The Actuarial Profession
making financial sense of the future

**FINANCE, INVESTMENT & RISK MANAGEMENT CONFERENCE**

15-17 JUNE 2008
HILTON DEANSGATE, MANCHESTER

The Actuarial Profession
making financial sense of the future

**SWAPS and SWAPTIONS**
**Interest Rate Risk Exposures**

Viktor Mirkin
vmirkin@deloitte.co.uk
17 JUNE 2008
HILTON DEANSGATE, MANCHESTER

## Outline

- Cash flow structure, optionality and pay-off
- Overview of the OTC market in 2008
- Valuation methodologies – models of the yield curve
- Black's formula
- Implied volatilities
- Data sources – the swap curve
- Hedging interest sensitive liabilities with swap and swaptions
- Rho, rhoga, vega, volga, rhova
- Exotic swaps
- GAO hedges: impact of mortality
- Liability valuation basis impact: marked to market calibration of yield curve and volatility. Evidence from WP insurers

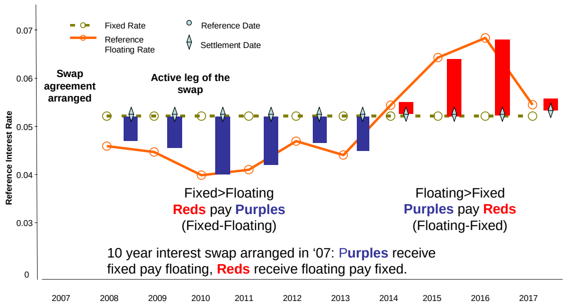
The Actuarial Profession
making financial sense of the future

# Relevance to Life Insurance Liabilities

- Insurers are structurally long in interest rates on marked to market liabilities
  - Guaranteed Annuity Options ⇔ Mortality linked receiver swaption with an asset linked nominal
  - Annuities ⇔ Mortality linked bond portfolios
- Liability matching strategies utilise opposite exposures
  - Gilts
  - Corporate Bonds
  - Receiver Swaps
  - Receiver Swaptions
- The relevance of swaps and swaption is therefore two fold
  - As hedging instruments
  - As reference instruments for marked to market calibration

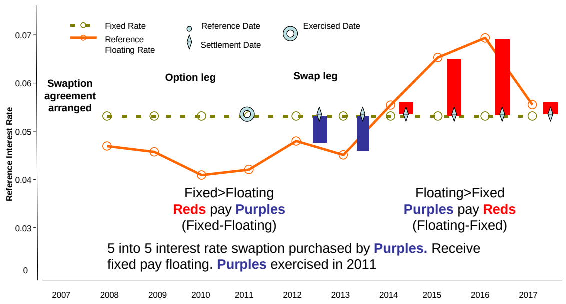


# Receiver Swap Cash Flows






# Receiver Swaption Cash Flows



## Receiver/Payer Relationships

- Payer Swap + Receiver Swap =0
- Receiver Swaption - Payer Swaption =Deferred Receiver Swap
- ATM: Receiver Swaption = Payer Swaption
- ATM Strike = Forward Swap Rate

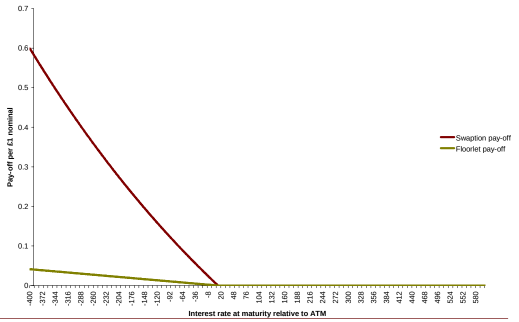


## Value Caps

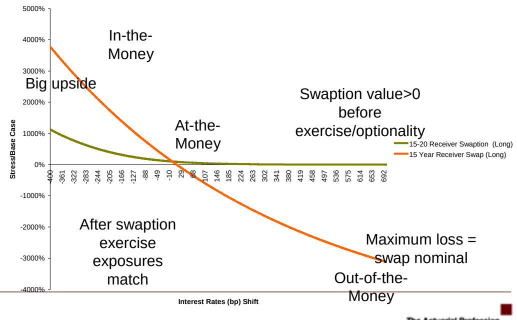
| | Max | Min |
|---|---|---|
| Receiver Swap | # Payments x Strike x Nominal | Nominal |
| Receiver Swaption | # Payments x Strike x Nominal | 0 |



## Swaption Pay-off at Exercise

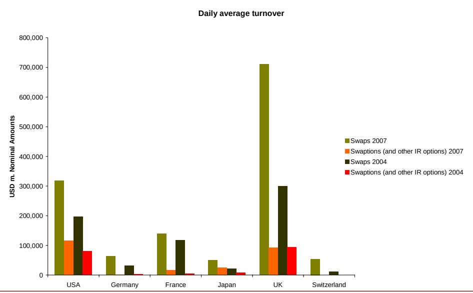


## Receiver Swap and Swaption: Parallel IYC Shift Response Function




The Actuarial Profession
making financial sense of the future

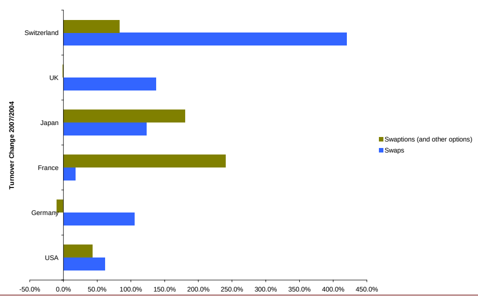
## Overview of the OTC Swap and Swaption Market




BIS Triennial Survey 07

The Actuarial Profession
making financial sense of the future

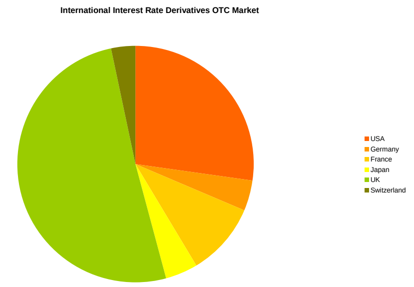
## Overview of the OTC Swap and Swaption Market




BIS Triennial Survey 07

The Actuarial Profession
making financial sense of the future

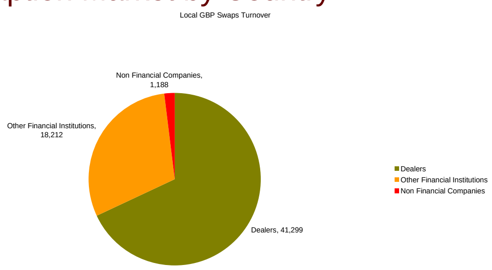
## Overview of the OTC Swap and Swaption Market by Country




BIS Triennial Survey 07



## Overview of the OTC Swap and Swaption Market by Country




BIS Triennial Survey 07



## Valuation Methods

| Swap | Swaption |
|---|---|
| Closed Form | Closed Form/Monte Carlo/Finite Differences |
| Fully Fitted Initial Yield Curve | Fully Fitted Initial Yield Curve |
| | Stochastic Yield Curve Model |
| | Arbitrage Free |
| | Market Consistent Calibration |

## Swap Valuation

$$V_{swap} = N * (g * [\sum_{i}^{i:s_i=S} ZCB(s_i)] + ZCB(S) - 1)$$

$$V_{swap} = 0$$

$$g_{ATM} = (1 - ZCB(S)) / [\sum_{i}^{i:s_i=S} ZCB(s_i)]$$



## Exotic Swap Valuation Example

- Receiver Swap with Asset Linked Nominal
- Designed to achieve a holistic hedge of a GAO liability

$$V_{swap} = \frac{Ne^{-q\tau}}{ZCB(\tau)} * \sum_{t=\tau}^{\tau+L} ZCB(0,t) * (k - g^f(t,t+1)) * \exp(\sigma_B^2 + \rho\sigma_N\sigma_B)$$



## Receiver Swaption Valuation in Closed Form

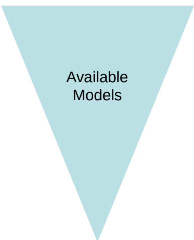
Short Rate Model

Bond Pricing Equation
Closed Form Solution

Closed Form Solution
for a Swaption

Analytic Calibration to Yield Curves
And Swaption Volatilities

Realistic Yield Curve Dynamics



## Receiver Swaption Valuation: Monte Carlo

$$Fix(s,s,t) = k\sum_{i=1}^{t} P_{s:s+i}$$

$$Float(s,s,t) = 1 - P_{s:s+t}$$

$$Swaption(s,s,t) = Max(Fix(s,s,t) - Float(s,s,t), 0)$$

$$Swaption(0,s,t) = \frac{1}{n}\sum_{j=1}^{n}\left( Swaption_j(s,s,t)\frac{D_j(s)}{D(0)}\right)$$

The Actuarial Profession
making financial sense of the future

## Receiver Swaption Valuation: Black's Formula

$$V_{swaption} = \sum_{i}^{i:s_i=S} ZCB(s_i)[g * N(-d_1 + \sigma\sqrt{T}) - f * N(-d_1)]$$

$$d_1 = \frac{\ln(f/g) + \sigma^2 T/2}{\sigma\sqrt{T}}$$

- Log-Normal swap rate
- Forward risk-neutral
- Universal quoting convention

The Actuarial Profession
making financial sense of the future

## Simple Forward Risk-Neutral Model for a GAO

$$\begin{pmatrix}\ln SA \\ \ln MA\end{pmatrix} = \begin{pmatrix}x_2 \\ x_3\end{pmatrix} \sim BivN\left\{\begin{pmatrix}\mu_2 \\ \mu_3\end{pmatrix}, \begin{pmatrix}\sigma_2^2 & \rho\sigma_2\sigma_3 \\ \rho\sigma_2\sigma_3 & \sigma_3^2\end{pmatrix}\right\}$$

$$E[SA * \max\{1, \frac{GA}{MA}\}] = E[SA] * N\left(\frac{\ln(\frac{GA}{E[MA]}) + \frac{1}{2}(2\rho\sigma_2\sigma_3 + \sigma_3^2)}{\sigma_3^2}\right) + \frac{E[SA] * GA}{E[MA]} * N\left(\frac{\ln(\frac{E[MA]}{GA}) + \frac{1}{2}(2\rho\sigma_2\sigma_3 + \sigma_3^2)}{\sigma_3^2}\right)$$

The Actuarial Profession
making financial sense of the future

## Receiver Swaption Valuation: Implied Volatilities

- Inverting closed-form solution for log-Normal swap rate

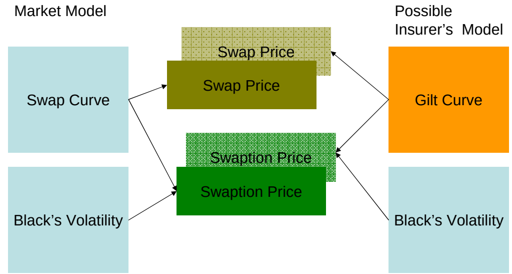
$$V^{Extended\ Model}_{swaption} = V^{Black's\ Formula}_{swaption}$$

$$V^{Market}_{swaption} = V^{Black's\ Formula}_{swaption}$$

- Infer corresponding implied volatility
- Important benchmark for cross product/quote comparison
- Important for extended model validation



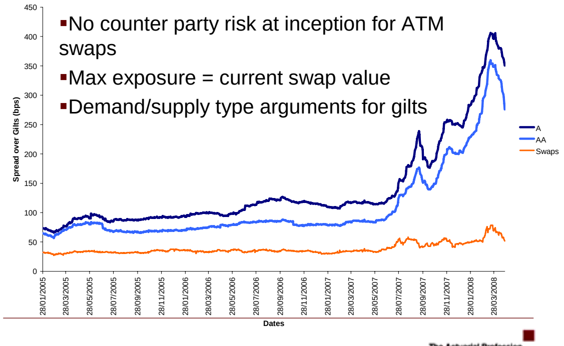
## Models and Prices



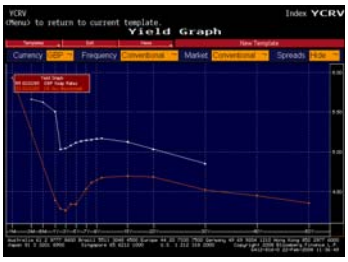



## Swap over Gilts Spread

- No counter party risk at inception for ATM swaps
- Max exposure = current swap value
- Demand/supply type arguments for gilts



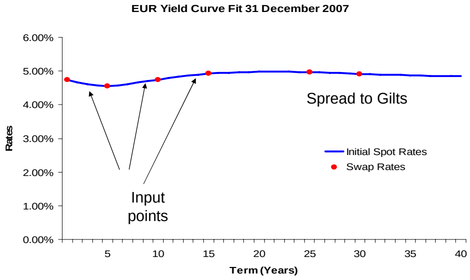
## Data Sources: Swap Curves




- Pit falls
  - Interpolation
  - Spot rates vs swap rates
  - Market compounding convention
  - Day counts

The Actuarial Profession
making financial sense of the future

## Yield Curves




Methodology: http://www-cfr.jbs.cam.ac.uk/archive/PRESENTATIONS/seminars/lent2001/asmithyield.pdf

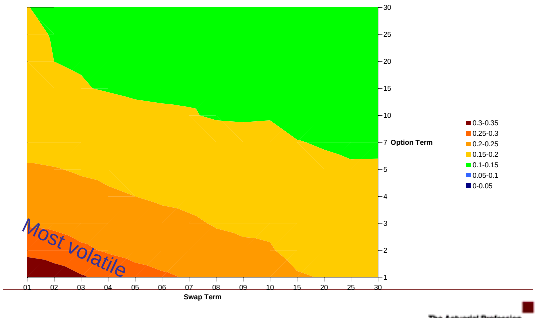
The Actuarial Profession
making financial sense of the future

## Data Sources: Implied Swaption Volatilities


- Pit falls
  - Log-Normal vs Normal vols

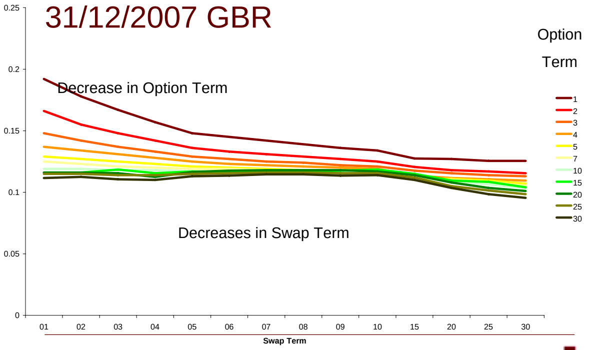
The Actuarial Profession
making financial sense of the future

## Implied Swaption Volatilities: 31/12/2007 GBR



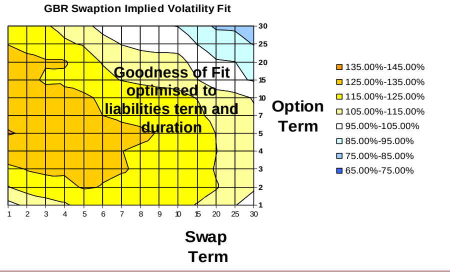

The Actuarial Profession
making financial sense of the future

## Implied Swaption Volatilities: 31/12/2007 GBR




The Actuarial Profession
making financial sense of the future

## Implied Swaption Volatilities: 31/12/2007 GBR 2nd Generation ESGs




The Actuarial Profession
making financial sense of the future

## Interest Rate Sensitive Exposures

| | | Long Position/ Asset | Short Position/ Liability | | |
|---|---|---|---|---|---|
| Receiver Swap | | ● | ○ | | |
| Payer Swap | | ○ | ● | | |
| Bonds/Annuities | | ● | ○ | | |
| Receiver Swaption/GAO | | ● | ○ | | |
| Payer Swaption | | ○ | ● | | |

● Net value moves in the opposite direction to interest rates
Losses when interest rates go up

○ Net value moves in the same direction as interest rates
Losses when interest rates go down

making financial sense of the future

## Interest Rate Volatility Sensitive Exposures

| | | Long Position/ Asset | Short Position/ Liability | | |
|---|---|---|---|---|---|
| Receiver Swap | | x | x | | |
| Payer Swap | | x | x | | |
| Bonds/Annuities | | x | x | | |
| Receiver Swaption/GAO | | ○ | ● | | |
| Payer Swaption | | ○ | ● | | |

● Net value moves in the opposite direction to volatility
Losses when volatility goes up

○ Net value moves in the same direction to volatility
Losses when volatility goes down

making financial sense of the future

## Exposure Hedging

| | Interest Rate Sensitivity | Interest Rate Volatility Sensitivity |
|---|---|---|
| **Liabilities** | | |
| Annuities | ○ | x |
| GAOs | ○ | ● |
| | | |
| **Assets** | | |
| Bonds (Long) | ● | x |
| Receiver swap (Long) | ● | x |
| Receiver swaption (Long) | ● | ○ |

● Losses when the driver goes up

○ Losses when the driver goes down

making financial sense of the future

## Hedged Position Construction

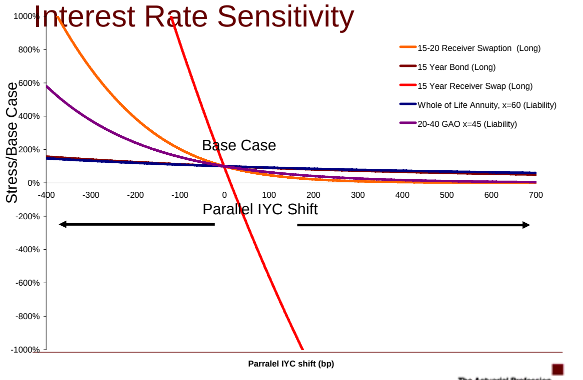
• Net Asset Value is a function of interest rate level and interest rate volatility. $NAV(r, \sigma)$

$$\Delta NAV = \frac{\partial NAV\ (r_0, \sigma_0)}{\partial r}\Delta r + \frac{\partial NAV\ (r_0, \sigma_0)}{\partial \sigma}\Delta \sigma$$
$$+ \frac{1}{2}\left(\frac{\partial^2 NAV\ (r_0, \sigma_0)}{\partial r^2}(\Delta r)^2 + 2\frac{\partial^2 NAV\ (r_0, \sigma_0)}{\partial r\ \partial \sigma}(\Delta r \Delta \sigma) + \frac{\partial^2 NAV\ (r_0, \sigma_0)}{\partial \sigma^2}(\Delta \sigma)^2\right)$$

• Net Asset Value will be unresponsive to changes in interest rate and volatility if the asset portfolio hedges liabilities. “Greeks” based hedge.

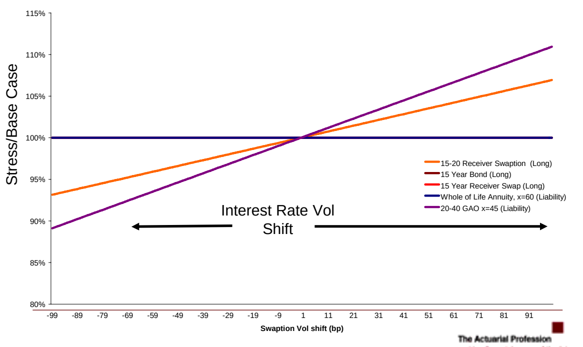


## Interest Rate Sensitivity

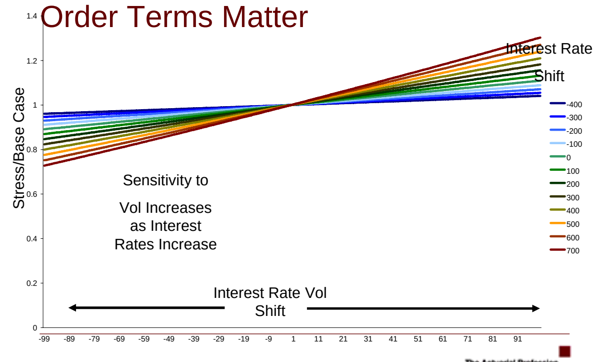




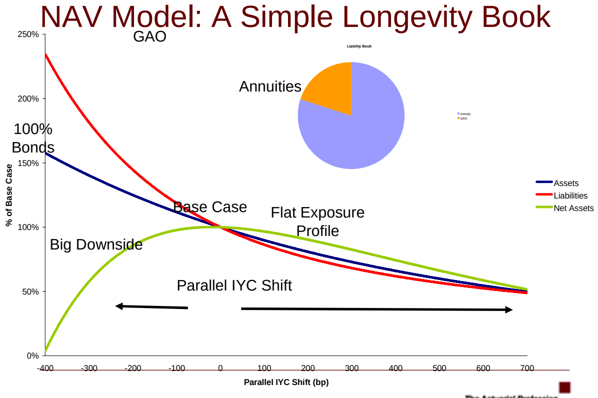

## Volatility Sensitivity



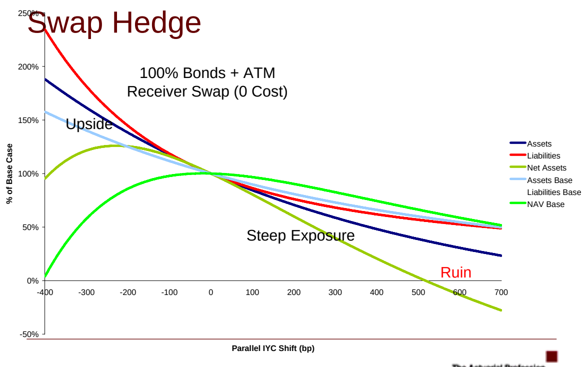
## Volatility Sensitivity: Why Second Order Terms Matter



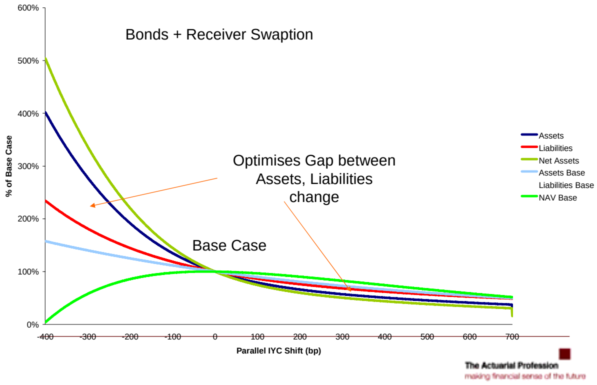

## NAV Model: A Simple Longevity Book



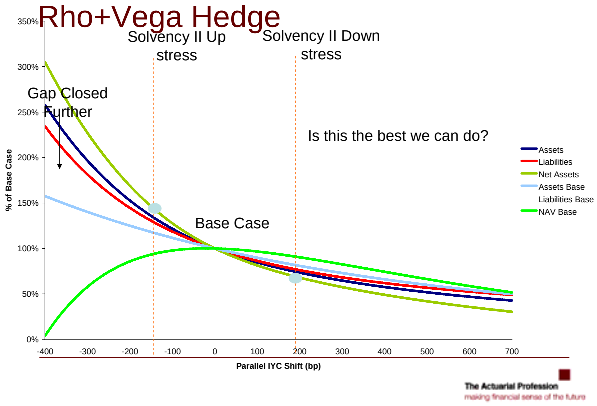

## NAV Model: Addition of a Downside Swap Hedge



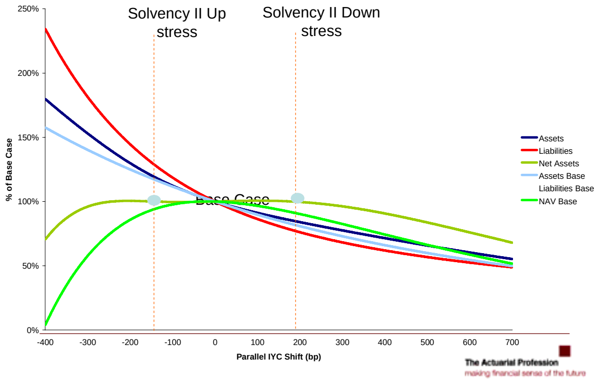
## NAV Model: A Swaption Rho Hedge



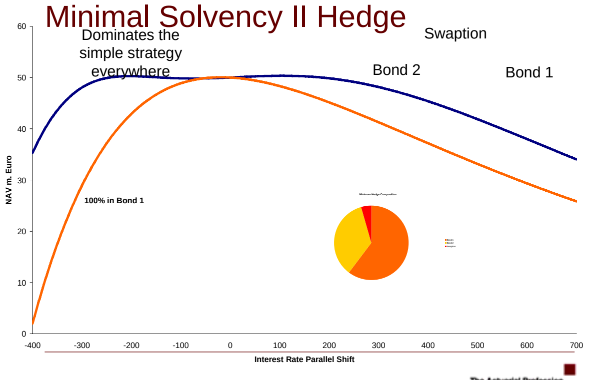

## NAV Model: Two Bond And Swaption Rho+Vega Hedge




## Minimal Solvency II Hedge

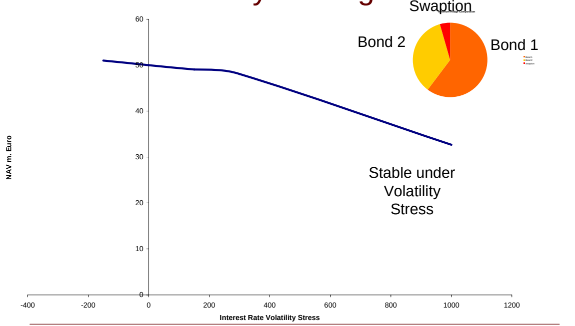


## Minimal Solvency II Hedge



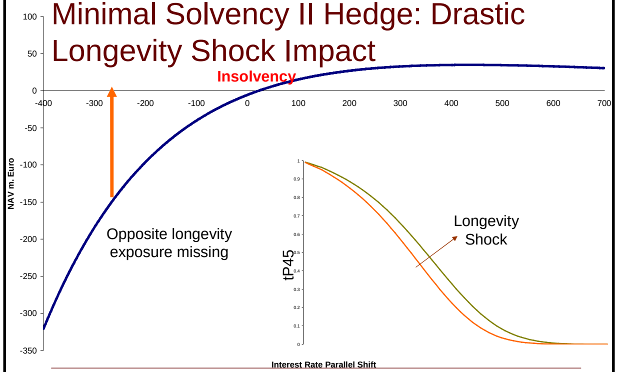

The Actuarial Profession
making financial sense of the future

## Minimal Solvency II Hedge




The Actuarial Profession
making financial sense of the future

## Minimal Solvency II Hedge: Drastic Longevity Shock Impact




The Actuarial Profession
making financial sense of the future

## Marked to Market Valuation: Significance of Assumptions

| | Yield Curve | Interest Rate Volatility |
|---|---|---|
| Annuities | | |
| GAO | | |
| Bonds | | |
| Swaps | | |
| Swaptions | | |



## Conclusions

- Non-dealer financial companies are important players in the UK swap market
- Insurers' interest rate sensitive exposures can be profoundly manipulated with OTC long term interest rate derivatives
- Hedging schemes must be carefully evaluated
- Yield curve is the single most significant valuation input