Subject ST6 — Finance and Investment
Specialist Technical B

EXAMINERS' REPORT

September 2008

Introduction

The attached subject report has been written by the Principal Examiner with the aim of helping candidates. The questions and comments are based around Core Reading as the interpretation of the syllabus to which the examiners are working. They have however given credit for any alternative approach or interpretation which they consider to be reasonable.

R D Muckart
Chairman of the Board of Examiners

December 2008

QUESTION 1

Syllabus section: (a)–(d)

Core reading: Units 2 & 3

(i)

(a)

Let the interest rate to the first maturity in t_1 years be rate R_1, and to the second maturity in t_2 years be R_2. Let the forward rate be f_{12}.

For no arbitrage, we require:

$$(1+R_1t_1)(1+f_{12}(t_2-t_1)) = (1+R_2t_2) \qquad (*)$$

Therefore:

$$f_{12} = \frac{R_2t_2 - R_1t_1}{(1+R_1t_1)} \cdot \frac{1}{(t_2-t_1)}$$

[*Note: The equivalent formula for continuously compounded rates is:*

$\exp(R_1t_1)\exp(f_{12}(t_2-t_1)) = \exp(R_2t_2)$, *which leads to* $f_{12} = \dfrac{R_2t_2 - R_1t_1}{(t_2-t_1)}$,

but the question specifically states that rates are annually compounding.

The above solution is correct, but was possibly an unfamiliar use of simple interest. Credit was given for a solution that used compound interest at each time point instead, i.e. used $(1+R_1)^{t_1}$ *rather than* $(1+R_1t_1)$ *etc. The same applies to part (b).*]

(b)

An FRA pays the difference between the actual money market rate and the agreed rate (k) for the period t_1 to t_2, discounted from date t_2.

Hence the current value of the FRA of principal L is:

$$L\ (f_{12}-k)(t_2-t_1)\frac{1}{(1+R_2t_2)}$$

[*Note: The equivalent formula for continuously compounded rates is:*

$$L\ \left[\exp(f_{12}(t_2-t_1)) - \exp(k(t_2-t_1))\right]\exp(-R_2t_2)$$

but the question specifically states that rates are annually compounding.]

(ii)

For forward rate F, at time t we have $F = Se^{r(T-t)}$.

Applying Ito's Lemma to F, we get:

$$\frac{\partial F}{\partial t} = -rSe^{r(T-t)}, \quad \frac{\partial F}{\partial S} = e^{r(T-t)}, \quad \frac{\partial^2 F}{\partial S^2} = 0.$$

Hence $dF = \left[e^{r(T-t)} \mu S - rSe^{r(T-t)} \right] dt + e^{r(T-t)} \sigma S dW = (\mu - r) F dt + \sigma F dW$.

Thus F follows the same geometric Brownian motion as S but with drift reduced by r.

(iii)

If x is the value of X in terms of Y, then $G = 1/x$ is the value of Y in terms of X.

Putting $G = 1/x$ into Ito's Lemma:

$$\frac{\partial G}{\partial t} = 0, \quad \frac{\partial G}{\partial x} = -\frac{1}{x^2}, \quad \frac{\partial^2 G}{\partial x^2} = \frac{2}{x^3}.$$

Hence $dG = \left[-\mu x \frac{1}{x^2} + \frac{1}{2} \sigma^2 x^2 \frac{2}{x^3} \right] dt - \sigma x \frac{1}{x^2} dW_t$

=> $dG = (-\mu + \sigma^2) G dt - \sigma G dW_t$

Putting $d\tilde{W}_t = -dW_t$ still gives a Wiener process:

$$dG = \left(-\mu + \sigma^2\right) G dt + \sigma G d\tilde{W}_t$$

so G is a geometric Brownian motion like x, but with growth $r_X - r_Y + \sigma^2$.

[*<u>Note</u>: It looks as though there is an asymmetry in the growth rates. The reason for this apparent paradox (known as Siegel's paradox) is that the growth is still being measured with respect to currency Y as the unit of account. Transforming to currency X as the unit of account by a change of measure (not in the syllabus, but see Hull p643) has the effect of reducing the growth by* σ^2*, so it becomes* $r_X - r_Y$ *as expected.*]

QUESTION 2

Syllabus section: (h) (i)-(iii)

Core reading: Units 8 & 9

(i)

(a)

The portfolio $\pi = (\phi, \psi)$ of asset and bond accumulates to

$$\pi_{\Delta t} = \begin{cases} \phi s_1 + \psi b e^{r\Delta t} & \text{if } s \mapsto s_1 \\ \phi s_2 + \psi b e^{r\Delta t} & \text{if } s \mapsto s_2 \end{cases}$$

This must be the same as the derivative payoffs, hence we require:

$$f(s_1) = \phi s_1 + \psi b e^{r\Delta t}$$

and $f(s_2) = \phi s_2 + \psi b e^{r\Delta t}$.

These two equations can be solved simultaneously to give

$$\phi = \frac{f(s_2) - f(s_1)}{s_2 - s_1}$$

and $$\psi = \frac{1}{b} e^{-r\Delta t}\left[f(s_2) - \frac{(f(s_2) - f(s_1))s_2}{s_2 - s_1} \right] = \frac{1}{b} e^{-r\Delta t}\left[\frac{s_2 f(s_1) - s_1 f(s_2)}{s_2 - s_1} \right]$$

(b)

The current value of the portfolio is $V = \phi s + \psi b$. This must be equal to the value of the derivative.

(ii)

To see that this is the only value that the derivative can have, suppose there is an option market maker who values the derivative at a price P different from V.

If $P < V$, a counterparty could buy from the market maker the derivative and sell the portfolio $\pi = (\phi, \psi)$ simultaneously. If $P > V$, they could do the opposite.

Hence the counterparty would have a net cash surplus of the absolute value of $V - P$.

This could be invested risk free to achieve a risk free profit of $(V - P)e^{r\Delta t}$

... since, after the time interval Δt, the cash flows from the portfolio exactly match the cash flows in respect of the derivative.

The only way this risk free profit is zero is if $V = P$.

QUESTION 3

Syllabus section: (g) & (i)

Core reading: Units 7 & 12

(i)

For each underlying instrument, let P_i be the sub portfolio relating to them.

By Taylor series expansion:

$$\Delta P_i \sim \frac{\partial P_i}{\partial S}\Delta S + \frac{\partial P_i}{\partial t}\Delta t + \frac{\partial P_i}{\partial \sigma}\Delta\sigma + \tfrac{1}{2}\frac{\partial^2 P_i}{\partial S^2}\Delta S^2 + \tfrac{1}{2}\frac{\partial^2 P_i}{\partial t^2}\Delta t^2$$

You are given that Delta_i = zero, hence:

$$\Delta P_i = \text{Theta}_i \Delta t + \tfrac{1}{2}\text{Gamma}_i \Delta S^2 + \text{Vega}_i \Delta\sigma$$

The overall change in portfolio value is therefore

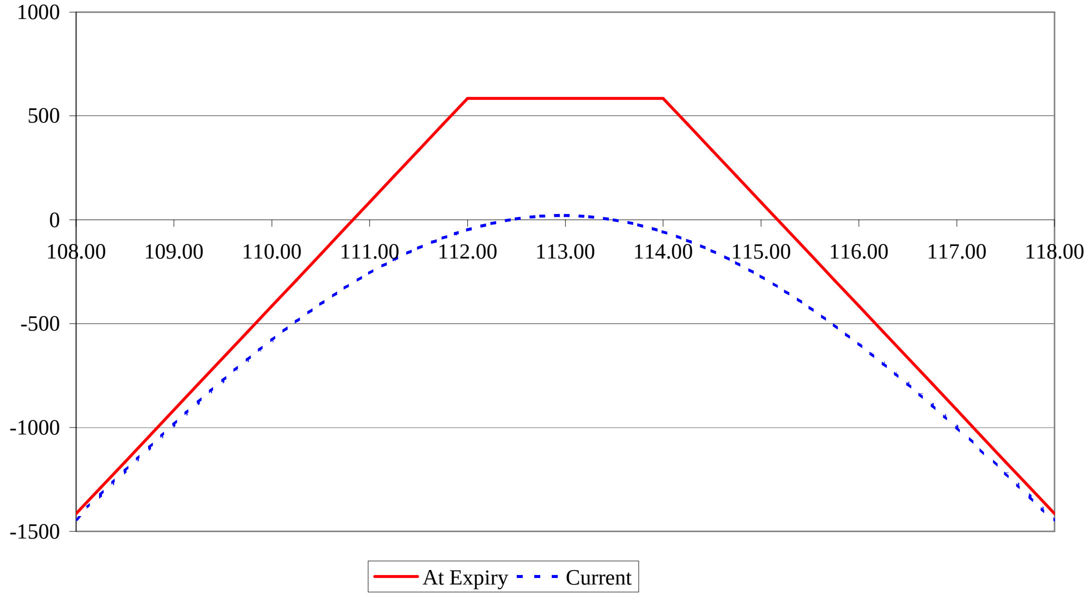
$$\Delta P = \sum \Delta P_i = \sum_i \text{Theta}_i \Delta t + \tfrac{1}{2}\sum_i \text{Gamma}_i (\Delta S_i)^2 + \sum_i \text{Vega}_i (\Delta\sigma_i)$$

Strictly, there is also a cross-gamma term $\tfrac{1}{2}\sum_{i,j} Gamma_{i\,j}\Delta S_i \Delta S_j$. The above is a simplified summation.

(ii)

The strangle diagram should show the expiry lines with 45% angles on the sides. The dotted line is the current time curve (at the date of taking the paper), which should pass through zero at the point 113.50. It assumes no prior P&L.

Eurex Bund Futures Sold Option Strategy
500 Dec 114 Calls & 500 Dec 112 Puts



[*The computer-generated example above illustrates what is required, but clearly the sketch expected would be less precise. The y-axis scale given is €m, but this detail is not required.*]

(iii)

(a)

The trader has sold volatility, so he is hoping for either or both of these outcomes:

- a decline in actual volatility in the market, resulting in time premium decay;
- a decline in implied volatility in the options he has sold, so he can buy them back for a profit.

[*Simply saying "hedging" is not correct, since such an activity would be revenue neutral.*]

(b)

Delta position = –500 (42% + –27%) = –75 contracts (i.e. 75 contracts short).

Therefore Delta hedge is to buy 75 contracts of December 2008 Bund future.

(c)

If the market falls sharply, the Call will be worthless and the Put will become almost identical to a future (all intrinsic value).

Hence the option position will be equivalent to being short 500 futures.

What actually happens to the trader's P&L depends on his next action, but he will almost certainly make a loss, since the volatility outcome is considerably higher than that assumed in the prices.

The trader will also lose because the options will increase in volatility. He may recoup some of this (unrealised) loss if the market calms down again before expiry.

The correct action for a Delta hedger is to sell contracts until the position is neutral. This could mean selling up to 500 futures, plus any ones he had bought as an earlier Delta hedge.

(d)

A week before expiry will exacerbate the Gamma effect. Hence:

- the final loss will be greater because the options will have little time value and hence little protection against sudden moves;
- there will be no loss due to volatility, since the time remaining is too small to allow any meaningful Vega exposure.

QUESTION 4

Syllabus section: (e) & (j)

Core reading: Units 5 & 13

The following table gives the answers to parts (i) and (ii):

			Cashflows		
(1) Time in ½ years	**(2) Forward rate %**	**(3) Discount (ZCB)**	**(4) Annual Bond %**	**(5) Margin only %**	**(6) Receiver Swap %**
0	4.000%				
1	4.354%	0.98039	0	0.500	–2.500
2	4.500%	0.95951	5.75	0.500	3.073
3	4.612%	0.93839	0	0.500	–2.750
4	4.707%	0.91724	5.75	0.500	2.944
5	4.791%	0.89615	0	0.500	–2.854
6		0.87518	105.75	0.500	2.855
Total		*5.56686*			

(i)

The factors d_i, i = 1 to 6, are in column (3), as decimal not %.

$d_1 = 1 / (1 + 0.04 / 2) = 1 / 1.02 = 0.98039$; $d_2 = d_1 / (1 + 0.04354 / 2) = 0.95951$ etc.

(ii)

Values are quoted in percentage of 100 nominal.

(a)

<u>FRN</u> – the LIBOR floating payments have a value of 100.

The value of the margins in column (5) = $\frac{1.00}{2} \times \sum_{i=1}^{6} d_i = 0.005 \times 5.56686 = 2.783$.

Hence total value of FRN = 102.783.

(b)

<u>Bond</u> – the cashflows are in column (4).

Value of bond = $5.75 \times (0.95951 + 0.91724) + 105.75 \times 0.87518 = 103.342$

(c)

<u>Swap</u> – the cashflows are given in column (6) and the swap can be valued as the product of these with the discount factors.

However, more easily the swap can be valued as the difference between (b) and (a).

Value of swap = 103.342 – 102.783 = 0.559

(iii)

In two years' time, the lower part of the table changes to:

			Cashflows		
(1) Time in ½ years	**(2) Forward rate %**	**(3) Discount (ZCB)**	**(4) Annual Bond %**	**(5) Margin only %**	**(6) Receiver Swap %**
4	3.707%	1.00000			
5	3.791%	0.98180	0	0.500	–2.354
6		0.96354	105.75	0.500	3.355

The value of the swap now = (–2.354 × 0.98180) + (3.355 × 0.96354) = 0.922

so the value is higher. [*But it will still fall to zero when t* = 3.]

QUESTION 5

Syllabus section: (h)(i)-(iii)

Core reading: Units 8 & 9

(i)

Assume $dS = \mu S dt + \sigma S dz$, with expiry at time T.

Using Ito's Lemma, $d(\ln S) = (\mu - \frac{1}{2}\sigma^2)dt + \sigma dz$, i.e. ln S is Normally distributed.

So the distribution of x = ln S_T is:

$$f(x) = \frac{1}{\sigma\sqrt{2\pi T}} \exp\left(-\frac{1}{2} \frac{(x - \ln S_0 - \mu' T)^2}{\sigma^2 T} \right)$$

where $\mu' = \mu - \frac{1}{2}\sigma^2$ and S_0 is the value of S at time 0.

Hence:

SPLurge value = $e^{-rT}\mathrm{E}_Q\left[X|F_0\right]$ where X is the payoff and F_0 the history to time 0

$$= M.e^{-rT}\int_{\ln K}^{\infty}\frac{(x-\ln K)}{\sigma\sqrt{2\pi T}}\exp\left(-\tfrac{1}{2}\frac{(x-\ln S_0-\mu' T)^2}{\sigma^2 T}\right)dx$$

Splitting up the two terms (x times the exponential, and $\ln K$ times the exponential), and by substitution of variables, the integral can be evaluated in the same way as the normal Black-Scholes formula.

(ii)

<u>Risk factors for SPLurge</u>

The Delta on a SPLurge is normally smaller than that of a European option.

The Gamma on a SPLurge is less peaked at the strike price but has a wider spread distribution ...

We know that $\frac{d\ln S}{dS}\approx\frac{1}{S}$, so that the Gamma becomes infinite at very low prices.

The Vega is similar to that of European options allowing for the proportionally lower price.

<u>Modelling issues</u>

The value of a Put SPLurge blows up towards zero stock price, but the stochastic model does not permit zero prices.

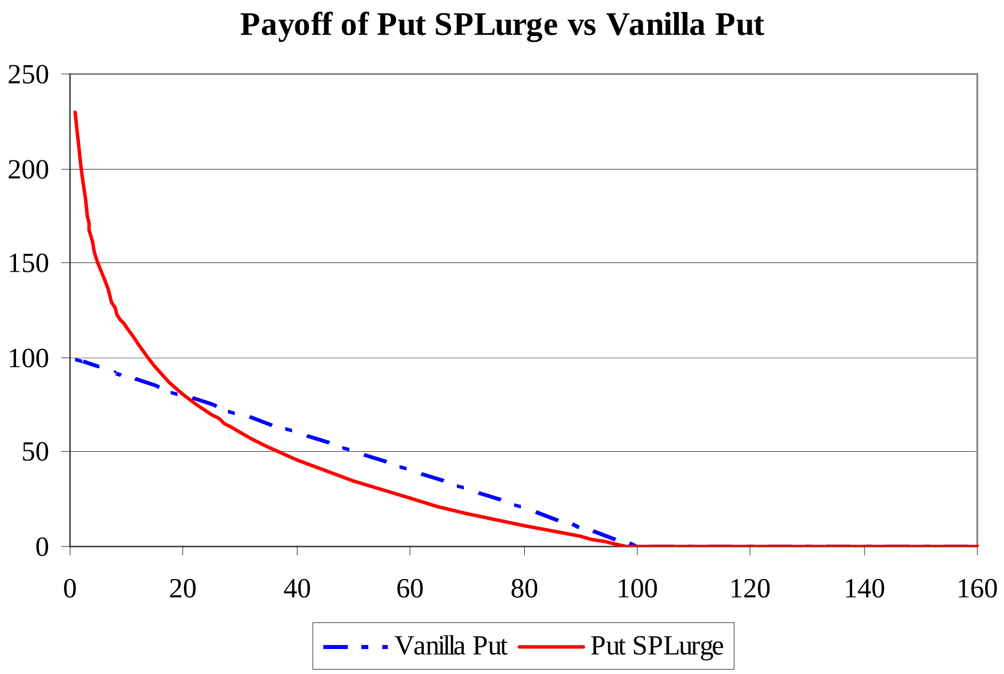
Only offer Put SPLurges on stocks where probability of zero price is negligible ...

… or package a Put SPLurge with a short very out of the money option (a sort of Put Spread) to eliminate payout problem if stock price falls towards zero ...

… or only offer Call SPLurges.

(iii)

Charting the payoff against possible values of S:



QUESTION 6

Syllabus section: (h)(iv)-(ix), (i)

Core reading: Units 10 - 13

(i)

Index has price J and volatility σ.

Let K be the strike, exercise at time T, with risk free rate r and dividend rate δ.

Black Scholes formula for value V of index with dividends:

$$V = e^{-rT}\left\{F\ \mathrm{N}\left(\frac{\ln F/K + \frac{1}{2}\sigma^2 T}{\sigma\sqrt{T}}\right) - K\ \mathrm{N}\left(\frac{\ln F/K - \frac{1}{2}\sigma^2 T}{\sigma\sqrt{T}}\right)\right\}$$

where $F = e^{(r-\delta)T} J$ is the forward price.

[*There are variations that expand the forward price within the logarithm.*]

So we set $d_1 = \dfrac{\ln F/K + \frac{1}{2}\sigma^2 T}{\sigma\sqrt{T}}$ and $d_2 = \dfrac{\ln F/K - \frac{1}{2}\sigma^2 T}{\sigma\sqrt{T}} = d_1 - \sigma\sqrt{T}$, and the formula becomes:

$$V = e^{-rT}\{F\,\mathrm{N}(d_1) - K\,\mathrm{N}(d_2)\}$$

(ii)

Starting level of FTSE J = 6,435

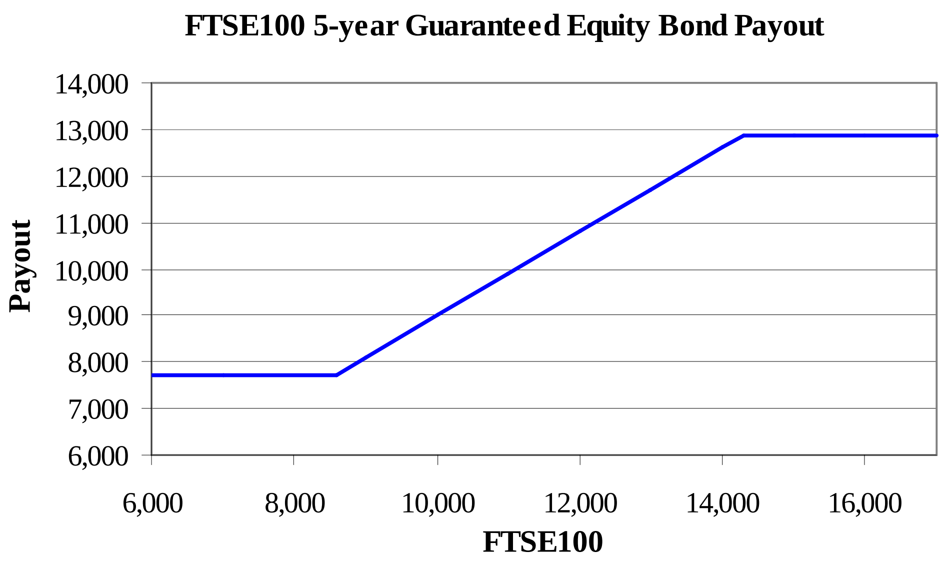
Min payout = 1.2 × 6,435 = 7,722

Max payout = 2 × 6,435 = 12,870

Given a 90% Delta (payout ratio vs the index), the min payout will apply when the index is below 7,722 / 0.9 = 8,580, and the max payout will apply when the index is above 12,870 / 0.9 = 14,300.

[*Alternatively, if the values above are labelled on the graph correctly, marks can also be given.*]

The payout relationship looks as follows:



(iii)

Dividend rate δ = 4% – because FTSE has 100 constituents, this is close to continuous.

Volatility σ = 0.20, r = 0.06, T = 5, so $\exp(-rT)$ = 0.74082 and $\sigma^2 T$ = 0.2.

Forward price F = $\exp((r - \delta)T)\,J$ = 7111.8

Looking at the payout graph above, we can see that it is the difference between two calls with strikes 8580 and 14300, plus some cash to provide the guarantee.

i.e. Payout = min (12870, max(7722, 0.9 * J))
= 7722 + 0.9 [max(J – 7722 / 0.9, 0) – max(J – 12870 / 0.9, 0)]
= 7722 + 0.9 [max(J – 8580, 0) – max(J – 14300, 0)]

<u>Option 1</u> (strike K_1 = 8,580)

$$d_1 = \frac{\ln F/K_1 + \frac{1}{2}\sigma^2 T}{\sqrt{\sigma^2 T}} = (-0.18768 + 0.1)/0.44721 = -0.19606 \text{ and } d_2 = -0.64327$$

so $V_1 = \exp(-rT)\,[F\,N(-0.19606) - K_1\,N(-0.64327)]$ = 572.0 index points.

<u>Option 2</u> (strike K_2 = 14,300)

$$d_1 = \frac{\ln F/K_2 + \frac{1}{2}\sigma^2 T}{\sqrt{\sigma^2 T}} = (-0.69850 + 0.1) / 0.44721 = -1.33831 \text{ and } d_2 = -1.78552$$

so $V_2 = \exp(-rT)\,[F\,N(-1.33831) - K_2\,N(-1.78552)]$ = 83.4 index points.

Hence value of Bond = $\exp(-rT) \times 7722 + 0.9\,[572.0 - 83.4]$ = 5720.6 + 439.8
= 6160.4 index points.

[*This part could also be answered by evaluating 0.9*(PV of fwd price + long put + short call). The put has strike K_1 and value 1,659.7. The call has strike K_2 and is Option 2 above.*]

QUESTION 7

Syllabus section: (k)

Core reading: Unit 14

(i)

(a)

<u>Reasons for choosing a full yield curve model</u>

The rationale for a term structure yield curve model is to be able to price simultaneously options spread across the entire range of maturities in a yield curve.

Exotic interest rate swaps and options include spread options, Bermudan swaptions and path-dependent options (knock-outs etc). These depend not just on the evolution of forward rates along the curve, but on the correlation between changes.

(b)

Desirable features

- model is arbitrage free, so produces prices consistent with the market
- flexible enough to cope properly with the required range of derivative contracts
- the current swap (or bond) curve should be reproduced by the model
- easy to specify and calculate (on a suitable computer)
- easy to calibrate – for example, a log-normal expression of the model will help fit to cap prices, which are traded based on the standard log-normal Black model
- enough degrees of freedom (parameters) to make the model flexible to cope with any yield curve shape, but not overly flexible so there is instability between parameters from one day to the next
- volatility of rates of different maturity should be different, with shorter rates usually being more volatile
- imperfect correlation between forward rates, although this is only needed when pricing certain types of option where correlation has a big effect on the price (e.g. yield spread options, callable swaptions)
- negative interest rates should not normally be allowed
- reasonable dispersion of rates over time (due to the Brownian motion) – too large a probability of getting hugely high or low values will distort the model
- one possible way of allowing for this is to make the model "mean-reverting", that is, when rates go too high (or low), they tend to revert back to some central level

(c)

No-arbitrage models

No-arbitrage, or "arbitrage free", models are a class of models that allow recovery of market prices of one set of securities given prices of another set. This creates a world of "relative" prices.

In a non-"arbitrage free" model, securities could be priced using the model and then traded at a different price in the real world, leading to persistent profit. In simplest terms, no-arbitrage is the absence of a "free lunch".

No-arbitrage is very important in yield curve models, since most complex structures are limiting cases of simpler structures (such as swaps, caps, floors) and ideally the model should recover the prices of the latter exactly.

Also, hedging is done using the simpler structures, so the accounting process will not be distorted by imaginary gains and losses.

(ii)

[*Only a description of the trinomial method is required here – numerical examples or algebraic derivations are not required. The solution is indicative only, as there are several other ways to describe the method.*]

A trinomial tree is a discrete-time representation of the continuous rate process, with the stochastic process defined by branches which can have three states: up, down and mid-way between.

If the probabilities of going up, down and middle are p_u, p_d and p_m respectively, then $p_u + p_d + p_m = 1$ and all values are strictly positive.

The time horizon is split into constant intervals of time Δt, so the rates are compounded Δt-period rates.

Accuracy is improved with smaller time-steps, up to a point. In computer calculations you would normally use over 100 time steps per year.

The tree for the one-factor model is built in two stages. The method is very similar to that of Explicit Finite Differences.

[*The b(t) term in the rate process:*

$$dr = a\left(b(t) - r\right)dt + \sigma dz$$

is the market-fitting function, which is used to precisely price the zero-coupon bonds in the market. So firstly we build the tree with only mean-reverting features by setting b(t) to zero, then add back in the term structure.]

<u>Stage 1</u>

Firstly, set $b(t)$ to zero:

$$dr^* = -ar^* dt + \sigma dz$$

which gives a rate process r^* that is initially zero, and whose evolution is governed by a constant mean reversion towards zero.

The value of σ is determined from the caplet volatilities.

There are three unknowns: p_u, p_d and Δr^* (since $p_m = 1 - p_u - p_d$).

It is convenient to set $\Delta r^* = \sigma\sqrt{3\Delta t}$, since this has been found to give the best numerical efficiency.

The other two equations come from the expectation and variance of Δr^* over the interval Δt, in which the tree must match the process.

There are three different forms of the mean and variance equations for the tree: when all the branches point upwards or horizontal, when all point downwards or horizontal, and when they straddle the line of unchanged rates.

There will be different probabilities for each situation.

There are also bounds (given by Hull & White in a paper on implementing their model) for the cross-over points at which the patterns must change for the probabilities to be always positive.

Since $E(\Delta r^*)^2$ contains the term $\sigma\Delta t$, the equations account for the volatility component.[½]

[*A diagram would be helpful, but it is not necessary to give the branching equations or derive these bounds.*]

Stage 2

Now add back in the term structure, to move back from r^* to r.

Let $\alpha(t) = r(t) - r^*(t)$. Then:

$$d\alpha = a\left[b(t) - \alpha(t)\right]dt \qquad (**)$$

which is a deterministic mean-reverting process (i.e. varies over time but not stochastically), which can easily be calibrated to the term structure.

The best method of solving the differential equation in (**) is by forward induction along the Δt scale using Green's functions. A Green's function $Q_{i,j}$ is the present value of 1 unit of cash payable at node (i, j) on the tree.

Now, $Q_{0,0}$ =1 and α_0 (= r) is set in terms of the the price of a zero-coupon bond maturing at Δt. [½]

Then $Q_{1,1}$, $Q_{1,0}$ and $Q_{1,-1}$ are set in terms of the tree probabilities p_u, p_d and p_m, $Q_{0,0}$ and α_0. There is a simple formula for α_1 based on the price of a zero-coupon bond maturing at $2\Delta t$ and these Q's.

And so on up to the time horizon.

[*More precise details are not required. Integration of (**) does not give exact enough values because we are approximating the continuous process.*]

(iii)

Does HW fulfil the desirable features?

The model is a suitable no-arbitrage full yield curve model, as it reproduces vanilla bond and swap prices exactly and prices options on these ...

... and has mean reversion and time-dependent parameters, so is flexible, behaves well and has little chance of producing negative rates.

It is relatively easy to specify mathematically, albeit with care required.

BUT

It only has one driving factor, so all forward rate moves have to be completely correlated – hence it cannot re-create complex yield curve changes ...

It is normal, not log-normal, so it can lead to negative rates [*theoretically, as in practice the probabilities of negative rates are negligible*] ...

... and it may be hard to calibrate to cap prices which are priced in the market with the Black log-normal model.

Differences arise away from the current level of rates. These effects, if not disentangled, will lead HW to have an incorrect balance between its volatility and mean-reversion parameters.

[*In some cases, caplet volatility which is sharply declining along the curve can lead to failure of the trinomial process further out in the tree.*]

Using constant values for a and σ limits the shapes of yield curve that can be modelled. Time-dependent mean reversion and volatility parameters can fit any current yield curve and forward volatility "hump" shape ...

… although they can imply an implausible evolution of the term structure of volatility.

QUESTION 8

Syllabus section: (k)

Core reading: Units 15 & 16

(i)

(a)

A CDS focuses on buying and selling protection against a "credit event", such as bankruptcy, failure to pay or restructuring of debt .

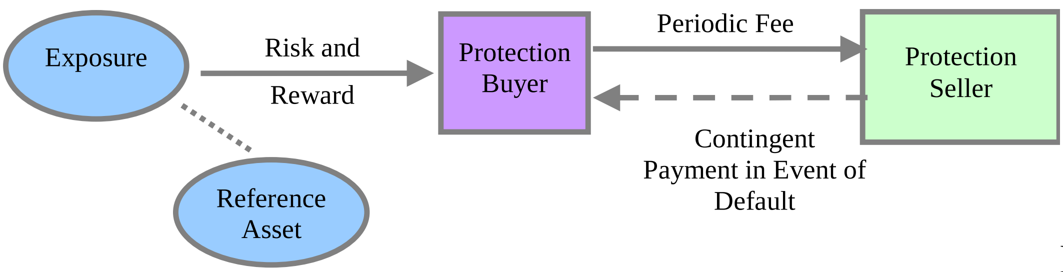
Two parties (a protection buyer and a protection seller) enter into an agreement whereby one party (the protection buyer) pays the other party (the protection seller) a fixed, periodic fee for the life of the agreement.

The protection seller makes no payments unless a specified credit event occurs on an underlying loan or bond (the "reference asset").

If such a credit event occurs, the protection seller either makes a cash payment to the first party or else delivers a specific physical security, and the swap terminates. The size of any cash payment is linked to the decline in the reference asset's market value following the credit event.

Payment of premiums is made in arrears, so a default part way through the year attracts an accrual adjustment.

[*<u>Note</u>: The CDS is by far the most widely used credit derivative currently. It essentially moves the credit derivative towards a form of insurance, although unlike insurance the protection buyer does not need first to be exposed to the underlying risk. In diagrammatic form:*



]

(b)

<u>Reasons for using CDS</u>

- Hedging a holding in a 3-year bond of that corporate name to reduce credit risk.
- Taking short position in that corporate name, expecting its credit spread to widen.
- Some other portfolio hedge either of a different corporate name or a different maturity, hoping to profit from the relative movement between the two.

[*Other reasons are possible – they must be relevant though.*]

(ii)

Let the spread on the CDS be S.

(A) <u>Periodic premium payment</u>

[*In this solution, premiums are assumed to be paid annually in arrears. Quarterly or semi-annual frequencies would also be acceptable.*]

			Periodic Premium Payment		
Years	**Rate**	**Defaults per 100**	**Survival per 100**	**Discount factor**	**Present value**
1	2.00%	2.000	98.000	0.94176	0.9229
2	2.25%	2.205	95.795	0.88692	0.8496
3	2.50%	2.395	93.400	0.83527	0.7801
				Total	2.5527

So periodic premium payment = 2.5527 *S*.

(B) <u>Principal payment on default</u>, assumed to take place at ½ year:

			Principal Payment on default		
Years	**Defaults per 100**	**Recovery**	**Payoff**	**Discount factor**	**Present value**
1	2.000	0.4	1.200	0.97045	0.0117
2	2.205	0.4	1.323	0.91393	0.0121
3	2.395	0.4	1.437	0.86071	0.0124
				Total	0.0361

so first discount factor = exp(–0.06 / 2) = 0.97045.

(C) Accrual adjustment per 1 unit of spread, assumed to take place at ½ year:

		Accrual Adjustment		
Years	**Survival per 100**	**Accrual**	**Discount**	**PV**
1	98.000	1.0000	0.97045	0.0097
2	95.795	1.1025	0.91393	0.0101
3	93.400	1.1975	0.86071	0.0103
			Total	0.0301

So accrual payment = 0.0301 S.

Hence (2.5527 + 0.0301)S = 0.0361, so S = 0.0361 / 2.5828 = 0.01398 or 139.8bps.

(iii)

Why would market quote a higher spread?

- Supply/demand – maybe some speculation, selling of spreads generally, or limited market liquidity
- Financing costs of market participants are not the same, introducing basis arbitrage between CDS spreads and the underlying cash bond market
- Your default analysis may be out-of-date, or perhaps not reflecting the market's current expectations
- The calculation is made in the "real world", so is not arbitrage free – there are other better models for obtaining CDS consistent with the market [*e.g. Jarrow Turnbull*]
- Discounting at the risk-free rate is inconsistent with using "real world" default probabilities.
- The recovery rate assumption is simplistic – the market may use a different one.
- There would be a bid/offer spread to cover the writer's costs and profit margin.

END OF EXAMINERS' REPORT