NOTES ON OTHER ACTUARIAL JOURNALS

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AUSTRALASIA

Transactions of the Actuarial Society of Australasia, **8**, 1953

Wickens, P. C. *Presidential address*, pp. 1–16. Refers to a possible combined investigation of Australian assured life mortality, the taxation of pension funds, life office investment yields and the decrease in the number of candidates for the Preliminary examination.

Oxby, L. G. *Overseas developments in life insurance*, pp. 17–35. 'Overseas' actually refers to the U.S. Discusses pension business (Deposit Administration and equity annuities), mechanization, sub-standard insurance and the *a*–1949 Table.

Fletcher, J. B. *Group life assurance and pension business*, pp. 37–47. The paper is sub-titled: Comparisons between underwriting methods in America and the United Kingdom.

Alder, M. C. *Notes on the investment of life assurance funds*, pp. 49–72. A review of recent papers on investments presented to the Institute and the Faculty.

Caffin, S. W. *Construction of mortality tables from population data*, pp. 73–94. Examines the effect of different age-groupings on King's method of graduation and considers the problem of errors of age in the exposed to risk.

Kensit, C. R. *Some elements of modern statistical theory*, pp. 95–134. An expository account.

Lane, G. C. *Accident mortality in Australia*, pp. 135–63. Analyses the data by periods, cause of accidental death, occupation, and State. Particular attention is paid to fatal automobile and motorcycle accidents.

Gray, J. C. and Cohn, L. J. *Group life and pension schemes—Office practice*, pp. 165–88. Discusses policy forms, applications for membership, office and valuation records, disability claims, surrender values and cash options on retirement.

FRANCE

Bulletin Trimestriel de l'Institut des Actuaires Français, **65**, Part 4

Risser, R. *Propos sur la formation de l'actuaire*, pp. 167–9. A plea for the more extensive study of Political Economy as a part of the training of actuaries.

Lamson, J. *Essai sur la comptabilité industrielle en assurance vie*, pp. 183–263. The methods of cost accounting are developed and applied, in great detail, to life assurance business. Shorter statement is contained in *Proc.* 13 *Internat. Cong.* 1, 62.

This part also contains a memoir of the Institute's former president Louis Weber (1866–1949) and a description of the fiftieth anniversary celebrations of the Norwegian and Swedish actuarial associations in 1954.

GERMANY

Blätter der Deutschen Gesellschaft für Versicherungsmathematik, Vol. 2, Part 1, 1954

LANG, R. *Bewertung von Sterblichkeitsbeobachtungen an kleinem Material*, pp. 7–37.

HELBIG, M. and STORCK, H. *Ditto*, pp. 39–59.

These two papers shared a prize offered for a monograph on the treatment of scanty mortality observations. Both are essentially expository and describe the application of confidence intervals and the χ^2-test in mortality statistics.

JECKLIN, H. *Lidstonesche Näherungsformel und Makehamsche Funktion*, pp. 61–70. Further notes on the re-graduation of mortality tables by the relation $\ddot{a}_{x\overline{n}|}=A_x+q_xB_n$ and on the valuation simplifications that result (see *J.I.A.* **80**, 116).

DENFFER, H. VON. *Über die zufälligen Sterblichkeitschwankungen bei Untersuchungen an erhöhten Risiken*, pp. 71–84. Since a subdivision of extra risks into relatively homogeneous categories results in small numbers exposed, it is important to decide how much of any deviation from a standard may be ascribed to chance. Poisson's law and the Normal approximation are used to provide confidence limits and χ^2-tests to assist in the classification of risks. There would not be general agreement about the author's views on the 'exactness' of the Poisson law for mortality observations.

LAH, I. *Die Ableitungen der Versicherungswerte nach einzelnen Zinsmassen*, pp. 85–92. The successive derivatives of a_x and allied functions with respect to i, δ and v are investigated by means of the four S-functions:

(*a*) Stirling's numbers of the first kind S_n^ν defined by

$$(x)_n=\sum_{\nu=1}^{n} S_n^\nu x^\nu, \quad \text{where} \quad (x)_n=x(x-1)(x-2)\ldots(x-n+1).$$

(*b*) Stirling's numbers of the second kind σ_n^ν defined by

$$x^n=\sum_{\nu=1}^{n}(x)_\nu\,\sigma_n^\nu.$$

(*c*) The numbers S_n^ν defined by

$$[x]_n=\sum_{\nu=1}^{n}(-1)^n(x)_\nu\,S_n^\nu,$$

where $[x]_n=x(x+1)(x+2)\ldots(x+n-1)$.

(*d*) The generalized commutation function

$$S_{x+1}^{(n)}=\sum_{t=1}^{\omega-x}\binom{n-1+t}{n}D_{x+t}.$$

These differential coefficients occur in the Taylor expansions of a_x at rate $i+h$. Numerical examples are given to show the accuracy obtainable when the expansion is broken off after each of the first few terms.

PÖTTKER, W. *Erweiterung einer von Jecklin angegebenen Formel zum Zinsfuss-problem*, pp. 93–96. The formula referred to is

$$\ddot{a}'_{x\overline{n}|}=\left(\frac{1}{\ddot{a}'_{\overline{n}|}}+\frac{1}{\ddot{a}_{x\overline{n}|}}-\frac{1}{\ddot{a}_{\overline{n}|}}\right)^{-1},$$

where dashes denote functions calculated at rate i' (see *J.I.A.* **78**, 135). The approximation is improved by expanding

$$\frac{1}{\ddot{a}_{x\overline{n}|}}-\frac{1}{\ddot{a}_{\overline{n}|}}$$

as a Taylor Series, and numerical examples are given.

MEIDELL, B. *Beitrag zur Anwendung des Mittelwertsatzes auf versicherungs-mathematische Probleme*, pp. 97–117. Develops the approximate formula

$$\frac{a'_x}{a_x}\simeq\left[\tfrac{3}{2}\,(1+hv)^{S_{x+1}/2N_{x+1}}+\frac{hv}{4}\,\frac{S_{x+1}}{N_{x+1}}-\frac{1}{2}\right]^{-1},$$

where a'_x is at rate i' and $i'=i+h$, and gives numerical examples. Other formulae are also derived.

ITALY

Giornale dell'Istituto Italiano degli Attuari, **17**, 1954

FINETTI, B. DE. *La compensazione tra rischi eterogenei*, pp. 1–21. Discusses, with particular reference to general insurance practice, the relative merits of homogeneous and heterogeneous portfolios. The former are better suited to the collection of statistics, but a miscellaneous portfolio of unusual risks of various types has the advantage that one can be sure that there will not be a high correlation among the claims.

MAZZONI, P. *Il metodo d'interpolazione del Lever*, pp. 22–35. Demonstrates the superiority of Lever's method (*J.I.A.* **52**, 171) over the method of linear interpolation.

Shows that

$$\frac{a_1-a}{a_2-a}=\frac{n''(c)}{a''(c)}\,e^{-n\delta-\theta\delta(n_1-n)}$$

where a_1 = approximate value of a_x^{δ} by Lever's method $=a_{\overline{n_1}|\delta}$, a_2 = linear interpolation between $a_x^{\delta_1}$ and $a_x^{\delta_2}$, a = true value $=a_{\overline{n}|\delta}$. $0<\theta<1$ and c is some intermediate value of the force of interest.

DIEULEFAIT, C. E. *Sulla legge di distribuzione degli zeri dei polinomi ortogonali classici di grado* n, *considerata al divergere di* n, pp. 36–46. The asymptotic values of the zeros of Legendre and Hermite Polynomials are obtained by probability methods due to Cantelli.

JECKLIN, H. *Sui fondamenti dell'interpolazione iperbolica nel calcolo delle riserve matematiche*, pp. 47–53. Further notes on the author's methods of calculating the approximate reserves of Endowment Assurances. (Cf. *J.S.S.* **10**, 119.)

GULOTTA, B. *Su alcune questioni riguardanti gli eventi compatibili e sulla loro applicazione a una questione di logica*, pp. 54–61.

FERRA, C. DE. *Tavola di mortalità per una popolazione inomogenea*, pp. 62–76. A composite group of lives is considered as made up of a number of sub-groups for each of which Makeham's Law applies. This means that the mortality law of the whole group is *not* that of Makeham. If

$$\mu(t) = A_i + B_i e^{\lambda t}$$

for the ith sub-group, and we assume that for the whole group the number living is given by

$$L(t) = e^{-[At + B(e^{\lambda t} - 1)/\lambda]},$$

then A, B and λ may be obtained by fitting the $L(t)$ curve to the 'mean values' of the individual $l_i(t)$ of the sub-groups (referred to a common radix). The article shows that by an ingenious choice of boundary conditions the fit can be achieved mathematically. Various interpretations of 'mean value' are used.

This issue also contains notes on the 14th International Congress of Actuaries, Madrid, 1954.

SCANDINAVIA

Skandinavisk Aktuarietidskrift, **37**, 1954, Parts 1 and 2

TEICHER, H. *On the multivariate Poisson distribution*, pp. 1–9. The so-called Correlated Poisson in two variates is extended to p variates and its properties studied.

STELSON, H. E. *Finding the root of an equation by iteration*, pp. 10–18. A development of ideas broached by Steffensen in two previous articles (see *J.I.A.* **73**, 441).

HOEL, P. G. *On a property of the sequential t-test*, pp. 19–22. Neither of the sequential t-tests proposed by Wold and Arnold, respectively, possesses the double minimax property usually ascribed to them.

MEIDELL, B. *Zur Abschätzung des Einflusses einer Zinsänderung auf die Leibrentenwerte*, pp. 23–37. Proposes the simple formula

$$\bar{a}_{x(\delta_1)} \doteq \left(1 + \frac{\bar{a}_{x(\delta_0)}}{\omega - x}\right)\left\{1 + \left(\delta_1 - \delta_0 + \frac{1}{\omega - x}\right) a_{x(\delta_0)}\right\}^{-1} \bar{a}_{x(\delta_0)},$$

where $\delta_1 - \delta_0$ is about ·005 and an error of a few units in the second decimal place is acceptable. A more exact formula is available when the function $\sum_{s=x}^{\infty} S_s$ is tabulated at the standard interest rate.

REIERSØL, O. *Tests of linear hypotheses concerning binomial experiments*, pp. 38–59. Applies Neyman's modified χ^2 (i.e. with actual, instead of expected, frequencies in the denominator of each term) to various tests of hypotheses about linear functions of probabilities that have observational analogues (i.e. relative frequencies). No numerical examples.

NORDBOTTON, S. *On the determination of an optimal sample size*, pp. 60–64. Each variate sampled is assumed to be measured with an error that varies inversely as the time spent in its observation. Subject to a fixed cost expressible in the form $C_0 + C_1 nt$, the sample size, n, and the total time spent, t, are determined so as to minimize the sampling variance of the measured mean.

RÅDE, L. *A note on a modified t-test*, pp. 65–70. The function $W_{m,k}^{-1}\sqrt{n}\,(\bar{x}-\xi)$, where $x_i\,(i=1, 2, \ldots, n=mk)$ is a random sample from a normal universe of mean ξ, $R_{m,k}$ is the mean range of the k sub-samples of size m, and $W_{m,k}=R_{m,k}/\xi(R_{m,k})$, is treated as the quotient of two normal variates and the power function calculated. Regarded as a substitute for the t-test of the mean the most powerful test is provided by $m=8$.

GURLAND, J. *On regularity conditions for maximum likelihood estimators*, pp. 71–76. Relaxes the condition that the third order derivative of $\log p(x)$ must exist and be bounded in Cramér's theorem about the asymptotic properties of maximum likelihood estimators.

SWITZERLAND

Mitteilungen der Vereinigung schweizerischer Versicherungsmathematiker, **54**, 1954, Parts 1 and 2

FRAUENFELDER, W. *Beobachtungen über die Sterblichkeit bei den Einzel-Kapitalversicherung der Schweizerischen Lebensversicherungs- und Rentenanstalt*, pp. 15–37. An investigation of the mortality experience of a large Swiss life office during the years 1940–50. The final graduated table (RAE 1940/50) aggregates all years of assurance, both sexes, industrial and ordinary business, and both medical and non-medical risks. However, figures are provided to show the mortality differences between medical and non-medical cases and between males and females. Secular trends are investigated for eight broad categories of cause of death.

LEEPIN, P. *Reserverechnung in der Volksversicherung*, pp. 39–46. Advocates the net premium valuation of industrial policies by punched cards that do not contain the net premium. This is effected by writing the net premium as a linear function of the gross, the two constants being calculated by (weighted) least squares.

VOGEL, W. *Die Faktorenmethode bei der individuellen Witwenrentenversicherung*, pp. 47–56. Considers the tabulation of reversionary annuity values by the formula

$$\breve{a}_{x|y}=f\,(x,\,\Delta)\;\breve{a}_{x|x-s},$$

where $\Delta=x-y$. The form of $f\,(x, \Delta)$ is investigated and the practical calculation of the table of values of $f\,(x, \Delta)$ used in the official Swiss group premium tables of 1953 is described.

ZWINGGI, E. *Ein Verfahren zur Bestimmung der Rendite von festverzinslichen Anleihen*, pp. 57–70. A direct method for calculating yields on securities. Writing i_0 for the nominal interest, i the required yield, and $1+i=\dfrac{1+i_0}{1+\epsilon}$,

the present value of the security is expanded in powers of ϵ as far as the second degree; this equation is then solved approximately for ϵ. Three cases with varying assumptions about capital repayment, interest payments, etc., are considered in detail and numerical examples given.

JECKLIN, H. and STRICKLER, P. *Eine Variante zur* F*-Methode der Reserveberechnung*, pp. 71–80. The *F*-method of hyperbolic interpolation for reserve values (see *J.I.A.* **75**, 114; **77**, 129; **78**, 138–39 and *J.S.S.* **10**, 119) expresses the reserves in the form ${}_tV = Gt/(1-\phi t)$, G and ϕ being functions of the portfolio but not of t. The present paper considers the derivative form ${}_tV = k + tg + th/(1-\phi t)$. Numerical examples include some figures for the A 1924–29 table with $\phi = \cdot 013$.

SACHS, W., STANISZEWSKI, J. and ROPER, G. *Vom Wesen der Auslese. Analytisch-Kritisches über die Grundlagen der Lebensversicherung erhöhter Risiken*, pp. 81–98. A closely reasoned and interesting discussion of the problem of selection and 'damaged lives' (*T.F.A.* **3**, 350 and 379). The authors distinguish (1) lives that are and remain 'select', (2) lives that die before the end of the 'select period', (3) substandard lives that become select during the 'select period' and (4) lives substandard throughout.

MAURER, W. and BOSS, M. *Eine verfeinerte* t*-Methode*, pp. 99–110. The *t*-method of valuation groups policies according to duration and uses an average age at entry, calculated by weighting mortality rates by sums assured. The author investigates improved methods of determining the average age, and gives numerical examples of the results.

KREIS, H. *Summation interpolierter Zahlenreihen*, pp. 111–16. The approximate evaluation of $s = f(a) + f(a+1) + f(a+2) + \ldots + f(b)$ by means of expressions like $\sum_{k=1}^{n} A_k f(x_k)$, $n \ll b - a + 1$. Numerical examples are given. Schobe's paper in the German *Blätter* (see *J.I.A.* **79**, 99) is more thorough.

ADRIAN, P. *Beziehungen zwischen den abhängigen und den unabhängigen Ausscheidewahrscheinlichkeiten bei besonderen Annahmen über den Verlauf der Ausscheideintensitäten*, pp. 117–23. Derives the relationships between the various functions of a multiple decrement table on the alternative assumptions (i) that the μ's are proportional to each other, and (ii) that the decrements of the single-decrement tables are uniformly distributed over the year of age.

JECKLIN, H. and STRICKLER, P. *Wahrscheinlichkeitstheoretische Begründung mechanischer Ausgleichung und deren praktische Anwendung*, pp. 125–61. What linear function of the $2r+1$ observed values of y_{x+j} symmetrically disposed about x minimizes the (unweighted) sum of squares of the deviations of y_{x+j} from $y\,(x+j) = a + bc^{x+j}$ and leaves y_x unchanged when, in fact, it equals $y\,(x)$? A solution is given which involves the (approximate) knowledge of c and a 13-term formula is illustrated on two Swiss mortality tables. The Austrian population table 1930/33 was graduated in a rather similar manner by Vajda and Boschan in 1937.

LEEPIN, P. *Sterbegesetze, welche eine exakte Darstellung der Leibrenten durch Zeitrentenwerte erlauben*, pp. 163–68. The general result of Jecklin and Leimbacher (see *J.I.A.* **80**, 116) is put into a more practical form for numerical work and an example is provided.

RUFENER, E. *Sterbegesetze, für welche sich der Leibrentenbarwert durch Zeitrenten darstellen lässt*, pp. 169–84. Detailed consideration of the equation $\bar{a}_{x\overline{n}|} = A_1(t)\Phi_1(x) + A_2(t)\Phi_2(x)$. If a certain function of the first and second differential coefficients of A_1 and A_2 does not vanish, the necessary and sufficient condition that $\bar{a}_{x\overline{n}|}$ should be expressible in this form is that l_x should satisfy a linear homogeneous differential equation of the second order with constant coefficients.

AMMETER, H. *La théorie collective du risque et l'assurance de choses*, pp. 185–204. An expository account of the collective theory of risk—including the author's own generalizations (see *J.I.A.* 75, 251)—as it applies to non-life insurance.

UNITED STATES AND CANADA

Transactions of the Society of Actuaries, 1954 Mortality and Morbidity Reports

Covers the mortality under standard ordinary policies during 1952 and 1953 and under individual immediate annuities during 1948–53. It also provides new aviation statistics, reviews group life conversion mortality (1947–53) and group weekly indemnity and hospital and surgical expense insurance, and gives the 1953 mortality experience under group annuities. A useful feature is the references made to earlier reports in the *Transactions* and elsewhere.