EXAMINATION

8 October 2009 (pm)

Subject ST6 — Finance and Investment
Specialist Technical B

Time allowed: Three hours

INSTRUCTIONS TO THE CANDIDATE

1. *Enter all the candidate and examination details as requested on the front of your answer booklet.*

2. *You have 15 minutes before the start of the examination in which to read the questions. You are strongly encouraged to use this time for reading only, but notes may be made. You then have three hours to complete the paper.*

3. *You must not start writing your answers in the booklet until instructed to do so by the supervisor.*

4. *Mark allocations are shown in brackets.*

5. *Attempt all eight questions, beginning your answer to each question on a separate sheet.*

AT THE END OF THE EXAMINATION

Hand in BOTH your answer booklet, with any additional sheets firmly attached, and this question paper.

In addition to this paper you should have available the 2002 edition of the Formulae and Tables and your own electronic calculator from the approved list.

NOTE: In this examination, you are never required to prove the use of an arbitrage-free methodology unless clearly stated in the question.

1 Let S_t be the price of an asset at time t which follows the geometric Brownian motion process:

$$S_t = S_0 \exp(\mu t + \sigma W_t)$$

under probability measure **P** in which W_t is a Brownian motion process, and μ and σ are constants.

Let B_t be a risk free asset whose price grows deterministically according to $B_t = e^{rt}$, where r is the continuously compounded risk-free rate, so that $Z_t = B_t^{-1} S_t$ represents the asset price process discounted to time 0.

Additionally, let X be a path-independent claim on S_t whose value is known at time T.

(i) By taking the logarithm of Z_t, or otherwise, derive the stochastic differential equation for Z_t. [4]

(ii) Explain what is meant by a self-financing and replicating strategy for X. [3]

(iii) Show how the Cameron-Martin-Girsanov and the Martingale Representation theorems can be used to construct a replication strategy for X. [6]

[Total 13]

2 In a zero interest rate environment, the price process S_t of a non-dividend paying equity at time t is:

$$dS_t = \sigma \, dW_t$$

where W_t is a standard Brownian motion.

A claim X exists on the stock at time T in the future such that:

$$X = \max[\exp(aS_T) - K, 0]$$

where a is an arbitrary constant and K is the strike price (i.e. the amount paid for the exponential payoff at time T).

Compute the arbitrage-free value of the claim using the statistical distribution of S_t directly. [8]

3 A fixed interest swap market denominated in Euros has a yield curve defined by the following continuously compounded interest rates:

Term (years)	*Zero Coupon Yield*
1	4.0%
2	4.4%
3	4.6%
4	4.8%
5	5.0%

A financial institution is committed to a forward agreement whereby it will pay 5.75% per annum interest on a nominal amount of €1million invested for three years from the end of year 2, with the interest being paid annually in arrears at the end of years 3, 4 and 5.

(i) (a) Calculate the fixed rate of a three-year forward-starting swap commencing in two years' time with annual payments.

(b) Hence state whether the forward agreement currently has a positive or negative value to the financial institution. [3]

The financial institution is considering alternative ways of hedging the interest rate risk within the forward agreement.

(ii) Calculate the price of a hedge that removes <u>all</u> interest rate risk. [2]

One possible contract being considered is a receiver swaption that would pay, at the end of year 2, the value of the forward agreement if it has a negative value to the financial institution at that time. The volatility of the forward rate is 12% per annum.

(iii) Specify the terms of this swaption and calculate its fair price. [6]

(iv) Describe an alternative strategy to that in (iii) which creates the same effect but uses a payer swaption. [2]

[Total 13]

4 In a risk-neutral world, zero coupon bond prices $P(t, T)$ maturing at time T follow a Heath-Jarrow-Morton (HJM) process:

$$dP = r(t)P\,dt + \sigma(T-t)^{\frac{1}{2}}P\,dz$$

where $z(t)$ is a Brownian motion with constant volatility coefficient σ.

(i) Define in terms of P:

(a) the forward rate $f(t, T_1, T_2)$

(b) the instantaneous forward rate $F(t, T)$

(c) the zero coupon rate $Z(t, T)$

[2]

(ii) (a) Derive a stochastic differential equation for f using Ito's Lemma.

(b) Using (ii)(a), derive a stochastic differential equation for F.

(c) Hence derive an expression for the instantaneous short term rate $r(T)$, given that $r(t) = F(t, t)$.

[7]

(iii) (a) State the general HJM result relating to the drift and volatility of $F(t, T)$. [*You do not need to derive this result.*]

(b) Explain the practical problems caused by the fact that the process for the short rate $r(t)$ is usually non-Markovian (i.e. path dependent).

[3]
[Total 12]

5 (i) (a) Describe convenience yield and the circumstances in which it might be negative.

(b) Explain how a Bermudan option differs from a European and an American option. [3]

A trader wishes to value a 3-year Bermudan Put option on a commodity with zero convenience yield. The commodity price is currently 20 and the strike price is 22.5. The option is exerciseable at the end of years 1, 2 and 3. The risk-free interest rate is 4%, continuously compounded, for all maturities.

The trader is thinking of using the least-squares approximation proposed by Longstaff and Schwartz. In this approach, the Put price V is approximated by the function:

$$V = a + bS + cS^2$$

where S is the commodity price and a, b and c are constants to be estimated. She has run her Monte Carlo model to generate sample paths for S, and has the following table of six sample results:

Path	*t = 0*	*t = 1*	*t = 2*	*t = 3*
1	20.0	23.1	23.6	21.4
2	20.0	18.2	15.9	15.1
3	20.0	24.1	18.4	19.7
4	20.0	18.6	19.8	15.6
5	20.0	22.2	26.1	29.2
6	20.0	18.0	15.6	17.0

[*Note: For parts (ii) and (iii) of this question, these six paths are assumed to be fully representative of the process, though of course in practice many more samples would be needed.*]

(ii) (a) Demonstrate that only paths 2, 3, 4 and 6 are in-the-money at time $t = 2$.

(b) For these paths, compare the payoff (exercise value) at time $t = 2$ with the payoff at time $t = 3$ discounted to time $t = 2$.

(c) Show numerically how the least-squares problem would be set up for time $t = 2$. [4]

Having run the least-squares estimator at time $t = 2$ for paths 2, 3, 4 and 6, the trader has the found the following values: $a = 272.4$; $b = -30.46$; $c = 0.86$.

(iii) Carry out the analysis at time $t = 2$, and hence show which paths lead to early exercise at time $t = 2$. [3]

(iv) Explain how the trader would use the method in practice to value the option, incorporating a proper sample size taken over all time points. [*You do not need to perform any calculations for this part.*] [2]

[Total 12]

6 A “cash-or-nothing” Call is a derivative that pays a fixed cash sum at maturity if the value of the underlying asset is above a specified threshold price on a fixed date, but zero otherwise.

A bank has just written a one-year cash-or-nothing Call on the FTSE 100 index with a threshold price of 5,000. The current price of the index is 4,200.

(i) Sketch simple graphs for this position against a suitable range of index values, showing the situation both now and almost twelve months from now, for each of the following three variables:

(a) Profit/Loss (to the bank)
(b) Delta
(c) Gamma

You may assume that implied volatilities are constant and you may ignore financing costs. [8]

(ii) Outline any practical difficulties the bank might have in hedging the derivative using a dynamic delta hedging methodology. [2]

A Call spread is made up of a long position in one Call option and a short position in another Call option against the same underlying asset with the same notional amount but a higher exercise price.

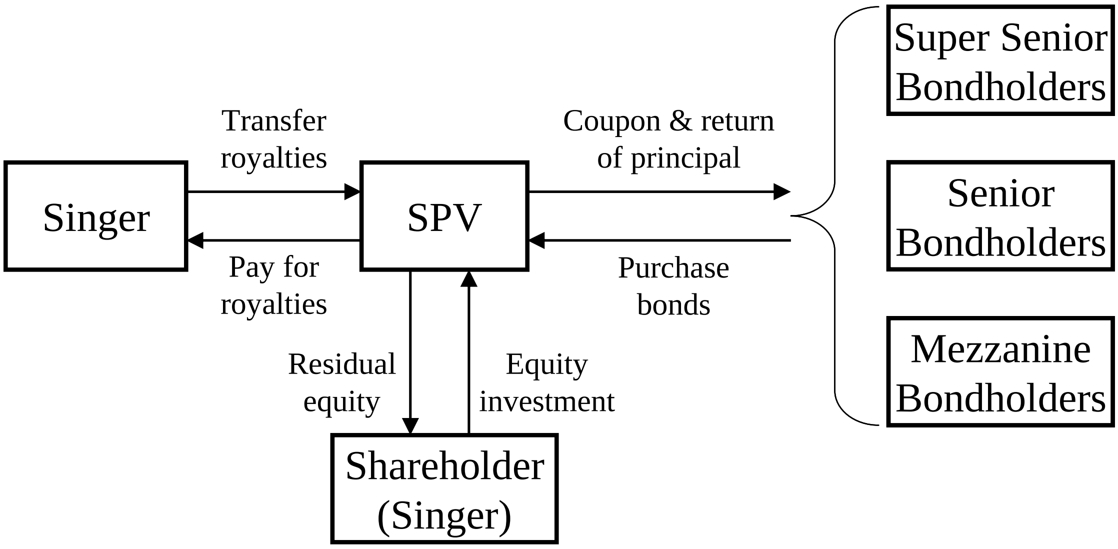
(iii) (a) Sketch a graph showing the payoff at maturity on a Call spread.

(b) Demonstrate a hedge that will perfectly replicate the payoff from the cash-or-nothing Call provided the FTSE 100 index in twelve months’ time is more than a given x% above or below 5,000. [6]

[Hint: Consider the cash-or-nothing Call as the limiting case of a sequence of Call spreads.]

[Total 16]

7 A popular singer is planning to securitise the royalties he will receive over the next ten years on all compact disc sales of his existing back catalogue of albums.



The securitisation will work as follows:

- A Special Purpose Vehicle (SPV) is established.
- The singer sells the royalties to the SPV in return for cash.
- The SPV raises cash by selling bonds secured on these royalties, tranched into three different levels of seniority and an equity share to the singer.
- The SPV pays annual coupon payments to the bondholders from royalties and uses any excess funds to repay the bonds, starting at the super-senior level and working down.
- After ten years, the SPV is wound down and any excess funds are returned to the shareholder.

(i) Describe the motives for the singer and various bondholders in entering into these transactions. [2]

(ii) Explain the purpose of establishing:

(a) an SPV
(b) the equity tranche [4]

The mezzanine bonds from this securitisation are proving difficult to sell because of a low credit rating. It has been suggested that they could be re-packaged with those of similar securitisations from other musical artists into a further securitisation. Effectively this would convert several issues of mezzanine bonds into senior bonds, a smaller portfolio of mezzanine bonds and some "junk" (non investment grade) bonds.

(iii) (a) Explain the likely reasons for the low credit rating.

(b) Discuss whether the proposal would work.

(c) Describe how the tranches in the re-packaged securitisation could be structured to increase the credit rating of the new mezzanine bonds. [5]

[Total 11]

8 A UK life insurance company has decided to hedge some of its exposure to the risk that UK equity markets fall sharply. It has been approached by an investment bank, which has recommended that it purchases a specified portfolio of FTSE index Put options with a variety of strikes and maturities.

The life insurance company's investment managers have presented a counter-proposal. Their idea is that the investment bank's recommended portfolio be designated the "benchmark portfolio" and that the fund managers attempt to replicate the benchmark using a dynamic delta hedging strategy.

The life insurance company's senior management team is not sure which alternative to choose, and has approached you as an independent consultant to help them reach an informed decision. As a first step, you have been asked to compare the benefits, costs and risks of adopting the delta hedging approach as opposed to buying options from the investment bank.

(i) Outline the points you would make in your report. [10]

(ii) Discuss the problems of incorporating dividends and volatility estimates in the dynamic hedging strategy. [5]
[Total 15]

END OF PAPER