RANDOM WALKS AND INVESTMENT THEORY

by

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§1. INTRODUCTION

THIS article discusses the random walk hypothesis, examines some of the evidence that has been put forward to support it, and draws some implications for the theory of investment analysis. There seem to be only three references to the random walk hypothesis in U.K. actuarial literature—Weaver and Hall (1967), Hemstead (1969) and Moore (1972), all of whom discuss it very briefly. The vast majority of the research in share prices, portfolio analysis, etc. has been performed in the U.S.A., but this article will attempt to discuss this problem in the light of current U.K. research, as well as its application to maturity guarantees for equity-linked policies.

§2. THE RANDOM WALK HYPOTHESIS

The sequence of random variables, $(Y_t, t = 1, 2, \ldots)$, is called a random walk if the increments

$$Y_t - Y_{t-1} = e_t$$

are independently distributed. Thus for a given initial value Y_0

$$Y_t - Y_0 = e_1 + e_2 + \ldots + e_t$$

is a partial sum of independent variables. The random walk hypothesis applied to share prices states that if p_t is the price of a share at time t, then

$$\ln p_t = \ln p_0 + \sum_{j=1}^{t} e_j$$

This means studying the variable

$$x_t = \ln p_{t+1} - \ln p_t$$

which can be expressed as

$$p_{t+1} = p_t \exp(\ln p_{t+1} - \ln p_t).$$

This has an interesting compound interest interpretation as the change in log prices is the yield, under continuous compounding, from holding that share. Thus the sequence of these variables is simply a succession of rates of return from the share.

The random walk hypothesis has been derived by regarding a share price as an ensemble of decisions in statistical equilibrium, analogously with an ensemble of particles in statistical mechanics.

The assumption of a random walk implies that the share price changes should be independent of each other and that they should conform to some probability distribution. Both of these features are important for discussion, but for historical reasons we will discuss independence first.

The best reference to this area is Cootner (1964) which is a collection of articles on this subject. The main U.K. work is by Dryden (1968, 1969, 1970 and 1970a). Other relevant articles are by Kendall (1953), Brealey (1970) and Kemp and Reid (1971).

The random walk hypothesis is essentially a short-term hypothesis as the intervals between prices have not been large (daily, weekly or monthly changes are usually studied). Thus it is a statement only about the short-term behaviour of the share market.

§3. THE INDEPENDENCE OF SHARE PRICE CHANGES

Independence in the random walk hypothesis is a very simple concept. It merely means that the members of a sequence of share price changes are independent of each other. Thus, future price changes and therefore future prices cannot be predicted from past price changes. At this stage, it does not say that future prices cannot be predicted from other past information or some other type of information.

The main tests used to detect dependencies in price changes have used statistical or financial-type tests. Typically the statistical tests have used serial correlation coefficients and runs tests to try and detect sequences of price changes which are positively or negatively correlated. In general, not a great deal of dependence has been found in sequences of share price changes.

Kendall (1953) analysed London share price indices by finding

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serial correlation coefficients for the first difference of weekly obser-
vations. In general, these coefficients did not differ significantly
from zero, and so supported the random walk hypothesis. Kendall
concluded that investors could not make money by watching price
movements and investing in shares which were apparently rising.
His paper is important because he attempted to analyse the indices by
the conventional time series method of separating the series into
trend, cycle, seasonal and residual components. This method, and a
more flexible approach using autoregressions, both broke down as the
random changes between terms were large enough to hide any
systematic effects so that the data behaved like wandering series.

The statistical testing of share price data can best be illustrated by
considering Dryden's (1970) work. He studied, for fifteen securities,
the daily closing price from *The Times*. The prices were adjusted for
script and rights issues, dividends, etc. The periods 1963–64 and
1966–67 gave two series each of 500 trading days.

The serial correlation coefficients, $r(a)$, $a = 1, 2, \ldots$, are defined by

$$r(a) = \sum_{t=1}^{n-a}(x_t - \bar{x})(x_{t+a} - \bar{x})/(n-a)s_x$$

where $\bar{x}$ and s_x are the mean and standard deviation of the $\{x_t\}$,
and n is the number of observations on the change in the log of
share price, x_t. These coefficients measure dependence between
price changes a periods apart. Assuming the population correlation
coefficients are 0, $r(a)$ has an approximate standard deviation of
$(n-a)^{-\frac{1}{2}}$, which lets us test if the observed values $r(a)$ are 0 or not.
Table 1 below gives values of daily serial correlation coefficients for
1963–64 for selected securities.

Table 1. *Daily serial correlation coefficients 1963–64*

Security	1	a (days) 3	12
Cater Ryder	·07	·13*	·02
Midland Bank	·24*	·11*	−·02
Dunlop Rubber	−·01	−·09*	·00
I.C.I.	·06	−·06	·01
Burmah Oil	·05	−·05	−·00
Consols (2½%) undated	·15*	−·07	−·04

*Means coefficient greater than two standard deviations (± ·09).

Most of these coefficients are small and not statistically significant. Even the largest, ·24, would not be a useful predictor as it could only explain 6% of the variance of prediction.

Working with the *Financial Times*—Actuaries 500 Share Index, Dryden (1970a) over January 1963 to April 1967, found $r(1) = ·29*$ and $r(9) = ·15*$ to be the only significant values—hardly large enough to be useful predictors.

Another simple method of seeking dependence in price changes is to study the number of runs of positive, negative and zero price changes. We define a run as a sequence of price changes of the same type. Thus we can record the frequency of runs by (i) type of run $(+, -, 0)$ and (ii) length of run (1, 2, . . .). Assuming price changes are independent, we can calculate the expected number of runs of each type and compare them with the actual number of runs as a test of the random walk hypothesis. Table 2 below contains actual and expected runs for I.C.I. over 1963–64.

Table 2. *Actual and expected runs, by length and type, I.C.I., 1963–64*

Length	Positive		No change		Negative	
	Actual	Expected	Actual	Expected	Actual	Expected
1	66	69·9	65	64·0	63	68·7
2	27	27·0	11	12·8	33	28·4
3	13	10·5	3	2·6	13	11·7
4	6	4·0	1	0·5	5	4·8
5	2	1·6	0	0·1	1	2·0
6	0	0·6	0	0·0	1	0·8
7	0	0·2	0	0·0	1	0·3
Total	114	113·8	80	80·0	117	116·7

Various comparisons can be made such as comparing actual and expected for the grand total of runs, the total runs of each type or the runs by length and type. In general, a certain small amount of dependence is discovered which is consistent with the significant serial correlation coefficients described earlier. It usually manifests itself as less actual runs than expected, i.e. the price changes tended to show a small amount of positive dependence which meant longer and therefore less actual runs.

Moderately similar results have been obtained by Brealey (1970) and Dryden (1970a), for the results of runs, tests and serial correla-

tion analysis. The picture is broadly similar for non-British data, except that even less dependence has been found in New York prices—Fama (1965) and Australian prices—Praetz (1969).

A non-statistical test of the random walk hypothesis is given by the filter method. An $x\%$ filter is defined as follows. If the daily closing price of a share moves up at least $x\%$, buy that share and hold it until the price goes down at least $x\%$ from a previous high, at which time simultaneously sell the share and go short. This short position is maintained until the daily closing price rises at least $x\%$ above a previous low, at which time one both covers and buys. Any move of less than $x\%$ in either direction is to be ignored. The aim of this technique is to filter away short-term price movements to enable profits to be made from longer-term movements.

Dryden (1970 and 1970a) has applied filter rules to series of share prices and share price indices. For convenience, the results are expressed in terms of annual per cent rates of return received from the filter strategy. These are presented in Table 3, for the securities of Table 1 over 1963–64 and the F.T.-Actuaries 500 Index for filters of size 1, 3 and 5%. For comparison, we also present rates of return for a Buy and Hold strategy, which is triggered off by the first 'buy' signal. These rates form a useful comparison for the filter tests as, if the random walk hypothesis holds, no mechanical trading rule applied to specific securities should consistently outperform a policy of simply buying and holding that security. However, if there is some degree of dependence, either positive (persistence) or negative (reaction) in successive price changes, then some system using this dependence should produce greater expected returns than a buy-and-hold policy.

The results of Dryden's work, of which a selection are presented in Table 3, are fairly strong support for the random walk hypothesis as the returns from buy-and-hold are usually greater than from the filter rules. These returns have not been adjusted for transactions costs due to the complex nature of such costs in the U.K. This naturally reduces the relative performance of the filter tests even more. The returns form the F.T.-Actuaries 500 Index have not been adjusted for dividends, which together with transactions costs, would reduce the returns from the small filters substantially. The work of Fama on U.S. data adjusted for dividends and transactions costs has provided very convincing evidence in support of the random walk hypothesis as the returns from the filter rules for his sample

Table 3. *Annual percentage rates of return from filter rules (F) and buy and hold (B)*

| | Filter size (%) | | | | | |
| | 1 | | 3 | | 5 | |
Security	F	B	F	B	F	B
F.T.-A 500 Index	14·2	2·3	−2·6	−0·5	2·2	−1·4
Cater Ryder	9·6	−1·1	7·9	1·4	1·1	0·4
Midland Bank	24·1	2·4	−4·5	2·9	−24·1	0·9
Dunlop Rubber	0·6	18·3	−14·2	18·3	−10·2	20·2
I.C.I.	−13·9	9·6	4·4	10·6	−3·4	10·0
Burmah Oil	9·3	21·9	−6·4	23·7	8·3	28·5
Consols (2%) undated	−4·1	0·6	−0·3	8·5	−6·6	6·3

of thirty shares were substantially less than the returns from the buy-and-hold strategy.

Having discussed certain methods of testing for independence in share price changes, let us consider briefly the second facet of the random walk hypothesis, the probability distribution of these changes. Naturally, we expect to find the normal distribution is a reasonable description of the distribution of price changes as these are changes in the logarithm of share price as defined in §2. The log transform should remove most of the skewness in the probability distribution.

Some empirical evidence on the shape of the distribution is provided by Dryden (1970 and 1970a). He studied the frequency distributions of his sample of shares and indices, including the F.T.-A 500 Index. These frequency distributions can best be described relative to the normal distribution with mean zero and unit standard deviation. It is apparent that they have a distribution which is highly non-normal. In general, there are far more observations than expected within $\pm \cdot 5s_x$, far less than expected in the range $\pm(1 \text{ to } 3)s_x$, and far more than expected at greater than $\pm 3s_x$. Descriptively, these distributions have centres which are high and peaked, tails which are fat, and are hollow in between, relative to the normal distribution. Many of these distributions are symmetrical, but some seem positively skewed to a small extent.

The implications of this shape of distribution for the risk conscious investor are quite remarkable. The fat tails mean that the chance of large deviations is far greater than intuition, perhaps based on the normal distribution, would lead us to expect. For example, Dryden's

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sample of shares shows that the number of price changes greater than $\pm 4s_x$ is about 50 times larger than expected!! As there is also the high, narrow peak, there is a greater probability of very small or no changes taking place. This partly reflects the large number of zero changes registered by individual shares. However, the high peak is also present in the frequency distributions of share price indices which have very few zero changes. Similar results for the distribution of price changes were found by Brealey (1970) for U.K. data, as well as for U.S. data and Australian data.

This characteristic distribution of share price changes has been studied extensively, and many unsuccessful attempts have been made to describe it by a known distribution function. However, the only successful attempt has been made by the author, Praetz (1972), who has represented changes in share price indices by a t distribution, suitably rescaled. The trouble with the data is that the mean and variance of the process which is generating it changes stochastically with the degree of activity in the share market.

The other work of Dryden (1968 and 1969) has been a study of the proportions of share prices rising, falling, or remaining unchanged from day to day. These proportions have been analysed by information theory and Markov process analysis.

The random walk hypothesis has recently been generalized by Fama (1970) to include a strong and a weak form. The weak form refers to the hypothesis as already stated, whereas the strong form says that all past information and expected future information is discounted and reflected in the price of the share under consideration.

The rest of this paper is concerned with discussing certain methods of prediction of share prices, both theoretical and those used by investment analysts. The random walk hypothesis should make it clear why in general these methods will not do any better than a simple buy-and-hold investment strategy.

§. 4. PREDICTION OF SHARE PRICES WITH THEORETICAL MODELS

A basic proposition of capital theory would equate the value of a capital asset to the discounted value of all future receipts. If we have a share valued at p_t at time t, giving rise to a perpetuity of receipts, $(r_{t+j}; j = 1, 2, \ldots)$ at the end of each time period, and discounted by an investor at an interest rate of i_t, then

$$p_t = \sum_{j=1}^{\infty} (1+i_t)^{-j} r_{t+j}.$$

Whilst theoretically valid for an investor ignoring transactions costs and all forms of taxes, the expression for p_t is intractable. A typical but simplified solution is given by assuming the relevant company is unlevered (its capital structure has no debt capital, i.e. only equity capital), that it earns a constant rate of return r on all its capital, and that it retains a constant proportion b of its profit after tax. Thus

$$r_{t+j} = r_t(1+br)^j$$

and so

$$p_t = r_t(1+br)/(i_t-br)$$

if

$$i_t > br.$$

Models like this can readily be extended to include leverage, risk and uncertainty, investor and corporate taxes, and outside equity financing if one is prepared to make simplifying assumptions in the functional forms of the relevant variables. This is necessary as we presumably wish to test statistically the significance of such variables. Thus we have to obtain p_t, or some transformation of it, as a function, preferably linear, of the variables which determine it. Typical work on this problem has used classical multiple linear regression on cross-section data of a sample of shares at a fixed time point to evaluate the importance of the specified variables. This approach outlined above has been manifest in a large number of works, including Weaver and Hall (1967). A typical model would be something like this:

price $= a+b$ (dividend)$+c$ (retained earnings)$+d$ (risk)$+ . . .$

In general, the regression results of such studies have been moderately successful. If we have relevant information on dividends, earnings, etc., we can obtain a warranted or expected price, which could be compared with the actual price of each share. However, if we misspecify the model by the omission of important variables, our expected price may be biased.

One advantage gained by working on cross-section data is that i_t

is fixed, as it is a uniform rate at time t with which an investor discounts future receipts. Thus we do not have to explain changes in i_t nor to attempt to predict it in the future. Very often, regressions performed on the same sample of shares at different points in time have unstable regression coefficients. As such regression coefficients are often functions of i_t, then changes in i_t from year to year will influence the changes in the coefficients. Even when working in this static sense, however, the best specification of the theoretical variables needed to solve the problem is far from clear. This static cross-section model can be used as a predictor perhaps if we used it to locate undervalued shares. The hypothesis that we would hope to exploit is that such shares perform better than the market as a whole. This approach has been tested by Malkiel and Cragg (1969) who found that it was not a useful predictor as sometimes shares selected in this way performed worse than the market average and sometimes better. In general, no clearly useful conclusions emerged from their study.

Turning to the more difficult problem of a time series study of a single share, we can use the static cross-section model used above with the modification that i_t, the investor interest rate used to discount future receipts, varies over time. We expect i_t to vary over time with the state of the economy and depending on expectations of inflation, the productivity of capital in the particular company, and the asset preferences structure of investors.

Prediction of share prices thus would need to predict dividends, earnings, risk, etc. as well as i_t, as now the coefficients a, b, c, d, are functions of i_t. However, the prospects for successful prediction of these variables are not good. Little and Rayner (1966) in *Higgledy, Piggledy Growth Again* found that future earnings were not predictable from past earnings. Cragg and Malkiel (1968) on U.S. data, found that the best estimates of future earnings produced by security analysts were little better as predictors. Dividends were a little more predictable, but the timing of dividend changes is very difficult to predict.

Any complete explanation of share prices over time is going to require an explanation of changes in investors' discount rates and of interest rates in general. This would, we expect, lead into the full field of macroeconometric models very quickly, if we wanted both to explain and predict interest rates. As far as we know, this field is almost completely unexplored due to its difficulty. Recent models of

the U.S. financial sector have ignored share prices. Thus a complete theory of share prices has both a micro and macro economic element. It is necessary to have a theory which will evaluate expected future earnings as well as a theory which will explain the interest rates with which investors will discount these expected future earnings.

§5. IMPLICATIONS FOR INVESTMENT THEORY

Strategies for price prediction of the 'chartist' (or 'technical') type would appear to have little chance of producing exceptional rates of return as they are based on past prices and other past information. As such, they are directly contradicted by the random walk hypothesis. Certain strategies appear to be unsuccessful in producing exceptional returns: filter systems, 'Dow Theory' systems and price-volume systems.

The Dow theory asserts that when a share peaks and then goes down, the peak is a 'resistance area'. If the price breaks through this resistance area, it is probable that it will keep on increasing, which is interpreted as a buying signal. Similarly for a trough, a price subsequently going lower is interpreted as a sell signal. The price-volume system asserts that when a share goes up with a large or increasing turnover, buying interest has increased and this should continue for some time, implying a buy signal. Similarly, a share moving down on large turnover implies a sell signal. Levy (1971), in a study of the usefulness of chartist patterns, found no predictive significance in five point charts.

These strategies do not always perform worse than simple buy-and-hold portfolios, as that is only an average result. Investors in these strategies are handicapped by extra transactions costs, the fact that they are out of the market for part of the time, and extra costs in watching the market.

A more powerful strategy for investment decisions is that of fundamental analysis. This uses published accounting information for the share under consideration. It uses a large amount of information about the share to assess its future earnings in order to decide whether or not the price of the share is less than its true value. If it is, it is interpreted as a buy signal. Conversely, a sell signal is inferred for shares with price in excess of value.

In general, the rates of return from this more intellectually satisfying method of approach will be less than returns obtained

from simple buy-and-hold strategies, once transactions costs are included.

The evidence for this has come from studies of portfolios of shares; Fama (1965), the Wharton Study (1962), and Jensen (1963). Overall, these portfolios have not performed any better than the market. Fama found that if the initial charges on mutual funds are ignored, the funds do about as well as a randomly selected portfolio. However, when the initial charges are included, the funds perform rather worse. This is consistent with the Wharton study of mutual funds as well.

That these fundamental methods of investment analysis do not appear to produce exceptional rates of return would seem to be in agreement with the strong form of the random walk hypothesis.

It is important that any method of investment analysis which is proposed should be subject to rigorous scientific testing to evaluate its usefulness. This would involve testing it over enough shares and different types of market conditions so that the expected value of such a method would emerge free of chance fluctuations.

There have been several references to the random walk hypothesis in actuarial literature. Weaver and Hall (1967) briefly mention it, but suggest it is equivalent to selecting shares using a pin. The splendid paper by Hemsted (1969) discussed the random walk hypothesis, but was clearly rather sceptical about it. In particular, Hemsted felt that useful predictions could probably be made using extra information on company and economy expectations and that unit trusts could do better than share price indices. He also felt that the share market is unsophisticated enough to tend to make share prices more likely to return to their intrinsic values when they were some distance away from these values. We feel that the evidence that is beginning to accumulate for the strong form of the random walk hypothesis, that we have already discussed, would tend to make it rather unlikely that any exceptional returns can be achieved from such features, after including transactions costs.

A puzzling feature of the work of Weaver and Hall is the result that shares selected by their ranking method perform better than the F.T.-A Index. However, the comparison ignored dividends and their sample excluded smaller companies and Insurance and Property shares. Their comparison was the percentage by which the performance of their portfolios exceeded that of the F.T.-A Index. A much more realistic measure would have been an annual rate of return,

which included dividends, capital growth and any other changes in the status of the share, such as new issues. As such, their evidence is hardly convincing.

§6. APPLICATION TO MATURITY GUARANTEES IN EQUITY-LINKED POLICIES

A direct application of the random walk hypothesis is given by Di Paolo (1970), Turner (1970) and Benjamin (1971) on the problem of maturity guarantees for equity-linked life assurance policies.

The simplest example of such a policy is given by an endowment policy with a premium of £1 per year for 10 years. The company invests the premiums in equity shares, so the maturity value of the policy depends on the level of these shares at the maturity date. To counter this investment uncertainty, the company offers a guaranteed minimum maturity value of, say, £10. As average rates of return from equity shares are positive, the company usually will not be called upon to exercise its guarantee. However, there is some risk that in times of falling share prices, its guarantee may be needed.

The approach used in the articles referred to above has been to obtain the probability distribution function of the rate of return of equity shares. This is based on actual historical data as it is highly non-normal in shape. These returns have been tested for randomness consistent with the random walk hypothesis. Then they have been used in a simulation exercise to provide future investment results to give the distribution of amounts needed under the maturity guarantee. Then a net risk premium can be calculated from the amounts required and their probabilities. Clearly the problem is a very complex one, especially in its legal and policy implications and the papers referred to above are just first attempts to render this new form of risk (stock market) amenable to conventional actuarial theory and practice.

§7. CONCLUSIONS

Most of the evidence for the random walk hypothesis has come from the U.S., where there does not seem to be much exploitable dependence in share prices. It is possible that there will be more dependence in U.K. prices, although we doubt if it will be all that profitable, given the evidence we have presented above.

Most of the conclusions that emerge from the random walk hypothesis are negative. Are we then reduced to share election using a pin? Certain rays of hope still remain, even though the investment strategy suggested by the random walk hypothesis is an inactive one.

The principle of diversification is an important one, as it can substantially lower the risk element in a portfolio. As we might emulate a buy-and-hold portfolio, we will try to minimize transaction costs. It is important to be conscious of the tax position of the portfolio owner, whether institutional or individual. In particular, shares should be sold to establish losses for taxation purposes. To avoid realizing capital gains, the portfolio could be borrowed against. These suggestions can alleviate the lot of the investor to some extent. The more difficult thing is to believe that there is a fair bit of truth in the random walk hypothesis.

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