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A Tale of Two Pension Plans: Measuring Pension Plan Risk from an Economic Capital Perspective

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Caveat and Disclaimer

Please note: Authors Douglas Andrews, Jaideep Oberoi and Pradip Tapadar are members of the United Kingdom's University Superannuation Scheme (USS), as referred to in this research paper.

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Executive Summary

This research project provides a risk analysis for two pension plans, one each representing the United States and the United Kingdom. We use publicly available information from a large U.K. defined benefits (DB) pension plan to construct a representation of the U.K. plan that matches its valuation, following and updating the work of Porteous et al. (2012). We also analyze a stylized U.S. plan with the same membership profile, but with provisions modified to reflect a typical U.S. DB plan. The risk assessment is carried out by estimating economic capital requirements using a Solvency II framework.

The analysis is carried out using a stochastic economic scenario generator calibrated to the U.K. and U.S. economies. The analysis also employs a stochastic mortality model, similarly calibrated to the United Kingdom and the United States.

The risk measure employed is economic capital, which is defined as follows:

> The economic capital of a pension plan is the proportion by which its existing assets would need to be augmented in order to meet net benefit obligations with a prescribed degree of confidence. A plan's net benefit obligations are all obligations in respect of current plan members, including future service, net of future contributions to the plan.

We employ a 99.5% degree of confidence, which is consistent with both the 2012 analysis and Solvency II. Results are shown for the full distribution of outcomes, but emphasis is given to the 0.5th percentile in line with the selected degree of confidence.

The main results of the study are as follows:

- As a percentage of starting assets, the U.S. stylized plan is more volatile than the U.K. plan. The U.S. plan requires more than three times its starting asset value as an economic capital buffer to provide 99.5% certainty of providing the pension benefits. The U.K. plan requires roughly half this percentage.
- The benefits of a larger allocation to long bonds are greater in the U.S. plan than in the U.K. plan. Largely, this is because the U.K. plan benefits increase completely in line with either wage increases or price inflation. The U.S. plan benefits reflect wage increases while individuals are accruing benefits but otherwise grant no inflationary increases.
- The effect on economic capital for either plan is much larger for changes in asset allocation than for changes to plan contributions.

Some implications of the results for various stakeholders are as follows:

- Plan sponsors should understand that there is a very large range of potential outcomes in a typical DB pension plan. This range can result in significant variation in contributions to the plan. To a certain extent, the range of outcomes can be narrowed by appropriate selection of asset allocation and plan provisions.
- The full distribution of results is shown in the body of this report. Pension practitioners should have discussions with plan sponsors to assist the latter in understanding the full range of uncertainty they are assuming in the financing of their DB plans.
- An economic capital framework provides pension regulators with another tool with which to consider their exposure to benefits guaranteed by the Pension Protection Fund and the Pension Benefit Guaranty Corporation. It also provides them with some guidance in circumstances where it is appropriate to expect plan sponsors to hold some degree of margin for adverse deviation within

their pension funds. The results clearly show that the appropriate degree of margin is materially affected by plan provisions, plan asset allocation and the desired degree of confidence that promised benefits will be provided.

- Economic capital frameworks may also be of interest to plan members. This framework can help them to understand the uncertainty the sponsor faces in financing DB pension plans. This approach can supplement other communications to plan members that educate them in plan financing.

Section 1: Introduction

Years of high inflation, good investment returns and profits during the 1970s and 1980s created the illusion that defined benefits (DB) pension plans are easily affordable. Due to the creation of large surpluses during those years, pension risks have generally been excluded from an organization's general risk management process. Over the past decade or more, however, increasing life expectancy and a steady fall in interest rates have meant that pension costs have increased. Consequently, many pension plans now have insufficient assets to cover all of their promised benefits. As a result, the security of members' benefits may be compromised.

Porteous et al. (2012) performed a risk assessment of a large United Kingdom defined benefit pension plan using publicly available data, albeit by determining the solvency capital requirement of the fund in a Solvency II framework.[1] This was in response to the interest in the new Solvency II rules at the time, although they were not applicable to pension funds. This research project updates that earlier work.[2] For comparison purposes, we also carry out an economic capital analysis for a stylized U.S. plan. The basic steps we follow are as follows:

- Step 1: Choose a representative pension plan for the United Kingdom and the United States for analysis.
- Step 2: Fix an appropriate start date and develop a model of the representative pension plan that adequately reflects the plan's membership and liability profile as of that date.
- Step 3: Choose a suitable, ideally stochastic, economic scenario generator (ESG) to project the plan assets and liabilities forward from the start date identified in Step 2.
- Step 4: Choose a suitable, possibly stochastic, mortality model to project forward the mortality experience of the plan members.
- Step 5: Quantify the pension plan risks using an appropriate risk measure.

For our analysis, we first need to fix a start date. This is typically driven by availability of data. At the time of beginning this research, the latest available published results for our chosen representative UK pension plan were in the triennial valuation from March 31, 2014, so we decided to use that as a start date.

The publicly available data from the actuarial valuation reports and other documents typically provide summarized data on membership profile, accrued benefits, average salary/pension, past service, age and gender distribution, and actuarial liability. As we do not have access to the full underlying membership data, we need to build a representative

[1] Though Solvency II also requires an assessment of operational risk, we do not reflect that in our analysis.
[2] The analysis has been updated to reflect the financial condition and membership structure in the most recently filed valuation report of the same fund, an updated calibration of the economic scenario generator and an updated model of stochastic mortality.

model, with appropriate model points for active members, deferred members and pensioners, to broadly match the published summarized data as of the chosen start date.

For the United Kingdom, we have decided to base our analysis on a representative model of the Universities Superannuation Scheme (USS) as of March 31, 2014, and project its assets and liabilities forward from that date onward. For the United States, we use a stylized U.S. pension plan using the same model points as the USS but with a number of changes to the benefit structure and contribution rates.

The risk measure employed is economic capital, defined as follows:

> The economic capital of a pension plan is the proportion by which its existing assets would need to be augmented in order to meet net benefit obligations with a prescribed degree of confidence. A plan's net benefit obligations are all obligations in respect of current plan members, including future service, net of future contributions to the plan.

We employ a 99.5% degree of confidence, which is consistent with both the analysis of Porteous et al. (2012) and Solvency II. Results are shown for the full distribution of outcomes, but emphasis is given to the 0.5th percentile in line with the selected degree of confidence.

This report is structured as follows:

- In Section 2, we provide a literature review relevant to this research.
- In Section 3, we provide an overview of the data we used to calibrate our models.
- In Section 4, we present our main assumptions.
- In Section 5, we present the methodology used to carry out the risk assessment of the pension plans.

In Section 6, we present the results of the U.K. plan.

- In Section 7, we present the results of the stylized U.S. plan.
- Section 8 concludes with a summary of anticipated future work.

Section 2: Literature Review

The literature dealing with measuring pension risk is extensive. As part of this project, a detailed literature review was prepared and has been posted as a separate document on the Society of Actuaries' website. In this section we provide a brief summary of the literature.

Several papers—including Porteous et al. (2012); Devolder and Piscopo (2014); Ai et al. (2015); and Yang and Tapadar (2015)—have used the Solvency II framework and Value at Risk (VaR) to estimate pension risk. There are also other methods to measure pension risks. For example, Boonen (2017) used expected shortfall (ES) to quantify pension risks; Devolder and Lebegue (2016) used ruin theory to measure the solvency capital requirement of pension products; Kemp and Patel (2012) used enterprise risk management techniques to measure the risks of pension plans; and Devolder and Lebegue (2017) used dynamic risk measures.

Some literature has compared the relative significance of factors driving pension risks such as equity risk, interest rate risk and longevity risk. Papers that have addressed these issues include Butt (2012), Liu (2013), Karabey et al. (2014) and Sweeting (2017). Other literature has compared the impact of different ESGs on pension risks (such as Abourashchi et al. 2016 and Devolder and Tassa 2016) and the impact of different mortality models on pension risks (such as Lemoine 2015 and Arik et al. 2018).

The literature comprises a broad range of research on managing risks from the sponsor's point of view. Some papers have used financial instruments to hedge or transfer the risk. Examples of the instruments used include natural hedging (Li and Haberman 2015); longevity hedges (Lin et al. 2014, 2015); and pension buyouts (Cox et al. 2018). Some papers have also focused on risk management based on the plan's structure, as with Kleinow (2011), Aro (2014) and Platanakis and Sutcliffe (2016). Moreover, some researchers have used optimization techniques to see the extent to which the sponsor's risk can be reduced. Some of the techniques they have discussed include dynamic asset allocation (Liang and Ma 2015) and automatic balancing mechanisms (Godinez-Olivares et al. 2016).

Finally, some authors have looked at pension risks from the point of view of plan members. Among these, a number of papers have focused on solving optimization problems to maximize the expected utility of plan members. For example, Devolder and Melis (2014) examined the benefits to plan members of having both funded and unfunded public pensions. Alternatively, Chen and Delong (2015) studied the asset allocation problem to maximize plan members' utility in a defined-contribution plan. Other papers have proposed innovative pension structures to reduce plan members' risks. Structures analyzed and examined included hybrid structures (Khorasanee 2012) and TimePension (Linnemann et al. 2014). Intergenerational risk sharing and the benefits to plan members/pensioners have also been areas of ongoing research interest (as with Chen et al. 2014 and Wang et al. 2018).

Section 3: Data

In this section we describe the data we use to build a pension model and carry out the risk assessment of a pension plan. The two main sources of pension plan risk come from uncertainty as to how assets will perform in the future (economic risk) and how mortality rates will change in the future (demographic risk).

To capture the economic risk, we need an ESG. For the United Kingdom, we adopt two different ESGs: the Wilkie model (Wilkie 1986, 1995; Wilkie et al. 2011) and the graphical model (Oberoi et al. 2018). The same underlying U.K. economic data is used for calibrating both models. The variables modeled are price inflation, salary inflation, dividend yield, dividend growth and long-term bond yield. Please refer to Wilkie et al. (2011) and Oberoi et al. (2018) for more details.

For the United States, the data come from two sources. The first is Robert Shiller, who provides online data for the Consumer Price Index (CPI), S&P 500 price index, S&P 500 dividend index and 10-year bond yield. The second is Emmanuel Saez, who provides online data for average wages in the United States. The data we use extend from 1913 to 2015.

To capture the demographic risk for both countries, we use model M7 from Cairns et al. (2009).[3] To parameterize model M7, we use data from the Human Mortality Database (HMD), which is a rich source of data based on population mortality rates for both the United Kingdom and the United States (among other countries).[4] For the United Kingdom, we use data from 1922 to 2016; for the United States, we use data from 1933 to 2016. The model is calibrated for both males and females, ages 30 to 100. Given that the data are sometimes unreliable above age 100, we do not include those lives for our analysis.

Section 4: Assumptions

In this section, we discuss our assumptions relevant to the United Kingdom plan based on the USS and how we modified the U.K. model for a stylized U.S. pension plan. The USS is one of the largest open DB plans operating in the

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[3] Details of model M7 are provided in Appendix B.1.

[4] The USS valuation report uses a deterministic table to carry the mortality projections (S1NA "light" mortality table and CMI_2014 projection table). As we want stochastic mortality rates, we use model M7, which is calibrated to HMD data. We then project paths for the future mortality rates and adjust these rates such that the central projection from our model M7 matches the mortality projections as per the USS valuation report.

United Kingdom, with more than 350 participating employers and approximately 400,000 plan members. The assumptions presented here are based on the valuation carried out for the plan as of March 31, 2014.[5] As we do not have access to the full underlying valuation data, we develop a USS "model" using model points that capture the broad membership profile. Additionally, we set out the plan provisions (and minor adjustments to valuation methodology) for a stylized U.S. pension plan.

4.1 Membership Profile

Table 1

Membership Profile

Active	Number	167,545
	Average pensionable salary	£42,729
	Average age	43.8
	Average past service	12.5
Deferred Members	Number	110,430
	Average deferred pension	£2,373
	Average age	45.1
Pensioners (including dependents)	Number	70,380
	Average pension	£17,079
	Average age	71.1

Table 1 shows the membership profile as presented in the 2014 USS valuation report. Only a single average age is provided for the active members, which is not sufficient to capture the overall risk characteristics of the plan. We need a range of model points to capture the intergenerational risk dynamics. The 2014 USS "Report and Accounts" provides information on the proportion of active members in different age bands, based on which we propose an age distribution of active members in Table 2. Table 2 also shows the past service and salary assumptions for active members for each model point. These have been set so the average past service and average salary of active members broadly match the figures from Table 1.

Table 2

Model Points, Past Service and Salary of Active USS Members

Age	Proportion	Number	Past Service	Salary
30	30%	50,264	7	£25,500
40	30%	50,264	11	£42,500
50	20%	33,509	15	£52,500
60	20%	33,509	19	£58,500
Total	100%	167,545		
Average			12.2	£42,600

For deferred members and pensioners, we use single model points to represent each of these membership categories. We also assume a 50:50 gender split and no salary differential between genders for all membership categories.

[5] Although a USS valuation report for 2017 has since been published, it has been the subject of significant public discussion. We continue to use the agreed 2014 information, in order to avoid the uncertainty surrounding the latest valuation, and because the nature of the analysis is not affected by this decision.

Table 3

Model Point and Accrued Pension of Deferred USS Members

Age	Number	Accrued Pension
45	110,430	£2,373

Table 4

Model Point and Accrued Pension of Retired USS Members

Age	Number	Accrued Pension
71	70,380	£17,079

4.2 Benefit Structure—USS

4.2.1 Pension Benefits

Pension and cash lump sum at retirement are calculated as follows:

Annual pension = Pensionable salary × Pensionable service × Accrual rate

Lump Sum = 3 × Annual pension

Until October 2011, the accrual rate was 1.25% (1/80th) and pensionable salary was on a final salary (FS) basis, defined as "the highest of either the best inflation-adjusted 12 months' salary over the last 36 months' membership; or the average of your best consecutive inflation-adjusted three years' salary during the last 13 years" for all members. For practical implementation purposes, we will assume that for FS, the pensionable salary is the member's salary in the final year of service.

From October 1, 2011, the FS plan was closed to new entrants. Instead, they joined a separate plan based on a Career Revalued Benefits (CRB) basis. On April 1, 2016, the FS plan was closed and all existing members were moved to the CRB plan, with an enhanced accrual rate of (1.33%).

To keep our model of USS simple, we assume that all members accrue benefits on the FS basis up to March 31, 2014. All members then move to the CRB basis from April 1, 2014, onward.

Annual pension is assumed to increase in line with the CPI, subject to a 5% limit.

Members' salaries increase in line with salary inflation. In addition to salary inflation, there is an explicit age-based promotional salary scale, as shown in Table 5.

Table 5

USS Assumptions

Age	Promotional Salary Scale		Withdrawal		Proportion Married	
	Male (%)	Female (%)	Male (%)	Female (%)	Male (%)	Female (%)
25	3.8	3.1	14.42	19.28	10.90	10.90
35	3.8	3.1	9.19	11.40	53.41	53.41
45	2.0	1.8	3.79	3.83	69.76	69.79
55	1.1	1.4	3.79	3.83	69.76	69.79

4.2.2 Withdrawal Benefits

For members who withdraw from the plan, a deferred inflation-linked pension is provided based on accrued service. Retail Price Index (RPI) indexation of salary is provided between the date the member withdraws from the plan and the date of retirement.

A sample of the withdrawal rates for the USS, which are 270% of the LG59/60 table for males and 113% of the LG59/60 table for females, is shown in Table 5.

4.2.3 Death Benefits

On the death of an active member, a lump sum payment of three times the annual salary is paid at the time of death, along with a spouse's pension of half the amount of pension the member would have received if he or she had survived until normal retirement.

On the death of a deferred pensioner, a lump sum equal to the present value of the deferred lump sum payable at normal retirement is provided along with a spouse's pension of half the amount of the deferred pension payable at normal retirement.

On the death of a pensioner, a spouse's pension of half the amount of the member's pension is payable.

Death benefits payable to the spouse of an active, deferred or retired member commence on the date of the member's death.

A sample of the married proportion, which is 109% of the 2008 Office of National Statistics table[6] for both males and females, is shown in Table 5.

4.3 Benefit Structure—U.S.-Style Plan

4.3.1 Pension Benefits

For the stylized U.S. pension plan, pensionable salary would best be determined as final three-year average salary. For practical implementation purposes, we will assume that pensionable salary is the member's salary in the final year of service. Pensionable service will be all years of service, and the accrual rate is set to 1.5%. There is no lump sum payment on retirement. For consistency, the promotional salary assumptions are assumed to be the same as for the USS (see Table 5). Also, there is no indexation during the payment period.

4.3.2 Withdrawal Benefits

For members who withdraw from the plan, a deferred pension is provided based on accrued service. No indexation of salary is provided between the date the member withdraws from the plan and the date of retirement. Also, there is no indexation during the payment period. The withdrawal rates are assumed to be the same as for the USS (see Table 5).

[6] The following link provides access to ONS 2008: https://webarchive.nationalarchives.gov.uk/20160107162445tf_/http://www.ons.gov.uk/ons/rel/pop-estimate/population-estimates-by-marital-status/mid-2010/index.html. Bear in mind that the table is updated from time to time.

4.3.3 Death Benefits

On the death of an active member, a lump sum equal to the present value of the pension the member would have received if he or she had survived until normal retirement is paid at the time of death.

On the death of a deferred pensioner, a lump sum equal to the present value of the pension the member would have received if he or she had survived until normal retirement is paid at the time of death.

On the death of a pensioner, a spouse's pension of half the amount of the member's pension is payable. The proportion of married members is assumed to be the same as for the USS (see Table 5).

4.4 Contributions

For the USS plan, employers contribute 16% of salary and employees contribute 8% of salary, amounting to a total contribution of 24% of salary.

For the stylized U.S. plan, employees do not contribute, while the employer contributes an amount equal to the current level of the normal actuarial cost, expressed as a percentage of salary. We have quantified from the data that a contribution rate of 10.8% of salary is sufficient to provide for the normal actuarial cost. An additional contribution is assumed for the first seven years to fund the initial deficit (see Section 4.6).

4.5 Valuation Method

The USS uses the Projected Unit Method (PUM) to estimate the liabilities of the plan. The PUM is a prospective valuation method in which liabilities are estimated based on the past service accrued on the valuation date, taking into account future salary inflation. We also use the PUM for the stylized U.S. plan.

4.6 Assets and Liabilities

For the USS, the starting values of assets and liabilities as on March 31, 2014, are

- L_0 = £46.9 billion (based on the valuation report, using a discount rate of 5.2% and decreasing linearly to 4.7% over 20 years); and
- A_0 = £41.6 billion (based on the valuation report).

This gives an initial valuation deficit of £5.3 billion. We assume there is no amortization of the initial deficit. The USS invests approximately 70% in real assets and 30% in fixed assets. For the purposes of our calculations, we assume an asset allocation of 70% equities and 30% bonds.

For the stylized U.S. plan, the starting values of assets and liabilities as on March 31, 2014, are assumed to be

- L_0 = \$32.6 billion (based on the valuation method described in Section 4.5, using a discount rate of 3.9%); and
- A_0 = \$26.1 billion (based on the assumption described below).

Assets are assumed to be 80% of the value of liabilities on March 31, 2014, to model a 20% initial deficit. We assume that the sponsor injects an additional annual cash flow over seven years in order to amortize the initial deficit. This annual cash flow is in addition to the contribution of the normal actuarial cost of 10.8% of salary. The base case asset allocation is assumed to be 50% equities and 50% bonds.[7] We will also investigate sensitivities to other asset allocations.

4.7 Economic Scenario Generator

To project assets and liabilities forward, we need an ESG. For the United Kingdom, we consider two ESGs. The first, the Wilkie model (Wilkie et al. 2011) is a well-established ESG within the actuarial literature. Two previous versions of the Wilkie model were published before the latest in 2011—the first in 1986 (Wilkie 1986) and the second in 1995 (Wilkie 1995). We give a brief overview of all three models in Section 4.7.1. For more detailed information on the three versions of the Wilkie model, please refer to Appendix A.1.

The second ESG we use is a graphical model developed by Oberoi et al. (2018) using U.K. economic data. For this report, we use the same methodology to develop and parameterize a model for the U.S. economy using U.S. data.

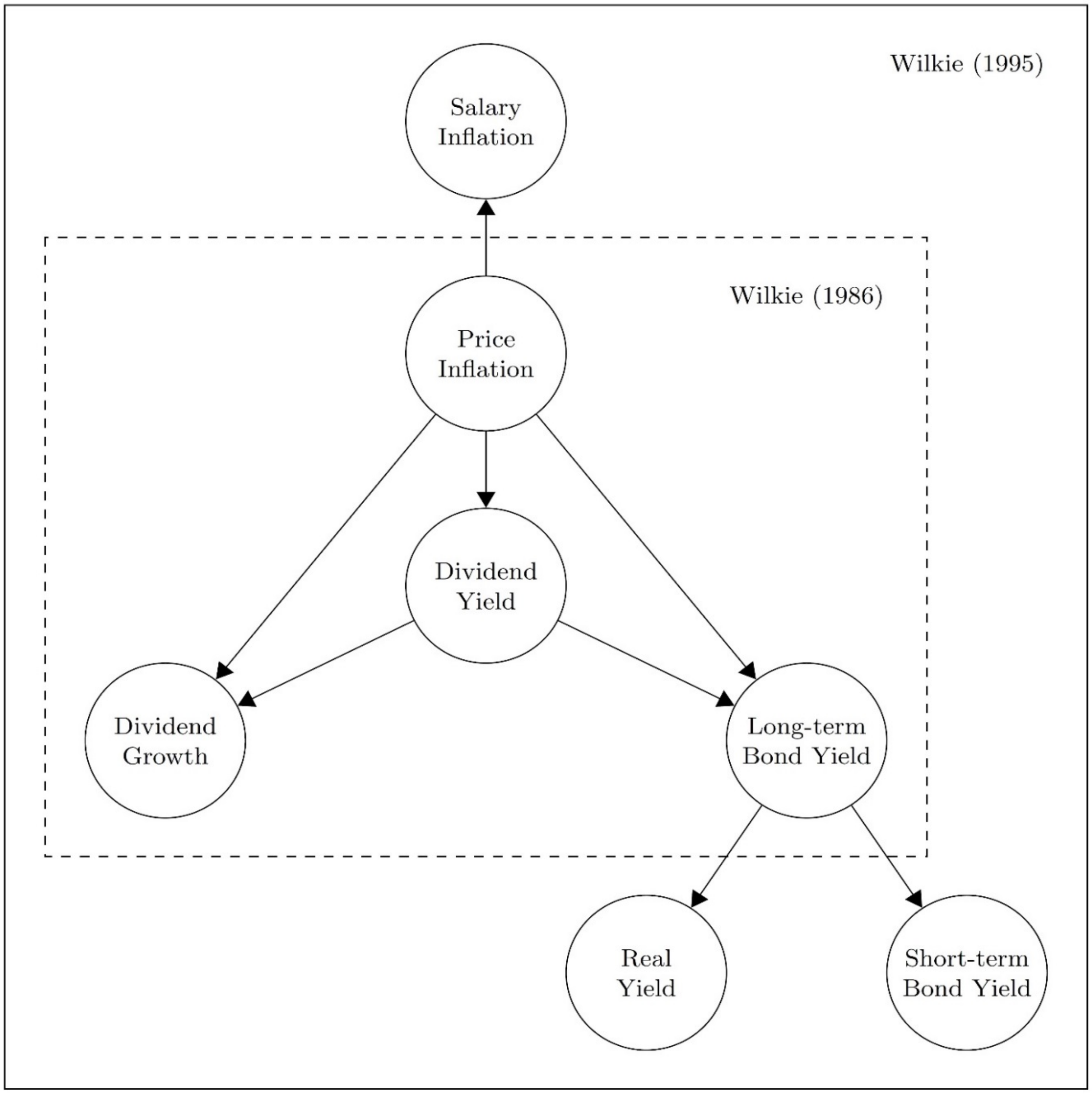
In the graphical approach, dependence between variables is represented by “edges” in a graph connecting the variables or “nodes.” This approach allows us to assume conditional independence between variables and to set their partial correlations to zero, providing for a more parsimonious specification of the model. Two variables may then be connected via one or more intermediate variables, so they might still be weakly correlated. The graphical approach is easy to implement, flexible, transparent and easy to apply to different datasets (e.g., countries). As the Wilkie model (Wilkie et al. 2011) is only parameterized to U.K. data, we will only use the graphical model as the ESG for the United States. We explain the graphical model in Section 4.7.2.

4.7.1 Wilkie Model

The Wilkie model is a multivariate autoregressive time series model that shows chosen economic variables using a cascade structure, as depicted in Figure 1. Price inflation impacts all the other variables in the model. Among the other variables are dividend yield, affected dividend growth and long-term government bond yields. The variables within the dashed area of Figure 1 are the original variables included in Wilkie (1986). The remaining variables were added in Wilkie (1995). The full model and relevant parameter values are shown in Appendix A.1.

[7] This asset allocation is meant to be illustrative of a typical U.S. pension plan.

Figure 1
Wilkie Models: Cascade Structure



Source: Wilkie (1986, 1995).

4.7.2 The Graphical Model

In a graphical model ESG, we first fit a univariate time series model, typically an AR(1) process, to each individual economic variable as follows:

$$Z_{(i,t)} = Y_{(i,t)} + \mu_{(i,t)}. \quad (1)$$

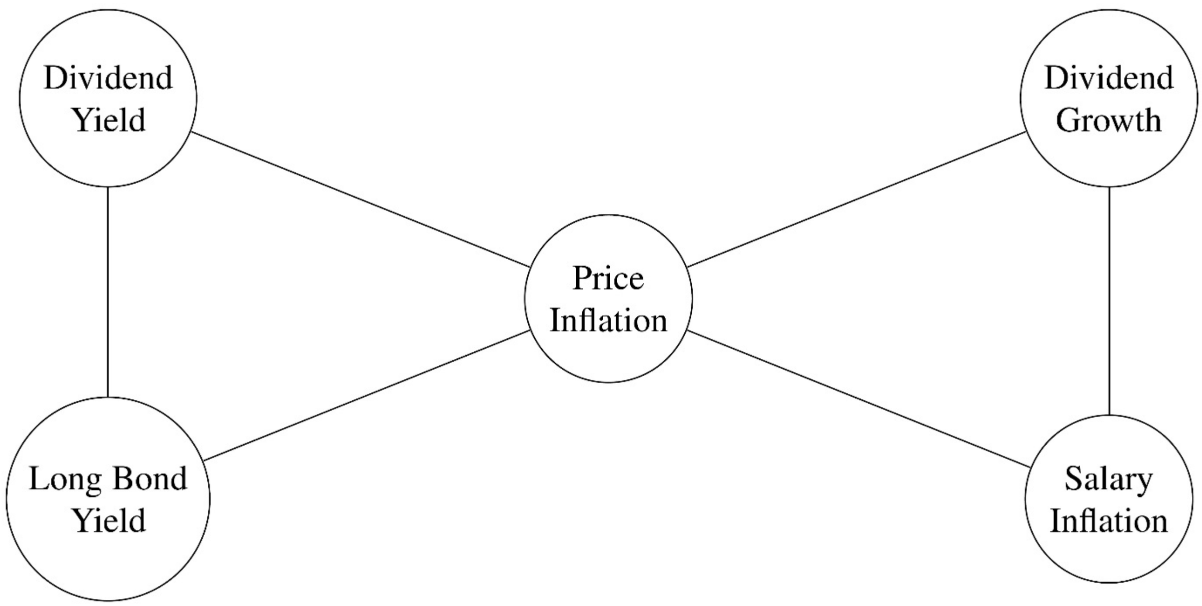
$Y_{(i,t)}$ is a first-order autoregressive time series with constant volatility:

$$Y_{(i,t)} = \beta_i Y_{(i,t-1)} + \varepsilon_{(i,t)}, \quad (2)$$

where $\varepsilon_{(i,t)}$ is shown as a graphical model. So, $Y_{(i,t)}$ represents the ongoing volatility around the economic variable's mean.

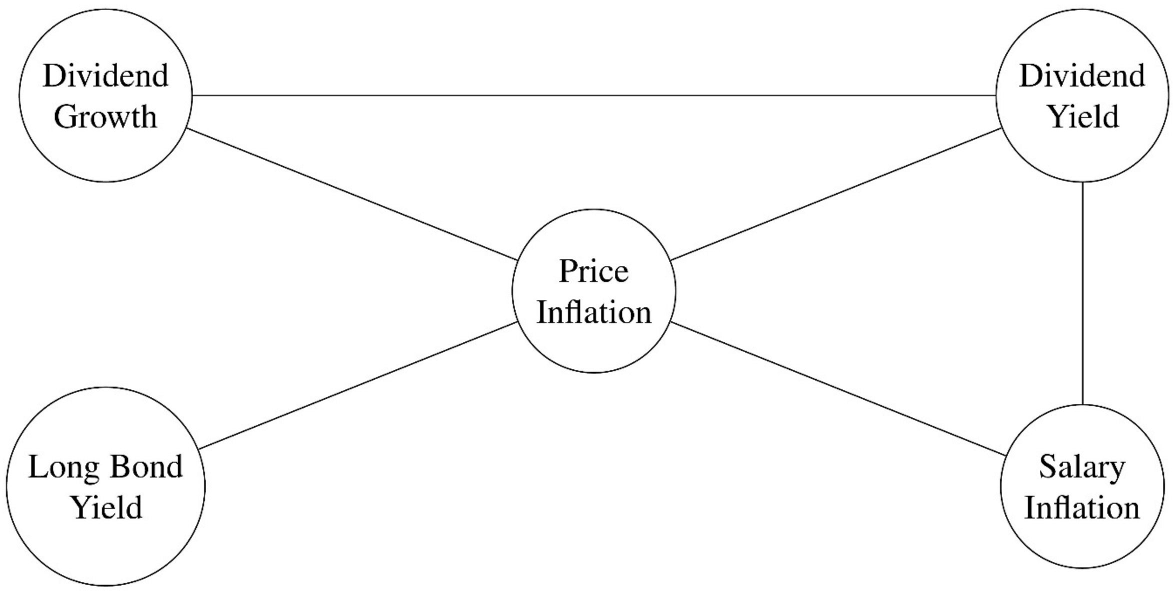
The innovations $\varepsilon_{(i,t)}$ are modeled using a graphical approach. The methodology is described in detail in Appendix A.2. A graphical model is a dimension reduction tool whereby all pairs of innovations need not be estimated. Instead, in this approach, we identify conditional independences and estimate the correlations between variables that are not conditionally independent. For example, Figures 2 and 3 show the structures that we use for the United Kingdom and the United States, respectively.[8] The methodology to determine the graphical structure for the two countries is identical. The different data in the two countries cause the differences in structure. The estimated parameter values are provided in Appendix A.2.

Figure 2
U.K. Graphical Structure



[8] Unlike the Wilkie model, there are no arrows in the graphical structure, meaning there is no assumption that any economic variable drives another, merely that they have some association.

Figure 3
U.S. Graphical Structure



4.8 Mortality Model

Future projections of the liabilities of a pension plan will also depend on the mortality assumptions. Recent advances in actuarial mortality modeling provide us with a range of alternative mortality models suitable for our purpose.

Cairns et al. (2009) provide a quantitative comparison of eight stochastic mortality models using data from England and Wales and the United States. An overview of that report is provided in Appendix B.1. For this report, we use model M7 to project stochastic mortality rates forward. Based on Cairns et al. (2009), model M7 represents a good fit for both U.K. and U.S. data.[9]

The structure of model M7, which models *q(t,x)*, the probability that an individual aged *x* at time *t* will die between *t* and *t+1*, is as follows:

$$\text{logit}\, q(t,x) = k_t^{(1)} + k_t^{(2)}(x-\bar{x}) + k_t^{(3)}[(x-\bar{x})^2 - \sigma^2] + \gamma_{(t-x)}^{(4)}, (3)$$

where

x is the age;

[9] Conceptually, model M7 projects future mortality improvements based on how many years into the future we are projecting and on the individual's year of birth (the cohort effect).

$k_t^{(i)}$ is the period effect;

$\gamma_{(t-x)}^{(i)}$ is the cohort effect; and

σ^2 is the average of $(x - \bar{x})^2$.

We parameterize model M7 using U.K. data from the HMD for both males and females from 1961 to 2014 for ages 30–100.

For the stylized U.S. plan, we parameterize model M7 using U.S. data from the HMD. As a sensitivity test, we also check the impact of using a deterministic projection of mortality rates using the RP-2006 mortality table and the MP-2018 projection table.[10]

Section 5: Methodology

In this section, we describe the methods used to update the economic capital analysis of the U.K. plan and apply the analysis to a stylized U.S. plan.

We will use the following notations:

A_t: Value of pension plan assets at time *t*.
L_t: Value of pension plan liabilities at time *t*.
X_t: Net cash flow of the plan at time *t* (excluding investment returns), meaning benefit payments net of contributions.
$I(s,t)$: Accumulation factor (accumulated value at time *t* of $1 invested at time *s*). These are obtained directly from the simulations of the underlying stochastic economic model.
$D(s,t)$: Discount factor, that is, $D(s,t) = I^{-1}(s,t)$.

Given the long-term nature of pension plan risks, we propose using a runoff approach, so the time horizon of our analysis is set until the time when the last of the current plan members dies. We assume that cash flows and valuations are carried out on an annual basis, so any surplus/deficit is determined at the end of each year. We define the profit vector, P_t , at time *t*, as

$$P_t = L(t-1) \,.\, I(t-1;t) - X_t - L_t, \ (4)$$

where *t* = 1,2, ..., *T* and

$$P_0 = A_0 - X_0 - L_0 . \ (5)$$

Under this setup, the current present value of future profits (PVFP) denoted by V_0 can be expressed as follows:

$$V_0 = \sum_{t=0}^{T} P_t \,.\, D_{(0,t)} , \ (6)$$

where *T* is the runoff time horizon. As there will be no residual liabilities after the last of the current members dies, $L_T = 0$.

[10] The Retirement Plan Experience Committee of the Society of Actuaries developed a U.S. pension plan mortality table using data with a central year of 2006. This base table was then projected to 2014 using the MP-2014 Projection Scale (called the RP-2014 table). For this study, we used the base table and called it the RP-2006 table. The latest Mortality Projection Scale (MP-2018) was then applied to project future mortality rates.

Using the relationship

$$I_{(0,t-1)} \,.\, I_{(t-1,t)} = I_{(0,t)} \;\; => \;\; I_{(t-1,t)} \,.\, D_{(0,t)} = D_{(0,t-1)}, \; (7)$$

along with the fact that $D_{0,0}$ = 1, Equation 6 can be rewritten as

$$V_0 = A_0 - \sum_{t=0}^{T} X_t \,.\, D_{(0,t)} \;.\; (8)$$

An intuitive interpretation of Equation 8 is that PVFP represents the present value of the final surplus/deficit—that is, whether the current level of assets, A_0, along with the future contributions, are adequate to pay all future benefits. Note that the value of the liabilities does not play a direct role in this measure; rather, the liabilities are reflected as part of the discounted cash flows, X_t.

Because future cash flows and asset returns are random variables that depend on the future random realizations of the underlying economic and mortality variables, the present value of the final surplus/deficit, V_0, is also a random variable. In contrast, a valuation actuary provides a single point estimate of the current value of future actuarial liabilities, L_0.

From this perspective, V_0 can be partitioned and expressed as

$$V_0 = (A_0 - L_0) + [L_0 - \sum_{t=0}^{T} \left(X_t \,.\, D_{(0,t)}\right)]. \;\; (9)$$

where the first component denotes the current valuation surplus or deficit, and the second component denotes emerging actuarial gains or losses.

Note that the point estimate of the value of actuarial liabilities, L_0, does not play a direct role in the calculation of V_0. For instance, a prudent valuation basis would produce a conservative high value for L_0, leading to a large current valuation deficit, but it will then be compensated by a corresponding rise in the emerging actuarial gains and vice versa.

For the U.K. plan, we do not employ amortization of deficits to ensure consistency with Porteous et al. (2012). However, for the stylized U.S. plan, we incorporate amortization in the following way. If the amortization period is 1—that is, there is an immediate cash injection from the sponsor to fully cover any deficit—then we have

$$V_0 = A_0 - \sum_{t=0}^{T} X_t \,.\, D_{(0,t)} + Y_0, \; (10)$$

where Y_0 is the cash injection at time 0, so that $Y_0 = L_0 - A_0$. We thus have the following equation:

$$V_0 = A_0 - \sum_{t=0}^{T} X_t \,.\, D_{(0,t)} + L_0 - A_0, \; (11)$$

which simplifies to

$$V_0 = L_0 - \sum_{t=0}^{T} X_t \cdot D_{(0,t)}. \quad (12)$$

If the amortization period is over *n* years, we have

$$V_0 = A_0 - \sum_{t=0}^{T} X_t \cdot D_{(0,t)} + \sum_{t=0}^{n-1} Y_t \cdot D_{(0,t)}, \quad (13)$$

where Y_t is the cash injection at time *t*. We will use Equation 13 for our analysis of a stylized U.S. plan.

We will use V_0 as a measure of risk in a DB pension plan. However, it would be helpful to use some form of standardization so the measure does not depend on the following:

- **Currency**, as one of our main goals in this research is to compare pension plan risks in different countries, namely the United Kingdom and the United States; and
- **Scale**, as different benefit structures would imply different magnitudes of plan assets and liabilities, comparing absolute values of the risks for different types of pension plans will not be meaningful.

Standardized PVFP, which we will denote by V_0^*, can be defined in many ways; here are two approaches:

- $V_0^* = \frac{V_0}{A_0}$: Conceptually, this amount can be interpreted as the proportional increase in assets required to meet all future benefit obligations in a particular scenario.
- $V_0^* = \frac{V_0}{L_0}$: Conceptually, this amount can be interpreted as the proportional loading that needs to be added to the liabilities so that if we had assets equal to the “loaded” liabilities we would be able to meet all future benefit obligations in a particular scenario.

The information contained in V_0^* is the same for either of these approaches, as long as the same standardization is used consistently throughout. We will use the standardization $\frac{V_0}{A_0}$ for the purposes of this report.

A risk measure in terms of economic capital can then be defined as follows:

> The economic capital of a pension plan is the proportion by which its existing assets would need to be augmented in order to meet net benefit obligations with a prescribed degree of confidence. A plan’s net benefit obligations are all obligations in respect of current plan members, including future service, net of future contributions to the plan.

This definition is designed to be consistent with our previous work on solvency capital calculations for many different financial services firms and conglomerates (Porteous and Tapadar 2005, 2008a, 2008b), so it is generic and flexible in terms of time horizons and liability valuation methods. However, due to the long-term nature of pension plans’ benefit obligations, it is important to use the entire runoff period as the time horizon.

The actual quantification of economic capital, using the distribution of the random variable V_0^*, can be carried out in one of the following ways:

- **VaR:** VaR is defined as $P[V_0^* \leq \mathrm{VaR}] = p$, for a given probability *p*. VaR represents the amount of additional initial assets required at time 0 (on top of existing assets) for the pension plan to meet all its future obligations with probability *p*, or confidence level (1 - *p*).

- **ES:** ES is defined as the average of all losses that are greater than or equal to the value of VaR for a given probability level *p*, that is, $E[V_0^* \mid V_0^* \leq \mathrm{VaR}]$. In other words, ES provides an estimate of the expected value of losses in the worst *p* proportion of cases.[11]

These definitions of VaR and ES are based on McNeil et al. (2015).

At the time of writing Porteous et al. (2012), there were indications that the regulators were considering a VaR measure at the 0.5th percentile level (or 99.5th percentile confidence level) over a one-year time horizon.[12, 13] Hence the 2012 research adopted VaR at the 99.5th percentile confidence level but used the entire runoff period as the time horizon. In the rest of this document, for all our results, we will present representative values of VaR and ES, with VaR at the 0.5th percentile level.

Section 6: Results for the U.K. plan

In this section, we present the results for the U.K. plan. As discussed in Section 4.7, we have two ESGs available for our analysis—the Wilkie model and the graphical model. In the first instance, we will run both for our base case to check the impact of alternative ESGs. However, for the sensitivity analysis, we will use only the graphical model. This is primarily because the Wilkie model is only calibrated to U.K. economic data. So, we will use only the graphical ESG for the stylized U.S. plan. Hence, focusing on and using the graphical model for the United Kingdom and the United States will produce consistent results.[14]

6.1 Base Case

Our base case results, using 100,000[15] simulations, are presented in Figure 4, which shows the full distribution of V_0^*. Representative values of VaR and ES are presented in Table 6.[16] Note that the ES measure is calculated based only on the simulated data and so will be underestimated, as the entire tail of the distribution cannot be captured through the simulations. There is an underestimation because the simulations do not fully capture the tails of the distribution. However, we do not employ any approximations, as doing this means we would have to choose a distribution (e.g., exponential distribution, Pareto distribution) to make the approximation. This would make the exercise more complicated, and we do not believe it would bring much added value.

- The differences in the results between the Wilkie and graphical ESGs reflect the different dynamics of the economic variables modeled.[17]
- The median value of V_0^* is 25% and 14% of A_0 for the graphical and Wilkie models, respectively. This reflects that, on average, both models suggest a positive present value of surplus (of about £10 billion and £6 billion under the graphical and Wilkie models, respectively).
- As expected, both Table 6 and Figure 4 show that for higher confidence levels (or equivalently lower percentiles), greater amounts of additional assets are required, and the ES increases substantially.

[11] ES and Conditional Tail Expectation (CTE) refer to the same concept.
[12] Note the difference in time horizon between our analysis, which is a full runoff of the liabilities, and this one-year horizon. The one-year horizon would require a much smaller amount of economic capital.
[13] Since the observations at the time of writing Porteous et al. (2012), there have been no updates to the regulators' quantitative thinking.
[14] Admittedly, the U.K. and U.S. models are slightly different in structure and are also calibrated to their respective economies.
[15] While the base case results for the U.K. plan use 100,000 simulations, all other results shown use 10,000 simulations. The reason for this is simply the amount of computer time involved.
[16] While the tables focus on the median and 99.5th percentile (the focus of this analysis), the reader can observe the full distribution of results shown in the figures.
[17] There will always be differences among different ESGs. If sufficient resources are available, analyzing a problem using multiple ESGs may provide valuable insights.

Table 6

Base Case Economic Capital (as a percentage of A_0 = £41.6 billion) at Different Probability Levels

	Graphical Model		Wilkie Model	
Percentile	VaR	ES	VaR	ES
50	25	-13	14	-14
10	-36	-74	-31	-55
0.5	-153	-198	-101	-126

6.2 Sensitivity to Asset Allocation

For the balance of the analysis, we use only the graphical ESG, the details of which are provided in Appendix A.2. We change the base case asset allocation strategy from 70% equities/30% bonds to 30% equities/70% bonds. Our findings are given in Table 7 and Figure 5, which show the base case results alongside the results for the changed asset allocation strategy for ease of comparison. All other assumptions are kept the same as that of the base case. We make the following observations:

- For increased bond investment, the distribution of V_0^* has moved to the left and has greater dispersion.
- The leftward shift of the distribution indicates a greater probability of larger deficits. This is reflected in the median (50th percentile) of V_0^*, which shows a loss of 21% of A_0 in terms of VaR (as compared to a surplus of 25% of A_0 for the base case results).
- The sensitivity patterns can be explained by the fact that the expected returns from bonds are lower in the long term compared to equities. So, a higher allocation to bonds can lead to potentially larger losses, which is reflected in the leftward shift and greater dispersion in the distribution.
- Moreover, fixed interest bonds are a poor match for real liabilities (the U.K. plan liabilities are fully inflation-protected). Hence, an increased allocation to nominal bonds has exacerbated the risk, producing a fatter-tailed distribution.

Table 7

Economic Capital (as a percentage of A_0 = £41.6 billion) for the Base Case and 30% Equities/70% Bonds Using the Graphical Model

	70% Equity, 30% Bonds (Base Case)		30% Equity, 70% Bonds	
Percentile	VaR	ES	VaR	ES
50	25	-13	-21	-72
10	-36	-74	-103	-149
0.5	-153	-198	-245	-296

6.3 Sensitivity to Contribution Rates

We analyze the impact of changes in the base case contribution rate of 22.5%. We consider two arbitrary additional cases: an increased contribution rate of 25% of salaries and a decreased contribution rate of 20%. All other assumptions are the same as for the base case, including the asset allocation strategy of 70% equities and 30% bonds.

We present our findings in Table 8 and Figure 6. Note that we have also included the base case results in Table 8 for ease of comparison. Similarly, in the two plots in Figure 6, we have included the distribution of V_0^* for the base case as the gray curve in the background. We make the following observations:

- Compared to the impact of change in asset allocation strategy, changes in contribution rates have a much smaller effect on the overall risk.
- As an example, at the 99.5% level of confidence (i.e., percentile level of 0.5%), a decrease in contribution of 2.5% (i.e., reduced from 22.5% to 20% of salary) results in an increase of loss from 153% to 160% of A_0 in terms of VaR. On the other hand, increasing the contribution rate to 25% produces a smaller loss of 146% compared to 153% for the base case.
- The left and right shifts of the V_0 distribution for decreased and increased contribution rates, respectively, can also be observed in Figure 6. However, note that the magnitudes of the shifts are relatively small compared to the impact of changes in the asset allocation strategy. To make the effect of the change in contribution similar to the effect of the change in asset allocation, the contribution rate would need to change by more than 50% (i.e., to alternative rates smaller than 11% or larger than 33%).

Table 8

Economic Capital (as a percentage of A_0 = £41.6 billion) for Different Contribution Rates Using the Graphical Model

Contribution Rate as a Percentage of Salary						
	20%		22.5% (Base Case)		25%	
Percentile	VaR	ES	VaR	ES	VaR	ES
50	21	-18	25	-13	29	-8
10	-41	-80	-36	-74	-30	-68
0.5	-160	-208	-153	-198	-146	-191

6.4 Graphs

Figure 4

Base Case Distributions of V_0^* (as a percentage of A_0 = £41.6 billion) for Both Graphical and Wilkie Models

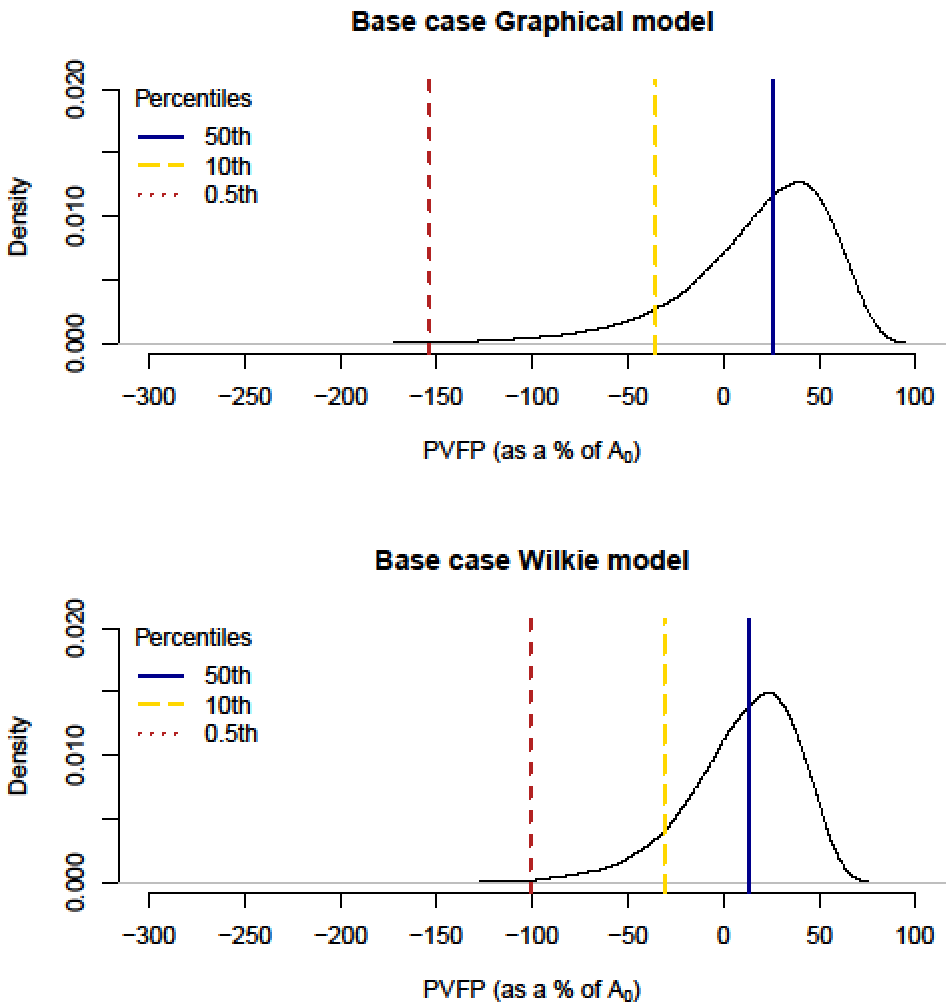


Figure 5

Base Case Distributions of V_0^* (as a percentage of A_0 = £41.6 billion) for the Base Case and 30% Equities/70% Bonds Using the Graphical Model

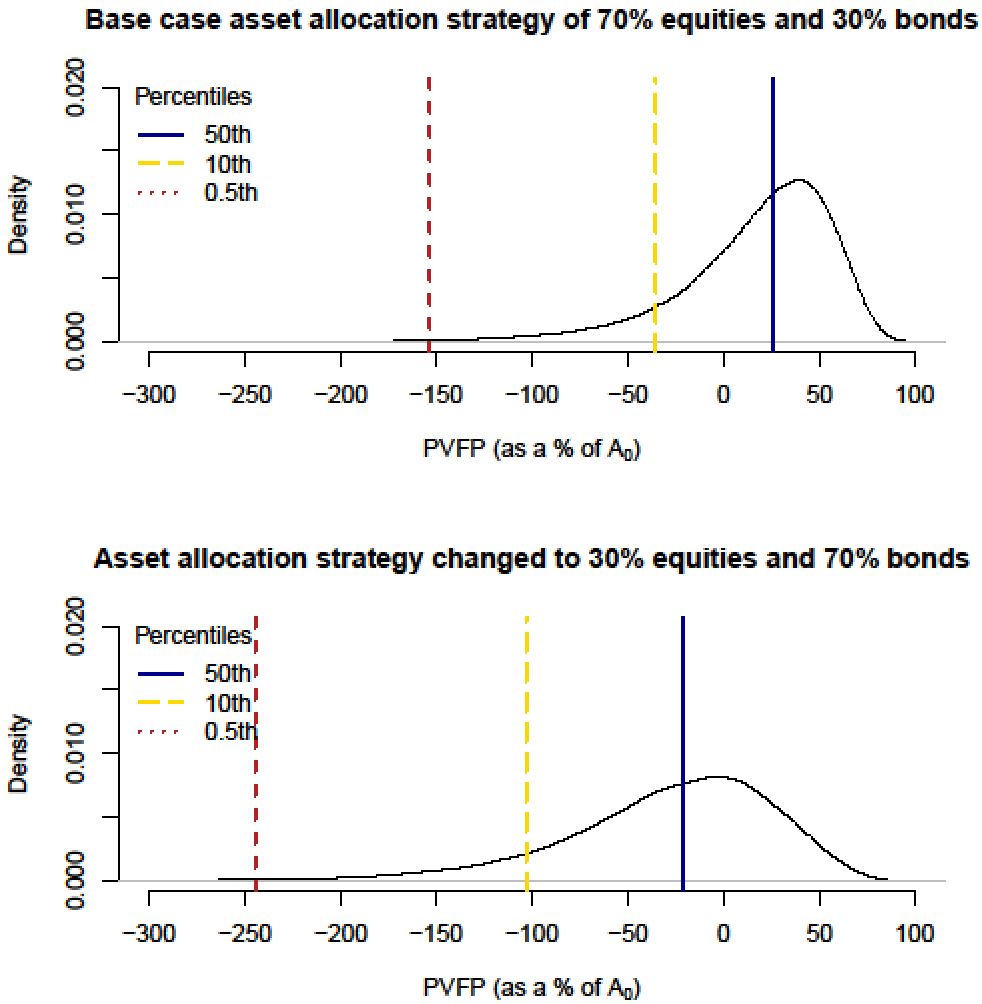
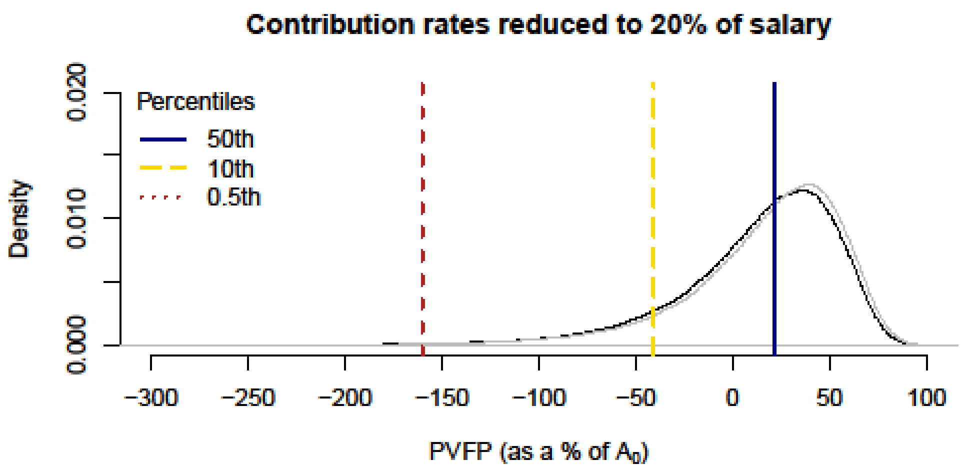
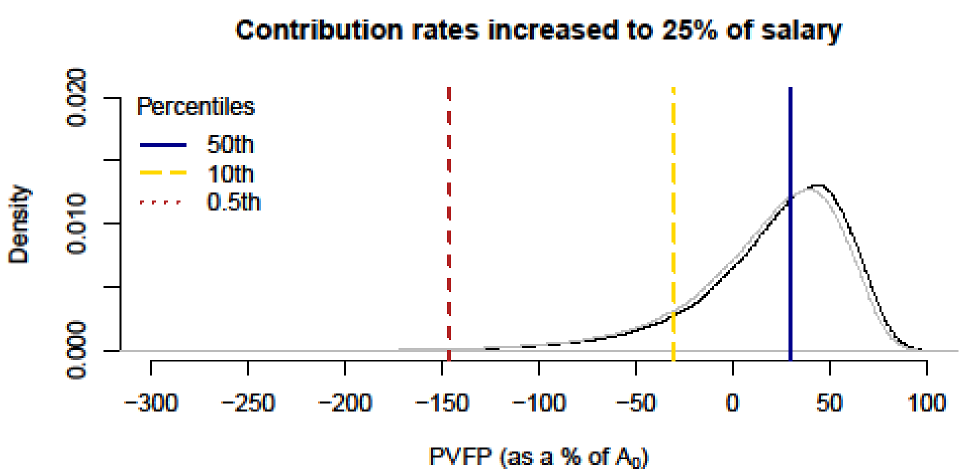


Figure 6

Base Case Distributions of V_0^* (as a percentage of A_0 = £41.6 billion) for 20% and 25% Contribution Rates Using the Graphical Model





The gray curve in the background shows the base case for comparison.

Section 7: Results for the Stylized U.S. Plan

In this section, we present results for the stylized U.S. plan. As previously discussed, only the graphical ESG is used to simulate future economic variables. Recall that the initial deficit of the stylized U.S. plan is 20% of the value of the liabilities.

7.1 Results with No Amortization

Our first results, using 10,000 simulations, are presented in Figure 7, which shows the full distribution of V_0^* based on the assumption that the initial deficit is not amortized. Representative values of VaR and ES are presented in Table 9. We make the following observations:

- The median value of V_0^* is -25% of A_0. This corresponds to a median deficit of $6.5 million, which is as expected.
- Also, as expected, both Table 9 and Figure 7 indicate that for higher confidence levels (or equivalently lower percentiles) greater amounts of additional assets are required, and the ES increases substantially.

7.2 Base Case

As discussed in Section 5, we assume that the initial deficit is amortized over a number of years. Hence, we have

$$V_0 = A_0 - \sum_{t=0}^{T} X_t \,.\, D_{(0,t)} + \sum_{t=0}^{n-1} Y_t \,.\, D_{(0,t)}. \quad (14)$$

For our base case, we assume an amortization period of seven years (i.e., *n* = 7), during which the sponsor injects a total of $L_0 - A_0$ spread evenly over those seven years—that is, $Y_t = \frac{1}{7}(L_0 - A_0)$. Note that for our case, this represents an additional contribution of approximately 4% of members' salaries.

Our base case results are presented in Figure 8, which shows the full distribution of V_0^*. Representative values of VaR and ES are presented in Table 9. We make the following observations:

- With the amortization cash flows, the distribution of V_0^* has moved to the right and has less dispersion.
- The right shift of the distribution indicates a smaller probability of larger deficits. This is reflected in the median (50th percentile) of V_0^*, which shows a loss of 1% of A_0 in terms of VaR (as compared to a deficit of 25% of A_0 for the starting case).
- Note that if the amortization period is 1, there is an immediate cover for the deficit amount and the average of V_0^* will be approximately zero because the base contribution is equal to the expected future benefit accruals. When the amortization period is seven years, however, there is a time lag in covering the deficit, so on average V_0^* is a small negative value. We will call this the base case.

Table 9

Base Case Economic Capital (as a percentage of A_0 = \$26.1 billion) at Different Probability Levels with and without Amortization

	No Amortization		With Amortization	
Percentile	**VaR**	**ES**	**Percentile**	**VaR**
50	-25	-88	-1	-60
10	-121	-187	-92	-156
0.5	-339	-444	-305	-415

7.3 Sensitivity to Asset Allocation Strategies

Recall that the base case asset allocation strategy is assumed to be 50% bonds and 50% equity. To test the impact of asset allocation strategies, we now consider two cases: 75% equity and 25% bonds, and 75% bonds and 25% equity. Table 10 and Figure 9 show the results for different asset allocation strategies. All other assumptions are kept the same as those for the base case. We make the following observations:

- For increased equity investment, the distribution of V_0^* has moved to the right, as equities are expected to generate higher returns in the long run. The distribution also has greater dispersion compared to the base case, as more exposure to equities leads to higher volatility.
- The right shift of the distribution is reflected in the median (50th percentile) of V_0^*, which shows a surplus of 6% of A_0 in terms of VaR (compared to a deficit of 1% of A_0 for the base case). The greater dispersion is reflected by the 0.5th percentile, which is much larger than the base case.
- For increased bond investment, the distribution of V_0^* has moved to the left. The dispersion is again greater than that of the base case but less dispersed than that with higher equity.
- The median of V_0^* under the increased bond investment shows a loss of 54% of A_0 in terms of VaR. The sensitivity patterns can be explained by the fact that the expected returns from bonds are lower in the long run compared to equities. So, a higher bond investment can lead to potentially large losses, which is reflected in the shift left.[18]

Table 10

Economic Capital (as a percentage of A_0 = \$26.1 billion) for the Asset Allocation Strategy at Different Probability Levels

	75% Equity, 25% Bonds		25% Equity, 75% Bonds	
Percentile	VaR	ES	Percentile	VaR
50	6	-58	-54	-119
10	-92	-169	-154	-224
0.5	-343	-478	-387	-505

[18] Note that for small shifts from equities to bonds, the VaR does improve. Beyond a certain shift from equities to bonds, the lower expected return of bonds (relative to equities) outweighs the matching benefits of bonds to the plan liabilities.

7.4 Sensitivity to Contribution Rates

In this section, we analyze the impact of changes in the base case contribution rate. As discussed in Section 4.4, the contribution rate for the stylized U.S. plan is 10.8%. We consider two cases for the sensitivity test; an increased contribution rate of 13.3% of salaries (an increase of 2.5%) and a decreased contribution rate of 8.3% (a decrease of 2.5%). All other assumptions are the same as the base case, including the asset allocation strategy of 50% equities and 50% bonds. We present our findings in Table 11 and Figure 10.

- Compared to the impact of change in asset allocation strategy, changes in contribution rates have a much smaller effect on the overall risk.
- For example, at 0.5% confidence level, a decrease in contribution of 2.5% (i.e., reduced from 10.8% to 8.3% of salary) results in an increase in loss from 305% to 326% of A_0 in terms of VaR. On the other hand, increasing the contribution rate to 13.3% produces a deficit of 286%.
- The left and right shifts of the distribution of V_0^* for decreased and increased contribution rates, respectively, can also be observed in Figure 10. However, the magnitude of the shift at the median is roughly an eighth of the impact of changes in the asset allocation strategy. The magnitude of the shift at the 0.5th percentile is roughly half of the impact of changes in the asset allocation strategy.

Table 11
Economic Capital (as a percentage of A_0 = $26.1 billion) for Different Contribution Rates at Different Probability Levels

	Increased Contribution		Reduced Contribution	
Percentile	VaR	ES	Percentile	VaR
50	7	-50	-9	-70
10	-80	142	-104	-170
0.5	-286	396	-326	-433

7.5 Sensitivity to Mortality Tables

We consider the sensitivity of changing the mortality assumptions to be deterministic in order to provide some comparison to common practice. We use the RP-2006 mortality table and the MP 2018 projection scale instead of model M7 calibrated to data from the HMD.[19] Unlike the other sensitivity tests, the mortality rates are deterministic in this case. All other assumptions, however, remain unchanged from the base case, and the economic assumptions are still stochastic. Note that the results presented are based on the same set of economic simulations as in the previous sections. Technically, using different assumptions would mean that L_0 and contributions would be slightly different. For consistency, we do not make any changes to the contribution rates or the liabilities when changing the mortality table. Note that RP-2006 has lower mortality rates compared to model M7 calibrated to HMD data. We present our findings in Table 12 and Figure 11.

- Compared to the base case, the median of the distribution has moved slightly to the right.
- Given that RP-2006 has lower mortality rates than M7, it has the following effects:
    - There is more cash inflow at the start, as benefit payments are smaller given fewer deaths among active members.

[19] Note that for consistency with model M7, the deterministic mortality assumptions are also truncated at age 100.

  - The cash outflow is higher toward the end, as pensions paid are higher given that pensioners survive longer.
  - As higher net contributions occur sooner than higher pension payments, the impact of contributions on V_0^* is larger. This is reflected in the median increasing to 6% of A_0 in terms of VaR.
- The dispersion has significantly reduced. The deficit at the 0.5% percentile level is 209% of A_0 in terms of VaR compared to 305% for the base case. This is due to
  - mortality rates being deterministic, and
  - higher stochastic positive cash flows at the beginning making the distribution less negatively skewed.

Table 12

Economic Capital (as a percentage of A_0 = $26.1 billion) Based on Deterministic RP-2006 Mortality Table and MP-2018 Projection Scale

	Base Case		RP-2006 Mortality Table	
Percentile	VaR	ES	Percentile	VaR
50	-1	-60	6	-44
10	-92	-156	-73	-117
0.5	-305	-415	-209	-261

7.6 Graphs

Figure 7

Starting Case with No Amortization Distributions of V_0^* (as a percentage of A_0 = $26.1 billion)

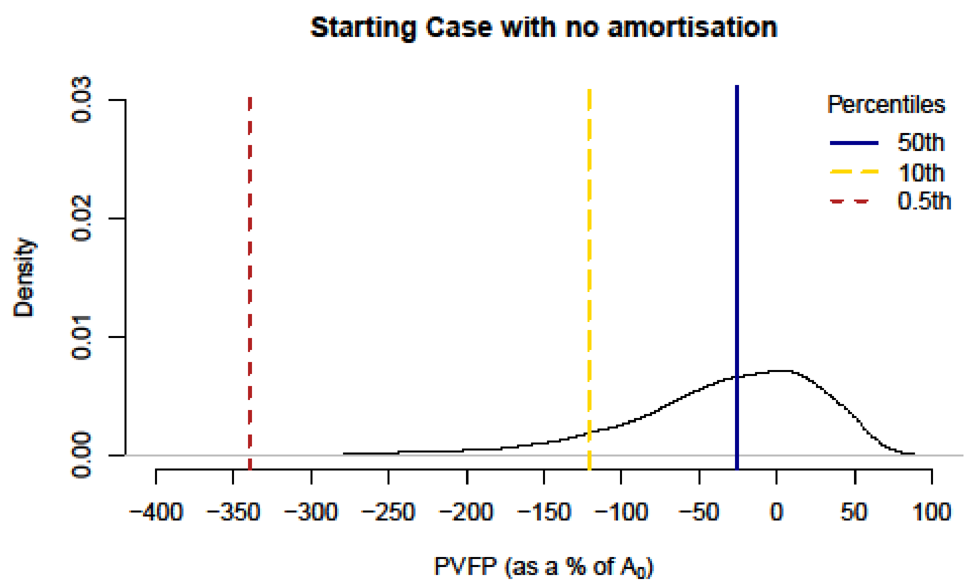


Figure 8

Base Case Distribution of V_0^* (as a percentage of A_0 = $26.1 billion)

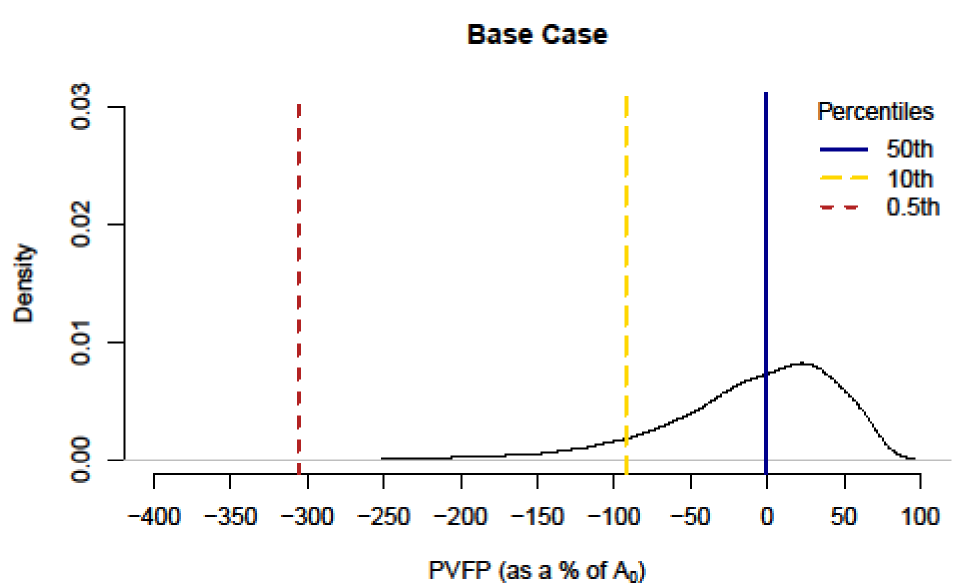
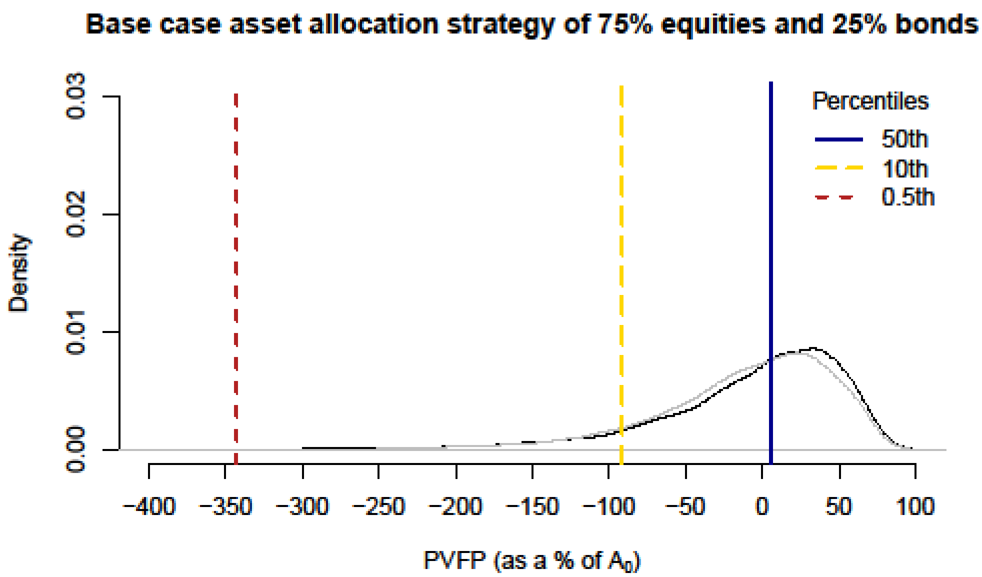
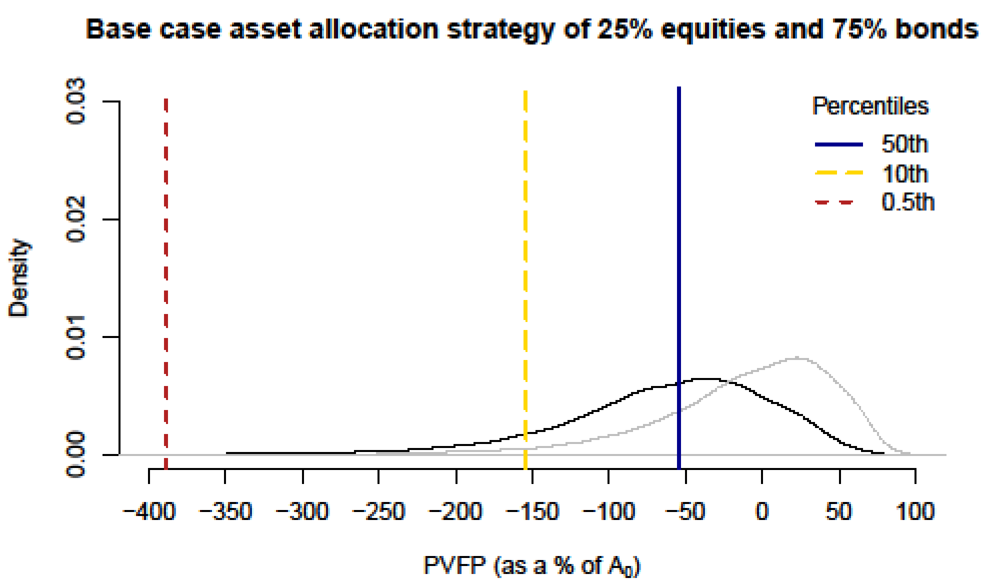


Figure 9

Distribution of V_0^* (as a percentage of A_0 = \$26.1 billion) for Different Asset Allocation Strategies

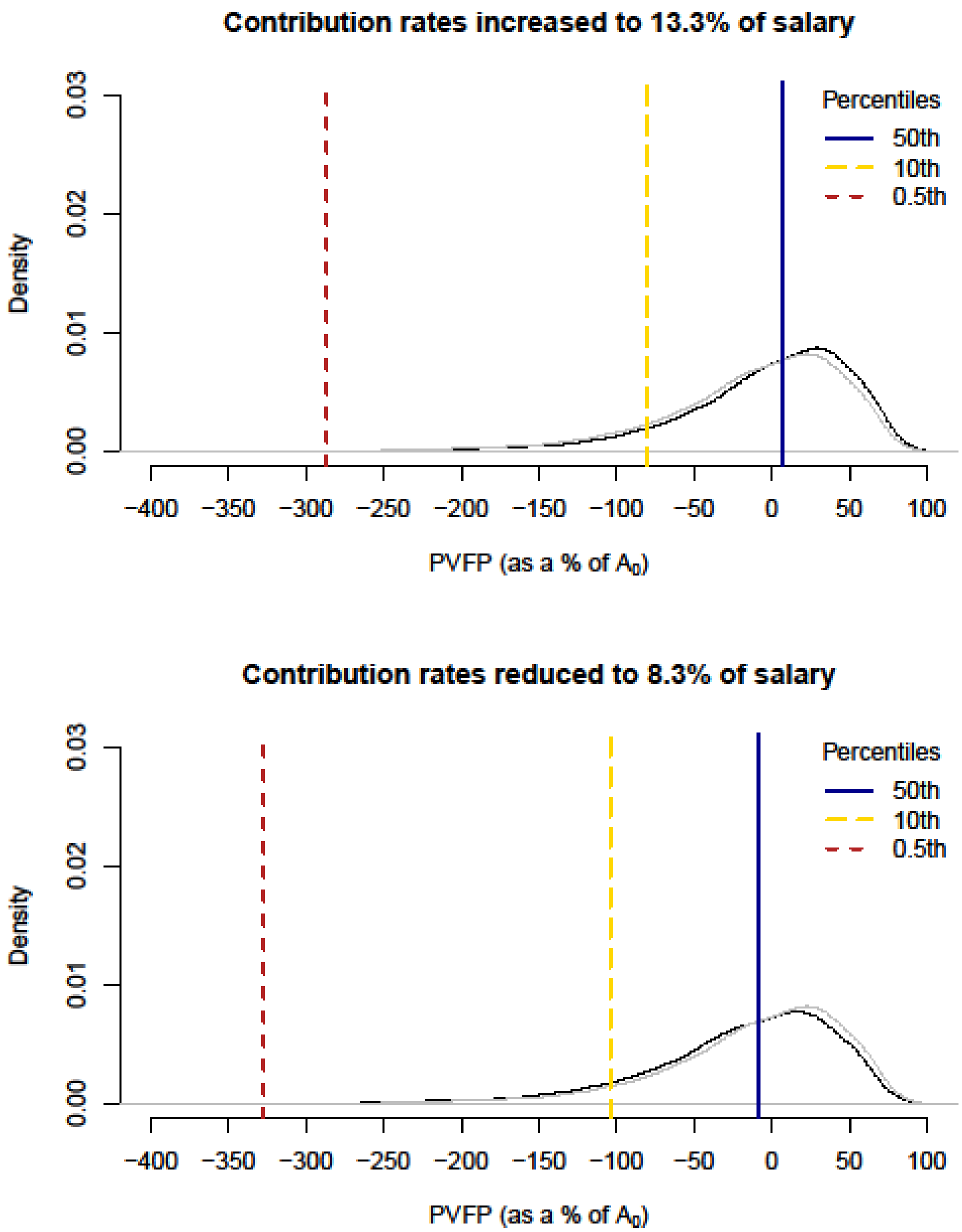




The gray curve in the background shows the base case distribution for comparison.

Figure 10

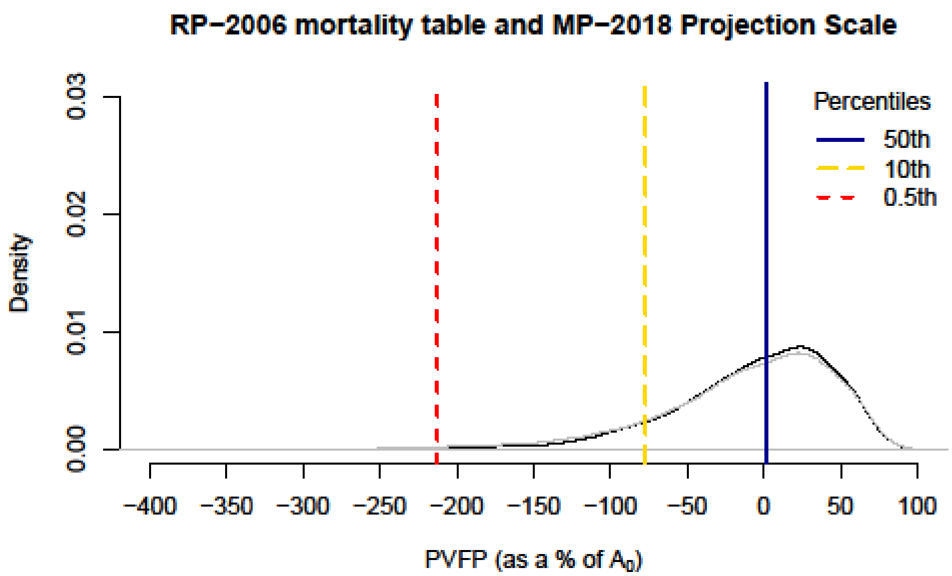
Distribution of V_0^* (as a percentage of A_0 = $26.1 billion) for Contribution Rates



The gray curve in the background shows the base case distribution for comparison.

Figure 11

Distribution of V_0^* (as a percentage of A_0 = $26.1 billion) Using the RP-2006 Mortality Table and MP-2018 Projection Scale.



The gray curve in the background shows the base case distribution for comparison.

Section 8: Conclusion and Future Work

Table 13 summarizes the results presented in Sections 6 and 7.

Table 13

Economic Capital (as a percentage of A_0 = £41.6 billion for the U.K plan and A_0 = $26.1 billion for the U.S. plan) for the Base Case and Various Sensitivities Using the Graphical Model

USS Plan				
	70% Equity, 30% Bonds (Base Case)		30% Equity, 70% Bonds	
Percentile	VaR	ES	VaR	ES
50	25	-13	-21	-72
10	-36	-74	-103	-149
0.5	-153	-198	-245	-296
	20% Contribution Rate		25% Contribution Rate	
50	21	-18	29	-8
10	-41	-80	-30	-68
0.5	-160	-208	-146	-191
U.S. Stylized Plan				
	75% Equity, 25% Bonds		25% Equity, 75% Bonds	
50	6	-58	-54	-119
10	-92	-169	-154	-224
0.5	-343	-403	-387	-505
	8.3% Contribution Rate		13.3% Contribution Rate	
50	-9	-70	7	-50
10	-104	-170	-80	-142
0.5	-326	-433	-286	-396

The main results of the study are the following:

- As a percentage of starting assets, the U.S. stylized plan is more volatile than the U.K. plan. The U.S. stylized plan requires over three times its starting asset value as an economic capital buffer to provide 99.5% certainty of providing the pension benefits. The U.K. plan requires roughly half this percentage of starting assets. Also, even though the U.S. stylized plan is smaller in currency terms, the absolute size of the required economic capital buffer is larger.
- The reduction in economic capital requirement of a larger allocation to long bonds is greater in the U.S. stylized plan than in the U.K. plan. Largely, this is because the U.K. plan benefits increase completely in line with either wage increases or price inflation. The U.S. stylized plan benefits reflect wage increases while individuals are accruing benefits, but otherwise the plan grants no inflationary increases.
- The effect on economic capital (for either of the plans) is much larger for changes in asset allocation than for changes to plan contributions.

Some implications of the results for various stakeholders are as follows:

- Plan sponsors should understand that there is a very large range of potential outcomes in a typical DB pension plan. This range can result in significant variation in contributions to the plan. To a certain extent, the range of outcomes can be narrowed by appropriate selection of asset allocation and plan provisions.
- The full distribution of results is shown in this report. Pension practitioners may have discussions with plan sponsors to assist them in understanding the full range of uncertainty they are assuming in the financing of their DB plans.

- An economic capital framework provides pension regulators with another tool to consider their exposure to benefits guaranteed by the Pension Protection Fund and the Pension Benefit Guaranty Corporation. It also provides them with some guidance in circumstances where it is appropriate to expect plan sponsors to hold some degree of margin for adverse deviation within their pension funds. The results clearly show that the appropriate degree of margin is materially affected by plan provisions, plan asset allocation and the desired degree of confidence that promised benefits will be provided.
- Economic capital frameworks may also be of interest to plan members. A framework can help them to understand the uncertainty the sponsor faces in financing DB pension plans. This approach can supplement other communications to plan members that educate them in plan financing.

While not specifically part of the current report, our project team anticipates extending this research in a couple of ways. First, we plan to examine a stylized Canadian pension plan to increase the geographic scope of our work.

We also plan to analyze the impact of changing population structure on investment returns. The analysis in this report considers investment uncertainty independent of mortality uncertainty. There is an argument (and a lot of academic literature) suggesting that increasing longevity will affect the returns on various asset classes. We will explore this relationship so we can comment on this interaction of investment and mortality uncertainty, rather than solely on their independent effects. To our knowledge, this area of study has not been addressed in the academic literature to date.

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Appendix A: Economic Scenario Generator

Projecting pension plan assets and liabilities requires the simulation of future economic scenarios. Typically, actuaries rely on ESGs to produce reasonable simulations of the joint distribution of variables relevant for asset and liability valuations.

A wide range of ESGs is currently used in the industry. These models have varying levels of complexity and are often proprietary. Among the few published models for actuarial use, the most well-known is the Wilkie model, first published by Wilkie in 1986. This reduced-form vector autoregression model for U.K. economic variables relies on a cascading structure, where the forecast of one or more variables is used to generate values for other variables, and so forth. This model has been periodically validated and recalibrated in Wilkie (1995) and Wilkie et al. (2011).

The Wilkie model provides time series models for price inflation, salary inflation, dividend yield, dividend growth, cash yield and long-term government bond (consol) yields in the United Kingdom. Although the model is limited in the sense that other asset classes, like property and corporate bonds, are not included, it does provide the basic variables required to project the U.K. plan assets and liabilities forward. A brief outline of the Wilkie model is given in the next section.

We also use the graphical model, discussed by Oberoi et al. (2018), to cross-validate the results. Graphical models rely on capturing the underlying correlation structure between the model variables in a parsimonious manner, making them useful for simulating data in high dimensions. In these models, dependence between variables is represented by edges in a graph connecting the variables or nodes. This approach allows us to assume conditional independence between variables and to set their partial correlations to zero. Two variables may then be connected via one or more intermediate variables, so they might still be weakly correlated. Graphical models have also been used in Porteous (1995); Porteous and Tapadar (2005, 2008a, 2008b); Porteous et al. (2012); and Yang and Tapadar (2015).

For the analysis of the U.K. pension plan, we have used both the Wilkie model and the graphical model to test the sensitivity of the results to the particular choice of stochastic economic model. As the Wilkie model is only calibrated to U.K. data, we will only use the graphical model for the analysis of the stylized U.S. plan. A brief outline of the graphical model is given later in this appendix.

A.1: The Wilkie Model

In 1984, David Wilkie first presented his work on a stochastic investment model for actuarial use in the United Kingdom. The work was formally published in 1986. Wilkie has periodically updated and recalibrated his model in Wilkie (1995) and Wilkie et al. (2011). He has also coauthored other recent papers with Sahin (2015, 2016a, 2016b, 2016c, 2017), which focus on certain specific aspects of the model. In this section, we will focus only on these joint papers to provide an overview of the Wilkie model.

The original purpose of the Wilkie model was to develop a minimal economic and investment model that actuaries could use for long-term simulations of future economic scenarios without being too concerned with short-term fluctuations. Model variables were specifically chosen with an eye toward the long-term nature of a life insurance company or pension plan's assets and liabilities. The actual constituents of the model and the model parameters have been updated periodically (Wilkie 1995, Wilkie et al. 2011), but the overall approach and structure has broadly remained the same.

A.1.1 Model Structure

Since the Wilkie model was first proposed in 1984, the notation has undergone some changes over time. We will present the notation used in Wilkie et al. (2011) to avoid confusion.

In the first paper, Wilkie (1986) presented a model for the following four variables:

$I(t)$: annual rate of price inflation;

$Y(t)$: dividend yield on an index of ordinary shares;

$K(t)$: annual rate of dividend increase;

$C(t)$: long-term yield on government bonds.

The variables were related to each other in a cascade structure, as depicted in Figure 12, where price inflation impacts all the other variables in the model. Among the other variables, dividend yield affects dividend growth and long-term government bond yields. The variables enclosed within the dashed area of Figure 12 are the original variables included in Wilkie (1986). The remaining variables were added in Wilkie (1995).

These original four variables were then used to define the following:

$I(t)$: RPI, $Q(t) = Q(t-1) \times exp[I(t)]$;

$D(t)$: index for dividends, $D(t) = D(t-1) \times K(t)$; and

$P(t)$: price index of ordinary shares, $P(t) = \frac{D(t)}{Y(t)}$.

Wilkie (1995) introduced a few more economic variables:

$J(t)$: annual rate of wage inflation;

$BD(t)$: “log-spread” between long-term and short-term bond yields; and

$R(t)$: real yields on index-linked stocks.

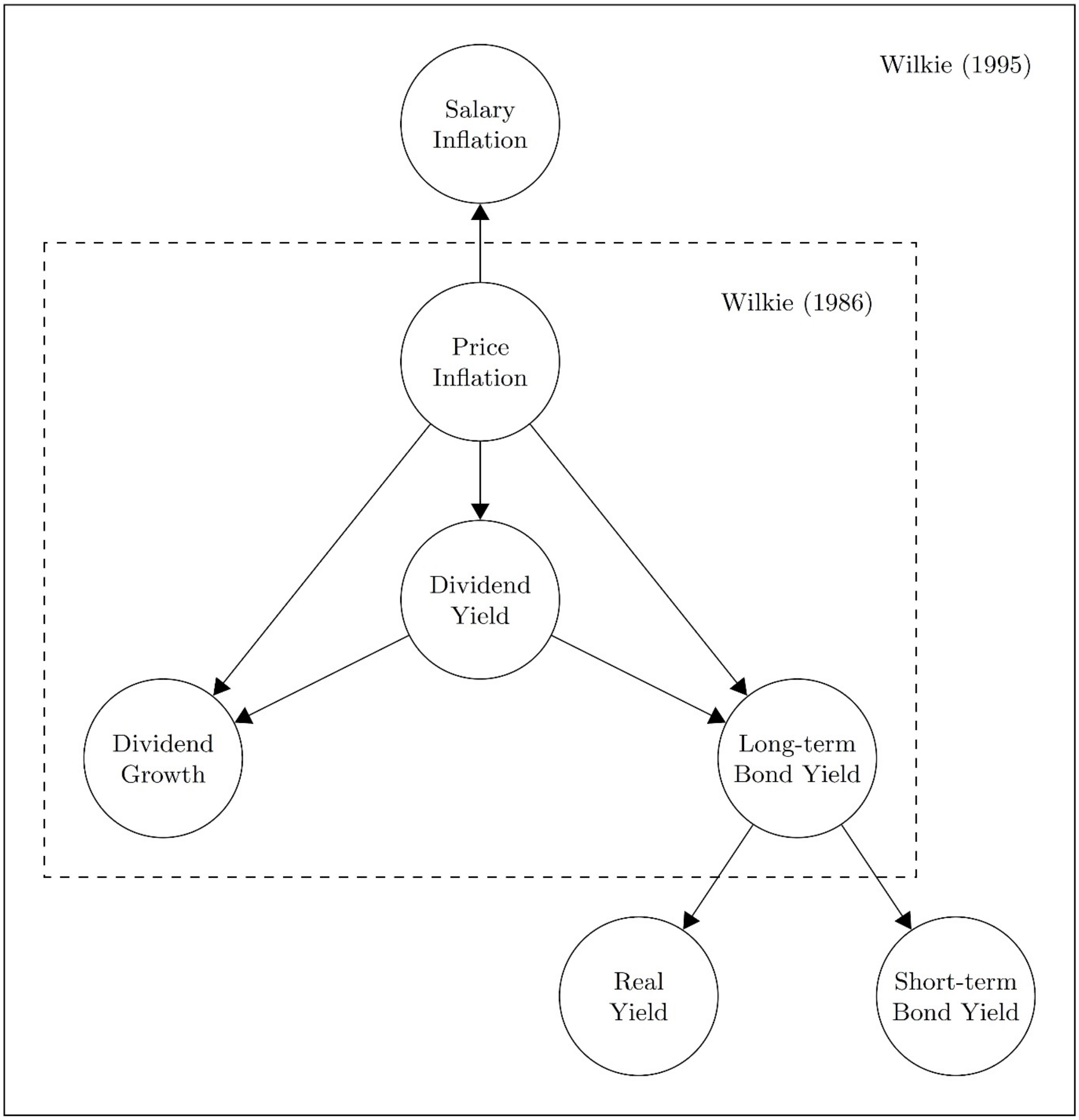
These new variables led to the following:

$W(t)$: index of wages, $W(t) = W(t-1)\ exp[J(t)]$; and

$B(t)$: short-term yields on government bonds, $BD(t) = \log C(t) - \log B(t)$.

Wilkie (1995) also proposed a model for property indices, but this was later discontinued as being unsatisfactory, so we have not included it here.

Figure 12
Wilkie Models: Cascade Structure



Source: Wilkie (1986, 1995).

A.1.2 Price Inflation

A simple autoregressive process is proposed for annual rate of price inflation:

$$I(t) = QMU + QA \times [I(t - 1) - QMU] + QSD \times QZ(t), \quad (15)$$

where $QZ(t) \sim N(0,1)$ and (QMU, QA, QSD) are the relevant parameters. QMU represents equilibrium inflation, QA controls for serial correlation, and QSD controls the volatility of inflation. The suggested parameter values are given in Table 14.

Table 14
Parameter Values for the Model for Price Inflation

Parameters	Wilkie (1986)	Wilkie (1995)	Wilkie et al. (2011)
QMU	0.0500	0.0470	0.0430
QA	0.6000	0.5800	0.5800
QSD	0.0500	0.0425	0.0400

A.1.3 Wage Inflation

For wages, it was proposed that an AR(1) process be combined with the effects of immediate past and present price inflation as follows:

$$J(t) = WW1 \times I(t) + WW2 \times I(t-1) + WMU + WN(t), \quad (16)$$

where

$$WN(t) = WA \times WN(t-1) + WSD \times WZ(t) \quad (17)$$

and

$WZ(t) \sim N(0,1)$ and ($WW1$, $WW2$, WMU, WA, WSD) are the relevant parameters.

In particular, a value of zero was proposed for WA, suggesting that the autoregressive part of the model $WN(t)$ could be omitted entirely. However, that would mean that the current rate of wage inflation is fully predictable using current and immediate past values of price inflation. As in the case of price inflation, WMU represents equilibrium real wage increase, and WSD controls for the volatility of real wage inflation. Serial correlation is controlled indirectly via the serial correlation in price inflation and the parameters $WW1$ and $WW2$.

Table 15
Parameter Values for the Model for Wage Inflation

Parameters	Wilkie (1986)	Wilkie (1995)	Wilkie et al. (2011)
WW1	—	0.6000	0.6000
WW2	—	0.2700	0.2700
WMU	—	0.0210	0.0200
WSD	—	0.0233	0.0219

A.1.4 Dividend Yield

The proposed model for dividend yield is as follows:

$$\log Y(t) = YW \times I(t) + \log YMU + YN(t), \quad (18)$$

where

$$YN(t) = YA \times YN(t-1) + YSD \times YZ(t), \quad (19)$$

$YZ(t) \sim N(0,1)$ and (*YW, YMU, YA, YSD*) are parameters of the model. This model says that the natural logarithm of the yield depends directly on the current rate of price inflation, as well as a first-order autoregressive model with serial correlation controlled by the parameter *YA* and volatility controlled by the parameter *YSD*. The equilibrium level of the dividend yield is the parameter *YMU*. The suggested parameter values are given in Table 16.

Table 16
Parameter Values for the Model for Dividend Yield

Parameters	Wilkie (1986)	Wilkie (1995)	Wilkie et al. (2011)
YW	1.3500	1.8000	1.5500
YMU	0.0400	0.0375	0.0375
YA	0.6000	0.5500	0.6300
YSD	0.1750	0.1550	0.1550

A.1.5 Dividend Growth

The model for the annual rate of dividend increase, $K(t)$, is made to depend on price inflation and the residuals from the dividend yield process. In addition, it also depends on its own lagged residual.

$$K(t) = DMU + DW \times DM(t) + DX \times I(t)$$
$$+ DY \times [YSD \times YZ(t-1)]$$
$$+ [DB \times DSD \times DZ(t-1)]$$
$$+ DSD \text{ x } DZ(t) \quad (20)$$

where

$$DZ(t) \sim N(0,1), \quad (21)$$

and

$$DM(t) = DD \times I(t) + (1 - DD) \times DM(t-1). \quad (22)$$

The parameter *DX* is constrained to be (1 – *DW*) so there is unit gain from inflation to dividends. So (*DMU, DW, DD, DY, DB, DSD*) are the relevant parameters. The second term is the inflation effect, the third term is the lagged dividend yield residual and the fourth term is the lagged “own” residual. The suggested parameter values are given in Table 17.

Table 17
Parameter Values for the Model for Dividend Growth

Parameters	Wilkie (1986)	Wilkie (1995)	Wilkie et al. (2011)
DMU	0	0.0160	0.0110
DW	0.0800	0.5800	0.4300
DD	0.2000	0.1300	0.1600
DY	–0.0300	–0.1750	–0.2200
DB	0	0.1550	0.4300
DSD	0.1000	0.0700	0.0700

The model proposed for the long-term bond yield consisted of two parts:

$$C(t) = CR(t) + CM(t), \quad (23)$$

where $CR(t)$ represents the “real” part and $CM(t)$ is an allowance for expected future inflation.

The model for $CR(t)$ is as follows:

$$log\ CR(t) = \ log\ CMU\ +\ CN(t), \qquad (24)$$

where

$$CN(t)\ =\ CA\ \times\ CN(t-1)\ +\ CY\ \times\ YSD\ \times\ YZ(t)\ +\ CSD\ \times\ CZ(t), \quad (25)$$

and

$$CZ(t) \sim N(0,1). \quad (26)$$

Note the dependence of $CN(t)$ on the residual of the current dividend yield. The other parameters are similar to the other models. *CMU* represents the equilibrium level of the real yield; *CA* controls for serial correlation; and *CSD* controls for volatility.

The model for $CM(t)$ is

$$CM(t)\ =\ max\,[CD\ \times\ I(t)\ +\ (1-CD)\ \times\ CM(t-1), CMIN - CR(t)]. \quad (27)$$

A floor of $CMIN$ = 0.005 is employed so that $C(t)$ cannot be negative in a simulation exercise. The relevant parameters are (*CD, CMU, CA, CY, CSD*). The suggested parameter values are given in Table 18.

Table 18
Parameter Values for the Model for Long-Term Yield

Parameters	Wilkie (1986)	Wilkie (1995)	Wilkie et al. (2011)
CD	0.0500	0.0450	0.0450
CMU	0.0350	0.0305	0.0223
CA	0.9100	0.9000	0.9200
CY	0	0.3400	0.3700
CSD	0.1650	0.1850	0.2550

A.1.6 Short-term Bond Yield

Short-term bond yield is indirectly modeled through the log-spread:

$$BD(t) = BMU + BA\ \times\ [BD(t-1) - BMU]\ +\ BSD\ \times\ BZ(t) \quad (28)$$

where

$$BZ(t) \sim N(0,1). \quad (29)$$

Then, the short-term bond yield, $B(t)$, is calculated using the following relationship:

$$BD(t) = \ log\ C(t) - \ log\ B(t) \quad (30)$$

The relevant parameters are *BMU, BA,* and *BSD*, where *BMU* represents the equilibrium level of yield spread, *BA* controls the speed of reversion to this equilibrium, and *BSD* controls the volatility of the spread. The suggested parameter values are given in Table 19.

Table 19
Parameter Values for the Model for Short-term Yield

Parameters	Wilkie (1986)	Wilkie (1995)	Wilkie et al. (2011)
BMU	—	0.2300	0.1700
BA	—	0.7400	0.7300
BSD	—	0.1800	0.3000

A.1.7 Index-Linked Bond Yields

The model for "real" interest rates on index-linked bonds is:

$$\log R(t) = \log RMU + RA \times [\log R(t-1) - \log RMU)] + RBC \times CSD \times CZ(t) + RSD \times RZ(t), \quad (31)$$

where

$$RZ(t) \sim N(0,1). \quad (32)$$

The relevant parameters are *RMU, RA, RBC,* and *RSD.* The presence of long-term bond yield residual represents simultaneous correlation between the residuals. *RMU* represents the equilibrium level of the real bond yield, *RA* controls the speed of reversion to this equilibrium, and *RSD* controls the volatility of the real bond yield. The suggested parameter values are given in Table 20.

Table 20
Parameter Values for the Model for Index-Linked Bond Yields

Parameters	Wilkie (1986)	Wilkie (1995)	Wilkie et al. (2011)
RMU	—	0.0400	0.0300
RA	—	0.5500	0.9500
RBC	—	0.2200	0.0080
RSD	—	0.0500	0.0030

A.2: The Graphical Model

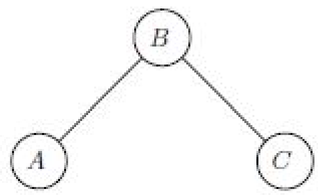
In this section, we provide a brief outline of the ESG developed by Oberoi et al. (2018) using a graphical model approach.

A graph, *G* = (*V, E*), is a structure consisting of a finite set of variables *V* (or vertices or nodes) and a finite set of edges *E* between these variables. The existence of an edge between two variables represents a connection or some form of dependence. The absence of this connection represents conditional independence.

For instance, if we have a set of three variables *V* = {*A,B,C*}, where *A* is connected to *B* and not to *C*, but *B* is connected to *C*, then *A* is connected to *C* via *B*. *A* is then conditionally independent of *C*, given *B*. Such a structure can be graphically represented by drawing circles or solid dots representing variables with lines between them representing edges. The graphs we consider here are called undirected graphs because the edges do not have a direction (which would otherwise be represented by an arrow). Such graphs model association rather than causation. The graphical model described here with three variables, *A*, *B* and *C*, is shown in Figure 13.

Figure 13

An Example of a Graphical Model with Three Variables and Two Edges



Graphical models enable us to represent the covariance structure, with dimension reduction, by effectively capturing conditional independence between pairs of variables and shrinking the relevant bivariate links to zero while allowing for weak correlations to exist in the simulated data.

For the example in Figure 13, the partial correlation matrix would look like this:

$$\begin{pmatrix} 1 & \rho AB & 0 \\ \rho AB & 1 & \rho BC \\ 0 & \rho BC & 1 \end{pmatrix},$$

where $\rho AB \neq 0$ and $\rho BC \neq 0$. So, variables *A* and *C* are independent, given variable *B*. Note that this could still generate nonzero unconditional correlation between *A* and *C*.

A.2.1 Modeling

The aim of a graphical model ESG is to give importance to long-run stable relationships and to generate a distribution of joint scenarios. This takes the approach of estimating the joint distribution of the residuals of individual time series regressions and focuses on the dependence between the residuals. For each variable, a time series model is fitted independently, and then a graphical model is fitted to the time series residuals across variables. For each time series X_t, the following AR(1) time series model formulation is used:

$$\mu_x = E[X_t] \quad (33)$$

$$Z_t = X_t - \mu_x \quad (34)$$

$$Z_t = \beta Z_{t-1} + e_t, \text{ where } e_t \sim N(0, \sigma^2). \quad (35)$$

The parameter estimates from the AR(1) regressions are provided in Table 21. All AR(1) coefficients are statistically significant at the 1% level, and there is no significant residual dependence in the errors.

A.2.2 U.K. Parameters and Structures

For calibration, Oberoi et al. (2018) use the same underlying data as were used for the calibration of the Wilkie model. The variables modeled are price inflation (I_t), salary inflation (J_t), dividend yield (Y_t), dividend growth (K_t) and long-term bond yield (C_t). Table 21 shows the relevant parameter estimates.

Table 21
U.K. Time Series Parameter Estimates

	μ	β	σ
I_t	0.0404	0.6102	0.0387
J_t	0.0528	0.7801	0.0282
Y_t	0.0468	0.6718	0.0085
K_t	0.0527	0.4263	0.0852
C_t	0.0617	0.9674	0.0083

The resulting partial correlation matrix is given in Table 22. Clearly, some of the partial correlations in the matrix are small. The goal is to identify the graph(s) with the minimum number of edges which describe the underlying data adequately.

Table 22
Partial Correlation Table for the United Kingdom

	I_t	J_t	Y_t	K_t	C_t
I_t	1	—	—	—	—
J_t	0.48	1	—	—	—
Y_t	0.16	0.11	1	—	—
K_t	0.18	0.15	-0.06	1	—
C_t	0.20	-0.09	0.37	0.06	1

As there are five variables in the model, there are $2^{10} = 1{,}024$ distinct models possible. Focusing only on those models that are optimally based on certain desirable features, Figure 14 shows the graphical structure of the following optimal models:

Model 1: optimal according to the Bayes information criterion (BIC);

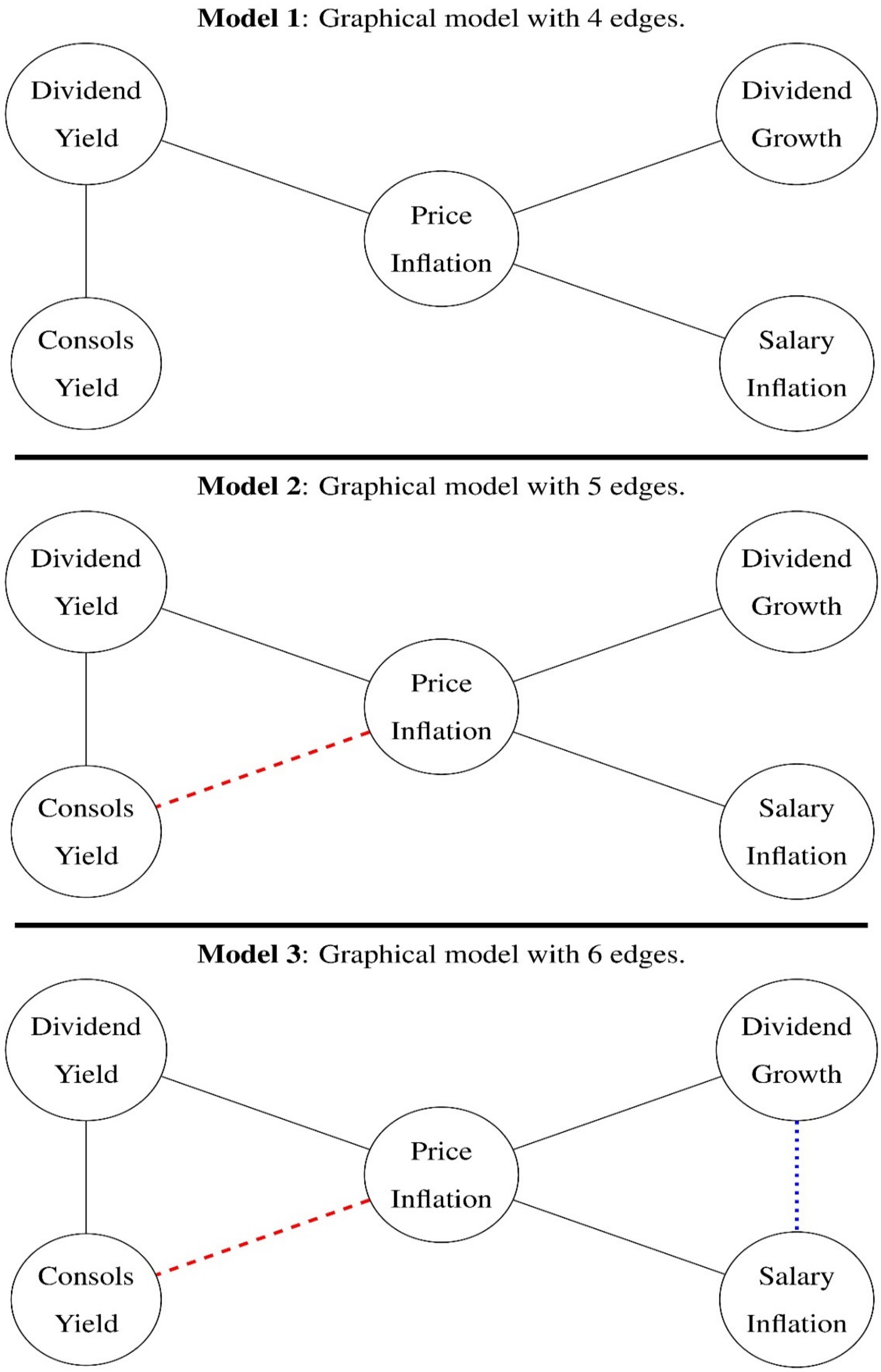
Model 2: optimal according to the Akaike information criterion (AIC);

Model 3: optimal using simultaneous *p*-values at confidence level α = 0.6.

Models 1, 2 and 3 produce qualitatively similar results, so in this report we only show results from Model 3.

Figure 14

Optimal Graphical Models Based on Different Selection Criteria for the United Kingdom



A.2.3 U.S. Parameters and Structures

We conduct a similar exercise with the U.S. data, which comes from two sources. The first is Robert Shiller, who provides online data for the CPI, S&P 500 Index, S&P 500 High Dividend Index and 10-year bond yield. The second source is Emmanuel Saez, who provides online data for average wages in the United States. The data we use extend from 1913 to 2015. Table 23 shows the relevant parameter estimates, and Table 24 shows the partial correlation matrix for the United States.

Table 23
U.S. Time Series Parameter Estimates

	μ	β	σ
I_t	0.0328	0.6211	0.0392
J_t	0.0464	0.4908	0.0643
Y_t	0.0413	0.8293	0.0100
K_t	0.0507	0.2746	0.1084
C_t	0.0489	0.9346	0.0091

Table 24
Partial Correlation Table for the United States

	I_t	J_t	Y_t	K_t	C_t
I_t	1	—	—	—	—
J_t	0.42	1	—	—	—
Y_t	0.20	-0.47	1	—	—
K_t	0.17	0.10	0.28	1	—
C_t	0.19	0.04	0.12	-0.06	1

As we did for the United Kingdom, we use BIC, AIC and simultaneous *p*-values to obtain the optimal graphical structures. Interestingly, all three methods produce the same structure, which is shown in Figure 15.

A.2.4 Scenario Generation

The process of simulating variables from the covariance structure generated by the graphical models involves a stepwise simulation based on the cliques and the edges connecting cliques. For every period, we start by simulating the innovations for one clique at a time, using the edges connecting cliques to build up the complete set of innovations. We then use the AR(1) process along with the innovations to update the value of each variable for the next period. We repeat the process for the next period and those that follow.

A.2.5 Simulations

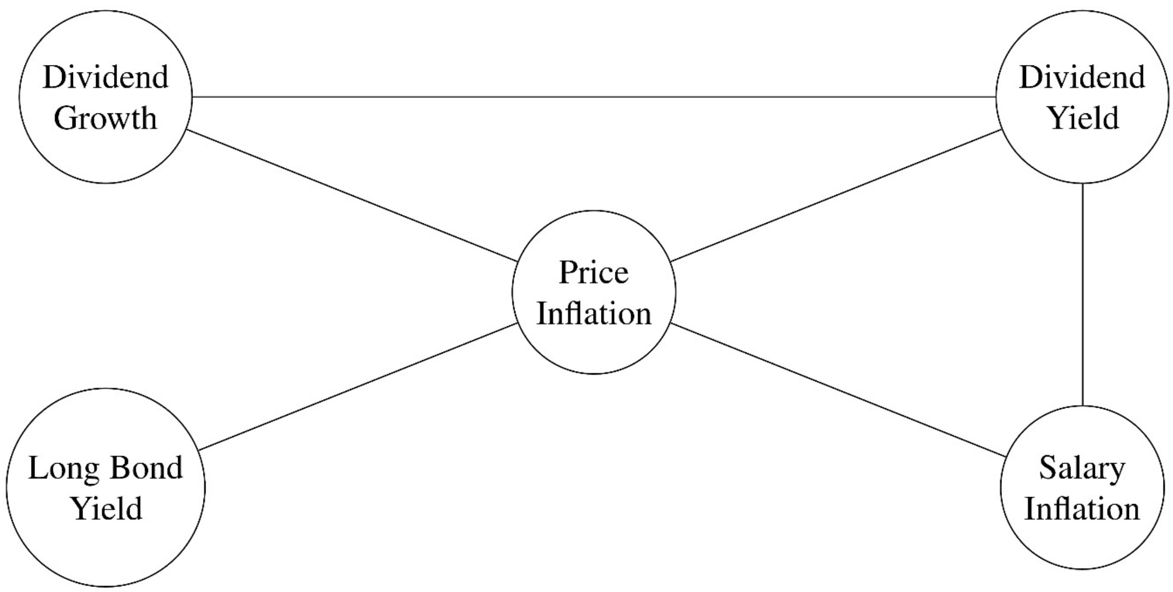
We simulate the same number of paths using the U.K. and U.S. graphical structures (Model 3 for the United Kingdom). We generate simulated values starting from the last data point available, which is 2017 for the United Kingdom and 2015 for the United States. We produce 10,000 paths for the joint set of variables for both countries.

A.2.6 Marginal Distributions

The simulation results can be viewed in terms of the marginal distributions of the variables and also in terms of their joint realizations. We look at "fan charts" of the distributions of the five variables over the length of the simulations. These are shown in Figures 16 and 17. The fan charts offer a useful sense check, as they can help identify potential violations of common-sense economic constraints that one would like to avoid in the simulations. For instance, due to the exceptionally low long-term bond yields in the current environment, we have imposed a constraint that the long-term yield does not fall below 0.05%. Based on the fan charts, the simulations from the graphical models look plausible when compared to the historical data.

Figure 15

Optimal Graphical Models Based on Different Selection Criteria for the United States



A.2.7 Bivariate Heat Maps

We also plot the bivariate heat maps generated by the simulations for each of the graphical models. The pairs we consider are, first, annual stock returns and annual bond returns, and second, annual price inflation and annual salary inflation. We overlay the map with annual observations of the relevant pairs from the historical data available and label the years when the inflation or returns were unusually high or low. These plots are provided in Figures 18 and 19, respectively. We note that for both the United Kingdom and the United States, the graphical model generates the right shape and applies appropriate mass to the relevant areas of the distribution by comparison to historical data.

Figure 16

Fan Plots of Simulations for Price Inflation, Salary Inflation and Dividend Yield

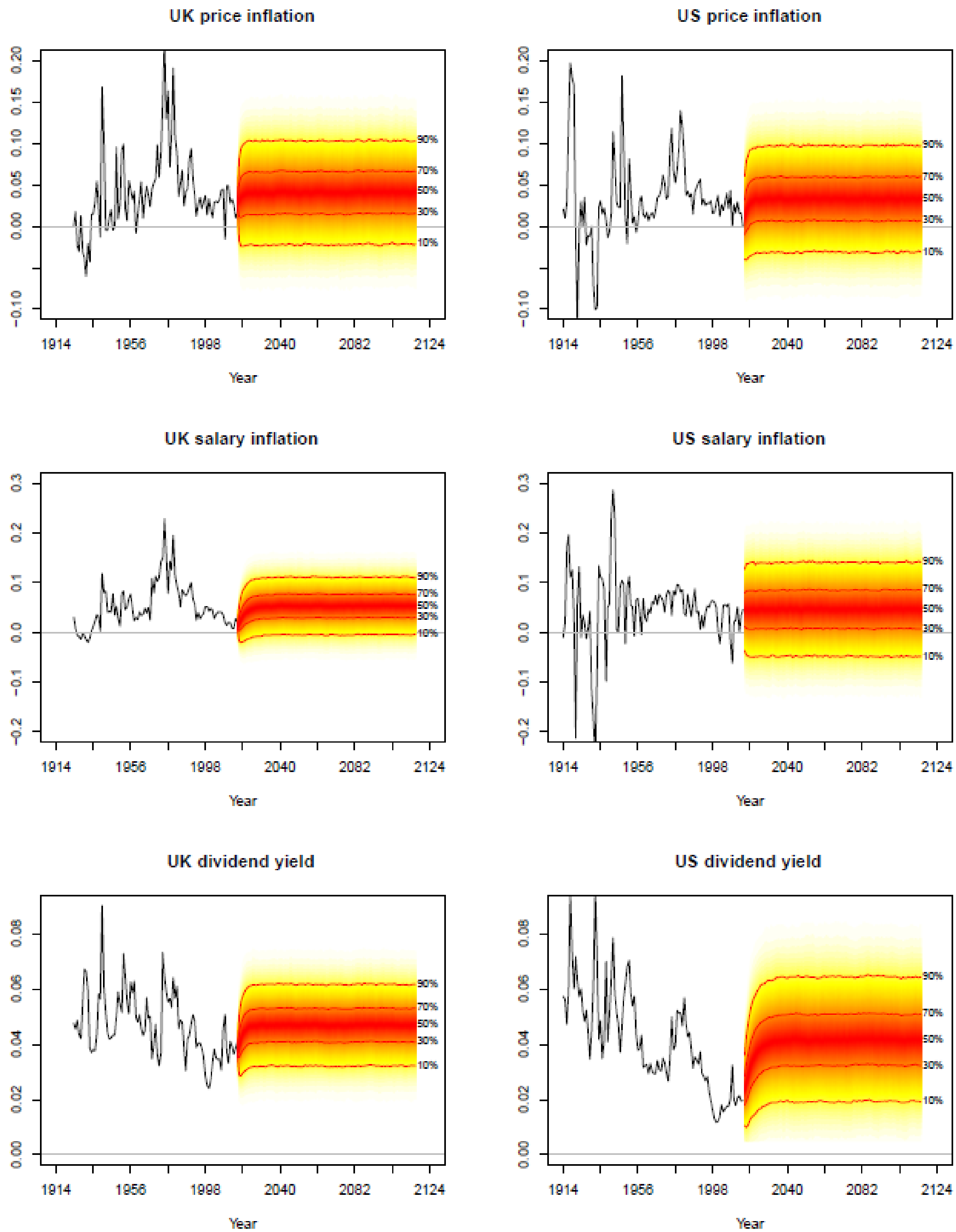


Figure 17

Fan Plots of Simulations for Dividend Growth and Long-term Bond Yield

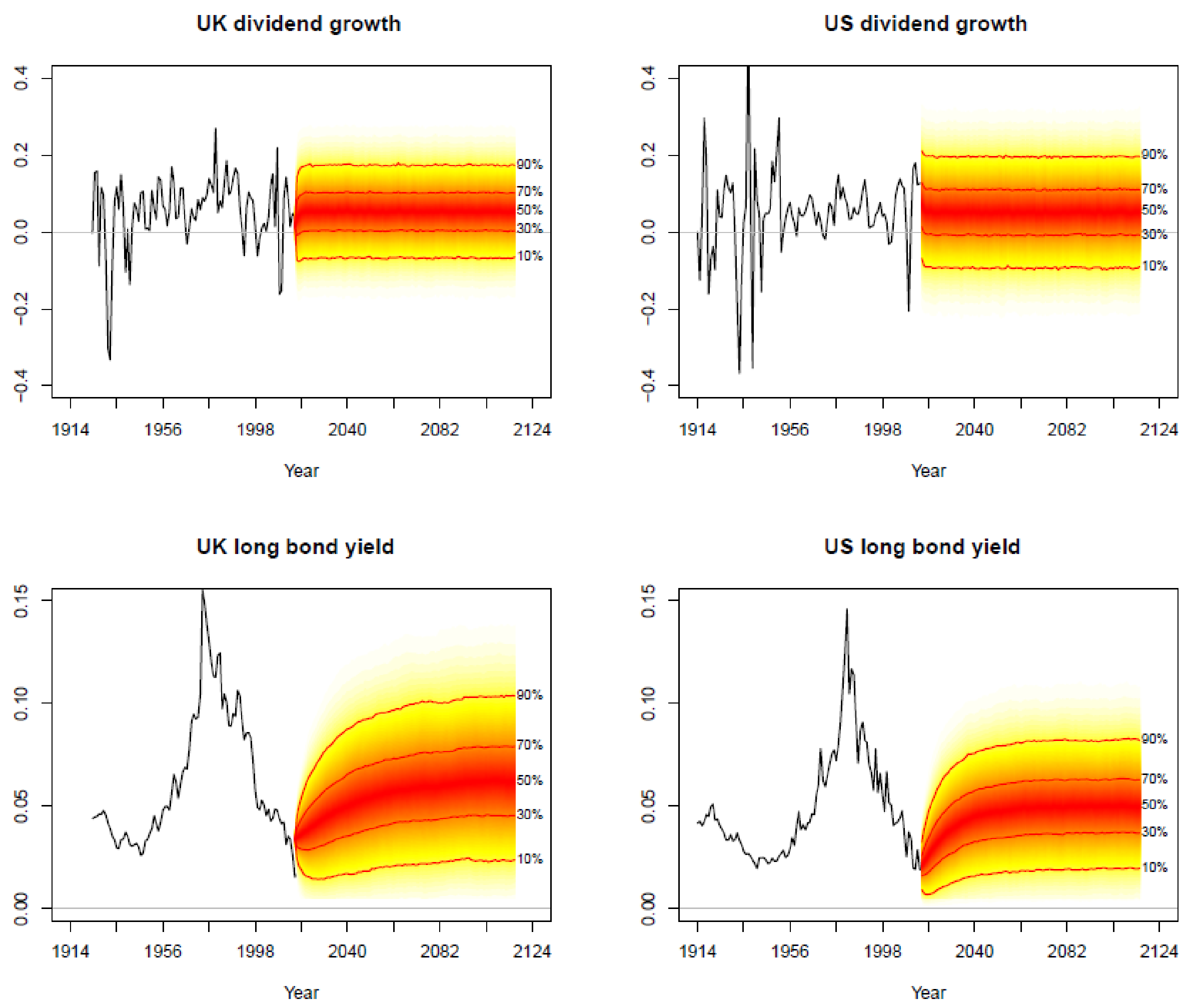


Figure 18
Plots of Simulated Share and Long Bond Return

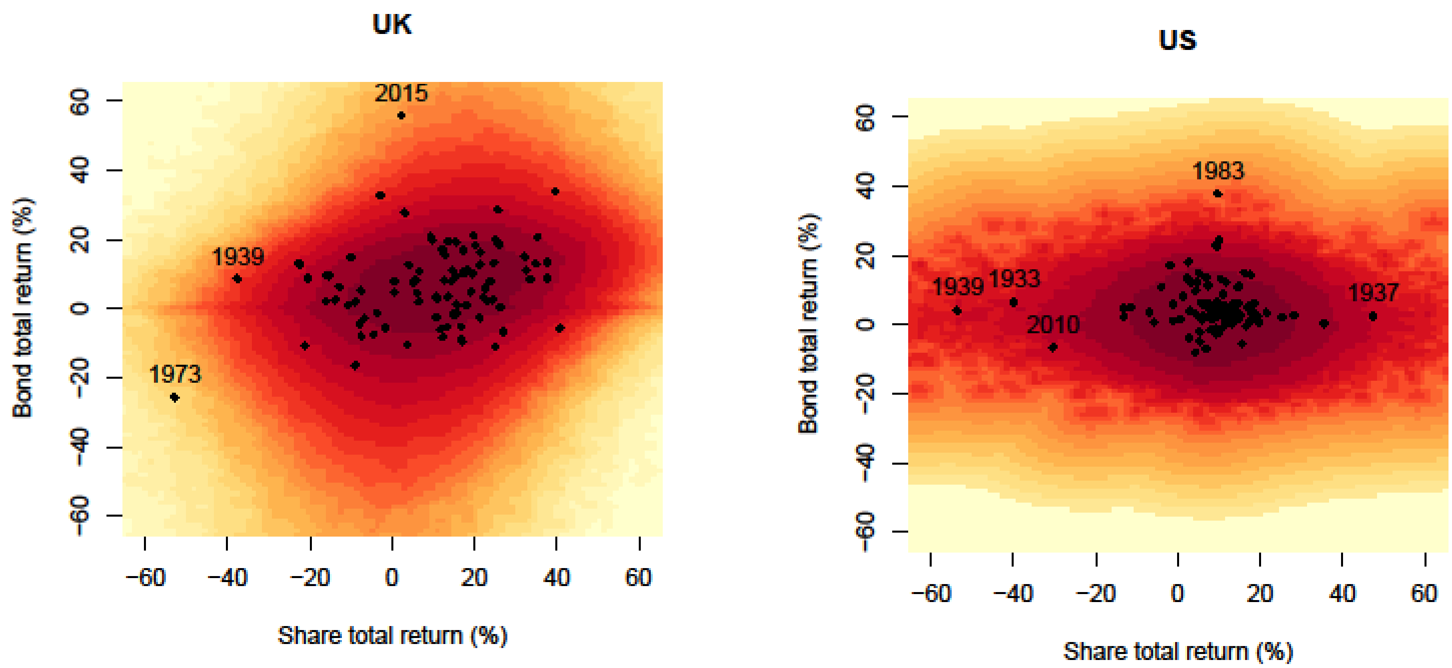
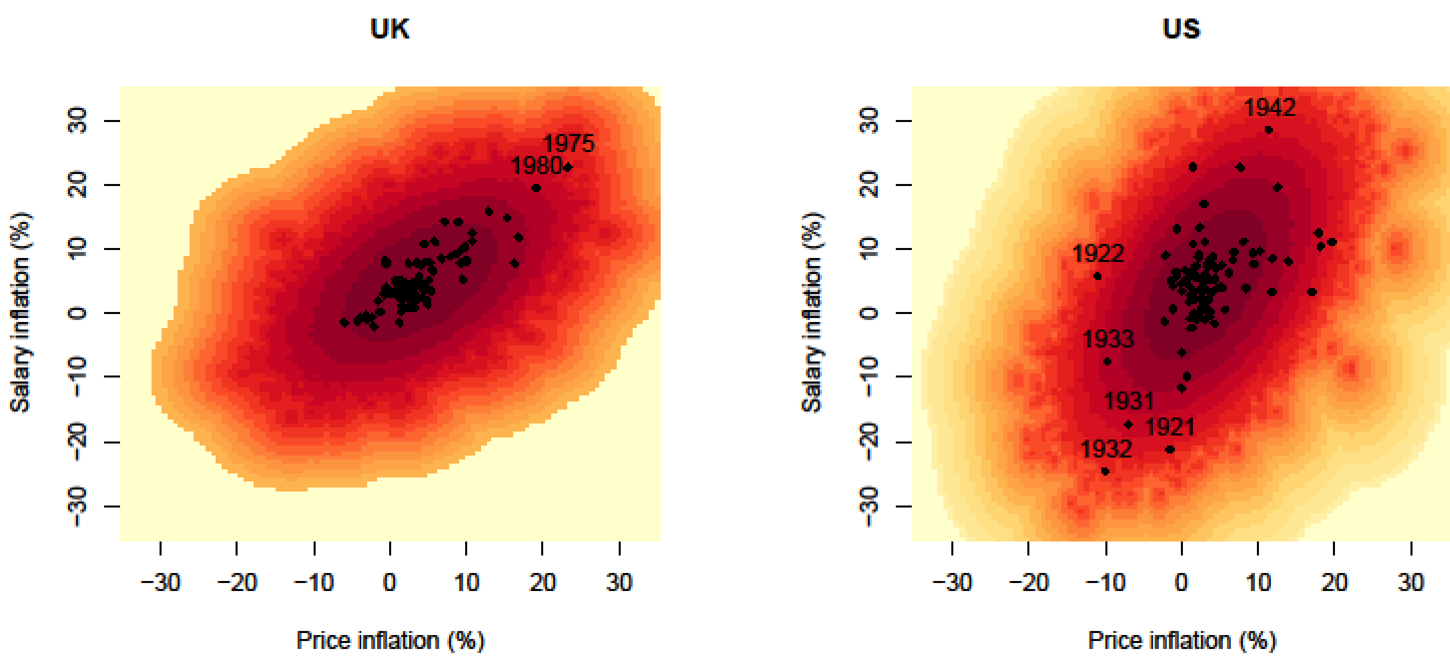


Figure 19
Plots of Simulated Price and Salary Inflation



Appendix B: Mortality Models

B.1 Cairns–Blake–Dowd (CBD) Models

Future projections of the liabilities of a pension plan also depend on the mortality assumptions. In the first instance, we use the assumptions in the latest actuarial valuation. This is primarily to ensure that our model of the USS broadly produces the same value of the actuarial liabilities as reported in the actuarial valuation. The mortality table used for the 2014 USS valuation was S1NA (light) adjusted down by one year for females and unadjusted for males. The Continuous Mortality Investigation (CMI) 2012 projections were used for mortality improvement factors.

Recent advances in actuarial mortality modeling also provide us with options to choose an alternative mortality model suitable for our purpose. Cairns et al. (2009) make a quantitative comparison of eight stochastic mortality models using data from the United Kingdom and the United States. We take a look at seven of those models.[20] Figure 20 shows the U.K. and U.S. crude death rates.

We will use the following notations:

***D*(*t,x*):** The number of deaths at age *x* and year *t*.

***N*(*t,x*):** The number of individuals alive aged *x* at the beginning of year *t*.

***E*(*t,x*):** The total exposure of individuals aged *x* during the calendar year *t*.

***q*(*t,x*):** The probability that an individual aged *x* at time t will die between *t* and *t + 1*.

μ(*t,x*): The force of mortality, defined as the instantaneous death rate, at exact time *t* for individuals aged exactly *x* at time *t*.

It is usually assumed that the force of mortality remains constant over each year of integer age and over each calendar year. Typical modeling approaches assume that the number of deaths at age *x* in year *t* follow one of the following models:

- $D(t,x) \sim \text{Binomial}\ [N(), q(t,x)]$
- $D(t,x) \sim \text{Poisson}\ [E() \times \mu(t,x)]$.

Under the assumption of a stationary population, the Poisson model can also be expressed in terms of $q(t,x)$, using the approximation $q(t,x) \approx 1 - exp[\mu(t,x)]$. Henceforth, for brevity, we will refer to $\mu(t,x)$ as the mortality parameter and the Poisson model for number of deaths.

Typical models used for fitting $\mu(t,x)$ take the form of an additive or a multiplicative (or combination) model of the following functions:

$\beta(x)$: capturing age-related effects;

$\kappa(t)$: capturing period-related effects; and

[20] Note that we did not include model M4 for our comparisons because it is very different from models M1 to M3 and M5 to M8. All of those models share the same underlying assumption that the age, period and cohort effects are qualitatively different in nature. In contrast, model M4 uses B-splines and P-splines to fit the mortality surface.

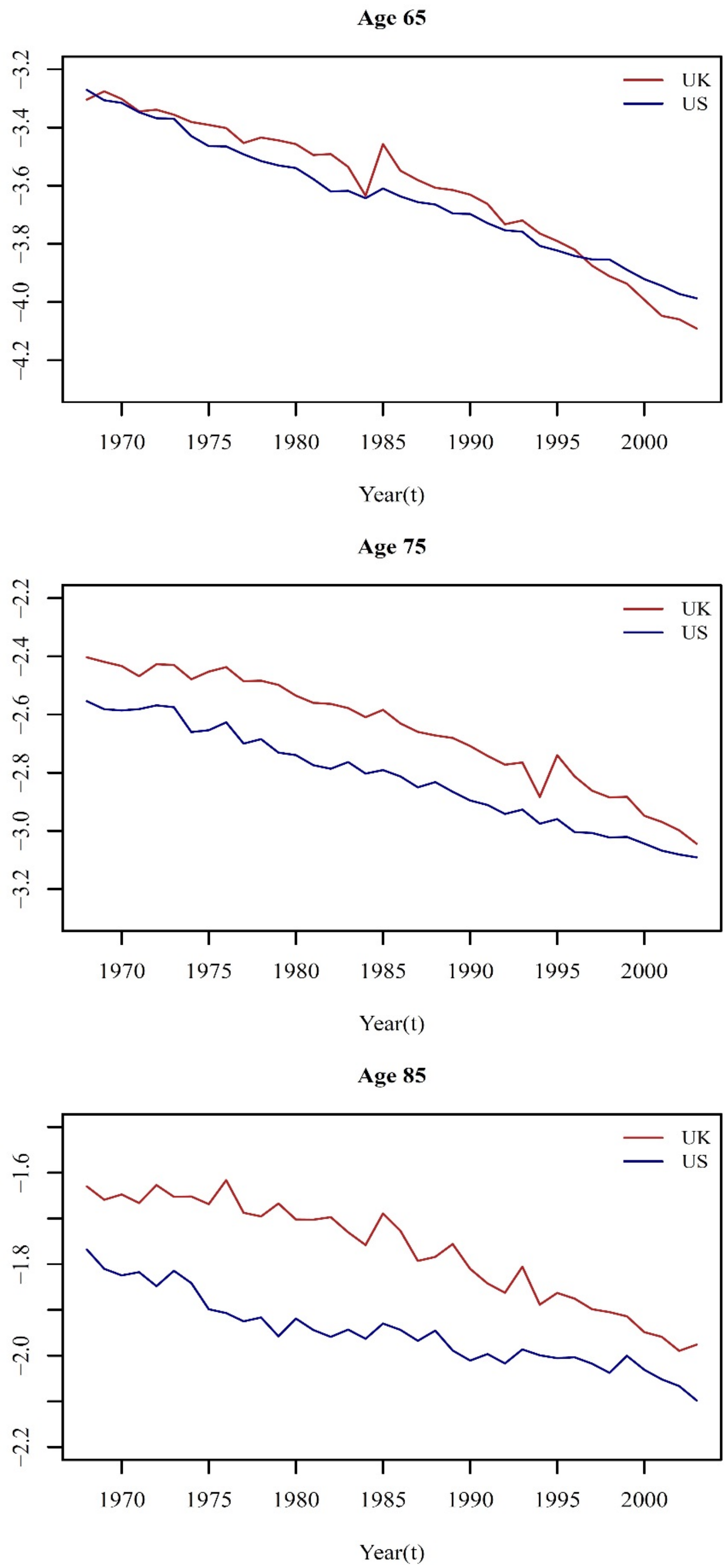
$\gamma(t - x)$: capturing cohort-related effects (note that $t - x$ gives the year of birth).

A simple example of one such model is model M3 in Cairns et al. (2009):

$$\log \mu(t,x) = \beta(x) + \kappa(t) + \gamma(t-x). \quad (36)$$

Figure 20

Log of U.K. and U.S. Crude Death Rates for Males Aged 65, 75 and 85



Historical data on deaths and exposures can then be used to estimate $\mu(t, x)$ for the period and ages relevant to the data. For example, in Cairns et al. (2009), data for ages 60 to 89 and years 1961 to 2004 were used to obtain estimates $\hat{\mu}(t, x)$ for t = 1961, 1962, …, 2004 and x = 60, 61, …, 89 using a maximum likelihood approach.

However, projecting pension plan liabilities forward in time also requires future projections of $\mu(t,x)$. Typically, this involves projecting the time series $\kappa(t)$ and $\gamma(t - x)$ forward. Cairns et al. (2011) suggest possible approaches to project mortality parameters forward based on the historical estimates of these parameters.

Whichever model we choose to implement, pension plans are exposed to two types of risks, specific risk and systematic risk, which arise from the actual mortality experience of the plan members being different from the expected.

Given the values of parameters $\mu(t,x)$, variation in the actual mortality experience is referred to as the specific risk. In other words, if we assume that $\mu(t,x)$ is known, then given the relevant exposures to risk, the number of deaths is a random variable. For example, if for a certain age and time the exposure to risk is 10,000 and the probability of death is 0.01, then the number of deaths can be …, 98, 99, 100, 101, 102, … with certain probabilities (with a mean of 100). This is specific risk. For a large pension plan like USS with 400,000 plan members, specific risk does not pose a significant threat, as it can be diversified away through pooling.

Systematic risk arises from the uncertainty surrounding the estimate of the underlying parameters $\mu(t,x)$. This is the uncertainty involved in projecting the time series $\kappa(t)$ and $\gamma(t-x)$ forward. For example, if the mortality rates improve faster than expected, then future $\mu(t,x)$ will be lower, which in turn will result in lower deaths. This risk cannot be diversified away and thus poses a bigger threat. So ideally the uncertainty, or randomness, in the projections of $\mu(t,x)$ needs to be recognized and incorporated in a stochastic mortality model.

Table 25
Mortality Models

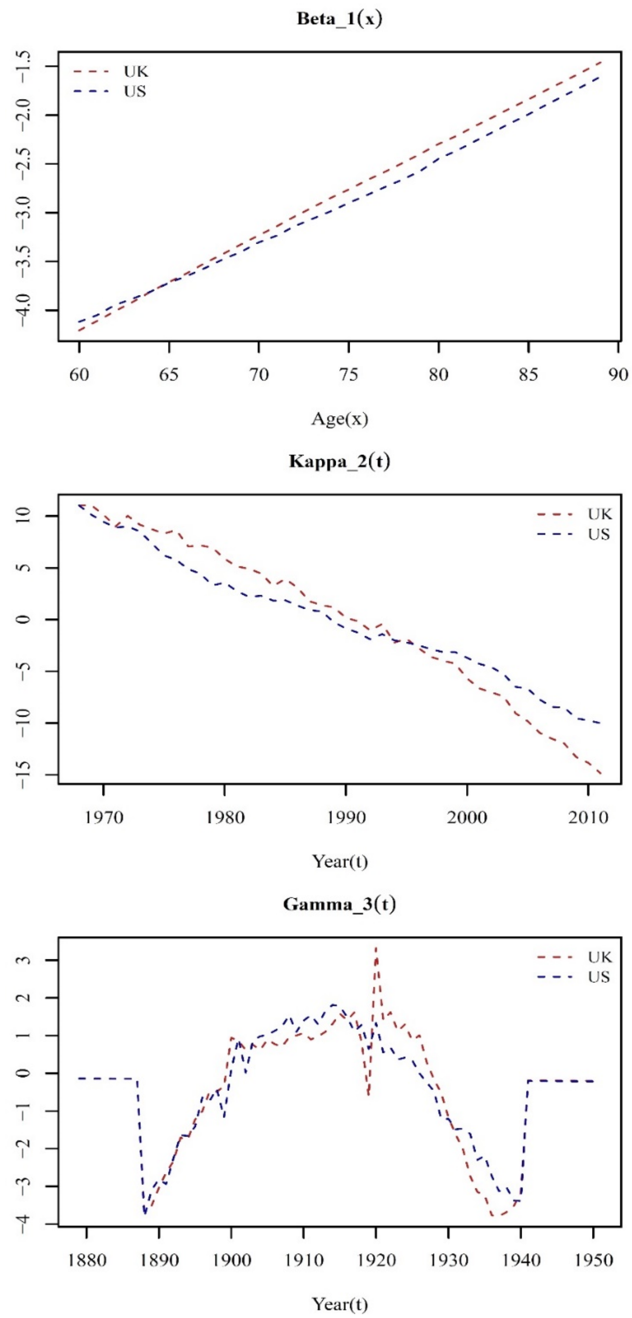
Model	Formula
M1	$\log m(t,x) = \beta_x^{(1)} + \beta_x^{(2)} k_t^{(2)}$
M2	$\log m(t,x) = \beta_x^{(1)} + \beta_x^{(2)} k_t^{(2)} + \beta_x^{(3)} \gamma_{(t-x)}^{(3)}$
M3	$\log m(t,x) = \beta_x^{(1)} + k_t^{(2)} + \gamma_{(t-x)}^{(3)}$
M5	$\operatorname{logit} q(t,x) = k_t^{(1)} + k_t^{(2)} (x - \bar{x})$
M6	$\operatorname{logit} q(t,x) = k_t^{(1)} + k_t^{(2)} (x - \bar{x}) + \gamma_{(t-x)}^{(3)}$
M7	$\operatorname{logit} q(t,x) = k_t^{(1)} + k_t^{(2)} (x - \bar{x}) + k_t^{(3)} [(x - \bar{x})^2 - \sigma^2] + \gamma_{(t-x)}^{(4)}$
M8	$\operatorname{logit} q(t,x) = k_t^{(1)} + k_t^{(2)} (x - \bar{x}) + \gamma_{(t-x)}^{(3)} (x_c - \bar{x})$

Table 25 shows seven of the eight stochastic models presented by Cairns et al. (2009). The model assumes that there is smoothness in the underlying mortality surface in the period effects as well as in the age and cohort effects.

The parameters are then estimated by maximum likelihood. As an example, we provide the parameter estimates for model M3, which are the easiest to explain. To fit the model, we use data from the HMD from 1968 to 2014 for males aged 60 to 89.[21]

Figure 21

Parameter Estimates for Model M3



21 This age range was selected because mortality rates for those under age 60 are close to zero and show little variation. Mortality rates for those over age 89 are based on very small exposures.

The parameter $\beta_x^{(1)}$ captures the age effect on mortality. As expected, as the age increases, the mortality rate (and hence $\beta_x^{(1)}$) increases.

The parameter $\kappa_t^{(2)}$ captures the period effect on mortality. Note that for both the United Kingdom and the United States the $\kappa_t^{(2)}$s become smaller with time, implying an improvement in mortality rates over time.

Finally, $\gamma_{t-x}^{(3)}$ captures the cohort effect on mortality. Unlike for the age or period effect, there is no specific trend for the cohort effect.

To compare the mortality models quantitatively, Cairns et al. (2009) use the BIC, which provides a mechanism for balancing the quality of fit and parsimony of the model. It also allows us to compare models that are not nested. Table 26 shows the BIC using the U.K. and U.S. fitted data. From the table, we see that model M7 provides a good fit for both U.K. and U.S. data, justifying its use for projecting future mortality rates for our research.

Table 26
BIC Rank for the Different Mortality Models Using Males

Model	U.K. (rank)	U.S. (rank)
M1	–10,925 (5)	–17,362 (5)
M2	–8,633 (4)	**–11,228 (1)**
M3	–14,153 (7)	–28,115 (6)
M5	–11,876 (6)	–30,134 (7)
M6	–8,607 (3)	–13,459 (4)
M7	**–8,488 (1)**	–12,781 (2)
M8	–8,503 (2)	–13,161 (3)

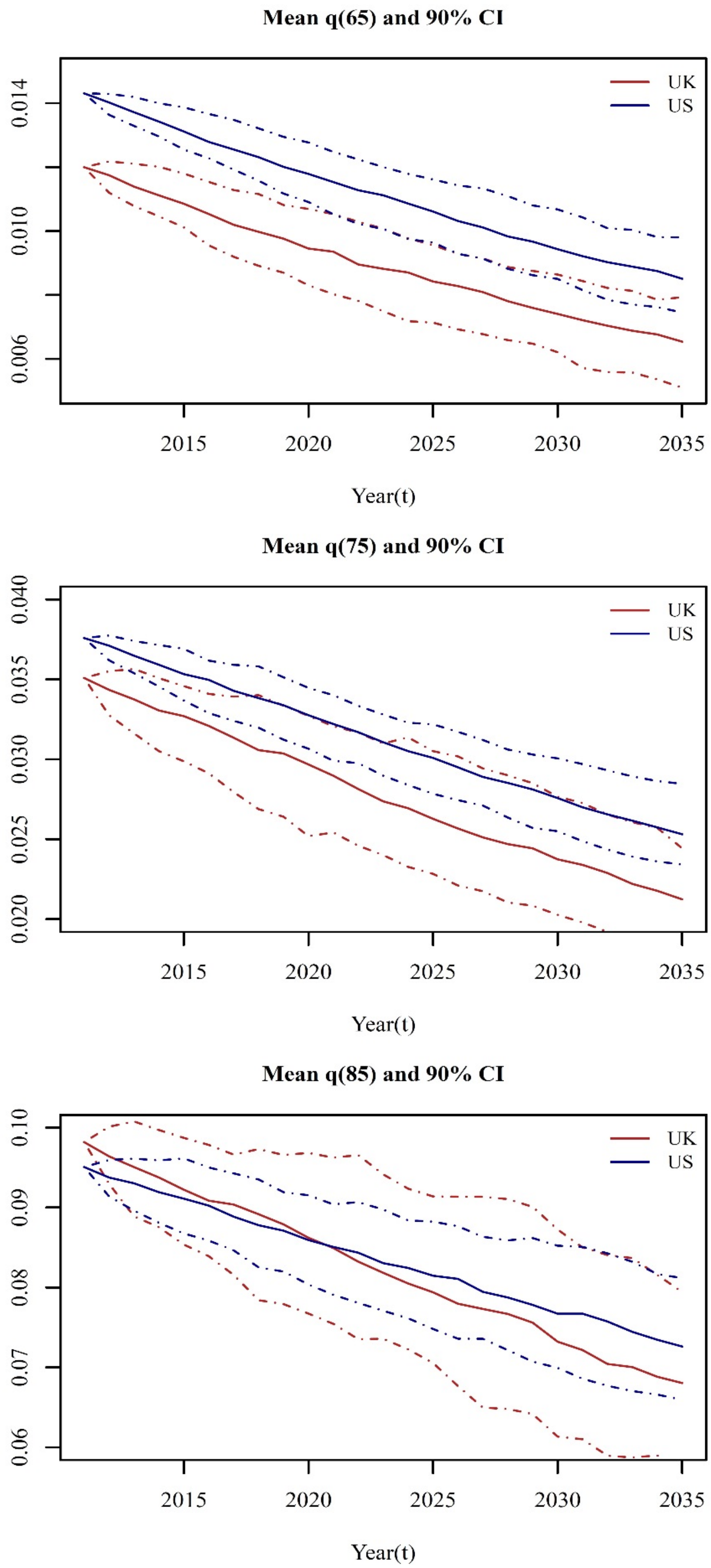
Using the fitted parameters, we project mortality forward. We show the simulated mortality rates under model M7 for U.K. and U.S. males aged 65, 75 and 85, together with the 90% confidence interval.

From Figure 22, we make the following observations:

- The mortality rates increase with age, showing that the age effect is captured in the simulations.
- The mortality rate goes down with time, showing that the period effect is also captured in the simulations.
- The longer the time horizon, the wider the fan charts. This shows the greater uncertainty when simulating over longer horizons.

Figure 22

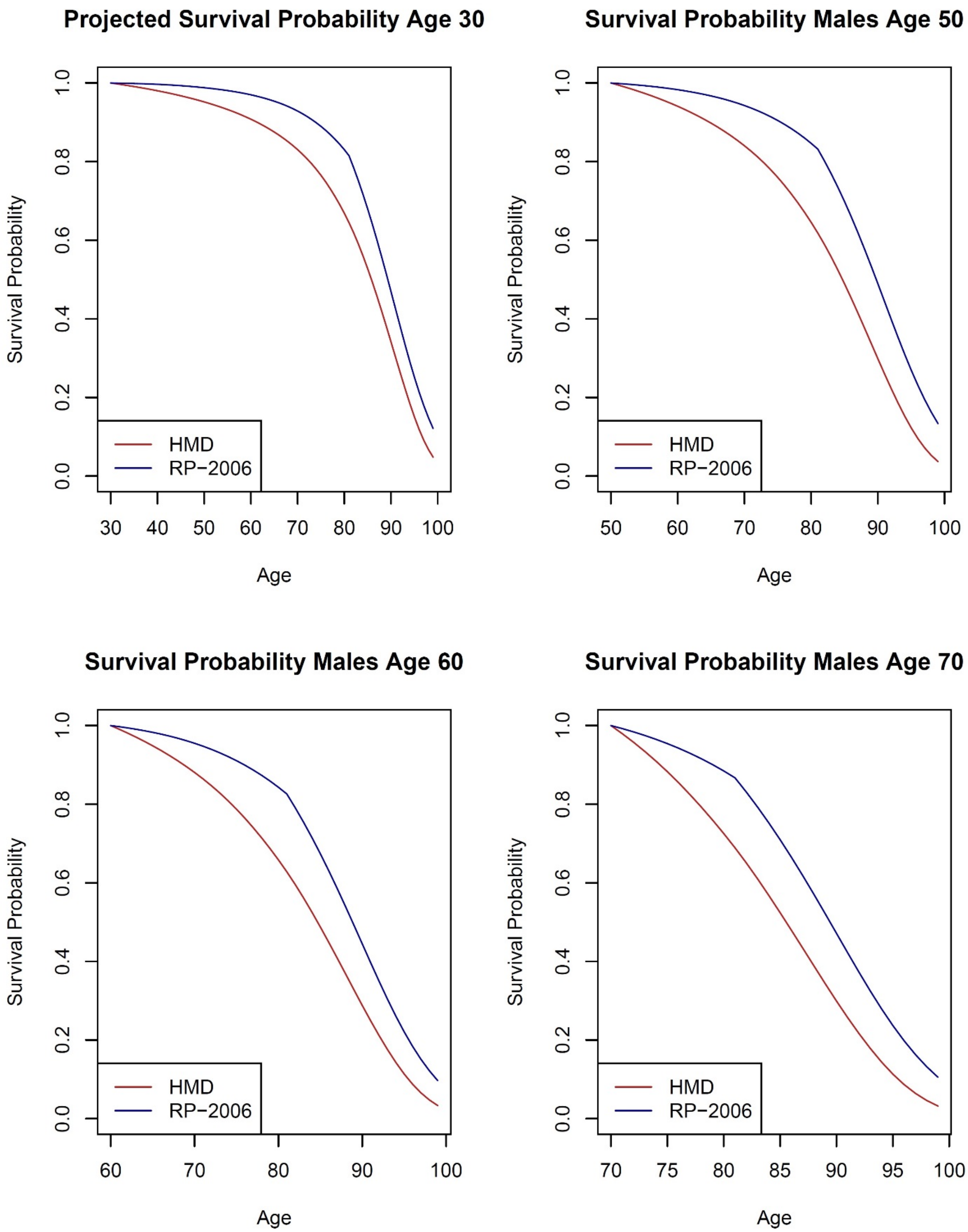
Simulated Mortality Rates from Model M7



B.2 RP-2006 Mortality Table and MP-2018 Projection Table

The RP-2006 is a deterministic mortality table, and MP-2018 comprises mortality projection tables used by some pension plans in the United States. We use RP-2006 and MP-2018 as a sensitivity test in our stylized U.S. plan analysis (see Section 7.5). Figure 23 compares the projected survival rates using the central projection of model M7 calibrated to U.S. HMD data and the RP-2006 alongside the MP-2018 projection table. The survival rates using the RP-2006 table are higher compared to the HMD. This is expected because the HMD is based on total population data, while the RP-2006 is based on pensioners' data.

Figure 23
Projected Survival Rates from Model M7 and RP-2006

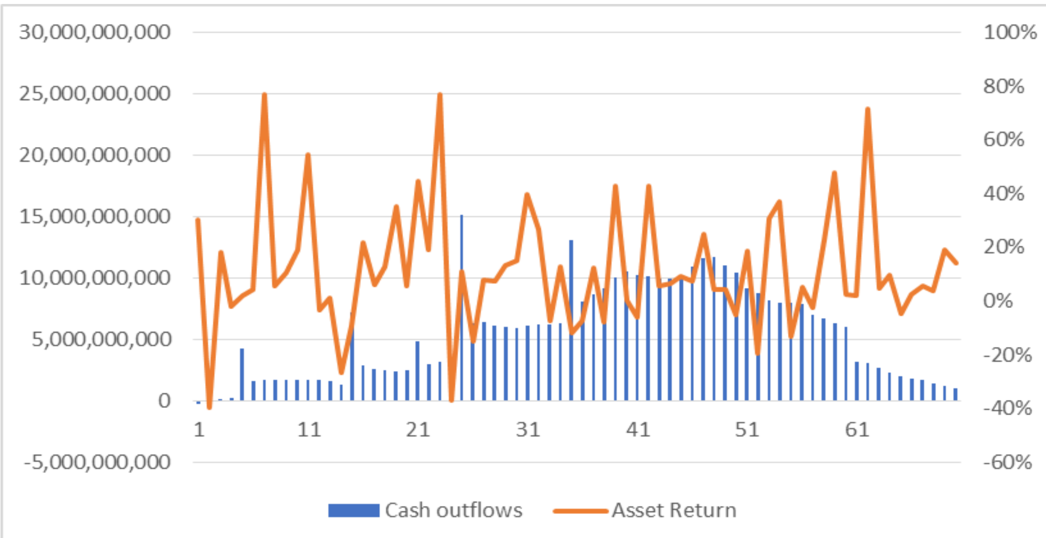


Appendix C: Simulation Example

We show two simulations of the cash flows and asset returns of the United Kingdom plan. Figure 24 shows the cash flows and asset returns that correspond to the 50th percentile of V_0. This is an example of one simulation.[22] The cash flows generated are based on the model points and benefit structures described in Sections 4.1 and 4.2, respectively, together with the mortality rates generated by stochastic mortality model M7. The asset returns are generated by the stochastic economic model, the graphical model assuming an asset allocation of 70% equity and 30% bonds. We then repeat this several times, say 10,000 or 100,000, to obtain a distribution of V_0.

Figure 24

Cash Flows and Asset Returns Corresponding to the 50th Percentile of V_0 for the U.K. plan



We note a peak in the cash flows at times t = 5, 15, 25 and 35. These correspond to the times when ages 60, 50, 40 and 30, respectively (i.e., the age of the model points used for active members), retire and receive a lump sum alongside their pension. The model is annualized, which means that pension payments and returns from assets are assumed to occur once every year. This is a shortcoming of the model, as in reality we expect pension payments to be made every month and returns from assets to be almost continuous.

[22] Note that the simulation generates the median level of V_0. However, it does not represent the median level of any of the other variables. Note that the return stream is highly variable.

Figure 25

Cash Flows and Asset Returns Corresponding to the 0.5th Percentile of V_0 of the U.K. plan

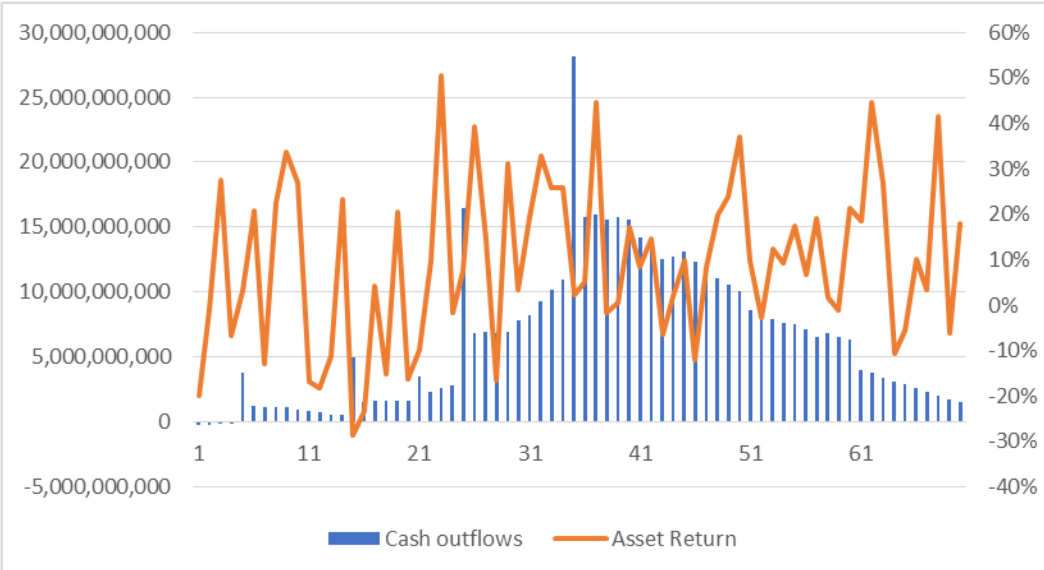


Figure 25 shows the cash flows and asset returns that correspond to the 0.5th percentile of the V_0. Note that in the long run, the asset returns for both simulations are quite similar and fluctuate around 10% per annum. However, in the early years, the returns from the 50th percentile are higher compared to the returns from the 0.5th percentile. This shows that returns in the early years have a big impact on the distribution of V_0.

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