountered with SPLurge compared with vanilla European options. [4]

(iii) Sketch the approximate expiry payoff of a Put SPLurge (with $K = 100$ and multiplier $M = 50$) together with a European Put of the same strike. [3]
[Total 11]

6 An equity index at current level J pays dividends at a continuously compounded rate δ and has price volatility σ.

(i) Write down the Black Scholes formula for valuing a T-year European Call option on this index with strike K, defining any symbols you use. [2]

A 5-year Guaranteed Equity Investment Bond pays the bondholder 90% of the FTSE100 equity index, subject to a minimum payout of 120% of the starting index level and a maximum payout of 200% of the starting index level.

The FTSE 100 index is currently trading at 6,435, with volatility of 20% and dividend yield of 4%. Interest rates in the UK are 6% continuously compounded. A new Bond is about to be launched.

(ii) Sketch the payout of this new Bond in five years, against possible values the index may take at that time. [4]

(iii) Calculate the fair price of the new Bond at inception. [9]
[Total 15]

7 You work for the Treasury division of a European bank which uses the Black model to value vanilla interest rate options (caps, floors and swaptions). The bank is looking to extend its range of products into more exotic interest-rate swaps and options denominated in Euro, and is considering building a term structure yield curve model for pricing and hedging these instruments.

(i) (a) Outline the reasons for choosing a full yield curve model in this situation.

(b) Set out the main criteria such a model must satisfy.

(c) Explain the particular relevance of a "no arbitrage" condition in this context. [6]

The Hull-White one-factor yield curve model projects the short-term rate r according to the formula:

$$dr = a(t)\big(b(t) - r\big)\,dt + \sigma(t)\,dz$$

where $\sigma(t)$ is the time-dependent short-rate volatility and z is a standard Brownian motion. Your bank has decided to use an implementation that sets parameters $a(t)$ and $\sigma(t)$ as constants a and σ respectively for all t.

(ii) Describe how you would construct a trinomial rate tree for this model, calibrated to the term structure of zero coupon bond prices and caplet volatilities. You do not need to derive the branching equations. [8]

(iii) Discuss the extent to which your Hull-White implementation will fulfil the desirable criteria you identified in (i)(b). [3]

[Total 17]

8 You are a risk analyst for an insurance company that has sold protection on a corporate reference entity via a 3-year credit default swap (CDS).

(i) (a) Describe the contract the insurance company has entered into, explaining how this operates and defining any terminology used.

(b) Give three reasons why a counterparty might purchase this CDS from the insurance company.

[5]

Your research leads you to believe that the 1 year default probability (conditional on no earlier default) for the corporate firm underlying the CDS is 2% during the first year, and increases by 0.25% each year thereafter. You also assume a recovery rate of 40%, and that any default will take place at the mid-point of the year. The risk-free interest rate is 6%, continuously compounded.

(ii) Estimate the theoretical price of the CDS using the information given. [8]

The actual price of the CDS in the market turns out to be higher (in spread terms) than your estimate suggests.

(iii) Give reasons (other than calculation error) why this difference could occur.

[3]
[Total 16]

END OF PAPER