Faculty of Actuaries | Institute of Actuaries

EXAMINATIONS

September 2005

Subject ST6 — Finance and Investment Specialist Technical B and Certificate in Derivatives

EXAMINERS' REPORT

Introduction

The attached subject report has been written by the Principal Examiner with the aim of helping candidates. The questions and comments are based around Core Reading as the interpretation of the syllabus to which the examiners are working. They have however given credit for any alternative approach or interpretation which they consider to be reasonable.

M Flaherty
Chairman of the Board of Examiners

29 November 2005

Please note that the model solutions provided are indicative, i.e. adequate to achieve full marks but without covering ever possible correct response. Several points made by candidates were equally valid, and these also achieved the allocated marks.

Q1
This was largely a straight bookwork question, at least for parts (i) and (ii), the latter incorporating some simple algebra.
Part (iii) produced a number of diverse responses, with quite a few candidates arguing that the arbitrages found did not exist. Note in the model solution how several relevant points, briefly made, are required and would achieve more marks than one extended point. (This has been a very common observation for ST6.)

Q2
This question was much better answered than the equivalent in April 2005. It is good to see candidates producing better answers to these graphical-type questions, which focus on applying knowledge of pay-offs, time values and sensitivities ("Greeks"). A good appreciation of these is essential when wanting to learn about derivatives.
For part (i), few explained well why a FTSE bull spread might be purchased. As in April, if an option structure is unfamiliar, candidates need to apply their general knowledge of options and to try to identify those aspects relevant to the question. In this case, limited risk and reduced cost are important.
For parts (ii) and (iii), the main requirement was to demonstrate the basic shape of the curves. A perfect standard is not expected for the "sketch". Rather, demonstration of understanding is the important factor. Full marks can be obtained with a neat diagram that broadly shows the patterns required (as illustrated in the model solution). It is not necessary to draw precise lines or label axes accurately, but the diagrams must be clear and representative – for example, the peak of the Delta graph in part (iii) must be at-the-money and less than 1 throughout.
The Gamma curve was more difficult, although many candidates managed to sketch it correctly. It is, of course, the derivative of the delta curve, which itself is the derivative of the P&L curve. Candidates can be successful here by deriving the curves for the combined strategy from basic option understanding.
Most candidates found the correct response to part (iv). It is always useful to consider separately the effects on intrinsic value and time value.

Q3
This question revolves around hedging a portfolio of options. Parts (i) and (ii) were mostly bookwork and were well answered. Part (iii) was seeking the response that the Delta of a put becomes more negative as the underlying market fall, with a value that approaches -1; most candidates stated this.
Part (iv) was not generally well answered. Here a discussion was required of the relative merits of replicating a put versus purchasing one outright. Issues to be covered were: cost, hedging horizon, volatility estimation and the sophistication of systems required. As mentioned above, it is always better for answers to these types of question to cover several relevant points rather than just one basic argument.

Q4
This was an application question manipulating discrete stochastic variables to find a valuation method for average rate options. It was badly answered by almost all candidates,

however, with several giving up after only a few paragraphs. Many of these could have done better if they had pressed on with their initial approach.
Part (i)(a) required splitting the stock price evolution into a series of discrete random jumps, then finding a simplified formula for the average. This was not trivial to do, but equally not hard with the hint given in the paper. Candidates should ensure that they understand how to represent discrete Brownian motion algebraically.
Part (i)(b) produced the well known result that average rate options can be considered equivalent to standard options with one-third of the volatility. Allowance was made if the candidate derived the correct limit in (b) from an incorrect formula in (a).
Part (ii) was poorly answered. There was often little sign of application to the specific question. As the text implied, the Black-Scholes model could be used to assess the value of average rate options, with the appropriate adjustment for the volatility, since the sum of normal distributions is also normal. Several candidates said Black-Scholes was not suitable at all, although they mainly focused on the weakness of certain assumptions rather than fundamental reasons.

Q5
Several candidates were daunted by this question, but those who pressed on with the algebra found it possible to obtain good marks.
Part (i) required a statement of pull-call parity. A disappointingly large number of candidates did not even mention this important concept. Early exercise breaks the link between puts and calls for American options.
Part (ii) contained some tricky algebra, but also gave very helpful hints, so unsurprisingly many candidates derived the correct results for both (a) and (b). In (c), a simple explanation was required that the algebra is similar, but the conclusion less clear, for American puts.
Part (iii), however, again exposed a reluctance of candidates, many of whom had correctly derived the results in part (ii), to employ a logical approach in applying a result. To obtain the full three marks for this part, successful candidates needed to make several succinct points, as per the model solution.

Q6
Part (i) was bookwork and well answered.
Part (ii) was generally covered well, involving as it did application of the reading material to three potential measures of market risk. There was a tendency in some responses to be too brief – seven marks over the three sub-parts implies the need for at least 2 to 3 good points (briefly stated) for each.
Part (iii) elicited good responses on credit – bought options and exercised swaps giving rise to credit risk in this case. Discussion on the market risk aspects was more disappointing – again, candidates needed to take their responses to the earlier parts and adapt them (with reasons) to the specific example.

Q7
Part (i) was bookwork and well answered.
Part (ii) presented a completely unfamiliar problem to most candidates, as was evident from their answers. When faced with something unusual, candidates need to consider these two important approaches: (1) to apply the principles they know, as appropriate, and (2) to use <u>all</u> the information given in the question.
In this case, firstly Ito's Lemma should be applied, as the question states – amazingly, several candidates did not do this for stochastic variables f and g, and several who did made errors.

Secondly, the relationship between the drift and volatility of f and g was given in the question, but hardly any candidates used this simple information.
Part (iii) required an educated guess, and was well answered.

Q8
This question on fixed interest yield curves was very straight-forward, as a most candidates appeared to find, since this was the best answered question in the paper. By far the largest volume of derivatives traded are in the fixed income area.
Parts (i) to (iv) posed few problems. In part (v), a few candidates made numeric errors in the swap calculation, but encouragingly almost all had the correct method. In (v)(b), the fundamental point is that swaps are priced in the interbank market, so the curve should be based on interbank rates not government bond rates. Other points were accepted, though.

1 ***Syllabus: (c)(i)***
Reading: U1 / Hull Ch 3

(i) <u>Risk-free rate of return</u> is the rate of return on an asset which has no default risk, e.g. a bond issued by a highly-rated government such as the US or UK, so repayment is absolutely guaranteed.

<u>Convenience yield</u> is the rate of return on an asset derived from holding an inventory of the physical commodity rather than owning a paper claim to it. This yield depends on the market's demand and requirement for a particular product, e.g. oil for refining, copper for electrical wiring, sugar for processing.

<u>Cost of carry</u> is the difference between the storage and financing costs of a commodity. For bonds, which typically have higher yields than commodities, the cost of carry can be negative.

(ii) Let F = forward price, S = spot price, r = risk-free rate, y = convenience yield, u = storage costs as a % of the price, all rates continuously compounded.

To finance up to time t a portfolio of the commodity valued with an initial spot price S_0, a cost of $S_0 e^{(r+u)t}$ must be incurred today.

However, a portfolio consisting of the future on its own will not earn the convenience yield, so the cost of this portfolio after time t will be $F_0 e^{yt}$.

Hence:

$$F_0 = S_0 e^{(r+u)t} e^{-yt} = S_0 e^{(r+u-y)t}$$

(iii) The point is about the forward price not always being theoretically in line with the spot price, i.e. the apparent absence of true arbitrage.

Arbitrage exists when actual market valuations are different from theoretical prices.

These can exist for long periods if the market is unable to correct them due to technical factors unconnected with the pricing mechanism.

Traders cannot always simply deliver a commodity to fulfil an arbitrage contract.

In the case of the oil market, for example, there are supply constraints such as the lack of availability of crude oil supplies to deliver into contracts, hoarding, unwillingness of sellers to release physical stock, ships not arriving in the right place at the right time, financing difficulties with suppliers.

Medium-term supply/demand mismatches can occur. If inventories are low and physical production is constrained (e.g. due to wars or shipping problems), prices for spot delivery may remain higher than forward prices indicate.

Equally, if supplies are suddenly plentiful, the spot price may fall temporarily.

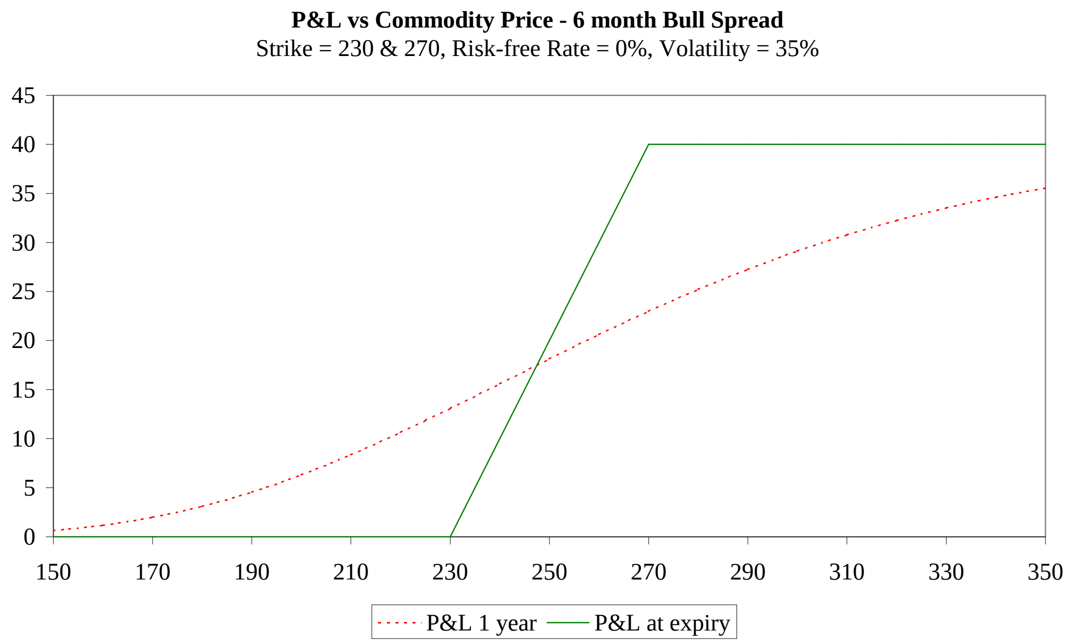
Changes in supply/demand effects are reflected within the convenience yield, which is a balancing item between theoretical and actual forward prices.

2 ***Syllabus: (f)(vi)(ix)***
Reading: U6 / Hull 14

(i) A bull spread provides a call at lower price than a single option, but with limited upside and no downside except the premium difference (see graph in part (ii)).

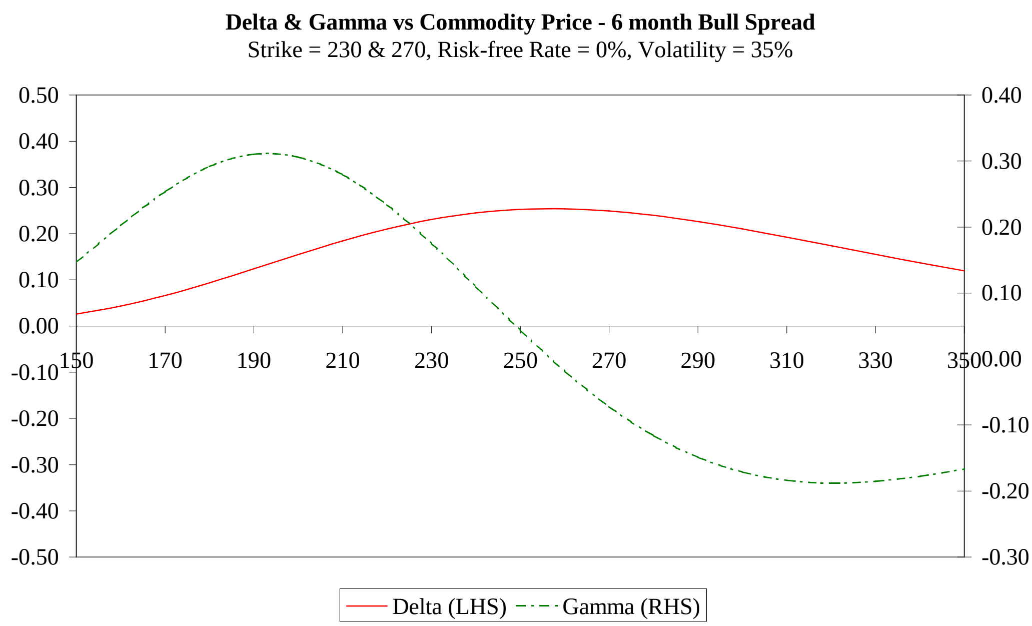
A fund manager might buy equity bull spreads at-the-money on the lower strike and out-of-the-money on the higher. This would enable him/her to enter the market for a limited risk but reduce the cost of entry.

(ii)



Note: "P&L 1 year" line is the "P&L today" line, i.e. with 1 year to go to expiry

(iii)



(iv) (a) If volatility falls, the time value will be lower, so the value will approach the expiry value …
... which implies that above ~250 the value will increase, below ~250 it will decrease.

(b) If time to expiry decreases, this has the same effect as (a), although the exact amplitudes will be different.

3 ***Syllabus: (f)(vi)(ix)***
Reading: U5 & 6 / Hull Ch 14, 15

(i) (a) Delta measures the rate of change of the value of the portfolio with respect to changes in the underlying asset; i.e. $\Delta_t = \frac{\partial V_t}{\partial S_t}$.

(b) Gamma measures the rate of change of the Delta with respect to changes in the underlying asset; i.e. $\Gamma_t = \frac{\partial \Delta_t}{\partial S_t} = \frac{\partial^2 V_t}{\partial S_t^2}$

(c) Theta measures the rate of change of the value of the portfolio with respect to the passage of time; i.e. $\Theta_t = \frac{\partial V_t}{\partial t}$

(d) Rho measures the rate of change of the value of the portfolio with respect to the risk free interest rate; i.e. $P_t = \frac{\partial V_t}{\partial r}$

(ii) The value of N put options with strike X and term T is

$$p = N\left\{Xe^{-rT}\Phi(-d_2(X,T)) - S_0\Phi(-d_1(X,T))\right\}$$

where $d_1(X,T) = \dfrac{\ln(S_0/X) + (r + \sigma^2/2)T}{\sigma\sqrt{T}}$ and $d_2(X,T) = d_1(X,T) - \sigma\sqrt{T}$.

The Delta of the put options is $\frac{\partial p}{\partial S} = N(\Phi(d_1(X,T)) - 1)$. (*)

The Delta of the whole portfolio is therefore

$$N_1(\Phi(d_1(X_1,T_1) - 1) + N_2(\Phi(d_1(X_2,T_2) - 1) + N_3(\Phi(d_1(X_3,T_3) - 1)$$

The Gamma is $\dfrac{\partial^2 p}{\partial S^2} = N \dfrac{\Phi'(d_1(X,T))}{S_0 \sigma \sqrt{T}}$, which for the whole portfolio becomes

$$N_1 \frac{\Phi'(d_1(X_1,T_1))}{S_0 \sigma \sqrt{T_1}} + N_2 \frac{\Phi'(d_1(X_2,T_2))}{S_0 \sigma \sqrt{T_2}} + N_3 \frac{\Phi'(d_1(X_3,T_3))}{S_0 \sigma \sqrt{T_3}}$$

(iii) From equation (*) we see that the delta of N put options with strike X, term T and current asset value S_0 is:

$$\frac{\partial p}{\partial S} = N(\Phi(d_1(X,T)) - 1)$$

If S_0 were to suddenly fall to $S_0^* = S_0 / 2$, the value of $d_1(X,T,S)$ would fall, and the value of $\Phi(d_1)$ would fall whence ($\Phi(d_1) - 1$) would become even more negative.

Hence the magnitude of the Delta will increase (becomes more negative).

This applies to all the put options in the portfolio, hence it applies to the portfolio Delta in aggregate.

(iv) Purchasing a put option from a counter-party requires payment of an option premium which reflects market implied volatility (plus whatever profit margins the counter-party can get away with in its pricing).

Synthesising a put option allows the cost of the option to flow through over time dependent on actual volatility experienced, rather than market implied volatility at inception of the hedge. The actual volatility experienced could be greater or less than the market implied volatility at hedge inception.

Synthesising a put option involves considerably more expertise and sophisticated systems in order to frequently rebalance the dynamic hedge position.

In practice, usually only the delta position can be rebalanced daily, and then transaction costs for these daily rebalancing trades could become prohibitive unless the portfolio is large enough to achieve economies of scale.

Delta hedging only protects the fund against small and immediate changes in the underlying asset values.

When markets fall very significantly over a short period of time (e.g. October 1987), the magnitude of the delta of the portfolio also changes significantly, and traders who have created put options synthetically are less likely to be able to adjust their positions fast enough to protect their portfolio.

In these extreme circumstances, delta hedging is less likely to be as effective as having invested in bought put options.

Insofar as the other Greeks are concerned, zero Gamma and zero Vega are more difficult to achieve because it can be difficult to find options or other non-linear derivatives that can be traded in the volumes required at competitive prices.
Hence a more practical approach involves monitoring the aggregate position, and seeking to rebalance the position only when the magnitudes of the Gamma and/or Vega become too large. If suitable options which neutralise the positions cannot be found at competitive prices, trading activities might have to be restricted.

Other points that could be made:
Rolling the hedge forward is easier under dynamic hedging, since the put has a fixed expiry.

A put has inbuilt rebalancing (similar to fourth point above)

4 ***Syllabus: (e), (f)(i)&(iv)&(iv)***
Reading: U3 & 5 / B&R 2, 3; Hull 19

(i) (a) Ave = $\frac{1}{n}\sum_{i=1}^{n} S_i$, where S_i refers to the stock price at time t_i.

$S_i = S_{i-1} + \Delta i$ where Δi is an independently distributed random increment from B_t at time point i.

Thus $S_1 = S_0 + \Delta_1 \qquad + \mu(t/n)$

$S_2 = S_0 + \Delta_1 + \Delta_2 \qquad + 2\mu(t/n)$

...

$S_n = S_0 + \Delta_1 + \Delta_2 \qquad + \Delta_n + \mu t$

Ave = $S_0 + \frac{1}{n}\left(n\Delta_1 + (n-1)\Delta_2 + ...1.\Delta_n\right) + \mu t$

The variance of each increment $\Delta_i = \sigma^2 t / n$

So that

Variance (Ave) = $\dfrac{1}{n^2}\left(\sigma^2 \dfrac{t}{n}\right)\left(n^2 + (n-1)^2 + .. + 1^2\right)$

Using the formula given

Variance (Ave) = $\dfrac{1}{n^2}\dfrac{\sigma^2 t}{n}\dfrac{1}{6}n(n+1)(2n+1)$

(b) In the limit $n \to \infty$, Variance (Ave) $\to \sigma^2 t / 3$

(ii) If Black-Scholes is to be applied to a realistic arithmetic average option consider the following:

- Since a sum of normal distributions is normal, it is valid to use the Black-Scholes model with the modified volatility as in (i)(b) above.

- Introduce an equivalent dividend yield to correctly forecast the forward price of the average.

- A realistic average is not continuous, therefore its variance will be larger than $\sigma^2/3$. Depending on the number of average points the variance may be significantly higher than $\sigma^2/3$.

- Need to keep a history of price fixes throughout life (or of the current level of the average) to give current pricing during life.

- Hedging parameters can be computed in the normal way, but the parameters follow a different profile over time. For example, near expiry the delta of an average rate option will generally be zero or 1 with very low gamma since further price changes will not affect the average.

Other points that could be made:
Obtain the value of σ from other vanilla options.
Discussion of the validity of the Black-Scholes assumptions in general.

5 ***Syllabus: (f)(vii)***
Reading: U6 / Hull Ch 8.4, 12.13

(i) (a) Consider two portfolios:

X: one European Call plus cash equal to Ke^{-rT} plus cash equal to the discounted dividends due before time T

Y: one European Put plus one unit of stock

(assuming the options are based on one unit of stock).

At expiry time T, both portfolios are equal to max[S_T, K] plus the rolled-up dividends.

Since there is no exercise before expiry, the portfolios must also have the same today.

Hence:

$$P_{\text{call}} + Ke^{-rT} + PV(\text{dividends}) = P_{\text{put}} + S_0$$

or:

$$P_{\text{put}} = P_{\text{call}} + Ke^{-rT} - (S_0 - PV(\text{dividends}))$$

[Alternatively, if the continuous dividend rate is q then

$$P_{\text{put}} = P_{\text{call}} + Ke^{-rT} - S_0 e^{-qT}$$

and candidates may use $T - t$ rather than t (i.e. at time t rather than 0).]

(b) This does not apply to American options because they can be exercised early, so the two portfolios may not both exist all the way to expiry.

(ii) (a) Consider the position at time t_n.

If the investor exercises the call just prior to t_n, he receives $S(t_n) - K$. If he does not exercise, the stock then drops to $S(t_n) - g_n$.

As with all options, it must be at least worth the discounted amount of its intrinsic value at expiry, i.e. $S(t_n) - g_n - K\exp(-r(T - t_n))$.

So the investor will not exercise early if he would receive less than this value, i.e. if:

$$S(t_n) - g_n - K\exp(-r(T - t_n)) \geq S(t_n) - K$$

or $$K(1 - \exp(-r(T - t_n))) \geq g_n$$

Now consider the position at time t_{n-1}.

If the investor exercises the call just prior to t_{n-1}, he receives $S(t_{n-1}) - K$. If he does not exercise, the stock then drops to $S(t_{n-1}) - g_{n-1}$.

Now the option must be at least worth the discounted amount of its (American style) exercise value at the next dividend date, i.e.

$$S(t_{n-1}) - g_{n-1} - K\exp(-r(t_n - t_{n-1})).$$

So the investor will not exercise early if he would receive less than this value, i.e. if:

$$S(t_{n-1}) - g_{n-1} - K\exp(-r(t_n - t_{n-1})) \geq S(t_{n-1}) - K$$

or $$K(1 - \exp(-r(t_n - t_{n-1})) \geq g_{n-1}$$

The general result follows by iteration.

(b) Using $\exp(x) \sim 1 + x$ in the result in (a) above, we have that early exercise is not optimal if:

$$Kr(t_{i+1} - t_i) \geq g_i$$

i.e. $$r \geq \frac{g_i}{K(t_{i+1} - t_i)}$$

which, for K near the current stock price, is equivalent to saying that the risk-free rate (LHS) is higher than the dividend yield (RHS), which it usually is.

This means that early exercise is not usually optimal except at the very last dividend date before expiry.

(c) The equivalent result for American puts uses identical arguments to the above, and produces the same equations but with the inequality sign the other way round. It is not as clear that the inequalities will apply, though, so the results are less conclusive.

(iii) Calculation 1: The option to expiry is the European value — the American value cannot be less than this.

Calculation 2: Using the option to the last dividend date values the option to exercise early at that dividend date. From the result in part (ii), we know that the last dividend date is the only time when it can be optimal to exercise early.

Hence, since the investor has the choice, it is approximately correct that the American value is the higher of the two calculations.

There are several limitations to this argument, though:

- the decision on early exercise is effectively made today, whereas in reality it will depend on the stock price and cannot be made until the time comes

- the volatility strictly only applies to the stock price with dividends discounted from today to expiry, but in the second calculation one less dividend is used *(giving a higher option value)*

- hence the true solution really requires a bivariate distribution to cope with the two separate assets

- the assumption $r > g / K$ must hold

6 ***Syllabus: (i)***
Reading: U9

(i) Market risk is the risk that the value of a portfolio will fall due to an adverse change in the level of market variables (prices, rates, volatilities etc.) applying to the valuation of instruments such as bonds, currencies, commodities and equities.

Credit risk is the risk that a counterparty to a transaction will default wholly or in part on its obligations, or that there will be a change in the market's perception of the likelihood of such a default occurring.

(ii) (a) The daily sensitivity report sets out the market risk of the portfolio in terms of sensitivity to key benchmark instruments.

It is obtained by perturbing the yield of each of the benchmark instruments used to construct the valuation curve by a small amount, thereby creating a new valuation curve. Changes in portfolio value from this perturbation give a "delta" sensitivity to each instrument, which can be summarised as a net basis point sensitivity of the entire portfolio, or parts of it.

The market risk sensitivity of the portfolio is therefore summarised in terms of a hedge using the benchmark instruments. The risk can be neutralised by taking opposite positions in each of these instruments.

The advantage of this method is that it is comprehensive and gives clear information about the market risks; its disadvantages are that it produces a whole range of numbers which can be hard to follow or interpret, and it does not indicate how the various benchmarks might move in a correlated way.

(b) Daily Value at Risk (DVaR) is measured as the potential level of loss with a specified confidence interval (say 99%) over one day. Hence, approximately one day in a hundred the potential loss may be expected to exceed the DVaR.

Correlations between different markets, instruments, currencies etc are taken into account, since if there is a fall in one market, there may well be a correlated fall in another market, so opposing positions will to some extent cancel.

This is a single number, which is easy to use and understand, but has the limitation that everything is compressed into one number, so information is lost.

It is also very sensitive to the correlation and volatility assumptions, relying on past data to assess future events.

(c) Stress tests are revaluations of the portfolio with large movements in rates and index values.

These are useful to show what might happen to an option portfolio which might have a large amount of convexity which would not show up on a delta sensitivity report.

(iii) (a) The swaption book will have sensitivities to interest rates and implied volatilities in three dimensions: by expiry, by strike and by term of underlying instrument.

These can be put into a matrix of sensitivities to show how the book would behave if rates or volatilities changed. This is often summarised by certain rows as "net" figures.

Particularly useful is sensitivity to implied volatility, which is a Vega matrix.

Interest rate DVaR can be calculated for the book by allowing correlations between different underlying swaps, and taking implied or historic volatilities from the market. There is also the volatility of the

Vega, which also have a DVaR. It is possible to look at the correlation between implied volatility and market rates, but this is less natural — the two parts of the DVaR could just be added.

A stress test will be useful due to the convexity effects. This would show if any big positions emerged far from the current levels of rates.

(b) Credit risk only arises on purchased swaption positions, and when any swaptions are exercised to become swaps.

7 ***Syllabus: (e)(iv)***
Reading: U2 & U4 / Hull Ch 11.6

(i) (a) Given a probability measure **P** and a history (filtration) of past events $\{F_t, t \le s\}$, then the stochastic process $\{X_t, t \ge 0\}$ is a <u>martingale</u> if:

$$E_P[X_t \mid F_s] = X_s \text{ for any } t \ge s.$$

In other words, the expected future value of the stochastic process X_t is its current value, i.e. it is driftless.

(b) The variable ϕ represents the value of f relative to the price of g. It can be thought of as *measuring* the price of f in units of g, rather than pounds (or any other currency).

The security g is said to be the numeraire asset, i.e. the asset by reference to which the worth of other assets (e.g. f, an arbitrary function) is measured.

(ii) We have:

$$\frac{\mu_f - r}{\sigma_f} = \lambda = \frac{\mu_g - r}{\sigma_g}$$

which implies

$$\mu_f = \sigma_f \lambda + r \text{ and } \mu_g = \sigma_g \lambda + r$$

Substituting $\lambda = \sigma_g$ gives

$$\mu_f = \sigma_f \sigma_g + r \text{ and } \mu_g = \sigma_g^2 + r$$

Substituting these equations into the SDE's for f and g gives

$$df = (\sigma_f \sigma_g + r) f dt + \sigma_f f dz \qquad (1)$$

$$dg = (\sigma_g^2 + r) g dt + \sigma_g g dz \qquad (2)$$

Using Ito's Lemma on (1) and (2) for the functions ln f and ln g respectively, we get:

$$d \ln f = (r + \sigma_g \sigma_f - \sigma_f^2 / 2)dt + \sigma_f dz$$
$$d \ln g = (r + \sigma_g^2 / 2)dt + \sigma_g dz$$

noting that $\frac{d}{df}(\ln f) = \frac{1}{f}$ and $\frac{d^2}{df^2}(\ln f) = -\frac{1}{f^2}$.

[*Alternatively, candidates may wish to use Ito on the original processes*

$$d \ln f = (\mu_f - \sigma_f^2 / 2)dt + \sigma_f dz$$
$$d \ln g = (\mu_g - \sigma_g^2 / 2)dt + \sigma_g dz$$

and then substitute for μ_f *and* μ_g *to get the same equations.*]

So that, taking differences,

$$d(\ln f - \ln g) = (\sigma_g \sigma_f - \sigma_f^2 / 2 - \sigma_g^2 / 2)dt + (\sigma_f - \sigma_g)dz$$

Equivalently, $d(\ln \frac{f}{g}) = -\frac{(\sigma_f - \sigma_g)^2}{2} dt + (\sigma_f - \sigma_g)dz$

Using Ito's Lemma in reverse (comparing to formulae (1) and (2) above), we can write down the process for f / g that gives such a result for $\ln(f / g)$

$$d(f / g) = (\sigma_f - \sigma_g)\frac{f}{g} dz$$

This is a driftless process, and is hence a Martingale.

(iii) A world which consists of a security (or stochastic process) g whose volatility σ_g is equal to the market price of risk, then the world is said to be forward risk neutral with respect to g.

If f is any other security, then in the world that is forward risk neutral with respect to g, f / g is a Martingale. It follows that the current value of f / g, viz f_0 / g_0 is equal to the expected value at time zero of all future values $E_g[f_t / g_t]$, where expectations are carried out using the probability density function underlying *g*.

8 ***Syllabus: (g)(ii)&(iii)***
Reading: U7 / Hull 5

(i) $P(t) = (1 + y(t))^{-t}$ for t = 1, 2, 3 etc
and $P(0) = 1$

(ii) $$f(t) = \frac{P(t-1)}{P(t)} - 1$$

$$1 = \sum_{s=1}^{t} g(t)P(s) + 1.P(t)$$

$$\Rightarrow g(t) = \frac{1 - P(t)}{\sum_{s=1}^{t} P(s)}$$

In the above, P(t) are in decimal (i.e. par = 1). The candidate could also have obtained full marks using par = 100 and putting the rates in %.

(iii) $$f(1) = \frac{P(0)}{P(1)} - 1 = \frac{1}{P(1)} - 1 = \frac{1}{(1 + y(1))^{-1}} - 1 = y(1)$$

$$g(1) = \frac{1 - P(1)}{P(1)} = \frac{1}{P(1)} - 1 = f(1) = y(1)$$

The next part is appears simple but requires the right approach, hence the hint.

Key is value $P(2)^{-1}$ in two different ways.

$$\frac{1}{P(2)} = (1 + f(1))(1 + f(2)) = (1 + f(1))(1 + f(1) + \Delta f)$$
$$= (1 + f(1))^2 + \Delta f(1 + f(1))$$

Also,

$$\frac{1}{P(2)} = (1+y(2))^2 = (1+y(1)+\Delta y)^2$$
$$= (1+y(1))^2 + 2\Delta y(1+y(1)) + (\Delta y)^2$$
$$\approx (1+y(1))^2 + 2\Delta y(1+y(1))$$

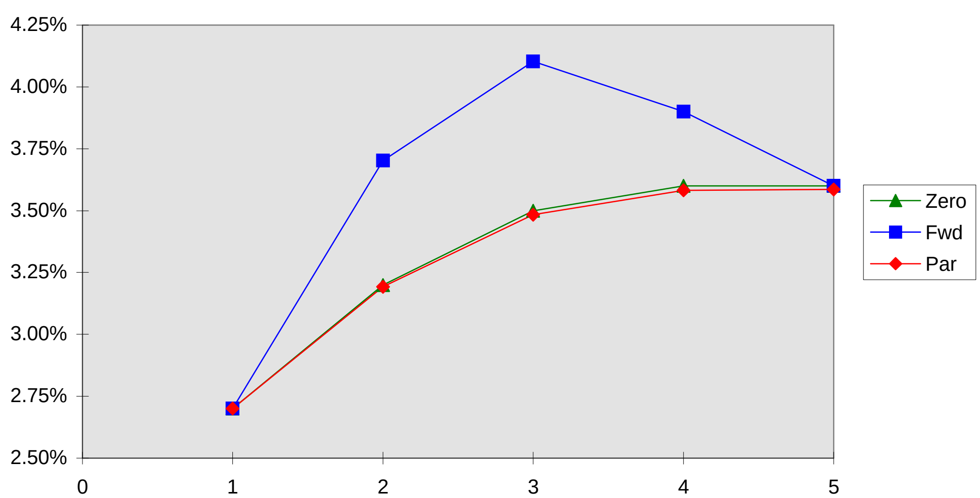
ignoring the last term which is considerably smaller than the others.

But $y(1) = f(1)$, so equating the two forms of $P(2)^{-1}$ gives the result.

(iv) (a) The following table should be created (the final column is only for section (v) below):

t	*y(t)*	*P(t)*	*f(t)*	*g(t)*	*Swap flows*
0		1			
1	2.700%	0.97371	2.700%	2.700%	−1.300%
2	3.200%	0.93895	3.702%	3.192%	−0.298%
3	3.500%	0.90194	4.103%	3.484%	0.103%
4	3.600%	0.86808	3.901%	3.582%	−0.099%
5	3.600%	0.83792	3.600%	3.585%	−0.400%

(b) Verify that the slope of $f(t)$ is twice that of $y(t)$: $3.702 - 2.7 \approx 2(3.2 - 2.7)$.



(c) The par and spot curves are virtually identical, with the spot slightly above the par.
The similarity is because the yield curve is valuing all cashflows on the same date at the same rate. To first order all the yields are very similar, so any bond of the same maturity will have approximately the same yield.

[*The reason for the slight difference is that, in an upward sloping yield curve, the par bond has a lower reinvestment rate for its coupon in the early years, whereas the zero has its yield throughout (by definition). This explanation is more detailed than was required to gain full marks.*]

(v) (a)

The swap floating coupons are the f(t), so the answer is:

$$\text{EUR } 100{,}000{,}000 \sum_{s=1}^{5} (f(s) - 4\%) P(s)$$

and the values of ($f(s)$ – 4%) are given in the last column of the table in (iv) above.

Hence value = EUR 100,000,000 × −0.01874 = EUR − 1,874,000.

Note that the swap value is negative. This is OK.

(b) The true market value will be different because the curve given was a government curve and swaps are traded on an interbank curve, which has higher yields (lower credit quality) – this would reduce the swap value.

Other points:
Supply / demand imbalances between markets, or liquidity differences.

END OF EXAMINERS' REPORT