EXAMINATION

29 March 2006 (pm)

Subject ST6 — Finance and Investment Specialist Technical B and Certificate in Derivatives

Time allowed: Three hours

INSTRUCTIONS TO THE CANDIDATE

1. *Enter all the candidate and examination details as requested on the front of your answer booklet.*
2. *You have 15 minutes at the start of the examination in which to read the questions. You are strongly encouraged to use this time for reading only, but notes may be made. You then have three hours to complete the paper.*
3. *You must not start writing your answers in the booklet until instructed to do so by the supervisor.*
4. *Mark allocations are shown in brackets.*
5. *Attempt all 8 questions, beginning your answer to each question on a separate sheet.*
6. *Candidates should show calculations where this is appropriate.*

Graph paper is required for this paper.

AT THE END OF THE EXAMINATION

Hand in BOTH your answer booklet, with any additional sheets firmly attached, and this question paper.

In addition to this paper you should have available the 2002 edition of the Formulae and Tables and your own electronic calculator.

NOTE: In this examination, you are never required to prove the use of an arbitrage-free methodology unless clearly stated in the question.

1 In the Vasicek model, the spot rate of interest is governed by the stochastic differential equation:

$$dr_t = a(b - r_t)dt + \sigma dB_t$$

where B_t is a standard Brownian motion and a, $b > 0$.

A stochastic process is defined by $s_t = e^{at} r_t$.

(i) Solve the equations above to find s_t, and hence show that:

$$r_t = b + (r_o - b)e^{-at} + \sigma \int_o^t e^{a(s-t)} dB_s$$ [4]

(ii) Find the probability distribution for:

(a) r_t , in the limit when t is large

(b) a zero coupon bond of maturity T [5]

[Total 9]

2 (i) Write down the criteria for a process $\{W_t : t \geq 0\}$ to be a standard Brownian motion. [2]

(ii) For a standard Brownian motion process W_t:

(a) By considering a discrete summation approximation for the integral $\int_0^t W_s dW_s$, or otherwise, show that the simple Newtonian differentiation of W_t^2 [i.e. $d(W_t^2) = 2W_t dW_t$] cannot be correct.

(b) Comment briefly on this feature of Brownian motion.

[5]

(iii) Consider two independent Brownian motions W_t and $W_t^{\%}$ and let ρ be any constant between –1 and +1. Demonstrate that $X_t = \rho W_t + \sqrt{1-\rho^2} W_t^{\%}$ is a Brownian motion. [3]

[Total 10]

3 In a Black-Scholes world, the price x of a single option on a non-dividend paying stock of price s is governed by the following differential equation:

$$\frac{\partial x}{\partial t} + rs\frac{\partial x}{\partial s} + \frac{1}{2}\sigma^2 s^2 \frac{\partial^2 x}{\partial s^2} = rx$$

where r is the risk-free rate of interest and σ is the stock's volatility.

(i) Define theta and gamma, and show the relationship between them in a delta neutral portfolio. [3]

(ii) Sketch the following three relationships graphically:

(a) theta against time to expiry for an at-the-money option
(b) gamma against time to expiry for an at-the-money option
(c) gamma against time to expiry for an out-of-the-money option

[5]

An options trader working for a UK bank provides a market-making service pricing interest rate options for his bank's clients. He hedges his option portfolio with other options in the same currency, but often these are of differing expiry dates and strikes.

(iii) Suggest reasons why the trader might wish to structure his positioning in this way. [3]

[Total 11]

4 As a commodity options dealer, you have access to a time series database of 10 years of daily opening and closing prices of a particular commodity. The commodity has relatively high storage costs and a short shelf life.

(i) Show how to calculate annualised volatility from historic price data for use in a log-normal based option model. [3]

(ii) Discuss the following issues in obtaining the volatility estimate:

(a) how much of your data it is appropriate to use, and why

(b) whether to use opening or closing prices, and why

(c) how to allow for jumps in the commodity price, both in the stored time series and in future market movements

[6]

(iii) Comment on the additional factors that might be relevant if you were buying an option on a futures contract based on the commodity. [2]

[Total 11]

5 (i) Define the following terms, and describe each of them in the context of a binomial model:

(a) stochastic process
(b) probability measures
(c) filtration
(d) previsible process

[8]

(ii) Describe how you would show that a discrete stochastic process S_i , i = 1, 2 ... , is a martingale under some given probability measure $\boldsymbol{Q}$. [4]

[Total 12]

6 An equity, which can be modelled by a simple binomial process, is currently priced at £35. Over each of the next two 6-month periods its price is expected to go up by 6% or down by 4%. The risk-free interest rate is 3% per annum, continuously compounded.

(i) Calculate, using a risk neutral valuation, the value of a 1-year at-the-money European Call, and estimate its current delta. [5]

(ii) Verify your answers to (i), using instead **only** a no-arbitrage argument. [7]

(iii) Show how you would adjust your model to value a 1-year at-the-money American Call option on the same equity. [2]

[Total 14]

7 Suppose the USD money market interest rate curve is currently 6% per annum for all maturities.

Consider an interest rate futures contract expiring in 5 years with an underlying term of 3 months (a “60–63 future”), and a forward rate agreement (FRA) of the same expiry date and term (a “60–63 FRA”).

(i) (a) Show that the value (variation margin) of USD 1 billion nominal of 60–63 futures, transacted when interest rates were 7%, is $2,500,000. You may ignore initial margin.

(b) Show that the value of USD 1 billion 60-63 FRA, transacted when interest rates were 7%, is $1,868,145.

[3]

Suppose the following combined trade was undertaken:

- Sell USD 10 billion 60-63 futures @ 93.00
- Sell USD 13.40 billion 60-63 FRA @ 7%

(Note: A sold FRA benefits if interest rates decline.)

(ii) (a) Give reasons why the combined trade might have been undertaken.

(b) Explain the relative sizes of the two "legs".

(c) Calculate the profit or loss for two different scenarios, when the interest rate curve changes instantaneously from 6% to 4%, and from 6% to 8%.

(d) Compare the two results in (c).

[6]

Two random variables s_1 and s_2 follow the Wiener Processes:

$$ds_1 = \mu_1 s_1 dt + \sigma_1 s_1 dz_1$$

$$ds_2 = \mu_2 s_2 dt + \sigma_2 s_2 dz_2$$

with $\mathbf{E}[z_1 z_2] = \rho$.

(iii) (a) Derive the process followed by the product $s_1 s_2$.

(Note: The generalised Ito's formula for n processes of the form $dx_i = \mu_i dt + \sigma_i dz_i$ is:

$$df = \frac{df}{dt}dt + \sum_{i=1}^{n} \frac{df}{dx_i} dx_i + \tfrac{1}{2} \sum_{i=1}^{n} \sum_{j=1}^{n} \frac{d^2 f}{dx_i dx_j} \sigma_i \sigma_j \rho_{ij} dt$$

where f is a function of the x_i and t, and ρ_{ij} is the correlation between z_i and z_j.)

(b) Describe how you would use this result to calculate an estimate for the (risk neutral) expected price differential between the contracts traded in (ii), stating any further information you would require.

[7]

[Total 16]

8 The current zero coupon yield curve for an interest rate market is as follows (all rates assume annual compounding):

Term	*Rate % per annum*
Overnight	3.75
1 year	4.25
2 year	4.40
3 year	4.50
4 year	4.55
5 year	4.60
6 year	4.65

(i) Calculate the following:

(a) The value of a zero coupon bond of maturity 2½ years.

(b) The continuously compounded 4-year zero coupon rate.

(c) The fixed leg coupon of a 4-year par value annual to annual interest rate swap.

(d) The fixed leg coupon of a (forward-starting) 4-year par value annual to annual interest rate swap commencing in two years' time. [6]

A constant maturity swap (CMS) is a swap where each payment on the floating leg of the swap is reset to be the fixed leg coupon of a par value interest rate swap of the given maturity current at each reset date.

(ii) (a) Calculate the fixed leg coupon of an annual to annual par value 1-year CMS which is based on the 4-year swap rate.

(b) Explain why the method used to calculate forward-starting swaps (see (i)(d) above) is not appropriate as a general method for valuing CMS. [5]

You are reviewing a large portfolio of swaps, swaptions, caps, floors, forward rate agreements and interest rate futures. The instruments have a range of maturities out to 30 years.

Options, all European in style, have been bought and sold, but there are no "exotic" instruments. Reports based on use of the Black model suggest that, at the overall level, the portfolio has minimal risk to interest rate or volatility changes. You are confident that all positions and third party valuations have been correctly recorded.

(iii) Discuss for this portfolio four important areas where the assessment of risk could potentially be incorrect. [6]

[Total 17]

END OF PAPER