EXAMINATION

20 April 2007 (pm)

Subject ST6 — Finance and Investment Specialist Technical B

Time allowed: Three hours

INSTRUCTIONS TO THE CANDIDATE

1. *Enter all the candidate and examination details as requested on the front of your answer booklet.*
2. *You have 15 minutes at the start of the examination in which to read the questions. You are strongly encouraged to use this time for reading only, but notes may be made. You then have three hours to complete the paper.*
3. *You must not start writing your answers in the booklet until instructed to do so by the supervisor.*
4. *Mark allocations are shown in brackets.*
5. *Attempt all 8 questions, beginning your answer to each question on a separate sheet.*
6. *Candidates should show calculations where this is appropriate.*

Graph paper is required for this paper.

AT THE END OF THE EXAMINATION

Hand in BOTH your answer booklet, with any additional sheets firmly attached, and this question paper.

In addition to this paper you should have available the 2002 edition of the Formulae and Tables and your own electronic calculator.

NOTE: In this examination, you are never required to prove the use of an arbitrage-free methodology unless clearly stated in the question.

1 (i) Consider two probability measures, **P** and **Q**.

(a) State the conditions under which the two measures are equivalent.

(b) State the necessary and sufficient conditions for a continuous process W_t to be a Brownian motion under measure **P** (called a **P**-Brownian motion). [3]

(ii) State the Cameron-Martin-Girsanov theorem for a **P**-Brownian motion process W_t and an **F**-previsible (and bounded) process γ_t, where $\mathbf{F}_t$ is a filtration (history) of events up to time t. [2]

(iii) Let W_t be a continuous **P**-Brownian motion process.

Let $X_t = \mu t + \sigma W_t$ be a drifting Brownian motion process, where μ and σ are both constant.

(a) Show that there exists a measure **Q**, equivalent to **P**, under which X_t is a driftless **Q**-Brownian motion.

[**Hint**: Consider the ratio $\gamma = \frac{\mu}{\sigma}$.]

(b) Find the volatility of X_t under both **P** and **Q**.

(c) Evaluate the Radon-Nikodym derivative $\frac{d\mathbf{Q}}{d\mathbf{P}}$ at time t. [6]

[Total 11]

2 Using the same notation as in the previous question, consider a **P**-Brownian motion W_t under a filtration $\mathbf{F}_t$.

(i) Show that the process $X_t = \frac{1}{\sqrt{a}} W_{at}$ is also a Brownian motion, where a is a constant. [3]

(ii) Obtain a stochastic differential equation for the process $Y_t = \exp\left(bW_t - \tfrac{1}{2} b^2 t\right)$, where b is a constant, and state with reasons whether or not it is a martingale. [3]

(iii) Obtain a stochastic differential equation for the process $Z_t = W_t^3 - 3tW_t$, and state with reasons whether or not it is a martingale. [3]

[Total 9]

3 (i) Explain, for a portfolio of derivatives, what is meant by:

(a) the net delta of the portfolio
(b) a delta-neutral hedge of the portfolio

[2]

Consider a ratio European Put option spread based on the FTSE100 index (current value 6,000), consisting of purchasing 1 unit of a 5,500 six-month Put option and selling 2 units of a 5,000 six-month Put option.

(ii) (a) Draw approximate graphs for each of the net Profit/Loss and net delta of the portfolio, for a suitable range of index values.

(b) Add to each graph a second line showing the net Profit/Loss and net delta of the portfolio immediately before expiry.

[8]

[**Note**: On the Profit/Loss graph you should include the effect of (but not calculate precisely) the purchase price of the options. Any curves may of course be drawn freehand.]

The observed implied volatility of the 5,000 Put option (as calculated using the Black-Scholes pricing formula) is higher than that of the 5,500 Put option. This is sometimes called a "volatility skew", or "smile".

(iii) Suggest reasons why this skew might occur. [2]

[Total 12]

4 Suppose the price of copper follows a log-normal stochastic process with annual expected return m and annual volatility v (both constants).

An investment bank has just announced that it is offering a new instrument called COLTS (COpper Log-price Tradable Security) that pays out at a single time T an amount equal to $\ln C_T$ (i.e. $\log_e C_T$), where C_T is the price of copper at time T.

(i) Obtain the probability distribution for C_T. [3]

(ii) Using risk-neutral valuation techniques, derive a formula for the price of COLTS at time t. [2]

(iii) Verify that your formula in (ii) satisfies the Black-Scholes-Merton partial differential equation. [3]

[Total 8]

5 Assume the price S of a stock paying a constant dividend yield of q follows the log-normal stochastic process with time-dependent drift and volatility:

$$dS = \mu(t)Sdt + \sigma(t)Sdz$$

where z is a standard Brownian motion.

A traded asset $V(S, t, T)$, which depends only on the price S and its evolution over time, satisfies the following parabolic partial differential equation (PDE):

$$\frac{\partial V}{\partial t} + (r-q)S\frac{\partial V}{\partial S} + \tfrac{1}{2}\sigma^2 S^2 \frac{\partial^2 V}{\partial S^2} = rV$$

where r is the risk free rate, t is the time since outset and T is the time to maturity of the asset.

(i) The "finite difference" (FD) method can be used to obtain an approximate numerical solution to the above PDE.

(a) Define and compare the two types of FD valuation.

(b) Outline, for **one** of these types only, how you would set up the necessary FD equations in the case where V is a one-year American Put option on the stock, allowing specifically for initial and boundary conditions. [9]

(ii) The binomial tree method is a widely-used alternative to the FD method.

(a) Explain how in practice the construction of these two methods achieves the same result.

(b) Comment on the suitability of each method for the valuation of stock options. [5]

[Total 14]

6 You are the Chief Risk Officer of a large European bank. The Board is concerned that its current trading activity is focused too heavily on products where margins are low (e.g. swaps, caps, and floors, FX and commodity options). It is therefore planning to expand into trading credit derivatives and inflation swaps, and is also considering setting up an advisory service for companies to tackle their pension deficits, including offering hedges for longevity risk.

The bank intends to hire a number of senior managers, who will build suitable models and set up the necessary support infrastructure. The Board has asked you to submit a report assessing the proposals from a risk perspective.

Set out the points you would cover in your report, which should address at least the following four areas:

- any additional risk factors to which the firm will be exposed
- how hedges of these risk factors might be achieved
- important control aspects of the new businesses
- potential modelling problems

You should focus entirely on the risk aspects of the proposal, and can assume that others have provided an underlying business strategy (explaining how the new products will be used), a detailed description of the workings of the products themselves and an assessment of any resourcing and system requirements. You should also bear in mind that the Board is already fully familiar with standard market and credit risk issues relating to the existing derivatives operation.

[14]

7 In a traditional risk-neutral world with one risk factor, the Heath-Jarrow-Morton (HJM) model of the yield curve implies a generalised process for zero-coupon bond prices $P(t, T)$ maturing at time T given by:

$$dP(t, T) = r(t)\, P\, dt + v(t, T, \Omega)\, P\, dz(t)$$

where $r(t)$ is the short-term interest rate at time t and $dz(t)$ is a standard Wiener process driving term structure movements. $v(t, T, \Omega)$ is the volatility of P, a generalised representation which also depends on Ω, the collection of past and future interest rates and bond prices.

(i) Define, in terms of P:

(a) the forward rate $f(t, T_1, T_2)$ applying between times T_1 and T_2 as seen at time t

(b) the instantaneous forward rate $F(t, T)$ applying at time T as seen at time t

[2]

(ii) Show that the process followed by the instantaneous forward rate $F(t, T)$ is entirely determined by the volatility function $v(t, T, \Omega)$. [5]

[**Hint**: Observe the process followed by $f(t, T, T+\Delta T)$ in the limit as $\Delta T \to 0$.]

(iii) (a) Discuss the advantages and disadvantages of using the HJM model compared with other one-factor models of the yield curve such as the LIBOR Market Model and the Hull-White model.

(b) Explain why it could be important to build models with more than one risk factor, and how suitable these models would be in multi-factor form.

[9]
[Total 16]

8 The bond market of a country consists of the following Government issued bonds, with their corresponding market prices per 100 nominal:

Term	*Coupon %*	*Price*	*Redemption Yield % p.a.*	*Discount factor*
1 year	5	99.876	5.130	0.95120
2 year	5.25	99.970	5.266	0.90239
3 year	6	101.882	5.305	0.85623
4 year	4.75	98.072	5.297	
5 year	4.5	96.880	5.225	

In each case, coupons are paid annually and the next one is due to be paid in a year's time. Redemption yields are annually compounded. The short-term (one-year) risk free rate is 5% per annum, continuously compounded.

(i) Calculate the annual discount factors applicable to this yield curve in years four and five. (A discount factor for year *n* is the present value of one unit of cash payable at the end of year *n*. The right-hand column above gives the values for years one to three.) [2]

(ii) (a) Using the values from (i), calculate the fixed coupon of a 5-year par value fixed to floating interest rate swap.

(b) Explain why this might not be the true market price. [4]

(iii) (a) Demonstrate that the market expects short-term interest rates to rise up to the end of year two, then fall thereafter.

(b) Explain the implication of this fact to investors in the 2- and 5-year bonds who are expecting to receive the redemption yield. [4]

(iv) Estimate using the Black model the price of a one-year European Call option on the 5-year Government bond. The price volatility of this bond is 6% per annum, and you may assume that the option is exerciseable immediately after the bond pays its coupon. [4]

(v) Explain why the Black model might not be entirely appropriate for the valuation in (iv). [2]

[Total 16]

END OF PAPER