EXAMINATION

9 April 2008 (pm)

Subject ST6 — Finance and Investment Specialist Technical B

Time allowed: Three hours

INSTRUCTIONS TO THE CANDIDATE

1. *Enter all the candidate and examination details as requested on the front of your answer booklet.*
2. *You have 15 minutes at the start of the examination in which to read the questions. You are strongly encouraged to use this time for reading only, but notes may be made. You then have three hours to complete the paper.*
3. *You must not start writing your answers in the booklet until instructed to do so by the supervisor.*
4. *Mark allocations are shown in brackets.*
5. *Attempt all 8 questions, beginning your answer to each question on a separate sheet.*
6. *Candidates should show calculations where this is appropriate.*

Graph paper is required for this paper.

AT THE END OF THE EXAMINATION

Hand in BOTH your answer booklet, with any additional sheets firmly attached, and this question paper.

In addition to this paper you should have available the 2002 edition of the Formulae and Tables and your own electronic calculator from the approved list.

NOTE: In this examination, you are never required to prove the use of an arbitrage-free methodology unless clearly stated in the question.

1 (i) Define the following terms in the context of a probability measure **P**:

(a) **P**-Brownian motion;
(b) **P**-Martingale. [3]

Let W_t be a **P**-Brownian motion.

(ii) Find and verify the (constant) value of α that makes the process $X_t = W_t + \alpha t$ a **P**-Martingale. [2]

(iii) In a world where all tradable assets are Martingales, show that it is not possible to construct a portfolio from these assets that will yield a guaranteed arbitrage profit. [3]
[Total 8]

2 (i) Explain:

(a) the difference between the price and the value of a forward contract

(b) the difference between a forward contract and a future (based on the same underlying asset)
[3]

A German auto manufacturer receives a significant amount of its revenue from the USA, and has been concerned for some time about the effect of the falling US dollar. It wishes to enter into a forward exchange rate contract to lock in the Euro value of certain US dollar cashflows due in six months time.

(ii) Derive a formula for the arbitrage-free forward exchange rate by constructing two portfolios, one involving a forward exchange rate contract, to replicate the value of a US dollar in Euros in six months time. Define any symbols you use. [3]

An alternative approach to using a forward currency contract would be to hedge using currency futures.

(iii) Explain how a futures hedge would operate and why it might be less suitable for the company. [3]
[Total 9]

3 (i) (a) Define the term “previsible” as applied to a stochastic process.
(b) State the Binomial Representation Theorem. [3]

A process S follows a discrete random walk under probability measure **P**. At each time step n (n = 1, 2, etc), it can either increase by 5 units or decrease by 3 units. Initially, $S_0 = 40$. A claim X exists on S at time $n = 3$ such that $X = \sqrt{S} - 6$ if this is positive, or zero otherwise. The risk-free rate is zero.

(ii) (a) Find the values of the probability measure **P**.

(b) Verify for $n = 1$ that the Binomial Representation Theorem applies to the expectation of X under **P**. [6]

(iii) Describe how the approach in (ii) could be used to price options on S. [3]
[Total 12]

4 You are observing bonds and swaps in a LIBOR based interest rate market.

Maturity (Years)	*A Current Zero Price*	*B Shifted Zero Price*
1	95.238	95.147
2	90.488	90.316
3	85.770	85.526
4	81.492	81.183
5	77.796	77.428

Column A gives the set of zero coupon bond prices that apply currently (each price is for a nominal of 100). Column B gives the prices of the same bonds after the yield curve has instantly shifted up by 10 basis points (0.10%) for all maturities. The modified duration of a bond priced at par is 4.299 on current rates.

(i) (a) Calculate the value of a 5-year 10% annual coupon bond at current rates.

(b) Hence derive the value of 5-year annual fixed-floating swap with 10% fixed coupon.

[3]

Let the "absolute yield sensitivity" of an instrument be defined as the instrument's value now less its value when rates have risen by a small increment Δr, all divided by Δr.

(ii) (a) Show how absolute yield sensitivity for a bond is related to its modified duration.

(b) Show numerically that the absolute yield sensitivities of the bond and swap in (i) are the same. [4]

[You may assume for this question that 0.10% is an appropriate small increment – in practice, with computers, a smaller value would be used.]

A "reverse floater" is a bond that pays a coupon equal to a fixed value X less the value of LIBOR at each fixing. So, for example, if LIBOR is 5.75% at the next fixing, the coupon becomes X – 5.75%.

Consider a 5-year annual coupon reverse floater where X = 10%.

(iii) (a) Show how the cashflows of the reverse floater can be decomposed into those of a fixed-floating swap and a zero coupon bond.

(b) Hence demonstrate numerically that the modified duration of the reverse floater is approximately twice that of a bond priced at par.

[6]

(iv) Explain how you would risk manage, in terms of duration, a portfolio of bonds that includes a reverse floater. [3]

[Total 16]

5 In an interest rate yield curve environment, a single factor is used to represent the stochastic evolution of rates. Let $P(t, t_k)$ represent the price at time t of zero coupon bonds maturing at future times t_1, t_2, … etc. and let $F_k(t)$ represent the forward rate from t_k to t_{k+1}.

(i) (a) Write down the relationship between P and F.

(b) Demonstrate that the forward rate $F_k(t)$ is a Martingale with respect to a measure that makes $P(t, t_{k+1})$ forward risk neutral. (You may assume the Equivalent Martingale Measure result.) [4]

(ii) Outline how the one-factor LIBOR Market Model is constructed using forward rates and their volatilities. [4]

You have been asked to determine the value and risk sensitivities of a "ratchet floater" bond of high credit quality. This instrument pays a semi-annual floating coupon of 6-month LIBOR plus a margin of 0.35%. In addition, at each reset date, the coupon can never be lower than the last coupon level (even if LIBOR falls), but can also only increase by a maximum of 0.50%.

(iii) Describe circumstances in which, despite the fact that its coupons can never decrease, the 'ratchet floater' might cost less than an equivalent standard floating coupon bond with the same margin. [3]

You propose building a two-factor LIBOR Market Model for your valuation, but your manager has asked why you cannot simply use a binomial tree version of the Black model, which is readily available.

(iv) Set out the points you would include in your response, comparing the merits of the two models in relation to the ratchet floater. [5]

[Total 16]

6 Consider a stock of price S which follows the stochastic process:

$$dS = \mu S\, dt + \sigma S\, dz$$

where μ, σ are positive constants and z is a standard Wiener process.

(i) (a) Using Ito's Lemma, derive the probability density function of S.

(b) Under a risk neutral measure, show how the drift μ relates to the constant risk-free rate r. [4]

Consider an option that pays a fixed amount K on a single date in the future, provided the stock lies within a specified range.

(ii) (a) Using your answer to (i)(a) above, derive a formula for the fair value of this option, without evaluating any integrals.

(b) Sketch the approximate behaviour of the Gamma of the option over a set of possible values of S, both now and near to expiry.

(c) Discuss any problems that you might encounter hedging this option with other market-traded instruments. [6]

[Total 10]

7 A commodity of price C is assumed to follow the process:

$$dC = \mu C\, dt + \sigma C\, dW_t$$

where μ, σ are positive constants and W_t is a standard Brownian motion. The continuously compounded risk free interest rate r is constant, and the convenience yield is zero.

You wish to value a special type of option. You construct a recombining binomial tree algorithm using a proportionate "up step" u and "down step" d for each small time interval Δt, and the commodity price at time 0 is S_0.

(i) Specify fully the first step of the binomial process, giving formulae for the up and down probabilities and step sizes u and d. [4]

The initial commodity price is 75, σ is 15% per annum and $r = 0$. You may assume that u can be approximated by $e^{\sigma\sqrt{\Delta t}}$ for small Δt.

(ii) For the tree specified in (i):

(a) Draw three steps of the tree with quarter-year time steps and calculate the commodity price at each node.

(b) Using this tree, calculate the price of a nine-month vanilla European call option with an at-the-money strike.

(c) By considering each possible path in the tree, evaluate the price of a nine-month European lookback call option, where the lookback period includes time 0. [9]

(iii) Compare your results in (ii)(b) and (ii)(c) with each other and in relation to the prices you would obtain from using an algebraic solution. [2]
[Total 15]

8 An exchange option gives the holder the right on exercise of the option to exchange one asset for another asset in some predetermined ratio.

Consider a European exchange option, with expiry in T years, based on two non-dividend paying equities. Let $S_1(t)$ be the price of the original equity and $S_2(t)$ the price of the equity to be received on exercise, with K units of S_1 being exchanged for one unit of S_2. S_1 and S_2 follow two separate (but correlated) geometric Brownian motions.

Let r be the risk-free rate, continuously compounded, and σ_i the volatilities of stock S_i (i = 1, 2), with ρ_{12} the correlation between the Brownian motions.

(i) (a) Write down the payoff of the option at time T.

(b) Derive a closed-form algebraic formula for option price V. [9]

[Hint: Consider a solution of the form $V(S_1, S_2, t) = S_1\Phi(X, t)$, where $X = \dfrac{S_2}{S_1}$, and compare with the standard Black-Scholes result. You may use the result that, for the change of measure that makes X a Martingale, the market price of risk is equal to the volatility of S_1.]

(ii) (a) Explain how you might use your answer to (i) to value a convertible bond.

(b) Assess how appropriate this approach might be in practice. [5]
[Total 14]

END OF EXAM