

THE CAPITAL LETTERS IN ACTUARIAL NOTATION

IN Great Britain it has hitherto been the custom to use upright roman type for the capital letters in actuarial notation, although the symbols in 'lower case' have been in italic type. The Council has decided that italic capitals should be adopted officially by the Institute as from the May 1952 examinations and the June 1952 number of *J.I.A.* The following historical information may be of interest to members.

In the early days of actuarial science, problems were mostly posed and solved numerically; such notation as was needed to give generality to the solutions was fashioned to the need, not fitted into a comprehensive and systematic body of notation. In consequence there was sometimes a lack of precision in the solutions which made itself more and more felt with the passage of time. Early in the nineteenth century three British authors, Baily, Milne and Gompertz, set themselves the task of systematizing the notation.

Baily and Milne, following the practice of earlier authors, used A, B, C and so on for the various lives, without special indication of age, and relied upon the use of the same letters in different styles of type to denote the various functions on a given life. Baily (1813) used roman capitals for the lives and italic capitals for the annuity values, but Milne (1815) adopted the reverse notation, using italics for the lives and small roman capitals for the annuity values. Milne also introduced Old English capitals for assurances, whereas Baily had used a bold cursive capital for the failing life under a survivorship assurance.

Gompertz (1820) has the germ of modern notation, for he used L_x to denote the living at age x —what would now be called l_x . He seems to have been the first to use a suffix for the age, though, otherwise, his notation had little influence.

In the early development of the columnar method, for example by Tetens (1785) and Barrett (published by Baily, 1813), the columns were headed with capital letters which appear to have been labels rather than symbols to represent mathematical functions of a variable x . Griffith Davies (1825), who originated the use of the letters D, N and S for commutation functions, may have chosen those letters because they were the initial letters of the words denominator, numerator and summation respectively, a suggested origin which is made the more likely by his use of the same letters D and N for the columnar calculation of the expectation of life. He placed the letters at the heads of the columns without adding the suffix x ; in his formulae he used the letter by itself to denote the value of the commutation function at the given age, whatever it might be, and added prefixes and suffixes to denote values at ages above and below the given age. Babbage (1826), working from the end of life as Barrett had done, added the suffix v to Barrett's A and B to indicate the function at age v . De Morgan (1840) seems to have been the first to add the suffix x to Davies's D and N to produce the commutation symbols in modern dress, though in 'terminal' form.

Milne's notation prevailed for many years, side by side with Davies's for the columnar method; Farren (1844) lists the known formulae in 'modern'—that is to say in Milne's—notation. However, sometime in the 1840's the practice of using one letter for a given life and all functions dependent on that life began to be abandoned for the use of one letter for a given function with suffixes to

represent age and duration. Thus David Jones (1843) adopted p for the probability of survival, a for the annuity value and A for the assurance single premium.

Davies's choice of upright capitals for the columnar method no doubt exerted an influence towards such capitals for the other symbols that began to be developed, for example P for the annual premium and V for the policy-value. It must be remembered, too, that the convention of italic type in mathematical notation had not then been established. The evolution of the symbols was a gradual process and the conventions with regard to suffixes for ages and durations did not at first follow present-day practice which is based on the systemization of T. B. Sprague and George King.

In the appendix to the Institute of Actuaries Life Tables (1872) and in the Key to the Notation in George King's Text-Book, Part II (1887), which formed the basis of the International Notation adopted at the Second International Congress (1898), the capital letters were printed in upright roman type. This practice has been followed in all publications of the Institute and Faculty (i.e. *J.I.A.*, *T.F.A.*, text-books and mortality tables) with a very few exceptions.

In text-books, such as *Mathematics for Actuarial Students*, which contain a large number of mathematical symbols but very few actuarial ones, italic type has been used in accordance with current mathematical practice, and the fact that this represented a break with tradition, so far as the occasional actuarial symbol was concerned, appears to have escaped notice. On the other hand, in older books such as the early editions of *Frequency Curves and Correlation*, the capitals in all the mathematical symbols are upright. Upright capitals were used in the memorandum on the International Notation which was published in July 1946. They were used again when the Memorandum was reproduced in *J.I.A.* LXXV and *T.F.A.* XIX. In the text-books which are now being published for the Institute and Faculty italic type is being used for all the capital letters.

In America, before the introduction of the International Notation in 1898, plain 'full-face' type was used for the capital letters. At that time, the Americans favoured the 'initial' definition of N_x :

$$N_x = D_x + D_{x+1} + \dots$$

(which was the form originally given by Tetens), whereas the Institute of Actuaries had adopted the 'Griffith Davies' or 'terminal' definition:

$$N_x = D_{x+1} + D_{x+2} + \dots$$

It was for this reason that the plain 'full-face' N was included in the International Notation as an alternative to Sprague's N which had been made necessary by the development of 'select' life tables. For some years after 1898 the Actuarial Society of America continued to use plain 'full-face' capitals for all the actuarial symbols, but in Volume 9 (1905) and all subsequent volumes of the *Transactions* of that Society the capitals are in italic type, and N_x conforms to the 1898 International Notation. This was confirmed by a special resolution of the Society passed in 1911. In all the volumes of the *Record* of the American Institute of Actuaries from Volume 1 (1909) onwards, and in the recently published *Transactions* of the Society of Actuaries, italic capitals are used.

On the continent of Europe, practice seems to favour italic capitals; these

are used in the Journal of the Italian Institute, the Bulletin of the Swiss Association, *Skandinavisk Aktuarietidskrift*, *Het Verzekerings Archief*, the pre-war German *Blätter* and the *Bulletin* of the Permanent Committee (though the reproduction of the July 1946 Memorandum in Bulletin No. 46 of the Permanent Committee adopts the upright capitals of the original). On the other hand, the capitals in the Bulletins of the French Institute and the Belgian Association and in the post-war *Blätter* are upright. Italic capitals have been used in the records of the proceedings of five of the twelve International Actuarial Congresses and are being used for the thirteenth.

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