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Trends in Canadian Mortality By Pension Level: Evidence From the CPP and QPP

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Caveat and Disclaimer

CONTENTS

Trends in Canadian Mortality By Pension Level: Evidence From the CPP and QPP

This paper looks at the mortality of Canadian pensioners subdivided by pension level, using data from the Canada Pension Plan (CPP) and Québec Pension Plan (QPP). Pension level is confirmed as giving rise to significant levels of mortality inequality at age 65, with a declining inequality gap as cohorts get older. We also find that levels of inequality have increased slightly over time, and that the QPP pensioners exhibit greater levels of inequality than the CPP pensioners. Additionally, we find strong, but indirect, evidence amongst the lowest pension groups for a healthy-immigrant effect.

We fit a range of multi-population stochastic mortality models to the CPP and QPP data and find that the Common Age Effect Model best satisfies a range of quantitative and qualitative criteria. The model allows us to distill further detail in the underlying mortality data and provide a coherent basis for forecasting mortality and assessing uncertainty in these forecasts.

There is no evidence to suggest that the recent slowdown in Canadian mortality improvements is more pronounced at one or other end of the socio-economic spectrum.

Keywords: Canadian pensioner mortality; CPP; QPP; pension level; healthy-immigrant effect; multi-population stochastic mortality model.

1. Introduction

A significant element of the work of life and pensions actuaries is the assessment of the historical and future mortality of a portfolio of lives. This paper discusses Canadian mortality, with an emphasis on socio-economic differences in the level of mortality and mortality improvement rates. The core objectives of this paper are:

- to establish
  - how wide the differences are in mortality between different socio-economic groups
  - how much these differences vary with age
  - whether these mortality inequalities have increased over time
- to identify potential reasons for the patterns that we observe in the data
- to provide actuaries with the opportunity to compare their own mortality data against benchmark Canadian sub-populations that share similar socio-economic characteristics
- to analyse the data within a model-based framework that can be used in future work on the projection of future levels of mortality and assess how much uncertainty there is around central forecasts.

This paper, therefore, focuses primarily on discussion and interpretation of historical mortality improvements, with a second paper planned on selection of a suitable stochastic model for forecasting.

Our discussion of socio-economic differences in the Canadian population is based on data provided by the Canada Pension Plan (CPP) and the Québec Pension Plan (QPP), with data grouped by pension level, and builds on earlier work by Adam (2012a, 2012b, 2016). However, before we look at that detailed data (Sections 3–8) we spend some time looking at national-level mortality in Canada and other countries (Sections 1–2) to consider the wider international context.

We begin with historical death rates in Canada, illustrated in Figures 1 and 2, using data from overlapping sources.

Figure 1 shows crude death rates for males and females using data from the Human Mortality Database (HMD, 2018) over the period 1921–2011. The y-axes for each of the eight sub-plots are, of course, different, but the ratio from top to bottom is the same in each case ($16\times$ from top to bottom), so that a flatter plot indicates lower rates of mortality improvements. Thus, females aged 25 have seen the greatest percentage improvements in mortality over the last 100 years (mostly between 1920 and 1950), while males and females aged 85 have seen the smallest improvements. As a group of plots, Figure 1 reveals:

- different rates of improvement over different time periods
- different rates of improvement at different ages

- different patterns of improvement for males and females of the same age
- volatility on a year-to-year basis around a varying underlying trend.

These observations point to uncertainty in both the short and long term when forecasting future mortality rates, as well as rates of improvement at different ages – points that we discuss further in Section 6.

Using data from Statistics Canada, Figure 2 shows mortality experience from 1981–2015. These plots allow us to focus in more detail on recent improvements. Similar comments can be made about differing patterns, but we can also identify a potential slowdown in mortality improvements from 2010 or 2011 at ages 45, 65 and 85.

The remainder of the paper is structured as follows. In Section 2 we discuss how to reduce sampling variation (noise) in the crude death rates and compare Canada with five other developed countries. Section 3 begins the focus on sub-population mortality. It begins with some brief background based on socio-economic data from England and Denmark, before moving on to a detailed analysis of the CPP and QPP data subdivided by pension level with a focus on age-standardised mortality rates (ASMRs). In order to understand better the CPP/QPP ASMR plots, we then look in more detail in Section 4 at the numbers of individuals attaining different levels of pension and how this has changed over time. Additionally, we consider how the cohort-dependent proportions in each group might have an impact on ASMRs. Section 5 then quantifies how migration in middle age might have an impact on the the composition of specific groups by pension level, and we introduce the *healthy-immigrant effect*, a factor that plays an important role in the subsequent modelling sections. Stochastic models are considered in Sections 6 and 7. Section 6 discusses why it might be important to use stochastic mortality models. Section 7 considers a range of potential multi-population stochastic mortality models before settling on a specific model (a special case of the Common Age Effect (CAE) Model of Kleinow, 2015) that best meets a list of desirable criteria. A detailed discussion of the modelling results follows with links to the earlier empirical sections of the paper. We then consider clustering (aggregating small groups with similar levels of mortality) in Section 8 as a further means of reducing noise in the underlying data. Noise reduction then allows us to make additional insights about specific sub-populations. Section 9 concludes.

Figure 1: Historical death rates for Canadian males and females aged 25, 45, 65 and 85 from 1921–2011. Y-axis: ratio of maximum to minimum is 16. Source: Human Mortality Database (HMD, 2018).

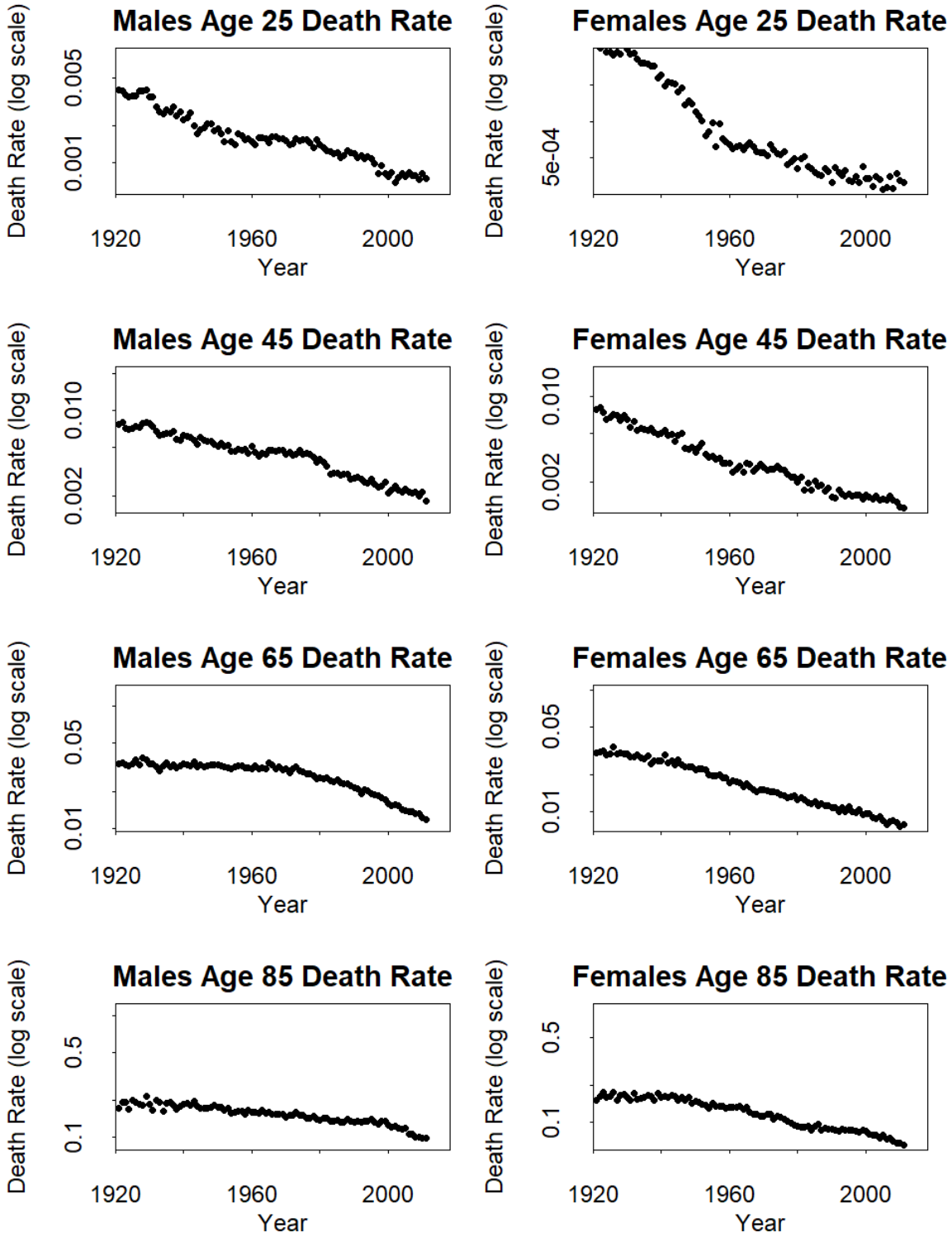
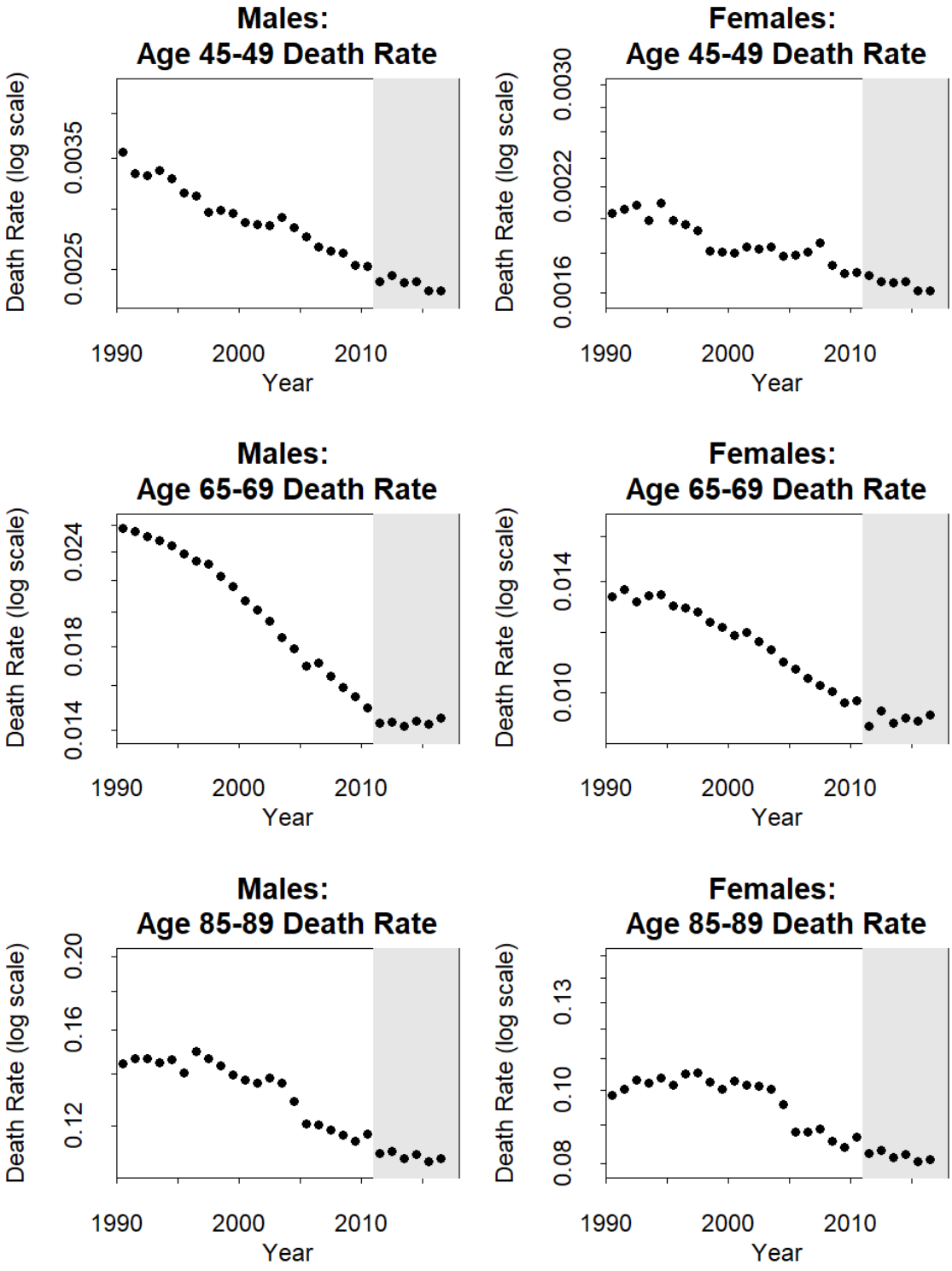


Figure 2: Historical death rates (mid-year to mid-year) for Canadian males and females in age groups 45–49, 65–69 and 85–89 from 1990–2015. Source: Statistics Canada, Tables 17-10-0005-01 and 17-10-0006-01.



2. Improving the Signal-to-Noise Ratio

Estimation of death rates typically makes use of observed numbers of deaths and exposures or population. Death counts, in particular, are inherently random, leading to sampling variation in estimated death rates, such as crude age-specific death rates (e.g. Figure 2). For example, let $D(x)$ be the observed number of deaths at age x last birthday during a year, and $E(x)$ be the central exposed to risk (which we shall refer to hereafter in an abbreviated form as the *exposure*). Then, under the Poisson assumption (see, for example, Macdonald et al., 2018), the crude age-specific death rate is $\hat{m}(x) = D(x)/E(x)$, with $S.D.(\hat{m}(x)) = \sqrt{m(x)/E(x)}$ giving a measure for the impact of sampling variation, or *noise*, in the data.

For smaller populations and especially some of the sub-populations considered in this paper, the level of noise can make it difficult to detect specific effects and trends in *underlying* mortality rates (the *signal*). To mitigate this, we use a variety of measures and methods that help to improve the signal-to-noise ratio:

- empirical measures (specifically ASMRs)
- model-based methods
- clustering.

ASMRs are outlined and illustrated below, while model-based methods and clustering are covered in Sections 7 and 8.

2.1. Age-standardised mortality rates

The ASMR is a year-t-specific, weighted average of the crude death rates that can be defined over a defined age range $x_0, \dots, x_1$

$$ASMR(t) = \frac{\sum_{x_0}^{x_1} \hat{m}(t,x)\tilde{E}(x)}{\sum_{x_0}^{x_1} \tilde{E}(x)}$$

where $\hat{m}(t,x)$ is the crude age-specific death rate in year t at age x, and $\tilde{E}(x)$ represents the "standard" exposure at age x (throughout this paper, we use the European Standard Population,[1] 2013). In some settings (e.g. demography) this might cover all ages ($x_0 = 0$ to $x_1 = 130$), but for many actuarial problems, it is better to restrict (x_0, x_1) to the age range of interest (e.g. 65–89).

In this paper the ASMR has three purposes: comparison of the level of mortality in different populations (Canada versus other national populations; and Canadian sub-populations), comparison of time trends in different populations, and improvement of the signal-to-noise ratio (especially over wider age ranges (x_0, x_1)).

[1] The use of alternative standard populations might push the ASMRs up or down, but the patterns of improvements that we observe and the relationships between different populations would be largely unaltered.

In Figure 3 we plot ASMRs for Canadian males and females for three distinct age bands over the period 2000–2015, using data from Statistics Canada. Each is plotted on a log scale and, although absolute values are different for each age band, to aid comparison, the maximum-to-minimum ratio on the y-axis is 1.6×. Proportionately, therefore, percentage improvements have been biggest in the 60–69 age band (steepest) and lowest in the 40–49 age band (least steep). We also see some genuine volatility from year to year in underlying mortality rates (e.g. low mortality in 2005 at higher ages, and high mortality in 2014 at all ages). Lastly we can detect graphically a slowdown in mortality improvements since 2010/2011.

Key questions for actuaries and other stakeholders are:

- At what pace are mortality rates likely to improve in the future?
- How might these improvement rates vary with age?
- Is the recent slowdown in mortality improvements in Canada genuine?
- Is the slowdown permanent or short-term?
- Is the slowdown widespread or specific to certain socio-economic groups?

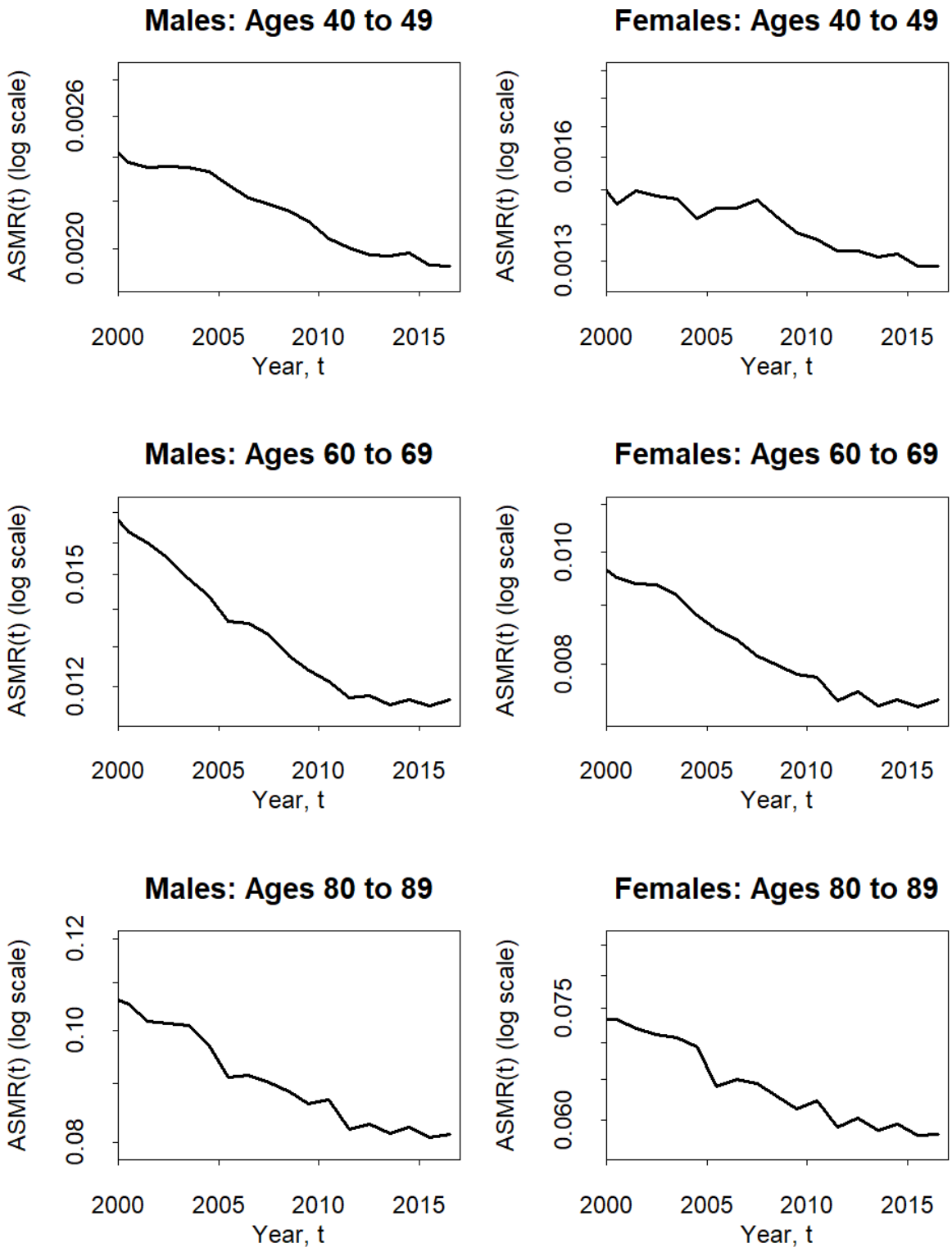
Here we will look at the question from two quite different perspectives:

- zooming out and compare Canadian mortality with that of other developed countries;
- zooming in and investigate mortality improvements amongst different socio-economic groups.

We consider the international perspective here, and the socio-economic perspective in the following sections. In Figures 4 and 5, we plot ASMRs since 1981 over two age bands (middle ages and higher ages) for six countries that are representative of developed countries worldwide.[2]

[2] Canadian death rates are for 1 July to 30 June using data from Statistics Canada and crude data are available for five-year age bands only. Death rates for other countries are for 1 January to 31 December using data from the Human Mortality Database (HMD, 2018). HMD data for Canada currently stops at 2011.

Figure 3: ASMRs for three age bands for Canadian males and females: 2000–2016. Source data: Statistics Canada.



A number of observations can be made:

- Males:
  - At both the middle and higher ages there is quite a wide spread of mortality rates in the six countries, with as much as a 90% difference in the middle ages and 35% at higher ages.
  - All countries exhibit a significant lowering of mortality over the 30+ years.
  - But the pattern of improvement is different in different countries.
- Females:
  - At both the middle and higher ages there is quite a wide spread of mortality rates in the six countries, with as much as a 120% difference in the middle ages and 75% at higher ages.
  - All countries exhibit a significant lowering of mortality over the 30+ years.
  - Japan, in particular, has experienced much faster improvements than other countries above age 65.

Out of the six countries, three exhibit a slowdown (Canada, the US, and England and Wales). But three have not experienced a recent slowdown (Japan, Sweden and Denmark). Denmark is in a catch-up phase, having fallen behind in the 1980s and early 1990s. But Japan and Sweden already have among the lowest mortality over a wide range of ages, alongside Canada at the higher ages. Both Japan and Sweden have seen fairly stable rates of improvement over the last 20 years.

There are two possible empirical conclusions from Figures 4 and 5. On the one hand, Japan and Sweden might be overdue a slowdown, matching Canada, for example. But equally, a possible conclusion is that (a) Japan and Sweden will continue to improve at the same rate as during the last 20 years and that (b) the slowdown in Canada is temporary, with a return to Japanese/Swedish rates of improvement in the long run.[3] Indeed, for females above age 65, if we take Japanese mortality as the best practice that other countries actively target, then in the long run female improvement rates in other countries could be substantially higher than they are at present.

[3] Underpinning (b) is an assumption that, in the long run, a developed country would not allow itself to fall further and further behind the world leaders in life expectancy.

Figure 4: ASMRs for males in six countries over age bands 45–64 (top) and 65–89 (bottom). Source data: Human Mortality Database; Statistics Canada.

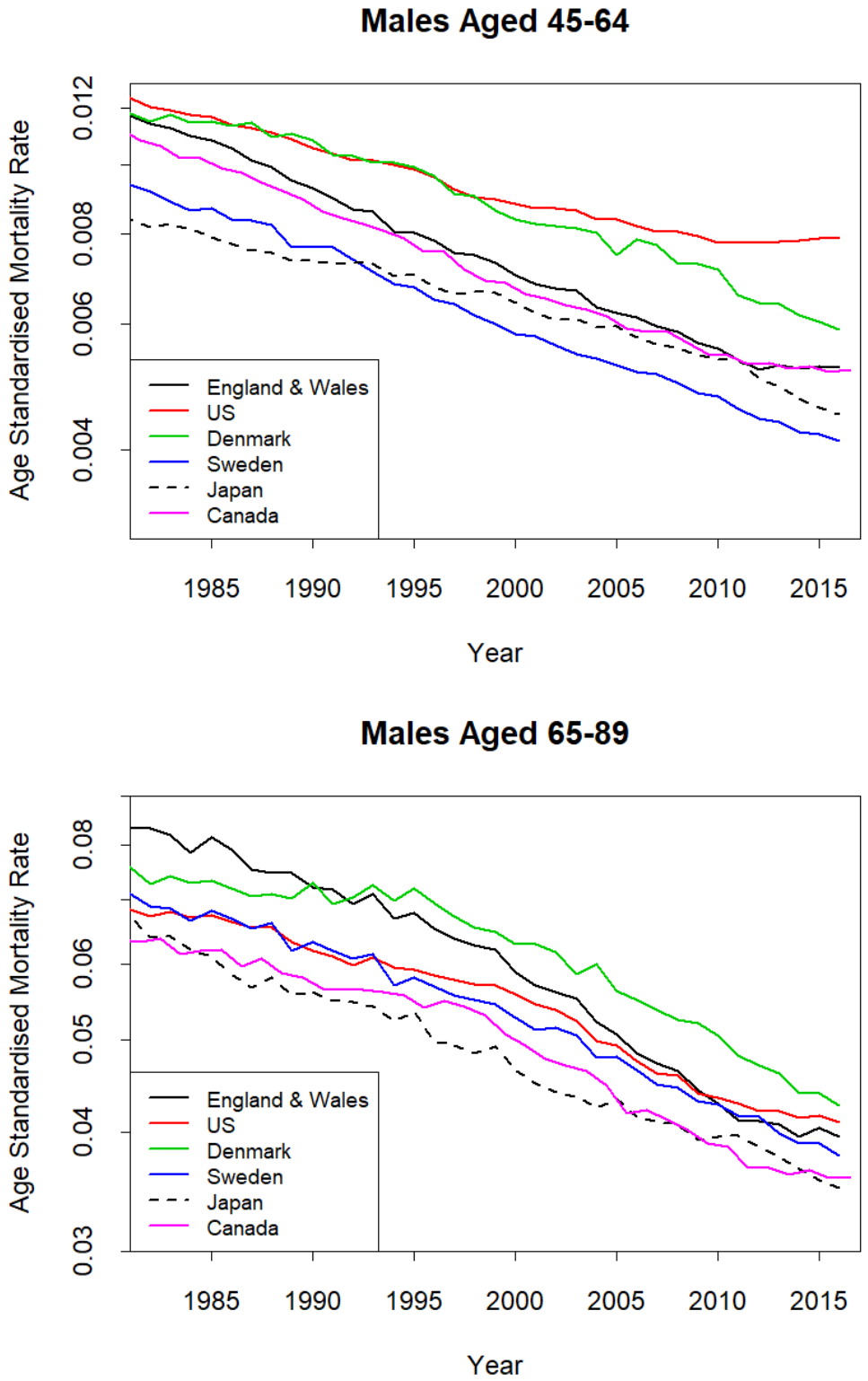
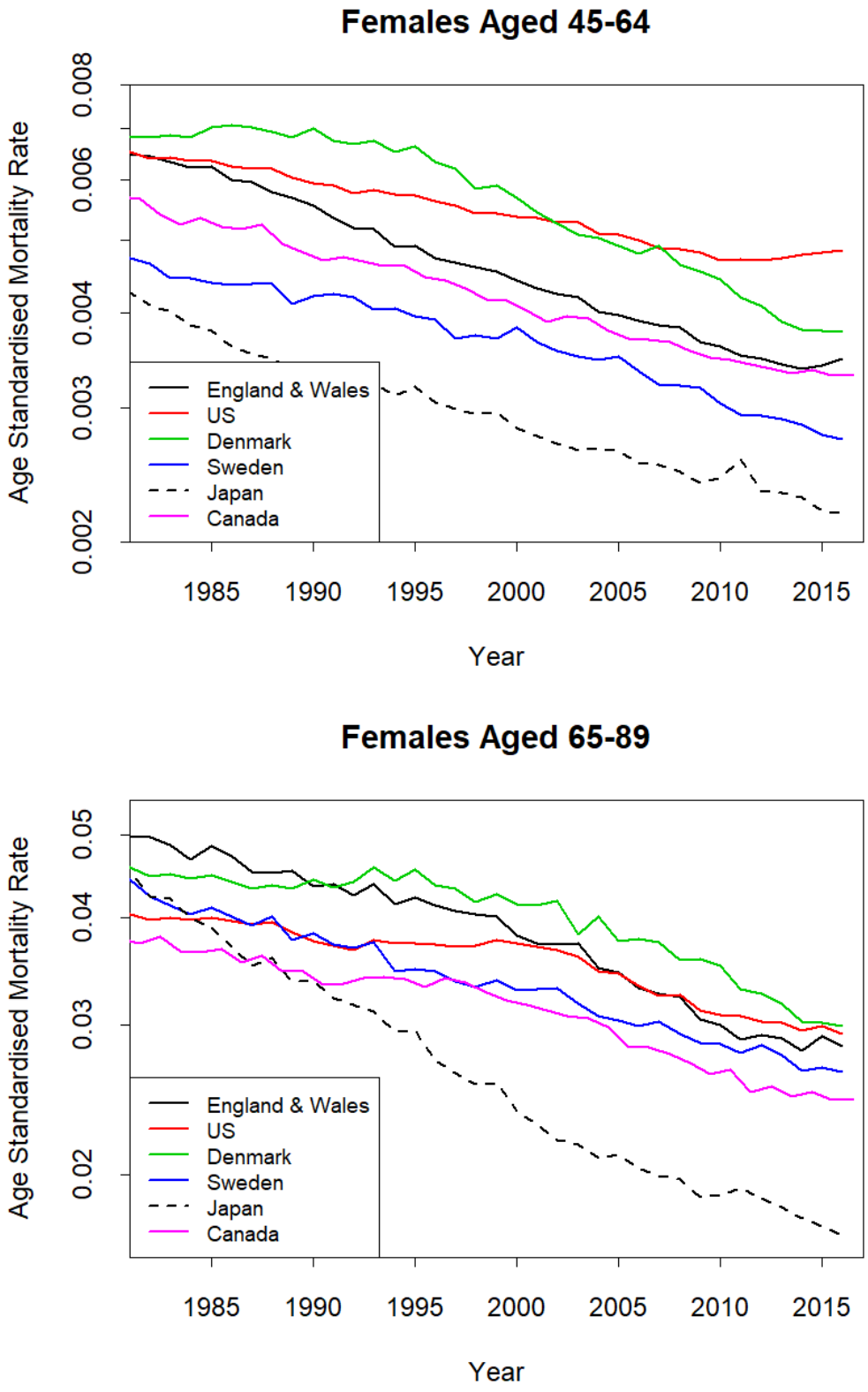


Figure 5: ASMRs for females in six countries over age bands 45–64 (top) and 65–89 (bottom). Source data: Human Mortality Database; Statistics Canada.



2.2. Can we say how big the slowdown is?

Graphically (e.g. Figure 3) there is reasonable evidence for a slowdown in Canada in recent years, but can we be precise about current versus older improvement rates? In reality, we have to be very cautious: there is sufficient volatility from year to year in mortality improvements that one will need many more than five or six years of data to get a good estimate. Additionally, if there has been a trend change, when was this? Graphically, the answer might seem obvious: 2011 (Figure 3). But one might also consider the following line of thought. 2011 was, by chance, a year with particularly low mortality and it emphasises what might actually be a more modest trend change that started a year or two earlier. To quantify this, Table 1 details mortality improvement rates for different age groups over different time periods. Improvements over the longer 10-year windows of 2000–2010 and 2001–2011 seem to be fairly stable. But then contrast improvements over the period 2011–2016 (encouraged by the position of the kink) versus improvements over 2010–2016. Differences between the two are quite substantial and highlight the heavy dependence on assumption setting on a trend change having happened in 2011 rather than, for example, 2010. Equally plausible is the idea that there has been a gradual rather than a sudden trend change (e.g. between 2009 and 2012), making estimation of the current improvement rate even more challenging.

Table 1: Annual mortality improvement rates by age group and over different periods of time for Canadian males and females. Source data: Statistics Canada, using mid-year to mid-year deaths and exposures. The improvement rate over the period t_0 to t_1 is defined as $\log(ASMR(t_0)/ASMR(t_1))/(t_1 - t_0)$

Age Group	2001–2011	2011–2016		2000–2010	2010–2016
Males 40–49	1.6%	0.7%		1.5%	0.9%
60–69	3.1%	0.1%		3.0%	0.6%
80–89	2.2%	0.2%		1.9%	1.1%
Females 40–49	1.2%	0.6%		0.7%	0.9%
60–69	2.3%	0.0%		2.0%	0.8%
80–89	2.0%	0.3%		1.6%	1.1%

3. Socio-Economic Differences in Mortality

We now zoom in rather than out to look at Canadian mortality by considering socio-economic sub-groups.

3.1. Introduction

By way of introduction, existing analyses of mortality differentials by socio-economic group are illustrated in Figures 6 and 7 for England (deciles by deprivation[4]) and Denmark (deciles by affluence[5]).

Both countries exhibit a considerable level of inequality: the least wealthy or most deprived in society experience much higher levels of mortality (and, hence, lower life expectancy) than the most well off. Also, in both countries we see relatively steady improvements in mortality from 2001–2012/2016 across all of the deciles. Income or wealth measured in some form is, of course, well known as a predictor of high or low mortality (see, for example, Mackenbach, 2003; Chetty et al., 2016; Longevity Science Panel, 2018; and Adam, 2012a, 2012b, 2016 in a Canadian context).

[4] Data are available for small geographical areas known as Lower Super Output Areas (LSOAs) (see www.ons.gov.uk/methodology/geography/ukgeographies/censusgeography). Each LSOA has an associated Index of Multiple Deprivation (IMD). The 32,844 individual LSOAs are then ranked using the IMD and grouped into deciles. Each LSOA will contain a mixture of people with different backgrounds and, therefore, differs from other socio-economic datasets that allocate individuals directly to socio-economic groups, such as Denmark.

[5] Data for each individual are recorded on the Statistics Denmark Register Database. Affluence is a simple combination of individual wealth and income. Individuals at each age are then ranked and grouped into deciles. (See Cairns et al., 2018.)

Figure 6: ASMRs for English males and females aged 65–89 subdivided into deciles using the Index of Multiple Deprivation.

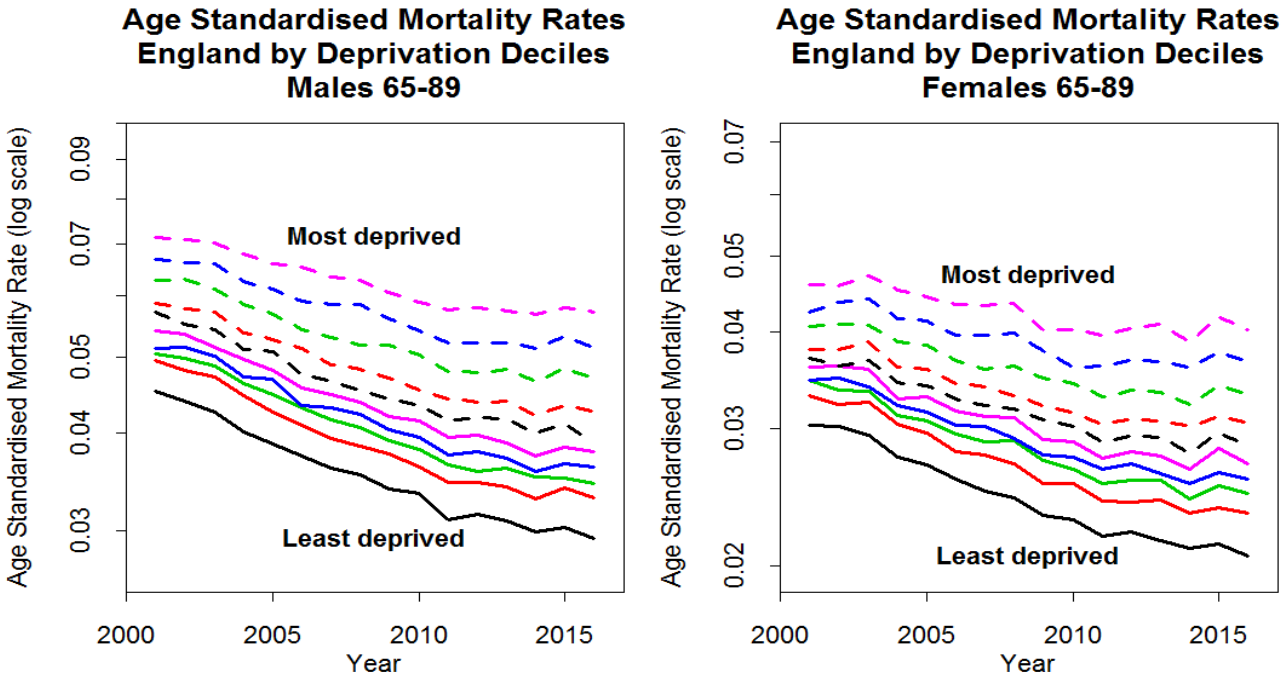
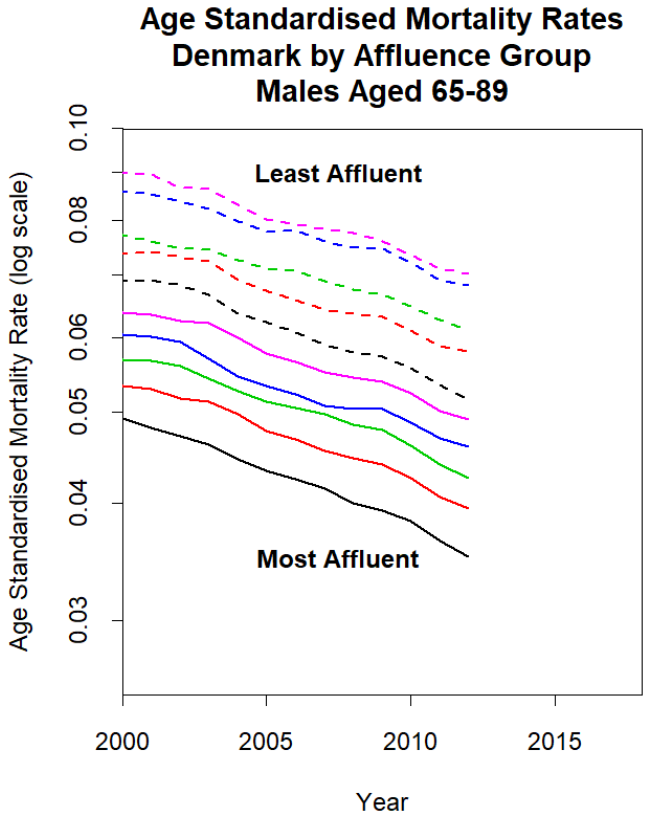


Figure 7: ASMRs for Danish males aged 65–89 subdivided into deciles using an affluence index.



Contrasting the two countries:

- England has had generally lower mortality than Denmark over this age range (echoing Figure 4).
- The gap between the least and most affluent in Denmark is wider than the English IMD deciles. However, this might reflect the way in which the deciles are formed in the two countries (see footnotes 7 and 8): in England there is a greater level of heterogeneity within each decile than the Danish deciles.
- Over the period 2001–2016, in England the gap between the least and most deprived has widened considerably. In Denmark, the gap has also widened, but only by a modest margin.

This raises the question: does Canada have similar levels of mortality inequality between socio-economic groups, and how has mortality in these different groups evolved over time?

To address this question, we consider mortality data for the CPP and the QPP. The QPP covers individuals resident in the province of Québec, while the CPP covers all other provinces in Canada. A key advantage is that, in combination, the CPP and QPP cover almost the entire population of Canada,[6] allowing us to compare the pensioners data with national and provincial mortality data.

3.2. CPP and QPP data

Data were provided in an aggregated form for pensioners only (that is, there were no data for pre-retirement members of the CPP and QPP).[7] Individual members were grouped according to their pension level expressed as a proportion of the maximum pension achievable by their cohort, as follows:

- Group 1: Pension level $= 0 - 9\%$ of the cohort maximum pension
- Group 2: Pension level $= 10 - 19\%$ of the cohort maximum pension
- ⋮
- Group 10: Pension level $= 90 - 99\%$ of the cohort maximum pension
- Group 11: Pension level $=$ cohort maximum pension.

Additionally, for the CPP only, the data exclude (a) individuals who had disability benefits converted into CPP pensions at retirement, and (b) individuals with a pre-existing survivor's pension at the time of retirement and who, as a result, have a pension level in excess of 100%.

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[6] In particular, the CPP and QPP cover Canadians who have participated in the workforce.

[7] Further details on an earlier CPP/QPP data extract can be found in Section III of Adam (2012a). Adam also discusses the advantages of using CPP/QPP data compared to data from private pension plans and insurers.

Death counts and central exposures[8] in the dataset are denoted by $D(g,i,t,x)$ and $E(g,i,t,x)$ respectively where

- g = gender
- i = pension group
- t = calendar year
- x = age last birthday (at date of death).

The corresponding crude age-specific death rate is then

$$m(g,i,t,x) = \frac{D(g,i,t,x)}{E(g,i,t,x)}.$$

The analysis here builds on earlier work by Adam (2012a, 2012b) where groupings are again by pension level as a percentage of the maximum by cohort. Adam initially subdivides into 21 bands (in 5% increments) but then reduces this to three (0–35%, 35–95% and 95–100%) for the purposes of further analysis, with an emphasis on the relevance of specific CPP/QPP groups as a proxy for modelling mortality in private pension plans. Here we persist for longer with 11 groups, giving further insights into mortality differentials and time trends.

3.3. Contributions and the cohort maximum pension

During the accumulation phase as a member of the CPP/QPP, individuals contribute a defined proportion of their earnings up to the Year's Maximum Pensionable Earnings (YMPE).[9] The YMPE increases each year in line with national average weekly wages, salaries and other earnings, and lies close to average earnings. As a result of the latter, a large proportion of the active membership of the CPP/QPP will be contributing at the maximum rate in any given year. However, to achieve the maximum pension, an individual must currently have contributed at the maximum rate in 83% of the eligible working years by cohort,[10] which is much more challenging than contributing the maximum in any single year.[11]

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[8] Central exposures are the total years of exposure during year t of persons in group (g,i) aged x last birthday. Equivalently, it represents the average number of people during calendar year t aged x last birthday.

[9] The YMPE was $55,900 in 2018.

[10] 83% applies to retirees from 2014 onwards; 85% in 2012 and 2013; 85% up to 2011.

[11] For example, individuals retiring in 2018 were eligible to contribute to the CPP from ages 18–64 (47 years). So they must have contributed at the maximum rate (i.e. earning above the YMPE) in 39 out of the last 47 years. More generally, the two plans started in 1966 with eligible contributions from the same year. So, for example, an individual retiring at age 65 at the end of 1985 would have had to contribute at the maximum for 17 (85%) out of the last 20 years (1966 to 1985) of their working life; that is, ages 45 to 64. For females the number of eligible years can be reduced from 47 years under the "child-rearing" provisions (e.g. a female retiring in 2018 with seven years approved under the child-rearing provision

The data cover all pensioners over the period 1967 (CPP) or 1968 (QPP) to 2015. The early years of the CPP and QPP did not permit retirement before age 65. The QPP allowed early retirement from age 60 from 1984 and the CPP from 1987, with a corresponding impact on exposures between ages 60 and 64 from these dates. Late retirement is also permitted, to allow members to accrue additional years (helpful for immigrants) or boost their best 83%. Currently, late retirement is permitted up to age 70 in order to improve the pension amount.[12] Prior to 1989, even later retirement was also possible. Late retirements can be detected in all 11 groups, but it is most obvious in Group 1.[13]

The data indicate that persons born before 1895 or 1896 were not eligible to receive a pension from the CPP or QPP, with resulting zero exposures for these cohorts. Bearing this in mind and the maturing nature of the CPP and QPP, as a compromise we used data from 1991–2015 and ages 65–89 in our modelling work.[14]

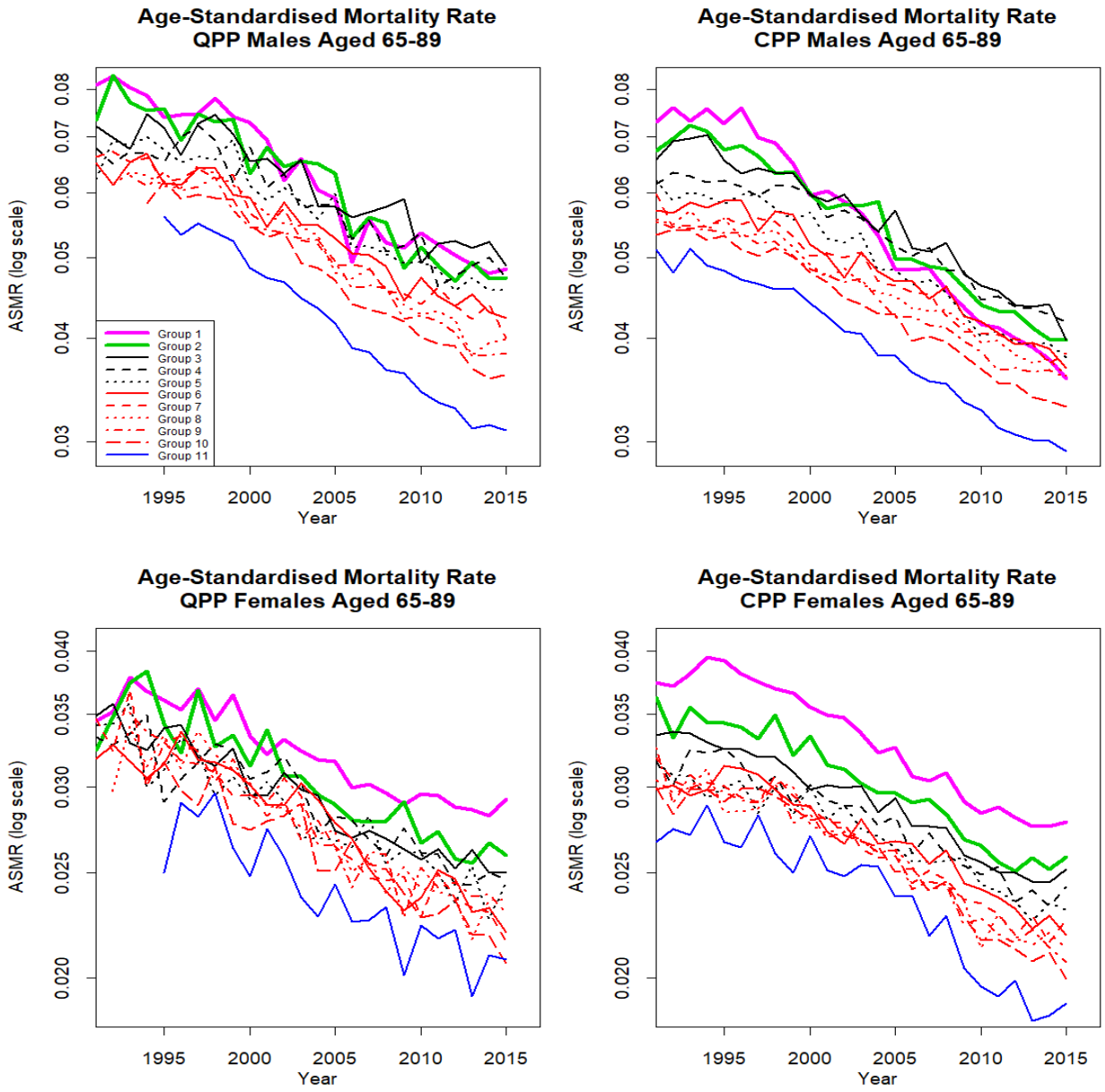
would have her pension calculated using the best 33 years (83% of $(47 - 7) = 40$) rather than the best 40, and the pension scaled appropriately).

[12] Technically, retirement after 70 is also possible, but there is no financial benefit to taking retirement after age 70.

[13] A late retirement factor is applied for late retirement. Group allocations for late retirees are made by comparing their pension with the age-65 maximum pension scaled up by the late retirement factor.

[14] We limit our investigations to age 65 and above, as the reasons for taking early retirement are varied and can be connected to health.

Figure 8: ASMRs for QPP and CPP males and females based on crude age-specific death rates for ages 65–89. Each plot shows the ASMR for groups 1 (low pension level) to 11 (maximum pension).



3.4. ASMRs by pension level

ASMRs for ages 65–89 based on crude age-specific death rates are plotted in Figure 8.[15] A number of observations can be made:

- The broad trends are similar to that for Canada as a whole.
- Although calculation of the ASMR dampens the impact of sampling variation, we can still see that smaller groups (e.g. males Group 1 or females Group 11) produce more volatile ASMR plots compared to larger groups (e.g. males Group 11).
- QPP ASMRs are mostly slightly above those for the CPP.
- Significant inequalities are evident between the 11 groups. In particular, the ASMRs for QPP Group 1 males are well over 50% higher than those for Group 11.
- In most years Group 11 stands well below the other groups.
- For females, Group 1 also stands clear of the other groups.
- Mostly, the ASMRs are ranked approximately in line with the group ordering: high mortality for Group 1 through to low mortality for Group 11. However, in terms of rankings, the data reveal one anomaly that needs some further investigation and discussion:
    - CPP males Groups 1 and 2: these start high, as one would expect, but then gradually drift down and cross over several mid-ranking groups (quite different from QPP males Groups 1 and 2).

There are a variety of reasons why Group 11 stands clear of the others. One reason is that within Group 11 there will be a potentially high degree of heterogeneity: some individuals consistently just above the threshold, others much more wealthy; and a mixture of occupation groups. Another reason that we now discuss is *conscientiousness*.

3.5. Conscientiousness

Conscientiousness is one of the five major character traits in the field of psychology. A conscientious individual will wish to do their work or duty well and thoroughly, and they will be careful, hard-working, diligent, dedicated and accurate in both their working and personal lives.

We can then conjecture that there will be a positive correlation between conscientiousness as a trait and sustained success in employment. In the Canadian context, diligence and working hard throughout one's lifetime might mean that a conscientious individual attains earnings above the YMPE in a greater number of years

[15] In the first few years, some ASMRs are missing for high-pension QPP groups. This is a result of zero exposures at high ages, meaning death rates required in the calculation of the ASMR are not available.

than a non-conscientious individual.[16] In particular, among, say, second-quartile earners (i.e. just above the YMPE), conscientious individuals are more likely to attain earnings above the YMPE in at least 83% of their working years (see Footnote 14).

Conscientiousness is important because it is the character trait that is most strongly correlated with life expectancy (see, for example, Kern and Friedman, 2008; Deary et al., 2010).[17] The conjecture that Group 11 in both the CPP and QPP data might contain a greater proportion of conscientious individuals compared to Group 10 would then, in part, explain why Group 11 has significantly lower mortality.

3.6. The slowdown in mortality improvements

We can also look at Figure 8 to investigate if the slowdown observed at the national level affects all groups or some subset. In fact, noise in the ASMRs makes it quite difficult to establish if any of the individual groups has experienced a slowdown, and, certainly, there is no consistent relationship between groups. Group sizes are also changing, making identification of a slowdown at the group level potentially trickier still.

3.7. Impact of group size

In interpreting Figure 8 we also need to be mindful that the proportions of each cohort in each group are changing over time. We discuss this further in Section 4. But here we can remark, by way of example, that if Group 1 was shrinking over time, that might have an impact on mortality rates[18] that interferes with other changes in the level of mortality.

3.8. Comparison of inequalities with other countries

We can also compare levels of health inequality (as manifested through the age 65–89 ASMR) between the CPP, QPP, England and Denmark by considering Figure 8 alongside Figures 6 and 7.

Visually, England and Denmark both exhibit a wider gap than the QPP and CPP. However, we need to be mindful that the Canadian data are subdivided using a different measure (pension level), which might be a less powerful predictor of mortality compared to deprivation (England) or affluence (Denmark). Also, Group 11 for CPP and QPP males is much larger than 10% in the earlier years, making the gap between Groups 1 and 11 smaller than it would otherwise be.

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[16] As an example, Egan et al. (2017) provide evidence that conscientious individuals will experience less unemployment in their working lifetimes. In the Canadian context, each period of unemployment makes it less likely that the individual's earnings will exceed the YMPE in a given year.

[17] For example, conscientious individuals are more likely to: adhere to a healthy diet, visit the doctor early when they develop symptoms related to ill health, and follow doctor's orders when given a diagnosis.

[18] Each group still contains a degree of heterogeneity. Everything else being equal, if Group 1 grows over time, then the average level of deprivation will be reduced, with a corresponding lowering of average Group 1 mortality.

Broadly, however, levels of health inequality in the QPP and CPP are consistent with what we see in England and Denmark.

4. Cohort Sizes

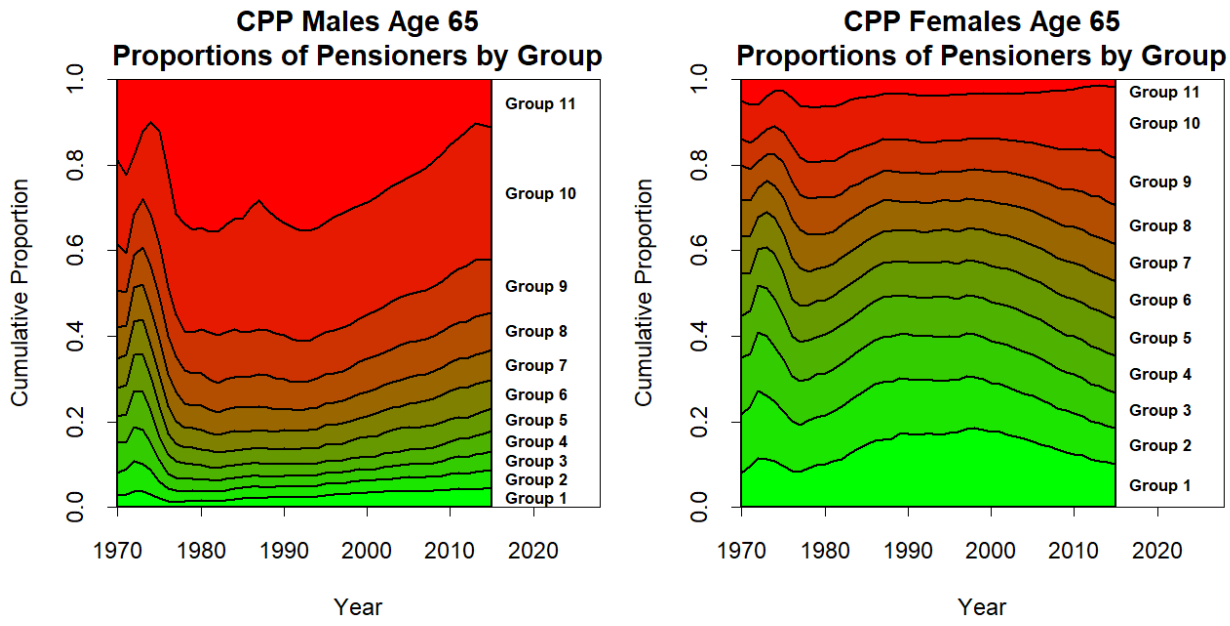
We now consider the relative sizes of each group and how these have changed over time. The proportions in each group by cohort are defined as

$$P(g,i,t,x) = E(g,i,t,x) / \sum_j E(g,j,t,x).$$

We focus our discussion on the normal retirement age $x = 65$, but the picture is broadly similar if, say, we use age 70.[19, 20]

Figures 9 and 10 illustrate in a heat map format for the CPP and QPP, males and females, how the proportions, $P(g,i,t,65)$, have changed over time. For example, in the left-hand panel of Figure 10, the proportion in Group 11 at age 65 in 1990 (running up and down the vertical line) was about 33% $(100 \times (1 - 0.67))$, and in Group 10 26% $(100 \times (0.67 - 0.41))$. But by 2015, the cohort aged 65 in Group 11 had fallen to just 8%, while Group 10 had grown to about 31%.

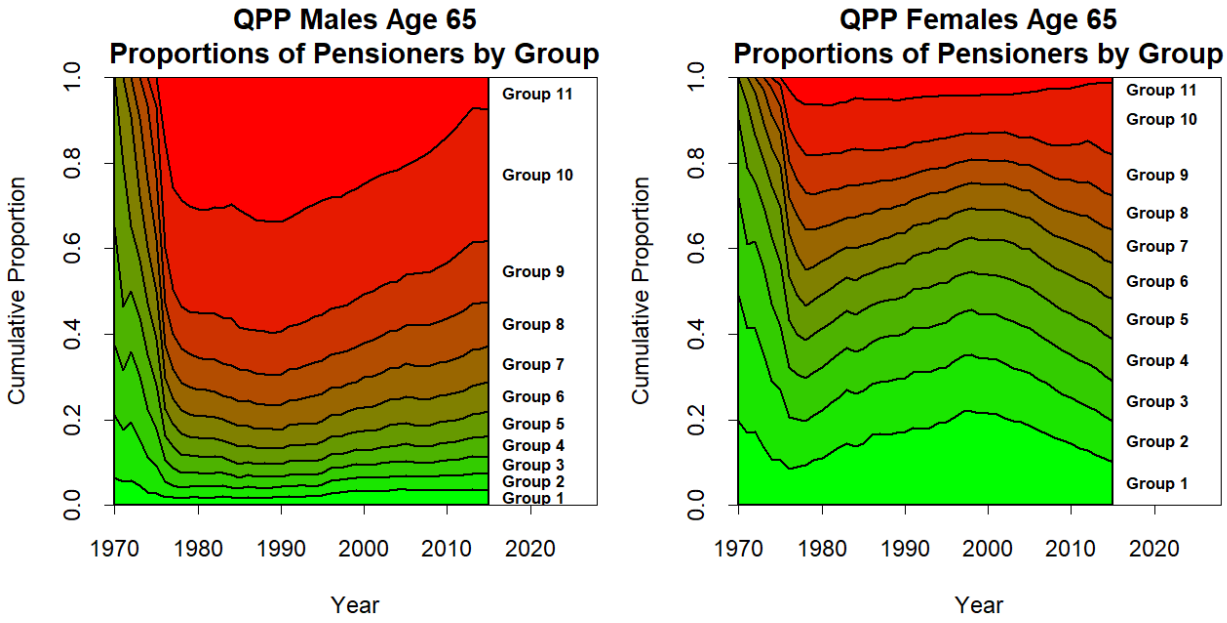
Figure 9: Proportions of pensioners aged 65 in each of Groups 1 to 11 (cumulative) by calendar year. The width of each band gives the proportion in each group. Left: CPP males. Right: CPP females.



[19] Proportions are slightly different at age 70 due to differential death rates between ages 65 and 70 by group, and late retirements between 65 and 70.

[20] Proportions by pension level are also illustrated in a different way in Appendix A of Adam (2012a).

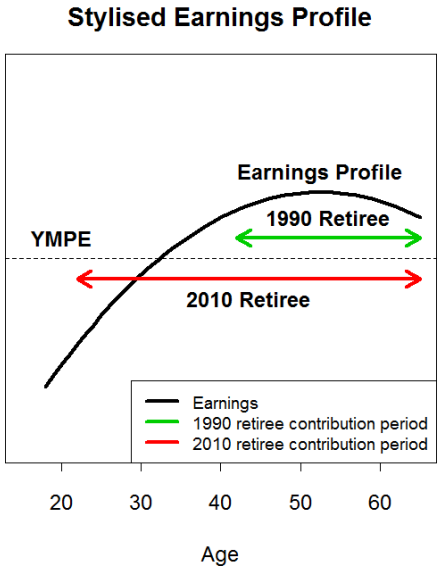
Figure 10: As Figure 9 but for QPP males and females.



We can comment as follows:

- For males, the heat maps for the CPP and QPP are broadly similar. Up to about 1977/78 there is considerable distortion, with it being much more difficult in the early years of the plans to attain higher pension levels. Around 1980, the proportions then settle down with the largest proportions falling into Groups 10 and 11.
- For females, similar comments apply except that a much smaller proportion of females attain higher pensions. In particular, Group 1 is generally the largest (although Group 10 has recently exceeded Group 1), reflecting the different work pattern of females compared to males (even after taking account of rules to mitigate the impact of taking some years out to raise a family). Lastly, the same YMPE applies to both males and females, so the extent to which there is a gender pay gap in Canada will be reflected in a lower proportion of females attaining higher pensions in CPP and QPP.
- For males, after 1990 a declining proportion of those reaching their 65th birthday attain the maximum pension (Group 11). The likely reason for this is that the number of qualifying number of years to attain the maximum has been changing (see Footnote 14). In combination with a typical career earnings path (Figure 11) starting low, peaking in middle age and declining slightly towards retirement (see, for example, Blake et al., 2007), this means it would have been easier to attain the maximum pension for someone retiring in 1990 compared to another retiring in 2010 (Figure 11).

Figure 11: A stylised but typical earnings profile for an individual over their working lifetime. An individual who reaches 65 in 1990 needs to have earned above the YPME in 20 out of 24 years from ages 41–64 (1966 to 1989) to get the maximum pension. An individual retiring in 2010 needs 37 out of 44 years (ages 21–64) above the YMPE.



- For females, we see a more complex picture. Group 11 declines in size (as for males). Group 1 gradually increases from 1980, peaks just before 2000 and then declines, probably reflecting changes in the underlying work patterns of females.
- For both males and females, the proportions in each group will also reflect historical patterns of immigration. Some individuals retiring at 65 might have lived their entire working lives in Canada, while others might have migrated to Canada during their careers, with a consequent impact on their CPP or QPP pension. For example, an individual who entered Canada at the age of 40 and retired at 65 in 2018 will only have been able to contribute to the CPP or QPP in 25 out of the required 47 years. We discuss this further in the next section.

5. Migration and Years of Residency

Given the preceding comments, we need to consider what proportion of each cohort retiring at 65 are immigrants. And how many years have immigrants been contributing to the CPP or QPP since they entered the country?

We define for each individual

$$Years\ of\ Residency\ (YR)\ = \begin{cases} 65 - \ Age\ on\ Arrival & if\ age\ on\ arrival\ > 18 \\ 47 & otherwise. \end{cases}$$

Out of each cohort we seek to estimate what proportion has $YR = 1,2, \dots ,47$. To achieve this we use data from the Canadian Human Mortality Database (CHMD, 2011) for the Canadian provinces (available up to 2011) and adopt a crude set of assumptions. The CHMD data can be used to obtain exposures, $E(i,t,x)$, by year, T, and single age, x, for Québec (i =Q) and Canada excluding Québec (i =CxQ).

- Following individual cohorts, the change from $E(i,t,x)$ to $E(i,t+1,x+1)$ is attributable to deaths and net emigration.
- We assume that:
  - In any year migration is either wholly out of or wholly into Q or CxQ.
  - There is no migration between Q and CxQ.
  - Emigrants do not return to Canada and therefore rejoin the CPP or QPP.
  - Plan members retire at 65.

These assumptions are very simplistic and could possibly be improved upon, but only with considerable additional effort. Furthermore, our resulting calculations of years of residency are not used in any further calculations; they are only used to help with qualitative interpretation of the results. More refined calculation of eligible years would be unlikely to alter these conclusions.

For each cohort, we use the assumptions above to estimate what proportion of the cohort entered Canada (Q or CxQ) one year before age 65, two years before 65, ..., 47 or more years before 65. Figure 12 show the results of these proportions as a heatmap. For example, for males, CxQ in 1990 (vertical line): about 75% of the cohort had at least 40 years of residency, 15% had $30 < YR < 40$, 3% had $20 < YR < 30$, 2% had $10 < YR < 20$, and 5% had $0 < YR < 10$.

- The relative sizes of the five groups and the pattern of their changes over time reflects the changing pattern of migration and age profile of immigrants over time.
- A much greater proportion of the Québec population have $YR > 40$, reflecting much lower levels of immigration in Québec at all ages compared to the rest of Canada.
- The black dots in the right-hand panel of Figure 12 pick out places where two boundary lines touch, meaning that there are no individuals within a particular 10-year band for YR in a particular retirement year. The sequence of three dots around 1980, 1990 and 2000 correspond to a sustained period of (net) emigration from Québec in the 1970s at all ages.
- For CxQ we see a more complex plot. However, there is a small but significant flow of immigrants into CxQ above age 55 that contributes to the persistent light green band ($0 < YR < 10$ years).

The size of the light green band ($0 < YR < 10$ years) for CxQ provides us with at least a partial explanation for the anomalous behaviour of male mortality in Groups 1 and 2 in the CPP mortality plots (Figure 8, top right). Groups 1 and 2 cover individuals with a pension of less than 20% of the maximum. Membership of these groups reflects either low earnings or a small number of contributing years. Figure 12 points to the CPP having greater numbers of immigrants arriving in their middle ages who would necessarily end up in Groups 1 or 2. Lastly, we have the healthy-immigrant effect: immigrants are

admitted to Canada on the basis that they are healthy and fit to work, and therefore healthier than the corresponding established population (see, for example, Vang et al., 2017). Over time, the healthy-immigrant effect fades, so, for our purposes it is strongest among the late-middle-age immigrants subsequently reaching age 65. On the other hand, although the selection effect associated with immigration fades within a particular group by pension level, more recent immigrants are likely to be more wealthy and, consequently, more healthy than the "archetypal" member of Groups 1 or 2 on very low earnings.[21]

So although we would expect Groups 1 and 2 to exhibit high mortality due to people on low earnings, headline mortality rates are reduced because of the presence of healthy immigrants (Group 1 more than Group 2).

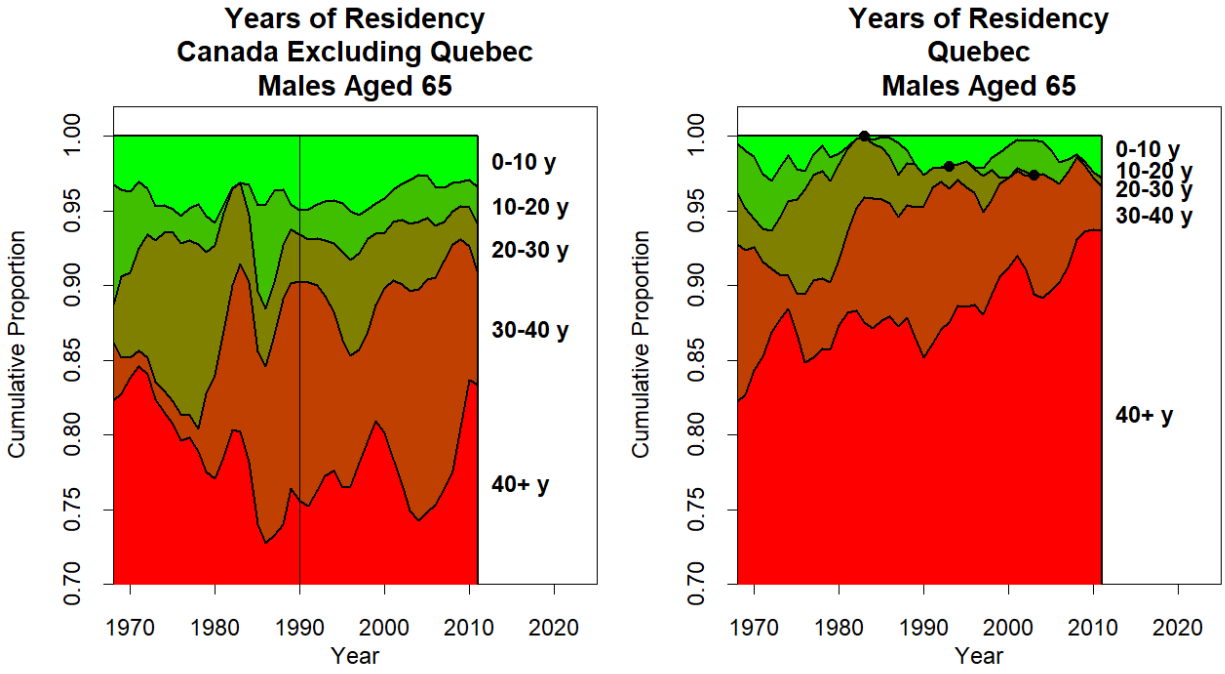
As a final point, we can postulate that the proportion of healthy immigrants in Group 1 has been growing over time. Figure 12 shows that the flow of immigrants over age 55 (the 0–10 year band) has been fairly stable over time (no obvious trend). But, for retirement in different years, how low does YR have to be to force an individual into Groups 1 or 2 at age 65? This depends on the number of *eligible* years for a native Canadian: among individuals reaching age 65 in 2018, an immigrant with $YR < 8$ is guaranteed to be allocated to Groups 1 or 2, whereas an individual reaching age 65 in 1985 with $YR < 3.4$ will definitely end up in Groups 1 or 2. Correspondingly, everything else being equal, Groups 1 and 2 will have fewer healthy immigrants among new retirees in 1985 compared to 2018. (See, also, Appendix B for further discussion.)

For QPP males in Figure 8, there is limited evidence for a healthy-immigrant effect: Groups 1, 2 and 3 overlap in an erratic way, with some convergence in the first 10 years.[22]

[21] Differences between the mortality of immigrants and the established population might also arise for cultural reasons (affecting diet and lifestyle).

[22] We do not detail this further in Section 7, but model-based estimates of the underlying mortality of QPP males reveal more clearly lower mortality for Groups 1 and 2, particularly at higher ages and less so at lower ages. This might be linked to late immigrants deferring retirement until age 70 or later.

Figure 12: Heat maps showing, for each cohort reaching age 65, estimates of how long people have been Canadian residents and eligible to contribute to the CPP/QPP. Proportions are built up cumulatively: $40+$ years; $30-40$ years; $20-30$ years; $10-20$ years; <10 years.



6. Stochastic Mortality Models: Motivation

The data in Figures 2 to 3 and the subsequent discussion all point to uncertainty in a number of features of future mortality:

- long-term improvement rates
- volatility from year to year
- improvement rates at different ages
- improvement rates in different populations.

This uncertainty leads to *longevity risk*: the risk that a cohort of people live longer *in aggregate* than anticipated. Longevity risk is of particular concern in a pensions and annuity context: if plan members live longer than anticipated then the plan or annuity provider will experience financial losses. Modelling and measurement of longevity risk is, therefore, important in a number of actuarial applications.

One approach to this is through the development of stochastic mortality models that seek to model the randomness from year to year and in the long run in *underlying* mortality rates. Such models can be used to tackle a variety of actuarial tasks, including:

- general risk assessment
- pricing: using the results of stochastic projections to establish margins for systematic longevity risk
- reserving: assessing and reserving for systematic risk in the runoff of a portfolio of liabilities
- reserving: assessing the systematic reserving risk over a one-year horizon linked to Value-at-Risk type of minimum reserving requirements (e.g. Solvency II)
- reserving: use of a two-population mortality model to quantify the diversification benefit between two populations
- assessment of risk reduction in longevity hedges.

In using stochastic mortality models to assess the problems above, uncertainty around the central mortality forecast is at least as important as the central forecast itself. Indeed, in some contexts (e.g. the UK Prudential Regulatory Authority's guidelines on assessing and reserving for longevity risk under Solvency II) the central forecast might be developed using non-stochastic methods (e.g. actuarial judgement) and uncertainty around that forecast is based on the outputs of a stochastic model.

7. A Family of Multi-population Stochastic Mortality Models

7.1. Modelling genealogy

The last 30 years have seen increasing interest in the use of stochastic mortality models, growing from the single-population models of Lee and Carter (1992), Renshaw and Haberman (2003), Cairns et al. (2006) and Plat (2009). Under the Lee and Carter model (LC), underlying death rates, $m(t,x)$, in year t at age x last birthday, are modelled as

$$\log m(t,x) = \alpha(x) + \beta(x)\kappa(t)$$

where $\alpha(x)$ and $\beta(x)$ are non-parametric age effects, and $\kappa(t)$ is a period effect. $\alpha(x)$ provides us with a form of base table (when $\kappa(t) = 0$), while $\beta(x)$ determines the relative rates of improvement at different ages. Improvements over time are driven by the single factor period effect, $\kappa(t)$.

This model was extended by Renshaw and Haberman (2003) (RH) to include a second period effect:

$$\log m(t,x) = \alpha(x) + \beta_1(x)\kappa_1(t) + \beta_2(x)\kappa_2(t).$$

A different style of model was proposed by Cairns et al. (2006) (CBD), who modelled the mortality rate as

$$logit\ q(t,x) = \kappa_1(t) + (x - \bar{x})\kappa_2(t),$$

where $logit\ z = \log(z/(1-z))$ for $0 < z < 1$, with $q(t,x)$ and $m(t,x)$ connected through $q(t,x) = 1 - \exp(-m(t,x))$, and $\bar{x}$ equal to the mean of the age range being modelled. By using predetermined parametric age effects ($\beta_1(x) = 1$ and $\beta_2(x) = (x - \bar{x})$), this reduced significantly the number of parameters to be estimated.

This was subsequently extended by Plat (2009), who introduced a non-parametric base table, $\alpha(x)$:

$$logit\ q(t,x) = \alpha(x) + \kappa_1(t) + (x - \bar{x})\kappa_2(t).$$

A key benefit of Plat's model and its various extensions was that it is suitable for fitting to a wider range of ages than is normally recommended for the CBD model. With the non-parametric base table a simple variant of the Plat model replaces $logit\ q$ with $\log m$:

$$\log m(t,x) = \alpha(x) + \kappa_1(t) + (x - \bar{x})\kappa_2(t).$$

7.2. Multi-population extensions

The CPP and QPP data each have 11 populations for each gender, so we seek to model $\log m(i,t,x)$ jointly for $i = 1, \ldots, 11$. Rather than focus on one specific model from the outset, we considered a family of models that have a multi-population version of the RH model as the most general case. Thus, model M1 is

$$\log m(i,t,x) = \alpha(i,x) + \beta_1(i,x)\kappa_1(i,t) + \beta_2(i,x)\kappa_2(i,t).$$

In this model:

- The age effect $\alpha(i,x)$ can be interpreted as a base table for Group i.
- $\beta_1(i,x)$ and $\beta_2(i,x)$ are group-specific age effects that we normally anticipate will allow for changes in the level (i.e. $\beta_1(i,x) > 0$ for all x) and slope (i.e. $\beta_2(i,x) > 0$ for lower ages and < 0 for higher ages) respectively of the log-mortality curve.
- $\kappa_1(i,t)$ and $\kappa_2(i,t)$ are period effects that, in combination with the $\beta_1(i,x)$ and $\beta_2(i,x)$ age effects, capture the group-specific variation in mortality from the base table over time.

All other models considered were special cases of M1 and are listed in Table 2.

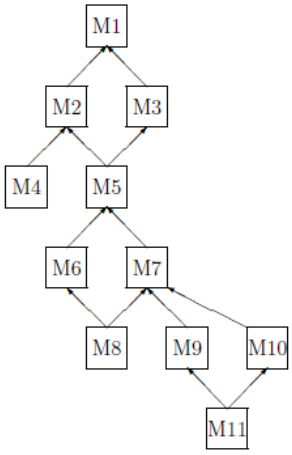
Table 2: Stochastic mortality models fitted to CPP and QPP males and females mortality data.

Model	$\log m(i,t,x)$	Comment
M1	$\alpha(i,x) + \beta_1(i,x)\kappa_1(i,t) + \beta_2(i,x)\kappa_2(i,t)$	Multi-population RH
M2	$\alpha(i,x) + \beta_1(i,x)\kappa_1(i,t) + \beta_2(x)\kappa_2(i,t)$	
M3	$\alpha(i,x) + \beta_1(x)\kappa_1(t) + \beta_2(i,x)\kappa_2(i,t)$	Li and Lee (2005)
M4	$\alpha(i,x) + \beta_1(i,x)\kappa_1(i,t)$	Multi-population LC
M5	$\alpha(i,x) + \beta_1(x)\kappa_1(i,t) + \beta_2(x)\kappa_2(i,t)$	CAE Model, Kleinow (2015)
M6	$\alpha(x) + \beta_1(x)\kappa_1(i,t) + \beta_2(x)\kappa_2(i,t)$	CAE Model with common $\alpha(x)$
M7	$\alpha(i,x) + \kappa_1(i,t) + (x - \bar{x})\kappa_2(i,t)$	Multi-population Plat
M8	$\alpha(x) + \kappa_1(i,t) + (x - \bar{x})\kappa_2(i,t)$	Plat with common $\alpha(x)$
M9	$\alpha(i,x) + \kappa_1(t) + (x - \bar{x})\kappa_2(i,t)$	Plat, common $\kappa_1(t)$
M10	$\alpha(i,x) + \kappa_1(i,t) + (x - \bar{x})\kappa_2(t)$	Plat, common $\kappa_2(t)$
M11	$\alpha(i,x) + \kappa_1(t) + (x - \bar{x})\kappa_2(t)$	Plat, common $\kappa_1(t)$, $\kappa_2(t)$

Some models are nested within others (e.g. all models are nested within M1) and the resulting hierarchy is illustrated in Figure 13.

In models M1, M5 and M6, the terms $\beta_1(i,x)\kappa_1(i,t)$ and $\beta_2(i,x)\kappa_2(i,t)$ are interchangeble with no impact on the model fit. The two components can normally be left as they are following model fitting. However, if we prefer that $\beta_1(i,x)\kappa_1(i,t)$ captures changes in the level of mortality across all ages, and $\beta_2(i,x)\kappa_2(i,t)$ captures changes in the slope (or a tilt), then, if required, we can swap round the two components.

Figure 13: Nested model hierarchy. Arrows indicate nesting: e.g. M2 is nested (i.e. is a special case) of M1.



A wider range of models is considered and reviewed by Villegas et al. (2017). Their focus is more on choosing an appropriate model for *two* populations (one dominant population and a second sub-population with specific characteristics) compared to 11 here (treated in a more balanced way). The list here also considers some models not considered by Villegas et al. (2017). Adam (2016) analyses several models using male and female CPP and QPP data simultaneously with three income classes and settles upon a model that is close to the Li and Lee (1995) model (here, M3).

We considered a range of quantitative and qualitative criteria to compare models and recommend which model is best for the multiple population dataset over ages 65–89.

From a quantitative perspective we use the Bayes Information Criterion,

$$BIC_M = -2l_M(\hat{\theta}_M) + k_M \log N,$$

where θ_M is the parameter vector for model M and $\hat{\theta}_M$ is its maximum likelihood estimator, $l_M(\theta_M)$ is the log-likelihood function, k_M is the number of parameters in M to be estimated (taking into account any identifiability constraints), and N is the number of observations.[23] The term $k_M \log N$ serves to penalise models that are over-parameterised. In aiming to minimise the BIC, we seek to include greater complexity

[23] Here $N = 11 \times 25 \times 25 = 6875$ (groups × years × ages).

(e.g. favouring M5 over M6) only if that greater complexity results in a *significant* improvement in the fit. In particular, where models are nested, the more complex model will always achieve a better fit (i.e. the maximum log-likelihood will be higher) but the improvement might be quite marginal and the extra parameters are simply overfitting the data. For further discussion of the BIC, see Appendix A.

We also consider the forward correlation term structure:

$$\rho(t,i,j,x_i,x_j) = cor\left(\log m(i,t,x_i), \log m(j,t,x_j)\right).$$

Our principal desirable criteria are as follows.

1. The BIC should not be significantly higher than other models.
2. The model should satisfy the principle of coherence. That is, for each (i,j,x), $m(i,t,x)/m(j,t,x)$ should not diverge as t gets large (see, for example, Hyndman et al., 2013).
3. The model should avoid significant crossovers in fitted mortality curves where these are not apparent in the raw data. For example, in the historical data and in forecasts, for a given t, is $m(1,t,x) > m(11,t,x)$ over all ages x?
4. Correlations between future mortality rates in different populations should be less than 1.
5. Correlations between future mortality rates at different ages should be less than 1.
6. Does the model produce a plausible forward correlation term structure? For example, for a given (t,i,j,x_i), does the shape of $\rho(t,i,j,x_i,x_j)$ as a function of x_j look reasonable: e.g. unimodal with a peak close to x_i? And is $\rho(t,i,j,x,y) < \rho(t,i,i,x,y)$ (i.e. the correlation between two ages in the same population is likely to be higher than the same ages in different populations)?[24]

Additional relevant criteria can be found in Cairns et al. (2009) and Villegas et al. (2017).

7.3. Model selection outcome: M6 – CAE with common $\alpha(x)$

There was no single model that satisfied all criteria better than all other models. In particular, the model with the lowest BIC did not completely satisfy some of the qualitative criteria.

On balance, we selected model M6 (CAE with common $\alpha(x)$) as being the most suitable for both the CPP and QPP males and females datasets: this model had one of the lowest BIC values (but not *the* lowest) as well as satisfying the qualitative criteria. For a full discussion of the model selection process, see Appendix A.

[24] See, Cairns et al. (2018) for further discussion of plausible forward correlation term structures.

Maximum likelihood estimates of the age and period effects for CPP and QPP males and females are presented in Figures 14 to 17. In each of the four cases (e.g. CPP males), the model is fitted to the 11 groups jointly resulting (for M6) in estimates for the common age effects, $\alpha(x)$, $\beta_1(x)$ and $\beta_2(x)$, and the group-specific period effects, $\kappa_1(i,t)$ and $\kappa_2(i,t)$.

Figure 14: Fitted age and period effects for QPP males.

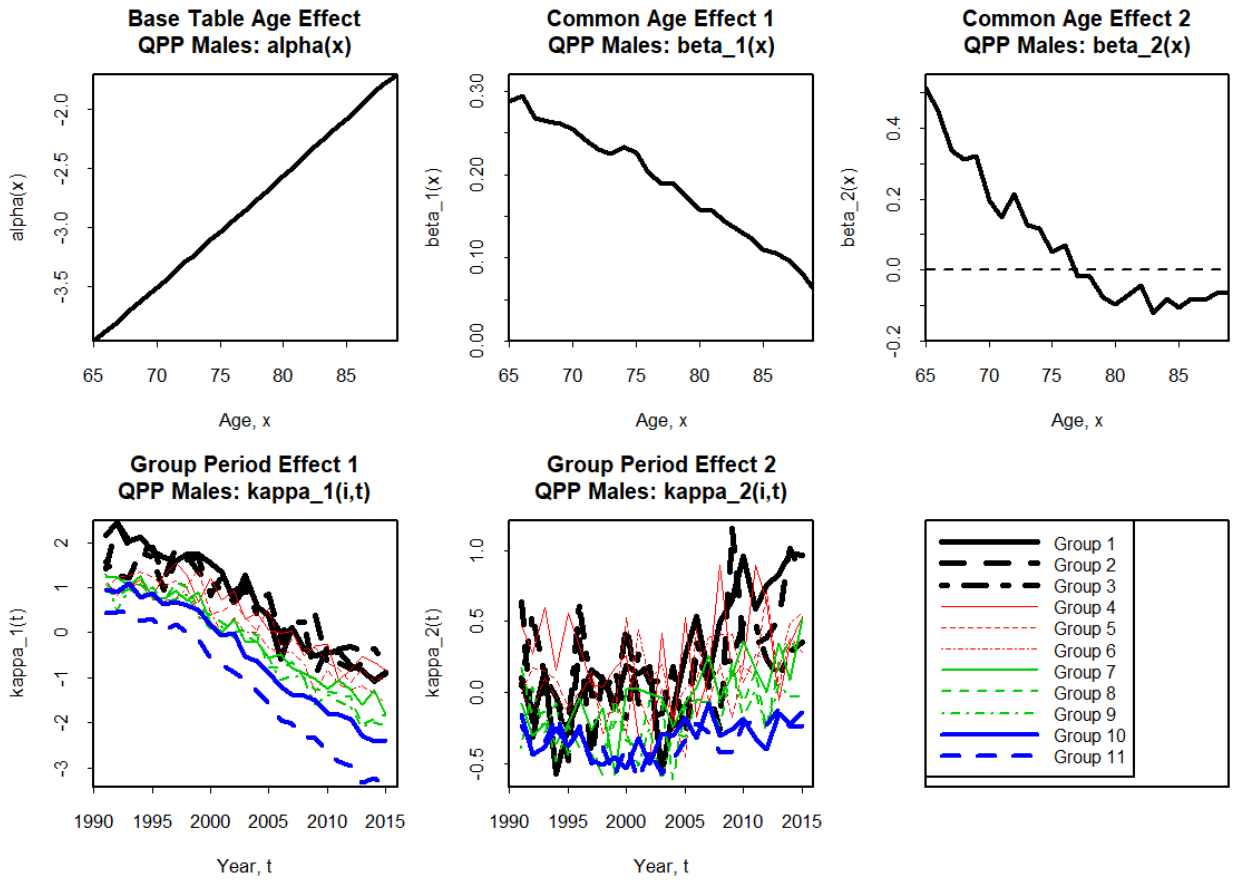


Figure 15: Fitted age and period effects for QPP females.

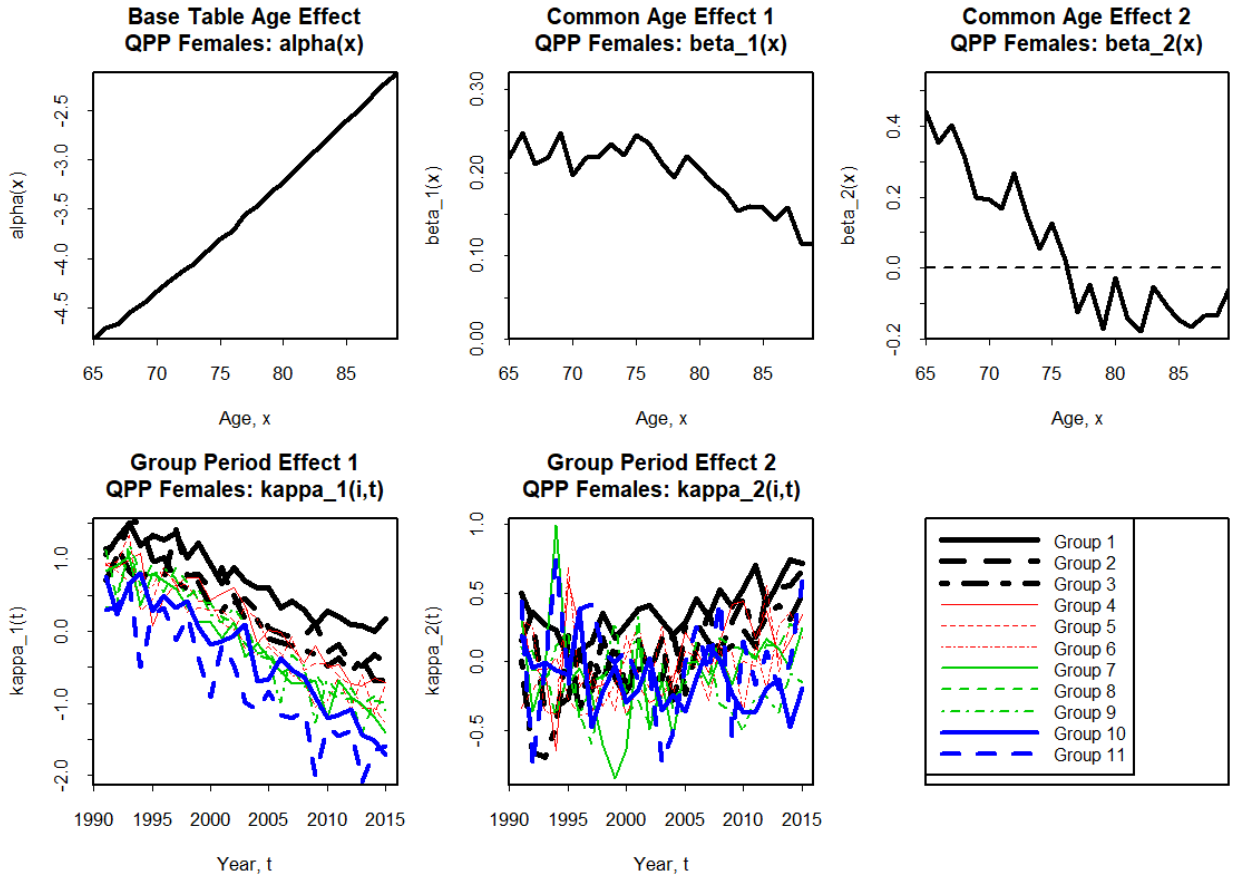


Figure 16: Fitted age and period effects for CPP males.

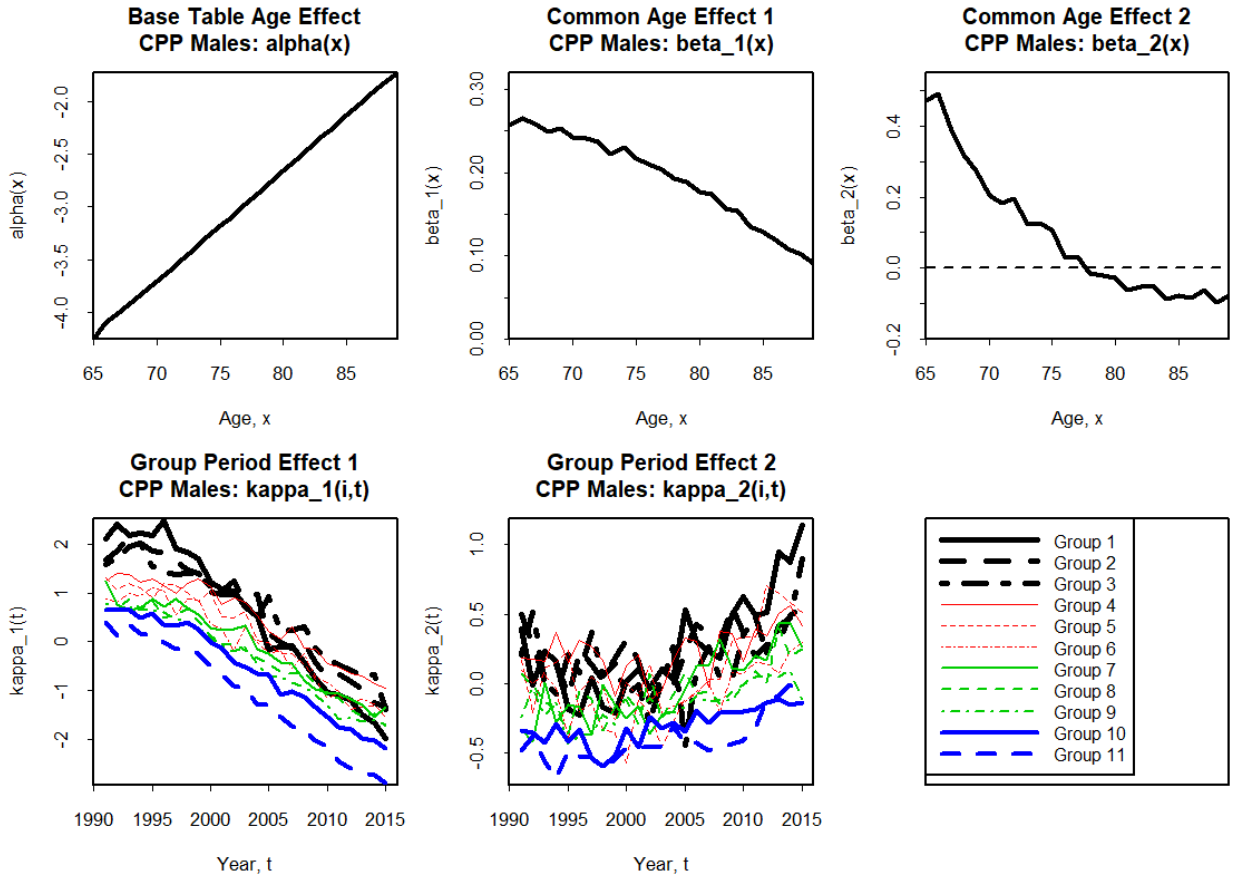
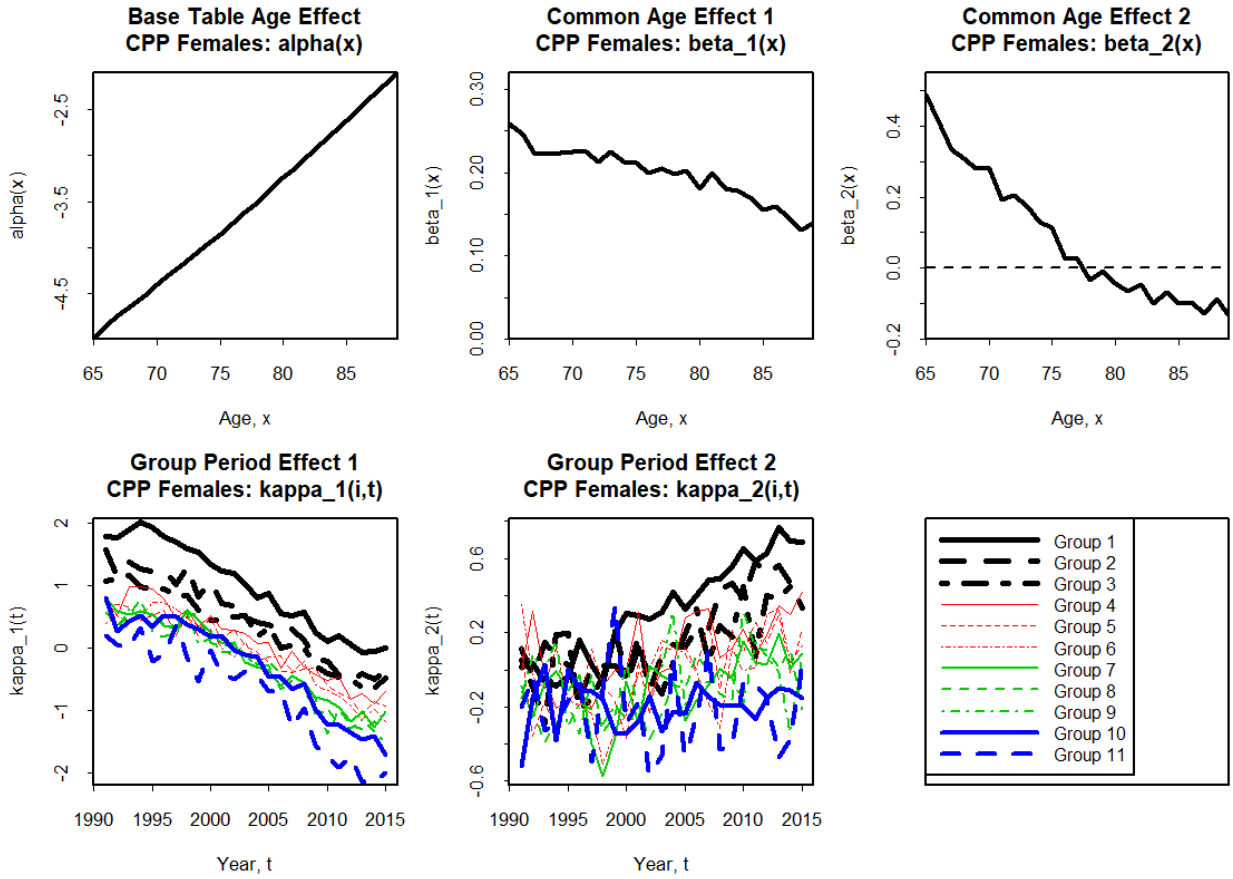


Figure 17: Fitted age and period effects for CPP females.



We can comment on the model and results as follows:[25]

- $\beta_1(x)$ is linked (in combination with the $\kappa_1(i,t)$) to changes in the level of mortality. Specifically, since $\beta_1(x)$ is positive at all ages, if $\kappa_1(i,t)$ falls then death rates fall at all ages and, given the shape of $\beta_1(x)$, fall by a larger percentage at younger ages than older ages.
- Since $\beta_2(x)$ is positive at younger ages and negative at older ages, it is linked to tilts. If $\kappa_2(i,t)$ falls then death rates *fall* at younger ages and *rise* at older ages; i.e. the death-rate curve tilts around, approximately, age 77 (where $\beta_2(x)$ crosses from positive to negative).
- The broad shapes of $\beta_1(x)$ and $\beta_2(x)$ are similar for all four main populations. The $\beta_2(x)$ curves, in particular, are very similar, with a characteristic shape that is fairly steep and then *gradually* flattens off when it turns negative. This specifically links $\beta_2(x)$ to the curvature that we observe in many log-mortality curves, which are less steep in middle age and become more steep in older age,

[25] Appendix C provides a simplified discussion of the role played by the age and period effects as they specifically affect Group 11.

converging to a Gompertz type of mortality curve.

- The similarity of the $\beta_1(x)$ and $\beta_2(x)$ for the four higher-level populations provides evidence that the underlying model is robust.
- For some groups, estimated curves for the period effects are relatively smooth (e.g. QPP males, Group 11) while others are quite volatile (e.g. QPP females, Group 11): both consequences of sampling variation in the observed deaths.
- Since M6 assumes a common base table, $\alpha(x)$, differences in the level of mortality between the 11 groups are modelled through the $\kappa_1(i,t)$ (in particular) and $\kappa_2(i,t)$. In each figure, therefore, we see that the $\kappa_1(i,t)$ estimates for high-pension groups tend to be lower (hence lower mortality) than lower-pension groups.
- The trend, shape and local volatility of the group-specific $\kappa_1(i,t)$ very closely match the shape of the corresponding ASMRs plotted in Figure 8. This indicates that the $\kappa_1(i,t)$ are the main drivers of headline mortality.
- The $\kappa_2(i,t)$ are generally more variable and consequently trends are less easy to detect. However, in some cases, there is an upwards trend in the $\kappa_2(i,t)$ after the late 1990s. This, in combination with decreasing values for the corresponding $\kappa_1(i,t)$,[26] is consistent with increased mortality improvement rates at high ages.
- In each figure we can also see that the $\kappa_2(i,t)$ are typically quite low for high-pension groups (e.g. $i = 10,11$) and high for low-pension groups. Consequently, log-mortality curves tend to be fairly linear for high-pension groups and more convex for low-pension groups.
- A widening gap between the $\kappa_1(i,t)$ reflects generally increasing levels of mortality inequality (e.g. QPP males and females).
- A widening gap between the $\kappa_2(i,t)$ normally means growing levels of inequality at the younger ages much more than higher ages (e.g. CPP females, Groups 1 and 10).

Model-fitting outputs also include estimates of the underlying death rates:

$$\log m(i,t,x) = \alpha(x) + \beta_1(x)\kappa_1(i,t) + \beta_2(x)\kappa_2(i,t).$$

Examples of these for 1995 and 2010 are plotted in Figures 18 and 19.

For the CPP males and females data (Figure 18) some aspects of the plots are as we might expect: higher pension groups experiencing lower mortality, and a wider gap at lower ages converging at higher ages. But there are a number of non-standard features or anomalies that were not evident in the earlier plots of the ASMRs (Figure 8).

[26] Note that the trend in the $\kappa_1(i,t)$ also changes around the same time.

- For CPP males, the 1995 mortality curves look fairly standard. On a more detailed level, we note that Groups 1–6 are tightly clustered at age 65, and then a spread develops by age 70 and beyond, perhaps connected to the healthy-immigrant effect.

  By 2010, the inequality gap has widened at age 65, but not at high ages. More importantly, we can see the anomalous Groups 1 and 2 in more detail than the previous summary ASMRs. At age 65, Groups 1 and 2 have among the highest mortality, but the mortality curves then drift well below Groups 3 and 4, ending up close to Groups 10 and 11. This behaviour is consistent with the healthy-immigrant effect and the heterogeneity that this creates within each group. Although this is a period rather than a cohort mortality curve, Group 1 will consist of a mixture of low-paid, long-stay/native Canadians and higher-paid, newer healthy immigrants. The former will die off at a much faster rate, leaving a much higher proportion of late immigrants in their eighties in Group 1 with low mortality.

For the QPP males and females data (Figure 19) some aspects of the plots are as we might expect: higher pension groups experiencing lower mortality, and a wider gap at lower ages converging at higher ages. But there are a number of non-standard features or anomalies that were not evident in the earlier plots of the ASMRs (Figure 8).

- For QPP males, 1995 looks reasonably "standard" with the exception of Group 4, where older individuals have relatively low mortality. This might be consistent with the healthy-immigrant effect: older pensioners in 1995 would have retired in the early years of QPP when the number of eligible years would have been quite short, allowing healthy, middle-aged immigrants entering the country in the 1970s to accumulate a reasonable pension (e.g. in the 30–40% band) by age 65.

  By 2010 we see a significant widening of the inequality gap at age 65. We also see a potential healthy-immigrant effect has emerged in Group 1 which is similar to, but smaller than, the effect in CPP Group 1 that we discuss further below.

- For QPP females we see some similar features to males: a widening inequality gap and Group 4 having low mortality in 1995.

Figures 18 and 19 can be compared to, and are consistent with, Adam (2016, Slides 12 and 13; 2009–2011 mortality). But, with the more detailed groupings here, we see how mortality inequalities continue to accumulate as we move right into the upper and lower tails of the income spectrum (similar to the patterns observed in Figures 6 and 7 for Danish and English deciles, and for US centiles: Chetty et al., 2016).

Figure 18: Fitted death rates by group for CPP males and females in 1995 and 2010.

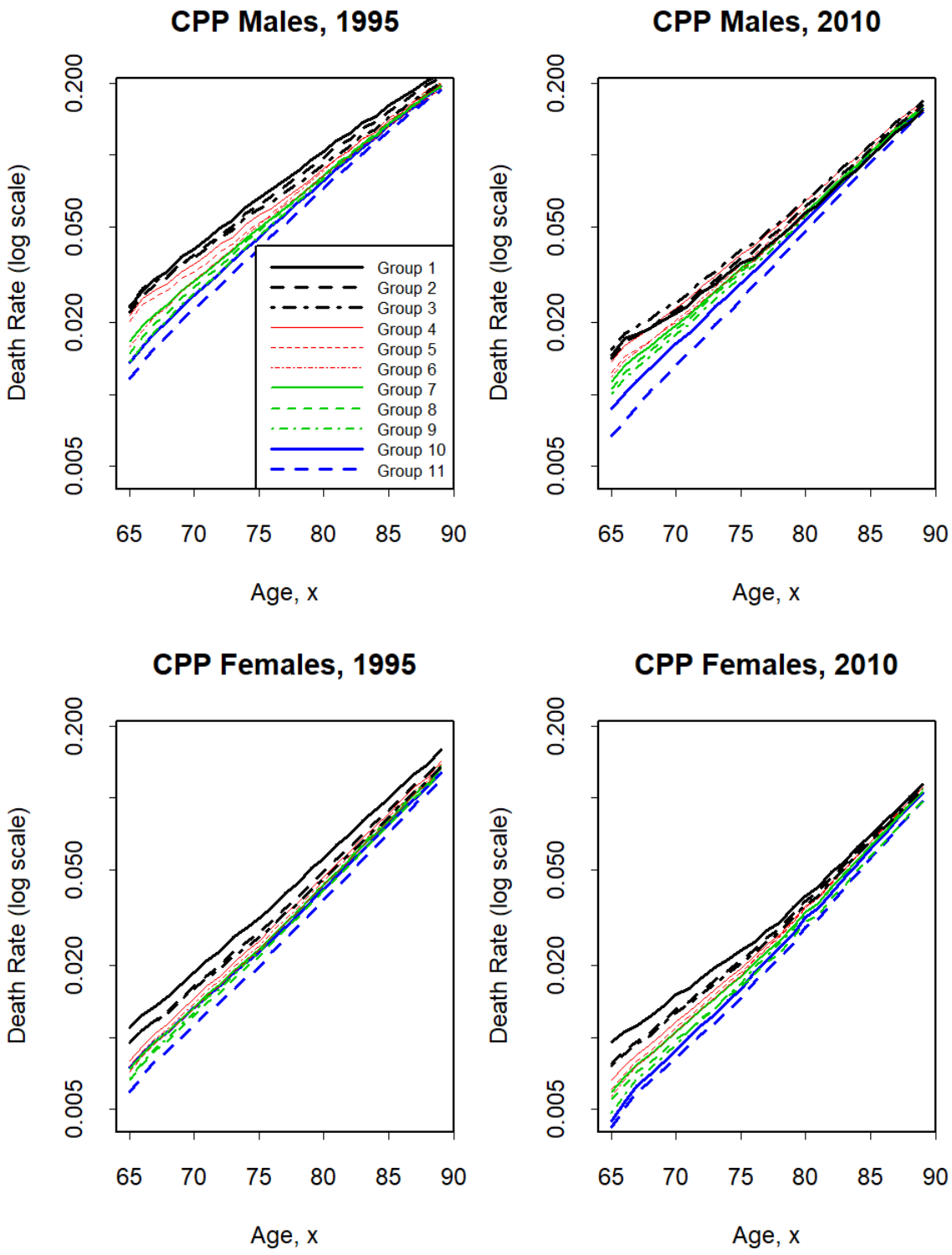
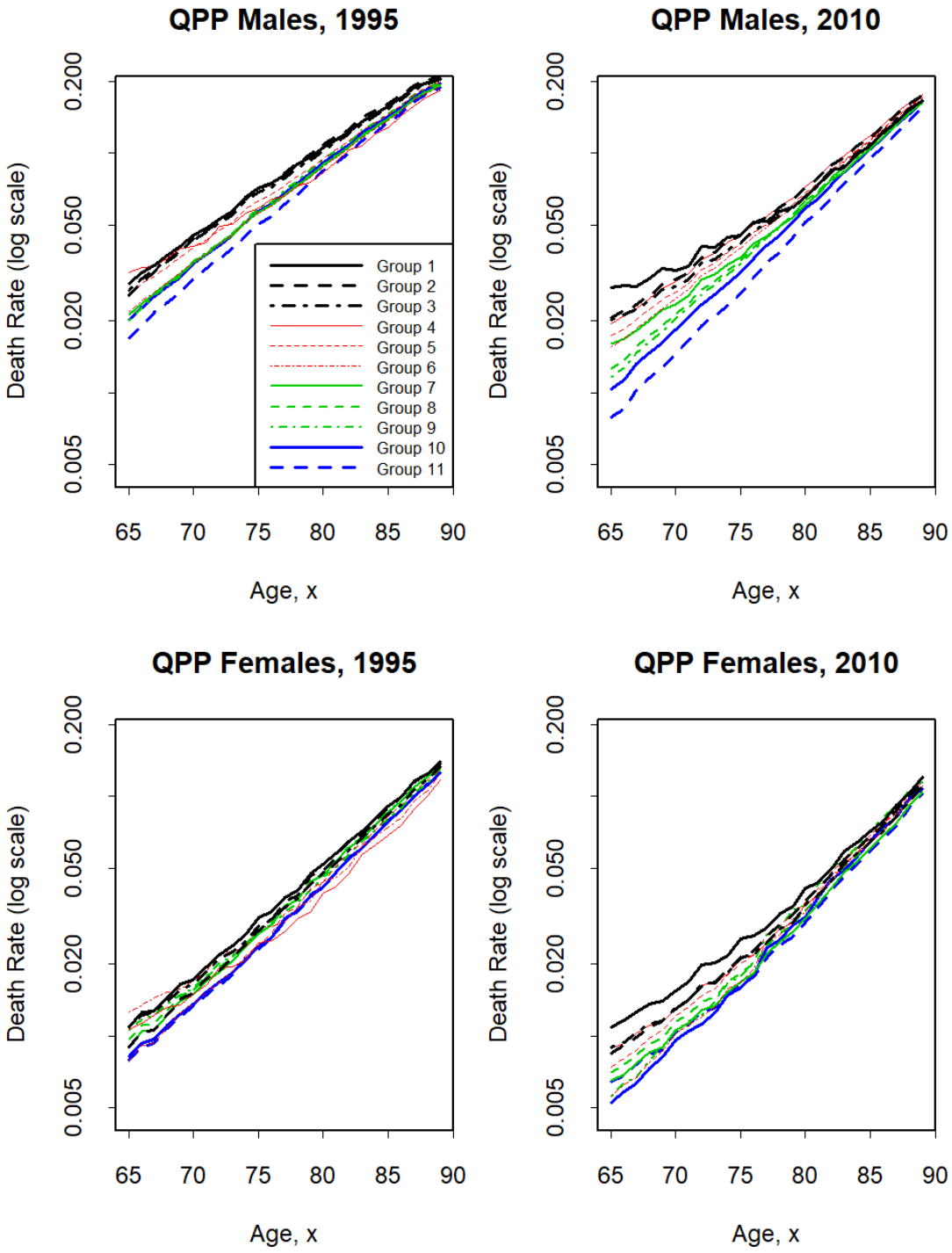


Figure 19: Fitted death rates by group for QPP males and females in 1995 and 2010.



Lastly, to illustrate the results in a different way, the fitted death rates can be used to calculate period life expectancies from age 65. Results are plotted in Figure 20 and give us greater insight into the potential impact of socio-economic differences in mortality on, for example, annuity pricing.[27]

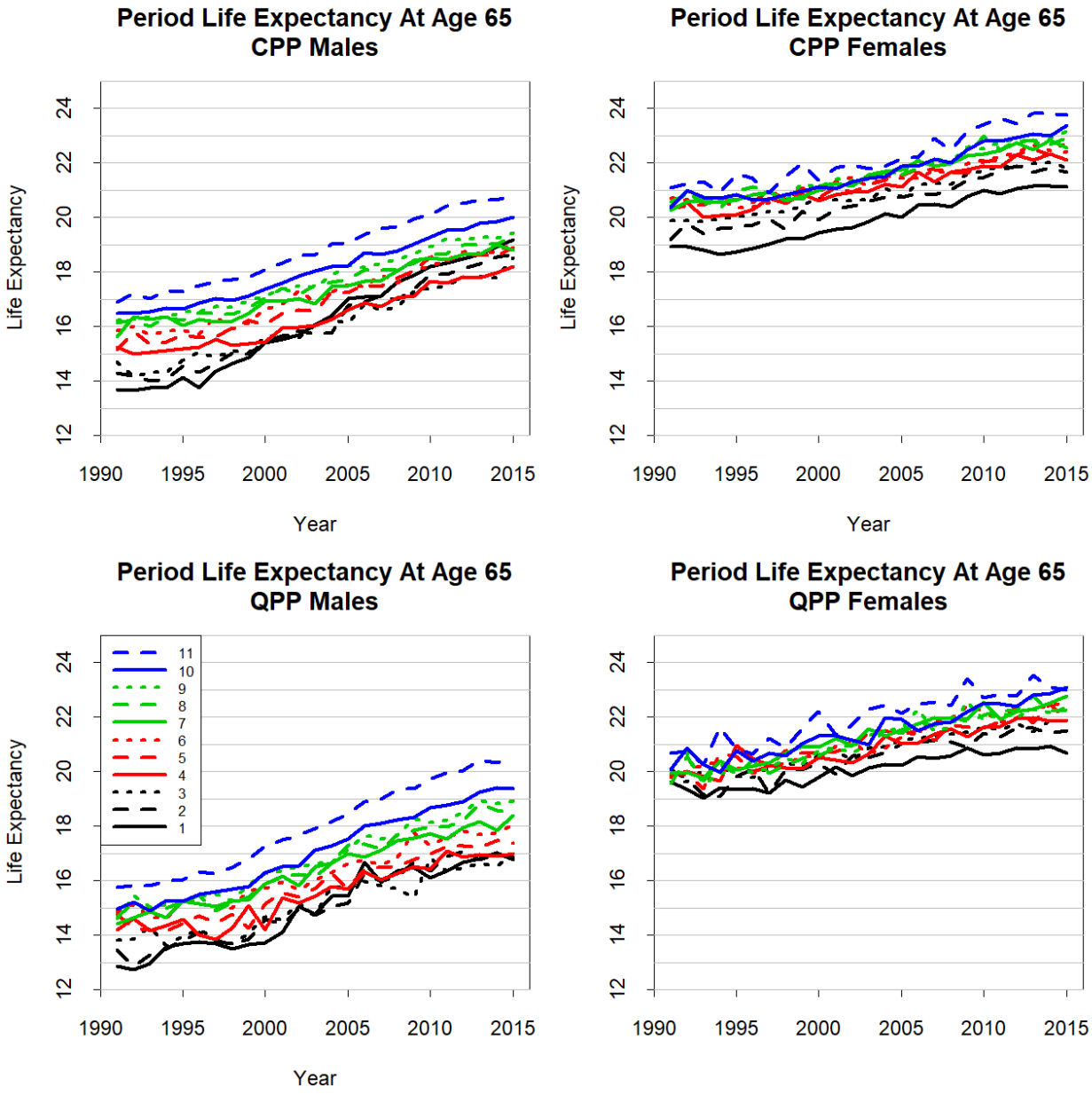
7.4. Evidence for a slowdown

The four panels in Figure 20 exhibit mixed evidence for a slowdown in mortality in recent years. Even with fitted mortality there is some noise in the life expectancies, making it difficult in many cases to detect trend changes. For CPP males and females, one can detect a trend change around 2010/11 in all groups. For QPP females, the trend change looks more pronounced (at least for some groups). For QPP males, the trend change seems more gradual, starting from around 2007, but also a clearer trend change compared to the other populations, perhaps because, prior to 2007, QPP males mortality was improving faster than CPP males.

The slowdown is considered briefly again in the next section, but, broadly, there is no evidence that the slowdown is more pronounced at one or other end of the socio-economic spectrum.

[27] Life expectancies for selected years are also tabulated in Appendix D.

Figure 20: Period life expectancies from age 65 for CPP and QPP males and females from 1991–2015 based on fitted mortality under M6. See also Appendix D.



8. Clustering

In the last section it was evident that large groups (e.g. QPP and CPP males, Groups 10 and 11) have relatively smooth estimates for the period effects (Figures 14 and 16, $\kappa_1(i,t)$, Groups 10 and 11). In contrast, some groups (see, for example, Figure 8, QPP males, Groups 1 to 4) typically represent only 2–4% of each cohort with more volatile estimates of the period effects (correspondingly, Figure 14, Groups 1 to 4). The reason for this is that, even after model fitting, greater sampling variation in death counts in small groups results in noticeably greater sampling variation in fitted period effects.[28] A consequence of this additional noise in fitted period effects is that forecast levels of uncertainty in future mortality can be artificially high for these small groups (Chen et al., 1997; Villegas et al., 2017).

At the same time, we can note that some adjacent groups typically have quite similar levels of mortality. Combining this with the slightly artificial choice of group boundaries (10%, 20%, etc., of the maximum pension), it would seem appropriate to consider combining adjacent groups into *clusters*. Clusters will typically improve the results if groups are small and have similar levels of mortality.

A systematic approach was taken to consider all possible clusters of the 11 groups. For QPP males using model M6, for example, we found that four clusters were optimal (optimal BIC), as detailed in Table 3. For the same population, other models typically, but not always, found the same four clusters to be optimal. Optimal clustering differed more for other populations, especially females (partly because of their very different group sizes).

[28] For further discussion of the impact of group size, see Chen et al. (2017).

Table 3: Optimal clustering for QPP and CPP males and females aged 65–89 and years 1991–2015.

	Cluster	Groups	Exposures
QPP Males	1	1–5	1.799 M
	2	6–8	2.124 M
	3	9–10	3.956 M
	4	11	2.826 M
QPP Females	1	1–2	3.138 M
	2	3–11	6.572 M
CPP Males	1	1–4	3.699 M
	2	5–8	6.326 M
	3	9–10	10.740 M
	4	11	8.277 M
CPP Females	1	1	4.832 M
	2	2–3	5.877 M
	3	4–6	6.813 M
	4	7–11	10.287 M

Fitted ASMRs for QPP males under M6 with and without clustering are plotted in Figure 21. Without clustering, we can clearly see how the smaller groups, 1–5, produce high levels of volatility in the ASMR from year to year. With clustering, we can see that the ASMRs for the four clusters all now exhibit similar levels of volatility: partly due to the larger sizes generally, partly because the clusters are all more similar in size (Table 3).

With clustering, the smoother ASMRs also allow us to see more clearly the different trends experienced by the different QPP clusters. In particular, we see a widening of the gap between the different clusters of QPP males. The reasons for this are not clear, although it might, in part, be due to the changing cluster sizes by cohort (Figure 10).

Figure 22 shows the fitted age effects, $\alpha(x)$, $\beta_1(x)$ and $\beta_2(x)$, without and with clustering. Importantly, the shift from 11 groups to four clusters has relatively little impact on estimates of $\beta_1(x)$ and $\beta_2(x)$: further evidence that model M6 is robust.[29]

Corresponding results for QPP females and CPP males and females are plotted in Figures 23–28.

For CPP males (Figures 25 and 26), the picture is much clearer with clusters compared to 11 groups, now that ASMRs have been smoothed out. Cluster 1 (Groups 1–4) shows the conjectured impact of the healthy-immigrant effect. Perhaps because of this, there is little evidence for a widening inequality gap. It is tempting to compare Cluster 4 for QPP and CPP males, the latter being lower. However, we need to recall that the QPP data include disability pensioners while the CPP data do not.

[29] The small shift in $\alpha(x)$ in Figure 22 is due to the application of identifiability constraints in the model fitting process.

Both QPP males and CPP males exhibit a slight widening of the gap between Clusters 3 and 4, potentially linked to the gradual shrinkage of Group 11 over time (Figures 9 and 10).

We can also look for evidence of a slowdown in mortality improvements. The ASMRs based on clusters are generally smoother than the 11 groups, making it easier to detect trend changes. However, similar comments and conclusions to those in Section 7.4 apply. Clustering does not reveal any further insights and we conclude that the slowdown seems to have affected all groups.

Figure 21: ASMRs based on fitted mortality using model M6 for QPP males from 1991–2015 for the original groups (thin lines) and the optimal clusters (thick lines).

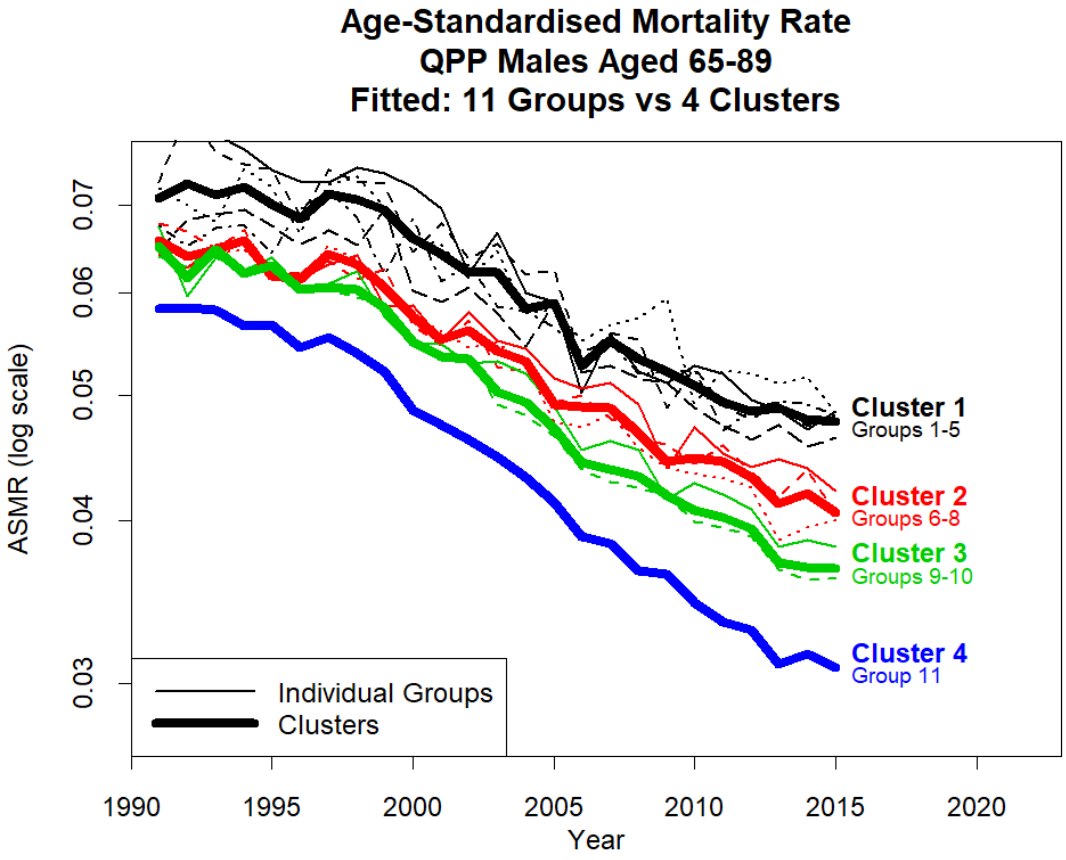


Figure 22: Common age effects, $\alpha(x)$, $\beta_1(x)$ and $\beta_2(x)$, for QPP males without (11 Groups) and with (four clusters) clustering.

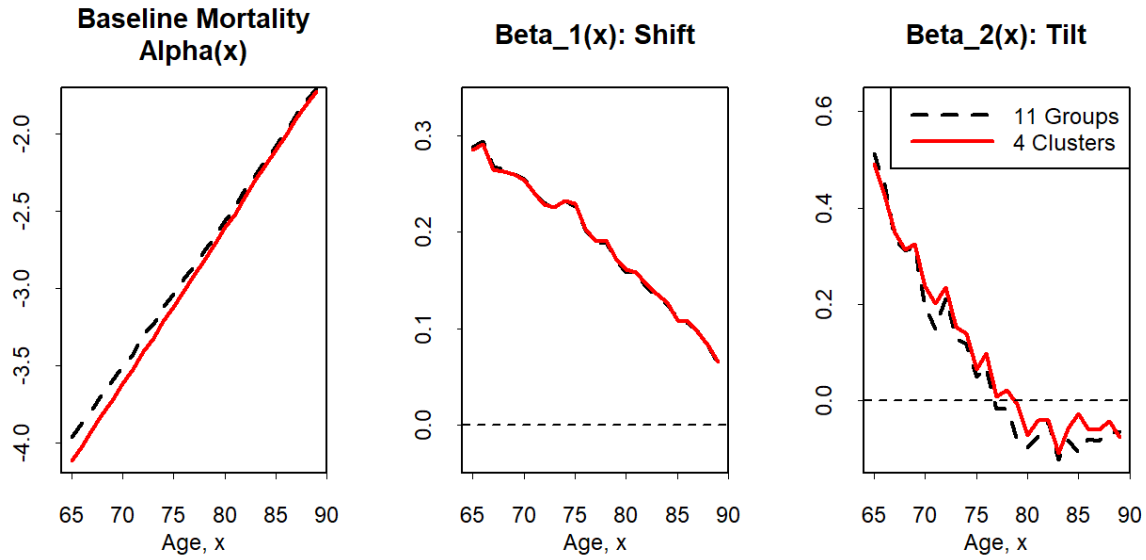


Figure 23: ASMRs based on fitted mortality using model M6 for QPP females from 1991–2015 for the original groups (thin lines) and the optimal clusters (thick lines).

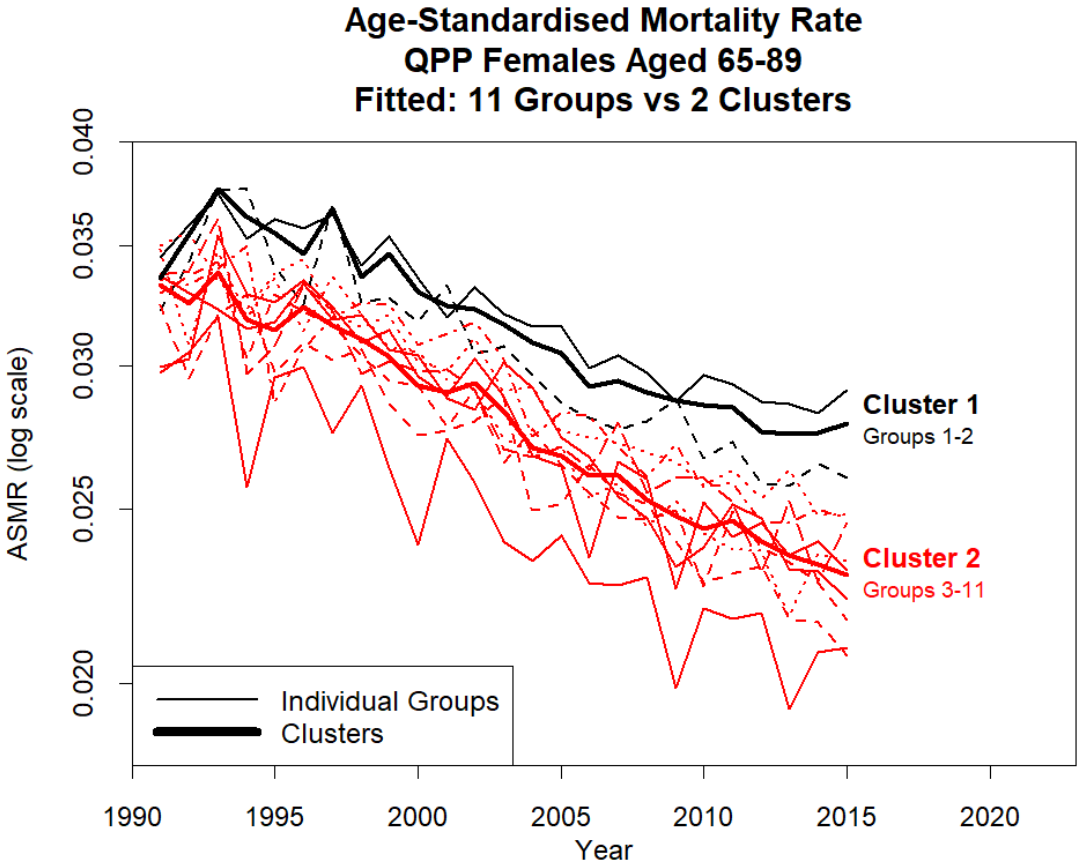


Figure 24: Common age effects, $\alpha(x)$, $\beta_1(x)$ and $\beta_2(x)$, for QPP females without (11 Groups) and with (two clusters) clustering.

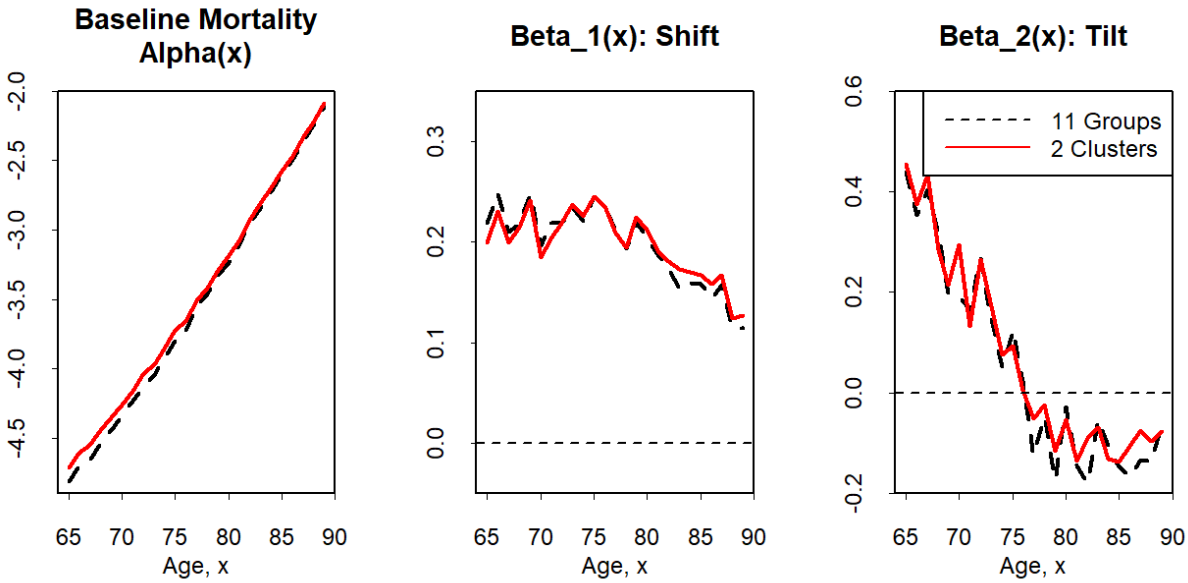


Figure 25: ASMRs based on fitted mortality using model M6 for CPP males from 1991–2015 for the original groups (thin lines) and the optimal clusters (thick lines).

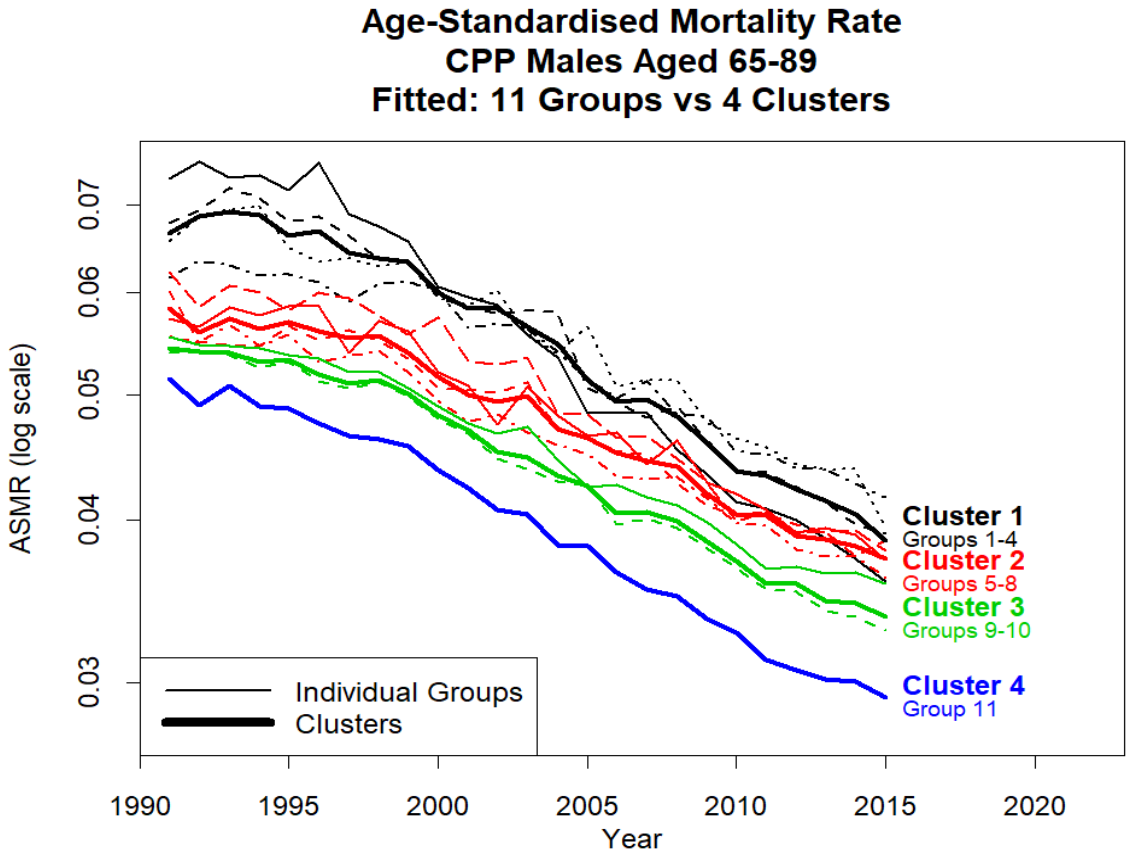


Figure 26: Common age effects, $\alpha(x)$, $\beta_1(x)$ and $\beta_2(x)$, for CPP males without (11 Groups) and with (four clusters) clustering.

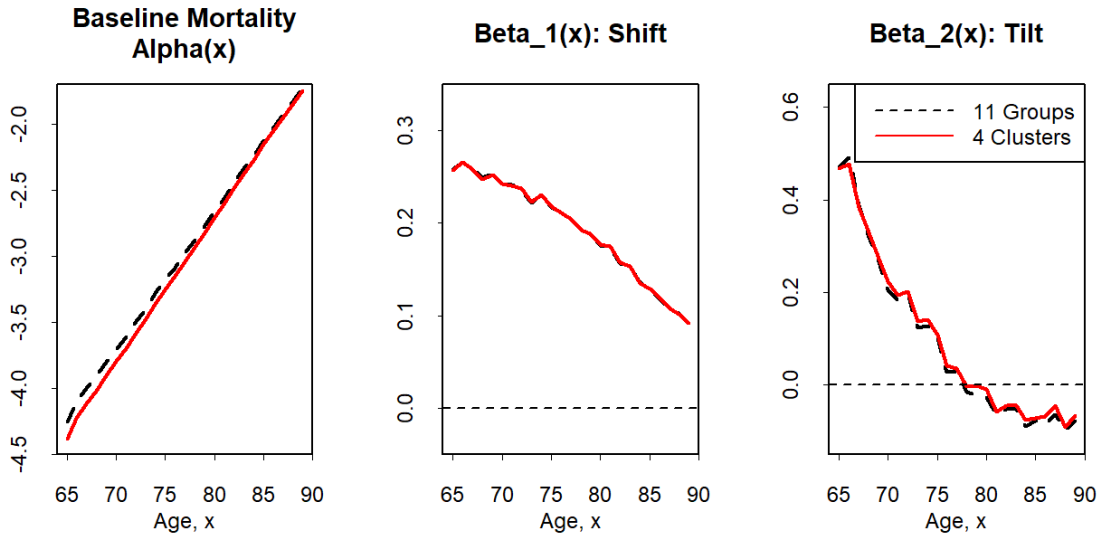


Figure 27: ASMRs based on fitted mortality using model M6 for CPP females from 1991–2015 for the original groups (thin lines) and the optimal clusters (thick lines).

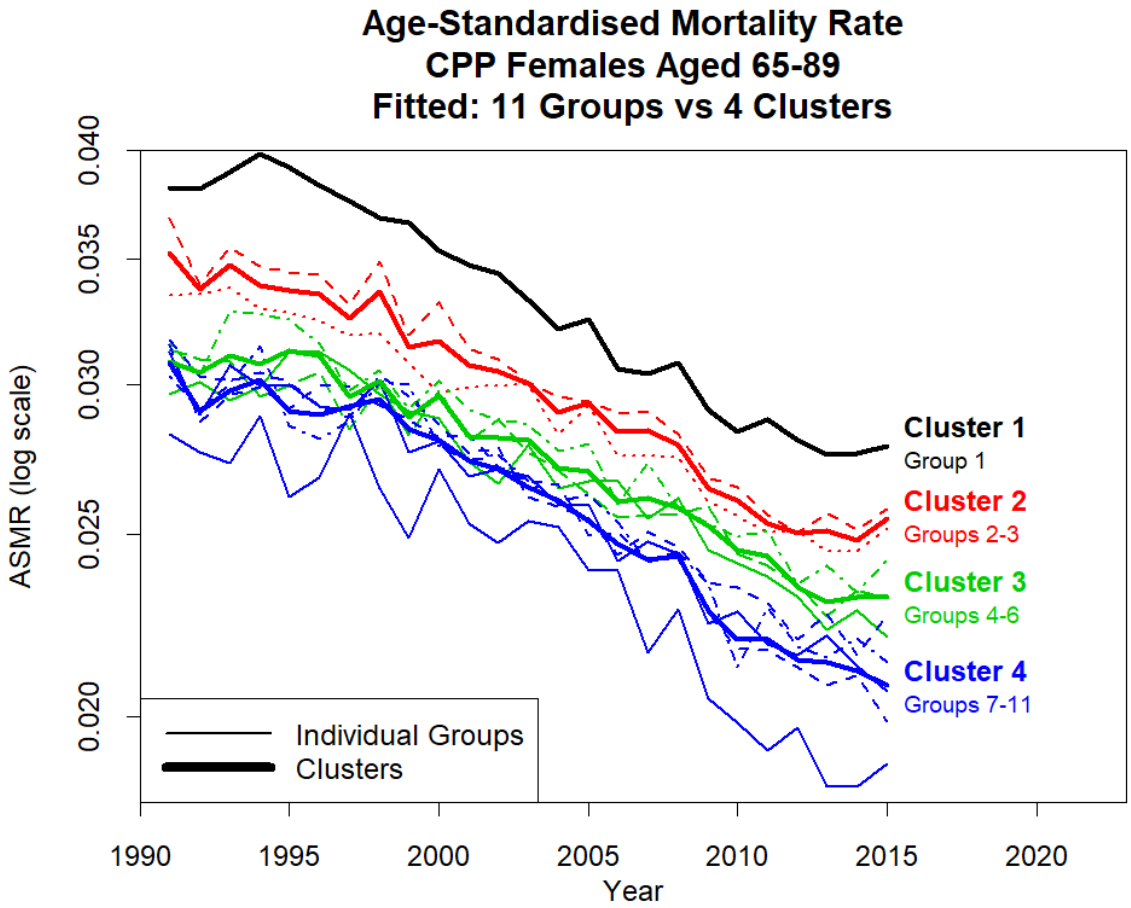
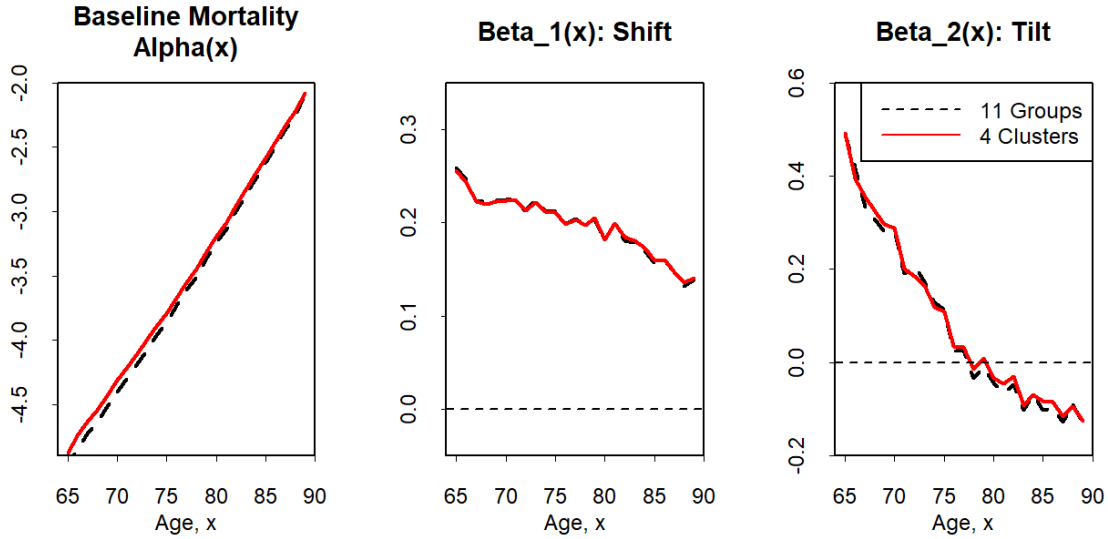


Figure 28: Common age effects, $\alpha(x)$, $\beta_1(x)$ and $\beta_2(x)$, for CPP females without (11 Groups) and with (four clusters) clustering.



9. Conclusions

We have carried out a detailed analysis of the mortality of CPP and QPP pensioners subdivided by pension level. Analysis has been backed up by a careful look at how the profile of each cohort by pension level has changed over time, as well as the impact of migration on mortality levels.

Headline conclusions were consistent with what we observe in other countries:

- significant variation in the level of mortality by pension level between all pension levels, especially at younger ages
- the inequality gap narrows with age.

Other important conclusions could not have been immediately anticipated at the outset:

- different patterns of inequality for males and females across the 11 groups
- greater levels of inequality in the QPP than the CPP
- a prominent healthy-immigrant effect that has a significant impact on the observed mortality of low-pension groups in the CPP
- a widening inequality gap between ages 65 and 75.

(The reasons for differing levels of inequality and the widening inequality gap are not clear and are likely to be complex.)

The second half of the paper considered how stochastic mortality models can be used to enhance our analysis of historical mortality, as well as provide a stepping stone towards projections. A wide variety of multi-population mortality models were considered and, through consideration of a mixture of quantitative and qualitative criteria, we found

that the CAE Model (M6) was best suited to the data being considered. The model indirectly provides a smoothing mechanism by pooling data over many more years than a traditional actuarial graduation. The model outputs then reveal further detail in the data not previously apparent, including an indication of the strength of the healthy-immigrant effect. Concerning the recent slowdown in Canadian mortality improvements, no evidence was found in the CPP or QPP grouped data for the slowdown being concentrated more at one or other end of the socio-economic spectrum.

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Appendices

A. Model Selection Criteria Assessment

Desirable criteria for a model to satisfy are:

- BIC: Model has a low BIC score, although not necessarily the best.
- GD: Graphical diagnostic tests[30] all or mostly satisfactory. Conclusions will depend on the fit of each model to the specific datasets.
- Coh: Multi-population model should satisfy the principle of coherence. Assessment considers whether or not the model prevents mortality rates in two populations from diverging over time and is not dependent on the datasets used.
- Cross: Model should not impose in-year mortality curve crossovers where these are not apparent in the raw data. Assessment is partly dependent on the fit of the model to the specific data, and partly based on the potential for crossovers to arise in future scenarios.
- Corr: Does the model produce a plausible forward correlation term structure, $\rho(t, i, j, x_i, x_j)$: within a population, non-trivial correlations between ages; between populations, non-trivial correlations; lower correlations between a pair of ages in different populations than within the same population; and lower correlations for ages that are further apart? Assessment is not dependent on the datasets used.

[30] Graphical diagnostic tests (GDs) offer a more informal alternative to more formal hypothesis tests; Figure 29 gives some examples. A GD will typically have an underlying hypothesis. If the hypothesis turns out to be valid then the GD should exhibit certain characteristics. For example, in the top-left panel of Figure 29 the hypothesis is that the residuals should be independent of each other, leading to a GD that should exhibit a random pattern of reds and blues. If the GD does not exhibit the anticipated characteristics (e.g. if the top-left panel of Figure 29 had clear clusters of reds and blues) then it is likely that the underlying hypothesis is not true. Additionally, the characteristics that we do observe can point to how the model might be improved.

Table 4: Assessment of model selection criteria for each model and dataset. BIC values should only be compared within the same column (same dataset). Ticks indicate that a specific model satisfies a particular criterion or scores well relative to other models.

	BIC				Criterion				
	CPP	CPP	QPP	QPP					
	Males	Females	Males	Females	BIC	GD	Coh	Cross	Corr
M1	63936	61415	56265	52401	✗	✓	✗	✗	✓
M2	62106	59638	54389	50598	✗	✓	✗	✗	✓
M3	60466	57854	52588	48733	✗	✓	✗	✓	✗
M4	60411	57737	52515	48658	✗	✓?	✗	✗	✗
M5	60598	58051	52775	48957	✗	✓	✓	✓	✓
M6	58823	56277	51030	47185	✓	✓	✓	✓	✓
M7	60502	57770	52526	48750	✗	✗	✓	✗	✓
M8	58764	56000	50798	46921	✓	✗	✓	?	✓
M9	59484	56244	51168	46965	✓	✗	✓	✗?	✓
M10	58774	56113	50835	46939	✓	✗	✓	✗?	✓
M11	57747	54546	49493	45139	✓	✗	✓	✓	✗

Table 5: Graphical diagnostic results for all underlying models. For CPP datasets Group 11 demonstrates significant non-random pattern in standardised residuals under all models (see, for example, Figure 30). Therefore any model that behaves well for Groups 1–10 and not obviously worse than other models for Group 11 is marked as 51.

Model	QPP males	QPP females	CPP males	CPP females
M1	✓	✓	✓	✓
M2	✓	✓	✓	✓
M3	✓	✓	✓	✓
M4	?	✓	✓	✓
M5	✓	✓	✓	✓
M6	✓	✓	✓	✓
M7	✓	✓	✓ ✗	✗
M8	✓	✓	✓ ✗	✗
M9	✗	✓	✗	✗
M10	✗	✓	✗	✗
M11	✗	✓	✗	✗

Our assessment of the models against these criteria, and BIC values, are presented in Table 4, with further detail on graphical diagnostics in Table 5. The nesting of models means that, for example, model M1 has a higher maximum log-likelihood than other models, but it is heavily penalised for being over-parameterised, leading to a poorer BIC than all of the other models.

Examples of graphical diagnostics are given in Figures 29–32. Standardised residuals are defined as

$$Z(i,t,x) = (D(i,t,x) - m(i,t,x)E(i,t,x))/\sqrt{m(i,t,x)E(i,t,x)},$$

and should be approximately independent and identically distributed standard normal random variables if we have a good model. Figure 29 is very typical for the great majority of groups, populations and models. Figure 30 highlights a potential cohort effect (see, for example, Cairns et al., 2009) that arises with most models for CPP males in Group 11 only.[31] Comparison of Figure 30 panels (c) and (d) reveals some similarity in the pattern from about 1918 onwards. Panel (c) is a standard plot of the residuals while panel (d) shows the proportion of each cohort by year of birth in Group 11 (dots at different ages, red line at age 65 only for younger cohorts). In particular, given that Group 11 is the top group, if it is smaller (so more concentrated on the most sustained high earners) then we might expect (even) lower mortality than would be the case if the proportion in Group 11 remained the same from year to year.

For these datasets (especially the age range of 65–89) we could equally opt for M8 rather than M6. However, our reasons for preferring M6 over, say, M8 are:

- M6 extends more easily to younger ages, whereas the linear age-period effects in M8 lead to a poorer fit over a wider age range.
- The linear age-period effects also lead to minor crossover problems in some years. This is implicit in a comparison of M6 and M8 for Group 1 in Figures 31 (M6) versus 32 (M8). In Figure 32(a) there are clusters of red cells in the later years at high and low ages. At the high ages this causes a crossover that is not evident in the data in these later years.

[31] In this context, cohort effects can arise when the individual groups still contain some residual degree of heterogeneity. If the balance between "sub-groups" changes by cohort, this has an impact on levels of mortality by cohort.

Figure 29: Model M6 graphical diagnostics for CPP males Group 6 using standardised residuals, $Z(i,t,x)$. (a) heat map of the $Z(i,t,x)$ (red – positive; blue – negative) (b) residuals by age (c) residuals by calendar year. (d) residuals by year of birth.

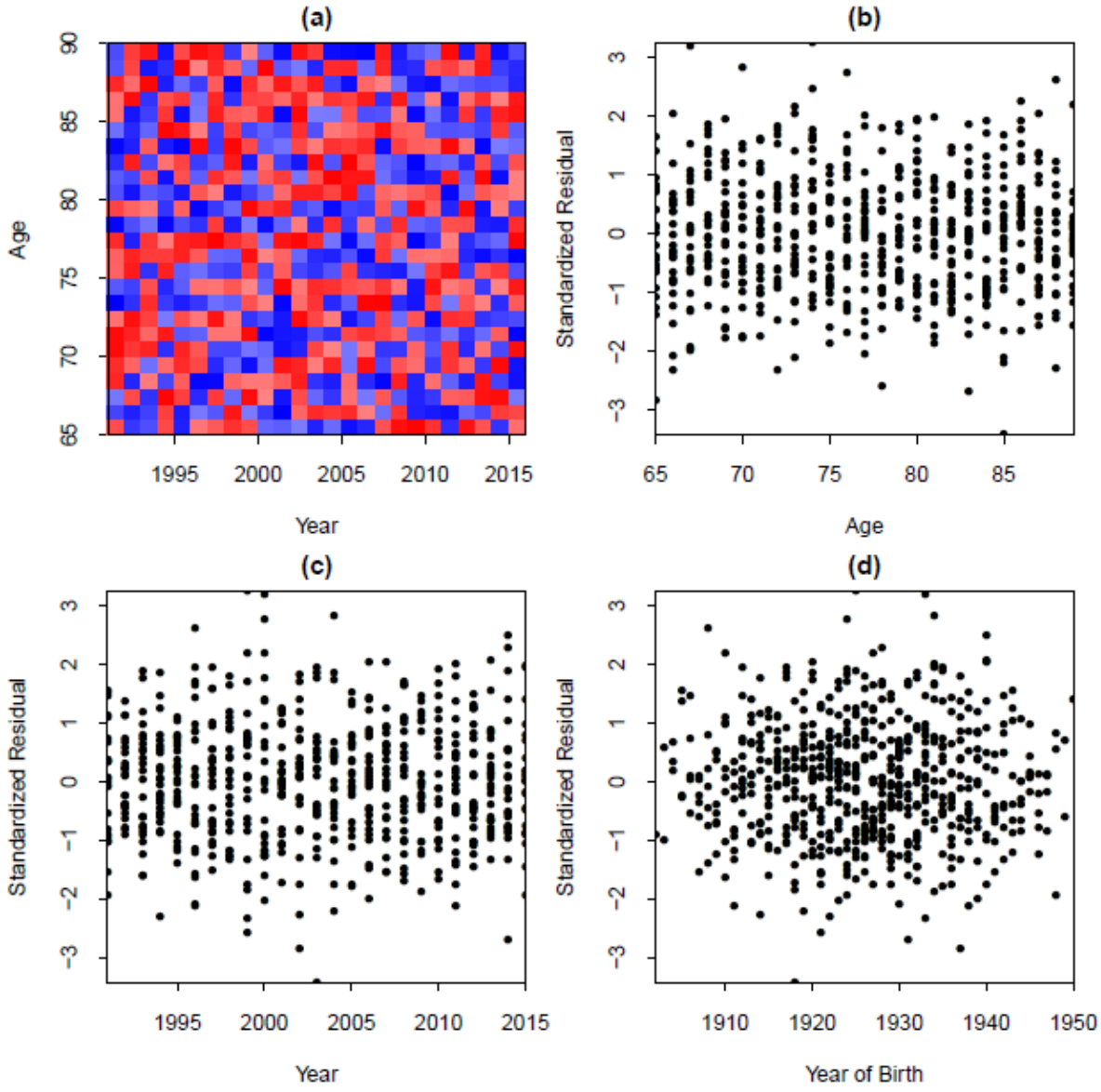


Figure 30: Model M6 graphical diagnostics for CPP males Group 11 using standardised residuals, $Z(i,t,x)$. (a) heat map of the $Z(i,t,x)$ (red – positive; blue – negative) (b) residuals by age (c) residuals by year of birth (d) proportion of total (t,x) exposures in Group 11 by cohort (red line: proportions at age 65 by cohort) with linear adjustment.

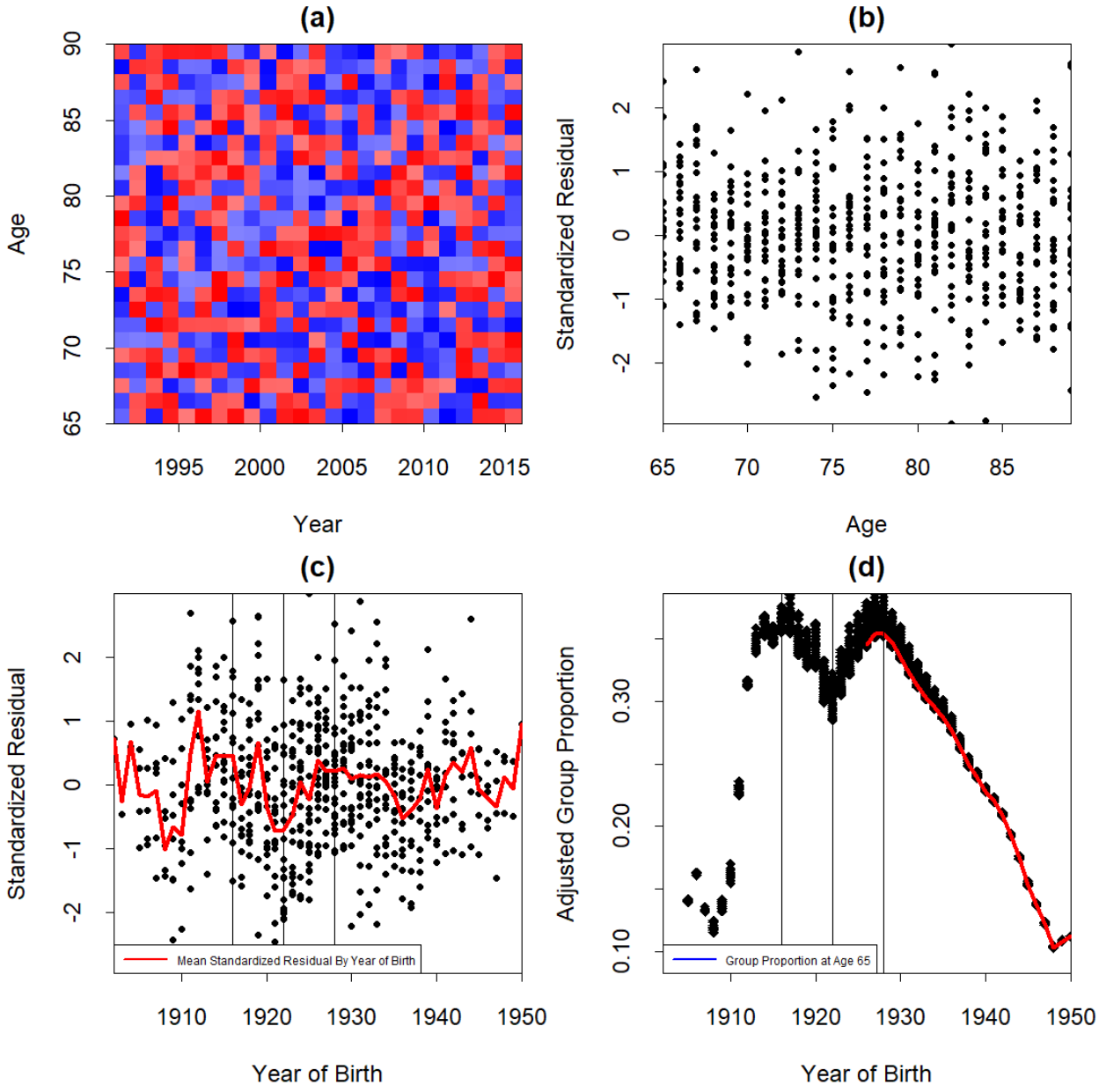


Figure 31: Model M6 graphical diagnostics for CPP males Group 1 using standardised residuals, $Z(i,t,x)$. (a) heat map of the $Z(i,t,x)$ (red – positive; blue – negative) (b) residuals by age (c) residuals by calendar year (d) residuals by year of birth.

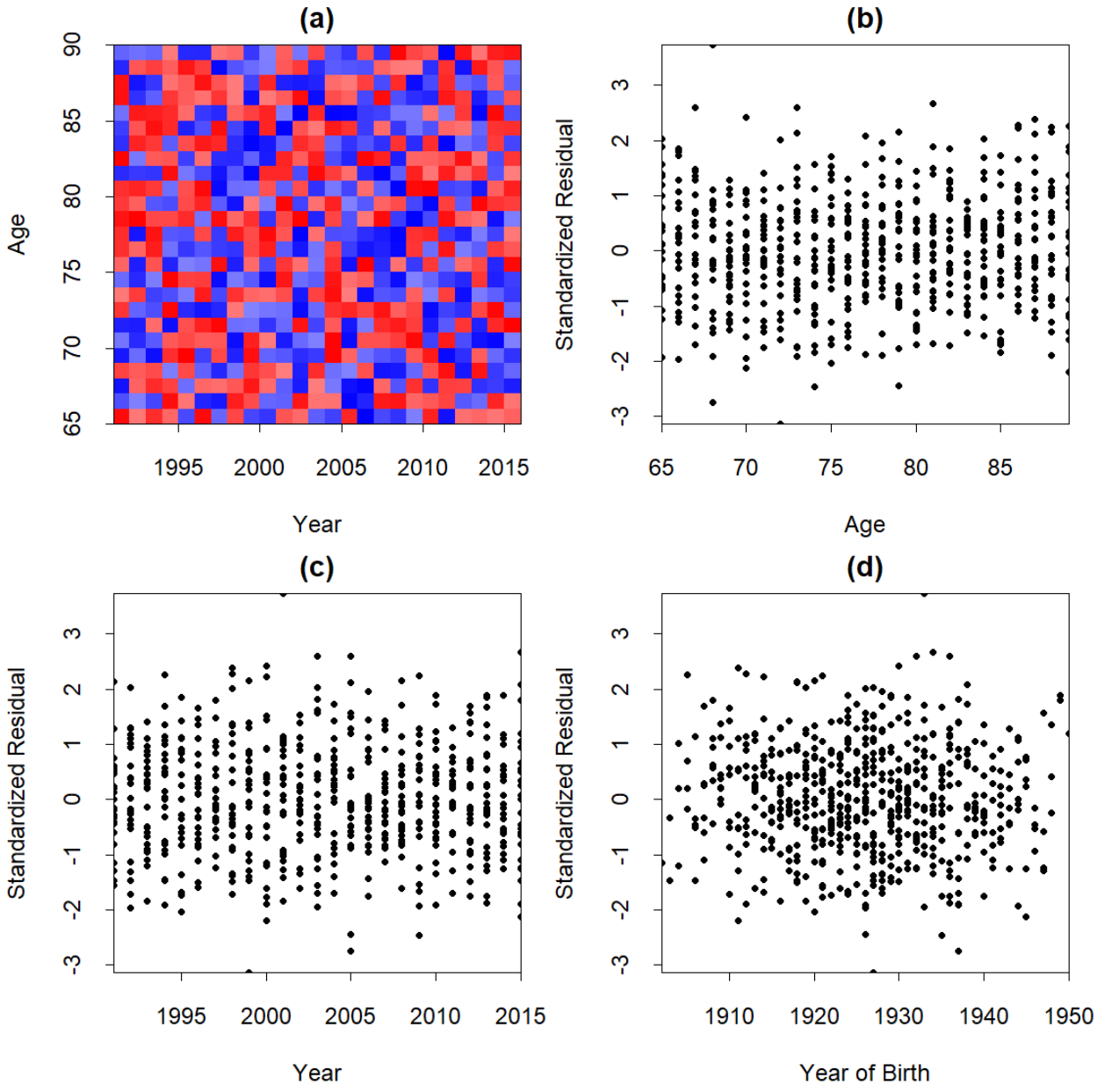
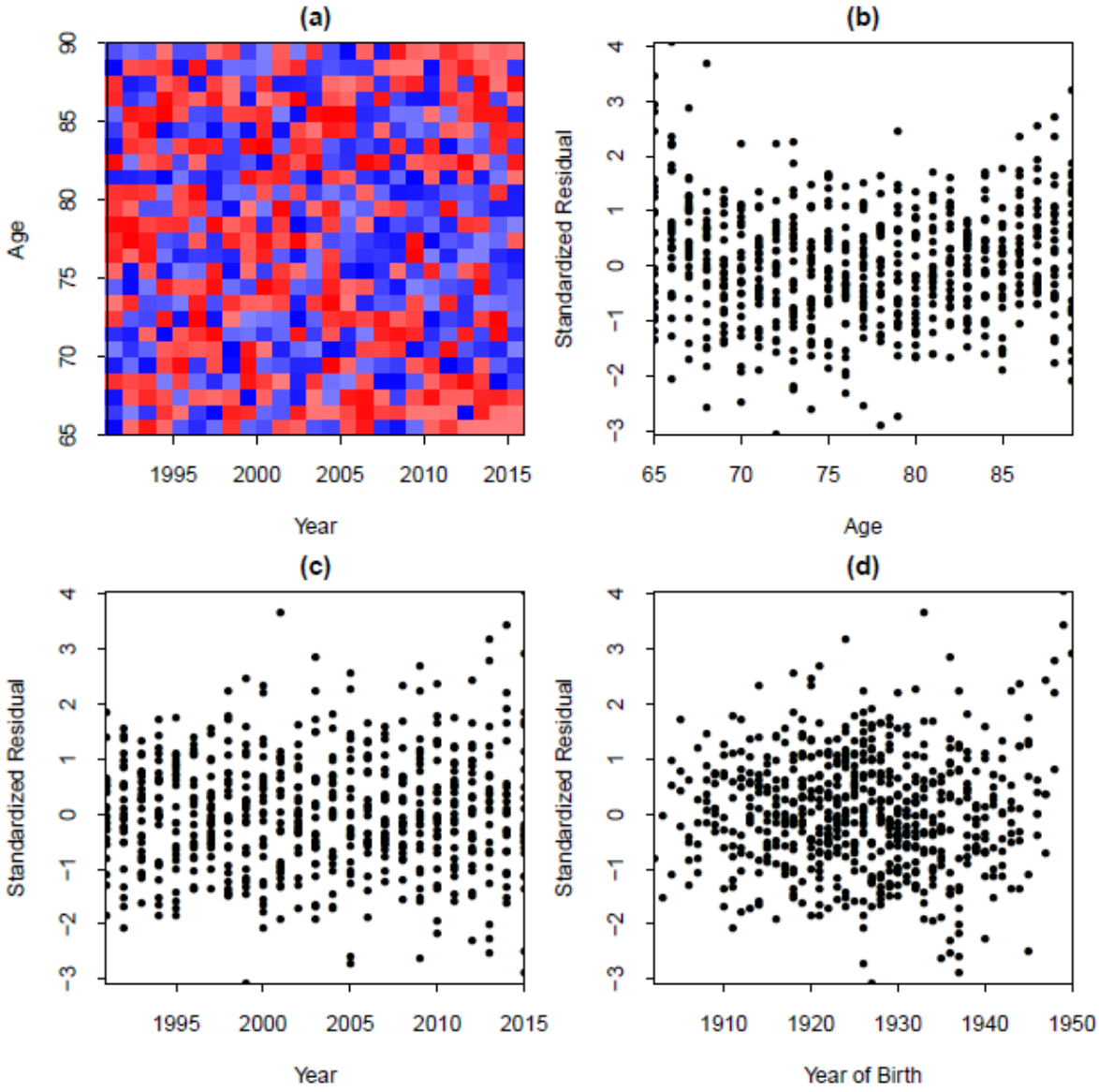


Figure 32: Model M8 graphical diagnostics for CPP males Group 1 using standardised residuals, $Z(i,t,x)$. (a) heat map of the $Z(i,t,x)$ (red – positive; blue – negative) (b) residuals by age (c) residuals by calendar year (d) residuals by year of birth.

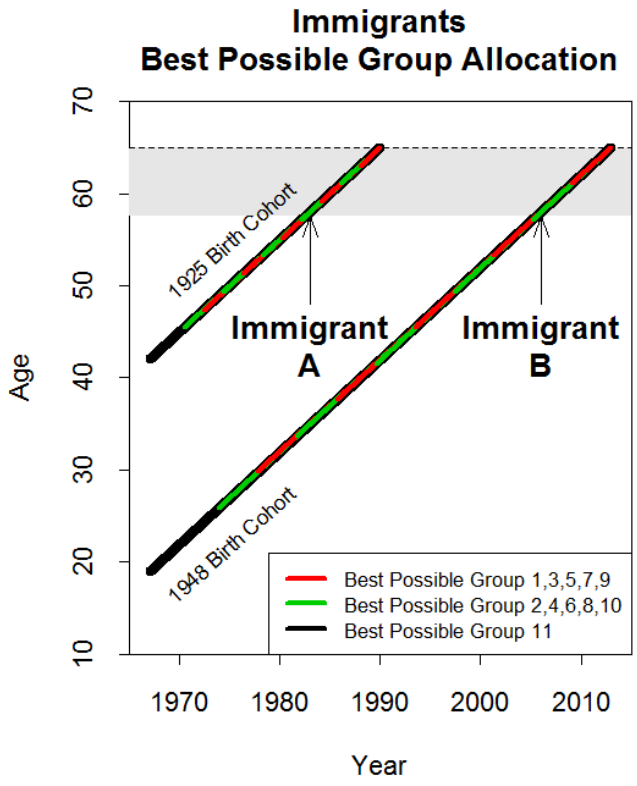


B. Growing Proportion of Immigrants in Low Groups

Figure 33 shows how, as the plan has matured, the balance of immigrants in the pensioners population will have shifted towards lower groups. Consider two immigrants A and B who both migrated to Canada at age 58. A belongs to the 1925 birth cohort, B to the 1948 cohort. If A earns above the YMPE from age 58 to 64 then he/she can retire at age 65 with a CPP pension that will be just under 40% of the maximum (Group 4) but no higher without deferring retirement. In contrast, immigrant B cannot do better than Group 2.

From the 1925 cohort, migration to Canada in their early 40s would be sufficient to make attainment of the maximum pension possible. From the 1948 cohort, migration to Canada would have had to be in their early 20s.

Figure 33: Attainability of different pension levels by cohort for immigrants at different ages. Immigrants A and B both arrive in Canada at age 58. If immigrant A (1925 birth cohort) earns consistently above the YPME then he/she will still only be in Group 4 on retirement at 65. Immigrant B (1948 birth cohort) cannot do better than Group 2.



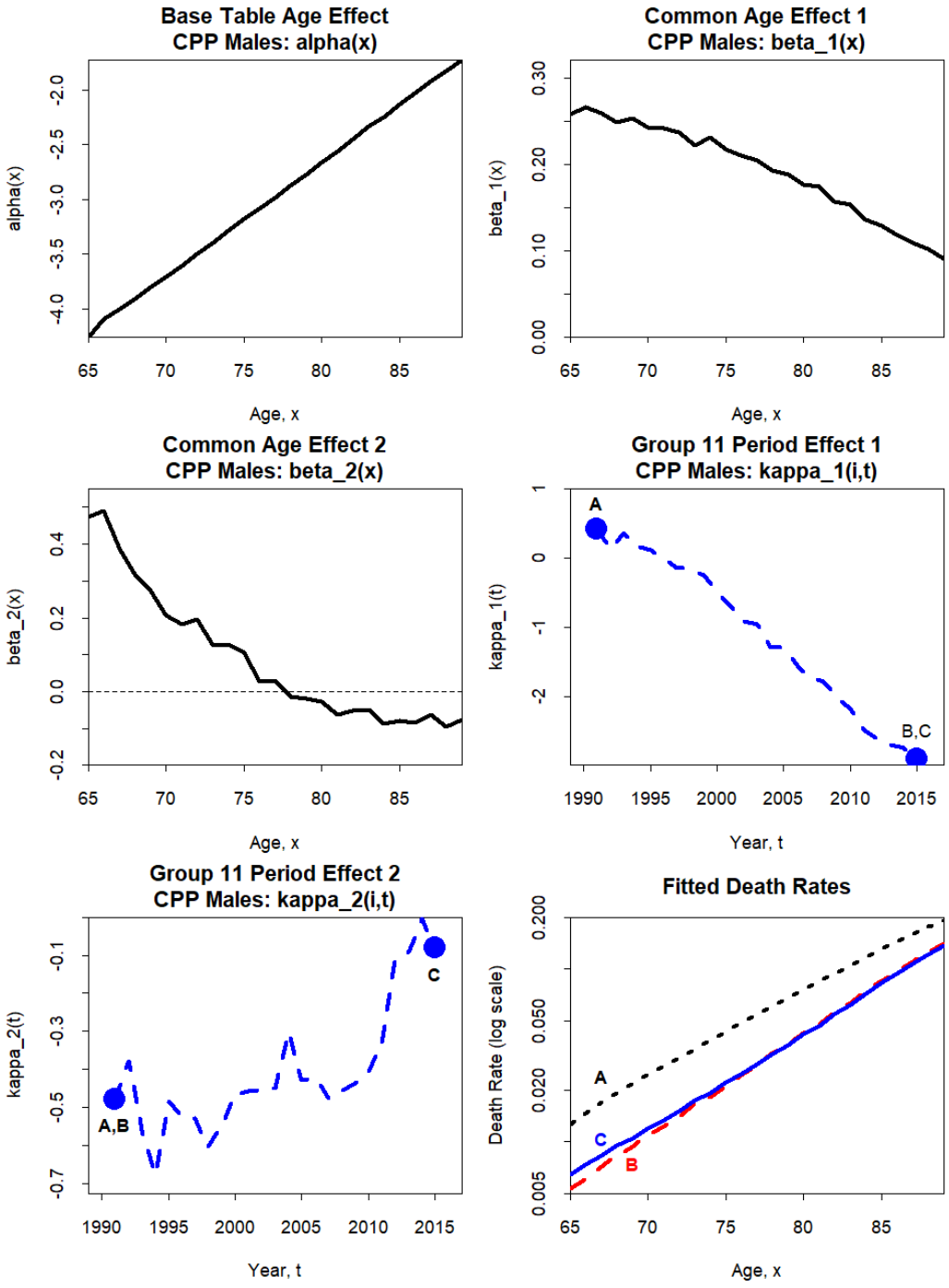
C. Basics of M6

In this appendix we illustrate how M6 works by reference to the results for Group 11, CPP males. In Figure 34, we have extracted the CPP males Group 11 age and period effects from Figure 16.

We now consider how the age and period effects impact on the derived death rates. Death rates are illustrated in the lower-right panel of Figure 34.

- The large dots in the middle-right (κ_1) and lower-left (κ_2) panels identify which values of κ_1 and κ_2 are used to construct the three curves A, B and C in the lower-right panel.
- Curve A (dotted black line) shows fitted death rates in 1991.
- The shift from curve A to curve B (dashed red line) shows the impact of the change in κ_1 *only* from 1991 to 2015 (κ_2 stays at its 1991 value). κ_1 (centre right panel) has fallen over this period. When multiplied by $\beta_1(x)$, the change in κ_1 results in a fall in the general level of mortality. However, since $\beta_1(x)$ is not flat, the percentage reduction is larger at younger ages.
- The shift from curve B to curve C (solid blue line) shows the additional impact of the change in κ_2 from 1991 to 2015. Since the matching age effect, $\beta_2(x)$, is positive at younger ages and negative at older ages, the rise that we observe in κ_2 over this period causes the curve to tilt around age 77: it rises at younger ages and falls (slightly) at higher ages.
- Comparing A, B and C, we can see that the impact of changes in κ_2 is much smaller than the impact of changes in κ_1.
- Changes in κ_2 for other groups are of a similar order of magnitude or smaller, and so a similar conclusion applies: the $\kappa_1(i,t)\beta_1(x)$ component of the model is the main driver of changes in the death-rate curve; the $\kappa_2(i,t)\beta_2(x)$ component of the model captures smaller, second-order effects.

Figure 34: Fitted age and period effects for Group 11 under model M6. Bottom right: fitted death rates for A: κ_1 and κ_2 at their 1991 values; B: κ_1 takes its 2015 value, but κ_2 stays at its 1991 value; C: both κ_1 and κ_2 take their 2015 values.



D. Tables of Period Life Expectancies

We tabulate here (Tables 6 and 7) period life expectancies for model M6 presented earlier in Figure 20. Additionally, we have tabulated period life expectancies for age 75 (Tables 8 and 9).

For comparison, age 65 period life expectancies using other tables are:

- HMD Canada,[32] 2010, males: 18.74
- HMD Canada, 2010, females: 21.68
- CPM-2014 base table,[33] males: 20.83
- CPM-2014 base table, females: 23.39

Corresponding age 75 period life expectancies are:

- HMD Canada, 2010, males: 11.54
- HMD Canada, 2010, females: 13.80
- CPM-2014 base table, males: 12.94
- CPM-2014 base table, females: 15.03

[32] See www.mortality.org, Canada, Life tables, "ex" column.
[33] Source data from www.cia-ica.ca/docs/default-source/2014/214013t1e.xls plus derived life expectancies.

Table 6: Fitted period life expectancies from age 65 under model M6 in selected years, Groups 1–11, for CPP males and females.

CPP Males Age 65	1995	2000	2005	2010	2015
Group 1	14.14	15.41	17.02	18.21	19.2
Group 2	14.55	15.44	16.77	17.89	18.65
Group 3	14.77	15.62	16.13	17.41	18.5
Group 4	15.2	15.44	16.6	17.65	18.19
Group 5	15.69	16.07	17.24	18.55	18.89
Group 6	15.86	16.63	17.53	18.24	19.05
Group 7	16.04	16.93	17.51	18.51	18.81
Group 8	16.24	17.05	17.72	18.65	19.31
Group 9	16.55	17.12	18.23	18.91	19.44
Group 10	16.63	17.36	18.24	19.27	20.02
Group 11	17.29	18.09	19.06	20.11	20.85
CPP Females Age 65	**1995**	**2000**	**2005**	**2010**	**2015**
Group 1	18.74	19.43	20.02	20.98	21.11
Group 2	19.65	19.93	20.75	21.48	21.65
Group 3	20	20.72	20.78	21.72	21.83
Group 4	20.09	20.61	21.1	21.89	22.09
Group 5	20.66	20.71	21.54	22.03	22.41
Group 6	20.37	20.94	21.43	22.11	22.73
Group 7	20.63	20.98	21.74	22.31	22.54
Group 8	21	21.16	21.55	22.99	22.9
Group 9	20.66	21.13	21.64	22.5	23.16
Group 10	20.84	21.11	21.89	22.8	23.38
Group 11	21.6	21.37	22.16	23.41	23.74

Table 7: Fitted period life expectancies from age 65 under model M6 in selected years, Groups 1–11, for QPP males and females.

QPP Males Age 65	1995	2000	2005	2010	2015
Group 1	13.7	13.74	15.43	16.11	16.77
Group 2	13.82	14.6	15.19	16.37	16.87
Group 3	13.97	14.68	15.73	16.82	16.98
Group 4	14.59	14.2	15.7	16.43	16.98
Group 5	14.47	15.16	15.74	16.99	17.36
Group 6	15.27	15.72	16.64	17.29	18.03
Group 7	15.24	15.88	17.01	17.72	18.39
Group 8	15.23	15.88	17.3	17.99	18.52
Group 9	15.09	16.28	17.15	18.15	18.93
Group 10	15.24	16.3	17.53	18.66	19.36
Group 11	16.04	17.25	18.47	19.71	20.49
QPP Females Age 65	**1995**	**2000**	**2005**	**2010**	**2015**
Group 1	19.33	19.78	20.23	20.61	20.68
Group 2	19.83	20.28	21.01	21.4	21.5
Group 3	19.81	20.62	21.2	21.62	21.84
Group 4	20.97	20.51	21.03	21.64	21.88
Group 5	20.56	20.66	21.35	21.6	21.92
Group 6	20.17	20.53	21.28	22.23	22.59
Group 7	20.06	20.91	21.53	22.54	22.76
Group 8	19.9	20.57	21.41	22.07	22.26
Group 9	20.07	20.77	21.56	21.87	22.37
Group 10	20.78	21.29	21.9	22.53	23.08
Group 11	20.83	22.21	22.14	22.72	22.98

Table 8: Fitted period life expectancies from age 75 under model M6 in selected years, Groups 1–11, for CPP males and females.

CPP Males Age 75					
Group 1	8.38	9.28	10.58	11.36	12.27
Group 2	8.69	9.49	10.24	11.02	11.59
Group 3	9.02	9.34	9.44	10.7	11.7
Group 4	9.24	9.36	10.2	10.74	11.22
Group 5	9.59	9.34	10.33	11.41	11.72
Group 6	9.32	10.03	10.51	11.07	11.71
Group 7	9.57	10.07	10.55	11.26	11.54
Group 8	9.57	10.22	10.75	11.3	11.85
Group 9	9.73	10.28	10.95	11.47	11.76
Group 10	9.79	10.31	10.86	11.61	12.15
Group 11	10.18	10.71	11.37	12.09	12.74
CPP Females Age 75					
Group 1	11.39	12.09	12.52	13.4	13.52
Group 2	12.09	12.24	12.94	13.62	13.78
Group 3	12.44	12.83	13	13.84	13.85
Group 4	12.3	12.78	13.29	13.84	14.1
Group 5	12.73	12.66	13.53	13.89	14.24
Group 6	12.51	12.9	13.43	13.9	14.44
Group 7	12.73	13.02	13.49	14.13	14.27
Group 8	12.98	13.21	13.43	14.73	14.42
Group 9	12.66	13.04	13.45	14.12	14.75
Group 10	12.96	12.98	13.62	14.35	14.83
Group 11	13.44	13.2	13.74	14.88	15.18

Table 9: Fitted period life expectancies from age 75 under model M6 in selected years, Groups 1–11, for QPP males and females.

QPP Males Age 75					
Group 1	8.34	8.67	9.64	10.49	10.83
Group 2	8.24	9.35	9.11	10.19	10.77
Group 3	8.48	8.92	9.5	10.56	10.54
Group 4	9.42	8.97	9.97	10.13	10.66
Group 5	8.86	9.69	9.34	10.45	10.71
Group 6	9.3	9.44	10.17	10.55	11.26
Group 7	9.22	9.74	10.32	10.98	11.45
Group 8	9.2	9.5	10.48	10.88	11.36
Group 9	8.98	9.77	10.26	10.91	11.47
Group 10	9.11	9.63	10.56	11.23	11.68
Group 11	9.57	10.29	11.05	11.88	12.37
QPP Females Age 75					
Group 1	11.96	12.41	12.75	13.16	13.31
Group 2	12.15	12.47	13.04	13.57	13.85
Group 3	12.41	12.75	13.43	13.85	14.01
Group 4	13.49	12.58	13.29	13.85	13.96
Group 5	12.7	13	13.46	13.59	13.95
Group 6	12.99	12.82	13.36	13.9	14.4
Group 7	12.39	12.76	13.54	14.34	14.56
Group 8	12.32	12.62	13.28	13.99	14.17
Group 9	12.62	12.84	13.49	13.55	14.09
Group 10	12.96	13.22	13.65	14.12	14.62
Group 11	12.96	14.1	14.01	14.5	14.89