Subject ST6 — Finance and Investment
Specialist Technical B

September 2009

EXAMINERS' REPORT

Introduction

The attached subject report has been written by the Principal Examiner with the aim of helping candidates. The questions and comments are based around Core Reading as the interpretation of the syllabus to which the examiners are working. They have however given credit for any alternative approach or interpretation which they consider to be reasonable.

R D Muckart
Chairman of the Board of Examiners

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Comments for individual questions are given with the solutions that follow.

1

(i) Let $Y_t = \log(Z_t)$.

Then

$$Y_t = \ln(B_t^{-1}S_t) = \ln(B_t^{-1}) + \ln(S_t) = \ln(e^{-rt}) + \ln(S_0 e^{\sigma W_t + \mu t}) = -rt + \ln S_0 + \mu t + \sigma W_t .$$

Hence $dY_t = (\mu - r)dt + \sigma\, dW_t$.

Since $Z_t = \exp(Y_t) = f(Y_t)$, say, apply Ito's formula to Z_t:

$$dZ_t = \left[(\mu - r) f'(Y_t) + \tfrac{1}{2}\sigma^2 f''(Y_t)\right] dt + \left[\sigma f'(Y_t)\right] dW_t$$

Substituting $f'(Y_t) = f''(Y_t) = \exp(Y_t) = Z_t$ leads to

$$dZ_t = (\mu - r + \tfrac{1}{2}\sigma^2)Z_t dt + \sigma Z_t dW_t$$

Note the term $\tfrac{1}{2}\sigma^2$ in the drift. This is very important and arises because of the stochastic nature of the process.

(ii) A portfolio is <u>self-financing</u> that neither requires cash injections nor generates cash withdrawals.

<u>OR</u> A portfolio is <u>self-financing</u> if and only if changes in its value depend only on changes in the prices of the assets constituting the portfolio.

Mathematically: If V_t denotes the value of the portfolio (ϕ_t, ψ_t), then the portfolio is self financing if and only if

$$dV_t = \phi_t dS_t + \psi_t dB_t$$

The amounts ϕ_t and ψ_t must be <u>previsible</u>, i.e. specifiable at time *t*.

A <u>replicating strategy</u> for X is a strategy which involves investing in previsible quantities ϕ_t and ψ_t of stock and risk free bonds respectively, such that the portfolio (ϕ_t, ψ_t) will have terminal value equal to the magnitude of the claim, i.e. $V_T = \phi_T S_T + \psi_T B_T = X$, which means that the portfolio's cashflows at claim exercise date match the cashflows under the claim.

Note that, by definition, any replicating strategy for the portfolio of (ϕ_t, ψ_t) of stocks and bonds will be self-financing.

(iii) We have, from (i), $dZ_t = (\mu - r + \tfrac{1}{2}\sigma^2)Z_t dt + \sigma Z_t dW_t$.

This process has a non-zero drift, i.e. $\mu - r + \tfrac{1}{2}\sigma^2$, so is not a Martingale.

The Cameron-Martin-Girsanov (CMG) theorem tells us that we can turn Z_t into a Martingale because exists a probability measure **Q**, equivalent to the measure **P**, under which Z is a Martingale.

To apply the CMG theorem, set $\gamma = \dfrac{\mu - r + \frac{1}{2}\sigma^2}{\sigma}$.

We note that γ is a bounded pre-visible process, because it is a constant, one of the conditions for CMG to apply.

The CMG theorem tells us that under **Q** the process Z_t is a Martingale:

$$dZ_t = \sigma Z_t \, d\tilde{W}_t$$

where $\tilde{W}_t = W_t + \gamma t$ is a **Q**-measure Brownian motion.

The next step in constructing the replication strategy is to form the discounted expected claim process $E_t = \mathrm{E}_{\mathbf{Q}}[B_T^{-1} X \mid F_t]$.

This too is a **Q**-measure Martingale, since:

$$\mathrm{E}_{\mathbf{Q}}[E_t \mid F_s] = \mathrm{E}_{\mathbf{Q}}[\mathrm{E}_{\mathbf{Q}}[B_T^{-1} X \mid F_t] \mid F_s] = \mathrm{E}_{\mathbf{Q}}[B_T^{-1} X \mid F_s] = E_s .$$

Since both Z_t and E_t are **Q**-Martingales, the Martingale Representation theorem gives us a pre-visible process ϕ_t such that $dE_t = \phi_t dZ_t$.

The replication strategy then consists of holding a portfolio of ϕ_t of stock and $\psi_t = E_t - \phi_t Z_t$ of risk free bonds.

The portfolio replicates the claim because the portfolio is self-financing, i.e. $dV_t = \phi_t dS_t + \psi_t dB_t$.

As the portfolio replicates the claim, the arbitrage-free condition requires that the value of the claim equals the value of the replicating strategy.

Syllabus section: (h) (i)-(iii)

Core reading: Units 8 & 9

This was a straightforward bookwork question. For part (i), few candidates obtained the required solution. Several candidates started with the wrong SDE for S(t). The main point here is the additional $\frac{1}{2}\sigma^2$ term which arises from the stochastic process.

Part (ii) was generally answered well, but part (iii) was not so well answered although CT8 students should be familiar with this. Some candidates mentioned CMG and MRT but did not show (as asked) how these can be used to construct the replication strategy.

2

We have $S_t = S_0 + \sigma W_t$, and since we know the distribution of Brownian motion W_t, we can state that $S_t \sim N(S_0, \sigma^2 t)$.

Hence the p.d.f. of S_t is $\dfrac{1}{\sigma\sqrt{2\pi t}} e^{-(s-S_0)^2 / 2\sigma^2 t}$.

The value of claim $X = \dfrac{1}{\sigma\sqrt{2\pi T}} \displaystyle\int_{-\infty}^{\infty} \max(e^{as} - K, 0) e^{-(s-S_0)^2/2\sigma^2 T} ds$

Substituting $s' = s - S_0$, this $= \dfrac{1}{\sigma\sqrt{2\pi T}} \displaystyle\int_{\frac{1}{a}\ln K - S_0}^{\infty} (e^{a(s'+S_0)} - K) e^{-s'^2/2\sigma^2 T} ds'$.

since $\max(e^{a(s'+S_0)} - K, 0) = 0$ when $\exp(a(s'+S_0)) < K$, i.e. $s' < \dfrac{1}{a}\ln K - S_0$.

i.e. the value of $X = \dfrac{1}{\sigma\sqrt{2\pi T}} \displaystyle\int_{\frac{1}{a}\ln K - S_0}^{\infty} \exp(a(s'+S_0)) \exp\left[\frac{-s'^2}{2\sigma^2 T}\right] ds'$

$$- \frac{1}{\sigma\sqrt{2\pi T}} K \int_{\frac{1}{a}\ln K - S_0}^{\infty} \exp\left[\frac{-s'^2}{2\sigma^2 T}\right] ds'$$

$= A - B$, say, taking the two terms separately.

Now $A = \dfrac{1}{\sigma\sqrt{2\pi T}} \displaystyle\int_{\frac{1}{a}\ln K - S_0}^{\infty} \exp(aS_0 + \tfrac{1}{2}a^2\sigma^2 T) \exp\left[\frac{1}{2\sigma^2 T}\left(-(s' - a\sigma^2 T)^2\right)\right] ds'$

by completing the square, so if we let $x = \dfrac{s' - a\sigma^2 T}{\sigma\sqrt{T}}$, with $dx = \dfrac{ds'}{\sigma\sqrt{T}}$,

then $A = \dfrac{1}{\sqrt{2\pi}} \displaystyle\int_{\frac{\frac{1}{a}\ln K - S_0 - a\sigma^2 T}{\sigma\sqrt{T}}}^{\infty} \exp(aS_0 + \tfrac{1}{2}a^2\sigma^2 T) \exp(-\tfrac{1}{2}x^2) dx$

$= \exp(aS_0 + \tfrac{1}{2}a^2\sigma^2 T)\left[1 - N\left(\dfrac{\frac{1}{a}\ln K - S_0 - a\sigma^2 T}{\sigma\sqrt{T}}\right)\right]$ where N is the cumulative Normal.

Also, if we let $y = \dfrac{s'}{\sigma\sqrt{T}}$, then $dy = \dfrac{ds'}{\sigma\sqrt{T}}$,

$$\text{then } B = \frac{1}{\sqrt{2\pi}} K \int_{\frac{\frac{1}{a}\ln K - S_0}{\sigma\sqrt{T}}}^{\infty} \exp(-\tfrac{1}{2}y^2)dy = K\left[1 - N\left(\frac{\frac{1}{a}\ln K - S_0}{\sigma\sqrt{T}}\right)\right].$$

So value of X

$$= \exp(aS_0 + \tfrac{1}{2}a^2\sigma^2T)\left[1 - N\left(\frac{\frac{1}{a}\ln K - S_0 - a\sigma^2T}{\sigma\sqrt{T}}\right)\right] - K\left[1 - N\left(\frac{\frac{1}{a}\ln K - S_0}{\sigma\sqrt{T}}\right)\right].$$

Syllabus section: (h) (i)-(iii)

Core reading: Units 8 & 9

This short question involved a direct integration of a pdf to obtain the value of a claim. Any complexity should have arisen not from the concept but from the evaluation of the integral, which had to be done in two parts and was algebraically a little fiddly.

Those who knew the techniques got high marks even if they slipped up a little on the detail of the integrals. A few candidates used a formula from page 18 of the 'Tables and Formulae' to obtain a closed form expression for the integrals.

3

(i) S

a. The forward swap rate Y (in %) satisfies:

$$100e^{-2\times 0.044} = Y\left[e^{-3\times 0.046} + e^{-4\times 0.048} + e^{-5\times 0.05}\right] + 100e^{-5\times 0.05}$$

Solving this, Y = 5.533%.

The question specifically states that interest rates are continuously compounded. If annually compounded rates are assumed instead, the swap rate becomes (incorrectly) 5.39%.

b. Because the 5.75% rate within the forward agreement is higher than the forward swap rate, the forward agreement has <u>negative</u> value to the financial institution.

(ii) The cost of a perfect hedge is equal to the liability that the forward agreement represents to the financial institution.

This is given by:

$$1{,}000{,}000 \times \frac{1}{100} \; 5.75\left[e^{-3\times 0.046} + e^{-4\times 0.048} + e^{-5\times 0.05}\right] + 100e^{-5\text{x}0.05} - 100e^{-2\times 0.044}$$

$$[\textit{or equivalently } 1{,}000{,}000 \times \frac{(5.75 - Y)}{100} \times \left[e^{-3\times 0.046} + e^{-4\times 0.048} + e^{-5\times 0.05}\right] \;]$$

= €5,371.

(iii) The financial institution needs to buy a receiver swaption with:

- Maturity 2 years
- Strike 5.75% pa
- Notional €1m
- Swap length 3 years
- Annual tenor

The swaption can be priced using Black's formula:

Price = $XA[KN(-d_2) - YN(-d_1)]$

Where

X = notional, K = strike, Y = forward swap (fixed rate),

A = value of payment of 1 for each period of the swaption life

$$d_1 = \frac{\ln \left. Y \middle/ K \right. + \frac{1}{2}\sigma^2 T}{\sigma\sqrt{T}}, \quad d_2 = d_1 - \sigma\sqrt{T}$$

In this case:

$$A = \left[e^{-3\times0.046} + e^{-4\times0.048} + e^{-5\times0.05} \right] = 2.47521$$

$$d_1 = \frac{\ln \left. 0.0553 \middle/ 0.0575 \right. + \frac{1}{2}(0.12)^2 2}{0.12\sqrt{2}} = -0.14183$$

so $N(-d_1)$ = 0.55639

and $d_2 = d_1 - 0.12\sqrt{2} = -0.31154$

so $N(-d_2)$ = 0.62230

Hence the price of the swaption is:

1,000,000 × 2.47521 × [5.75% × 0.62230 – 5.533% × 0.55639] = €12,369

(iv) Rather than buying a receiver swaption, the financial institution could:

- Pay a premium to enter into a forward-starting swap commencing at end year 2, running for 3 years paying floating rates but receiving a fixed rate of 5.75%
- Buy a payer swaption with:
  - Maturity 2 years
  - Strike 5.75% pa
  - Notional €1m
  - Swap length 3 years
  - Annual tenor

Syllabus section: (e) & (j)

Core reading: Units 5, 13

This question, which focused on the fixed income swap market, is typical of several on hedging strategies which compare using a future (or forward) and using an option (or options). It appears that many students did not fully understand the differences.

In part (i), several candidates lost marks by not reading that the interest rates in the question were continuously-compounded.

Parts (ii) and (iii) were answered well, although not many candidates got the right numerical solution. Part (iv) invited the application of Put-Call parity but was not well attempted. One key point is that the Put-Call parity equivalent of buying an option must still involve <u>buying</u> an option, not selling one, because overall Gamma and Vega sensitivities are maintained, and cash / futures / forwards do not contribute any (or at most only second order amounts).

4

(i) K $f(t,T_1,T_2) = \dfrac{\ln P(t,T_1) - \ln P(t,T_2)}{T_2 - T_1}$

$$F(t,T) = -\frac{\partial \ln P(t,T)}{\partial T}$$

$$Z(t,T) = -\frac{\ln P(t,T)}{T-t}$$

(ii)

a. Using Ito's lemma, $d(\ln P) = (r - \frac{1}{2}\sigma^2 (T-t))dt + \sigma(T-t)^{1/2} dz$, so

$$df = \frac{\frac{1}{2}\sigma^2 (T_2 - T_1)dt + \sigma\{(T_1 - t)^{1/2} - (T_2 - t)^{1/2}\}dz}{T_2 - T_1}$$

$$= \frac{1}{2}\sigma^2 dt + \frac{\sigma\{(T_1 - t)^{1/2} - (T_2 - t)^{1/2}\}}{T_2 - T_1} dz \qquad (*)$$

b. Let T_2 tend towards T_1 in (*):

$$dF = \tfrac{1}{2}\sigma^2 dt - \sigma \frac{\partial (T-t)^{1/2}}{\partial T} dz = \tfrac{1}{2}\sigma^2 dt - \tfrac{1}{2}\sigma (T-t)^{-1/2} dz$$

c. Hence:

$$r(T) = F(T,T) = F(0,T) + \int_0^T dF(t,T)$$

$$= F(0,T) + \int_0^T \tfrac{1}{2}\sigma^2 dt - \int_0^T \tfrac{1}{2}\sigma (T-t)^{-1/2} dz(t)$$

$$= F(0,T) + \tfrac{1}{2}\sigma^2 T - \tfrac{1}{2}\sigma \int_0^T (T-t)^{-1/2} dz(t)$$

(iii)

a. If $m(t, T)$ is the instantaneous drift of F and $s(t, T)$ the instantaneous standard deviation of F, i.e. $dF(t,T) = m(t,T)dt + s(t,T)dz$ then m is <u>entirely determined</u> from s by the relationship:

$$m(t,T) = s(t,T)\int_t^T s(t,u)du \, .$$

Could add Ω to the brackets, representing a dependency on other state variables.

b. If the process for r is path-dependent, then the HJM implementation will create non-recombining trees ...

... which will be calculatively onerous and intense, particularly if the model uses more than one factor of uncertainty.

Syllabus section: (k)

Core reading: Units 14

It was pleasing that many candidates recalled the bookwork in part (i) of this question on forward and zero coupon rates as applied to HJM. However, they did not work methodically with Ito's Lemma and simple integration to derive the results asked for in part (ii).

Part (iii) seemed more familiar and was answered well.

5

(i) <u>F</u>

a. <u>Convenience yield</u> is the benefit from ownership of the physical commodity over and above any financing and storage costs, usually expressed as a continuously compounded return.

Hence the convenience yield is negative when the forward price is above the rolled up spot price. Typical circumstances in which this occurs are:

- The commodity deteriorates in storage
- There is plentiful spot supply
- New supply is anticipated for future delivery.

Note: high (or low) financing cost is not a reason why convenience yield might be negative.

b. A Bermudan option is exerciseable on several separate dates before expiry of the option ...

c. ... whereas a European option is exerciseable on only one expiry date, and an American option on all dates up to and including expiry.

(ii) Discount at 4% = exp(–0.04) = 0.96079

Strike = 22.5

a. (a)Comparing payoffs at t = 2 against the strike price ⇒ paths 2, 3, 4 and 6 lead to at-the-money options.

b. Calculate payoffs at t = 2 and t = 3:

Path	***t* = 2**	**Payoff *t* = 2**	***t* = 3**	**Payoff *t* = 3**	**Discounted payoff *t* = *3***
1	23.6	0	21.4	0	0
2	15.9	6.6	15.1	7.4	7.11
3	18.4	4.1	19.7	2.8	2.69
4	19.8	2.7	15.6	6.9	6.63
5	26.1	0	29.2	0	0
6	15.6	6.9	17.0	5.5	5.28

Note: paths 1 and 5 are shown for completeness; they are not strictly required.

c. Setting up the least-squares problem means setting the values of $V = a + bS + cS^2$ to the values of S at t = 2 for each of those paths:

For path 2: $V_1 = a + b\,(15.9) + c\,(15.9)^2$

For path 3: $V_2 = a + b\,(18.4) + c\,(18.4)^2$

For path 4: $V_3 = a + b\,(19.8) + c\,(19.8)^2$

For path 6: $V_4 = a + b\,(15.6) + c\,(15.6)^2$

and then minimising the square of the differences between the V_i and the discounted
t = 3 payoffs, i.e. 7.11, 2.69, 6.63 and 5.28.

(iii) Using the values of a, b and c:

For path 2: $V_1 = 5.503$

For path 3: $V_2 = 3.098$

For path 4: $V_3 = 6.446$

For path 6: $V_4 = 6.514$

If we are going to exercise early at $t = 2$, we need the value at exercise to be higher than the option value.

The respective values for exercise early at $t = 2$ are: 6.6, 4.1, 2.7 and 6.9, so will exercise early on paths 2, 3 and 6 but not 4.

(iv) In general, the full process will follow the method in (ii) and (iii) above.

It will need many more Monte Carlo samples – hundreds if not thousands.

The least-squares is still performed as above at time $t = 2$.

In a similar fashion, it is also performed at time $t = 1$, the other possible exercise date.

A table is created of cash-flows from the option at $t = 1$, 2 and 3 – those that are not exercised early are the values of the European option.

Syllabus section: (a)-(d), (f)

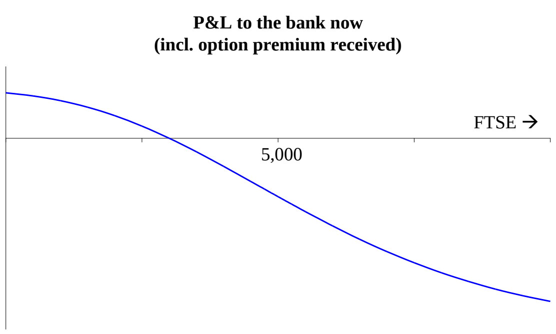
Core reading: Units 1 – 4, 6

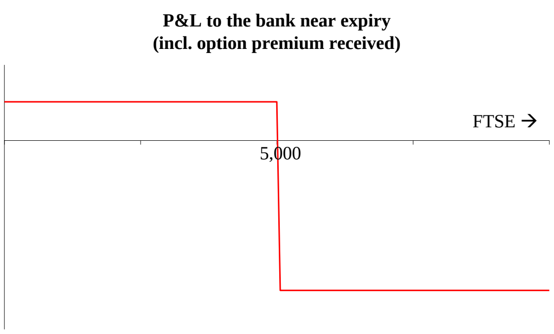
This question was generally well answered, the Longstaff-Schwartz least-squares approximation for an option price.

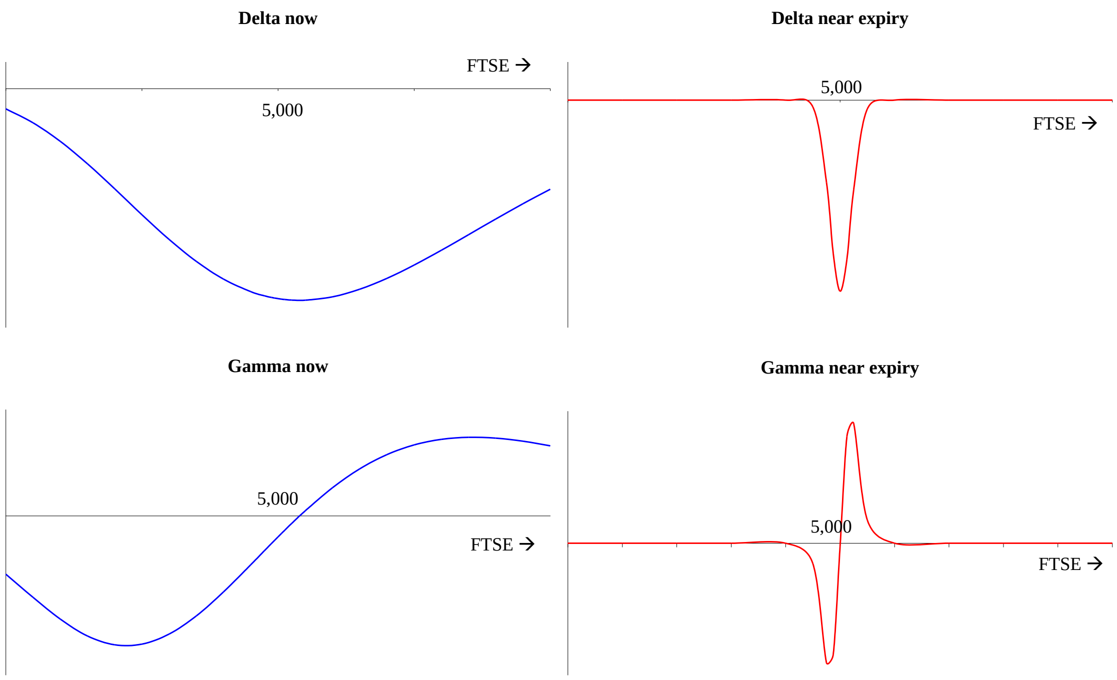
After some initial bookwork, the question took the student through six paths in a discrete option pricing example, then asked how a realistic application would differ from the simple approach. Several candidates received full marks as they worked methodically through each part.

6

(i)







(ii) The graphs illustrate the problems that arise in delta hedging a derivative with a discontinuous payment.

Around the strike price, the delta of the option changes rapidly ...

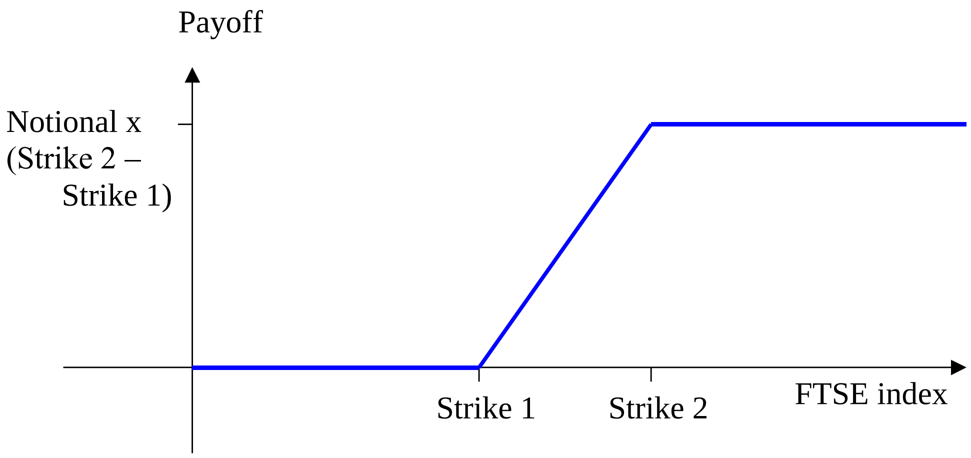
... so delta-hedging could be expensive if the oscillations around the strike are frequent.

Close to maturity, the delta hedge involves holding a huge amount in equities if the FTSE is close to the strike price but only a very small amount in equities if the FTSE is slightly further away on either side.

Hence the dynamic delta hedger is expected to invest heavily in equities just before the FTSE passes through the 5,000 barrier. To achieve this, the hedger needs to be <u>continuously</u> trading in FTSE futures.

(iii)

a.



b. As the two strikes get closer together, the Call spread tends towards a cash-or-nothing Call.

What remains is to specify what strike prices and notional are required to meet the requirements of the question.

Consider a Call spread with:

- Strike 1 = 5,000 * (1 – x / 100)
- Strike 2 = 5,000 * (1 + x / 100)
- Notional = P / (Strike 2 – Strike 1) = P / (100x) units of FTSE

where P is required payoff from the cash-or-nothing Call.

If the FTSE in one year's time is less than Strike 1, both Calls are worthless and the Call spread pays off zero.

If the FTSE in one year's time is greater than Strike 2, then:

- option 1 pays (FTSE – Strike 1) × P / (100x)
- option 2 pays –(FTSE – Strike 2) × P / (100x)

so total payoff is P.

So call spread exactly replicates cash or nothing option provided FTSE is more than x% away from 5,000, as required.

So the hedge consists of

- Bought twelve month European call option on P / (100x) units of FTSE with strike 5,000 – 50x, and
- Sold twelve month European call option on P / (100x) units of FTSE with strike 5,000 + 50x

Syllabus section: (g) & (i)

Core reading: Units 7, 12

This was a typical option strategy question involving sketching the sensitivities, and was answered reasonably. It showed how a binary or digital option could be approximated using a Call spread, which actually occurs in practice because it is easier with Call spreads to incorporate volatility skews and smiles.

For part (iii), quite a few students drew a profit/loss diagram, rather than a payoff diagram. This is possibly because they still had in mind the wording of part (i)(a) and included the option premiums.

7

(i) For the singer, this is the opportunity to crystallise future royalties into cash ... ... which provides the certainty of a principal amount now that could be used for a major purchase or to fund other business ventures.

For the bond holders, this is the opportunity to purchase bonds with:

- better yields than can be obtained on other similar assets
- secured by solid underlying cashflows
- tranched to tailor risk participation (the yields on the different tranches compensate the bondholders for the different risks of loss)
- diversified risks and cashflows vs other corporate bonds in their portfolios

(ii)

a. <u>SPV</u>

The SPV is established solely for the purpose of facilitating the securitisation.

The SPV separates the securitisation from the singer's personal credit position, so that the bonds are backed by the royalties rather than the singer's finances.

This is beneficial because:

- the singer does not need to worry about being saddled with debt if royalties fall much shorter than expected
- investors only need consider the royalties in assessing credit risk, not the singer's personal finances

b. <u>Equity tranche</u>

The equity tranche is established because (unlike a mortgage portfolio, say) the royalties have an unlimited upside ...

... and somebody needs to own the residual funds after all the bondholders have been paid off.

There is also an incentive for the holder (the SPV owner) to keep collecting payments.

The equity tranche also enhances the credit rating of the bonds being issued.

The larger the equity tranche, the more security there is for the bondholders.

The equity tranche is in any case more difficult to package and sell.

(iii)

a. A low credit rating reflects the fact that the rating agencies think there is a significant probability that CD sales (and royalties) will be so low that the bonds will default.

There may even be a default by an agent (or agents) supplying royalty payments.

CD sales could be low either because:

- this particular singer loses his popularity (specific risk), or
- CD sales (or prices) generally fall, perhaps due to other musical formats becoming more popular or a flood of competing CDs (generic risk).

b. Packaging together mezzanine bonds across a number of artists will diversify away the specific risk but not the generic.

However, the credit risks for bonds from all the different artists may be more correlated with each other than might first be thought.

Because of this, the idea in question may not work.

c. The credit rating of the mezzanine bonds could be increased by

- increasing the size of the equity tranche unilaterally, or
- increasing the equity tranche and/or reducing the sizes of the senior and super senior tranches with the mezzanine tranche being adjusted to keep the total funding fixed

In practice, the whole securitisation will probably be structured so that each tranche of bonds has just enough security to qualify for a particular credit rating.

Syllabus section: (l) & (m)

Core reading: Units 15, 16

This was a new type of question on securitising a singer's royalties using an SPV, which seemed to capture the imagination of many candidates. Interest in the themes of the question offset a general unfamiliarity with some of the concepts involved in securitisations and SPVs.

8

(i) <u>Benefits</u>

The life insurance company can always assess where it stands, since the strategy is simply investing in equities which the firm is experienced at already ...

... whereas it may not be so easy to account for and risk assess bought option hedges.

It retains control of its portfolio.

The hedging strategy is flexible and can be tailored precisely to the portfolio.

There is no up-front premium or hidden charges.

There is no close-out cost – it can cease the hedging strategy immediately if it wishes.

There is no counterparty credit risk exposure to an investment bank as there is with buying options ...

... which increases if the options hedge is beneficial (i.e. if the market falls), although the risk can be mitigated by the investment bank posting mark-to-market cash collateral.

<u>Costs</u>

Delta hedging appears to be cheaper than buying options because there is no immediate outlay ...

... but it is less easy to estimate the all-in cost ...

... and there is potentially more downside if the strategy does not work as expected.

However, there will be a cost to the delta hedging from gamma losses and the final cost depends on how the FTSE moves in future and the frequency of hedge rebalancing or narrowness of the trigger points that generate rebalancing.

Computer run scenarios will be needed to support the dynamic hedging. In order to understand the possible costs, Monte Carlo runs are required.

There will be ongoing dealing and administration costs ...

... with expertise and internal controls required to ensure the hedging is done effectively.

Delta hedging will probably result in the life insurance company having higher capital requirements.

<u>Risks</u>

Gamma risk: Whenever the market moves and the hedge is rebalanced, there will be a cost.

The more volatile the market is and either (i) the bigger the market movements before hedges are rebalanced, or (ii) the more the market goes up and down without going anywhere making the hedge go backwards and forwards, the bigger the gamma losses, the bigger the cost of delta hedging ...

... and hence there are several scenarios where delta hedging will prove to be more expensive than buying options.

Vega risk: The delta hedge provides no protection against changes in market implied volatilities (this can work both ways).

If the life insurance company has liabilities that are sensitive to implied volatilities, then this sensitivity is not hedged under a delta hedging arrangement.

Liquidity risk: The delta hedging strategy assumes that there will always be a deep and liquid market in FTSE futures.

If the market dries up (during a crash, for example), then the delta hedge cannot be operated properly and significant gamma losses could result.

Basis risk: (index vs actual)

Different equity positions vs index.

This risk is present whichever strategy chosen, i.e. options or futures ...

... but the investment bank could tailor the options more precisely to the portfolio.

Operational risk: Delta hedging is a task which relies heavily on a small number of highly technical staff.

There will be risks associated with e.g. staff retention, sickness, possibility of human error, lack of senior management understanding of the hedge.

(ii) <u>Dividends</u>

The theoretical way to allow for dividends in the Black-Scholes formula is to create a forward price that is free of dividends by rolling up the dividends at the risk-free rate.

Dividends should be estimated from historical trends or, if there is not sufficiently reliable or long enough history then a representative dividend index can be used.

In practice, it can be difficult to estimate dividends for some equities since dividends are not constant over time ...

... particularly where profitability in that firm or sector is changing rapidly.

Also, dividends are not continuous as assumed.

There is also as a second order effect the variability of the risk-free rate for reinvestment of dividends.

<u>Volatility</u>

The theoretical way to incorporate volatility in a dynamic hedge is to use implied volatility for the period of the hedge.

The implied volatilities can be extracted from liquid option prices using interpolation.

However, if implied volatility is low and actual volatility turns out higher (or vice versa), the dynamic hedge will be mismatched and therefore less effective.

If the market has had a period of extreme volatility, the implied volatility levels might be high and not so reliable as forward predictors. This apparent "imperfect market" effect can occur due a "fear" factor, or market participants reluctant to sell volatility when they have lost money in the volatile period. Often in these circumstances implied volatility will stay higher than is justified by subsequent price action.

There will also be correlation effect between the index and the portfolio constituents.

Generally, traders like to choose a mixture of historic and implied for their estimates.

Syllabus section: (h)(iv)-(ix), (i)

Core reading: Units 10 - 13

This question dealt with alternative strategies of buying options outright or using dynamic hedging. Quite a few students misread the preamble and thought that the "delta hedging strategy" was to ensure the delta of the insurer's portfolio was to be kept at zero, rather than tracking the delta of the recommended Put options.

Many students did compile clearly presented solutions based on bullet points.

END OF EXAMINERS' REPORT