EXAMINATION

14 September 2005 (pm)

Subject ST6 — Finance and Investment Specialist Technical B and Certificate in Derivatives

Time allowed: Three hours

INSTRUCTIONS TO THE CANDIDATE

1. *Enter all the candidate and examination details as requested on the front of your answer booklet.*
2. *You have 15 minutes at the start of the examination in which to read the questions. You are strongly encouraged to use this time for reading only, but notes may be made. You then have three hours to complete the paper.*
3. *You must not start writing your answers in the booklet until instructed to do so by the supervisor.*
4. *Mark allocations are shown in brackets.*
5. *Attempt all 8 questions, beginning your answer to each question on a separate sheet.*
6. *Candidates should show calculations where this is appropriate.*

Graph paper is required for this paper.

AT THE END OF THE EXAMINATION

Hand in BOTH your answer booklet, with any additional sheets firmly attached, and this question paper.

In addition to this paper you should have available the 2002 edition of the Formulae and Tables and your own electronic calculator.

NOTE: In this examination, you are never required to prove the use of an arbitrage-free methodology unless clearly stated in the question.

1 (i) In the context of forward contracts on commodities, explain the concepts of "risk-free rate of return", "convenience yield" and "cost of carry". [3]

(ii) Derive a formula for the forward price of a consumption commodity. [3]

(iii) A friend who works in the oil market is questioning the theory behind commodity futures. He points out that arbitrages appear to persist between spot prices and futures prices for long periods. Give a response to the point he is making. [4]

[Total 10]

2 A bull spread is the simultaneous purchase of a call option of a given strike and maturity and a sale of a second call option of the same maturity but higher strike.

A bull spread is purchased on a commodity priced at 250, with the two strikes of 230 and 270, and expiry date six months from today. Implied volatility is at 35% and the risk-free rate may be assumed to be zero.

(i) Discuss the conditions under which a trader or hedger might wish to purchase a bull spread on a commodity. [2]

(ii) Sketch the value of the bull spread against a range of commodity prices, showing the situation as at today and just before expiry. [2]

(iii) Sketch the value of the Delta and Gamma against the commodity price as at today. [4]

(iv) Describe (separately for each case) how the value of the bull spread at a given commodity price would change:

(a) if volatility falls to 25%
(b) a month from today [2]

[Total 10]

3 Consider a portfolio of derivatives on an equity total return index. The portfolio consists of:

- N_1 sold put options, with strike X_1 and outstanding term T_1
- N_2 sold put options with strike X_2 and term T_2, and
- N_3 sold put options with strike X_3 and term T_3

The portfolio value is V_t at time t. You may assume that all the options are adequately valued using the Black-Scholes pricing formula.

(i) Give definitions for the Delta, Gamma, Theta and Rho of the portfolio. [2]

(ii) Derive formulae for the Delta and Gamma of the portfolio, given the current index value of S_0. [2]

(iii) Describe what would happen to the value of Delta if the index were to suddenly fall by 50%. [2]

(iv) Discuss the relative merits of seeking portfolio insurance by purchasing a put option from a third party, as opposed to pursuing a dynamic hedging strategy. [6]

[Total 12]

4 A stock follows the stochastic process $ds = \mu dt + \sigma dB_t$, where B_t is a standard Brownian motion.

(i) (a) Compute the variance of a discrete average:

$$\text{average} = \frac{1}{n}\sum_{i=1}^{n} S_i \quad \text{where } S_i \text{ is the stock price at } t_i \text{ and } t_i = i.\frac{t}{n}.$$

(b) Show the variance in the limit of continuous sampling. [7]

(ii) Discuss the issues involved in using the Black-Scholes model to price a realistic option on an arithmetic average through out its life. [5]

[Note: The following may be useful: $\sum_{1}^{n} r^2 = \frac{1}{6}n(n+1)(2n+1)$ *.]*

[Total 12]

5 A dividend-paying stock has current price is S_0. Dividends on the stock due to be paid before time T are $g_1, g_2, ..., g_n$, at respective times $t_1, t_2, ..., t_n$. (You may ignore ex-dividend periods.) The risk-free interest rate, continuously compounded, is r.

Consider American and European call and put options on this stock, all with strike K and time to maturity T years.

(i) (a) Show that the value of the European put option can be derived from the value of the European call option.

(b) Explain why this result does not apply to the American options. [3]

(ii) (a) Prove that it is not optimal to exercise the American call option early if:

$$K(1-\exp(-r(T-t_n)) \geq g_n$$

and $K(1-\exp(-r(t_{i+1}-t_i)) \geq g_i$ for $i = 1, 2, ..., n-1$

(b) Using an approximation for the exponential function, show that the inequalities above for $i < n$ normally apply, and thereby conclude at what point an American call is most likely to be exercised early.

(c) Comment on the equivalent result for early exercise of an American put option.

[7]

(iii) It has been found that an approximation for the value of the American call option is obtained by evaluating the European call option at expiry times T *and* t_n, and to take the greater of the two values.

Discuss why this might be a close approximation, and what limitations there might be in its application.

[3]

[Total 13]

6 You are the risk manager of a major UK bank which trades a wide range of fixed income products, including bonds, swaps and options.

(i) Define "market risk" and "credit risk". [2]

(ii) Outline how you would measure market risk using:

(a) a daily report of risk sensitivities against limits
(b) a daily Value-at-Risk
(c) a weekly stress test

[7]

(iii) (a) Illustrate your answer to (ii) with reference specifically to a multi-currency portfolio of swaptions, i.e. options on LIBOR-based fixed-floating swaps.

(b) Identify the source of credit risk in this portfolio.

[4]
[Total 13]

7 Let f, g be two non-income producing securities which depend on a single source of uncertainty, with:

$$df = \mu_f f dt + \sigma_f f dz$$
$$dg = \mu_g g dt + \sigma_g g dz$$

Define λ, the market price of risk; as:

$$\lambda = \frac{\mu_f - r}{\sigma_f} = \frac{\mu_g - r}{\sigma_g},$$

where r is the risk-free rate, and let $\phi = f/g$.

(i) (a) Define a martingale.

(b) Explain the concept of numeraire assets in the context of the securities ϕ, f and g.

[3]

(ii) Using Ito's formula on $\ln \phi$, show that if $\lambda = \sigma_g$, then ϕ is a Martingale. [10]

(iii) Suggest what might be meant by a security f being "forward risk neutral" with respect to a security g. [2]

[Total 15]

8 For a Eurozone government bond yield curve, let $P(t)$ be the price and $y(t)$ the yield of a zero coupon bond of length t, where $t = 1, 2, 3$ ... years. Further, let $f(t)$ be the one-year forward rate from $t - 1$ to t, and $g(t)$ be the yield of a par coupon bond of maturity t years. All yields are annually compounded.

[Note: The par coupon yield is the annual coupon on a bond priced at par. You may assume in this question that we are not interested in intermediate points along the curve.]

(i) Derive a formula for $P(t)$ given $y(t)$. [1]

(ii) Derive formulae for $f(t)$ and $g(t)$ in terms of $P(t)$ for $t = 1, 2, 3 \ldots$. [2]

(iii) Prove that $f(1) = g(1) = y(1)$ and that the slope of $f(t)$ is approximately twice that of $y(t)$ at $t = 1$. [2]

[Hint: Let $\Delta f = f(2) - f(1)$ and $\Delta y = y(2) - y(1)$.]

Maturity (years)	*Spot yield*
1	2.7%
2	3.2%
3	3.5%
4	3.6%
5	3.6%

(iv) (a) Using your formulae in (i) and (ii), calculate the values of the forward rates and par yields given the above table of spot yields, and hence verify numerically the result in (iii).

(b) Sketch the three curves on a single graph.

(c) Comment on the similarities between the par and zero curves.

[7]

(v) (a) Use your curve to value a EUR 100 million 5-year interest rate swap, paying 4% fixed rate annually and receiving floating rate annually.

(b) Suggest why the true market value of such a swap is likely to be different.

[3]
[Total 15]

END OF PAPER