EXAMINATION

3 October 2007 (pm)

Subject ST6 — Finance and Investment
Specialist Technical B

Time allowed: Three hours

INSTRUCTIONS TO THE CANDIDATE

1. *Enter all the candidate and examination details as requested on the front of your answer booklet.*
2. *You have 15 minutes at the start of the examination in which to read the questions. You are strongly encouraged to use this time for reading only, but notes may be made. You then have three hours to complete the paper.*
3. *You must not start writing your answers in the booklet until instructed to do so by the supervisor.*
4. *Mark allocations are shown in brackets.*
5. *Attempt all 8 questions, beginning your answer to each question on a separate sheet.*
6. *Candidates should show calculations where this is appropriate.*

Graph paper is required for this paper.

AT THE END OF THE EXAMINATION

Hand in BOTH your answer booklet, with any additional sheets firmly attached, and this question paper.

In addition to this paper you should have available the 2002 edition of the Formulae and Tables and your own electronic calculator.

NOTE: In this examination, you are never required to prove the use of an arbitrage-free methodology unless clearly stated in the question.

1 (i) Consider a discrete stochastic process $X = X_i$ that evolves over an integer time sequence i, for $0 \le i \le T$. Explain the following terms:

(a) A filtration $\mathbf{F}_i$ at time i.

(b) A binomial measure $\mathbf{Q}$.

(c) The conditional expectation $E_{\mathbf{Q}}(X \mid \mathbf{F}_i)$

(d) A previsible process $\phi = \phi_i$ [4]

(ii) Explain how, using conditional expectations, you can construct a stochastic process M based on any process X, such that M satisfies the martingale condition:

$$E_{\mathbf{Q}}(M_j \mid \mathbf{F}_i) = M_i \quad \text{for all } i \le j$$ [3]

(iii) (a) State the Binomial Representation Theorem for a binomial measure $\mathbf{Q}$ and martingale process M.

(b) Explain briefly how this could be used to price contingent claims on a stock. [4]

[Total 11]

2 Consider a basket of three independent tradable non-dividend paying equities in a Black-Scholes world, with respective volatilities 20%, 25% and 30% per annum. Each equity is currently priced at 100. Risk-free interest rates for the period are 6% per annum, continuously compounded.

An investment product pays a return of £1 million if one or more of the three equities rises by at least 20% at the end of a three month period.

(i) State, using the Black-Scholes European option formula, the probability that an option is in the money at expiry, defining all terms. [2]

(ii) Calculate, for each equity separately, the probability that it rises by at least 20% at the end of a three month period. [3]

(iii) Calculate the value of the investment product under a risk neutral measure. [4]

(iv) Explain what would happen to your answer in (iii) if instead the equities were positively correlated. [2]

[Total 11]

3 The US dollar/sterling exchange rate is trading at \$1.90. One-year interest rates in both US dollars and sterling are 5.5%. Option implied volatilities have been at historic lows, but have just risen by about 1 percentage point as the exchange rate has become more volatile.

The following table of one-year option prices has just been supplied by a currency options broker (the convention is to quote prices in hundredths of a cent):

Relative Strike	**Strike**	**Call Price (x 10,000)**	**Put Price (x 10,000)**	**Implied Volatility**
ATM	1.9000	537.9	537.9	7.50%
+5 cents	1.9500	354.5	827.7	7.70%
+10 cents	2.0000	232.6	1,179.1	8.00%
–5 cents	1.8500	794.0	320.8	7.40%
–10 cents	1.8000	1,124.7	178.3	7.45%

(i) (a) Demonstrate algebraically that Put-Call parity applies for the currency options version of the Black-Scholes formula.

(b) Verify Put-Call parity numerically for the option pair with strike \$1.80. [4]

(ii) Explain why the implied volatilities of the options shown in the table above are not all the same. [2]

A *butterfly spread* is the combination of two simultaneous option trades in equal amounts:

- the sale of an at-the-money straddle (one European Call plus one European Put, each with the same strike K), and
- the purchase of an out-of-the-money strangle (one European Put with out-of-the-money strike K_1 plus one European Call with out-of-the-money strike K_2, where $K_1 < K < K_2$).

(iii) By considering the payoff pattern at expiry, show that a butterfly spread is equivalent to the simultaneous purchase and sale of two (different) Call spreads. [3]

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(iv) (a) Calculate the net premium of the 5 cents out-of-the-money butterfly spread (i.e. $K_1 = 1.85$ etc.) and the 10 cents out-of-the-money butterfly spread (i.e. $K_1 = 1.80$ etc.), ignoring transaction costs.

(b) Give reasons, using your knowledge of the factors that affect option prices, why a currency dealer might decide to take a position in such a spread.

(c) Suggest which of the two spreads calculated in (a) might be preferred. [5]

[Total 14]

4 Consider a non dividend paying equity of price S_t and a risk-free savings account of value B_t, whose evolution at time t follows these processes:

$$dS_t = \mu S_t dt + \sigma S_t dw_t$$

and

$$dB_t = rB_t dt$$

where w_t is a standard Brownian motion, and μ, σ and r are constants.

Further, consider a portfolio consisting of an amount ϕ_t of the equity and ψ_t of the savings account, such that $\phi_t = S_t$. Initially, the portfolio is entirely invested in the equity.

(i) Explain, in words and algebraically, what it means for a portfolio to be self-financing. [2]

(ii) Find the value of μ that makes $\frac{B_t}{S_t}$ a martingale under a suitable risk neutral probability measure. [3]

(iii) Derive a formula for ψ_t such that the portfolio is self-financing. [4]

[Total 9]

5 The Cox-Ingersoll-Ross (CIR) model is a one-factor arbitrage free interest-rate model of the short rate $r(t)$ of the form:

$$dr = \lambda(\mu - r)dt + \sigma\sqrt{r}\,dw_t$$

where $r(0) > 0$, λ, μ and σ are constants, and w_t is Brownian motion.

(i) (a) Define the term “arbitrage free”.

(b) Outline the significance of such a property for an interest-rate model. [3]

Under the CIR model, the partial differential equation (PDE) for the price $P(t, T)$ of a zero coupon bond of maturity T valued at time t is given by:

$$\frac{\partial P}{\partial t} + (\sigma - \lambda r)\frac{\partial P}{\partial r} + \tfrac{1}{2}\sigma^2 r\frac{\partial^2 P}{\partial r^2} - rP = 0$$

and this PDE is solved by a function of the form:

$$P(t,T) = \exp\left[A(t,T) - B(t,T)r(t)\right].$$

(ii) (a) Show, using Ito’s Lemma, what this PDE implies for the expected value of $d(\ln P)$.

(b) Derive from the PDE a differential equation for $B(t,T)$. [5]

(iii) Assess the advantages CIR has over the simpler Vasicek approach as a model of the yield curve. [4]

[Total 12]

6 (i) Write down formulae for the payoff and price (based on Black’s model) of a European interest rate caplet, defining any terms you use. [4]

(ii) Calculate the price of a six-month 6% binary caplet based on the one-month LIBOR rate that pays £1,000,000 on exercise. You may assume that the zero coupon yield curve is currently flat at 5.35% (continuously compounded), and that implied volatility is 16%. [4]

(iii) Discuss the merits of using “spot” or “flat” implied volatility curves in the pricing of caps. [4]

[Total 12]

7 Bank A wishes to swap payments of a 5.75% annual coupon US dollar-denominated bond with a 4.5% annual coupon Euro-denominated bond held by Bank B. Each bond has a remaining life of three years.

The two banks set up a cross-currency swap arrangement, which involves exchanging the coupons and final maturity principal as they become due. They agree to exchange \$200 million nominal of the dollar-denominated bond for the currency equivalent amount of the Euro-denominated bond based on the spot (current) exchange rate.

The spot exchange rate is 1.28 US dollars to 1 Euro. Rates are flat in the US at 6% and in Europe at 4% (both continuously compounded).

(i) (a) Calculate the forward exchange rates for the each of next three years.

(b) Hence derive the present value to bank A of the swap arrangement. [6]

Bank A wishes to use similar types of cross-currency swap to transfer the payoffs of complex interest rate options from foreign currencies back to US dollars. In such cases, if the option were exercised, the cross-currency swap would have to be cancelled.

(ii) Discuss what valuation problems the addition of a cancellation feature would add to the pricing of the cross-currency swap. [2]

(iii) (a) Describe briefly the Monte Carlo simulation method for valuing complex interest rate and currency options.

(b) Indicate how it could be used to value the option in (ii). [5]

(iv) Discuss two methods that can be used to improve the efficiency and convergence speed of the Monte Carlo method. [4]

[Total 17]

8 You are the new head of risk management for a life office. The office's most profitable business lines at present are annuities and guaranteed equity bonds.

The office has a portfolio of assets supporting the annuity business. There is a policy to mismatch the liabilities by investing in equities and corporate bonds as well as fixed income instruments of various maturities.

The guaranteed equity bond business is supported by a combination of derivatives with major investment banks that provide the required product set (an equity option and guarantee) matched to the underlying investment terms.

There are a few risk controls currently in place. The principal one comprises a Value-at-Risk calculator for the annuity mismatches using a one-day 95% confidence variance-covariance method. In addition, longevity risk — using mortality experience, trends, forecasts and back-tests — is adequately assessed. However, there is little else — for example, at present there is no measurement of credit risk on any of the derivative contracts undertaken.

The Chief Executive Officer has asked you to propose relevant improvements to the quality and scope of the risk management capability, to be evidenced by regular reports produced for management and the Board.

(i) (a) Explain how the Value-at-Risk calculator would measure market risk on the office's portfolio.

(b) Outline what limitations Value-at-Risk might have as a risk measure. [5]

(ii) Discuss how and why you would wish to monitor credit risk on your portfolio of derivatives. [3]

(iii) Set out some suggestions for improving the risk assessment process. Your notes should mention where further investigation is required. [6]
[Total 14]

END OF PAPER