NOTES ON FOREIGN ACTUARIAL JOURNALS

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AMERICA

Transactions of the Actuarial Society of America, Vol. XLVII, Part II, 1946

W. A. JENKINS. *Annuity premiums and reserves based on an assumption of decreasing mortality*, pp. 265–285. Discusses forecast tables on 'year of issue' and 'year of exposure' bases and develops Fraser's scheme of simplifying the 'year of exposure' method.

M. GELLES. *Overhead and unit costs*, pp. 286–303.

K. LANG. *Simplifying the valuation of annuities certain and instalment-refuna annuities by assuming continuous payments*, pp. 304–310.

R. P. COATES and D. D. CODY. *Observations on exposure sampling procedures*, pp. 311–325. Extends and suggests modifications of formulae given by G. C. Campbell in Vol. XLVI of the same journal (see also discussion, pp. 528–540).

D. C. BAILLIE. *Actuarial note: On testing the significance of mortality ratios by the use of χ^2*, pp. 326–344. Develops (a) the ordinary normal curve theory for a single year, (b) the χ^2 theory for several groups subject to differing q's, and discusses various special points in appendices (see also discussion, pp. 541–561).

Transactions of the Actuarial Society of America, Vol. XLVIII, 1947

E. W. MARSHALL. *Presidential Address*, pp. 1–9. The Actuarial Society and the Sherman Act.

H. F. GUNDY. *War mortality and its effect on life insurance in Canada*, pp. 10–35. Gives insurance experience and also figures relative to the Canadian armed forces arranged in various periods.

E. C. BERKELEY. *Electronic machinery for handling information and its uses in insurance*, pp. 36–52. The machines mentioned are M.I.T.'s differential analyser No. 2, Harvard's sequence controlled calculator, Moore School's ENIAC and Bell Laboratories' relay calculator.

H. S. BEERS. *Premium interpolation*, pp. 53–75.

E. OLIFIERS. *The evolution of the exposure formulae*, pp. 76–94. A careful study which will be helpful to those who prefer to express exposed to risk in algebraic form, but others would have been helped by arithmetical examples.

D. N. WARTERS. *Group insurance on level premium plans*, pp. 95–115.

Report of Joint Committee on Mortality, pp. 133–165. Deals with the experience under settlement options and deferred annuities (1940–45). The non-refund annuities show lighter mortality than the refund annuities during the first few years but little difference thereafter. The indications of a select period are more pronounced with the non-refund annuities.

J. S. ELSTON. *Analysis of Joint Mortality Experience*, 1939–45, pp. 239–266. The analysis is as regards (1) the experience for the first fifteen policy years and for the sixteenth and subsequent policy years, (2) corresponding war deaths, (3) the experience for issue years 1931–38 observed from issue to the policy anniversaries in 1939. The relative experiences of companies are shown.

C. L. WEAVER. *Actuarial note: Allowance for rounding errors in the summation check*, pp. 267–269.

Report of Joint Committee on Mortality, pp. 314–339. This relates to the experience between policy anniversaries in 1945 and 1946 and discusses war deaths between 1941 and 1946.

Report of Committee to prepare Mortality and Morbidity Studies on Group Insurance, pp. 340–366. Continuation of previous work.

Aviation Statistics, pp. 367–373. The 17th report of the Committee of the Society.

The Record. American Institute of Actuaries, Vol. xxxv, Part II, 1946

S. P. ADAMS. *Actuarial note: Mortality tables and $2\frac{1}{2}$% commutation columns based on multiples of 1941 Commissioners' Standard Ordinary table*, pp. 207–231. The rates of mortality adopted range from 125% to 500% of the normal.

E. A. ARCHIBALD. *Volunteer State Life mortality experience*, pp. 232–234. The Company operates only in the southern states and the investigation is in terms of death strain. The experience is small. The mortality appears to be light.

J. E. HOSKINS. *The application of the standard non-forfeiture law to supplemental term insurance*, pp. 235–257. A general discussion with appendices giving formulae for premiums, values, etc.

W. O. MENGE. *Commissioners' Reserve Valuation Method*, pp. 258–300. An explanatory paper with formulae, numerical examples, etc.

Informal discussions on various subjects, pp. 345–415.

BELGIUM

Bulletin de l'Association Royale des Actuaires Belges, No. 50, 1945

F. DE CREEFT. *Au sujet des aménagements de la sécurité sociale en matière de pensions*, pp. 2–14. An account of some of the steps taken to improve old-age pensions in Belgium owing to war-time rises in cost of living.

J. H. BAPTIST. *Étude de la dépendance stochastique*, pp. 15–36. A review of fundamentals. The distinction between functional and stochastic independence is clearly made.

R. CONSAEL. *Sur les formules 'en Z' et sur quelques équations fonctionnelles qui s'y rattachent*, pp. 37–61 and (1946) pp. 5–30. Develops a large number of relations in respect of annuities and assurances of the joint-life and survivor and survivorship types.

E. Franckx. *Note sur un mode de contre-assurance*, pp. 62–68. The retrospective policy value at the end of t years of an assurance under which premiums are returned with interest at the valuation rate subject to a waiting-period of k years $(k < t)$ can be written as

$$c_1/D_{x+t} + c_2/v^{x+t} - c_3 a_{\overline{w-x-t}|},$$

where $c_3 = P$, $c_2 = P \sum_{j=x+k}^{w} v^j$, and $c_1 = P(N_x - N_{x+k})$,

w being chosen arbitrarily but $> x + k$.

Bulletin de l'Association Royale des Actuaires Belges, No. 51, 1946

E. Franckx. *L'évolution des collectivités*, pp. 31–40. Reprint of an article in the Swiss *Bulletin* of 1945.

H. Maurice. *Étude sur le bénéfice de mortalité et le bénéfice d'intérêt*, pp. 41–60. An algebraic analysis of mortality and interest profits on the lines of Lidstone's paper in *J.I.A.* Vol. XXXIX. The numerical value of the 'remainder' is traced throughout the existence of a 20-year endowment assurance on (30), the mortality and interest bases conforming to current Belgian practice and experience.

Bulletin de l'Association Royale des Actuaires Belges, No. 52, 1947

L. J. Smid. *Baisse de la mortalité aux Pays-Bas et ses conséquences pour l'assurance sur la vie*, pp. 5–21. An interesting account of a forecast of mortality and its effect on actuarial functions; generation mortality is considered.

J. Brieussel. *La réassurance à la prime de risque combinée avec la réassurance à la prime normale*, pp. 22–35. Suggestions for overcoming objections to the risk-premium method of reassurance.

J. De Moor. *Aperçu sur l'assurance à double risque*, pp. 36–42. Formulae covering invalidity and death.

FRANCE

Bulletin Trimestriel de l'Institut des Actuaires Français, No. 177.

The papers were read in 1939 and during the earlier part of the occupation. There is an obituary notice of Henri Galbrun.

J. Petit. *Note sur les bénéfices de mortalité*, pp. 1–8. Formulae based on the retrospective method are given and examples of the profit resulting in endowment assurances from the use of the A.F. table, assuming actual mortality to be represented by that of the P.M.F. 1931 table.

P. Razous. *L'état actuel de la prévention des accidents du travail*, pp. 9–19. An account of the present conditions in industry, agriculture, etc. The main causes of accidents are defective machinery, defective lay-out of machinery, and human shortcomings. Some of the plans adopted to counteract the last mentioned, in France and elsewhere, are given.

E. Krauss. *Applications de la statistique mathématique à la biologie*, pp. 1–9.

P. SUMIEN. *Le régime du contrôle par l'État des sociétés d'assurances et de capitalisation, à la suite des réformes de* 1939, pp. 1–10. A lecture given to the French Institute on 2 June 1939. The new law enjoined the licensing of all insurance associations, and the mere fact that they were constituted in accordance with existing laws and regulations did not automatically confer a licence. The new act was modelled to some extent on Swiss law. Note that 'capitalization' includes life assurance as well as sinking-fund assurance.

G. RAILLARD. *Sur l'extension du théorème de Tchebycheff au cas de deux moments,* pp. 1–12.

G. BAUDEZ. *Le plein dans des compagnies d'assurances,* pp. 1–88. Divided into four parts: (1) It is assumed that the chances of yearly death or survival follow the normal law, the 'maximum at risk' being based on the assumption that deviations will fall within remote limits (giving a chance against of the order of 1/10,000); (2) The problem is investigated on the hypothesis that assurance transactions are a gambling contest between assurer and assured; (3) It is emphasized that too large a gambling fund (i.e. capital) militates against profits and that this fact and other considerations must be admitted in order that a reasonable balance may be struck; (4) Special points that must be considered in connexion with life assurance are dealt with.

Bulletin Trimestriel de l'Institut des Actuaires Français, Nos. 178, 179, 1947

C. DUC. *Nomogrammes et abaques financiers,* pp. 1–53. Diagrams are given for ordinary interest functions and for loans.

J. BOUTHILLIER. *Étude du contrôle technique d'une compagnie d'assurances sur la vie,* pp. 55–121. The author discusses the analysis of profits and the control of the expenditure of a company by three methods: (1) the actuarial method involving checks on rates and valuations at assumed true rates of mortality and interest at regular intervals, (2) the accounting method whereby profit is traced and analysed from year to year in each group, (3) the statistical method, only approximate. In the numerical examples a 20-year endowment assurance is chosen, age 35, table A.F., true mortality being assumed to follow the recent experience of an important French company. Valuations are on the Zillmer plan owing to large initial commissions. Unfortunately the author uses a notation of his own in which there is lack of precision.

HOLLAND

Het Verzekerings-Archief, Vol. XXVII, 1947

J. ENGELFRIET. *Une théorie générale de récurrence en matière d'assurance sur la vie et contre l'invalidité,* pp. 1–78. The basis of the very general theory developed here is that at any future date each status involved in an assurance (life or invalidity) will be found in one of a number of conditions (e.g. various degrees of invalidity, death). The object is to derive recurrence formulae for (i) assurance values, (ii) policy values, and (iii) mean square risks, in respect of status in condition i at time t from those in all possible conditions at time $t+1$. One of the explicit survivorship probabilities derived has close analogies with that given by Greenwood and Yule (*J.R.S.S.* 1920) for the relative frequencies of individuals with 0, 1, 2, ..., accidents when each mishap affects the liability to further accident.

TH. C. L. KOK. *Het intrestbestanddeel van de levensverzekering*, pp. 79–94. An examination of the fundamental principles which should underlie taxation of interest in financial transactions. A set of 'obvious' axioms leads to unique and 'sensible' results in the pure interest case but the proper taxation of life assurances may be defined in several ways.

J. HAGE. *De* 3–3½% *Nederlandsche Staatsleening* 1947, pp. 95–98. Tables to facilitate the calculation of the value of the Dutch National loan of 1947 at various future dates and rates of interest.

B. GROOTENBOER. *Groepsgewijze reserveberekening, in het bijzonder voor rentever-zekeringen op twee levens*, pp. 99–114. Consideration of two methods (one British, the other French) suggested in 1939 for valuing last-survivor annuities in groups. Preference is given to Derrien's (see *J.I.A.* Vol. LXX, p. 416) partly owing to the results of a model office valuation based on a Makeham table. It would be useful to extend these trials to a more general case.

F. SALA. *Sur une méthode approximative pour le calcul des primes pures pour quelque position caractéristique, sans connaître autre que les probabilités annuelles de survie pour les âges de* 10 *en* 10 *ans*, pp. 128–134. Uses Lubbock's formula to fourth differences.

C. VAN EBBENHORST TENGBERGEN. *Invloed van toekomstige sterfteverlaging op de vereischte premies voor eigen- en weduwepensioen*, pp. 135–147. Numerical examples, involving some interesting approximations and using 'generation' life tables, to estimate the effect of future mortality improvement on single- and joint-life and survivorship annuity-values.

ITALY

Giornale dell' Istituto Italiano degli Attuari, Vol. XX, 1942

F. P. CANTELLI. *I fondamenti matematici della tecnica delle assicurazione*, pp. 1–27. Recapitulates and extends earlier work.

B. DE FINETTI. *Impostazione individuale e impostazione collettiva del problema della reassicurazione*, pp. 28–53. Sets out the different ways in which the problem of the 'optimum' of reassurance may be approached, according to whether a company decides for itself how to deal with each individual case, or whether decisions are co-ordinated and agreed with the reassurer. It is only under the second plan that it is possible to arrive at an 'optimum', which consists in the distribution of risks according to the principle which the author calls 'equipartition', and which might be substituted for the method in general use with some advantage.

R. INVREA. *Ancora a proposito del rischio medio*, pp. 54–56. Further demonstration of formulae given in the same journal, 1941, pp. 167 *et seq.*

L. LORDI. *Sulle tavole di mortalità che portano alle stesse riserve matematiche*, pp. 57–65. An investigation, on the continuous basis, of tables of mortality which give identical reserves. The condition known to be necessary for equal reserves is also sufficient. Moreover, varying the rate of interest used, it is shown that there does not exist a second table of mortality giving the same reserves at more than one rate of interest.

G. OTTAVIANI. *Sulle tavole di mortalità*, pp. 66–76. Shows mathematically that the exact ages method is preferable to a formula due to Gini based on a constant force of mortality and suggests ways in which the latter can be improved.

The number also contains short summaries of three lectures by:

B. DE MORI on the relative merits of individual and global reassurance,

M. CASTELLANI on payments to the heads of families varying with the size of family, etc.,

R. OTTAVIANI on war risk in connexion with life assurance.

SCANDINAVIA

Skandinavisk Aktuarietidskrift, Vol. XXX, 1947.

A. RENBERG. *Une méthode pour calculer les réserves mathématiques à l'inventaire*, pp. 1–7.

L. WILHELMSEN. *On the valuation of life policies*, pp. 8–17.
These papers suggest that calculations be made by the assumed mortality table of the total sums assured expected to become payable in each year and also the total premiums expected to be received. The resulting figures are then valued by interest functions. Renberg gives details of the appropriate punched-card procedure. Wilhelmsen points out that a varying rate of interest could be used, discusses changes in mortality, and gives a numerical example.

II. L. SEAL. *A probability distribution of deaths at age x when policies are counted instead of lives*, pp. 18–43. Assuming that the probability distribution of deaths at age x is Poisson and that the distribution of duplicate, triplicate, etc., policies is given, a complicated expression is obtained for the probability of y 'policy claims' from E policies exposed to risk. Asymptotic expressions are obtained for the first seven ordinates of this distribution and for the first four moments. Numerical examples are given to illustrate the high degree of skewness of the distribution and its slow tendency towards normality.

C.-E. QUENSEL. *The validity of the z-criterion when the variates are taken from different normal populations*, pp. 44–55. By considering the form assumed by the characteristic function of the variables,

$$y_1 = \sum_{i,j} (x_{ij} - \bar{x}_i)^2 = \text{sum of squares within classes},$$

$$y_2 = \sum_i n_i (\bar{x}_i - \bar{x})^2 = \text{sum of squares between classes},$$

where the distribution function of x_{ij} is

$$\frac{1}{\sqrt{(2\pi)}\,\sigma_i} \exp\left[-x_{ij}^2 / 2\sigma_i^2\right] \quad (j = 1, 2, \ldots, n_i),$$

it is shown that the variance of the distribution of $z = y_2/y_1$ is increased in comparison with the usual case where $\sigma_i = \sigma$. A slightly more general case is also considered.

G. ELFVING. *A simple method of deducing certain distributions connected with multivariate sampling*, pp. 56–74. A useful compendium of multivariate sampling distributions derived in a uniform manner.

T. PENTIKÄINEN. *Einige numerische Untersuchungen über das risikotheoretische Verhalten von Sterbekassen*, pp. 75–87. A numerical application of collective risk theory appropriate for small funds. The relative unimportance of variations in the distribution of sums at risk on individual lives is once again demonstrated.

SWITZERLAND

Mitteilungen der Vereinigung schweizerischer Versicherungsmathematiker,
Vol. XLVII, 1947, Part II

M. HALDY, E. DÉCASTEL and M. BALLENEGGER. *De l'influence d'une diminution de l'invalidité sur les primes et les réserves mathématiques des caisses de retraite.* pp. 281–320. Continues and supplements Urech's study in Vol. XXV (*J.I.A.* Vol. LXI, p. 378) concerning change of bases in health and pension insurance. In this new study the effects of a halving of the invalidity rate on contributions and reserves for individuals and for a model fund are considered. Unlike Urech, the authors suppose the contributions to remain unaltered when the basis of a fund's reserves is changed. The effects of changing the pension age from 65 to 60 or 70 are also investigated.

A. MARET. *De la fonction d'événement d'un ensemble ouvert variable*, pp. 321–327. Using a notation invented for the purpose, relations are developed to link the 'constant' and the 'unstationary' communities. These complete the cycle of relations connecting stationary, constant, and unstationary communities.

ED. BURNENS. *Die Erfahrungsnachwirkung bei Wahrscheinlichkeiten*, pp. 329–352. A contribution to the problem of 'predecessor influence' improbability. In the continuous case the probability of a success at time t is given by $p(t) = A_1(t)/[A_1(t) + A_2(t)]$, where 'predecessor influence' is defined by

$$A_j(t) = A_j(0) + a_j \int_0^t [1 - p(\xi)] \, d\xi, \quad j = 1, 2 \quad (a_1 \geqslant 0, \, a_2 < 0),$$

and an explicit relation is obtained for $p(t)$. The discontinuous case is treated by means of two particular cases, the first of which consists of a series of drawings from an urn initially containing w white and b black balls, in which only white balls drawn are replaced. Ingenious and accurate approximations are found for the exact results, which are very complicated.

GUNTHER ALBERS. *Ein vereinfachtes Verfahren der Lebensrückversicherung gegen natürliche Prämien*, pp. 353–358. An approximate method for calculating in bulk the sums to be reassured and the premiums payable under an automatic risk-premium contract.

E. MICHALUP. *Theorie und Anwendung der 'oskulatorischen' Interpolationsformeln*, pp. 359–407. A review of available formulae of osculatory interpolation together with several new formulae designed to ensure smoother run of the fifth differences of the function graduated. The actuary now has a choice of 52 published formulae before him.

E. Zwinggi. *Über Darstellungsformen der Prämien und Reserven der Todes-fallversicherung*, pp. 409–413. A number of representations are obtained of P_t and $_tV$, the (continuous) level premium and policy value at time t after entry, respectively, for a whole-life assurance with variable sum assured. An example is

$$_tV = \int_0^t (1 - {_\tau}V)\,P'_\tau\,a_\tau\,d\tau,$$

where a_t and P_t are related by $P_t = \dfrac{1}{a_t} - \delta$ and P' is the differential coefficient of P.