JIA 70 (1939) 0111-0118

REVIEWS

Karl Pearson. An appreciation of some aspects of his life and work. By Professor E. S. PEARSON.

[Pp. viii + 170. Cambridge University Press. 1938. Price 10*s.* 6*d.* net.]

THIS is a fascinating book. Compiled originally for publication in *Biometrika*, these articles are now reproduced in volume form, and while not professing to be a biography in the ordinary sense, they are in reality much more. For the volume constitutes an appreciation, by Karl Pearson's son and successor, of the main aspects of his father's life-work, warmed by personal knowledge and affection, but tempered throughout by critical acumen. We are given glimpses of a happy domestic life: of care-free holidays: of the born fighter refreshed and ever eager to renew the strife: of a personality which combined extraordinary charm with pugnacity developed to an unusual degree. In the preface the author modestly reminds his readers "that this is in no sense a Life of Karl Pearson. Nevertheless it was important, while memories were fresh and records easy to trace, that some account of facts should be put down on paper, leaving to a later date and to other hands the full task of relating Karl Pearson's life and work to the history of scientific thought and achievement."

It is fitting that reference be made here to the illustrations. The frontispiece, showing K. P. at the age of sixty-seven, is arresting. Like Gladstone and Huxley—to name no others—K. P.'s features improved with the intellectual developments of age. We have portraits of him also at the ages of twenty-five and of thirty; a strong fighting face: and again at fifty-three, in his Biometric Laboratory, with his beloved Brunsviga beside him. Nor must one omit to refer to the reproduction of his penmanship—written, of course, with the familiar quill.

Turning now to Karl Pearson's work, the first thing that must strike every reader is its enormous range. While Freethought, Socialism, studies of Spinoza, and legal studies occupied their places in his earlier days, it is recorded that "his mathematical talent was not lying entirely fallow". At that period he also learnt "the meaning of thoroughness and patience in research", which was to be his guiding principle throughout life. In 1884, at the early age of twenty-seven, he was appointed to the chair of Applied Mathematics and Mechanics at University College, London, and there he worked for nearly fifty years. His teaching methods, it is stated by a contemporary, were secondary to his personality, despite his wonderful clearness as a lecturer and the extraordinary conscientiousness that ran through all that he did. In 1891 came the founding of the science of biometry and the development of

this line of study constituted the leading feature of Karl Pearson's life-work. The introduction of mathematical methods into biology was then novel and aroused bitter controversies; in which, naturally, the personal element played no small part. The gradual developments of K. P.'s mathematical processes are described in some detail, and these culminated in the decision of the leaders of his school of thought to publish a journal in support of their views. Thus was born the famous *Biometrika*, the first number of which appeared in 1901. *Biometrika* is still running successfully.

The progressive developments of K. P.'s special lines of work—the Biometric and Eugenics Laboratories—belong partly to these earlier years but mainly to the whole period which followed until his retirement in 1933. Lack of money continuously cramped his efforts at extensions, despite the generous subsidy which, year after year, the Drapers' Company provided, supplemented as it was by the Galton and other legacies. But in the last resort the College had to provide—up to definite limits and so far as its funds permitted—the sum needed to balance departmental income and expenditure. In this connexion reference may here be profitably made to the peculiar position which Karl Pearson occupied in the University College machinery. His department, for all practical purposes, was independent. The Galton Laboratory possessed its own buildings, and out of its own income could provide for the major part of its expenditure. It is true that a Sub-Committee of the College occasionally met to deal with these adverse financial margins and with other troubles. At such Committees, Karl Pearson always attended by invitation, though in theory, not by right. Apart from finance, where College exigencies had to intervene, Karl Pearson always got his way and these meetings eventually resolved themselves into the simple financial problem of deciding the maximum subsidy that the College could afford to contribute.

So Karl Pearson worked on year after year, his international reputation growing continuously, while the world-wide circulation of *Biometrika* appealed more and more to biological and mathematical students. The war came as a disaster to his work, but of *Biometrika* he claimed "that it should be as well done as ever". And on the very day of the Armistice, Karl Pearson himself gave a first lesson on a Brunsviga calculating machine to a wounded New-Zealander. Here one sees his human touch in perfection. For however acid and controversial he might prove to be in penmanship, his personal charm and devotion to duty came uppermost in this and in many other characteristic aspects of his life.

It has already been noted that Karl Pearson occupied a virtually independent position as regards his connexion with University College. Incidentally—and very happily—he was thereby exempt from the customary age limit of retirement. Thus when a generous donor provided the new buildings for the Galton Laboratory in 1920, Karl Pearson

at sixty-three still had some thirteen years of work before him. But active and full of productivity as were these years, he began at last to feel the pressure of his age. At seventy-one he wrote "it is absurd as one grows old to have to take so many precautions". He added, a little later, "when we mind labour, then, then only we're too old". In 1929 he completed his monumental *Life of Galton* and in the succeeding years he published in *Biometrika* "some dozen contributions to statistical theory". In 1933 he resigned his professorship but still continued his life-long activities. It was typical of the man that when a testimonial fund was raised on his retirement, part of the sum collected was, at his request, devoted to the purchase of a new Brunsviga. Up to the very end he was working at *Biometrika* and he died just before he had corrected the final proofs of the current number. At his funeral, Browning's lines were recited:

"This man decided not to Live but Know."

There could be no better epitaph. A. D. B.

A Study in the Analysis of Stationary Time Series. By HERMAN WOLD.

[Pp. 214 + viii. Almqvist and Wiksells Boktryckeri-A.-B., Uppsala. 1938. Price kr. 6.]

THE mathematical model of a game of chance, which was proposed by Pascal in 1654, has served as the foundation for a highly abstract mathematical superstructure which appears, more and more, to lose touch with the realities of experience. Occasionally, however, the practical statistician is able to find an application even for some of the higher flights of the mathematician's fancy. Thus it is that Herman Wold has been able to apply the methods of random processes, which have been developed theoretically by Khintchine and Kolmogoroff, to Yule's empirical system of time series analysis.

As is well known, a time series is a sequence of observations ordered in time; for example, the rate of mortality at age thirty experienced by a certain office on its whole-life non-profit policies considered year by year from, say, 1890 until 1938, would constitute a time series with, probably, a noticeable secular trend. Consider only a series with no trend, or alternatively suppose the secular trend component removed; then, in the mathematical model, if x_t ($t = 0, \pm 1, \pm 2, \ldots$) is a series of random variables representative of a time series and the law of the series is independent of time, i.e. if the joint probability distributions of $x_{t_1+k}, x_{t_2+k}, \ldots x_{t_n+k}$ and $x_{t_1}, x_{t_2}, \ldots x_{t_n}$ are identical, the series is "stationary". Khintchine had already proved some fundamental theorems relating to stationary processes where t varied continuously, and now Wold extends those theorems and develops others relative to a discrete

time series. The most important particular cases are those where x_t can be written in the form

$$x_t = y_t + ay_{t-1},$$

i.e. as the sum of a random impulse and a fraction of a previous random impulse, and where we write

$$x_t + bx_{t-1} = y_t$$

(in this case y_t plays the part of a random shock acting on a pendulum swinging with damped oscillations). These are simple examples of the general linear relations dealt with by the author. They are called "moving averages" and "linear autoregression" respectively. The classical instance of the periodic function masked by random components appears as a special case of linear autoregression.

An inherent characteristic of these approaches is that long-term forecasts are useless—for the forecast merely reduces to the mean of the observations or to a damped oscillation tending towards this mean—but that short-term forecasts, and these are usually of more importance, are fairly efficient. This is to be contrasted with the forecasts of periodogram analysis which extend indefinitely into the future.

In the practical application of these schemes the so-called auto-correlation coefficients appear as determining criteria. The auto-correlation coefficient of the k'th degree is defined as the correlation coefficient between every $(k+l)$'th term of the time series under consideration, i.e. the variables x_i and y_i $(i = 1, 2, 3, \ldots)$ of the usual correlation formula are replaced by the i'th and the $(k+i)$'th term of the time series $(i = 1, 2, \ldots \overline{n-k})$, n being the number of terms in the series analysed. Actually the term "auto-correlation coefficient" is reserved for the hypothetical model, and "serial coefficient" is the name applied to the value derived from the data. A graph of the series of auto-correlation coefficients (serial coefficients), from $k = 0$ up to about $k = 20$, is drawn as the preliminary step in the analysis, and the form of this "correlogram" is used to determine which of the above types of relation is more likely to lead to a good hypothetical representation of the observed series.

The approach is frankly mathematical, but the author shows us his sound practical views in the fourth and last chapter of the book. If fault there is, it lies, in our opinion, in the extreme generality of the theoretical demonstrations. The subject is an important practical one and the methods developed should be made available, without insistence on generality, to all workers in economic and meteorological statistics. In the examples of the application of the methods to actual data three or four parameters are all that is found necessary, and it would seem that the whole theory could be expressed more simply with this in view. However, the book sets out to give a complete mathematical vindication of the proposed methods and, as such, appears to be impeccable.

In an Appendix, Wold suggests a modification of the Cramer-Mises ω^2 test of goodness of fit which has the advantage of reducing the numerical calculations required. But he omits to derive the corresponding standard error of the new criterion, w^2, and having, in an example, obtained a value $w^2 = 0{\cdot}0265$ he merely compares it with the expected value of w^2, namely 0·1107, and remarks that this "indicates a very nice fit". If an independent verification of the test of fit is made by Karl Pearson's P_λ method (v. *Biometrika*, XXV, 379, and XXX, 134) a probability of obtaining a fit as bad or worse is found to be about 0·2, i.e. in one graduation in five a worse agreement of observed and hypothetical values would be obtained. This certainly confirms the author's conclusion but is, we submit, a sounder type of statistical judgment.

Whilst the grammatical constructions are not always perfect, for a book printed and published abroad the English is excellent. The author systematically introduces apt, though novel, English equivalents to certain French and German expressions used in treatises on probability (for example, *aleatory*, *conditioned expectation*, *probabilistic*, *aprioristic*).

H. L. S.

Tables of $\tan^{-1} x$ *and* $\log(1+x^2)$, *to assist in the calculation of the ordinates of a Pearson Type IV curve.* By L. J. COMRIE, Ph.D.

[Pp. 18. No. XXIII of *Tracts for Computers*, edited by E. S. PEARSON, D.Sc. Cambridge University Press. 1938. Price 3*s.* 9*d.*]

THE tables occupy seven pages and give $\tan^{-1} x$ for values of the argument from 0·00 by intervals of 0·01 to 5·00, from 5·0 by intervals of 0·1 to 20·0, and from 20 by intervals of 1 to 164; they also give $\log(1+x^2)$ for the same values of the argument up to 15·0. The inverse tangent is shown of course in circular measure, and the logarithm to base 10. Both functions are tabulated to 7 decimal places. Second central differences of the functions are also given, thus facilitating interpolation by Bessel's formula. The tables are printed clearly and conveniently.

The equation of the Type IV curve may be written in the form:

$$y = y_0 (1+x^2)^{-m} e^{-\nu \tan^{-1} x},$$

whence $\log_{10} y = \log_{10} y_0 - m \log_{10}(1+x^2) - \nu \log_{10} e \tan^{-1} x,$

from which the purpose of the tables is apparent. The introductory pages contain a note by the author on the preparation of the tables, and an illustrative example by H. L. Seal of their use.

Life Assurance from Proposal to Policy. By H. HOSKING TAYLER, F.I.A., A.C.I.I., and V. W. TYLER, F.I.A. Third Edition by H. NOEL FREEMAN, B.A. (Cantab), F.I.A.

[Pp. 191. London: Sir Isaac Pitman and Sons, Ltd. 1939. Price 6*s.* 0*d.* nett.]

THE first edition of this book was reviewed in *J.I.A.* Vol. LVI, pp. 233. On this, the occasion of the issue of a third edition, it is unnecessary to do more than call attention to the main changes embodied in the revision. The changes consist of introduction of new paragraphs dealing with group pension policies, policies of the family protection type, the modification of Joint Life Assurances to meet the change in income-tax practice concerning partnership assurances, paragraphs dealing with Aviation Risks, the 1929 Industrial Assurance and Friendly Societies Act, and a slightly modified suicide clause. The paragraph on p. 127 concerning suicide is not clear. It seems to say that in the absence of policy conditions, an assignee of a life assured who commits suicide while sane can successfully sue the office: and to suggest that a suicide clause is needed, not to protect assignees, but to protect the office. Modifications in the text consequent upon the change in the basic income tax rates have been carried out throughout the book.

Distribution of Surplus. By J. B. MACLEAN and E. W. MARSHALL.

[Pp. 154. Actuarial Studies No. 6, The Actuarial Society of America, New York.]

THE actuarial societies of Great Britain and North America traditionally require their members to be trained in practical studies as well as in purely mathematical subjects. The foundations of such a training are the papers and discussions at sessional meetings, and this must necessarily be so in matters which remain the subject of controversy. For tuition purposes these papers are not altogether satisfactory, and the organization of classes has led to a demand for more suitable teaching material. In this country the demand has been met by the series of Consolidation of Reading books, the scope of which has developed as the series has grown. At first care was taken to point out that the books were not text-books and were intended to supplement but not to supplant the ordinary course of reading. Thus in the first book we read "Consolidation of reading can never mean condensation of reading", but in a later book the aim is stated to be "to map out a course of study which shall reduce to a minimum the number of papers to be read". The American Actuarial Societies are served by a series of "Actuarial Studies" of which No. 6 is the subject of this review. It is interesting to compare the aims of these studies with those quoted above. "Each volume is intended to bring together within a reasonable space the more important

points of information on the subject discussed." Lest there should be any bias the work was undertaken by two persons "a principal contributor who would discuss the subject as a whole from one general viewpoint, and a supplementary contributor who would approach the subject from another angle". Thus the Actuarial Studies serve a purpose different from that of the Consolidation of Reading series, and in the present case at least the result is more nearly akin to a text-book. It must be admitted that the result appears to be more helpful to the student, although there is the danger that students may be led to trust too much to "the book" and not to read critically, comparing the opinions expressed with others set out elsewhere.

Naturally, in a book intended for American students, distribution of surplus is discussed mainly from the point of view of the contribution method. One of the objections of British actuaries to this method is its supposed rigidity, but Mr Maclean (the principal contributor) brings out clearly that if the method is properly applied it will be flexible. The contribution methods used by some British Offices attempt to reduce the distribution of surplus to a formula which, whilst it may be suitable at the outset, is too rigid for use over a long period. Time and again history has shown that equity is not attained by rules formulated in advance; changes in experience vitiate the rules.

Many actuaries will remember the paper submitted to the Institute in 1931 by the present principal contributor, Mr J. B. Maclean, in which he put forward a view of the contribution method which was new to most of us. In some respects this book goes further even than the paper. To him the method implies (1) that the scale of participating premiums is not much in excess of the rates estimated to be required for the contractual liability, i.e. without definite bonus loading, and that valuations are normally on the same basis, (2) that surplus is traced to its sources by analysing the experience of the fund in as small groups as are necessary or practicable, (3) that distributions follow the general average experience and are not necessarily related to the surplus of a particular year, (4) that the distributions over a period of years are tested to see if they are equitable. Thus he suggests (p. 33) that a rate of interest lower than the average yield should be assumed for single premium and other contracts of an investment type during periods of low market rates of interest, and (p. 19) that there should be set aside a contingency fund for an amount representing an appreciable proportion of the liabilities, and the supplementary contributor (p. 138) describes a method of testing the general equity of the distributions over a period of years.

It is stated that the contribution method probably arose partly from the fact that the charter of the company which originated the system required an equitable distribution of surplus. But on p. 80 less than justice is done to the British Office which first introduced reversionary bonuses. The office was founded for the transaction of "equitable

assurances". For the greater security of the fund the actuary decided to distribute surplus at 10-year intervals only and to carry forward one-third of the surplus at each valuation. It was a stroke of genius on the part of that actuary, William Morgan, to devise a bonus system which was equitable in those conditions (now unusual) and which remains so after 150 years.

Although a large part of the book is concerned with the contribution method Mr Maclean is at pains to show that it is not universally practicable. Thus he shows that participating immediate annuities can be dealt with more satisfactorily by a floating bonus method similar to the method for life assurance discussed recently at the Institute by Dr K.-G. Hagstroem of Stockholm. Further, in the case of industrial assurance Mr Maclean's sympathies apparently lie with some form of reversionary bonus system.

This review is too brief to describe at length the contents of the book, but the authors and the society may be congratulated upon an informative and stimulating study. It is suggested that British and American actuaries are fundamentally not so far apart as would appear from a superficial knowledge of the differences in practice in the two countries. We should all be better actuaries if there were more opportunities of studying the work of others overseas. Could we not have a common educational policy among the actuarial societies of the English-speaking world by publishing one series of actuarial studies in which the general principles adopted in the solution of practical problems would be briefly presented to students—and others—at home and abroad?

M. E. O.