ROC/GIRO Working Party

Best Estimates and Reserving Uncertainty

Paper for GIRO 2007

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Many thanks also to those other members of the profession who have participated in the work of this working party for some of its life, and to those who responded to the survey, provided data and provided other useful insights.

Many thanks to Insureware for providing a working group and a submission to this working party.

Contents

Appendices

1. Summary and Introduction

1.1. A Word of Thanks

A great many people have contributed a lot of hard work to this working party – beyond the members named, including former working party members, their colleagues and other members of the profession. I am very grateful to all who have taken part and have helped contribute to a wide ranging paper.

1.2.This Report

This report has four main components:

1.2.1 Best Estimates

Some brief thoughts on what we mean by best estimate. This is not intended to be a comprehensive treatise on the subject but rather to point out that when thinking about reserving uncertainty, and the range of potential outcomes for the ultimate claims payments, it is as well to understand what one means by a best estimate. Our thoughts on this subject are contained in Section 2, and we provide a suggested standard definition, based on a subjective estimate of the mean of possible outcomes.

We note that some of the commonly used reserving methods, such as the basic chain ladder, do not provide an estimate of the statistical mean, although it can be tempting to describe the estimates so derived as though they were estimates of the mean. We have not probed this matter further. It has been addressed in previous papers and there is another ROC working party looking at the performance of reserving methods.

1.2.2 Survey

We have carried out a survey of the profession to determine what methods are being used by members now in investigating reserving uncertainty. This is described in Section 3 and Appendix E. We see that the over-dispersed Poisson (ODP or Bootstrap) method is widely used. We note that there was a previous informal survey of the profession at GIRO in 1993 and we show high level results of that survey for an interesting comparison.

1.2.3 Applying Methods to Real Data

The third area of work, and the one which has involved dozens of volunteers, is in the application of a numbers of methods and models to some real claims development data, which was kindly provided to the working party for this purpose. The working party split into a number of sub-groups who each carried out testing. We also opened the doors to submissions from any professional service firm who wanted to test a method or model of their choosing and we received one such submission from Insureware.

Sections 4 to 8 deal with this phase of the work. Section 4 explains the background to this exercise and how we set about the work. Section 5 describes the data and the issues we faced. Section 6 illustrates in a table the key features of the methods we tested and Appendix A gives a summary of the methods. Section 7 contains a numerical summary of the results of the working group testing and Section 8 summarises the qualitative comments about using the different methods, such as ease of use. Appendix F contains the complete Insureware submission.

It is interesting to note that there are areas of significant overlap between the distributions of outcomes produced by the various methods – and there are also important and noteworthy areas of non-overlap. Further, as the results in Section 7 show, it is not unusual for there to be significant differences between indications from various stochastic reserving techniques and not every method will give a reasonable result for every set of data.

It is also interesting to look at the full set of results, say for the Employers' Liability class (Graph 1 in Section 7). Some actuaries may wonder whether many or any of the method generate a 95^{th} centile value which is as high in relation to the mean as seasoned actuaries might expect, based on their experience of reserving such classes.

Having discussed these findings within the working party and with the Reserving Oversight Committee, we believe that – at least in the short-term - there is a clear need to apply a variety of methods rather than relying heavily upon the output of one, or even two methods. There also continues to be a need for careful professional judgement. In the longer term, as a profession, we feel there is an imperative to have a deeper understanding of the causes of variation between the results produced by the methods, and to add an intuitive understanding of the appropriateness of applying particular stochastic methods to the actuary's tool-kit. Since quantifications of reserving uncertainty will be becoming part of the statutory returns in the near future, the results of these methods are likely to attract greater scrutiny from outside the profession.

1.2.4 Numerical Simulation Testing of Stochastic Methods

This area of work had originally been intended as somewhat of an aside. We intended to use artificial simulated data to test how well the various methods handle unusual circumstances. We decided to begin by creating a baseline – testing the performance of the stochastic methods when all of their conditions are met. In theory, the simulations should show that 25% of outcomes fall above the method's 75^{th} centile result, and that 1% of outcomes fall above the method's 99^{th} centile result. In fact, we found some interesting results, with the methods most commonly used, applied in the way they are generally applied, tending to under-estimate the tails of the distribution. Considerably more than 1% of observations were falling above the 99^{th} centile produced by the methods in our testing. The work is described in Section 9 and the detailed calculations can be found in Appendix B.

On the one hand these could indicate that we have an important and urgent issue to deal with as a profession; on the other hand, there could be aspects of our analysis which are partly or wholly responsible for generating these anomalous results.

We have thought of three issues in relation to our testing work which could be contributing to the anomalous results at the extremes of the distributions and these are shown below.

- We may have made a calculation error in our testing. To seek to reduce this risk we have had a number of working party members reproducing the calculations independently.

- There may be a fundamental flaw in our approach to testing stochastic methods (for example, the method for generation of the artificial data may have introduced anomalies or inaccuracies at the extremes of the distribution). To reduce this risk we need careful consideration from the wider profession.

- When testing methods that produce only the first two moments of the predictive distribution for reserves (i.e. mean and standard error) we obtained percentiles by fitting particular analytic distributions (Log-Normal and Inverse Gauss) to the mean and standard error. We found that the Log-Normal and Inverse Gauss results did not differ materially, so we have presented detailed results only for the Log-Normal. However, it is still possible that using other analytic distributions would have led to different conclusions. (The use of the Log-Normal for this purposes was recommended by Thomas Mack in his 1993 paper, and we believe is quite common practice).

An alternative explanation for some or all of the anomalies is:

- The stochastic methods tested (including over-dispersed Poisson and Mack) may genuinely not work well at the tails of the distributions (for example, because functions of unbiased estimators, which are themselves approximately unbiased, may not be sufficiently unbiased in these extremes).

If this alternative is a true explanation for some, or all, of these anomalies then such would be a serious finding, because these methods are in widespread use across the profession, and in particular, are used for capital modelling where the extremes tails of the distributions are potentially in play.

We have set out our work here so that more members of the profession can consider the way we have approached it, and the potential implications, in the hopes that together we can better understand the true position and work together towards resolving any issues which may arise.

1.3 Judgement as a Method

1.3.1 Is Judgement a Method?

One of the methods we looked at and thought about was “Judgement”. We had some interesting discussions about what this meant, and whether it was a meaningful method, both within the working party and with the Reserving Oversight Committee. As you will see in Section 8 the working groups had differing views on this.

We thought it would be helpful to add some words about what we mean by judgement as a method, how it can work in combination with other methods, and why it is important.

1.3.2 What is the Judgement Method?

Actuaries apply professional judgement in most of their work, from choosing development factors to exclude from averages, to interpreting data and softer information. That is not what we have in mind as the judgement method, but rather the following.

The judgement method, in the context of estimating quantifications of reserving uncertainty, is a use of the actuary's own experience as an implicit benchmark for expected results. In fact this definition may work in wider contexts.

The actuary may have implicit experience-based benchmarks for many possible values, including for example, tail factor extremes, the ratio of tail centiles to means, "worst case" loss ratios for certain classes of business, exposure based method parameters.

There are two methods related to the judgement method, being benchmarking and scenario testing.

Benchmarking is the use of explicit data to create benchmarks which will guide the selected estimates, or parameters involved in their derivation. Judgement is similar because the actuary is essentially using his or her own, potentially complex and interacting, set of benchmarks based on personal experience. When an actuary has become sufficiently confident in their implicit benchmarks they may become "rules of thumb" and be adopted by other actuaries and spread through the profession's consciousness.

Scenario testing is the creation of possibly hypothetical scenarios to be used as deterministic parameterisations for certain calculations. The choice of scenarios to consider may be based on the actuary's own experience and hence a manifestation of the judgement method.

1.3.3 How does Judgement Work in Combination with other Methods?

Actuaries may carry out a number of methods in estimating the distribution of possible ultimate claims outcomes. In so doing they may combine these methods with the judgement method. For example, the actuary may believe that the 95th centile of the distribution of outcomes is being underestimated by his or her chosen method. This could be because in his or her experience they may feel they have seen sufficiently many examples of adverse development to suggest a higher level of deterioration for a 1 in 20 event. They therefore adjust the results of their method to align the aberrant parts of the distribution more in line with their own experience.

1.3.4 Why is the Judgement Method important?

The judgement method is important because it is so often used in practice, whether or not this is stated explicitly. Given that different actuaries have different past experiences it seems likely that they may at times have different implicit benchmarks which they have developed from their respective experiences. It is arguably therefore all the more important that we disclose when we are using this method, and seek to articulate the implicit benchmarks we are using.

There are clear risks with the judgement method, especially if it is not used in a transparent manner. Some actuaries may have limited experience, or their experience may not be typical, and so their personal suite of benchmarks may be unreliable. The ability to assimilate experiences in such as way as to differentiate between different types of circumstances to create sufficiently intricate implicit benchmarks may vary from person to person, so that some develop more robust implicit benchmarks than others. Peer review can clearly reduce these risks.

1.3.5 Development of the Profession's Judgement on Reserving Uncertainty

One of the objectives of this working party is to add some material to assist in developing the profession's judgement on estimating reserving uncertainty.

We are acutely aware of the need to resolve the issues raised in 1.2.4 so that we can be sure that the profession's growing experience and rapidly forming implicit personal benchmarks, especially for the tails of the distributions, are based on a robust foundation.

1.4 Next Year

While we have carried out a great deal of work this year there remains a lot still to do. In Section 10 we set out some objectives for next year's working party to carry on the work.

2. Best Estimates

2.1 Definitions of "Best Estimate"

What is an actuary's "best estimate" of an outcome, for example, of ultimate losses?

It is her/his subjective derivation of the mean of all possible outcomes, taking into account all available information about the business being analysed.

This definition allows for the subjective interpretation by the actuary of the available data and the choice of models and methods used. As a default it also excludes allowance for events not reflected in the data such as unanticipated major new types of latent claims, although in some cases it could be argued that all available information includes knowledge about the risk of such new claims potentially arising.

Subjectivity is included in the definition because it emphasises that even with a standard definition of "best estimate" one would still expect different parties to produce different estimates. The uncertainty about the random process being estimated and the data provided gives grounds for different judgements to be made about how the future will unfold. Also, different actuaries may understand the "available information" to include or exclude different matters. Therefore a variety of reasonable best estimates is possible, even with a common definition.

However, if the actuary diverges from the above definition then we would recommend that a detailed description be given, defining exactly what is being estimated. This should include comment on:

- Why the above standard definition is not being used in a particular instance and why an alternative definition is thought to be a more appropriate single point estimate, "best" summarising the true underlying random probability distribution. Other summary statistics which might be favoured include the mode and the median
- Whilst the actuary's view of the mean of the whole distribution of all possible outcomes should not vary depending on the purpose of his or her work, the purpose may influence which areas of divergence from this complete mean may be acceptable or desirable. The actuary should therefore comment on the purpose of the best estimate and how this influences the choices made about what, if any, contingencies to exclude from the set of possible outcomes.
- How prudent, optimistic or pessimistic the estimate is intended to be. In this case it is preferable to define any deviation from the standard definition explicitly; for example "The best estimate is the mean of the underlying distribution of the claims allowing for all possible outcomes, plus a margin of prudence." (Giving reasons for the prudent stance)
- Whether unlikely outcomes are included or excluded (if excluded explain why). It is worth differentiating between remote events that are significant to a particular insurer and those that could cause significant proportions of the industry to collapse
- Whether an attempt has been made to provide for latent events

2.2 Mean versus Median

There is sometimes a debate about whether best estimates should be means or medians.

The mean definition of a best estimate accords with UK professional guidance in GN20 and it corresponds to equivalent definitions used internationally – albeit using different terminology (i.e. "expected value estimate" of a reasonable provision in the US – ASOP 36 and "central estimate" in Australia – PS300). It accords with the anticipated reserving requirements under Solvency 2 and IFRS.

The mean is arguably a more intuitive choice and it has the benefit that the expected average surplus equals the expected average deficit. That said some of our stakeholders may assume the best estimate is "the midpoint", or that it is "equally likely to be too high as too low", which indicates a median rather than a mean.

The US system, albeit using an "expected value" terminology, does not necessarily require the actuary to anticipate future contingencies and so, if it is a mean, it is perhaps a mean of a distribution with some of the extreme tail removed. Given the typically skewed nature of distributions of ultimate claims outcomes, such an estimate may be nearer to the median of the whole distribution than the mean.

This working party has come down in favour of the mean in this debate.

3. Survey and Results

3.1 Survey of GI Actuaries

Given the ROC brief to investigate reserving uncertainty methodology, it seemed appropriate to include practical evidence of the methods and views of the industry towards reserving uncertainty.

A questionnaire was written – see ***Appendix E1***, and distributed to members on the GIRO mailing list by Peter Stirling on 17th May.

The accompanying text follows:

"To General Insurance Actuaries on behalf of GI ROC

As you probably know, four work streams of GI ROC (General Insurance Reserving Oversight Committee) were announced at last year's GIRO convention. The '**Best Estimates and estimating uncertainty**' and '**Effectiveness of reserving methods'** working parties would like your help in completing a brief on-line survey as follows:

The GI ROC working parties will be reporting on their investigations into the assessment of best estimates and reserving uncertainty at this year's GIRO conference. A survey has been developed to put the results into a practical perspective and to assess where the industry is up to in terms of the methods currently in use. Your participation in this survey would be very much appreciated. All responses will be anonymised and only the combined results in summary form will be publicly disclosed following analysis of the raw data. Please access the survey by clicking on the link below. The survey is open for responses until Friday 25th May.

http://wam.actuaries.org.uk/eforms/eForm.aspx?TAG=ROC
Many thanks in anticipation of your help."

3.2 A Previous Survey in 1993

A 1993 GIRO working party produced a paper entitled "Variance in Claims Reserving" that discussed the principles of stochastic claims reserving and tested several methods current at the time on actual datasets. That paper is available under "general insurance convention papers archive" on the Institute of Actuaries website.

An informal survey on use of stochastic reserving methods was also carried out at the 1993 GIRO. 40% of those present who were involved in claims reserving said they had used a stochastic method at one time or another. The table below shows numbers who had used various classes of stochastic method.

Table 3.2.1 - Numbers of attendees at 1993 GIRO who had previously used stochastic reserving methods in practice

	Log incremental regression (static)	Log incremental regression (dynamic)	Bootstrap	Operational time
Consultants	3	3	1	7
Lloyds/RI	2	0	0	0
Insurers	7	1	2	3
Total	12	4	3	10

Log incremental regression (static) refers to regression methods where the dependent variable is the log of the aggregate incremental paid amounts and the underlying run-off pattern is assumed to be the same for all origin years. Log-incremental regression (dynamic) refers to similar methods in which run-off pattern is allowed to vary across origin years using the Kalman filter (this method was advocated by Insureware at the time). Mack's method was not considered by the 1993 working party as it had only just been published.

3.3 The Results of our 2007 Survey

Please see ***Appendix E2*** for the detailed results.

There were 47 respondents, mostly based in the UK, from a mixture of Insurance companies, Reinsurance companies, Lloyd's syndicates and consultancies. Just over half represented insurance companies. Please see Chart 1 and Chart 2 of ***Appendix E2***.

Respondents were invited to report on the classes of business for which their organisation performed reserving work. Reinsurance companies covered most of the classes of businesses - see Chart 3. The largest number of responses related to Personal lines – property, Personal lines – motor, Commercial lines – property and Commercial lines – liability, although for Lloyd's syndicates Marine was also significant.

For the 3 largest classes, respondents were asked which methods and models they used to calculate best estimate values and to determine uncertainty.

The results from question 3a (best estimates) indicate that the Basic Chain ladder – incurred is most often the key method, followed by Bornhuetter Fergusson, paid chain ladder and then judgement. Alternative approaches included using loss ratios from pricing work.

The results from question 3b (reserving uncertainty) indicate that the key methods were: Overdispersed Poisson (ODP) stochastic chain ladder/ bootstrap, judgement, scenarios and then basic chain ladder paid and incurred methods. Alternative methods included using benchmark information from the market.

Question 4 asked about the key requirements for determining uncertainty.
They were:
1) to identify variability around the best estimate and,
2) to identify the tail of the distribution (above 95th percentile).

Question 5 asked what were the factors influencing a choice of method/model.
The choice was decided based upon
1) Practical issues relating to the data
2) The quality of results that the model/method will give.

Interestingly, “knowledge of the actuarial staff” was listed as the least important factor. This indicates a confidence within the industry that actuarial staff understand the relevant models being used.

Question 6 asked what systems/software were used to model uncertainty.
When modelling uncertainty organisations were evenly split between using in-house models and a combination of in-house and off-the-shelf packages. The packages mentioned included ResQ, Igloo, RMS Risklink and @Risk. ResQ was most frequently mentioned.

Question 7 asked about the methods used to communicate uncertainty to senior management/ executives.
The most frequent methods used were:
1) Quantitative using percentiles.
2) Quantitative using everyday English.
A satisfyingly low number didn’t communicate uncertainty at all.

Question 8 asked about what aspects of uncertainty were covered in communications. Communication of uncertainty most commonly covered process and parameter uncertainty, but less often covered model uncertainty.

Questions 9 to 13 asked for additional information was collected relating to the groupings by time periods commonly used in reserving to calculate best estimates and for calculating uncertainty.

For calculating best estimates, the majority grouped data using annual origin periods. The most common grouping by development period was quarterly.

For calculating uncertainty, the majority grouped data using annual periods and annual development periods.

Reserving, in most instances was done quarterly using a full analysis. Where reserving was done monthly, a reduced analysis was more often performed.

3.4 Our Thoughts and Conclusions

When drafting the survey, the extent to which different institutions would use different methods was not known. Since the GRIT paper was issued in 2006, it was hoped that its key recommendations had been adopted. This survey indicates that most respondents do communicate and quantify uncertainty.

The survey shows reliance on a small number of standard methods for producing estimates of reserve uncertainty, and the key requirements of such methods are to identify variability around the best estimate and identify the tail of the distribution. Other parts of the ROC paper give an independent assessment of several of the methods that can be used and illustrate their limitations.

The answers to question 5 reveal a confidence from respondents that actuarial staff members are equipped with sufficient knowledge to deal with whichever method/model is chosen to measure uncertainty. This may be owing to high quality actuarial training, or may indicate a degree of misplaced complacency – particularly if the limitations associated with the methods used are not fully understood.

The above results are representative of only a small proportion of the industry, and represent the views at a particular point in time. It is hoped that this survey (or a similar one) could be re-issued on an annual basis to detect changes that the industry are making in reserving methodologies.

Ideally the results would have been split and analysed by line of business. This has been done, but not published in ***Appendix E2***, owing to the small sample sizes and similarities between the responses. Where the responses were not very similar, they are detailed below.

Judgement and scenarios were more used commonly to calculate best estimates and reserving uncertainty used in reinsurance lines than was the case for direct writers.

Reinsurers also were more interested than primary insurers in identifying the complete distribution of reserve variability than just the tail of a distribution. The key requirement though was variability around the best estimate for both types.

Personal lines insurers chose a method to identify reserve uncertainty based more upon the quality of results than the quality of the data. Commercial lines and reinsurers chose a method based more upon the available data.

The survey questions about the groupings of data suggest that different groupings by origin/development year are used when determining best estimates compared to reserve uncertainty. Perhaps further studies could be performed to indicate whether this impairs the accuracy of the best estimates/reserve uncertainty results.

4. Method Testing: Our Scope and Approach

4.1 Background

There have been many developments in recent years that have generated increased interest in the assessment of uncertainty within the reserving process. These have largely been driven by the Morris review, which identified the lack of a consistent approach or set of approaches relating to both the estimation and communication of uncertainty within point estimates for claims reserves within formal actuarial reports.

The work of GRIT, which was intended to pre-empt and also respond to the Morris report investigated the calculation of reserve uncertainty to a limited extent, focussing largely on the way such concepts are understood within the profession, as well as how they are communicated outside, and how such information is received and interpreted.

Further impetus has been provided by the re-interpretation of the GN12 guidance on formal actuarial reports, which has underlined the need for actuaries to indicate both the source and amount of uncertainty within their work, where practicable.

Additionally various authors continue to investigate the statistical credibility of various models, and develop increasingly sophisticated methods to derive more reliable and consistent results to compare to best estimates and standard actuarial reserving tools.

Separately from reserving uncertainty alone, the introduction of the Individual Capital Assessment by the FSA, and to a lesser extent the discussions relating to IFRS, have brought the attention of parties both inside and outside the profession to bear on uncertainty in general, with reserving uncertainty being a strong element of these discussions.

These discussions have focussed on the definition of adequacy around particular centiles of probability, implicitly implying that a full distribution of outcomes is required to derive the associated processes.

The above sources of interest in this area are relatively well reported, however there are other, less public, areas that have increased the need for research and discussion in this field. These are through the proliferation of more advanced computing power and the increasing use of off-the-shelf reserving packages that include “standard” reserving uncertainty methods, which can be run almost as an afterthought to the reserving process.

This has increased the ease of obtaining reserve uncertainty information, but also has opened the question of how much the explicit and implicit assumptions relating to the models are understood by the reserving actuary, let alone the user of the report.

The above issues have raised the level of awareness of reserving uncertainty as a topic within the profession, and we believe there is a need to broaden the general knowledge base within the profession on the methods available, their strengths and limitations and to generally demystify the area for the general insurance population at large.

4.2 GRIT Conclusions

One of the key recommendations from GRIT was: "Providing more information on uncertainty in our reserve estimates. In particular, we recommend that actuaries provide a quantitative indication of the range of outcomes for future claim payments…".

The 2005/6 Estimating Reserving Uncertainty Working Party following GRIT also showed that only a limited number of actuaries were using defined methods within the reserving process or assessing reserve uncertainty at all, and that those that did tended to rely on a single method.

This paper is intended to help members of the Profession understand more about the tools at their disposal, as well as giving some information about the relative merits and limitations of such methods. It is hoped that this may help in the fulfilment of the GRIT recommendation, and also to ensure that a follow-up to the Working Party next year would indicate better understanding and use of such methods.

4.3 What the paper is attempting to do

The key objectives of this paper are:

- to educate the majority of general insurance actuaries so that they are aware of the wide variety of methods available;
- to educate the majority of general insurance actuaries so that they are able to understand at a basic level what the differences are between these models are;
- to give a high-level review of the practical and theoretical aspects of a few of the more common models based on application to real data sets;
- to provide resources of data, models and references with which to accustom actuaries to these models.

4.4 What the paper is not attempting to do

We are definitely not attempting to suggest that a particular model is somehow "better" than any other, nor are we trying to review all possible methods. In particular we are not attempting to bring the reader up to date with the latest developments in the field, as we are concentrating on educating from the most basic level.

We expect future working parties to extend the work presented here in response to feedback from the profession on this paper, and this is discussed in more detail in Section 10.

Before we get into more detail about the work we have done, we would like to draw the reader's attention to the work of the ROC best estimate working party, which is reviewing more traditional actuarial models, and assessing their effectiveness in various circumstances. We have shared data with them, and intend eventually to present combined results to compare best estimates and ranges. This will follow next year.

4.5 Our Approach

Briefly, our approach centred on selecting a few methods we could look at for a number of datasets, and could compare the observations of a number of sub-groups of the working party for each method employed. This process would not only look at the technical results, but also assess each method for ease of use, understanding and interpretation. The results of these analyses have been combined to present an assessment of the methods under various categories to help identify those with strengths and weaknesses in particular areas.

The following paragraphs set out this process in more detail.

We selected a number of methods for assessment. The selection was largely based on the knowledge within the working party itself as to the most common methods within the profession, as well as through literature searches and discussions with colleagues.

This "long" list was then reduced to a selection of eight, including purely subjective approaches and methods that required a level of detail within the data that is not usually available, particularly for consultancy work or for older data sets. We also provided our data to consultancies to allow them to provide results on their own models for comparison, we received a completed response from one consultancy, which is included in full in Appendix F. These methods are briefly discussed in Appendix A.

The data we applied these methods to was sourced anonymously from a number of companies. We attempted to cover "normal" lines of business, as well as some more unusual classes such as Marine. The classes included long and short tail, and had data of different periodicities and levels of detail. In particular we unfortunately were not able to obtain transaction level information, which reduced the number of available methods by one.

The data was adjusted such that the source was not easily identifiable, however this process was not well defined, and hence the consistency of some data sets was questionable. However, as the key objective was to test the methods against each other, such inconsistencies did not necessarily result in a data set being rejected.

To concentrate the results of the models we selected three core data sets for analysis. These represented an employers' liability class, a commercial property class and a motor class. Other classes were modelled for some methods, but they did not form the core of our work. The data is discussed in more detail in Section 5.

The working party was split into a number of sub-groups, each of which was given two methods to investigate, as well as considering the methods of scenario testing and judgement.

Each method was assessed using a standard questionnaire, which investigated various aspects of the method, both numerical and practical. Although we tried to avoid using off-the-shelf packages for all groups working on a particular method, where they were available use was made of them to investigate the effects of implicit assumptions relating to ensuring that the method could operate on the data sets provided.

It was therefore intended that each of the methods was investigated by a number of groups independently. However, there were fewer responses than originally envisioned, as discussed in Section 7. The results of the investigations for each were compared to identify any sources of user error or where off-the-shelf packages had used a more advanced version of the model than from the primary reference. In addition the practical aspects of the models were discussed to assess the less technical areas for investigation. Sections 7 and 8 discuss the numerical and qualitative comparisons respectively, with Section 6 giving a high-level comparison of the results.

We also asked for input from Professional Service firms, and Insureware made a submission using some of their own methods applied to our datasets. In particular, in Section 7 we show the results for the Employers' Liability class which compares the methods tested by the Working Party subgroups with modelling frameworks utilised by Insureware. The full submission from Insureware can be found in Appendix F.

A separate strand of our work focussed on assessing the theoretical accuracy of a number of models under ideal conditions. This was achieved using stochastic simulation whereby the models were applied to a large number of simulated triangles, each with a known ultimate. These data sets have been derived such that the assumptions required by those models are fulfilled. The aim of these tests was to calculate the accuracy of the model in estimating the probability of events at particular centiles. This investigation is set out in Section 9.

5 The Data

5.1 Introduction

This section considers the data requirements for the application of the various methods and the limitations imposed by the data available to the working party.

5.2 Required data characteristics

5.2.1 Real or artificial data

The working party set out to test a range of methods for estimating uncertainty by considering the merits and disadvantages associated with their practical application.. This "real world" testing suggested that the methods should be tested using real data. This presented a number of problems. Firstly there is no central repository of insurance data in the UK and secondly distinguishing how different features of the data are impacting different methods can be extremely challenging.

The Institute of Actuaries has arranged the collection and rescaling of real data from companies who were willing to contribute. The rescaling methodology has not been disclosed to the members of the working party but it is understood that the approach used is sufficient to ensure that individual data is not recognisable without distorting the variability within that data. The working party recommends that the resulting data is not appropriate for reaching conclusions around the relative performance of any sectors of the insurance industry as premium income and loss information from the same entity may have been rescaled on different bases.

The amount, timing and uncertainty of emerging cash flows is affected by a wide range of factors arising from policyholder attitudes and behaviour, the companies' own management strategies and practices and the wider economic and legislative environment. It is often difficult to interpret the results of analyses when using familiar, well understood actuarial techniques such as the chain ladder or Bornhuetter-Ferguson technique. Understanding the results is an even greater challenge when the techniques being applied are not widely used and understood.

Artificial data can be designed to avoid the distorting features of actual live insurance data. An alternative approach to testing using real data is to develop artificial data tailored to the characteristics of the method being tested to produce a "control" set of results. Understanding of the method's response to some of the features of general insurance can be achieved by adding new features to the artificial data and comparing the results with the control.

The working party has tested the Mack and ODP methods (both analytic and bootstrap versions) using artificial data. An overview is given in Section 9, and details in Appendix B.

5.2.2 Aggregate or transactional data

Ideally the working party would have preferred to work with transaction level data and aggregated it according to the requirements of each method. Although this approach is time consuming it means that data can be aggregated in a variety of different ways and a deeper level of investigation and understanding is possible. In practice aggregated data is more readily available and, as described above, easier to alter to ensure the source of the information is not recognisable.

The working party, in drawing up a list of methods for measuring uncertainty, identified techniques that required transactional data and could not be run using aggregated triangulation data. Although we understand that access to transaction data is still being sought, this data has not been available to the working party and consequently the techniques that rely on such data have not been tested.

Aggregate data has been made available to the working party. Paid and incurred claims triangulations have been provided for all except one data set which excluded paid data. A reasonable number of years of exposure, varying from 8-20 years, have been available for most classes. For longer tail liability risks this volume of data is not sufficient to show fully mature historical loss development and it is necessary to make assumptions regarding future development to ultimate. Development intervals varied by data set between monthly, quarterly and annual. This has not presented a problem for testing conducted to date but does prevent consideration of how the techniques would respond to quarterly analyses for many of the data sets provided. Exposure information is available for 11 out of the 30 data sets and separate large loss information has been provided for 13 of the data sets. Claim counts are available for half the data sets although for two of these claims counts are only in relation to large losses.

Some very limited qualitative information has also been supplied in respect of each data set namely:

- Class and sub-class of business;
- Gross or net of reinsurance (not known for all data sets).

The sub-class information is useful in separating out some specific risks from the underlying data. For example subsidence and weather losses are separately identified from other property losses. Separate data for bodily injury and non-bodily injury motor claims are also available.

The table attached in Section 5.3 below summarises the aggregate data available to the working party.

Whilst the working party acknowledged from the outset that access to high quality data was likely to be a problem, the information available is not ideal. We anticipate most actuaries would be able to access better quality information in the normal course of their work. It is noted, however, that in the testing of the methods it is useful to know which methods can be applied when there is restricted or less than ideal information. The practical constraints of applying each method, including data requirements, are considered in Section 8.

5.3 Classes of Business

The working party has been relying on information donated to the institute so has not been able to specify its own data requirement. A range of classes of business have been provided including:

- Property – personal lines and commercial lines
- Motor – personal lines and commercial lines
- Marine
- Construction
- Liability – employers' and public

In order to ensure maximum comparability of the results it was decided to test each of the selected methods on a limited number of classes of data in the first instance. A review of the data resulted in the recommendation to use the following three classes:

- Commercial Property
- Personal Motor Non-comprehensive
- Employers' Liability

The Commercial Property and Personal Motor Non-Comprehensive data sets both provide triangulations by accident quarter and quarterly development period for payments, incurred losses and reported claim counts with corresponding earned premiums and earned exposure. The data is gross of reinsurance. The underlying business is UK risks sourced through intermediaries, direct sales and corporate partners. Large claims are included in the underlying loss experience and are not available separately.

Apart from one anomalous figure, assumed to be a data error, the loss development for the Commercial Property business is very stable. In contrast the Personal Motor Non-comprehensive data demonstrates a greater degree of variation around the average development as would be expected for a liability class of business. Comparing the results for these two classes under a number of different methods will assist in considering the extent to which the various methods respond to the variability in the underlying loss data.

The Employers' Liability data set provides annual development triangulations by annual exposure period for payments, incurred losses, settled claim counts and incurred claim counts. No premium or exposure data is provided. The data is provided for three sub-classes and is further sub-divided between large losses and attritional claims. No qualitative information is provided on the nature of the underlying sub-classes (hence these are not identified separately in the table below).

The sub-classes demonstrate very different features. One subclass appears fully developed after 10 or 11 years whilst the other two are still developing after twenty years (the limit of the data), one of which is demonstrating latent development after 12 years and relatively high variability compared with the other two subsets. This data is particularly interesting in terms of testing the models but the eventual results will be highly dependent on individual assumptions in relation to the tail development beyond the 20 years development history provided.

The following table provides a summary of all the data sets available to the working party.

Class	Sub Class	Paid / Incurred?	Origin Period	Development Period	Large Losses?	Exposure?	Net of Reinsurance?
Construction	Employers' Liability	P & I	1985 - 2005	Annual	By Class. Not by sub Class	No	?
Construction	Public Liability	P & I	1985 - 2005	Annual	By Class. Not by sub Class	No	?
Construction	Damage	P & I	1985 - 2005	Annual	By Class. Not by sub Class	No	?
Employers' Liability	Employers' Liability	P & I	1991 - 2005	Annual	By Class. Not by sub Class	No	?
Public Liability	Public Liability	P & I	1991 - 2005	Annual	By Class. Not by sub Class	No	?
Employers' Liability	Employers' Liability	P & I	1985 - 2005	Annual	Yes	No	No
Home	Direct Subsidence	P & I	1995 - 2005	Monthly	No	No	?
Home	Int Subsidence	P & I	1995 - 2005	Monthly	No	No	?
Home	Direct Non subsidence	P & I	1998 - 2005	Monthly	No	No	?
Home	Int non Subsidence	P & I	1998 - 2005	Monthly	No	No	?
Comm Property	Comm Property	P & I	1997 - 2006 Q3	Quarterly	No	Yes	No
Household Weather	Household Weather	P & I	1997 - 2006 Q3	Quarterly	No	Yes	No
Property	Weather	P & I	1985 - 2005	Annual	By Class. Not by sub Class	No	No
Property	Ex Weather	P & I	1985 - 2005	Annual	By Class. Not by sub Class	No	No
Marine	Cargo	P & I	1988 - 2005	Annual	Yes (for Gross)	Yes	Yes
Marine	Liability	P & I	1988 - 2005	Annual	Yes (for Gross)	Yes	Yes
Marine	Hull	P & I	1988 - 2005	Annual	Yes (for Gross)	Yes	Yes
Motorcycle	Motorcycle	P & I	1997 - 2006 Q3	Quarterly	No	Yes	No
Motor Fleet	Bodily Injury	P & I	1997 - 2006 Q3	Quarterly	No	Yes	No
Motor	Direct injury	P & I	1990 - 2005	Monthly	No	No	?
Motor	Int injury	P & I	1998 - 2005	Monthly	No	No	?
Motor	Direct Non injury	P & I	1990 - 2005	Monthly	No	No	?
Motor	Int non injury	P & I	1998 - 2005	Monthly	No	No	?
Pers Motor Non Comp	Pers Motor Non Comp	P & I	1997 - 2006 Q3	Quarterly	No	Yes	No
Pers Motor Non Comp	Pers Motor Non Comp	P & I	1997 - 2006 Q3	Quarterly	No	Yes	No
Business Interuption	Weather	P & I	1985 - 2005	Annual	By Class. Not by sub Class	No	?
Business Interuption	Ex Weather	P & I	1985 - 2005	Annual	By Class. Not by sub Class	No	?
Personal Creditor	Personal Creditor	P & I	1997 - 2006 Q3	Quarterly	No	> 2003	No
Prof Indemnity	Prof Indemnity	P & I	1997 - 2006 Q3	Quarterly	No	Yes	No
Prof Indemnity	Prof Indemnity	I	1989 - 2005	Annual	No	No	No

5.4 Items Beyond the Data

Assessments of uncertainty need to also consider the potential for differences in future claims experience when compared with past experience as depicted by the historical loss data. The working party considered that appropriate methods for evaluating uncertainty would need to be able to take into account anticipated future changes relative to past experience.

The working party considered the potential sources of uncertainty which may not be reflected or may only be partially reflected in the data sets used and came up with the following non-exhaustive list:

- Inflation
- Policy data e.g. lack of historical split by risk factors
- Rating indices
- Latent claims
- Case estimation practice
- Data quality
- Circumstances of business
- Staff turnover
    - Claims handlers
    - Underwriters
    - Actuaries
- Product bundles
- Aggregate policies – combined data
- Legal changes e.g. Ogden tables
- Pricing change data
- Underwriting cycle
- Terms and conditions
- Period claims order
- Known events not in data
- Social and economic factors
- Knock-for-knock
- Contract certainty
- Large catastrophes (extreme events or just catastrophes?)
    - Impact on claims philosophy
    - Demand surge
- Climate change
- Return period of large claims
    - What should be stripped out
    - Potential impact on UPR
- Distribution methods
- Geographical exposure changes
- Changes in reinsurance
- Influence of rating agencies
- Influence of market views
- Reserving process

- Anchored knowledge
- Personal independence

Each of these factors not only has the potential to change the expected amount and timing of future cash flows but the uncertainty around the amount and timing of those cash flows. A challenge for measuring uncertainty will be how actuaries can reflect these and other factors affecting the future performance of the business within the methods available.

6 Key Features of the Methods Tested

	ODP/Bootstrap	Mack	Bayesian/BF Method	Judgement	Scenarios	Regression/Curve Fitting
Description	Most common bootstrap model. Potential to use different distribution for the residuals	Calculation of standard error with and without tail factors.	Uses ODP model with a series of prior ULR estimates defined by a distribution	Based on professional experience	Can include any variation such as changing development patterns or single events	Fits Craighead curve to each origin year to derive initial estimate of ULR, then smoothes across origin years using regression
Data required	Cumulative claims triangles (paid or incurred)	Cumulative claim triangles (paid or incurred)	Cumulative claim triangles (paid or incurred)	Any	Any	Premium and claim amounts triangles
Is the method acceptable to the Profession?	Yes	Yes	Yes	Yes	Yes	Depends on purpose
Is the method easy to use and is it practical?	Yes	Yes	No	Yes	Yes	Yes
Can judgement or amendments be applied?	Yes	Amendments needed where gaps in published method	Requires prior distribution of ultimate position of each origin year	Yes - essential	Yes via choice of scenarios and manual adjustments or tweaks	Yes, perhaps too easily
Is the method easy to explain?	Principles easy to explain	No	Very difficult	Yes	Yes	Yes
When is method good? (Or not?)	Good if little negative development and residuals are iid and run-off pattern is same for all years.	Good only if run-off pattern is same for all years	Good if little negative development and residuals are iid and run-off pattern is same for all years.	Good if actuary has additional knowledge; bad if not experienced	Not good if volatile datasets or inexperienced actuary	Good if run-off pattern varies across origin years. Not good if there is much negative development

	ODP/Bootstrap	Mack	Bayesian/BF Method	Judgement	Scenarios	Regression/Curve Fitting
Are extreme events included?	Only if in data	Only if in data	Yes, if in data and/or in prior distributions	Yes if desired	Yes if desired	Yes, if in data but can exclude if desired
Produce complete distribution of outcomes?	Yes if process error is simulated in addition to bootstrapping for parameter error	Produces mean and standard error only	Yes	Yes as any required percentile can be estimated using judgement	No – produces a few possible outcomes to which probabilities can be judgementally applied	No, just an approximate range
Type of uncertainty measured	Bootstrap method gives parameter uncertainty, process uncertainty can be simulated in addition	Process and parameter uncertainty	Process and parameter uncertainty	Potentially model error as well as parameter and process error	Usually just parameter uncertainty	Parameter uncertainty only (dependent variable in regression is expected ULR)
Time to program and complete	Easy to program in Excel though long time to run	Easy to program and quick to run	Specialist software required and very slow to run			Easy to do in Excel
Comparison of class results to aggregated	Automatic consistency between origin year and aggregate results	Automatic consistency between origin year and aggregate results	Automatic consistency between origin year and aggregate results	Should be consistent given enough care, but this not guaranteed	Does not produce separate assessment of aggregate uncertainty	Does not produce separate assessment of aggregate uncertainty

7 Numerical Summary of Results

7.1 Summary of Approach

The Working Party has tested and compared a variety of methods to quantify reserving uncertainty. The working party members were split into groups with each group applying a method on 3 datasets. These datasets were chosen for their dissimilarity – Employers' Liability, Personal Motor and Commercial Property. More than one group attempted some of the methods.

7.2 Comparison of Methods

A key aim of this analysis was to assess how comparable the results were from different method frameworks. The following Sections show a graphical comparison of the mean, inter-quartile range and 5^{th}-95^{th} percentile range of reserve estimates for each of the methods by class.

Possible causes of the variation in estimates produced by the methods could include differences in the:

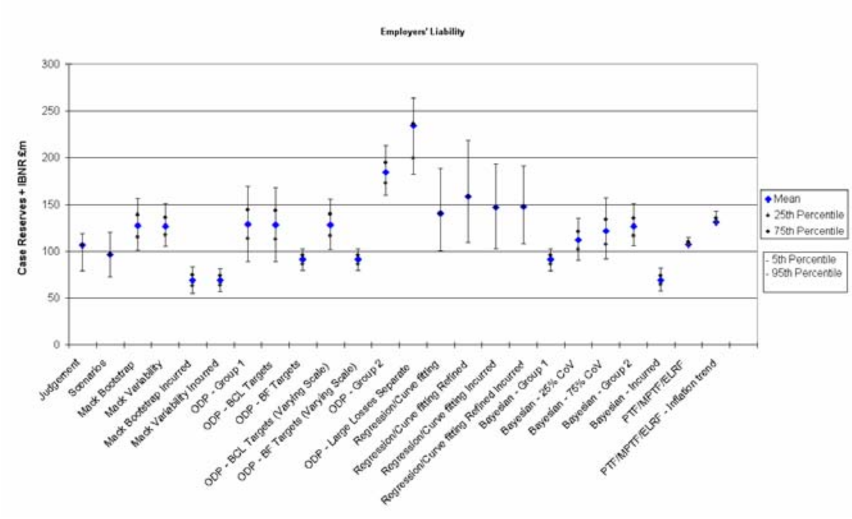
- theoretical framework of the methods
- practical application of the methods
- subjective choice of underlying parameters/data used

Although the data being used in the trial was real insurance company data, anonymity was given to the source of the data. Therefore, the groups applying the methods did not have a detailed knowledge of the company supplying the data, or have the facility to investigate the causes of any apparent anomalies in the development. A key message from the GRIT report was the importance of understanding the data when applying any actuarial method. Some of the results of the analysis described below highlight this issue, as different treatment of anomalies in the data produced a wider range of results than that seen from applying different methods.

A further difficulty in drawing conclusions from the analysis was the relatively low number of groups involved. There were few cases where more than one group applied the same method to the same data set, making it impossible to reliability adjust for the variation in estimates arising because different people were applying the methods. However, from the few observations possible, it appears that the variation produced by different groups was very significant compared to the variation between methods. This may have arisen from some groups applying the methods automatically, with little judgemental adjustment, whilst others may have incorporated market knowledge into the analysis.

Given all the difficulties outlined above, any conclusions from this analysis about the relative results from the different methods could be spurious.

Employers' Liability Results Comparison



Graph 1: Employers' Liability Method Comparison

The graph above shows the results produced for the reserves at the various centile points we discussed above, for a number of methods and modelling frameworks. The majority of results shown are those produced by the working groups. The two right most results shown, (PTF/MPTF/ELRF, PTF/MPTF/ELRF – Inflation trend) are from the submission from Insureware. This is the only class for which we received a distribution from Insureware.

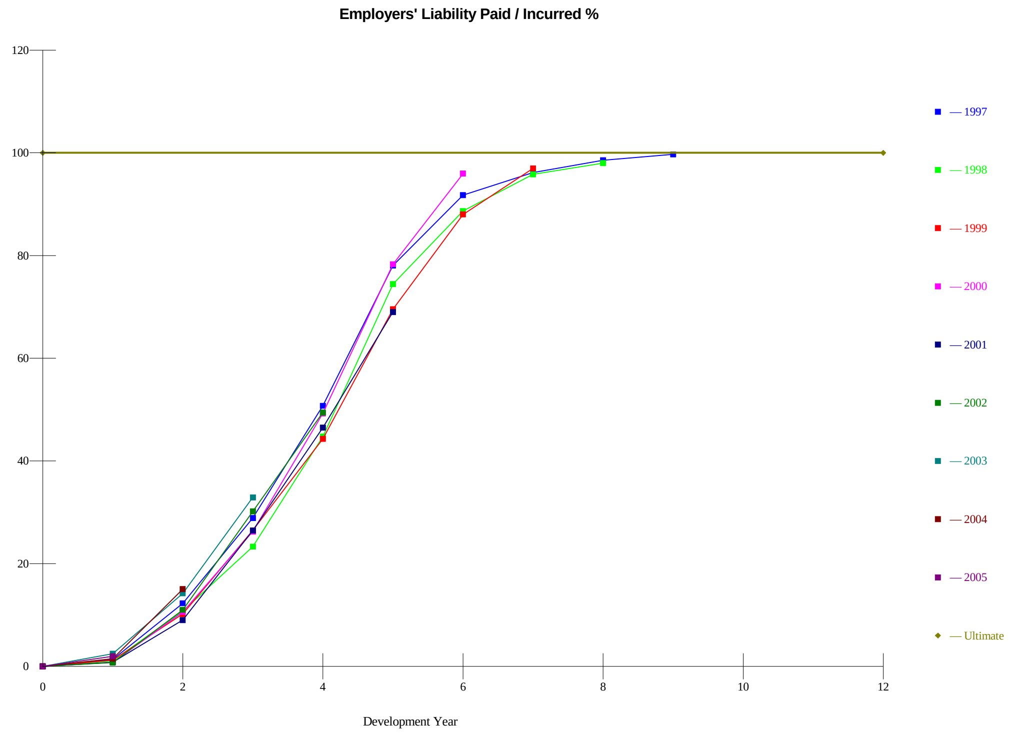
We have discussed the results produced for this class with the Reserving Oversight Committee. We would observe that some actuaries may be concerned that the 95th centiles produced by most of the methods in Graph 1 appear surprisingly low for this class of business when benchmarked against their personal reserving experience. It would be interesting to hear from the profession whether this is so. We refer the reader to the discussion about the judgement method in Section 1.

Some of the methods and modelling frameworks produce fairly narrow ranges in this example – such as Mack Bootstrap Incurred, Mack Variability Incurred, ODP BF Targets, ODP BF Targets (Varying Scale), Bayesian - Group 1, Bayesian Incurred, PTF/MPTF/ELRF, PTF/MPTF/ELRF – Inflation trend.

Some of the methods produce fairly wide ranges in this example, such as ODP – Group 1, ODP – BCL Targets, ODP – Large Losses Separate and the four Regression/Curve fitting methods.

There is a wide variety of means produced across the various methods, the highest being around three times the lowest.

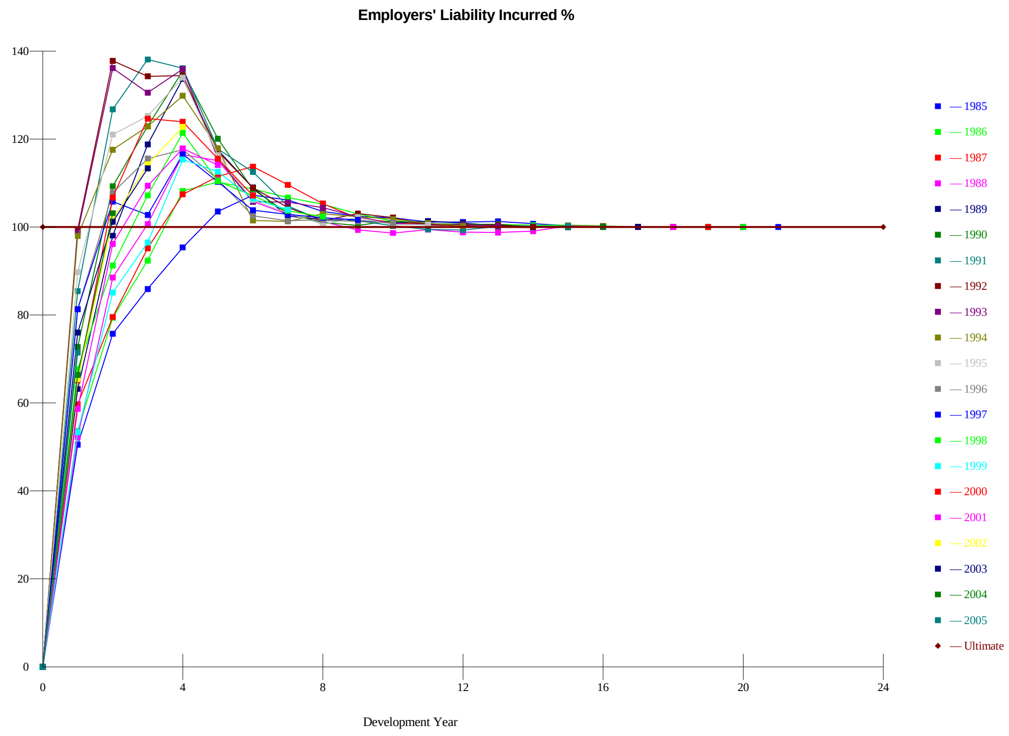
One cause of the difference in the results appears to arise from the choice of paid versus incurred triangles. The majority of groups used paid triangles but for the Mack and Bayesian approaches incurred data was also used. The relationship between Paid and Incurred data changed in more recent origin periods and gave rise to noticeably different results with the same method. The graph below shows the development of paid claims as a proportion of incurred claims.



Graph 2: Employers' Liability Paid to incurred Data Comparison 1997 to 2005

It can be seen that the paid claims as a proportion of incurred claims increased for more recent origin periods, possibly indicating either a relative weakening in case reserves or a speeding up of payment patterns, or a combination of both. The assumption made about which of these explanations underlies the change in development significantly affects the results produced.

When applying the methods to incurred claims, it is implicitly assumed that the development pattern of incurred claims is stable. As the graph below shows, the older years had shown a substantial proportion of redundancy within outstanding claims case estimates being released after about development year 4.



Graph 3: Employers' Liability Incurred Claims Development - All Years

When applying the methods to incurred claims, the level of this release was implicitly assumed to continue. The mean estimates produced by the incurred methods appear unreasonably low, as in some cases they are below paid claims to date, with little historical evidence for recoveries on paid. It is notable that the automatic application of these methods produced estimates where even the 95th percentile was significantly below the mean produced by methods applied to paid claims. However, the systematic decrease in incurred claims invalidates the assumptions for some of the methods used.

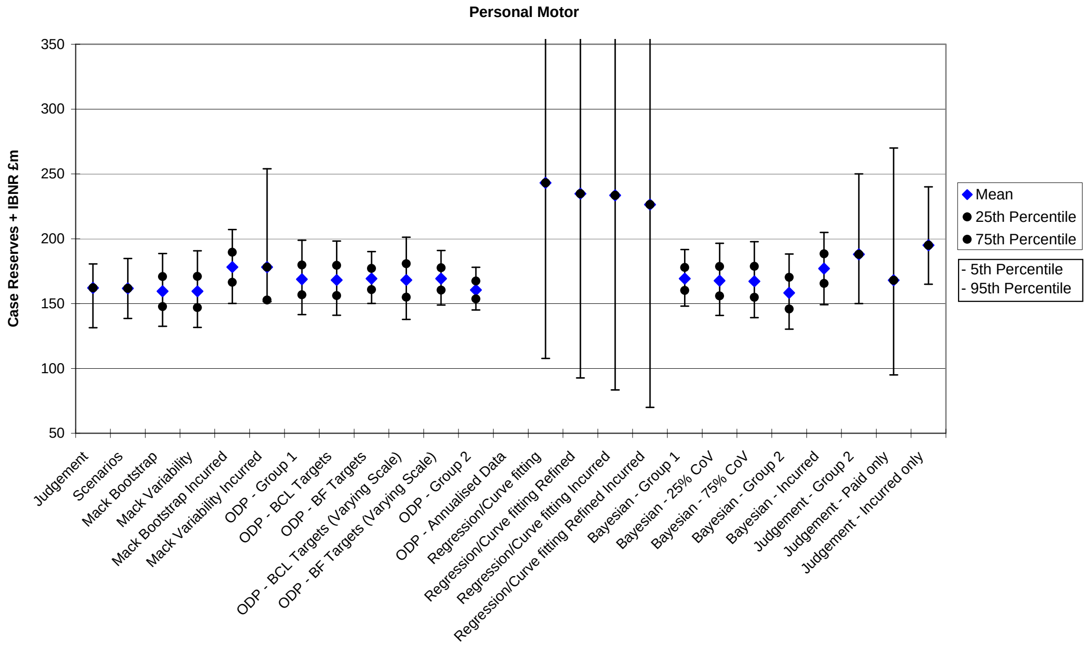
The decision to analyse large losses separately for the ODP method also gave rise to very different results.

The choice of approach within each method was also influential. For example the choice of informed prior and scaling approach in the Bayesian method led to quite different results. Group 1 applied the deterministic basic chain ladder method to both Paid and Incurred data and then based the informed prior on the average of the ultimates by origin year. Group 2 did not use an informed prior. Neither group incorporated their own external judgement to influence the informed prior.

The Insureware team did not believe in a method as such but rather a modelling framework to identify the structure in the data. The modelling frameworks used were the Probabilistic Trend Family (PTF), Multiple Probabilistic Trend Family (MPTF) and Extended Link Ratio Family(ELRF). Please note that the graph for this approach does not show the 5th and 25th percentiles.

Comparing the 95th percentile to the mean the Regression/Curve Fitting method gave the highest value at 138% compared to the Insureware approach giving the lowest value at 107%. This is quite a significant variation in the estimated uncertainty.

Personal Motor Results Comparison

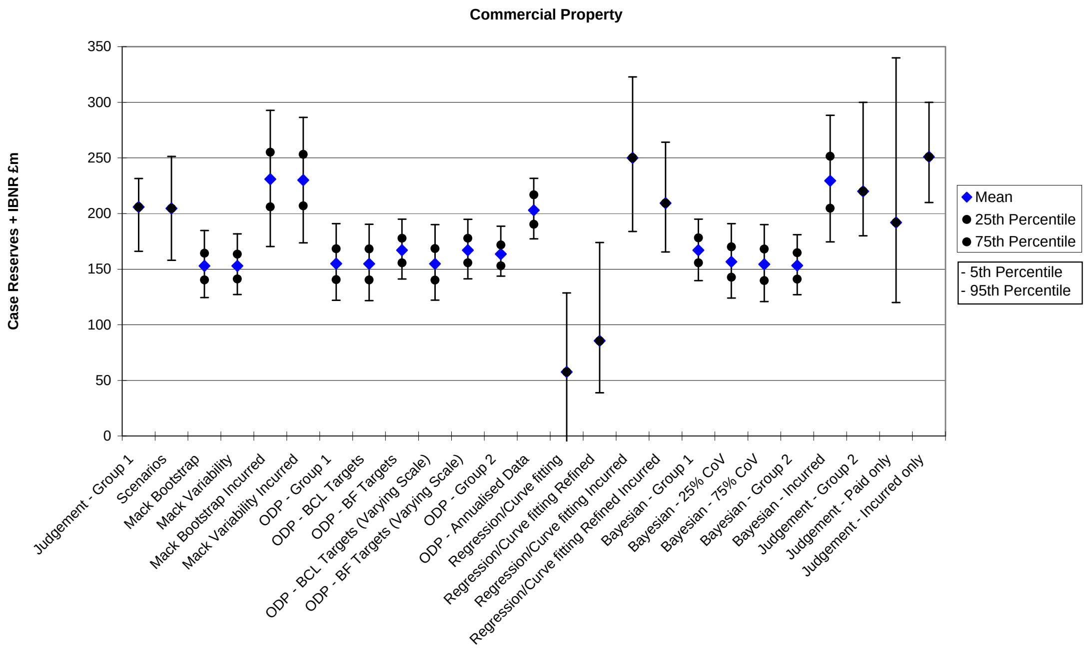


Graph 4: Personal Motor Method Comparison

The majority of methods gave consistent results for this dataset. The ratio of the 95th percentile to the mean mostly varied between 110% and 120%. The exceptions were the Regression / Curve fitting methods (varying between 186% and 243%) and the Judgement method on Paid Data from Group 2 giving a ratio of 133%. It is noticeable that these exceptions were the methods where intermediate percentiles were not calculated.

The Mack Variability Incurred projection was significantly skewed due to the high coefficient of variation and the decision to use a lognormal distribution.

Commercial Property Comparison



Graph 5: Commercial Property Method Comparison

The estimates seemed to be split into two main groups with the results being quite consistent within these groupings. This was partly explained by the choice of Paid or Incurred data. As for the Employers' Liability class, there was a change in the relationship between paid and incurred claims, albeit in this case paid claims reduced as a proportion of incurred claims for recent origin periods leading to estimates from incurred claims exceeding those from methods applied to paid claims.

The Regression / Curve Fitting results on paid data were noticeably different to the remaining methods.

8. Summary of Qualitative Comments

8.1 Introduction

In applying the selected methods to the real datasets, sub-groups of the working party considered the following qualitative questions:

- Would the method be acceptable to the Profession?
- Ease of use and practicality of method
- How difficult is it to apply judgement and / or amendments to the results
- How easily would you be able to explain the method to non technicians?
- Does the method include extreme events? (By this we mean can you allow for the sudden emergence of large individual losses, late tail kicks in incurred, surprising developments on known large losses, etc)
- When is the method good, when is it not good, and when does the method fail?

Comments from the sub-groups are summarised in the remainder of this section for each method in turn. In some cases, these comments were collected from more than one-subgroup and the sub-groups may have had differing views: these have been retained.

*8.*2 ODP Bootstrap Method

8.2.1 Would the method be acceptable to the Profession?

Three sub-groups looked at this method. The consensus was that this method would be acceptable (and our survey results, Section 3, show that it is in fact one of the most widely used methods). However, the new simulation results presented in Section 9 and Appendix B of the present paper suggest that a renewed debate on this would be appropriate: published versions of the ODP bootstrap method seem to be inaccurate in the extremes of the predictive distribution.

8.2.2 Ease of use and practicality of method.

A group that used an existing proprietary implementation commented that the bootstrap ODP method is easy to use once set up, and as the algorithms are relatively simple, the program runs quickly. This group added that the method is relatively straightforward and can be run in an Excel spreadsheet; but that it is better suited to specialist software without the limitations of Excel.

Another group who used Excel commented that it is easy to programme, taking 1-2 days to programme for a generic application including tests. However, this group found it quite slow to run in Excel (about 30mins for 200 simulations). This group commented that a significant number of simulations is required to generate distributions without "roughness" in the tail, as use of scale parameter and integer

Poisson results can give clumpy results on an individual underwriting year basis, particularly where minimal reserves are expected.
A third group programmed the method in C++ and found that 1,000 simulations took a fraction of a second, so running enough simulations to obtain a smooth predictive distribution was not a problem.

8.2.3 How difficult is it to apply judgement and / or amendments to the results

Judgement is required on details of the method where the published literature is silent or inconclusive. If using a proprietary implementation, some of these judgements have been made by the software providers and users should be aware of this.
One group commented that checks are needed to ensure that individual development factors are greater than one. The group that used a C++ implementation did not impose this constraint but instead imposed the weaker constraint that all cumulative paid amounts should be positive in pseudo-data (as described in Section B2.4.1 of the present paper).

The group that used the proprietary application commented that the ODP Bootstrap results can be scaled to achieve any desired mean or coefficient of variation. Judgement is required on whether or not to do this, and in the choice of target mean and/or variance. The mean could be chosen to match results attained by a deterministic statistical method, such as a deterministic chain ladder. The variance could be chosen to match that of another, perhaps more complicated, stochastic method. Otherwise the variance could be scaled (either additively or multiplicatively) in proportion to the scaling of the mean.

However, it is unclear what the bootstrap ODP method achieves if the bootstrap predictive distribution is shifted and scaled in this way. This presupposes that the mean and predictive standard error have been obtained by some other method, in which case an analytic distribution (instead of a bootstrap distribution) could be fitted to give the required mean and standard error.

The group that programmed the method in Excel commented that the method has limited usability on a pure basis as it needs all individual development factors (idfs) to be greater than one, adding that judgement can be applied to select a set of idfs, but this will increase the residuals and hence uncertainty. This group also commented that other amendments to the base model are possible, including changing the assumptions on the predictive distribution.

8.2.4 How easily would you be able to explain the method to non technicians?

One group commented that the bootstrapping part of the method (in respect of parameter uncertainty) would be easier to explain to non-technicians than the forecasting part: it would be possible to explain the principles of bootstrapping in layman’s terms. Another commented that principles would be easy to explain, but technical details more difficult. This concurs with experience of other members of the working group who have had to explain the method in practice.

8.2.5 Does the method include extreme events?

One group commented that the ODP bootstrap can produce some very extreme outliers that may need to be capped in order for the distribution to be used. Judgement is needed on whether/how to include extreme outcomes in the bootstrap results. Another group commented that extreme events are only included as much as they exist in existing data.

8.2.6 When is the method good, not good, and when does it fail?

One group commented that the ODP bootstrap is good if there are few negative developments, and it is reasonable to assume that the residuals are identically distributed (subject to scaling) and independent.

The group that used the proprietary implementation commented that the method will not work if the sum of a column in the incremental triangle is negative. Another group commented that for similar reasons, the method may not work well on incurred data. It was generally recognised that the method requires the underlying run-off pattern to be the same for all origin years, and will not work well if this is not the case. This is confirmed by the simulation results presented in Appendix B (Section B.4.5) of the present paper.

8.3 Mack’s Method

Two sub-groups tested and commented on this method.

8.3.1 Would the method be acceptable to the Profession?

As for the bootstrap ODP method, the general view was that the method is currently quite widely accepted by the profession. However, the new simulation results presented in Section 9 and Appendix B of the present paper suggest that a renewed debate on this would be appropriate because the method can be very inaccurate in the extremes of the predictive distribution.

8.3.2 Comment on difficulty of method to program and run

One group used an existing implementation and commented that software was readily available for the method and was quick to run. Another group found it straightforward to programme the method in both Excel and C++.

8.3.3 How difficult is it to apply judgement?

The group that used an existing implementation said that adjustments could not be made. The group that programmed the method in accordance with Thomas Mack’s original papers found that nearly all details are specified by Mack, but there is some scope for judgement where there are too few data-points to estimate a variance parameter for each development period, and where tail factors are required. Mack also does not fully prescribe how a complete predictive distribution should be obtained from the mean and standard error that are produced by the method.

8.3.4 How easily would you be able to explain the method to non technicians?

One group commented that it would probably be difficult to explain this to non technicians but no more so than most methods. Another group commented that it is a relatively difficult method to explain to non-technicians.

8.3.5 Does the method include extreme events?

It was generally recognised that extreme events are only included to the extent that they occur in the past data.

8.3.6 When does the method fail or not work well?

It was generally recognised that the method requires the underlying run-off pattern to be the same for all origin years, and will not work well if this is not the case. The method will not fail if the run-off pattern varies, but will give poor results. This is confirmed by the simulation results presented in Appendix B (Section B.4.5) of the present paper.

8.4 Bayesian Bornheutter-Ferguson (BBF)

Two subgroups tested this method – both used proprietary specialist software (Igloo).

8.4.1 Would the method be acceptable to the Profession?

The method is a formalisation of the widely used Bornheutter-Ferguson method, so in principle should be acceptable to the profession, provided it is correctly implemented and the judgements required understood.

8.4.2 Comment on difficulty of method to program and run

The BBF method has some advanced theory behind it and requires some advanced sampling algorithms. The method requires sampling several parameters from a non-standard multivariate distribution. This has been done using Adaptive Rejective Metropolis Sampling (ARMS). The Gibbs method is used to reduce the multidimensional sampling problem to an iterative one dimensional problem. This would be very difficult to perform without using specialist software.

Both groups commented that the specialist software used is currently slow to run. One group said that running 50,000 simulations on a 10 by 10 triangle takes over one hour using a PC with processor speed 3 GHz and 2GB of RAM. The other said that run time can be decreased by reducing the sampling rate, but that when unadjusted, 10 scenarios for one triangle takes 20mins to run.

8.4.3 How difficult is it to apply judgement?

The model is flexible as it can accept prior information in the form of a distribution. The prior distributions used in the BBF method influence the results. The output distribution can be adjusted to achieve a desired mean and variance, however this would undermine the decision to use the BBF method in the first place.

The method requires a prior distribution for the ultimates separately for each origin period. Judgement is required in choosing these distributions. The levels of certainty of the prior estimates are reflected in the choice of the coefficients of variation for the prior distributions. If a high CoV is chosen for the prior distributions then the BBF predictive distribution resembles that produced by the ODP Bootstrap method. If the CoV of the prior distributions are small then the ultimates will have a smaller CoV. The mean of the distribution tends towards that of a deterministic BF model as the CoV of the priors tend towards zero.

8.4.4 How easily would you be able to explain the method to non technicians?

One group commented that the BBF method is difficult for a technician to understand so it would be very difficult to explain the method to a non-technician. Even to explain the principles of the BBF method in layman's terms would be very challenging. The other group said it would probably be difficult to explain this to non technicians but no more so than most methods.

8.3.5 Does the method include extreme events?

It was generally recognised that extreme events are only included to the extent that they occur in the past data.

8.4.6 When is the method good, when is it not good and when does it fail?

The BBF is useful to use when there is a prior assumption of the ultimate values. It is not so good when time or processor speed is an issue. One group said the method will not work if the sum of a column in the incremental triangle is negative. The other said the method does not seem to work if there is no movement in the upper right of the triangle.

8.5 Judgement

Two sub-groups considered this approach.

8.5.1 Would the method be acceptable to the Profession?

We refer the reader to this discussion of this method in Section 1. The following discussion is from the working groups.

The first sub-group said: Yes. "Actuarial judgement" is probably the most widely used concept when setting reserves. It does rely solely on the judgement of the person setting the reserves, but it allows that person full control and flexibility to adapt to any trends they may see in the data without much effort. This method could be criticised as the accuracy of the best estimates is dependent on the experience of the actuary applying the judgement. There is always a risk that two reasonable actuaries would generate two very different results through taking a different view around emerging trends or allowances for distortions.

The other sub-group said: I don't think so. It is very subjective and difficult to do. However this second group also recognized that this method does have the merit of potentially allowing for model error as well as parameter and process error.

8.5.2 Comment on difficulty of method to program and run

The first group considered the use of the basic chain ladder method as a starting point for the application of judgement, and commented that to generate a range of results using judgement it can be somewhat laborious tweaking development factor assumptions to allow for distortions within the data. They added that automatically generated development factors can be easily obtained however as a starting point by sampling across varying numbers of accident periods. Generally, initial selection can be made quickly (several minutes). Generating a sensible range of results through adjustment of factors naturally adds to time taken.

They also commented that input from several modellers provided greater variety of answers. In this way, the subjectivity of the approach could be viewed as an advantage when assessing reserve uncertainty. However, they also commented that it can be difficult to avoid being influenced by an expectation of what reserves should be. For example, the actuary could be 'anchored' to previous valuation results. They commented that the method potentially allows for the many complexities of different classes of business, but taking all factors into account by judgement is demanding and time consuming to do well. Additional information (e.g. large claims data) or discussions with claims handlers can provide valuable insight leading to a better application of judgement.

The other group said it is generally very difficult to translate judgement into percentiles.

8.5.3 How difficult is it to apply judgement?

See response to 8.5.2 above.

8.5.4 How easily would you be able to explain the method non technicians?

It is easy to explain how certain factors have been taken into account when presenting results to non technicians. It is also relatively easy to explain the basic chain ladder approach to non technicians to expand on where judgement has been applied.

8.5.5 Does the method include extreme events?

Extreme events can be suitably allowed for through additional margins within the provisions by making reasonable assumptions about likelihood and severity. However, there may be much subjectivity over what is ‘reasonable’ here.

8.5.6 When is the method good, when is it not good and when does it fail?

It is good to apply judgement when the actuary believes he or she has relevant knowledge or experience about the expected future development of the claims, and is adept enough to make suitable allowances. The application of judgement is less necessary for standard short-tail classes where past performance remains a good guide to the future.

Lack of experience or inaccurate supporting data/information could lead to unsuitable application of judgement. Peer review can help to mitigate these risks.

8.6 Scenarios

There are several ways to employ scenario testing. This method can be applied to many underlying reserving methods including chain ladder, Bornheutter Ferguson and exposure methods.

One sub-group considered the use of scenarios generated from the basic chain ladder method and commented as follows:

8.6.1 Would the method be acceptable to the Profession?

Yes. Application of judgement can play a major role in selecting scenarios to obtain a range of results. A more mechanical process of simply selecting the most optimistic and pessimistic development factors is unlikely to be suitable, especially where large distortions exist within the data.

8.6.2 Comment on difficulty of method to program and run

The basic chain ladder method is very simple and easy to use. A simple mechanical process of selecting the most optimistic and pessimistic development factors is easy to apply. Other considerations should be made however to avoid generating meaningless results. Inherent uncertainty / volatility in smaller datasets e.g. from large claims, ideally need to be considered and allowed for in the scenarios selected.

8.6.3 How difficult is it to apply judgement?

It is easy to apply amendments to results via manual adjustments and other tweaks when using a simple, well designed model.

8.6.4 How easily would you be able to explain the method to non technicians?

It is relatively easy to explain the basic chain ladder approach to non technicians and describe the choice of scenarios generating the range of results.

8.6.5 Does the method include extreme events?

Extreme development factors can only be selected to the extent they are generated from the underlying dataset. A mechanical selection of factors, with no consideration to the distorting effect of extreme events, would not be sensible.

(The working party would add that the use of scenarios can be a helpful approach to considering extreme events, and can be usefully employed, for example, in exposure reserving for asbestos claims).

8.6.6 When is the method good, when is it not good and when does it fail?

A purely mechanical application of selecting best/worst scenarios would not be advised, especially when used for volatile datasets. Conversely, this type of method works well with large stable datasets. Lack of experience could lead to inappropriate selection of scenarios.

8.7 Craighead Curve followed by Regression

This was considered by one sub-group who commented as follows:

8.7.1 Would the method be acceptable to the Profession?

It depends on the purpose. It gives a rough indication of uncertainties of reserves based on past development observations. The method would need heavy adaptations and more research to be used for stochastic simulations.

8.7.2 Comment on difficulty of method to program and run

The method is easy to implement using Excel. The regression method does not give estimation for the first two origin periods (because one needs more than two points to perform regression)
Curve fitting could be an issue if there is much negative developments: the Craighead curve is increasing, and other curves could be vulnerable to over-fitting.

8.7.3 How difficult is it to apply judgement

Judgments are required at several points in the methods (e.g. the weight to give in the curve fitting for each data point, the curve to use, the weight to give in the regression for each data point, etc.) But (at least for the curve fitting), it seems that there isn't that much more judgment required than in, say, the chain ladder. The paper (by Benjamin and Eagles) proposed several modifications to the method - very easy to "break into the method" and make modifications

8.7.4 How easily would you be able to explain the method to non technicians?

Should be easily explained to colleagues in other professions.

8.7.5 Does the method include extreme events?

The mechanical application of the model does include them - but one may want to exclude (or put less weight) on them for curve fitting or regression.

8.7.6 When is the method good, when is it not good and when does it fail?

No comment provided.

8.8 (Multiple) Probabilistic Trend Family (M)PTF

The comments in this subsection have been taken from the submission to the working party provided by Insureware Pty Ltd. Insureware produces software (ICRFS-plus) for carrying out these methods. Their complete submission is given in Appendix F.

8.8.1 Would the method be acceptable to the Profession?

Yes.

8.8.2 Comment on difficulty of method to program and run

It is extremely difficult to program (but theoretically possible) in a spreadsheet form, but is easy to run in ICRFS-Plus as all the programming has been pre-specified. ICRFS-Plus is a point and click system with extremely fast algorithms. Individual tests do not need to be programmed. It takes seconds for individual tests to be completed.

8.8.3 How difficult is it to apply judgement within the program

It is not difficult. Judgement is made on the basis of accurate information about the volatility in the business.

8.8.4 How easily would you be able to explain the method to non technicians?

No comment provided.

8.8.5 Does the method include extreme events?

No comment provided.

8.8.6 When is the method good, when is it not good and when does it fail?

No comment provided.

9. Assessing Performance of Stochastic Reserving Methods by Numerical Simulation

9.1 Introduction

The work described in previous sections has given us an initial impression of the main features of several stochastic methods. Although we have given an initial assessment of these methods (as described in previous sections) we cannot formulate definitive conclusions on the performance of these methods from the experience of applying them to a handful of data-sets.

Towards this end, we have also used numerical simulation methods to assess the performance of various stochastic methods. The basic idea is to apply the methods to a large number of simulated run-off triangles for which the 'true' ultimate is known (by simulating it), and to compare the predictions produced by the various methods to the true ultimate.

While this approach can be criticised on the grounds that simulated data may not exhibit all the complications and variety or real-world data, it does have merits:

- It enables methods to be tested on very large numbers of triangles.
- Predictions can be compared to 'true' ultimates without waiting years for the true ultimate position to be reached.
- Triangles can be constructed such that they exactly satisfy the assumptions underlying a method: the performance of a method in this ideal situation indicates the limits of the method's potential performance on real data. (If a method does not perform well on ideal data, it is unlikely to perform well on real data.)
- The robustness of a method to violations of its underlying assumptions (which may be met in practice) can be tested in a controlled way.

We have made a start with this but have not yet done enough to formulate definitive conclusions. We propose that the working party should continue pursuing this approach next year.

In practice, the assumptions of a stochastic method are never perfectly satisfied, and (even if the stochastic assumptions approximate the past run-off data reasonably closely) there is the added risk of shocks and changes in the future unlike anything observed in the past.

9.2 What constitutes "good performance" for a stochastic reserving method?

9.2.1 Three main types of uncertainty

In the following discussion, it is worth bearing in mind the three main categories of uncertainty when forecasting outstanding claims liabilities:

- model uncertainty
- parameter uncertainty
- process uncertainty

9.2.2 Why different stochastic methods give different results

Some stochastic methods tend to give wider confidence intervals for the eventual outcome than others. Some possible reasons are listed below (examples follow this list):

a) Inappropriate stochastic model: if a method is used where the underlying stochastic assumptions are not appropriate for the data, then the results - both best estimates and uncertainty assessments – are unlikely to be good.

b) Failure of some methods to take account of both parameter and process uncertainty.

c) In assessing parameter uncertainty, failure of some methods to take account of uncertainty in all parameters.

d) Failure to make use of all available data.

e) Failure to take account of correlations between different components of the overall forecast.

f) Use of inadequate mathematical approximations in the formulation of a stochastic method (so that even if the method is applied where its stated assumptions are perfectly satisfied, it may not give reliable results).

An example of (a) is misuse of any method based on an assumption that the underlying run-off pattern across development time is the same for all origin years. Where this assumption is a reasonably good approximation to reality, such methods may give realistic results. But if applied in situations where this assumption is unreasonably wide of the mark, such methods may tend to understate uncertainty, that is, give predictive ranges that are unrealistically narrow.

While there will always be some model uncertainty, we should attempt to minimise this by being aware of all model assumptions and of the claim settlement processes involved in the lines of business concerned, so we are in a position to judge whether the model assumptions are reasonably realistic in each application of any stochastic method. The run-off data itself can often be used to check for gross violations of the model assumptions.

An example of (b) is where bootstrapping is used to determine a 'range of best estimates' but no attempt is made to include future process uncertainty.

An example of (c) occurs where run-off data is pre-adjusted for inflation as if the rate of claims inflation were known precisely.

An example of (d): given a triangle of aggregate amounts paid and a corresponding triangle of counts of claims closed, some methods may not make use of the counts triangle. It is tempting to think that using more relevant information necessarily reduces uncertainty, leading to narrower predictive ranges. However this is not necessarily the case: the additional information might indicate that there is more apparently random variation than previously thought. For example, suppose we are using a stochastic chain ladder method and all available diagnostics indicate that the model is appropriate. That is, past data shows no statistically significant departures from a constant run-off pattern and all residual variation apparently satisfies the stochastic assumptions. The quantum of parameter and process variation is assessed from this residual variation (e.g. using Bootstrap, or analytic methods). Then, two diagonals later, there is no material change in the run-off pattern, but residual variation is much greater. This new information indicates that process variation is potentially much greater than it previously appeared: as a result future process uncertainty may increase, despite there being a reduced period of future development.

An example of (e): a method may produce forecasts and standard errors for each origin year separately and in combining these, fail to take proper account of the possibility that over-estimation of a single parameter of the model may cause forecasts for all origin years to be simultaneously overstated.

An example of (f) occurs where a stochastic method relies on asymptotic unbiasedness and efficiency of maximum likelihood estimates, but the number of estimated parameters is so high compared to the number of data-points that these asymptotic approximations are poor.

9.2.3 Choice of stochastic method

It is hopefully clear from the above discussion that, in choosing between stochastic methods, the width of predictive ranges produced by the methods should not be the primary consideration.

Users of our results from outside the profession may be forgiven for judging methods in this way, because they may make the implicit assumption that all methods used by actuaries will be ‘correct’ in the sense of not omitting any major source of uncertainty (such as those listed above). However, this will only be true if we within the profession ensure this is the case by critically examining all stochastic methods we use. For us, the primary considerations should be that stochastic methods:

- attempt to assess all sources of uncertainty,
- are technically accurate in this assessment given the assumptions of the method,
- are applied only where the assumption of the method have a reasonable chance of being approximately true.

If these points are all satisfied, then we have to accept the resulting ranges regardless of their width. (Of course, this may be an iterative process: if ranges come out surprisingly narrow or wide, the above points should be critically examined and the method adjusted and refined if necessary. But when we are eventually satisfied that a valid method has been validly applied, the results should not be rejected merely on the basis that the width of predictive ranges was initially surprising, or may be surprising or inconvenient to others.)

With reference to model assumptions being satisfied, a related and more difficult question concerns the robustness of a method to violations of its assumptions. That is, how reliant is a method on the correctness of its assumptions? This is an important question as it is rare for model assumptions to be perfectly satisfied in reality.

For example, one key assumption underlying all stochastic chain-ladder methods is (as in the basic chain-ladder) that the underlying run-off pattern is the same for all origin years. If a stochastic chain ladder method is used where this assumption is false, are the results likely to be materially misleading? Perhaps not, because if a method involves assessing the quantum of process variation from residual variation in the data, then systematic departures from this key assumption will be partly taken into account through an increase in the residual variation.

9.3 Outline of numerical simulation approach to testing stochastic methods

To test stochastic methods, we can use the well known fact that if X is a random variable, and F(x) is its cumulative distribution function, then the random variable F(X) has a uniform distribution on the unit interval [0,1].

In the context of stochastic reserving, X represents the total of future claim payments (i.e. ultimate less amount paid-to-date). A stochastic method produces a function F(x) that purports to be the distribution function of X. (Many stochastic methods produce only a best estimate reserve and a root-mean-square predictive error, but these can be extended by using some class of analytic distributions, e.g. Log-Normal, to produce a complete distribution F(x).)

If a stochastic method is reasonably good, F(X) should therefore have approximately a uniform distribution. To test this, we need a number of independent instances from this supposedly uniform distribution to see if they are indeed uniformly distributed. The larger the number of instances, the more powerful will be the test.

Having carried out the stochastic method on a particular triangle, we can obtain one instance of the random variable F(X) by waiting for the triangle to reach its ultimate position: this gives us one instance (x_0 say) of the random variable X, hence one instance $F(x_0)$ purportedly from the uniform [0,1] distribution.

By applying the same stochastic method many times to independent triangles, and then waiting for each to develop to ultimate, we could gradually accumulate an increasingly large sample that can be tested for uniformity.

Clearly, if we use only real-world data, it is likely to take a very long time to obtain a large enough sample for a reasonably powerful test of uniformity. The entire process can be accelerated by using a large number of artificial triangles. This approach also has the advantage of allowing us to test the robustness of a method to violations of its underlying assumptions.

So, for each stochastic method we can do two things:

a) Check the performance of the method in situations where its underlying assumptions are perfectly satisfied.
b) Test the robustness of the method to violations of its assumptions.

The main steps of (a) are:

(i) Generate a large number I of artificial datasets (i = 1…I) that follow the assumptions of the method: each dataset to contain run-off arrays (on which to apply the stochastic method) and 'true' reserves (denoted r_i).

(ii) For each artificial dataset, apply the stochastic method to obtain the predictive distribution function $F_i(x)$ (where x is any possible value of aggregate future payments).

(iii) For each artificial dataset, calculate $u_i = F_i(r_i)$ (where r_i is the simulated 'true' total of future payments).

(iv) Test the dataset $\{u_i$: i = 1…I$\}$ for uniformity. If this is not significantly different from uniform, then the method is 'stochastically correct' (in other words, it appears to correctly assess parameter and process error, given the underlying model assumptions).

For (b) (the assessment of robustness) the steps are as for (a) except that at step (i) the artificial datasets would be generated to have some feature that departs from the assumptions underlying the stochastic method. For example, if the chosen stochastic method assumes that settlement delay is unrelated to the size of a claim, we could generate datasets in which larger claims tend to take longer to settle than smaller claims. If the distribution obtained at step (iv) still does not depart significantly from a uniform distribution, then we can conclude that the method is robust to the particular aspect modelled. In other words, no material additional uncertainty arises from this particular aspect of model uncertainty.

9.4 Overview of simulation analysis carried out so far and provisional conclusions

We have done some initial work applying the approach described in the previous section to:

- Thomas Mack's method as described in his 1993 paper.
- The over-dispersed Poisson (ODP) method as described by Arthur Renshaw and Richard Verrall (1998), Peter England and Richard Verrall (1999) and Peter England (2001).
- Tom Wright's operational time average-cost-per-claim method as described in his 1992 paper.

Appendix B gives a detailed description of the simulations carried out and the results obtained. Here we give just a brief overview and summarise the main findings.

For each of these stochastic methods, we simulated at least 30,000 triangles using algorithms that perfectly satisfy the stochastic assumptions underlying each method. We then applied the stochastic method to each simulated triangle to obtain a predictive probability distribution function F(x) (where x the total of future payments). We then calculated u = F(r) where r is the 'true' outcome (simulated along with each triangle). As discussed, in the previous section, this quantity should be uniformly distributed in the range 0 to 1. However, for all the stochastic methods tested, too many value of F(r) were very close to 1. The table below shows the proportion of simulations in which F(r) exceeded 0.99. In other words, this is the proportion of simulations in which the true outcome exceeded the 99th percentile of the predictive distribution obtained from the triangle. Clearly, if the predictive distribution is accurate, this should occur in 1% of simulations.

Table 9.4.1 Proportion of simulations in which 'true' outcome exceeded 99th percentile

Mack 1993 (with Log-Normal)	8% to 13%
Analytic ODP (Renshaw & Verrall, 1998), Pearson dispersion	2.6%
Analytic ODP (Renshaw & Verrall, 1998), deviance dispersion	2.7%
Bootstrap ODP (England & Verrall, 1999)	3.1%
Bootstrap ODP (England 2001)	2.6%
Operational time (Wright 1992), Pearson dispersion	4.0%

Bootstrap ODP (England 2001) is the only one of these methods that produces a full predictive distribution F(x): the others produce just a best estimate and a standard error (or "root-mean-square predictive error"). For these methods we used a Log-Normal predictive distribution F(x) (as suggested by Mack 1993) in order to calculate u = F(r). (We also tried using the Inverse Gauss distribution for F(x) but found this made no material difference to the main results summarised above.)

It is emphasised that for each of the methods, the above results are based on at least 30,000 triangles generated in a way that perfectly satisfies the stochastic assumptions of the respective method.

For most methods we have so far used only a single set of parameters to generate triangles that perfectly satisfy the assumptions. Other parameters would no doubt lead to different results. Only in the case of Mack's method have we tried more than one set of parameters for generating the triangles, which is why the table above shows a range of results (from 8% to 13%). For all stochastic methods the simulated triangles had 10 origin years and annual development. We would expect better performance on larger triangles but have not yet tested this.

We should generally expect stochastic methods to perform worse than this in practice because their assumptions will never be perfectly satisfied. We have done some testing of robustness to violations of assumptions only for Mack's method and the ODP methods so far. The results for all variants of the ODP method indicate, as expected, a deterioration in performance if applied where the ODP assumptions are violated. Surprisingly, results obtained so far for Mack's method show it performs better where its assumptions are not satisfied than where they are perfectly satisfied (details and a possible explanation are given in Appendix B).

9.5 Conclusions to Date

The main conclusion of the simulation work carried out so far is that all these stochastic reserving methods tend to understate the chance of extreme adverse outcomes, even in situations where their underlying assumptions are perfectly satisfied.

In Appendix B we outline some possible explanations of why these stochastic methods do not work well at the extremes. To be fair to the developers of these methods, the goal-posts have moved. The aim at the time these methods were developed was to improve on the usual practice of providing just 'best estimate' reserves, or perhaps estimates based on just a few scenarios (to give perhaps low, medium and high reserve estimates). It is only in more recent years that regulators have begun specifying particular extreme percentiles for capital requirements. Our results indicate that further work is now needed to develop and refine stochastic reserving methods to better meet this new challenge.

10. Objectives for the Working Party for Next Year

There are three main areas of objectives for next year:

10.1 Objective 1: Statistical Testing When Conditions Met

We would like to continue the consideration of the performance of statistical methods when all their underlying conditions are met. We would like to better understand to what extent and in what circumstances these methods can be reliable indicators of ultimate claims outcomes at the tails of the distribution. Do we need to alter the way we use such methods and are there any “quick wins”? We see this as a key priority in the context of reserving risk assessment for capital purposes.

Part of the work in this area is likely to include liaison between this working party and the one concentrating on best estimate reserving and the performance of methods in that context.

10.2: Objective 2: Test More Methods

We would like to expand the review and testing of methods on “real” data to include additional methods, if possible those which operate on transactional data. This would ideally include further quantitative and qualitative review.

10.3: Objective 3: Robustness of Methods in Real Life

We would like to expand the work on simulated data to test the response of methods to circumstances when there underlying conditions are not met – which is the case in most real life scenarios. How do methods respond when there are new trends, changes in claims processing speeds or underwriting cycles, for example. How should the actuary make use of information beyond the triangle? How should an actuary test whether methods are appropriate or not – and how might these tests vary depending on the purpose and the part of the distribution of outcomes which is of particular interest?

Appendix A: Summary of the Methods Tested

Appendix A – Summary of each of the methods tested

The following methods are described briefly below:
1. Judgement
2. Scenario testing
3. Mack
4. Over-dispersed Poisson stochastic chain ladder
5. Transaction level modelling
6. Operational time
7. Regression/ Curve fitting
8. Bayesian/BF method
9. Probabilistic Trend Family (PTF) - ICRFS

Note that Prof. Richard Verrall has a web-lecture on a number of these methods at: https://talk.city.ac.uk/stochasticreserving. In addition the paper by England and Verrall provides a detailed review of the differences between many methods and approaches (England and Verral, BAJ (2002)).

Additional references to these methods are contained in the bibliography, the references provided here represent the actual methods applied in each case.

1. Judgement

Description: This method is described in Section 1 of the report.

Key assumptions: This will vary depending on exactly how the actuary applies judgement. In some cases the only assumptions will be the selected results themselves.

Data required: Any – but relevant experience is also needed.

Reference: n/a

2. Scenario testing

Description: although not the only methodology that can be defined under this heading, this is generally perceived as selecting alternative parameters from the best estimate projection that reflect the user's view of the extreme ends of the range to be defined.

Key assumptions: the assumptions under this method are similar to those under the judgement method. However, they are generally more specific and relate to changes in one or more parameters to investigate the effect such changes have on the projected reserves. Note that any assumptions that relate to the underlying model(s) used to calculate the best estimate will still be applicable in this case.

Data required: Any.

Reference: n/a

3. **Mack**

Description: This method calculates the standard error involved in the application of the Chain-ladder method for the reserve estimation for each origin year and for all years combined, allowing for both parameter and process uncertainty. The 1999 paper allows for tail factors within the method.

The method can be applied both as a deterministic calculation, or using a bootstrap approach. It is suggested that standard distributions (LogNormal; Gamma) can be used to derive ranges based on the results of the method.

Key assumptions:

- The underlying run-off pattern is the same for all origin years (as in the basic chain ladder method)
- Future development factors are independent of past development factors (i.e. the size of the next factor is independent of the size of the last factor).
- Variance of the next cumulative claims amount is proportional to the current cumulative claims amount.

Data required: Cumulative claims triangles (paid or incurred).

Reference: ASTIN Bulletin vol. 23 (1993) (and update in 1999 - ASTIN vol.29)

4. **Over-dispersed Poisson method**

Description: This method calculates the distribution of outcomes for each origin year and for all years combined. It models incremental claims using an Over-dispersed Poisson distribution. The calculation of a distribution of outcomes is performed using a Bootstrapping calculation.

Key assumptions:

- The underlying run-off pattern is the same for all origin years (as in the basic chain ladder method)
- Incremental claims amounts are stochastically independent
- The variance of incremental claim amounts is proportional to the mean (process error).
- Incremental claims are positive for all development periods (though there are adjustments to the method that can allow for data where this is not the case).

Data required: Cumulative claims triangles (paid or incurred).

Reference: "Stochastic Claims Reserving", England and Verrall, BAJ (2002) and others

5. **Transaction level modelling**

Description: This method uses Bootstrapping techniques to model individual claims events, based on investigation of individual claim data. GLM methods are used to fit the claims development factors to each claim's data, based on all available information e.g. credit rating or exposure measure contained within the policy or claims data to derive the best estimate. Similar techniques are applied to derive estimators for reserve ranges.

Key assumptions:

- Individual claim development is a function of characteristics relating to that policy and claim.
- IBNR claims originate in a similar response to the predicted claims development factors.
- A known mix of "good" and "bad" policies is contained within each origin year to allow for changes in business, i.e. the reserving cycle.

Data required: Policy and developmental claim data for each claim.

Reference: "Loss reserving using claim level data", Guszcza and Lommele, CAS Fall Forum (2006)

6. Operational time

Description: An average-cost-per-claim method in which the average-cost-per-claim closed is assumed to depend on the point in operational time at which a claim is closed. 'Operational time' is defined as the number of claims closed expressed as a proportion of the ultimate number of claims in an origin year (so it increases from zero at the beginning of each origin year to one when an origin year reaches ultimate). The main advantage of using operational time is that the method does not require the underlying run-off pattern (across real development time) to be the same for all origin years. Generalized linear modelling is used to find the best fitting formula relating mean claim amount to operational time. Best estimate reserves are determine by evaluating this fitted formula for all future operational times. Predictive standard errors are calculated that allow for both process and parameter uncertainty.

Key assumptions:

- The probability distribution for the amount of an individual claim depends on the point in operational time (between 0 and 1) when the claim is settled.
- The variance of the probability distribution for individual claim amounts is proportional to the mean raised to some power (determined from residual analysis).
- Aggregate incremental claim amounts are stochastically independent (given the number of claims closed).

Data required:
The method requires the usual run-off triangle of aggregate paid amounts, and also a triangle that counts the number of claims closed. Optionally, the method can also make use of counts of reported claim numbers (used to improve estimates of ultimate claim numbers).

Reference: "Stochastic reserving when past claim numbers are known", Wright T. S, Proceedings of Casualty Actuarial Society 1992

7. Regression/ Curve fitting

Description: Estimates ultimate claims by fitting a Craighead (or similar) curve to paid and incurred development data (either as ratios of ultimate premium or absolute values) using a weighted least squares fitting algorithm. The range of reserve estimates is derived by plotting IBNR vs. incurred claims for each development [period within a given origin year.

This plot is examined for potential uncertainty for fitting a regression curve to the data for each origin year separately. This method does not therefore explicitly generate a particular statistical property of the reserve distribution; the extent of the range is defined by the fitting process and the implicit assumptions used therein.

This method does not, therefore, produce reserve ranges for all origin years combined.

Key assumptions:
The fitting curve is a good description of the claim development

Data required: Ultimate premium, paid and incurred cumulative claims.

Reference: Benjamin, S. & Eagles, L. (1997). A curve fitting method and a regression method. Claims Reserving Manual volume 2. London: Institute of Actuaries.
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8. Bayesian/BF method

Description: Bayesian method in which prior probability distributions are specified for the ultimates of each origin year. Using very vague priors yields the same results as the over-dispersed-Poisson chain-ladder method: using exact priors produces the same results as the Bornheutter-Ferguson method. So the method allows for a complete spectrum of results between these two extremes. The appropriate point on this spectrum is determined by the prior distributions, which must be set by the user on the basis of judgement and/or information from other sources (eg industry data).

Key assumptions:
Same as for over-dispersed Poisson model:

- The underlying run-off pattern is the same for all origin years (as in the basic chain ladder method)
- Incremental claims amounts are stochastically independent
- The variance of incremental claim amounts is proportional to the mean
- Incremental claims are positive for all development periods (though negatives can be removed or adjusted if not too many).

And in addition:

- Assumptions for mean and variance of prior distributions (by judgement, or from industry data, or both).

Data required:
Usual run-off triangle of aggregate amounts (paid or incurred). However, as non-positive increments must be removed or adjusted, the method may break-down if there are too many negative increments, so it is often unsuitable for use with incurred data.

In addition, information (or prior experience with other datasets) is needed to inform the judgemental selection of prior means and variances.

Reference: Verrall R. J (2001): A Bayesian generalized linear model for the Bornhuetter-Ferguson method of claims reserving. Actuarial Research Paper No. 139, Department of Actuarial Science and Statistics, City University.

9. Probabilistic Trend Family (PTF) (as implemented in ICRFS-plus)

Description: Normal theory linear regression models for the logarithm of aggregate increments. Regression parameters represent origin year effects, and differences ('trends') between successive development years and successive calendar years.

Key assumptions:

- Same development pattern for all origin years after allowing for calendar year effects.
- Aggregate incremental paid data are approximately log-normally distributed. (Note this can never be exact because the sum of log-normals is not log-normal, but it might sometimes be a reasonable approximation: residual analysis can be used to check.)
- Incremental paid amounts are stochastically independent.

Data required:
Aggregate development data with few negative increments. (Because the dependent variable in the regression is the log of the incremental data, negative or zero increments have to be removed, so the method is often unsuitable for use with incurred data.)

If available, the method can also make use of some measure of exposure (eg premium, or number of claims reported in first development period).

Reference: Best Estimates for Reserves, Glen Barnett and Ben Zehnwirth, PCAS 2000 Volume LXXXVII Part 2

Appendix B: Numerical Simulation – Detailed Results

B Assessing performance of stochastic reserving methods by numerical simulation

B.1 Assessment of Mack's 1993 method using simulation

B.1.1 Mack's assumptions

As discussed in Section 9, we aim to use simulation to:

a) Determine whether Mack's method gives correct results when its assumptions are perfectly satisfied, and

b) Determine the robustness of the method to its assumptions not being perfectly satisfied. In other words, to answer the question: does the method continue to give reasonable uncertainty assessments when the claim payment process deviates (in ways that may be met in reality) from the assumptions underlying the method?

In Mack's 1993 paper he considers the basic chain ladder method (BCL) and sets out to find stochastic assumptions under which the BCL should give good point estimates, and then to find formulas (derived from these stochastic assumptions) for the root-mean-square predictive error of the BCL. (We will use the term 'standard error' to mean the root-mean-square predictive error.) The assumptions identified by Mack are as follows (where C_{jk} denotes the cumulative amount paid in origin year j by the end of development period k):

1. There exist parameters f_k such that: $E(C_{j,k+1} \mid C_{j1},\ldots C_{jk}) = f_k.C_{jk}$
2. There exist parameters α_k such that: $Var(C_{j,k+1} \mid C_{j1},\ldots C_{jk}) = \alpha_k^2.C_{jk}$
3. Accident years are stochastically independent.

B.1.2 Artificial data satisfying Mack's assumptions (Algorithm A)

To test whether Mack's formulas give a correct assessment of uncertainty when these assumptions are perfectly satisfied, we generate artificial run-off triangles satisfying these assumptions. Since the assumptions concern only the conditional mean and variance of the aggregate run-off data (saying nothing about higher moments), we are free to use any probability distribution in generating the artificial data (provided the above mean and variance assumptions are satisfied). Because of its widespread acceptance in modelling loss data and its ease of use, we have used the Log-Normal distribution initially.

The steps used to generate an artificial run-off triangle (and corresponding 'true' ultimates) satisfying Mack's assumptions are as follows:

Algorithm A for artificial run-off data:

1. Decide on the dimensions of the triangle. We have used 10 origin years with annual development: the number of development years reducing from 10 in the first origin year to 1 in the last.

2. Decide on values of the parameters f_k and α_k: the values we used are given in Table B-1 below.
3. For each origin year j, generate a value for C_{j1} (representing the amount paid in the first development year). Mack's assumptions say nothing about how these values are generated so we are free to use any method. We used random sampling from a Log-Normal distribution: the same Log-Normal distribution for all origin years (mean = variance = 1.0) but independent random sampling for each origin year.
4. For each origin year, generate C_{jk} (for k>1) recursively using Mack's assumptions. We generated C_{j2} by random sampling from the shifted Log-Normal distribution that gives values greater than C_{j1}, with mean equal to $f_1.C_{j1}$ and variance equal to $\alpha_1^2.C_{j1}$. We then generated C_{j3} by random sampling from the shifted Log-Normal that gives values greater than C_{j2}, with mean equal to $f_2.C_{j2}$ and variance equal to $\alpha_2^2.C_{j2}$. We continued recursively in this way, using independent random sampling at each stage, until we obtained a value for $C_{j,10}$, which is the 'true' ultimate figure for origin year j. This was repeated for each origin year, using independent random sampling for each one. The triangle was then constructed by discarding the lower right part of the development array (except the $C_{j,10}$ values which were kept as the 'true' ultimates for comparison with estimates produced by applying Mack's method to the upper left triangle). Note that we assumed complete development after 10 years, so no tail-factors were necessary when producing forecasts by Mack's method.

Table B-1 – Parameter values used at Step 2 of Algorithm A

Dev-yr (k)	1	2	3	4	5	6	7	8	9
f_k	4.289	2.064	1.502	1.268	1.150	1.085	1.048	1.027	1.015
α_k	1	1	1	1	1	1	1	1	1

B.1.3 Performance of Mack's method where its assumptions are true

B.1.3.1 Details of simulation method

We created 10,000 artificial triangles (and corresponding 'true' ultimates) by Algorithm A, and applied Mack's method to each triangle. For each triangle, we considered only the reserves for the entire triangle (all origin years combined) and we compared the forecast produced by Mack's method to the 'true' reserve.

Mack's method produces a best estimate (equal to the basic chain ladder or BCL estimate) and an estimated root-mean-square prediction error (or just 'standard error'). We used these to calculate the 'standardised predictive error' as [BCL estimate - true reserve] / (Mack's standard error). If Mack's standard errors are correct, then this should have a mean close to zero and mean square close to one.

We also followed Mack's recommendation of using a Log-Normal distribution for the reserve, with mean equal to the BCL estimate and standard deviation as given by Mack's formula. From this, we found the probability (based on knowledge of the upper left triangle only) that the ultimate outcome would be less than what it turned out to be. In other words, if F(x) denotes the

Log-Normal cumulative distribution function (with mean equal to the BCL estimate and standard deviation as given by Mack), we calculated F(true reserve). If, in addition to Mack's standard errors being correct, the Log-Normal is a reasonable distribution for the predictive error, then the quantity F(true reserve) should be uniformly distributed on the unit interval. Mack also suggests using a Normal distribution (instead of the Log-Normal) when the coefficient of variation is less than 50%, so we have also investigated this.

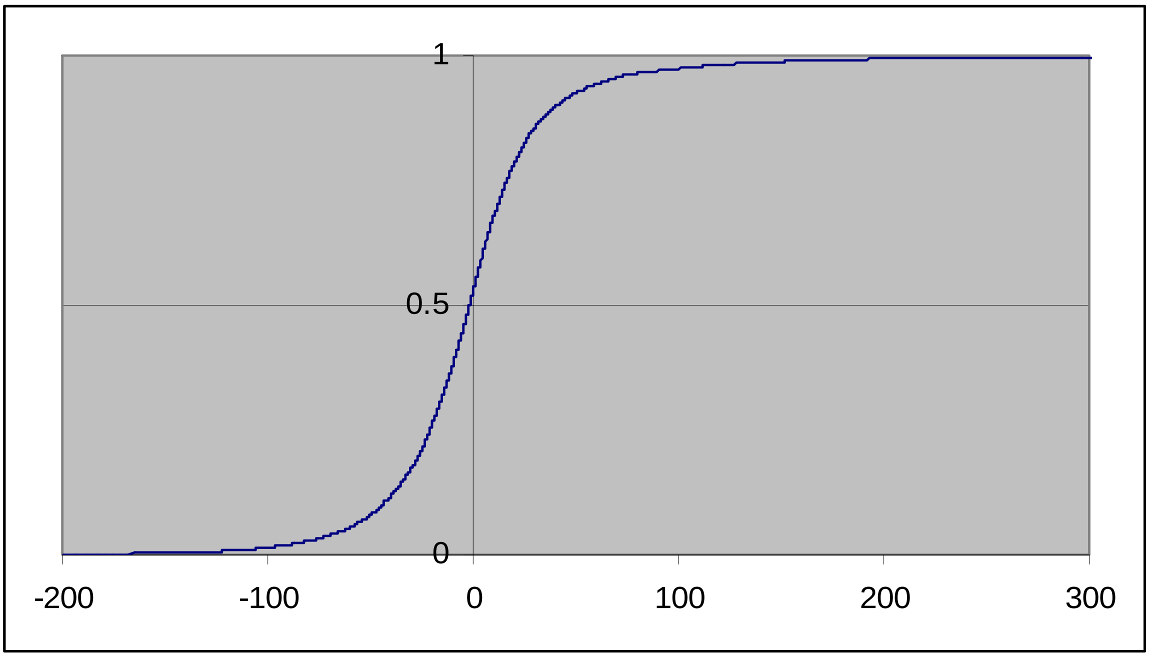
B.1.3.2 Results for first 10,000 simulations

Table B-2 shows results from the first 10,000 simulations carried out. Each value in the second column is the mean (over all 10,000 simulations) of the quantity indicated in the first column.

Table B-2 – Results of applying Mack's method to data generated using Algorithm A

Simulated quantity	Mean from 10,000 simulations
BCL estimated reserve	77.74
Indicator that (BCL > True)	47.25%
BCL estimate - True reserve	0.68
Mack standard error	27.88
(BCL – True) / (Mack std error)	-0.60
Square of the above	6.6
Log-Normal F(true reserve)	0.571

The graph below shows the cumulative distribution function for the quantity (BCL estimate – True reserve).



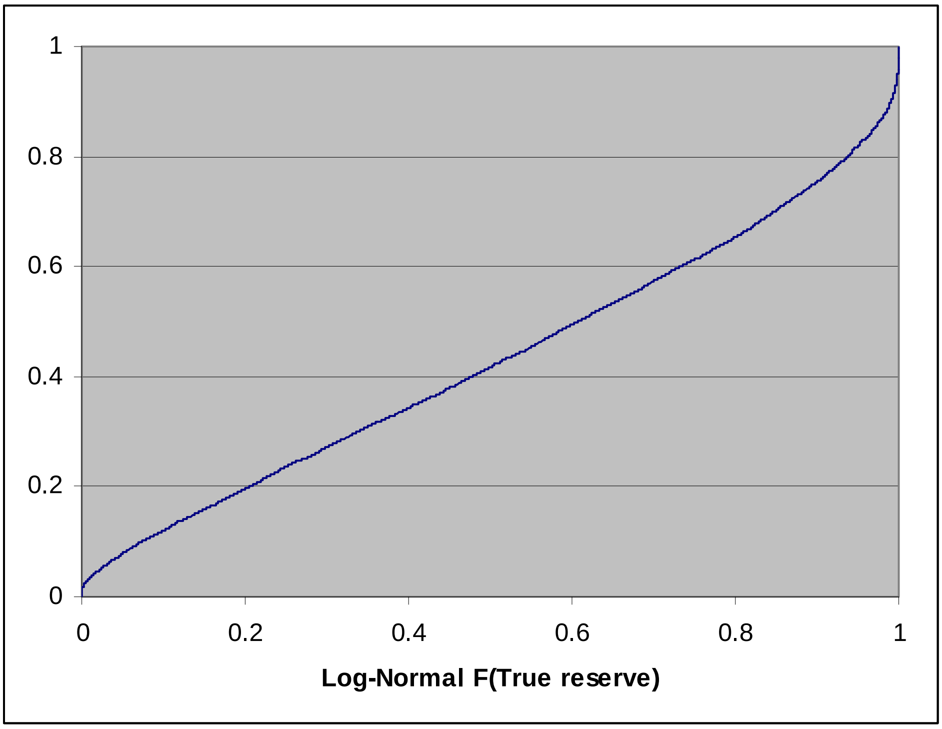
The figure 47.25% means that in 4,725 out of the 10,000 simulations, the BCL estimate was greater than the 'true' reserve (in the other 52.75% of simulations the BCL estimate was less than the true reserve). So BCL reserves are smaller than 'median' reserves in this case.

The fact that the mean predictive error (0.68) is such a small proportion (0.9%) of the mean estimated reserve (77.74) supports the belief that the basic chain ladder method is unbiased when the assumptions given by Mack hold true (ie, that BCL reserves are 'mean' reserves).

However, the fact that the mean standardised predictive error appears (at -0.60) to be significantly below zero, and its mean square (6.6) significantly greater than one, suggests that when the BCL gives an underestimate, the Mack standard error also tends to be understated. So it seems that Mack's method tends to understate the chance of extremely high outcomes. This is confirmed by analysis of the quantity F(true reserve) based on the Log-Normal predictive distribution.

The mean value of F(true reserve) being substantially higher than 0.50 indicates that this quantity is not uniformly distributed on the unit interval as it should be, but tends to be too high. In other words, it seems that actual outcomes (total future claim payments) tend to be higher than indicated by Mack's method.

We have investigated this further by looking at the empirical distribution function of this quantity which is shown below. The vertical axis shows the proportion of simulations in which F(true reserve) was less than the value shown on the horizontal axis. If this quantity were uniformly distributed (as it should be), then this graph should not differ significantly from a straight line from (0,0) to (1,1). Clearly it does: the shape confirms that Mack's method tends to understate reserve uncertainty, particularly the chance of extremely high outcomes (right end of graph below).



In 3,467 simulation out of the first 10,000 simulations (34.67%), the quantity F(true reserve) exceeds 0.80, and in 2,455 of these (24.55%) exceeds 0.90. If the Log-Normal distribution based on Mack's standard error were correct, these events should occur in only 20% and 10% of simulations respectively. Mack's method clearly tends to understate the chance of very high outcomes even when its underlying assumptions hold precisely (at least, this is so for the particular set of parameters selected to generate the artificial data).

At the other extreme (left hand end of the above graph): F(true reserve) is less than 0.20 in 19.58% of simulations and less than 0.10 in 12.08% of simulations, so Mack's method seems better at assessing the chances of very low outcomes than the chances of very high outcomes.

These and similar results are summarised below:

Table B-3 – Further results of applying Mack's method to data generated using Algorithm A

p	Chance that true out-turn exceeds $F^{-1}(1-p)$
1%	10.1%
5%	18.02%
10%	24.55%
20%	34.67%
30%	42.66%
50%	58.30%
70%	72.84%
80%	80.42%
90%	87.92%
95%	92.18%
99%	96.48%

To make absolutely clear what these results mean: the first row of the table shows there is approximately a 10% chance that the outcome will exceed the value that Mack's method (with a Log-Normal distribution for the reserve) indicates is the 99th percentile. In other words: while Mack's method indicates there is only a 1% chance that the outcome will exceed a certain value, there is actually a 10% chance.

B.1.3.3 Results for further sets of 10,000 simulations

To check that the above results are not unduly affected by sampling error, we repeated the calculations for several additional sets of 10,000 simulations, each time using the same parameters for the artificial data but a different seed for the random number generator. The results are summarised below: the first seven rows give mean values (over 10,000 simulations), the remaining rows give the actual proportion of simulations for which the 'true' ultimate exceeded the Mack percentile indicated.

Table B-4 – Results of applying Mack's method to data generated using Algorithm A

	Results from different sets of 10,000 simulations			
	Set 1	Set 2	Set 3	Set 4
BCL estimate	77.74	76.78	77.55	78.18
Indicator that (BCL > True)	47.25%	45.82%	46.44%	46.52%
(BCL estimate – True reserve)	0.68	-1.08	0.28	0.55
Mack standard error	27.88	27.41	27.88	28.46
(BCL - True) / (Mack std error)	-0.60	-0.66	-0.62	-0.63
Square of the above	6.6	7.7	5.2	5.7
Log-Normal F(true liability)	0.571	0.582	0.577	0.577
1%	10.1%	10.3%	10.5%	10.4%
5%	18.0%	18.8%	18.0%	18.9%
10%	24.6%	26.0%	25.6%	25.2%
20%	34.7%	35.9%	35.6%	35.0%
30%	42.7%	44.5%	43.6%	43.8%
50%	58.3%	59.7%	58.6%	59.2%
70%	72.8%	73.9%	73.5%	73.4%
80%	80.4%	81.0%	80.1%	80.8%
90%	87.9%	88.2%	87.7%	87.9%
95%	92.2%	92.1%	92.0%	92.0%
99%	96.5%	96.2%	96.2%	96.2%

These results show that the apparent anomalies in the initial 10,000 simulations do not result from sampling error: there is little difference between results from the four independent sets of 10,000 simulations.

B.1.3.4 Results based on alternative parameters in Algorithm A

The particular parameters that gave the above results are the first set of parameters that we tried for constructing artificial triangles satisfying Mack's assumptions. The use of a Log-Normal distribution with mean = variance = 1.0 to generate C_{j1} (Step 3 of the data generation algorithm) tends to produce greater variation between origin years than in a typical real-world triangle. To test the sensitivity of the results to this aspect, we repeated the simulations, but this time with the variance of the Log-Normal distribution used at Step 3 reduced to 0.01. This produces more realistic looking triangles. Results are summarised in Table B-5 below.

In this set of results, we have also calculated percentiles using a Normal predictive distribution (instead of a Log-Normal) in those simulations for which Mack's standard error is less than 50% of the BCL reserve (as suggested by Mack).

Table B-5 – Further results of applying Mack's method to data generated using Algorithm A

Variation coefficient of C_{j1} reduced from 100% to 10%.	Results from 10,000 simulations	
BCL estimate	78.0	
Indicator that (BCL > True)	48.1%	
(BCL estimate – True reserve)	0.93	
Mack standard error	29.52	
(BCL - True) / (Mack std error)	-0.51	
Square of the above	4.2	
F(true liability)	Log-Normal 0.569	Normal when (Std Error / BCL) < 50% 0.555
1%	8.4%	12.0%
5%	16.3%	18.2%
10%	22.5%	23.0%
20%	32.6%	30.7%
30%	41.2%	37.9%
50%	57.8%	53.0%
70%	73.9%	71.7%
80%	81.9%	82.2%
90%	90.0%	92.6%
95%	93.8%	97.0%
99%	97.8%	99.2%

This set of results (using the Log-Normal predictive distribution) shows a slight improvement compared to the previous results. Using the Normal distribution for predictions when the predictive coefficient of variation is less than 50% produces a less accurate assessment of the chances of extreme adverse outcomes: the proportion of simulations in which the true outcome exceeds the 99th percentile increases from 8.4% when the Log-Normal is always used to calculated the 99th percentile, to 12.0% when the 99th percentile is sometimes calculated using a Normal distribution. The main conclusions are unchanged: even where its assumptions are perfectly satisfied, Mack's method does not necessarily give a correct assessment of reserve uncertainty. For triangles constructed with the particular parameter values that we have used, Mack's method significantly understates the chance of very adverse outcomes.

B.1.3.5 Independent checking of the above results

Our main finding – that Mack's method can substantially understate the chance of extreme adverse outcomes even if its assumptions are perfectly satisfied – is clearly important given the quite widespread use of Mack's method and (with the advent of ICA and Solvency II) the increasing importance of accurately assessing the chances of extreme adverse outcomes. For this reason we carried out thorough checking of these results to ensure that they are genuine and not the result of errors in our implementation of Mack's method or in our simulation of triangles that satisfy Mack's assumptions.

To provide a final comprehensive check, another member of the working party, who was not involved in the work described above, carried out a completely independent simulation exercise (working only from Mack's 1993 paper and preceding sections of the present paper). All the simulation results described in preceding and later sections of the present paper were obtained using C++ programs (written by Tom Wright) to generate the artificial triangles and to apply Mack's method (and other stochastic methods) to those triangles. For the independent exercise, Mack's method and Algorithm A (as described above) were implemented by Gary Dunne in Miscrosoft Excel using VBA macros. This independent implementation was used to carry out a further 10,000 simulations with the same parameter values as those used (in the C++ programs) to produce the results in Table B-5 above. A Log-Normal predictive distribution was used in every simulation. The results are summarised below.

Table B-6 –Results obtained from an independent simulation exercise (Mack's method applied to data generated using Algorithm A, with same parameters as in Table B-5)

	Results from 10,000 simulations
BCL estimate	76.8
Indicator that (BCL > True)	47.3%
(BCL estimate – True reserve)	-0.34
Mack standard error	29.12
(BCL - True) / (Mack std error)	-0.53
Square of the above	3.7
1%	8.4%
5%	16.7%
10%	23.5%
20%	33.1%
30%	41.7%
50%	58.1%
70%	74.2%

80%	82.6%
90%	90.3%
95%	94.8%
99%	98.0%

Differences between the results in the above two tables are not statistically significant: they are consistent with sampling variation resulting from the two sets of results being based on independent sets of 10,000 artificial triangles. (This is clear from the fact that the differences are of the same order of magnitude as differences in results from different sets of 10,000 triangles shown in Table B-4).

Having obtained essentially the same results in two quite separate and independent simulation exercises, we are confident that these results are genuine.

B.1.3.6 Results for triangles where not all increments are positive

At Step 4 of Algorithm A, we chose initially to use shifted Log-Normals (such that the mean and variance after shifting were as required by Mack's assumptions) so that the artificial data had only positive increments. To test the sensitivity of the results to this aspect, we have also generated artificial triangles using Log-Normal distributions directly for the cumulative amounts C_{jk} at Step 4 (so the increments will sometimes be negative, as might occur in incurred data). Results for 10,000 simulations are shown below. Note that the triangles generated in this way still perfectly satisfy Mack's assumptions (as do all triangles analysed in Section 2.4.3).

Table B-7 – Further results of applying Mack's method to data generated using Algorithm A

Simulated data with negative increments.	Results from 10,000 simulations
BCL estimate	77.8
Indicator that (BCL > True)	47.2%
(BCL estimate – True reserve)	0.76
Mack standard error	55.4
(BCL - True) / (Mack std error)	-0.47
Square of the above	4.3
Log-Normal F(true liability)	0.582
1%	13.4%
5%	23.2%
10%	30.6%
20%	41.0%
30%	48.3%

50%	60.8%
70%	70.8%
80%	76.0%
90%	81.9%
95%	85.6%
99%	90.7%

These results show the performance is worse here than when Mack's method is applied to triangles that only have positive increments. Here, (with the possibility of negative increments) the method significantly understates the chances of extremely low outcomes as well as understating the chance of very high outcomes: in 9.3% of simulations the eventual outcome was lower than what was supposed to be the 1st percentile, and in 13.4% the eventual outcome was higher than what was supposed to be the 99th percentile.

B.1.3.7 Possible explanation of our findings

It is beyond the scope of this paper to definitively explain where Mack's method goes wrong. One possibility is that significant bias is introduced by taking non-linear functions of unbiased parameter estimates. Mack's formulas for the standard error have terms with f_k^2 in the denominator. Mack uses unbiased (chain ladder) estimators for f_k, but an unbiased estimate of f_k does not give an unbiased estimate of $1/f_k^2$. Given the relatively large number of parameters estimated from the data (9 f-parameters and 9 α-parameters from 55 data points), the reliability of some of these will be relatively low, which could result in substantial bias in non-linear functions (such as $1/f_k^2$). In addition, the square root of an unbiased estimate of the mean-square prediction error is not an unbiased estimate of the root-mean-square prediction error (or 'standard error') because square root is not a linear function. Further, even if the method did produce an unbiased estimate of the standard error, it would not necessarily give correct percentiles because the predictive distribution function is not linear (ie the predictive distribution is not uniform).

We expect to find that other stochastic methods suffer from similar problems, but this should be less so for Bayesian methods because these do not focus on unbiased estimation of parameters of the predictive distribution: instead they aim to calculate the predictive distribution function directly.

It would be interesting to investigate how Mack's method performs on triangles constructed (by Algorithm A) using a wider range of parameter values (ie different values of f_k, α_k, and parameters of the loss distribution).

B.1.4 Performance of Mack's method where its assumptions are false

Given the findings of the previous section, we should not expect that Mack's method will perform well where its underlying assumptions are false. In his 1993 paper Mack advises against applying the method where the assumptions are believed to be false, and gives suggestions for checking the assumptions using the data. However, these diagnostic checks may not always be carried out, and even where they are, they may show no clear evidence that the assumptions are false where this is in fact the case: this is quite possible given the small volume of data in many run-off triangles. Therefore, it is of interest to explore how the method performs when its assumptions are false.

Testing of Mack's method when its assumptions are false is carried out in later sections by applying Mack's method to artificial triangles generated to satisfy the assumptions of other stochastic methods.

B.2 Assessment of over-dispersed Poisson (ODP) methods by simulation

B.2.1 Assumptions of the ODP model

The over-dispersed Poisson (ODP) model (as described, for example by Renshaw and Verrall 1998) is based on the following three assumptions (we use Y_{jk} to denote the aggregate incremental amount paid in development period k of origin year j):

1. $E(Y_{jk}) = x_j.p_k$

2. $Var(Y_{jk}) = \varphi.x_j.p_k$

3. The Y_{jk} are stochastically independent (across all j and all k).

If the parameters p_k are normalised so that $\Sigma_k\ p_k = 1$, then they represent the proportion of the ultimate paid amount expected to fall in each development year k, and x_j represents the expected ultimate amount for origin year j (where all expectations here are those applying before any Y_{jk} have been observed).

Renshaw and Verrall's version of the ODP method uses quasi-likelihood maximisation to estimate the parameters of the model and produces 'best estimate' reserves that are identical (provided the run-off array has no missing values) to those produced by the basic chain ladder method. Renshaw and Verrall give formulas for predictive standard errors of the BCL reserves incorporating parameter and process error. England and Verrall (1999) and England (2001) also consider the ODP model, and they describe a bootstrap procedure that is also intended to give predictive standard errors incorporating parameter and process error. Here, we consider both these methods which we call respectively the 'analytic ODP method' and the 'bootstrap ODP method'. Both these methods are based on the same stochastic model, which is fully described by the above three assumptions.

B.2.2 Artificial data satisfying ODP assumptions (Algorithm B)

To test the ODP methods when the underlying assumptions are true, we generate artificial data that satisfy the above three assumptions. Since the assumptions say nothing about the distribution of the Y_{jk} other than the constraints on the first two moments represented by assumptions 1 and 2, we are free to use any distribution for this purpose. We have chosen to use a compound Poisson/Log-Normal distribution for each Y_{jk} because it is possible that actual paid run-off data may approximate this. In other words, each Y_{jk} is constructed as the sum of a number (N_{jk} say) of independent Log-Normal amounts, where the number N_{jk} is generated from a Poisson distribution. The parameters of the Log-Normal distribution used for the individual claim amounts are constant across all cells (j,k) of the run-off triangle: this ensures that the quantity $Var(Y_{jk})$ / $E(Y_{jk})$ is the same in all cells (as required by assumptions 1 and 2, where this ratio is denoted φ).

The algorithm we have used to generate the artificial data is as follows:

Algorithm B for artificial run-off data:

1. The ultimate number of claims in an origin year is generated by random sampling from a Poisson distribution (same parameters for each origin year, but independent sampling).
2. Each claim is assumed to be settled by a single payment, and the development year of the payment determined by independent random sampling from a Multinomial distribution (same parameters for each origin year: these are the parameters denoted p_k in the above assumptions of the ODP method).
3. The amount of each individual claim payment is determined by independent random sampling from a Log-Normal distribution (same parameters in every cell of the triangle).
4. The amounts of claims settling in the upper left triangle of the run-off array are accumulated to create this run-off triangle, and all claim amounts (regardless of the development year when settled) are accumulated to obtain the 'true' ultimate position for each origin year.

It is perhaps not immediately obvious that Steps 1 and 2 produce stochastically independent Poisson numbers (N_{jk}) across all development years of a given origin year (which is required by Assumption 3 of the ODP method). However, this is quite a well known result: a proof is given in the last section of this appendix, which also shows that the constant φ of Assumption 2 is related to the mean μ and variance σ^2 of the loss distribution used at Step 3 by $\varphi = (\mu^2 + \sigma^2) / \mu$.

Algorithm B is clearly simpler in many ways than what typically occurs in reality, but is nevertheless more realistic than Algorithm A (used to generate data strictly in accordance with Mack's assumptions). For this reason, as well as using data generated by Algorithm B to test the performance of the ODP methods where their assumptions are true, we have used the same artificial data to test Mack's method where its assumptions are false. (It is easily proved that triangles generated using Algorithm B do not satisfy Mack's assumptions – see the last section of this appendix.)

B.2.3 Performance of analytic ODP method when its assumptions are true

We carried out three sets of 10,000 independent simulations. In each simulation, a run-off triangle was generated using Algorithm B, and the analytic ODP method applied to produce reserve estimates (equal to BCL estimates) and root-mean-square prediction errors (called 'standard errors' below) calculated as described by Renshaw and Verrall 1998. Again, we looked at only the total reserve for all years combined (rather than the estimates for separate origin years). In quasi-likelihood modelling, there are two commonly used methods for estimating the dispersion parameter (the parameter φ of ODP Assumption 2): using the sum of squared Pearson residuals, or using the deviance. As there is little theoretical basis for favouring one over the other, we have tested both methods: in each set of results below, the first column relates to the Pearson residual method, the second column to the deviance method. Since the ODP method does not produce a full predictive distribution, we have used a Log-Normal distribution fitted to the best estimate and standard error to obtain percentiles (in the same way as described earlier for Mack's method). Results are summarised below.

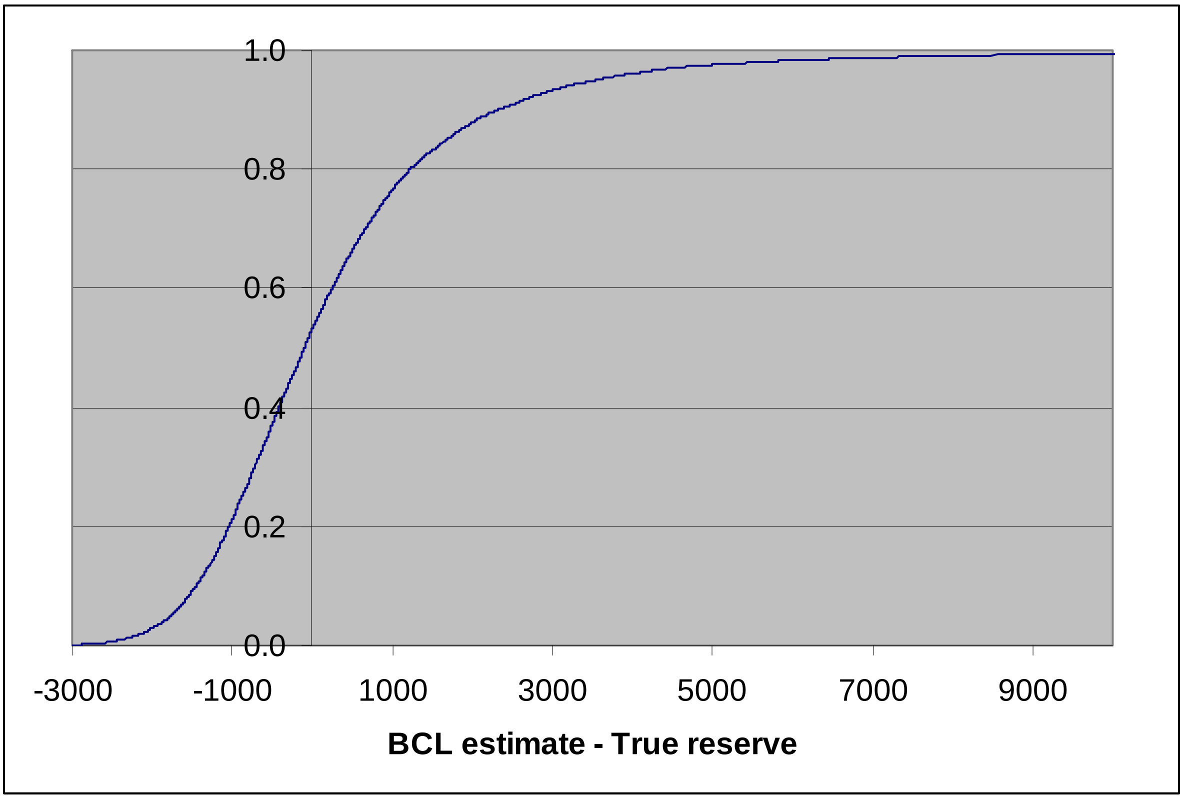
Table B-8 – Results of applying the analytic ODP method to data generated using Algorithm B

	Results from different sets of 10,000 simulations					
	Set 1		Set 2		Set 3	
Best estimate reserve	3,668		3,654		3,632	
Best estimate > True	46.9%		46.8%		46.4%	
(Best estimate – True)	303.7		292.7		268.4	
ODP standard error	1,511	1,491	1,514	1,494	1,502	1,482
(Best - True) / (std error)	-0.271	-0.276	-0.263	-0.267	-0.280	-0.285
Square of the above	1.48	1.51	1.46	1.50	1.45	1.49
Log-Normal F(True)	0.571	0.570	0.568	0.568	0.574	0.573
1%	2.4%	2.6%	2.7%	2.7%	2.6%	2.7%
5%	9.5%	9.8%	9.1%	9.3%	9.9%	10.2%
10%	17.1%	17.4%	16.4%	16.7%	17.2%	17.4%
20%	29.8%	30.0%	29.0%	29.2%	29.7%	29.8%
30%	41.0%	41.1%	40.3%	40.4%	41.1%	41.1%
50%	60.4%	60.2%	60.3%	60.1%	60.9%	60.8%
70%	77.3%	77.1%	76.6%	76.3%	77.6%	77.3%
80%	84.1%	83.8%	84.1%	83.8%	84.8%	84.5%
90%	90.8%	90.7%	91.0%	90.8%	91.4%	91.2%
95%	94.4%	94.4%	94.8%	94.7%	94.7%	94.6%
99%	97.9%	97.8%	98.1%	98.1%	98.2%	98.2%

These results show that the analytic ODP method is biased upwards when applied to triangles generated using Algorithm B. Over all 30,000 simulations the ODP reserve overstates the true reserve by about 7.9% of the mean true reserve, and this occurs fairly consistently across the three sets of 10,000 simulations. Since the best estimate reserves produced by the ODP method are identical to those produced using the basic chain ladder (BCL) method, this means that the BCL method is biased upwards when applied to data of this type. The results in Table B-8 also show that, although the BCL estimate is positively biased, it exceeds the true reserve in only about 47% of simulations. It would be interesting to investigate to what extent this positive bias of the BCL depends on the particular parameter values used in generating the triangles.

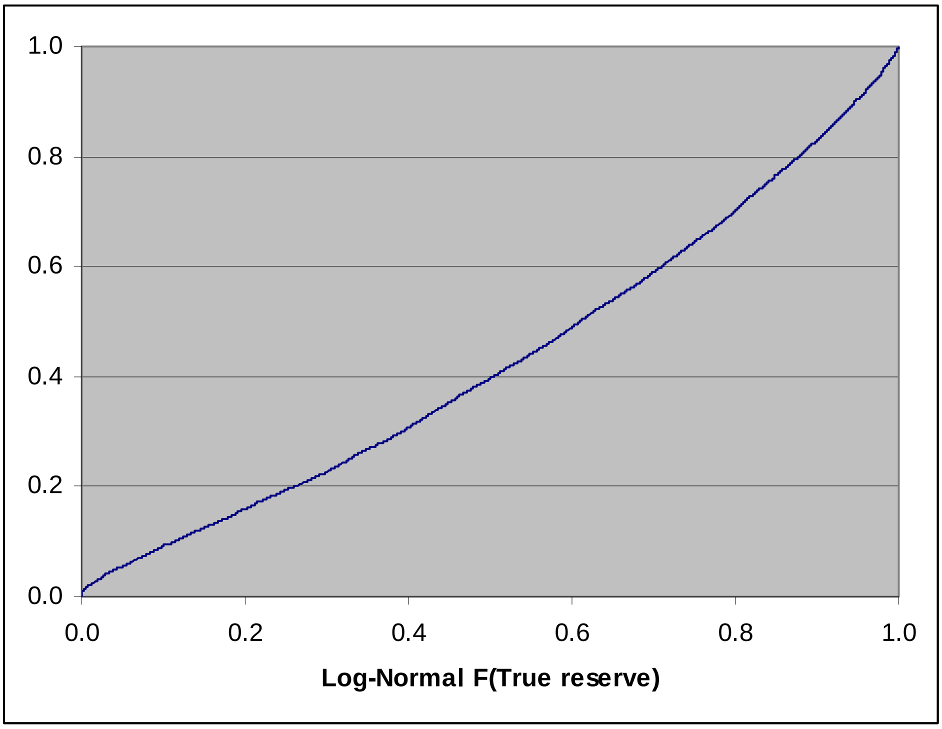
The fact that there is bias should not be surprising given that the assumptions Mack showed to be necessary for unbiasedness of the BCL are known to be violated here. The bias of the BCL method when applied to triangles with independent increments has previously been noted and studied by Stanard (1985) (who used a numerical simulation method similar to that described here) and by Taylor (2001 and 2002) who derived approximate formulas for the bias.

The positive bias of the BCL (and the less than 50% chance that the BCL estimate exceeds the true reserve) is evident from the following graph, which is the cumulative distribution function of the quantity (BCL estimate – True reserve):



The negative mean value of the 'standardised predictive error' implies (given that the 'best estimate' is positively biased) that when the method understates the true outcome, the analytic ODP standard error also tends to be understated.

The distribution of the quantity F(true reserve) (where F(x) is the Log-Normal predictive distribution) shows that this method understates the chance of very adverse outcomes. For example, in about 2.6% of simulations the true outcome exceeded what was supposed to be the 99^{th} percentile. The full empirical distribution function of this quantity for the first set of 10,000 simulations is shown below: it clearly differs significantly from a uniform distribution (which would be a straight line from (0,0) to (1,1)).



The analytic ODP method appears to perform slightly better when the dispersion parameter is estimated from the Pearson residuals (first column for each set of 10,000 simulations) than when it is estimated from the deviance (second column).

B.2.4 Performance of bootstrap ODP method when its assumptions are true

B.2.4.1 Two variants of bootstrap ODP method

We have tested the performance of two variants of the bootstrap ODP method as described by England & Verrall (1999) and England (2001).

In their 1999 paper England & Verrall describe a bootstrap procedure that is intended to give a probability distribution in respect of BCL parameter uncertainty only (ie not including future process uncertainty). They point out that the predictive standard error (including process as well as parameter uncertainty) can be obtained by adding the estimated process variance to the variance of the bootstrap distribution (representing parameter uncertainty only) then taking the square root. They propose estimating the process variance as φ times the BCL reserve, where φ is the estimated dispersion parameter of the ODP assumption. They found that, in the case of one particular triangle, this method gave a predictive standard error close to that given by the analytic ODP method. The 1999 paper does not describe a procedure for obtaining a full predictive probability distribution incorporating process and parameter uncertainty.

In his 2001 paper, England outlines an ODP bootstrap procedure for obtaining a full predictive probability distribution (incorporating both parameter and process uncertainty). In this procedure, the allowance for future process variation is based on BCL projections from bootstrap pseudo-data (not on BCL projections of the original data as in the 1999 paper) and is simulated by adding process variation to each future cell of the pseudo-data triangle. In their 2002 paper, England and Verrall applied this method to one particular triangle and obtained a predictive standard error somewhat higher than given by the analytic ODP method: the values were 19,267 and 18,193 respectively (which they described as close). They also found that the mean of the bootstrap predictive distribution was higher than the BCL reserve: the values were 53,210 and 52,135 (described as reassuringly close).

We have tested both these variants of the bootstrap ODP method: referred to below as the “1999 bootstrap ODP method” and “2001 bootstrap ODP method”.

Our implementations of these methods are exactly as described in the papers by England and Verrall except in one respect: we ensure that all pseudo data triangles have no negative cumulative paid amounts. This is done by applying the algorithm to produce each pseudo triangle exactly as described by England and Verrall, but then rejecting any triangle that has any negative cumulatives. Each time a triangle is rejected another one is created and the final bootstrap predictive distribution is based on the outcomes from 1,000 pseudo-triangles in which all cumulative amounts are positive (1,000 is the number of bootstrap simulations suggested by England and Verrall). The reason we have made this modification is that we found the performance of the bootstrap method is sometimes extremely poor if pseudo-data triangles with negative cumulatives are allowed.

For example, consider the following cumulative run-off triangle. This is an artificial triangle generated using Algorithm B. The final column gives basic chain ladder (BCL) ultimates.

11.1	122.1	206.5	353.6	534.7	547.0	581.9	621.4	802.3	807.8
22.1	79.0	902.4	1,037.5	1,450.9	1,521.3	1,556.5	1,604.3	1,604.8	1,615.9
22.7	115.1	233.4	408.6	546.0	616.8	692.8	712.0		775.3
115.5	340.3	491.4	622.6	716.5	870.9	954.0			1,077.9
0.0	324.2	488.3	810.7	996.7	1,073.5				1,291.2
105.1	222.8	392.5	629.1	743.9					975.8
18.2	120.2	365.3	452.1						766.0
34.3	386.7	461.5							1,095.3
130.7	249.4								1,225.5
90.4									1,892.9

The triangle below is a pseudo-data set produced by applying the bootstrap algorithm described by England and Verrall (1999) to the above triangle. The final column gives the BCL ultimates for this pseudo triangle.

32.248	242.0	481.5	666.0	893.3	937.1	1,029.6	1,116.9	1,117.2	1,115.2
2.194	323.8	607.6	790.2	1,136.9	1,166.7	1,542.3	1,646.7	2,109.3	2,105.5
-4.485	-50.4	231.9	462.7	554.2	584.4	654.3	661.6		771.0
-42.552	85.5	215.7	498.7	505.8	728.7	820.0			1,014.5
32.145	95.6	232.1	381.9	528.5	617.7				905.1
-45.767	142.1	204.4	299.0	520.3					850.0
-10.300	97.9	220.4	297.1						648.4
27.423	176.9	517.4							1,747.8
9.118	133.5								1,098.4
192.355									77,700,000.0

There is nothing to prevent negative increments in pseudo data-triangles generated as described by England and Verrall, and negative increments may produce negative cumulatives. In this example, the pseudo data has several negatives in the first development period. As a result, the first BCL development factor is extremely large: (1246.9 / 0.025 = 49,097.8) which produces a very high BCL ultimate for the last origin year (77.7 million). If something like this occurs in one of 1,000 bootstrap simulations, it can have a massive impact on both the mean and the variance of the bootstrap distribution. Given that the original triangle has only positive increments, it would in most circumstances be unrealistic to allow for such extreme possibilities in bootstrap simulations. For this reason, in our implementation of the bootstrap methods, we reject any pseudo triangles in which the cumulative amounts are not all positive. Pseudo triangles with negative increments are accepted provided all cumulative figures are positive.

To check that our bootstrap procedures are correct implementations of the methods described by England and Verrall we replicated their results as given in the table on page 293 of their 1999 paper and in Appendix A of England 2001. Our bootstrap results differed from theirs by an amount consistent with bootstrap sampling error. The modification described above (rejection of pseudo-triangles with negative cumulatives) makes no difference to the results obtained for the particular two triangles analysed in the papers by England and Verrall.

For each of the two variants of the bootstrap ODP method (1999 and 2001 variants) we have carried out three sets of 10,000 independent simulations, each based on a run-off triangle generated using Algorithm B. For each of the 30,000 artificial run-off triangles, we used 1,000 bootstrap simulations (as suggested by England & Verrall).

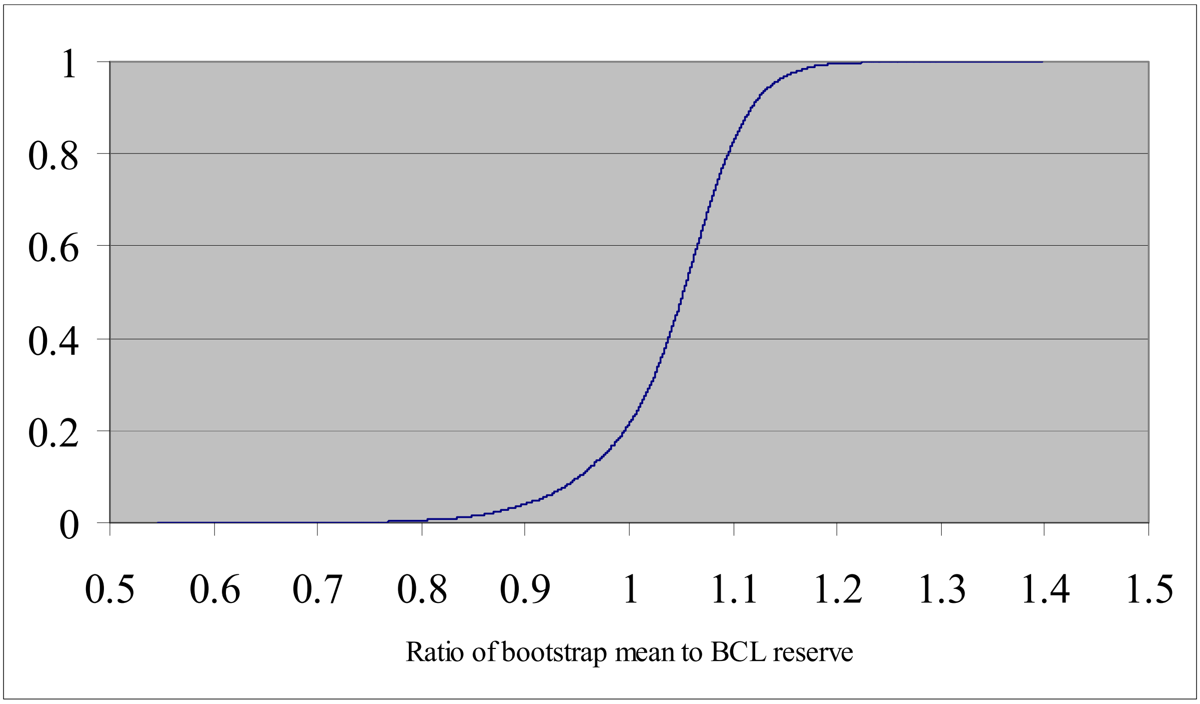
B.2.4.2 Performance of 1999 bootstrap ODP method when its assumptions are true

Table B-9 below gives results of applying the 1999 bootstrap ODP method to data generated using Algorithm B. The 1999 bootstrap ODP method does not give a full predictive distribution so we have used a Log-Normal with mean equal to the BCL reserve and variance equal to the predictive variance (calculated as the bootstrap variance representing parameter uncertainty, plus φ times the BCL reserve). Note that the mean of the Log-Normal predictive distribution was set equal to the BCL estimate (that is, the ODP best estimate), not to the mean of the bootstrap distribution, because our purpose here is to assess the performance of published methods. Although England & Verrall do not suggest the use of a Log-Normal predictive distribution, they make it clear that the predictive variance calculated as described above is supposed to be the predictive variance of the ODP best estimate reserve (ie the BCL reserve): they ignore the difference between the BCL reserve and the mean of the bootstrap distribution on the grounds that these two quantities are close.

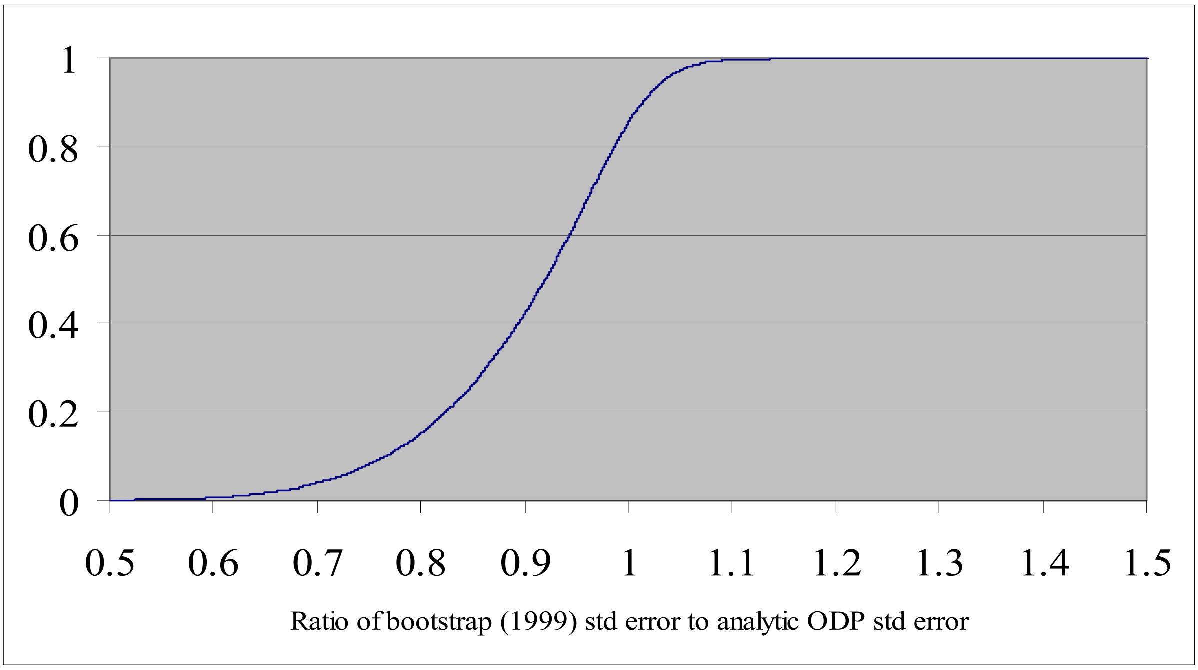
Table B-9 – Results of applying 1999 bootstrap ODP method to data generated using Algorithm B

	Results from different sets of 10,000 simulations			
	Set 1	Set 2	Set 3	All combined
BCL estimate	3,668	3,654	3,632	3,651
Best estimate > True	46.9%	46.8%	46.4%	46.7%
(BCL estimate – True reserve)	303.7	292.7	268.4	288.3
Bootstrap mean	3,727	3,719	3,695	3,714
Bootstrap standard error	1,304	1,302	1,295	1,300
(BCL - True) / (BS std error)	-0.251	-0.242	-0.261	-0.251
Square of the above	1.727	1.722	1.703	1.717
Log-Normal F(true liability)	0.559	0.556	0.562	0.559
1%	3.1%	3.1%	3.0%	3.1%
5%	10.6%	10.2%	11.1%	10.6%
10%	18.4%	17.8%	18.6%	18.3%
20%	31.1%	30.0%	30.8%	30.6%
30%	41.4%	40.8%	41.4%	41.2%
50%	58.8%	58.6%	59.3%	58.9%
70%	74.3%	73.6%	74.4%	74.1%
80%	80.9%	80.6%	81.5%	81.0%
90%	87.7%	87.8%	88.2%	87.9%
95%	91.8%	91.7%	92.1%	91.9%
99%	96.1%	96.4%	96.4%	96.3%

These results show that there is in fact a statistically significant difference between the BCL reserve and the mean of the bootstrap distribution in this case. Over all 30,000 artificial datasets, the mean of the bootstrap distribution exceeds the mean BCL reserve by about 1.7%. This difference occurs consistently across all three sets of 10,000 triangles, showing that it is statistically significant (ie is not just caused by bootstrap sampling error). The graph below shows the cumulative probability distribution (over all 30,000 triangles) for the ratio of bootstrap mean to BCL reserve. The bootstrap mean exceeds the BCL reserve for approximately 79% of the triangles we generated using Algorithm B.



Comparing Tables B-8 and B-9 we see that the 1999 bootstrap standard error tends to be lower than the analytic standard error, and percentiles based on the bootstrap standard error are less accurate. The graph below shows the cumulative probability distribution (over all 30,000 triangles) for the ratio of bootstrap standard error to analytic ODP standard error. The bootstrap standard error is smaller than the analytic standard error for approximately 86% of the triangles generated using Algorithm B.



In summary it seems that the bootstrap ODP method (as described by England & Verrall 1999) does not perform quite as well as the analytic ODP method when the ODP assumptions are perfectly satisfied. (At least, this is the case for triangles generated using the particular set of parameters in Algorithm B that we used here.)

B.2.4.3 Performance of 2001 bootstrap ODP method when its assumptions are true

The table below gives results of applying the 2001 bootstrap method to data generated using Algorithm B. The 2001 bootstrap method does give a full predictive distribution so we have not used a Log-Normal distribution for percentiles: we have calculated percentiles directly from the full predictive distribution given by the bootstrap method.

Table B-10 – Results of applying 2001 bootstrap ODP method to data generated using Algorithm B

	Results from different sets of 10,000 simulations			
	Set 1	Set 2	Set 3	All combined
BCL estimate	3,668	3,654	3,632	3,651
Best estimate > True	46.9%	46.8%	46.4%	46.7%
(BCL estimate – True reserve)	303.7	292.7	268.4	288.3
Bootstrap mean	3,727	3,719	3,695	3,714
Bootstrap standard error	1,307	1,309	1,299	1,305
(BCL - True) / (BS std error)	-0.245	-0.237	-0.255	-0.246
Square of the above	1.701	1.692	1.670	1.688
F(true liability)	0.527	0.523	0.530	0.527
1%	2.6%	2.8%	2.5%	2.6%
5%	8.3%	8.0%	8.4%	8.2%
10%	14.3%	13.7%	14.5%	14.2%
20%	24.8%	24.4%	25.3%	24.8%
30%	34.8%	34.2%	35.3%	34.8%
50%	53.9%	53.4%	54.4%	53.9%
70%	71.8%	71.2%	71.9%	71.6%
80%	80.1%	79.8%	80.8%	80.2%
90%	88.6%	88.4%	89.0%	88.7%
95%	93.0%	93.1%	93.2%	93.1%
99%	97.3%	97.5%	97.6%	97.5%

The bootstrap 2001 standard error is smaller than the analytic ODP standard error in 82% of simulations. However, comparing the above results with Table B-8 we see that the assessment of uncertainty for adverse outcomes, while not perfect, is about as good as provided by the analytic standard error. (Using either method the true outcome exceeds what is supposed to be the 99th percentile in approximately 2.6% of cases.) Since the standard error tends to be smaller but does not understate the chance of extreme adverse events any more than the analytic standard error, the 2001 bootstrap method appears to be the better method for assessing the reliability of BCL reserves when the ODP assumptions are perfectly satisfied. This may be because the method gives a full predictive distribution so it is not necessary to use an analytic predictive distribution (such as the Log-Normal) which may not have an appropriate shape at the extremes.

B.2.5 Performance of Mack's method when its assumptions are false

We applied Mack's method to the same artificial data (generated using Algorithm B) as used to test the performance of the ODP method when its assumptions are true. It is easily shown that data generate in this way do not satisfy Mack's assumptions (see the last section of this appendix). Results are summarised below. The final column gives results for all 30,000 simulations combined.

Table B-11 – Results of applying Mack's method to data generated using Algorithm B

	Results from different sets of 10,000 simulations			
	Set 1	Set 2	Set 3	All combined
BCL estimate	3,668	3,654	3,632	3,651
Best estimate > True	46.9%	46.8%	46.4%	46.70%
(BCL estimate – True reserve)	303.7	292.7	268.4	288.3
Mack standard error	1,961	1,960	1,951	1,957
(BCL - True) / (Mack std error)	-0.222	-0.210	-0.227	-0.220
Square of the above	1.11	1.10	1.08	1.10
Log-Normal F(true liability)	0.586	0.583	0.589	0.586
1%	1.4%	1.5%	1.4%	1.4%
5%	6.8%	6.4%	6.7%	6.6%
10%	13.1%	12.8%	13.5%	13.1%
20%	27.2%	26.5%	27.4%	27.0%
30%	41.2%	40.5%	41.2%	41.0%
50%	64.7%	64.4%	64.5%	64.5%
70%	81.4%	81.0%	82.1%	81.5%
80%	87.8%	87.6%	88.5%	88.0%
90%	93.4%	93.5%	93.7%	93.5%
95%	96.0%	96.2%	96.3%	96.2%
99%	98.3%	98.5%	98.4%	98.4%

The best estimate reserve (and the difference between the best estimate and true reserve) is the same as shown in the previous Table B-10: this is because the ODP method and Mack's method give identical best estimates when fitted to triangles with no missing values (both give the BCL estimates).

The negative mean value of the 'standardised predictive error' (-0.220) implies (given the positive bias) that when the BCL understates the true outcome, the Mack standard error also tends to be understated. The same occurred when Mack's method was run on triangles that satisfy its assumptions.

Despite the positive bias of the BCL estimates (or perhaps because of this bias), Mack's standard errors appear to give a better assessment of overall predictive error here than in our simulations on triangles that satisfy Mack's assumptions: the mean square of the 'standardised predictive error' is not much greater (at 1.10 over all 30,000 simulations) than the ideal value of one.

The distribution of the quantity F(true reserve) (where F(x) is the Log-Normal predictive distribution) shows that the method slightly understates the chance of very adverse outcomes. For example, in 1.4% of simulations the true outcome exceeded the 99^{th} percentile as given by Mack's method. Clearly the performance of Mack's method here is much better than on triangles constructed (using Algorithm A) to perfectly satisfy Mack's assumptions!

B.2.6 Performance of ODP method when its assumptions are false

For completeness, we have also tested the ODP methods (analytic and bootstrap) on data produced using Algorithm A (satisfying the assumptions of Mack's method). It is easily proven that data generated in this way do not satisfy the assumptions of the ODP method (see the last section of this appendix). Results (based on the same 10,000 data-sets as used for the results in Table B-4) are given below. The results obtained using Mack's method are repeated here for comparison. As before, the results in the first column for the analytic ODP method are based on estimating the dispersion parameter using the Pearson residuals, and the second column using the deviance. For the bootstrap ODP results, the first column relates to the 1999 method, the second column to the 2001 method.

Table B-12 – Results of applying Mack and ODP methods to data generated using Algorithm A

	Mack	analytic ODP		bootstrap ODP	
BCL estimate	78.0	78.0		78.0	
BCL estimate > True reserve	48.1%	48.1%		48.1%	
(BCL estimate – True reserve)	0.93	0.93		0.93	
Standard error	29.52	30.8	31.02	26.81	27.07
(BCL - True) / (Std error)	-0.51	-0.40	-0.40	-0.41	-0.41
Square of the above	4.2	2.9	2.8	3.5	3.5
F(true liability)	0.569	0.569	0.570	0.558	0.526
1%	8.4%	5.4%	5.4%	6.5%	5.9%
5%	16.3%	12.8%	12.7%	14.7%	13.7%

10%	22.5%	19.1%	19.0%	21.2%	20.1%
20%	32.6%	30.7%	30.6%	32.0%	29.9%
30%	41.2%	40.5%	40.5%	40.9%	38.1%
50%	57.8%	59.1%	59.1%	57.4%	52.9%
70%	73.9%	75.7%	75.9%	72.7%	68.3%
80%	81.9%	83.6%	84.0%	79.9%	75.6%
90%	90.0%	91.1%	91.4%	87.8%	84.5%
95%	93.8%	94.5%	94.8%	92.0%	89.8%
99%	97.8%	97.8%	97.9%	96.3%	95.4%

Although these results indicate that the analytic ODP methods perform better than Mack's method when applied to triangles that satisfy Mack's assumptions (and the analytic ODP method performs better than the bootstrap ODP methods), the results also show that the ODP methods do not perform particularly well on this type of data. However, it is difficult to imagine any real-life claims settlement process that would approximate Algorithm A: we think it is of greater value to test the ODP method on data that violate the assumptions of the method in more realistic ways. For this purpose, we have generated data by Algorithm C below. This is just like algorithm B except that the Negative Binomial distribution is used at Step 1 instead of the Poisson. It is easily shown (see the last section of this appendix) that data generated in this way do not have independent increments (Y_{jk}) so Assumption 3 of the ODP method is violated. (In fact, the Y_{jk} values generated in this way are positively correlated within any origin year j).

Algorithm C for artificial run-off data:

1. The ultimate number of claims in an origin year is generated by random sampling from a Negative Binomial distribution (same parameters for each origin year, but independent sampling).
2. Each claim is assumed to be settled by a single payment, and the development year of the payment determined by independent random sampling from a Multinomial distribution (same parameters for each origin year: these are the parameters denoted p_k in the above assumptions of the ODP method).
3. The amount of each individual claim payment is determined by independent random sampling from a Log-Normal distribution (same parameters in every cell of the triangle).
4. The amounts of claims settling in the upper left triangle of the run-off array are accumulated to create this run-off triangle, and all claim amounts (regardless of the development year when settled) are accumulated to obtain the 'true' ultimate position for each origin year.

The table below gives results for the analytic ODP method. As before, for each set of 10,000 simulations, the first column gives results based on estimating the dispersion parameter using the Pearson residuals, and the second column using the deviance (this has been done for the first set of 10,000 simulated triangles only). For the bootstrap ODP results, the first column relates to the 1999 method, the second column to the 2001 method.

Table B-13 – Results of applying analytic ODP method to data generated using Algorithm C

	Results from different sets of 10,000 simulations					
	Set 1		Set 2		Set 3	
Best estimate reserve	3,612		3,600		3,574	
Best estimate > True	45.5%		45.8%		44.8%	
(Best estimate – True)	244.1		229.0		211.8	
ODP standard error	1,487	1,467	1,489		1,483	
(Best - True) / (std error)	-0.31	-0.31	-0.31		-0.32	
Square of the above	1.53	1.56	1.51		1.48	
Log-Normal F(True)	0.579	0.579	0.580		0.584	
1%	2.6%	2.8%	2.8%		2.5%	
5%	10.2%	10.3%	9.8%		9.7%	
10%	17.7%	17.9%	17.3%		17.7%	
20%	30.8%	31.0%	30.7%		30.7%	
30%	42.4%	42.3%	42.4%		42.8%	
50%	61.8%	61.6%	61.6%		62.3%	
70%	77.6%	77.4%	78.2%		78.3%	
80%	84.9%	84.6%	85.0%		85.4%	
90%	91.5%	91.4%	91.9%		92.0%	
95%	94.9%	94.8%	95.0%		95.3%	
99%	98.1%	98.1%	98.2%		98.3%	

Table B-14 below gives results for the bootstrap ODP method. As before, for each set of 10,000 simulations, the first column relates to the 1999 method, the second column to the 2001 method.

Table B-14 – Results of applying bootstrap ODP methods to data generated using Algorithm C

	Results from different sets of 10,000 simulations					
	Set 1		Set 2		Set 3	
Best estimate reserve	3,612		3,600		3,574	
Best estimate > True	45.5%		45.8%		44.8%	
(Best estimate – True)	244.1		229.0		211.8	
Bootstrap mean	3,675	3,676	3,668	3,668		
ODP standard error	1,284	1,289	1,284	1,290		
(Best - True) / (std error)	-0.287	-0.281	-0.289	-0.284		
Square of the above	1.77	1.75	1.76	1.73		
F(True)	0.568	0.535	0.549	0.535		
1%	3.2%	2.8%	3.3%	2.9%		
5%	11.3%	8.8%	11.1%	8.7%		
10%	18.8%	14.7%	18.8%	14.7%		
20%	31.9%	25.6%	31.8%	25.8%		
30%	42.7%	36.2%	42.8%	36.1%		
50%	60.4%	55.2%	60.1%	55.4%		
70%	74.8%	72.6%	75.1%	72.4%		
80%	81.5%	80.9%	81.7%	80.7%		
90%	88.6%	89.4%	88.5%	89.4%		
95%	92.2%	93.5%	92.3%	93.7%		
99%	96.3%	97.6%	96.5%	97.7%		

These results show that the ODP methods performs nearly as well here as where the underlying assumptions are satisfied (ie when applied to data generated using Algorithm B). The results for the analytic ODP method again show slightly better performance when the dispersion parameter is estimated from Pearson residuals (first column) than when it is estimated from the deviance (second column). These results also show that the 1999 bootstrap ODP method performs worse than the analytic method, but the 2001 bootstrap method performs slightly better than the analytic method (which is the same ranking as when the ODP assumptions are satisfied). Clearly more testing is needed to see whether the performance of the analytic ODP method is generally insensitive to the ultimate claim number distribution.

B.2.7 Further results for Mack's method when its assumptions are false

We have also tested Mack's method on the same data (generated using Algorithm C) and the results (below) again show that it performs surprisingly well given its poor performance on data that satisfies its assumptions precisely.

Table B-15 – Results of applying Mack's method to data generated using Algorithm C

	Results from different sets of 10,000 simulations			
	Set 1	Set 2	Set 3	All combined
BCL estimate	3,612	3,600	3,574	3,595
BCL estimate > True reserve	45.5%	45.8%	44.8%	45.4%
BCL estimate – True reserve	244.1	229.0	211.8	228.3
Mack standard error	1,917	1,923	1,903	1,914
(BCL - True) / (Mack std error)	-0.248	-0.249	-0.256	-0.251
Square of the above	1.15	1.14	1.12	1.14
Log-Normal F(true liability)	0.593	0.594	0.597	0.595
1%	1.7%	1.7%	1.5%	1.6%
5%	6.8%	7.1%	6.8%	6.9%
10%	13.8%	13.8%	13.9%	13.8%
20%	27.9%	28.1%	28.0%	28.0%
30%	42.4%	42.4%	43.1%	42.6%
50%	65.8%	65.4%	65.8%	65.7%
70%	82.0%	82.2%	82.4%	82.2%
80%	88.5%	88.4%	88.9%	88.6%
90%	93.7%	93.9%	94.0%	93.9%
95%	96.4%	96.4%	96.6%	96.5%
99%	98.5%	98.6%	98.6%	98.6%

B.3 Summary of results so far for Mack and ODP methods

Both Mack's method and the ODP methods can be applied to any triangle and give exactly the same reserve estimates as the basic chain ladder (BCL) method. However, the two sets of assumptions (those underlying Mack's method and those underlying the ODP methods) cannot both hold true. If Mack's assumptions hold true, then the BCL estimates are unbiased, but Mack's standard errors tend to understate the chance of extreme outcomes: substantially in the simulations we have carried out. If the ODP assumptions hold true, then the BCL estimates usually understate the true ultimates but when they overstate they do so on average by much more than the average understatement, so that overall the BCL estimates are positively biased (by about 8% in simulations carried out so far). ODP standard errors (whether calculated analytically or by bootstrapping) understate the chances of extreme outcomes when the ODP assumptions are perfectly satisfied. Although the analytic and bootstrap methods of calculating ODP standard errors sometimes give very different results for the same triangle, there is little difference in the overall performance of the two methods in assessing reliability of BCL estimates when the ODP assumptions hold true.

Given its poor performance when its assumptions are true, Mack's method performs surprisingly well in the scenarios we have tested where its assumptions are false. Curiously, in the simulations carried out so far: Mack's method performs better than the ODP methods when applied to triangles that satisfy the ODP assumptions (Algorithm B), and the ODP methods perform better than Mack's method when applied to triangles that satisfy Mack's assumptions (Algorithm A).

Further results for Mack's method and the ODP methods applied where their assumptions are false are given later (by applying these methods to triangles that satisfy the assumptions of Wright's method).

B.4 Assessment of Wright's 1992 (operational time) method

B.4.1 Assumptions of Wright's method

The original paper (Wright 1992) describes several related stochastic methods for use in various different circumstances. What they all have in common is that they are average-cost-per-claim methods in which the mean and variance of individual claim payments are assumed to be functions of 'operational time' (the same functions of operational time across all origin years). Operational time is defined as the proportion of the ultimate number of claims that have been closed, so it increases from zero at the start of an origin year to one when a year reaches full development.

Given a development triangle of aggregate claim amounts, and a corresponding triangle of the numbers of claims closed, the original paper describes how generalized linear modelling can be used to determine the relationship between mean claim amount and operational time, and how this can be used to forecast the total of future payments and its root-mean-square predictive error (allowing for both future process variation and parameter estimation uncertainty).

Some of the different circumstances covered by the original paper are:

- The individual payment amount distribution can be assumed to have the same coefficient of variation (ratio of standard deviation to mean) at all operational times, or this can be assumed to vary across operational times in a defined way.
- Claims inflation can be assumed to be absent (on the grounds that the claim amounts triangle has been pre-adjusted perhaps) or the average rate of claims inflation can be estimated as one of the parameters of the generalized linear model.
- The ultimate number of claims can be assumed known with certainty (as would be the case with reporting year cohorts or if there were no reporting delays in accident year cohorts) or uncertainty in ultimate numbers of claims can be taken into account.
- Each claim payment in the amounts triangle can assumed to be separately counted in the claim numbers triangle (as when every claim is settled with a single payment) or there may be partial payments in the amounts triangle that are not separately counted in the claim numbers triangle.

To date, we have tested the method by numerical simulation in only the simplest of these situations:

- The coefficient of variation of individual claim payments is the same at all operational times.
- There is no claims inflation.
- The ultimate number of claims is fully known for each origin year.
- All claims are settled by a single payment (so each payment contributing to the aggregate claim amounts triangle is counted in the triangle of numbers of claims closed).

B.4.2 Artificial data satisfying Wright's assumptions (Algorithms D and E)

To test the performance of the method when these assumptions are true, we generated artificial run-off datasets as follows:

Algorithm D for artificial run-off data:

1. The ultimate number of claims in an origin year is generated by random sampling from a Negative Binomial distribution (same parameters for each origin year but independent sampling).
2. Each claim is assumed to be settled by a single payment, and the development year of the payment determined by independent random sampling from a multinomial distribution (same parameters for each origin year).
3. The amount of each individual claim payment was determined by independent random sampling from a Log-Normal distribution. The mean of the Log-Normal distribution was determined, for each claim, from the operational time at which the claim settles. We used the formula: mean = $\exp(b_0 + b_1.t + b_2.t^2)$ where t is operational time (for example, the 86th claim to settle in an origin year in which the ultimate number of claim is 112 has t = 85.5 / 112). For the parameters, we used the values $b_0 = b_1 = b_2 = 1.0$: the mean amount of individual payments then rises from 2.718 at t = 0 to 20.09 at t = 1. The shape of the Log-Normal distribution was kept the same for all claims: the dependence of the mean on operational time was achieved by scaling the entire distribution. We used Log-Normal distributions all with coefficient of variation equal to 2.528 and skewness coefficient equal to 23.7.
4. The amounts of claims settling in the upper left triangle of the run-off array were accumulated to create this run-off triangle, and all claim amounts (regardless of the development year when settled) were accumulated to obtain the 'true' ultimate position for each origin year.

Note that it is only at Step 3 that this differs from Algorithm C (used in the previous section for testing the Mack and ODP methods). In Algorithm C the same probability distribution for individual payments is used at all stages of development: in Algorithm D this distribution is multiplied by a scaling factor that is a function of operational time. We used the same parameters in Steps 1 and 2 of Algorithm D as we used when generating data by Algorithm C to test the ODP and Mack methods.

Because Wright's method does not assume that the underlying run-off pattern (over real development time) is the same for all origin years, we have also generated data (by Algorithm E) in which the mean delay to settlement decreases across the origin years:

Algorithm E for artificial run-off data:

Algorithm E is the same as Algorithm D except that the probabilities of the multinomial distribution used at Step 2 are not the same for all origin years. The probabilities we used for each origin year are shown below (one row for each origin year: each row adds to 100%).

Table B-16 – Multinomial probabilities of claim settlement delay used in Algorithm E

	1	2	3	4	5	6	7	8	9	10
OYr 1	4.3%	14.3%	19.8%	19.3%	15.5%	11.0%	7.2%	4.4%	2.6%	1.5%
2	4.6%	15.0%	20.4%	19.5%	15.3%	10.7%	6.8%	4.1%	2.4%	1.3%
3	5.0%	15.7%	20.9%	19.6%	15.1%	10.3%	6.4%	3.8%	2.1%	1.2%
4	5.3%	16.5%	21.5%	19.6%	14.8%	9.9%	6.0%	3.5%	1.9%	1.0%
5	5.7%	17.3%	22.0%	19.7%	14.5%	9.4%	5.6%	3.2%	1.7%	0.9%
6	6.2%	18.2%	22.5%	19.7%	14.1%	9.0%	5.2%	2.9%	1.5%	0.8%
7	6.6%	19.1%	23.1%	19.6%	13.7%	8.5%	4.8%	2.6%	1.3%	0.7%
8	7.2%	20.0%	23.6%	19.5%	13.3%	8.0%	4.4%	2.3%	1.2%	0.6%
9	7.7%	21.0%	24.1%	19.3%	12.8%	7.5%	4.1%	2.1%	1.0%	0.5%
10	8.4%	22.1%	24.5%	19.1%	12.3%	7.0%	3.7%	1.8%	0.8%	0.4%

Assuming origin years are accident years, this implies that mean delay between loss event and claim settlement steadily reduces from about 3.4 years in the first origin year to 2.6 years in the latest origin year.

B.4.3 Performance of Wright's method when its assumptions hold

For testing Wright's method, each dataset generated by Algorithm D or Algorithm E comprises:

- Ultimate number of claims in each origin year (from Step 1)
- Upper left triangle of numbers of claims closed (from Step 2)
- Upper left triangle of aggregate claim amounts (from Step 4)
- Ultimate aggregate claim amount for each origin year (from Step 4).

The first three of these items were treated as if known, and were used to estimate the ultimate aggregate amount (and its standard error) using Wright's operational time method. The last item is the 'true' ultimate aggregate amount which was compared to the estimate produced by Wright's method. We compared only the estimated and true totals for the entire triangle, not the results for individual origin years. As in Mack's method, we used a Log-Normal distribution (with mean equal to the best estimate reserve and standard deviation equal to the estimated predictive standard error) to determine the values F(true reserve) for comparison with a uniform distribution.

The original paper (Wright, 1992, pages 270 to 272) describes two methods for estimating the dispersion parameter (the squared coefficient of variation of individual payments, denoted φ^2 in the original paper). This can be estimated using the mean-squared Pearson residual or the mean-squared deviance residual. As neither has a clear theoretical advantage, we have used both methods in these simulations to try to establish which performs better.

Results are summarised below for three independent sets of 10,000 simulations. For each set of 10,000 simulations, the first column gives results obtained using Pearson residuals to estimate the dispersion parameter; the second column gives results based on deviance residuals.

Table B-17 – Results of applying Wright's method to data from Algorithm D (constant run-off pattern)

	Results from different sets of 10,000 simulations					
	Set 1		Set 2		Set 3	
Best estimate reserve	3,434		3,436		3,440	
Best estimate > True	49.6%		49.2%		49.5%	
Best estimate – True	37.4		21.7		39.4	
Wright standard error	763.7	673.2	764.8	673.0	765.8	674.1
(Best - True) / (std error)	-0.222	-0.227	-0.242	-0.248	-0.214	-0.218
Square of the above	1.58	1.86	1.59	1.85	1.53	1.78
Log-Normal F(True)	0.540	0.533	0.545	0.538	0.540	0.533
1%	4.1%	5.0%	3.7%	5.1%	3.7%	4.5%
5%	10.3%	11.7%	9.4%	12.2%	9.9%	11.3%
10%	15.8%	17.5%	14.6%	18.3%	15.8%	17.7%
20%	25.9%	27.7%	24.8%	28.6%	26.1%	27.7%
30%	35.7%	36.7%	33.8%	37.6%	35.7%	36.6%
50%	54.4%	53.5%	52.1%	54.0%	54.7%	53.8%
70%	73.2%	70.6%	71.4%	70.8%	73.1%	70.5%
80%	82.4%	79.3%	81.1%	79.6%	82.6%	79.2%
90%	91.5%	88.6%	90.9%	89.0%	91.8%	88.8%
95%	96.3%	94.2%	95.7%	94.3%	95.9%	94.0%
99%	99.3%	98.5%	99.1%	98.7%	99.3%	98.6%

These results suggest there may be a small positive bias in the best estimate of the reserve produced by Wright's method in this case: over all 30,000 simulations, the mean estimation error is about 0.96% of the mean true reserve. This is not very consistent across all three sets of 10,000 simulations (the mean error in Set 2 is about half the value in Set 1 or 3) so it might be caused by sampling error rather than being a bias. As well as being close to unbiased, estimates exceeded the true reserve in close to half (49.4%) of simulations.

As with Mack's method and the ODP method, there is clear evidence that Wright's method tends to understate the predictive standard error when it understates the true reserve: the mean value of the 'standardised predictive error' is consistently negative (and its mean square significantly greater than one). The performance is slightly better when the dispersion parameter is estimated from Pearson residuals (first column for each set 10,000 simulations) than when it is estimated from deviance residuals: Pearson residuals tend to give a higher estimate of the dispersion parameter, hence a higher predictive standard error (about 764 instead of 673 in this example) which more closely approximates the reliability of the best estimate reserve.

The tendency of the method to understate the predictive standard error when it understates reserves is also apparent in the results based on a Log-Normal predictive distribution: in 3.8% of the simulations the true reserve exceeded what was supposed to be the 99^{th} percentile (based on dispersion parameter estimated from Pearson residuals: this rises to 4.9% if deviance residuals are used).

One possible explanation of why Wright's method seems to understate the chance of adverse outcomes is that the method relies on asymptotic results for unbiasedness and standard errors of the parameter estimates of generalized linear models, and the volume of data may not be sufficiently large for the asymptotic results to work well. To investigate this further, we looked at the mean estimated values of the parameters (b_0, b_1, b_2) across all simulations. It seems that the estimates are biased: the true values of these parameters were all exactly one, but the mean estimated values were (0.975, 1.113, 0.873) in the first set of 10,000 simulations, (0.977, 1.118, 0.864) in the second set and (0.979, 1.107, 0.876) in the third: the differences between the estimated values and true values appear to be consistent and not just caused by sampling error.

The table below only shows results with the dispersion parameter estimated from Pearson residuals (because the earlier results indicate this is slightly better than using the deviance).

Table B-18 – Results of applying Wright's method to data from Algorithm E (varying run-off pattern)

	Results from different sets of 10,000 simulations			
	Set 1	Set 2	Set 3	All combined
Best estimate	3,048	3,057	3,061	3,055
Best estimate > True reserve	50.4%	49.6%	50.2%	50.1%
Best estimate – True reserve	29.7	24.9	38.9	31.2
Wright standard error	697.5	700.7	704.7	701
(Best - True) / (Std error)	-0.213	-0.222	-0.203	-0.213
Square of the above	1.58	1.57	1.50	1.55
Log-Normal F(true liability)	0.539	0.541	0.537	0.539
1%	4.0%	4.0%	3.9%	4.0%
5%	9.9%	10.2%	9.6%	9.9%
10%	15.7%	16.5%	15.3%	15.8%
20%	25.7%	26.8%	25.8%	26.1%
30%	35.2%	36.2%	35.2%	35.5%

50%	53.9%	54.3%	54.0%	54.1%
70%	73.6%	72.8%	72.9%	73.1%
80%	82.7%	81.9%	82.2%	82.3%
90%	92.0%	91.4%	91.8%	91.7%
95%	96.1%	96.0%	96.1%	96.1%
99%	99.1%	99.3%	99.2%	99.2%

These results are very similar to those based on data from Algorithm D, and the same comments apply. This confirms that Wright's method is not sensitive to whether or not the underlying run-off pattern (across real development time) is the same for all origin years.

Further work is needed to investigate:

- The extent to which these conclusions depend on the particular set of parameters chosen in simulating the artificial data by Algorithms D and E.
- How more general versions of Wright's method perform in situations where the underlying assumptions are perfectly satisfied (for example, in situations of non-zero claims inflation, or where not all claims are settled by a single payment).
- The performance of the method if applied where its underlying assumptions are not true.

B.4.4 Performance of Mack's method on data from Algorithms D and E

As a further test of Mack's method in situations where its underlying assumptions are false, we applied Mack's method to the same claim amounts triangles (generated using Algorithms D and E) as using in testing Wright's method when its assumption hold true. Results are summarised below:

Table B-19 – Results of applying Mack's method to data generated using Algorithm D

	Results from different sets of 10,000 simulations			
	Set 1	Set 2	Set 3	All combined
BCL estimate	3,654	3,659	3,666	3,660
BCL estimate > True reserve	47.7%	46.4%	47.6%	47.2%
BCL estimate – True reserve	257	245	265	256
Mack standard error	1,856	1,855	1,865	1,859
(BCL - True) / (Mack std error)	-0.227	-0.241	-0.231	-0.233
Square of the above	1.17	1.18	1.15	1.17
Log-Normal F(true liability)	0.580	0.586	0.581	0.582
1%	1.6%	1.8%	1.7%	1.7%

5%	6.8%	7.1%	7.3%	7.1%
10%	13.8%	13.5%	13.7%	13.7%
20%	27.5%	27.2%	27.0%	27.2%
30%	40.3%	41.0%	40.1%	40.5%
50%	62.4%	63.8%	62.9%	63.0%
70%	80.5%	81.5%	80.8%	80.9%
80%	87.7%	88.1%	88.2%	88.0%
90%	93.5%	93.7%	93.9%	93.7%
95%	96.4%	96.3%	96.5%	96.4%
99%	98.7%	98.6%	98.7%	98.7%

These results are very similar to those obtained by applying Mack's method to data generated using Algorithm C. Compared to the results of applying Wright's method to data generated using Algorithm D, we see that Mack's method gives much higher predictive standard errors (mean of around 1,860 for Mack, compared to around 765 for Wright). This is to be expected because Mack's method makes use of the claim amounts triangle only, whereas Wright's method makes use of the claim amounts and claim numbers triangles. Clearly, in situations where claim numbers data are available, more reliable reserve estimates will usually be obtained by making use of this information rather than ignoring it.

The following results show that Mack's method (and the BCL method) breaks down badly if applied where the underlying run-off pattern is not the same for all origin years. The data simulated using Algorithm E has an accelerating development pattern. In this case, the BCL estimates exceed the true outcome in 84.5% of simulations, and on average overestimate the true reserves by 61%. In only 0.1% of simulations does the true ultimate exceed what Mack's method indicates is the 99th percentile. None of this is at all surprising – a key assumption of all chain-ladder methods (including Mack's) is that the underlying run-off pattern is the same for all origin years. These results confirm that such methods should not be used where this is not the case.

Table B-20 – Results of applying Mack's method to data generated using Algorithm E

	Results from 10,000 simulations (Set 1)
BCL estimate	4,873
BCL estimate > True reserve	84.5%
BCL estimate – True reserve	1,854
Mack standard error	2,257
(BCL - True) / (Mack std error)	0.665

Square of the above	0.985
Log-Normal F(true liability)	0.295
1%	0.1%
5%	0.5%
10%	1.4%
20%	4.3%
30%	8.8%
50%	22.3%
70%	42.1%
80%	54.9%
90%	70.9%
95%	80.6%
99%	92.1%

B.4.5 Performance of ODP methods on data generated using Algorithms D and E

As a further test of the ODP methods (analytic and bootstrap) in situations where the underlying assumptions are false, we have applied these methods to the same claim amounts triangles (generated using Algorithms D and E) as used in testing Wright's method when its assumptions hold. We have done this for only the first set of 10,000 triangles produced by each algorithm. Results are summarised below. For the analytic ODP method, since earlier results show that estimating the dispersion parameter from Pearson residuals is generally better than estimating it from the deviance, results are given below for the Pearson residual method only.

Table B-21 – Results of applying ODP methods to data generated using Algorithm D

	Analytic ODP	Bootstrap 99	Bootstrap 01
BCL estimate	3,654	3,654	3,654
BCL estimate > True reserve	47.7%	47.7%	47.7%
BCL estimate – True reserve	257	257	257
Bootstrap mean		3,723	3,724
ODP standard error	1,723	1,259	1,261
(BCL - True) / (Std error)	-0.278	-0.270	-0.270
Square of the above	1.38	1.95	1.93
F(true liability)	0.578	0.553	0.522
1%	2.3%	3.6%	2.8%
5%	8.5%	11.8%	9.2%
10%	16.0%	19.7%	15.9%
20%	28.6%	31.0%	26.4%
30%	40.1%	41.0%	35.9%
50%	61.2%	57.6%	52.9%
70%	79.7%	72.4%	69.3%
80%	87.0%	79.7%	78.4%
90%	93.6%	87.3%	87.3%
95%	96.8%	91.2%	92.2%
99%	99.0%	96.1%	96.9%

These results show that, while it does not perform as well as Mack's method on triangles generated by Algorithm D, the analytic ODP method does perform almost as well here as where its assumptions are perfectly satisfied (Algorithm B, Table B-8). The parameters we have used in Algorithm D produce individual claim amounts with a mean value that increases by a factor of about seven between development periods 1 and 10, which implies that the parameter φ (defined as ratio of variance to mean of aggregate increments) increases by a similar factor. One of the ODP assumptions is that the value of φ is constant across the entire triangle. This assumption is far from true here, so it seems that the analytic ODP method is quite robust to violations of this assumption. The bootstrap methods appear to be less robust to violations of this assumption.

In tests of the ODP methods where their assumptions are true (data generated using Algorithm B) we found that the 1999 bootstrap method gave a predictive standard error smaller than the analytic ODP standard error in 86% of simulations. Here (date generated using Algorithm D), the

1999 bootstrap method gave a smaller standard error than the analytic ODP standard error in 96.2% of simulations, and the 2001 bootstrap standard error was smaller than the analytic standard error in 95.7% of simulations. It seems that the bootstrap methods tend to understate the predictive standard error when data are generated by Algorithm D.

Another of the ODP assumptions is that the underlying run-off pattern across development periods is the same for all origin years: this assumption is satisfied in triangles generated by Algorithm D. However, the following results show that the analytic ODP method (like Mack's method) are not robust to violations of this assumption: it performs very poorly when applied to data generated by Algorithm E.

Table B-22 – Results of applying analytic ODP method to data generated using Algorithm E

	Results from 10,000 simulations (Set 1)
BCL estimate	4,873
BCL estimate > True reserve	84.5%
BCL estimate – True reserve	1,854
ODP standard error	2,154
(BCL - True) / (ODP std error)	0.663
Square of the above	0.990
Log-Normal F(true liability)	0.287
1%	0.1%
5%	0.7%
10%	1.8%
20%	4.6%
30%	8.8%
50%	21.0%
70%	39.4%
80%	52.7%
90%	69.2%
95%	80.4%
99%	93.0%

B.5 Provisional conclusions

Much more work is needed before we can formulate definitive conclusions, but based on the simulations carried out so far it seems (provisionally) that:

- When the basic chain ladder method is applied to triangles where the underlying run-off pattern is the same for all origin years, it understates the true reserve more often than not. However, the mean value of BCL reserves exceeds the mean value of true outcomes: in other words, the BCL is positively biased. These results apply when the data are generated using a range of simplified but reasonably realistic algorithms (subject to the constraint that the underlying run-off pattern is the same for all origin years).
- When calculated from triangles where the underlying run-off pattern is the same for all origin years (this being an example of when the conditions are NOT satisfied), Mack's standard errors, used in conjunction with positively biased BCL estimates, provide reasonably accurate assessments of the chances of very adverse outcomes. This applies when the data are generated using a range of increasingly realistic algorithms that do not satisfy Mack's explicit assumption (but do have the same underlying run-off pattern across all origin years).
- Strangely however, Mack's method performs very poorly on data generated in such a way that Mack's explicit assumptions are perfectly satisfied. (This is perhaps because the BCL estimates are not biased in this case, but the Mack standard errors tend to be understated.) As it is difficult to imagine any real-world claim settlement process that would approximate Mack's assumptions, this may not be a major drawback of the method.
- Given the above, there seems to be little value in trying to check (through diagnostics) that Mack's explicit assumptions hold true when applying Mack's method (or perhaps the assumptions should be checked and the method used only where the assumptions appear to be false!) It is however important to check that the run-off pattern does not differ significantly between origin years.
- Standard errors for BCL estimates calculated using the over-dispersed Poisson (ODP) method can understate the chance of adverse outcomes even where the explicit assumptions of the ODP method are perfectly satisfied: in simulations carried out so far, what was supposed to be the 95th percentile was exceeded in nearly 10% of simulations, and what was supposed to be the 99th percentile was exceeded in 2.6% of simulations.
- The performance of the analytic OPD method is insensitive to the probability distribution of the ultimate number of claims in an origin year (even if this is such as to violate the assumptions of the method) and insensitive to the relationship between mean size of claim payment and delay to settlement (provided this is the same for all origin years).
- The bootstrap ODP method does not in general give the same predictive standard error as the analytic ODP method, even where the ODP assumptions are perfectly satisfied.
- The bootstrap ODP method is more sensitive than the analytic ODP method to violations in the ODP assumptions: its performance deteriorates if there is a strong relationship between the mean claim size and the delay to settlement (such that the ratio of variance to mean is not the same for all aggregate increments in the paid claims triangle).

- When applied to triangles for which the underlying run-off pattern is not the same for all origin years, the BCL method can give very poor estimates, and both Mack's method and the ODP methods give very unreliable standard errors. (Neither method was ever intended to be used in these circumstances, of course.)
- Wright's operational time method also tends to understate the chances of adverse outcomes: in simulations carried out so far, what was supposed to be the 95th percentile was exceeded in about 10% of simulations, and what was supposed to be the 99th percentile was exceeded in 3.9% of simulations.
- Provided the assumptions of the method continue to hold, the performance of Wright's method is not sensitive to variation in the underlying run-off pattern between origin years.
- We have not yet done any testing of the sensitivity of Wright's method to violations of its assumptions.

B.6 Technical results for Mack and ODP methods

B.6.1 Mutually exclusivity of assumptions

Here we show that the assumptions of Mack's (1993) method and the over-dispersed Poisson (ODP) method are mutually exclusive, that is, the two sets of assumptions cannot both be true simultaneously. This explains how it is that - when applied to triangles with no missing data - both methods produce 'best estimates' identical to those given by the basic chain ladder (BCL) method, yet the two methods give different root-mean-square predictive errors (or 'standard errors' for short). Since the two methods rely on two mutually exclusive sets of assumptions, both have the potential to give correct standard errors in different circumstances, but they can never both be correct.

Mack's assumptions are:

M1: There exist parameters f_k such that: $E(C_{j,k+1} \mid C_{j1},\ldots C_{jk}) = f_k.C_{jk}$

M2: There exist parameters α_k such that: $Var(C_{j,k+1} \mid C_{j1},\ldots C_{jk}) = \alpha_k^2.C_{jk}$

M3: Accident years are stochastically independent.

The ODP assumptions are:

P1: $E(Y_{jk}) = x_j.p_k$

P2: $Var(Y_{jk}) = \varphi.x_j.p_k$

P3: The Y_{jk} are stochastically independent (across all j and all k).

First we show that if P1 and P3 hold, then M1 necessarily does not hold.

From the definitions of C_{jk} and Y_{jk} as respectively cumulative and incremental aggregate paid amounts:

$E(C_{j,k}| C_{j,k-1}) \quad = C_{j,k-1} + E(Y_{j,k} \mid C_{j,k-1})$

$= C_{j,k-1} + E(Y_{j,k})$ by assumption P3 (independent increments)

$= C_{j,k-1} + x_j .p_k$ by assumption P1

$= (1 + x_j .p_k / C_{j,k-1}).C_{j,k-1}$

Comparing this to assumption M1 we see that assumption M1 cannot hold because the quantity in parentheses in the last equation is a function of the random variable $C_{j,k-1}$ so cannot be a constant f_k (the same for all origin years).

Next we show that if M1 holds, then P3 necessarily does not.

Assumption M1 implies that $E(Y_{j,k+1} | C_{j1}\ldots C_{j,k}) = (f_k -1).C_{jk}$

Putting k = 1, this implies: $E(Y_{j2} | Y_{j1}) = (f_1 -1).Y_{j1}$

which implies that Y_{j2} and Y_{j1} are not stochastically independent, contradicting P3.

B.6.2 Algorithm B satisfies the ODP assumptions

We next show that Algorithm B satisfies the ODP assumptions (and therefore, by the above results, does not satisfy Mack's assumptions).

Algorithm B is:

1. Generate the ultimate number of claims N_j from a Poisson distribution with parameter λ say.
2. Find numbers N_{jk} settled in each development year k by splitting N_j using a multinomial distribution, with probabilities p_k, where sum is 1.
3. Generate the amount of each claim Z_{jki} (i = 1 to N_{jk}) by independent sampling from a distribution with mean μ and variance σ^2 say.
4. Calculate $Y_{jk} = \Sigma_i Z_{jki}$

First we show that Steps 1 and 2 produce independent Poisson numbers N_{jk}. We consider just two development years (the argument easily generalises to more than two). The joint probability density function of N_{j1} and N_{j2} is given by:

$f(N_{j1}, N_{j2}) = f(N_{j1}, N_{j2} | N_j) . f(N_j)$

= multinomial * Poisson

= product of two Poisson pdfs for N_{j1} and N_{j2}.

Since their joint pdf factorises, N_{j1} and N_{j2} are stochastically independent.

Clearly Y_{j1} and Y_{j2} generated by Steps 3 and 4 of Algorithm B are therefore also independent, so assumption P3 is satisfied.

To show that P1 and P2 are also satisfied we must consider $E(Y_{jk})$ and $Var(Y_{jk})$:

$E(Y_{jk}) = E(E(Y_{jk} | N_{jk}))$

$= E(\mu.N_{jk})$ from Step 4 of Algorithm B

$= \mu.\lambda.p_k$ because N_{jk} is Poisson($\lambda.p_k$)

from which we see that P1 holds, with $x_j = \mu.\lambda$ (ie expected ultimate is expected number of claims λ multiplied by expected amount μ of each one).

$$\begin{aligned} Var(Y_{jk}) &= E(Var(Y_{jk} \mid N_{jk})) + Var(E(Y_{jk} \mid N_{jk})) \\ &= E(\sigma^2.N_{jk}) + Var(\mu.N_{jk}) && \text{from Step 4 of Algorithm B} \\ &= \sigma^2.\lambda.p_k + \mu^2.\lambda.p_k && \text{because } N_{jk} \text{ is Poisson}(\lambda.p_k) \\ &= (\sigma^2 + \mu^2).\lambda.p_k \end{aligned}$$

from which we see that P2 holds with $\varphi = (\sigma^2 + \mu^2) / \mu$.

If algorithm B were generalised to have a different ultimate expected number of claims λ_j for each origin year, the above argument is unaffected and the ODP assumptions continue to hold (with $x_j = \mu.\lambda_j$).

B.6.3 Algorithm C does not satisfy ODP or Mack assumptions

Algorithm C is the same as Algorithm B except that the distribution used in Step 1 is Negative Binomial instead of Poisson. It is easily shown (by replacing the Poisson pdf with the Negative Binomial pdf in the above) that in this case, the join distribution $f(N_{j1}, N_{j2})$ does not factorise, which implies that N_{j1} and N_{j2} are stochastically dependent, which in turn implies that assumption P3 does not hold. It is also easy to show that assumption M1 does not hold in this case.

Appendix C: Long List of Methods

Appendix C

Long List of Methods

No.	Category	Paper / Workshop / Reference name	Link
1	Mack	Mack, T. (1993). Distribution Free Calculation of the Standard Error of Chain Ladder Reserve Estimates. ASTIN Bulletin, 23, pp. 213-225.	http://www.casact.org/library/astin/vol23no2/213.pdf
2	Mack	Mack, T. (1999). The Standard Error of Chain Ladder Reserve Estimates: Recursive Calculation and Inclusion of a Tail Factor. ASTIN Bulletin, 29, pp. 361-366.	http://www.casact.org/library/astin/vol29no2/361.pdf
3		Over Dispersed Poisson, including GLM	
4		Benchmarking	
5		McLennan, A. & Murphy, K. P. (2006). A Method For Projecting Individual Large Claims. Casualty Actuarial Society, Fall Forum 2006, pp. 205-236.	http://www.casact.org/pubs/forum/06fforum/209.pdf
6	Individual Level Claim Reserving	Guszcza, J. C. & Lommele, J. (2006). Loss Reserving Using Claim-Level Data. Casualty Actuarial Society, Fall Forum 2006, pp. 111-140.	http://www.casact.org/pubs/forum/06fforum/115.pdf
7		Patel, C. C., & Raws, A. (1998). Statistical Modeling Techniques for Reserve Ranges: A Simulation Approach. Casualty Actuarial Society, Fall Forum 1998, pp. 229-255.	http://www.casact.org/pubs/forum/98fforum/patel.pdf
8		Extrapolation - Curve Fitting for the Tail	
9		Taylor (check CAS - Australian)	
10		Halliwell, L. J. (1996). Loss Prediction by Generalized Least Squares. Proceedings of the Casualty Actuarial Society 83, pp. 436-489.	http://www.casact.org/pubs/proceed/proceed96/96436.pdf
11		Deterministic - Curve Fit	
12		Historical Ults - Restating	
13		Stressing - Best Estimate and Scenario	
14		McLennan - Fitting Curves to Loss Development Factors	
5		Panning, W. H.(2006). Measuring Loss Reserve Uncertainty. Casualty Actuarial Society, Fall Forum 2006, pp. 237-267.	http://www.casact.org/pubs/forum/06fforum/241.pdf
16		Scenarios Using Exposure Based Methods	
17		Meyers, G. G. (2006). Estimating Predictive Distributions for Loss Reserve Models. Casualty Actuarial Society, Fall Forum 2006, pp. 159-203.	http://www.casact.org/pubs/forum/06fforum/163.pdf

18		Verrall, R. J. & England, P. D. (2005). Incorporating Expert Opinion into a Stochastic Model for the Chain-Ladder Technique. Insurance: Mathematics and Economics Vol. 37, pp. 355-370.	
19		Verall/England - New Paper to be released Annals of Actuarial Science	
20		Renshaw, A. E. & Verrall, R. J. (1994). The Stochastic Model Underlying the Chain-Ladder Technique. Proceedings of the ASTIN Colloquium.	
21		Murphy, D. M. (1994). Unbiased Loss Development Factors. Proceedings of the Casualty Actuarial Society 81, pp. 154-222.	http://www.casact.org/pubs/proceed/proceed94/94154.pdf
22	Bootstrap / Overdispersed Poisson	England, P. D. & Verrall, R. J. (2002). Stochastic Claims Reserving in General Insurance. British Actuarial Journal Vol. 8, pp. 443-544.	
23	Operational Time	Wright, T. S. (1992). Stochastic Claims Reserving When Past Claim Numbers Are Known. Proceedings of the Casualty Actuarial Society 79, pp. 255-361.	http://www.casact.org/pubs/proceed/proceed92/92255.pdf
24		Reid, D. H. (1997). Reid's Method: Claims Reserving Manual Vol. 2. London: Institute of Actuaries.	
25		Reid, D. H. (1997). Operational Time and a Fundamental Problem of Insurance in a Data-Rich Environment: Claims Reserving Manual Vol. 2. London: Institute of Actuaries.	
26	Regression /Curve Fitting	Benjamin, S. & Eagles, L. (1997). A Curve Fitting Method and a Regression Method. Claims Reserving Manual Vol. 2. London: Institute of Actuaries.	
27		Gogol, D. F. (1995). Using Expected Loss Ratios in Reserving. Casualty Actuarial Society, Fall Forum 1995, pp. 241-244.	http://www.casact.org/pubs/forum/95fforum/95ff241.pdf
28		Brickman, S. et al (1993). Variance in Claim Reserving. General Insurance Convention 1993.	http://www.actuaries.org.uk/files/pdf/library/proceedings/gen_ins/gic1993/0001-0125.pdf
29	Operational Time	Lowe, J. (1994). A Practical Guide to Measuring Reserve Variability Using: Bootstrapping, Operational Time and a Distribution-Free Approach. General Insurance Convention 1994.	http://www.actuaries.org.uk/files/pdf/library/proceedings/gen_ins/gic1994/0157-0196.pdf
30		Larsen, C. (1995). The Bootstrap Method and Some Reserving Applications. General Insurance Convention 1995.	http://www.actuaries.org.uk/files/pdf/library/proceedings/gen_ins/gic1995/0223-0232.pdf

31		England, P. D. & Verrall, R. J. (1998). Standard Errors of Prediction in Claims Reserving: A Comparison of Methods. Analytic and Bootstrap Estimates of Prediction Errors in Claims Reserving. General Insurance Convention 1998.	http://www.actuaries.org.uk/files/pdf/library/proceedings/gen_ins/gic1998/0459-0478.pdf
32		Waszink, H. & van der Wardt, M. (1998). A Stochastic Model to Determine IBNR Reserves. General Insurance Convention 1998.	http://www.actuaries.org.uk/files/pdf/library/proceedings/gen_ins/gic1998/0149-0174.pdf
33		Lyons, G. et al (2002). Claims Reserving Working Party Paper. General Insurance Convention 2002.	http://www.actuaries.org.uk/files/pdf/giro2002/Lyons.pdf
34		Skinner, J. (2002). Practical Stochastic Reserving (Summary). General Insurance Convention 2002.	http://www.actuaries.org.uk/files/pdf/giro2002/Skinner.pdf
35		Brix, A. et al (2000). Stochastic Reserving. General Insurance Convention 2000.	http://www.actuaries.org.uk/files/pdf/library/proceedings/gen_ins/gic2000/0347-0349.pdf
36		England, P. D. & Aghadiuno, C. (1997). An Update on Stochastic Reserving Methods and Associated Measures of Reserve Variability. General Insurance Convention 1997.	http://www.actuaries.org.uk/files/pdf/library/proceedings/gen_ins/gic1997/0081-0084.pdf
37		Mohammad Khan et al (2005). Estimating Reserving Uncertainty. General Insurance Convention 2005.	http://www.actuaries.org.uk/files/pdf/proceedings/giro2005/Khan.pdf
38		England, P. D. & Verrall, R. J. (2005). More on Stochastic Reserving in General Insurance. General Insurance Convention 2005.	http://www.actuaries.org.uk/files/pdf/proceedings/giro2004/England.pdf
39	Bayesian & Bornhuetter Fergusson	England, P. D. (2002). A Beginner's Guide to Bayesian Modelling. General Insurance Convention 2002.	http://www.actuaries.org.uk/files/giro2002/England_Bayesian.ppt
40		Blum K. A. & Otto D. J. (1998). Best Estimate Loss Reserving: An Actuarial Perspective. Casualty Actuarial Society, Fall Forum 1998, pp. 55-102.	http://www.casact.org/pubs/forum/98fforum/blumotto.pdf

Appendix D: Bibliography

Appendix D: Bibliography

Basic Chain Ladder

Paper/Link

Stanard, J. N. (1985). A simulation test of prediction errors of loss reserve estimation techniques. Proceedings of the Casualty Actuarial Society, LXXII. 124-148. http://www.casact.org/pubs/proceed/proceed85/85124.pdf

Taylor, G. (2003). Chain ladder bias. ASTIN Bulletin, 33 No. 2, 313-330. http://www.casact.org/library/astin/vol33no2/313.pdf

Mack

Paper/Link

Mack, T. (1993). Distribution free calculation of the standard error of chain ladder reserve estimates. ASTIN Bulletin, 23, 213-225. http://www.casact.org/library/astin/vol23no2/213.pdf

Mack, T. (1994). Which stochastic model is underlying the chain ladder method? Insurance: Mathematics and Economics, 15, 133-138.
Mack T. (1994). Measuring the variability of chain ladder reserve estimates. Casualty Actuarial Society, Spring Forum, 101-182. http://www.casact.org/pubs/forum/94spforum/94spf101.pdf

Mack, T. (1997). Measuring the variability of chain ladder reserve estimates. Claims Reserving Manual volume 2. London: Institute of Actuaries. http://www.actuaries.org.uk/files/pdf/library/crm2-D6.pdf

Mack, T. (1999). The standard error of chain ladder reserve estimates: Recursive calculation and inclusion of a tail factor. ASTIN Bulletin, 29 No. 2, 361-366. http://www.casact.org/library/astin/vol29no2/361.pdf

Bootstrap/Overdispersed Poisson

Paper/Link

Renshaw, A. E. & Verral, R. J. (1998). A stochastic model underlying the chain ladder technique. British Actuarial Journal, 4, 903-923.
England, P. D. & Verrall, R. J. (1999). Analytic and bootstrap estimates of prediction errors in claims reserving. Insurance: Mathematics and Economics, 25, 281-293.
England, P. D. (2001). Addendum to `Analytic and bootstrap estimates of prediction errors in claims reserving'. Actuarial Research Paper No. 138, Department of Actuarial Science and Statistics, City University, London, EC1V 0HB.
England, P. D. & Verrall, R. J. (2002). Stochastic claims reserving in general insurance. British Actuarial Journal, 8, 443-544.
Online web lecture for stochastic claim reserving models including the over-dispersed Poisson: https://talk.city.ac.uk/stochasticreserving

Individual level claim reserving

Paper/Link

Guszcza, J. C. & Lommele, J. (2006). Loss reserving using claim level data. Casualty Actuarial Society, Fall Forum, 111-140
Taylor, G. C., McGuire, G. & Sullivan, J. (2006). Individual claim loss reserving conditioned by case estimates. Taylor Fry Consulting Actuaries.
Wright T S (1997). "Probability distribution of outstanding claim payments from individual payments data." Claims Reserving Manual volume 2. London: Institute of Actuaries.
http://www.actuaries.org.uk/files/pdf/library/crm2-D7.pdf

Operational time

Paper/Link

Wright, T. S. (1992). Stochastic Claims Reserving When Past Claim Numbers Are Known. Casualty Actuarial Society. Proceedings. 150 & 151, 255-361.
http://www.casact.org/pubs/proceed/proceed92/92255.pdf

Bain, D. (2003). A practical implementation of Wright's Operational Time Model. Society of Actuaries in Ireland.
Lowe, J. (1994). A practical guide to measuring reserve variability using: bootstrapping, operational time and a distribution-free approach. General Insurance Convention.

Regression/Curve Fitting

Paper/Link

Benjamin, S. & Eagles, L. (1997). A curve fitting method and a regression method. Claims Reserving Manual volume 2. London: Institute of Actuaries.
http://www.actuaries.org.uk/files/pdf/library/crm2-D3.pdf

Bayesian & Borhuetter/Fergusson

Paper/Link

England, P. D. & Verrall, R. J. (2002). Stochastic claims reserving in general insurance. British Actuarial Journal, 8, 443-544.
England, P. D. (2002). A beginner's guide to Bayesian modelling. General Insurance Convention.
Verrall, R. J. (2001). A Bayesian generalised linear model for the Bornhuetter-Ferguson method of claims reserving. Actuarial Research Paper No. 139, Department of Actuarial Science and Statistics, City University, London.

Appendix E: Detailed Survey Results

This appendix has two components: Appendix E1 and Appendix E2.

Appendix E1 - ROC Working Party Questionnaire

The following survey has been developed to assist the Reserving Oversight Committee (ROC) in the formulation of its conclusions relating to the working parties 'Best estimates and estimating uncertainty' and 'Effectiveness of reserving methods'.

1) In order to classify responses, please identify the type of organisation you represent:

Insurance company
Reinsurance company
Lloyd's syndicate
Consultancy

1a) Where is your company based?

UK
Overseas

2) What are your key products/classes of business that you perform reserving for? (Please select a maximum of three)

Personal lines - property
Personal lines - motor
Commercial lines - property
Commercial lines - motor
Commercial lines - liability
Health insurance
Credit insurance
Marine/aviation/transport
Reinsurance - Property
Reinsurance - Motor
Reinsurance - Liability
Other - please state

3) For the classes above please check the appropriate box in relation to the following questions:

a) Method not used
b) Method partially used (for background)
c) Method is a key supplementary method
d) Key/principal method

3a) What are the methods and models you use to determine best estimate values?
(In respect of each of the lines of business selected above).

Name of method/model	a)	b)	c)	d)
Mack				
Overdispersed Poisson stochastic chain ladder/				
Bootstrap				
Transaction level/ individual claim				
Operational time				
Regression/Curve fitting				
Bayesian/BF method				
Basic Chain Ladder - paid				
Basic Chain Ladder - incurred				
Bornhuetter Fergusson				
Average cost per claim				
Judgement				
Scenarios				
Other - please state				

3b What are the methods and models you use to determine uncertainty?

Name of model

Mack
Overdispersed Poisson stochastic chain ladder/
Bootstrap
Transaction level/ individual claim
Operational time
Regression/Curve fitting
Bayesian/BF method
Basic Chain Ladder - paid
Basic Chain Ladder - incurred
Bornhuetter Fergusson
Average cost per claim
Judgement
Scenarios
Other - please state

4 When determining uncertainty what are your key requirements - please state in order of importance

Identifying variability around the best estimate
Identifying the complete distribution
Identifying the tail of the distribution (above 95th percentile).

5 When determining which model/method to use in assessing uncertainty, what are the key criteria you use (in order of importance)?

Ease of use
Quality of results
Communicability
Practical issues - relating to available data
Time/ resource constraints
Knowledge of actuarial staff

6 When modelling uncertainty do you use?

In-house models
Off-the shelf packages
A combination of in house models/ off the shelf packages
If "off the shelf" packages are used - please list them:

7 When communicating uncertainty in the reserves to excutives/senior managers within the organisation, what methods are used (in order of importance)

Quantitative using percentiles
Quantitative using everyday English
Whole account scenarios with indication of likelihood
Whole account scenarios without indication of likelihood
Scenarios relating to parts of the reserves with indication of likelihood
Scenarios relating to parts of the reserves without indication of likelihood
Stress tests
Other (please specify)
Uncertainty only communicated qualitatively
Uncertainty not communicated

8 Does your communication of uncertainty cover? (select all that apply)

Process uncertainty
Parameter uncertainty
Model uncertainty

9a When calculating best estimates what grouping of data do you use by origin period?

Monthly
Quarterly
Half-yearly
Annual

9b When calculating best estimates what grouping of data do you use by development period?

Monthly
Quarterly
Half-yearly
Annual

10a When calculating uncertainty what grouping of data do you use by origin period?

Monthly
Quarterly
Half-yearly
Annual

10b When calculating uncertainty what grouping of data do you use by development period?

Monthly
Quarterly
Half-yearly
Annual

11 How frequently do you reserve each class of business mentioned in 2) above?

Monthly
Quarterly
Half-yearly
Annually

12 If you reserve monthly - do you carry out a full analysis or a reduced analysis each month?

Full analysis
Reduced analysis

13 If you reserve quarterly - do you carry out a full analysis or a reduced analysis each quarter?

Full analysis
Reduced analysis

Thank you for completing this survey. Your results will be kept anonymous and used to assist the ROC working party's work.
This questionnaire may be repeated annually to monitor the take-up of new methodology.

Appendix E2 - Detailed Survey Results

Chart 2. Where based

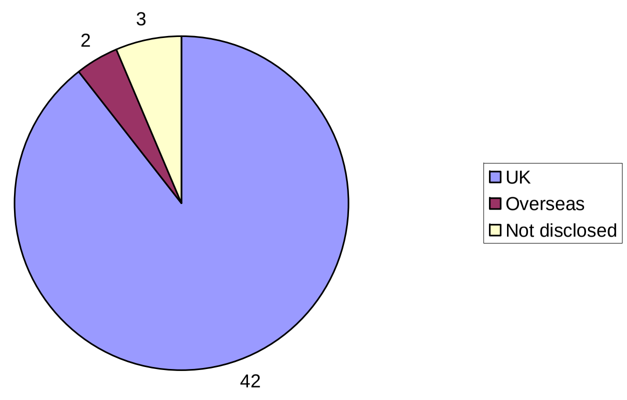


Chart 1. No of respondents

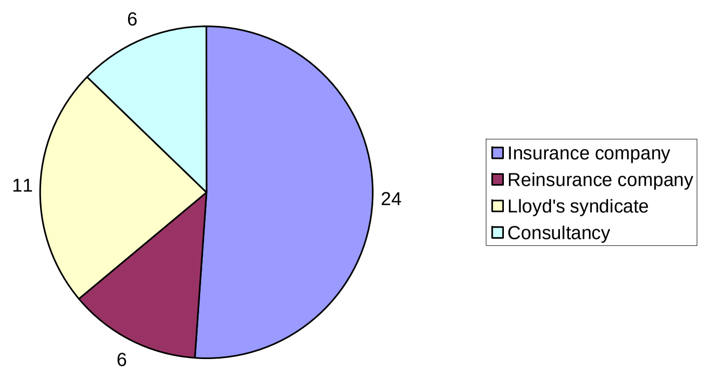
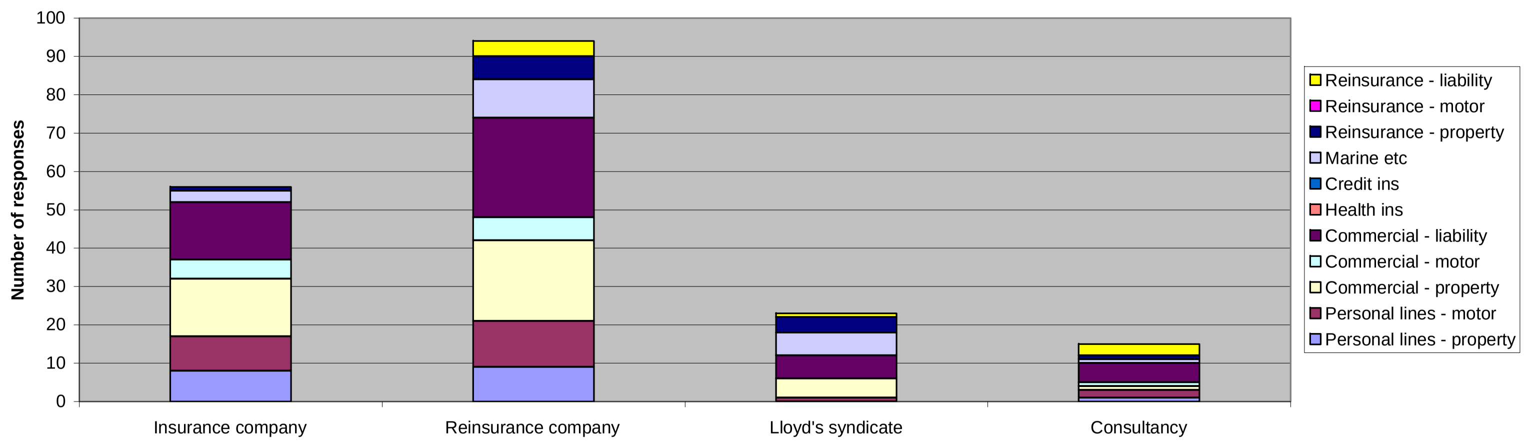


Chart 3. Key products/classes that reserving is performed for



Analysis of data using all data combined

Data has been combined for all classes

3a) What are the methods and models you use to determine best estimate values?

All lines

Method	a	b	c	d	Total
	Counts				
Mack	93	2	7	3	**105**
Overdispersed Poisson stochastic chain ladder/ Bootstrap	85	7	10	6	**108**
Transaction level/ individual claim	43	24	19	21	**107**
Operational time	100	1	1	3	**105**
Regression/Curve fitting	55	13	26	11	**105**
Bayesian/BF method	64	2	15	22	**103**
Basic Chain Ladder - paid	15	19	45	32	**111**
Basic Chain Ladder - incurred	8	1	9	94	**112**
Bornhuetter Fergusson	11	11	16	67	**105**
Average cost per claim	43	19	26	19	**107**
Judgement	4	30	36	38	**108**
Scenarios	30	34	31	7	**102**
Other - please state	0	0	0	0	**11**

Other responses

Simple stochastic models

Per contract loss ratio from pricing

Implied from pricing adequacy

Use of market 3rd party data to asses level of subsidence claims

Main methods: Generalised cape cod & ICRFS

Key

a) Method not used

b) Method partially used (for background)

c) Method is a key supplementary method

d) Key/principal method

3b What are the methods and models you use to determine uncertainty?

All lines

Method	a	b	c	d	Total
	Counts				
Mack	54	6	14	12	**86**
Overdispersed Poisson stochastic chain ladder/ Bootstrap	36	7	6	47	**96**
Transaction level/ individual claim	69	3	7	9	**88**
Operational time	82	0	0	3	**85**
Regression/Curve fitting	58	11	9	6	**84**
Bayesian/BF method	73	6	0	7	**86**
Basic Chain Ladder - paid	43	5	16	21	**85**
Basic Chain Ladder - incurred	48	7	16	18	**89**
Bornhuetter Fergusson	60	3	11	12	**86**
Average cost per claim	60	8	6	13	**87**
Judgement	11	15	31	38	**95**
Scenarios	33	14	20	34	**101**
Other - please state	0	0	0	0	**15**

Other responses

Simulation Models

Simple stochastic models

Benchmark info from external consultants

ICRFS

Either completely judgemental or fitting a distribution to a number of projections (eg low, best est and high). Alternatively model future claim numbers, assume a distribution, assume a distribution for cost of each claim, assume a distribution for IBNER

Market benchmarks

Bespoke methodology

Key

a) Method not used

b) Method partially used (for background)

c) Method is a key supplementary method

d) Key/principal method

4 When determining uncertainty what are your key requirements - please state in order of importance

All lines	**Count of preferences**			
Key requirements	**1**	**2**	**3**	**Total**
Identifying variability around the best estimate	35	5	5	**45**
Identifying the complete distribution	8	11	27	**46**
Identifying the tail of the distribution (above 95th percentile).	6	26	13	**45**
Total	**49**	**42**	**45**	0

Key
1) Most important
3) Least important

5 When determining which model/method to use in assessing uncertainty, what are the key criteria you use (in order of importance)?

All lines	**Count of preferences**						
Key requirements	**1**	**2**	**3**	**4**	**5**	**6**	**Total**
Ease of use	6	3	12	12	10	3	**46**
Quality of results	12	16	9	4	1	4	**46**
Communicability	6	7	7	13	7	6	**46**
Practical issues - relating to available data	18	8	5	5	6	4	**46**
Time/ resource constraints	4	8	8	5	13	7	**45**
Knowledge of actuarial staff	4	4	5	4	8	21	**46**
Total	**50**	**46**	**46**	**43**	**45**	**45**	0

Key
1) Most important
6) Least important

6 When modelling uncertainty do you use?

All lines				
	Combination	**Off shelf**	**In house**	**Total**
Count of usage	17	11	17	**45**

Packages mentioned
ResQPro
ResQ
RMS RiskLink
Resq Professional
ResQ Pro Bootstrap
Igloo
ResQ
Resq Pro
ResQ Professional (Mack inbuilt), @Risk Professional used for assessing distributions, Simulum used for DFA/ICA type analysis
ResQPro ResQ Bootstrapping Add-on
ResQ
Res-Q IGLOO
Igloo
In-house models based around @Risk software. Supplemented by benchmark info from external consultants
ICRFS
ResQ
RESQ
ResQ - bootstrapping
ResQ
ResQ
Igloo
ResQ
Igloo

7 When communicating uncertainty in the reserves to excutives/senior managers within the organisation, what methods are used (in order of importance)

All lines	**Count of preferences**									
Methods used	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**Total**
Quantitative using percentiles	21	8	3	2	0	1	0	0	1	**36**
Quantitative using everyday English	10	15	4	2	1	1	0	2	0	**35**
Whole account scenarios with indication of likelihood	1	2	6	4	2	4	3	1	0	**23**
Whole account scenarios without indication of likelihood	1	2	0	7	5	4	3	0	0	**22**
Scenarios relating to parts of the reserves with indication of likelihood	3	3	4	2	5	3	2	2	0	**24**
Scenarios relating to parts of the reserves without indication of likelihood	4	1	3	4	2	4	4	1	1	**24**
Stress tests	2	3	3	2	4	2	6	4	0	**26**
Uncertainty only communicated qualitatively	5	1	2	0	1	1	2	11	1	**24**
Uncertainty not communicated	3	1	0	0	0	1	0	0	17	**22**
Other (please specify)	0	0	0	0	0	0	0	0	0	0

Complete distribution of outcomes with reserves for given scenarions and at various exceedence probabilities

This seems to be the best place to comment on the questions on this page. Q7 depends on the context. I will be looking at the best estimate when recommending a balance sheet but at the tail when looking at a transaction. Q8 and Q10 again depend on

Key for question 7
1) Most important
9) Least important

8 Does your communication of uncertainty cover? (select all that apply)

All lines	**Number**
Uncertainty coverage	
Process uncertainty	34
Parameter uncertainty	38
Model uncertainty	23

9a When calculating best estimates what grouping of data do you use by origin period?

All lines	**Frequency**				
	M	**Q**	**H**	**A**	**Total**
Numbers by frequency	2	11	2	31	**46**

9b When calculating best estimates what grouping of data do you use by development period?

All lines	**Frequency**				
	M	**Q**	**H**	**A**	**Total**
Numbers by frequency	3	31	1	11	**46**

10a When calculating uncertainty what grouping of data do you use by origin period?

All lines	**Frequency**				
	M	**Q**	**H**	**A**	**Total**
Numbers by frequency	1	8	2	33	**44**

10b When calculating uncertainty what grouping of data do you use by development period?

All lines	**Frequency**				
	M	**Q**	**H**	**A**	**Total**
Numbers by frequency	2	8	2	33	**45**

11 How frequently do you reserve each class of business mentioned in 2) above?

All lines	Frequency				
	M	Q	H	A	Total
Numbers by frequency	5	85	4	19	**113**

12 If you reserve monthly - do you carry out a full analysis or a reduced analysis each month?

All lines	Type of analysis		
	R	F	Total
Numbers by type	14	1	15

13 If you reserve quarterly - do you carry out a full analysis or a reduced analysis each quarter?

All lines	Type of analysis		
	R	F	Total
Numbers by type	22	67	89

Appendix F: Insureware Submission

This appendix contains the Insureware submission in the full form received by the working party. It is written by Insureware, not by the working party. The approaches are described in more detail than we have described the methods tested by the working groups, although we have referenced relevant papers in relation to those methods. Some of the material is beyond the scope of the working party.

This appendix has six components as follows:

F1. Uncertainty Working Group Test Template.doc

F2. Variability and Uncertainty.doc

F3. Employers Liability_UC_O110.xls

F4. ELRF study.xls

F5. EmployersLiabilityUC01100012.xls

F6. MarineCargoGrossvNet.xls

Appendix F

Best Estimate & Uncertainty Working Group

Method Evaluation Template

Name of Method:

We do not advise a method. We use modelling frameworks to identify the structure in the data. In actuarial parlance, we design a method that is appropriate for the data.

There are three modelling frameworks we typically use, Probabilistic Trend Family (PTF), Multiple Probabilistic Trend Family (MPTF), and the Extended Link Ratio Family (ELRF).

The PTF and ELRF frameworks are discussed in the paper 'Best Estimates for Reserves' (available on the CAS website at: www.casact.org/pubs/proceed/proceed00/00245.pdf). The paper has been on the CAS syllabus since January 2005.

PTF Modelling Framework

In the PTF modelling framework an optimal model is identified, equivalently, built or designed that captures the variability (volatility) in the incremental loss development array. The variability is described using four components of interest. Namely, trends in the three directions: **development period**, **accident period** and **calendar period**, and the variability of the data about the trend structure. The (process) variability is an integral part of the model.

A PTF model is succinctly described by four graphs; three graphs describe the trend structure in the three directions, the fourth graph depicts the process variability. The identified model is tested to ensure that the model assumptions are consistent with the data; including validation testing (by removal of years). The triangle is regarded as a sample path from the fitted model. Thus data simulated from the model should not be distinguishable from the original data in respect of trend structure and volatility about the trend structure.

MPTF Modelling Framework

The MPTF modelling framework is used to design (build) an optimal composite model for multiple incremental loss development arrays. The identified composite model captures (describes) the variability in each loss development array (a la PTF) and the relationships between them.

This has applications to modelling **multiple lines of business, multiple segments, multiple layers** and **credibility modelling**.

Relationships between lines of business, for example, involve two types of correlations; process correlation and parameter correlation.

ELRF Modelling Framework

The ELRF modelling framework in the first instance formalizes average link ratios (Age-to-age development factors) as regression estimators through the origin. It is also extended to include intercepts and constant trends for each development period across the accident periods. The framework provides statistical tests of link ratio methods including Mack, Murphy, and many extensions thereof.

The ELRF regression framework provides the ability to determine statistically whether the selected link ratios quantify salient features of the data and whether they have any predictive power.

It is very important to recognize that apart from the many failings of link ratio methods the actuary has no control over assumptions going forward in respect of projections. Indeed, there are no simple descriptors of the volatility in the data including how much inflation has been captured by the method!

By contrast, a PTF or MPTF has simple descriptors of the volatility in the data and assumptions made in deriving reserve distributions by accident year, calendar year and total are explicit and can be controlled (chosen).

Datasets used:

We have used the Employers' Liability and Marine Cargo data sets.

The Employers' Liability data is sub-divided into three data sets. As noted in Section 5, no information is available on the nature of these data sets. We have used two of these datasets to demonstrate the application of the modelling frameworks.

We have also tested a composite model created by combining the two subclasses of Employers' Liability data already used.

Marine Cargo paid loss data was available gross and net of reinsurance.

We have only modelled a small number of datasets to illustrate the rich statistical ideas incorporated in the above mentioned modelling frameworks.

General Comments on the Method

Does the Method produce a complete distribution of Outcomes?

Yes. The PTF and MPTF identified models forecast distributions for every cell in the future and their correlations, conditional on an explicit set of easily interpretable assumptions. These distributions include both process variability and parameter uncertainty. Distributions of aggregates across accident periods and calendar periods can also be obtained.

Would the Method be acceptable to the Profession?

Yes.

What Uncertainty has been measured by the method?

PTF and MPTF models include both process and parameter uncertainty.

There is an important distinction between variability and uncertainty and the two should not be used interchangeably.

"***Variability is a phenomenon in the physical world to be measured, analyzed and where appropriate explained. By contrast uncertainty is an aspect of knowledge***."

Sir David Cox.

For further details see:
http://www.insureware.com/Library/Technical/VariabilityandUncertainty.doc

Comment on difficulty of method to program & run

It is extremely difficult to program (but theoretically possible) in a spreadsheet form, but is easy to run in ICRFS-Plus as all the programming has been pre-specified. ICRFS-Plus is a point and click system with extremely fast algorithms.

How long did the tests take to program and complete.

Individual tests do not need to be programmed. It takes seconds for individual tests to be completed.

How difficult is it to apply judgement and / or amendments to the results within the program

It is not difficult. Judgement is made on the basis of accurate information about the volatility in the business.

How do the Class by Class results compare to the Aggregated results.

We do not believe in aggregating data across classes (and usually not even segments). We design a composite model that captures the variability and trend structure in each class and the relationships between classes. For example, see the composite model: Appendix F5

However, if two segments exhibit the same trend structure then it might make sense to aggregate the data.

Qualitative Results:

Qualitative results are provided for each dataset as follows.

Appendix F3: Employers Liability subclass “a”.

Appendix F4: Employers Liability subclass “b”.

Appendix F5: Employers Liability Composite Model.

Appendix F6: Marine Cargo Gross and Net.

The Employers’ Liability data used was provided sub-divided between three sub-classes. Two of those subclasses have been used in developing the results shown in this appendix. For the purpose of this appendix only these two sub-classes have been referred to as “a” and “b”. No information has been available regarding the nature of these Employers’ liability sub-classes as explained in Section 5.

Quantitative Results

All quantitative results are included in the same workbooks as the qualitative results. Please see corresponding Quantile and model diagnostic spreadsheets.

Tests to establish if model is reasonable

Comment on validity of underlying model on data set.

All the models we have produced for the datasets on the accompanying spreadsheets are statistically valid. The model assumptions have been tested and met. In one case we simulated three triangles from the (PTF identified) model for the real data. It is almost impossible to distinguish the simulated triangles and the real data in respect of salient features. See: Appendix F3

Comments on results by accident year

The forecast means and standard deviations of distributions by accident year have been included. They are based on explicit easily interpretable assumptions that can be related to the historical experience.

Comments on Coefficient of Variations (COV) & standard deviation measured by accident year

The coefficient of variations decrease as the accident year increases for all our PTF and MPTF based models as should often be expected of a good model. Similarly, the standard deviation of the reserves also increases. HOWEVER, this is not true for an Extended Link Ratio Family (ELRF) based models that also include average link ratios!

Comments on correlations of standard error & COV when aggregating classes together compared to class by class results.

As mentioned above, we do not aggregate the data. We design a composite model for the individual classes that captures the volatility in each class and their inter relationships (two types of correlations). For an example of a composite model: See: Appendix F5

Comment on how stable results are when development factors or residuals are changed

A PTF or an MPTF based model captures the volatility in the data in respect of trend structure and volatility about the trend structure, called process variability. Assumptions in respect of future trend (and uncertainty thereof) and process variability are explicit and need to be argued for in respect of past volatility. To illustrate this very important point, suppose the calendar year trend has been relatively stable in the last seven years and its estimate is 10%+_3%. Typically when doing a projection of reserve distributions you would assume that the true trend is a random number from a normal distribution with mean .10 and std. dev. 0.03. That is, all the reserve distributions by Calendar year, accident year and Total are based on this explicit assumption. Suppose we now change the assumption to 15%+_ 3%, all the resulting distributions will be different.

Incidentally, if the trend has been relatively stable for the last seven years (as in this example) then removal of years (diagonals) should yield statistically stable reserve distributions beyond the last year and the volatility of the numbers in the cells removed should have been predicted years ago by the then-estimated model.
See: Appendix F3

Tests for Goodness of Fit and Prediction Error.

Comment on tests of Standardised Residuals

Normality tests were applied to all residuals including during validation analysis where normality tests were applied to the residuals not used during model estimation.

Comment on tests of residual patterns

In model identification various discriminatory statistical criteria are used to ensure parsimony. All remaining patterns in the residuals are random.

We also perform validation analysis and simulate triangles from fitted models to ensure that the real data is indistinguishable from the simulated data.

Test the Amended Model Fit by Removing 2 diagonals

Removal of calendar years (diagonals) is called validation analysis, and is part of extracting information from the data. In order to observe stability in the reserve distributions the most recent trends must be stable. If you do have stability of most recent trends, then the most important test is whether the model years ago predicted the volatility of the numbers in the years that have been removed. Results of these kinds of critical tests are done shown in Appendix F3

Appendix F2.1

Variability and Uncertainty

There is an important distinction between variability and uncertainty and the two should not be used interchangeably.

"***Variability is a phenomenon in the physical world to be measured, analyzed and where appropriate explained. By contrast uncertainty is an aspect of knowledge***."

Sir David Cox.

Uncertainty and variability are philosophically very different and it is common for them to be kept separate in risk analyses modeling.

Variability is the effect of chance and a function of the system. It is not reducible through either study or further measurement (may be reduced through changing the system).

Uncertainty is the assessor's lack of knowledge (level of ignorance) about the parameters that characterize the physical system that is being modeled. It is sometimes reducible through further measurement or study.

Uncertainly has also been called "fundamental uncertainly" or "degree of belief".

Simple examples

Suppose a symmetric coin is tossed 100 times and X denotes the no. of heads.

The mean number of heads (the mean of X) is 50. The SD of X is 5. The Binomial probability of each possible outcome of X (0, 1, 2,..100) is known precisely. There is <u>no uncertainty</u> about the coin's <u>variability</u>.

A 100% **confidence interval** for the mean is [50, 50]. There is no uncertainty in the mean and indeed in any of the probabilities of the outcome X. The probability that X=50 (the mean) is approximately 0.08. A 95% **prediction interval** for the outcome X is [40,60]. This 95% prediction interval <u>cannot</u> be shortened.

Suppose we do not know the true probability of a head, p, because the coin is mutilated. Suppose also that before the coin is tossed 100 times, it is tossed 10 times to get an estimate of the probability p. Suppose for the sake of argument 5 heads are observed. Now the estimate of .5 of the probability of a head (in one toss) is uncertain.

We can create a confidence interval (CI) for p, and also for the mean 100p of the number of heads in 100 tosses. The CI is an interval around the estimated mean, namely, 50=100*.5. The CI is not the same as a prediction interval for the outcome in 100 tosses.

A prediction interval includes process variability and is wider than a confidence interval. A confidence interval is for a parameter, which is a constant. A prediction interval is for a random variable. [The loss reserve is a random variable].

Now, a 95% prediction interval for the number of heads in 100 tosses will be wider than [40, 60], say, [35, 65]. This interval can only be reduced to at best [40, 60] by reducing the parameter uncertainty through more (prior) sampling. But you **cannot** make a 95% prediction interval narrower than [40, 60].

Consider another example with the same mean. A symmetric roulette wheel, numbered, 0, 1, 2, 3,....., 100 that is turned only once, and let X be the random variable that represents the outcome. The mean of X is 50, SD is 29. There is no uncertainty about the variability in the outcome X. The probability that X=50 is 1/101.

A 100% **confidence interval** for the mean is [50, 50] (just like the coin). A 95% prediction interval for the outcome X is [2, 97], for example.

Each process (symmetric coin and symmetric roulette wheel) has the same mean, or if you like the same "best estimate". Which one requires more capital?

So "best estimate" is pretty useless and a "range of estimates”, also pretty useless, notwithstanding the fact that how do you know if an estimate is "best"?

In general, only in the presence of a probabilistic framework can you assess "best". Indeed it is only in a probabilistic framework that you identify (build) a model that represents the variability in the data.

Probabilistic Trend Family (PTF) of models described in the paper *“Best Estimates for Reserves”*

In the PTF modeling framework a model is identified (built) that quantifies the variability in the data.

Variability is decomposed into Trends + Process Variability. You cannot reduce process variability. You can reduce parameter uncertainty (for the past) by having a larger triangle or a related triangle so that you can do some credibility modeling. Future variability on a <u>log scale</u>= parameter uncertainty + process variability.

In general, parameter uncertainty increases the length of a prediction interval, alternatively, the skewness (both mean and coefficient of variation of the loss reserve distribution). In the case of no parameter uncertainty the prediction distribution’s skewness is determined by only the process variability inherent in the data that cannot be reduced.

Summary of Appendix F3

Appendix F3 is outputs from the model for Paid Losses adjusted by number of claims reported in first Development year for Employers' Liability subclass "a". (Note Appendix F4 is based on a different subclass of Employers' Liability data referred to as subclass "b".)

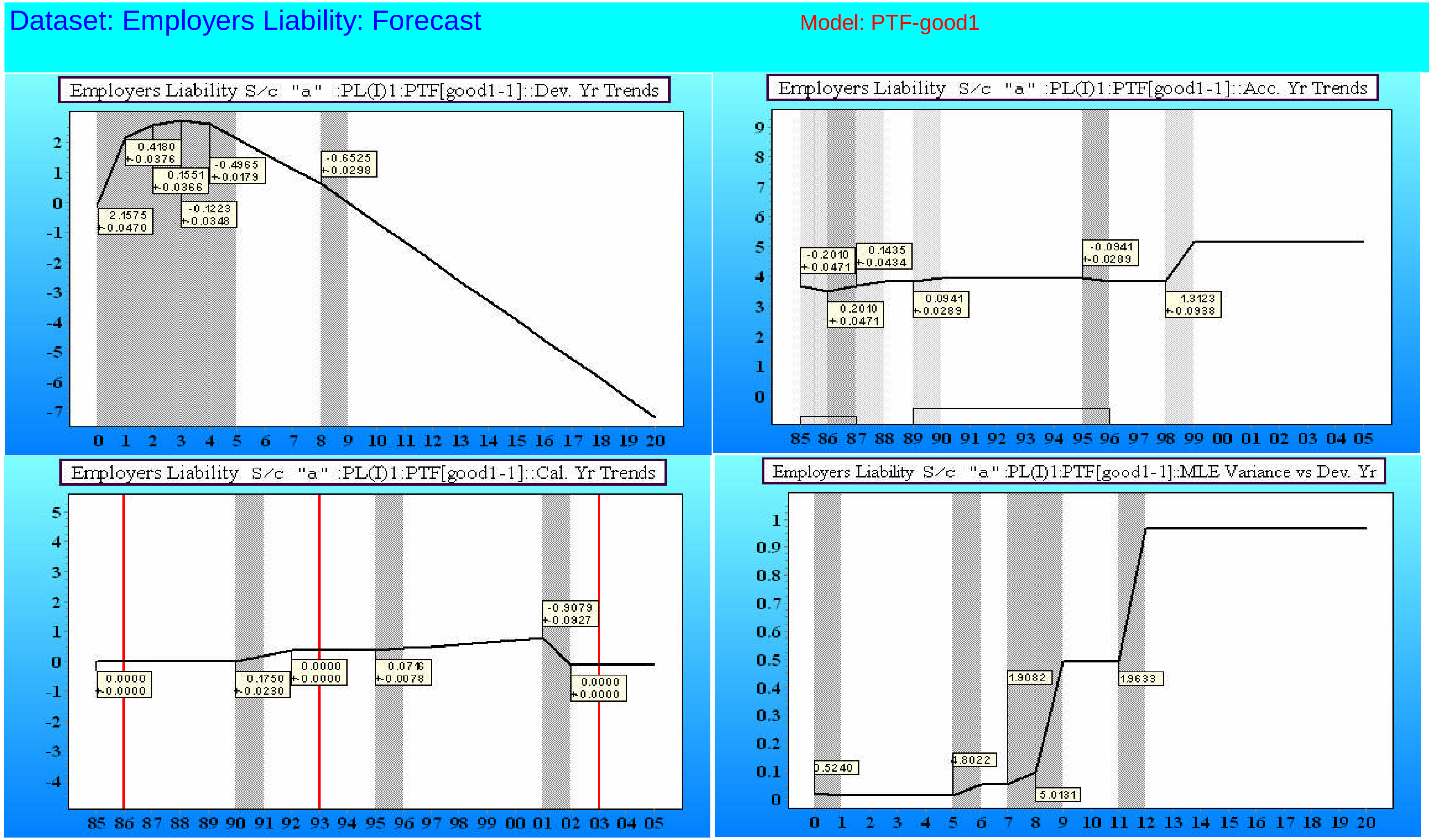
Appendix F3.2 provides the model displays

Appendices F3.3 and F3.4 give forecast distributions under two different future scenarios, based on the volatility found in the data.

Appendix F3.5 summarises validation analysis based on removal of the last three Calendar years. Whether a model validates well or not depends on the trend structure in the data.

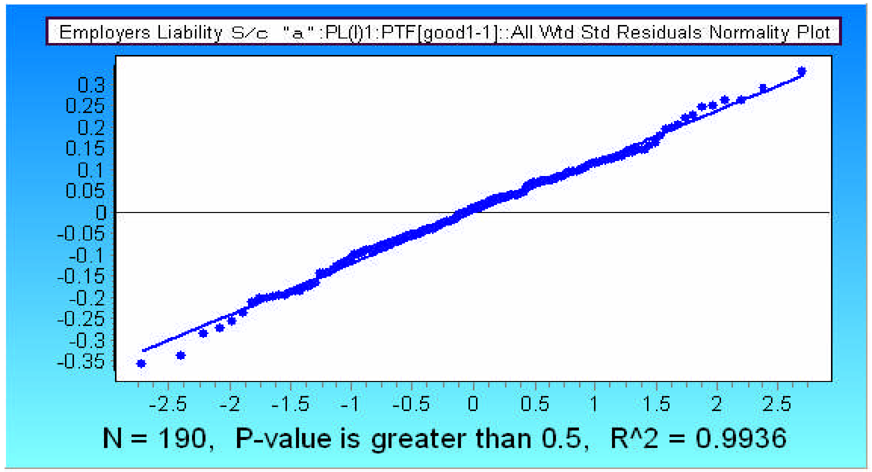
Appendix F3.6 demonstrates that the fitted model captures accurately the volatility in the data. Three triangles are simulated from the fitted model and are seen to be indistinguishable from the real data. Moreover forecast distributions are the same.

Appendix F3.7 shows the relationship between the Paid Losses and the Case Reserve Estimates.



The model display depicts the trends in the three directions plus the process variability about the trend structure.

Note that Weighted Std Residuals are normally distributed - see Normality display below



The volatility of the residuals (process variability) is an integral part of the model.

Note that in respect of Calendar year trends, there is a huge drop in respect of 2001/2002 and the more recent trend is zero.

To assume that the trend will be zero for the next 20 years is very optimistic. We call this scenario 1.

Scenario 2 assumes that the trend for the next six years is 7.16%+-0.78% and thereafter reduces to zero.

Note that all forecasts distributions are conditional on an explicit, easily interpretable set of assumptions. Two key assumptions involve the future Calendar year trend volatility and the process variability in the past data depicted by the bottom right graph in the model display.

Another scenario that can easily be considered is the 17.5%+-2.3% at least for the next two years.

More importantly it would be important to investigate what might have caused the huge drop from 2001-2002 and the relatively constant trend from 1995-2001

Dataset: Employers Liability: Forecast Distributions

Model: PTF-good1

Scenario 1: Inflation continues with zero trend

Reserve Forecast Table

Accident Period vs Development Period

	Cal. Per. Total	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	Reserve	Ultimate
1985	327,998	327,998	2,822,314	4,286,674	5,005,747	4,429,542	2,695,789	1,994,968	1,447,601	902,569	572,356	298,319	167,149	118,820	66,699	37,476	21,076	11,864	2,506	1,315	691	363	0	25,580,529
	363,421	363,421	2,823,353	3,740,983	4,851,694	4,302,127	3,337,231	2,176,450	1,561,371	1,390,771	628,642	87,377	200,919	39,315	23,958	29,482	9,494	11,340	0	3,845	0	-1,245	0	0
1986	3,137,155	314,841	2,709,078	4,114,650	4,804,828	4,251,711	3,082,343	2,282,197	1,389,616	866,408	549,417	307,566	172,337	122,512	68,775	38,644	21,734	4,588	2,405	1,262	663	348	348	23,570,729
	3,165,803	342,451	2,787,749	3,771,543	4,460,001	4,101,358	2,822,690	2,145,689	1,196,979	827,302	537,642	372,893	48,850	-7,546	123,409	51,765	-13,445	215	0	237	600	487	487	487
1987	7,323,749	327,998	2,822,314	4,286,674	5,005,747	5,276,625	3,826,934	2,377,407	1,447,601	902,569	614,748	344,160	192,853	137,106	76,972	43,252	9,126	4,780	2,506	1,315	691	363	1,054	26,164,111
	6,900,955	372,223	2,597,547	4,597,842	4,439,529	4,171,869	4,073,713	2,327,095	1,894,814	807,301	428,835	165,943	100,930	94,769	1,811	88,567	269	0	0	0	951	507	1,103	1,103
1988	12,304,967	362,256	3,117,370	4,735,250	6,584,540	6,941,315	4,224,345	2,624,591	1,598,294	1,070,395	729,103	408,206	228,757	162,641	91,313	19,252	10,076	5,278	2,767	1,452	763	401	2,616	32,732,389
	11,547,134	326,349	3,172,571	5,176,011	5,379,296	6,387,889	4,521,772	2,811,563	1,323,411	1,445,179	311,985	562,611	405,192	31,637	434,136	433,892	7,580	2,729	-4,029	1,973	1,050	560	2,388	2,388
1989	16,990,391	351,978	3,028,922	5,478,566	7,621,744	6,744,373	4,104,489	2,550,125	1,667,891	1,117,061	760,927	426,045	238,766	169,766	35,773	18,706	9,790	5,128	2,689	1,411	741	390	5,230	34,488,631
	16,930,957	398,416	3,384,938	6,261,944	7,232,344	6,056,747	4,172,737	2,638,001	1,999,745	1,280,583	518,592	432,369	47,274	15,294	16,795	199	4,548	22,876	3,611	1,917	1,020	544	4,416	4,416
1990	20,097,157	379,738	3,890,728	7,039,194	8,219,945	7,273,483	4,426,533	2,953,994	1,932,258	1,294,265	881,725	493,729	276,725	73,872	38,592	20,180	10,561	5,532	2,900	1,522	799	420	11,174	39,101,530
	20,777,343	338,276	3,981,902	7,269,814	7,273,202	6,764,962	4,663,626	3,778,364	1,786,086	2,035,300	966,986	143,383	21,026	-117	20,750	28,458	18,336	7,353	3,896	2,069	1,101	587	8,988	8,988
1991	26,697,014	389,243	3,989,913	6,059,969	7,076,465	6,261,666	4,093,134	2,731,640	1,786,901	1,196,962	815,477	456,656	96,124	63,595	33,224	17,372	9,092	4,763	2,497	1,310	688	362	18,712	33,795,441
	25,151,751	357,601	3,930,777	6,254,476	7,270,856	5,543,963	4,385,023	2,271,060	2,163,980	1,131,551	314,255	-113,095	-18,502	267,189	15,518	2,076	11,973	6,330	3,354	1,781	948	505	14,566	14,566
1992	33,503,376	354,477	3,050,078	4,632,526	5,409,584	5,141,467	3,361,048	2,243,179	1,467,447	983,024	669,757	140,906	73,482	48,615	25,398	13,280	6,950	3,641	1,909	1,002	526	277	27,585	27,187,447
	33,007,909	406,312	3,485,198	4,787,544	6,016,700	4,772,276	3,259,134	1,978,110	1,388,877	645,655	305,919	90,952	22,476	12,200	-11,490	17,351	9,153	4,839	2,564	1,361	724	386	20,975	20,975
1993	33,263,505	295,203	2,540,061	3,857,901	4,838,880	4,599,349	3,006,808	2,006,857	1,312,915	879,548	225,216	117,345	61,195	40,486	21,151	11,060	5,788	3,032	1,590	834	438	230	44,123	24,957,383
	32,793,140	286,900	2,500,351	4,252,026	5,428,687	4,340,034	3,008,771	2,731,563	1,181,851	728,782	371,273	68,942	10,783	3,295	27,456	14,450	7,622	4,030	2,135	1,134	603	322	32,974	32,974
1994	31,473,817	335,836	2,889,680	4,714,267	5,913,229	5,620,882	3,674,814	2,452,831	1,604,758	404,177	256,216	133,496	69,618	46,059	24,062	12,582	6,585	3,449	1,808	949	498	262	96,255	27,930,657
	32,018,107	359,335	2,940,485	4,749,103	5,360,802	5,962,373	4,131,260	2,780,882	1,056,343	386,307	226,250	-15,475	-103,263	59,485	31,235	16,439	8,671	4,584	2,429	1,290	686	366	71,008	71,008
1995	29,114,365	370,278	3,422,255	5,583,414	7,003,692	6,657,867	4,352,989	2,905,638	715,087	445,629	282,493	147,187	76,757	50,782	26,530	13,872	7,260	3,803	1,994	1,046	550	289	182,884	29,632,347
	29,082,604	272,072	3,351,803	5,088,547	7,048,157	6,656,030	3,895,374	1,595,575	794,846	513,843	47,316	185,902	62,026	65,586	34,439	18,125	9,561	5,054	2,678	1,422	757	404	101,046	101,046
1996	29,365,438	373,692	3,453,624	5,634,678	7,067,685	6,718,980	4,392,885	1,103,615	671,648	418,554	265,331	138,246	72,095	47,698	24,919	13,030	6,819	3,572	1,873	983	516	271	310,023	32,887,118
	31,282,166	369,338	3,373,646	5,580,846	7,733,248	9,576,139	4,414,667	466,863	777,454	204,599	80,294	111,281	58,242	61,592	32,342	17,022	8,979	4,747	2,515	1,336	711	379	147,766	147,766
1997	30,508,236	452,194	4,179,321	6,819,032	8,553,572	8,132,086	2,002,289	1,243,249	756,628	471,512	298,902	155,738	81,217	53,733	28,072	14,679	7,682	4,024	2,110	1,107	582	306	648,151	32,400,477
	27,665,268	339,282	3,732,991	5,227,094	9,228,797	8,435,768	2,888,546	1,135,434	547,640	216,773	240,279	125,361	65,611	69,385	36,434	19,175	10,115	5,348	2,833	1,505	801	427	294,613	294,613
1998	32,778,418	457,495	4,228,506	6,899,649	8,655,029	3,100,593	1,885,782	1,170,908	712,602	444,076	281,510	146,676	76,491	50,607	26,438	13,824	7,235	3,790	1,987	1,043	548	288	1,054,514	39,515,120
	32,987,794	278,722	3,494,212	5,806,965	11,247,156	10,692,710	4,906,059	1,673,282	361,501	147,905	226,298	118,066	61,793	65,347	34,314	18,060	9,526	5,036	2,669	1,417	754	402	318,921	318,921
1999	36,465,760	1,566,708	14,482,146	23,629,812	11,083,082	9,807,436	5,968,885	3,708,648	2,258,555	1,408,419	893,178	465,557	242,881	160,753	84,015	43,948	23,009	12,058	6,324	3,320	1,744	917	5,604,680	46,760,396
	34,807,630	261,587	3,279,725	6,885,099	10,432,638	11,101,285	6,269,954	2,925,429	532,779	463,153	716,414	374,293	196,167	207,678	109,162	57,510	30,367	16,070	8,523	4,530	2,413	1,288	1,155,713	1,155,713
2000	49,001,211	1,397,800	12,921,401	7,882,100	9,204,080	8,144,705	4,956,933	3,079,892	1,875,645	1,169,639	741,750	386,627	201,704	133,499	69,771	36,497	19,109	10,013	5,252	2,757	1,449	762	7,734,366	41,191,749
	37,162,551	167,544	3,509,353	6,962,743	9,221,279	9,569,081	4,027,383	718,487	442,452	384,631	594,954	310,836	162,909	172,468	90,655	47,760	25,218	13,346	7,078	3,762	2,004	1,070	1,223,561	1,223,561
2001	66,251,939	1,389,714	4,802,771	7,293,884	8,517,209	7,536,891	4,587,013	2,850,049	1,735,671	1,082,352	686,396	357,774	186,651	123,537	64,564	33,773	17,683	9,266	4,860	2,551	1,341	705	11,744,187	37,391,773
	38,780,673	158,189	2,663,632	6,601,908	8,424,381	7,799,475	498,312	664,868	409,434	355,928	550,555	287,639	150,752	159,598	83,890	44,196	23,336	12,350	6,550	3,481	1,854	990	1,268,375	1,268,375
2002	32,258,341	557,264	4,794,893	7,281,920	8,503,238	7,524,529	4,579,489	2,845,374	1,732,824	1,080,577	685,270	357,188	186,345	123,334	64,458	33,718	17,654	9,251	4,852	2,547	1,338	704	19,249,453	38,546,594
	36,338,872	155,809	3,402,607	7,405,743	8,332,982	821,511	497,495	663,778	408,762	355,344	549,652	287,168	150,504	159,336	83,752	44,123	23,298	12,329	6,539	3,476	1,851	988	1,548,518	1,548,518
2003	36,334,581	401,468	3,454,371	5,246,093	6,125,964	5,420,875	3,299,188	2,049,885	1,248,374	778,477	493,687	257,328	134,248	88,853	46,438	24,291	12,718	6,665	3,496	1,835	964	507	19,993,792	29,515,092
	38,753,674	476,574	3,196,497	5,848,228	669,867	591,839	358,409	478,204	294,483	255,999	395,984	206,883	108,427	114,790	60,337	31,788	16,785	8,882	4,711	2,504	1,334	712	1,339,824	1,339,824
2004	36,625,332	313,423	2,696,799	4,095,582	4,782,489	4,232,032	2,575,649	1,600,328	974,595	607,751	385,418	200,894	104,806	69,367	36,253	18,964	9,929	5,203	2,729	1,433	753	396	19,704,570	22,920,584
	37,600,773	203,281	3,012,732	448,295	522,960	462,044	279,807	373,330	229,901	199,857	309,141	161,512	84,648	89,616	47,105	24,816	13,104	6,934	3,678	1,955	1,041	556	1,168,228	1,168,228
2005	34,703,794	284,888	2,451,278	3,722,712	4,347,083	3,846,740	2,341,157	1,454,631	885,866	552,420	350,328	182,604	95,265	63,052	32,953	17,238	9,025	4,729	2,481	1,302	684	360	20,361,907	20,635,105
	32,991,052	273,198	270,806	407,481	475,349	419,978	254,333	339,341	208,970	181,661	280,997	146,808	76,942	81,457	42,816	22,557	11,911	6,303	3,343	1,777	946	505	1,119,926	1,119,926
	Total Fitted/Paid		2006	2007	2008	2009	2010	2011	2012	2013	2014	2015	2016	2017	2018	2019	2020	2021	2022	2023	2024	2025	Total Reserve	Total Ultimate
Cal. Per. Total	598,526,543		31,181,087	25,252,076	18,851,459	12,677,799	7,692,458	4,668,475	2,777,393	1,633,673	940,569	509,561	280,703	159,942	83,535	43,601	22,373	11,337	5,581	2,562	1,080	360	106,795,624	666,905,202
	560,109,578		1,751,257	1,534,726	1,457,655	1,198,589	976,256	859,996	641,822	492,459	397,906	233,529	146,915	105,717	56,116	29,878	15,888	8,435	4,441	2,275	1,127	505	4,607,172	4,607,172

1 Unit = $1

Inside the table the blues are the observed, the black are means of log-normal distributions and the reds are the corresponding Std Devs of the log-normals.

Given that on the log-scale we have normal distributions in every cell about the trend structure, the corresponding distributions on the dollar scale are log-normals (that are correlated via the parameter estimates).

The predictive log-normal distributions include parameter uncertainty and process variability.

Summaries by Year

Accident Yr Summary

Acc. Yr	Mean Reserve	Mean Ultimate	Standard Dev.	CV Reserve	CV Ultimate
1985	0	25,580,529	0	****	0
1986	348	23,570,729	487	1.4	0
1987	1,054	26,164,111	1,103	1.05	0
1988	2,616	32,732,389	2,388	0.91	0
1989	5,230	34,488,631	4,416	0.84	0
1990	11,174	39,101,530	8,988	0.8	0
1991	18,712	33,795,441	14,566	0.78	0
1992	27,585	27,187,447	20,975	0.76	0
1993	44,123	24,957,383	32,974	0.75	0
1994	96,255	27,930,657	71,008	0.74	0
1995	182,884	29,632,347	101,046	0.55	0
1996	310,023	32,887,118	147,766	0.48	0
1997	648,151	32,400,477	294,613	0.45	0.01
1998	1,054,514	39,515,120	318,921	0.3	0.01
1999	5,604,680	46,760,396	1,155,713	0.21	0.02
2000	7,734,366	41,191,749	1,223,561	0.16	0.03
2001	11,744,187	37,391,773	1,268,375	0.11	0.03
2002	19,249,453	38,546,594	1,548,518	0.08	0.04
2003	19,993,792	29,515,092	1,339,824	0.07	0.05
2004	19,704,570	22,920,584	1,168,228	0.06	0.05
2005	20,361,907	20,635,105	1,119,926	0.06	0.05
Total	106,795,624	666,905,202	4,607,172	0.04	0.01

1 Unit = $1

Calendar Yr Summary

Calendar Yr	Mean Reserve	Standard Dev.	CV Reserve
2006	31,181,087	1,751,257	0.06
2007	25,252,076	1,534,726	0.06
2008	18,851,459	1,457,655	0.08
2009	12,677,799	1,198,589	0.09
2010	7,692,458	976,256	0.13
2011	4,668,475	859,996	0.18
2012	2,777,393	641,822	0.23
2013	1,633,673	492,459	0.3
2014	940,569	397,906	0.42
2015	509,561	233,529	0.46
2016	280,703	146,915	0.52
2017	159,942	105,717	0.66
2018	83,535	56,116	0.67
2019	43,601	29,878	0.69
2020	22,373	15,888	0.71
2021	11,337	8,435	0.74
2022	5,581	4,441	0.8
2023	2,562	2,275	0.89
2024	1,080	1,127	1.04
2025	360	505	1.4
Total	106,795,624	4,607,172	0.04

1 Unit = $1

In order to find distributions of aggregates by accident year, calendar year and total of all years, we simulate from the predictive log-normal distributions of each cell, incorporating their correlations.

We do this because there is no analytical form for the sum of log-normals.

However, note that the means and Std Devs of the distributions of aggregates are computed via the model using second order moments,

The sample quantiles (percentiles) below are based on 10,000 simulations of the forecast triangle.

The Kernel is fitted to the sample frequency plot whereas the log-normal and gamma are just based on the means and std. devs. of the actual forecast distributions.

Note that the distributions are slightly skewed.

Below are the quantiles (percentiles) for the Aggregate Reserve Distribution predicted by the model.

Quantiles and VaR   Mean = 106.796, S.D. = 4.607, Provision = 106.796, 1 Unit = $1,000,000

Quantile Statistics and Value at Risk (Acc Year: Total)

%	Sample			Kernel			LogNormal			Gamma		
	Quantile	# S.D.'s	V-a-R	Quantile	# S.D.'s	V-a-R	Quantile	# S.D.'s	V-a-R	Quantile	# S.D.'s	V-a-R
99.995	124.733	3.893	17.937	125.664	4.095	18.869	126.185	4.209	19.39	125.664	4.095	18.868
99.99	124.519	3.847	17.723	125.259	4.007	18.463	125.256	4.007	18.46	124.786	3.905	17.99
99.98	124.349	3.81	17.554	124.752	3.898	17.957	124.293	3.798	17.498	123.874	3.707	17.078
99.97	124.185	3.774	17.389	124.386	3.818	17.59	123.713	3.672	16.918	123.323	3.587	16.528
99.96	124.104	3.757	17.309	124.083	3.752	17.287	123.294	3.581	16.498	122.924	3.501	16.129
99.95	123.393	3.603	16.598	123.815	3.694	17.02	122.963	3.509	16.168	122.61	3.432	15.814
99.94	123.114	3.542	16.319	123.58	3.643	16.784	122.69	3.45	15.894	122.349	3.376	15.553
99.93	123.108	3.541	16.313	123.366	3.597	16.571	122.456	3.399	15.66	122.126	3.328	15.33
99.92	122.869	3.489	16.073	123.172	3.555	16.377	122.252	3.355	15.456	121.931	3.285	15.135
99.91	122.833	3.481	16.037	122.995	3.516	16.199	122.07	3.315	15.274	121.758	3.248	14.962
99.9	122.422	3.392	15.626	122.831	3.481	16.036	121.906	3.28	15.11	121.601	3.214	14.805
99.8	121.548	3.202	14.753	121.605	3.214	14.809	120.797	3.039	14.001	120.539	2.983	13.744
99.7	120.51	2.977	13.714	120.732	3.025	13.937	120.12	2.892	13.324	119.889	2.842	13.094
99.6	119.768	2.816	12.972	120.095	2.887	13.299	119.625	2.785	12.829	119.414	2.739	12.618
99.5	119.334	2.722	12.539	119.619	2.783	12.824	119.232	2.699	12.437	119.036	2.657	12.24
99.4	119.034	2.656	12.239	119.238	2.701	12.443	118.905	2.628	12.11	118.721	2.588	11.925
99.3	118.738	2.592	11.943	118.92	2.632	12.124	118.624	2.567	11.829	118.45	2.53	11.654
99.2	118.52	2.545	11.725	118.644	2.572	11.848	118.377	2.514	11.582	118.211	2.478	11.416
99.1	118.24	2.484	11.445	118.401	2.519	11.606	118.157	2.466	11.361	117.998	2.432	11.202
99	117.946	2.42	11.15	118.184	2.472	11.388	117.957	2.423	11.161	117.805	2.39	11.009
98	116.635	2.136	9.839	116.785	2.168	9.989	116.578	2.123	9.783	116.469	2.1	9.674
97	115.812	1.957	9.017	115.93	1.983	9.134	115.712	1.935	8.916	115.627	1.917	8.832
96	115.125	1.808	8.33	115.271	1.84	8.475	115.065	1.795	8.269	114.996	1.78	8.201
95	114.638	1.702	7.842	114.728	1.722	7.933	114.541	1.681	7.745	114.485	1.669	7.689
94	114.156	1.598	7.361	114.266	1.621	7.47	114.096	1.585	7.301	114.051	1.575	7.255
93	113.735	1.506	6.94	113.867	1.535	7.071	113.708	1.5	6.913	113.671	1.492	6.876
92	113.411	1.436	6.615	113.513	1.458	6.718	113.362	1.425	6.566	113.332	1.419	6.536
91	113.101	1.369	6.305	113.196	1.389	6.401	113.048	1.357	6.252	113.024	1.352	6.228
90	112.795	1.302	5.999	112.907	1.326	6.111	112.759	1.294	5.964	112.741	1.29	5.945
89	112.542	1.247	5.747	112.64	1.268	5.844	112.492	1.236	5.696	112.478	1.233	5.683
88	112.319	1.199	5.524	112.39	1.214	5.594	112.242	1.182	5.446	112.233	1.18	5.437
87	112.097	1.151	5.302	112.154	1.163	5.358	112.007	1.131	5.211	112.001	1.13	5.206
86	111.863	1.1	5.068	111.929	1.114	5.134	111.784	1.083	4.989	111.782	1.082	4.987
85	111.637	1.051	4.841	111.716	1.068	4.92	111.573	1.037	4.777	111.574	1.037	4.778
84	111.445	1.009	4.65	111.51	1.023	4.714	111.371	0.993	4.575	111.375	0.994	4.579
83	111.278	0.973	4.483	111.311	0.98	4.516	111.177	0.951	4.382	111.184	0.953	4.389
82	111.095	0.933	4.3	111.119	0.938	4.324	110.991	0.911	4.196	111.001	0.913	4.205
81	110.88	0.887	4.084	110.934	0.898	4.138	110.812	0.872	4.017	110.824	0.874	4.028
80	110.705	0.849	3.909	110.754	0.859	3.958	110.639	0.834	3.843	110.653	0.837	3.857
79	110.503	0.805	3.707	110.579	0.821	3.783	110.471	0.798	3.675	110.487	0.801	3.691
78	110.351	0.772	3.556	110.408	0.784	3.612	110.308	0.762	3.512	110.325	0.766	3.53
77	110.182	0.735	3.386	110.242	0.748	3.446	110.149	0.728	3.354	110.168	0.732	3.373
76	110.051	0.707	3.255	110.079	0.713	3.284	109.995	0.694	3.199	110.015	0.699	3.22
75	109.894	0.673	3.099	109.921	0.678	3.125	109.844	0.662	3.048	109.866	0.666	3.07
74	109.728	0.636	2.932	109.765	0.645	2.97	109.696	0.63	2.901	109.72	0.635	2.924
73	109.573	0.603	2.777	109.613	0.612	2.818	109.552	0.598	2.756	109.577	0.604	2.781
72	109.435	0.573	2.639	109.464	0.579	2.669	109.41	0.567	2.615	109.436	0.573	2.641
71	109.285	0.54	2.49	109.318	0.548	2.523	109.271	0.537	2.476	109.298	0.543	2.503
70	109.153	0.512	2.357	109.175	0.516	2.379	109.135	0.508	2.339	109.163	0.514	2.367

TABLE CONTINUED ON NEXT PAGE

Quantiles and VaR Mean = 106.796, S.D. = 4.607, Provision = 106.796, 1 Unit = $1,000,000

Quantile Statistics and Value at Risk (Acc Year: Total)

%	Sample			Kernel			LogNormal			Gamma		
	Quantile	# S.D.'s	V-a-R	Quantile	# S.D.'s	V-a-R	Quantile	# S.D.'s	V-a-R	Quantile	# S.D.'s	V-a-R
69	108.999	0.478	2.203	109.035	0.486	2.239	109	0.479	2.205	109.029	0.485	2.234
68	108.823	0.44	2.028	108.897	0.456	2.101	108.868	0.45	2.072	108.898	0.456	2.102
67	108.698	0.413	1.903	108.762	0.427	1.966	108.738	0.422	1.942	108.768	0.428	1.973
66	108.565	0.384	1.769	108.629	0.398	1.834	108.609	0.394	1.813	108.64	0.4	1.845
65	108.454	0.36	1.658	108.499	0.37	1.703	108.482	0.366	1.686	108.514	0.373	1.718
64	108.346	0.336	1.55	108.37	0.342	1.574	108.356	0.339	1.561	108.389	0.346	1.593
63	108.209	0.307	1.414	108.243	0.314	1.447	108.232	0.312	1.436	108.265	0.319	1.469
62	108.094	0.282	1.299	108.118	0.287	1.323	108.109	0.285	1.313	108.142	0.292	1.347
61	107.976	0.256	1.18	107.995	0.26	1.199	107.987	0.259	1.192	108.021	0.266	1.225
60	107.861	0.231	1.066	107.872	0.234	1.077	107.866	0.232	1.071	107.9	0.24	1.105
59	107.734	0.204	0.938	107.751	0.207	0.955	107.747	0.206	0.951	107.781	0.214	0.985
58	107.611	0.177	0.815	107.631	0.181	0.835	107.628	0.181	0.832	107.662	0.188	0.866
57	107.5	0.153	0.705	107.511	0.155	0.716	107.509	0.155	0.714	107.544	0.162	0.748
56	107.389	0.129	0.594	107.392	0.129	0.597	107.392	0.129	0.596	107.426	0.137	0.631
55	107.265	0.102	0.47	107.274	0.104	0.478	107.275	0.104	0.479	107.309	0.111	0.514
54	107.151	0.077	0.356	107.156	0.078	0.36	107.158	0.079	0.363	107.193	0.086	0.397
53	107.045	0.054	0.249	107.038	0.053	0.242	107.042	0.054	0.247	107.076	0.061	0.281
52	106.915	0.026	0.119	106.92	0.027	0.124	106.927	0.028	0.131	106.961	0.036	0.165
51	106.799	0.001	0.004	106.802	0.001	0.007	106.811	0.003	0.016	106.845	0.011	0.049
50	106.684	-0.024	-0.112	106.684	-0.024	-0.111	106.696	-0.022	-0.099	106.729	-0.014	-0.066

Below are qunatiles and VaR statistics for the next calendar year 2006

Note that the distributions here are more skewed than for the aggregate reserves.

Quantile Statistics and Value at Risk (Cal. Yr: 2006)

%	Sample			Kernel			LogNormal			Gamma		
	Quantile	# S.D.'s	V-a-R	Quantile	# S.D.'s	V-a-R	Quantile	# S.D.'s	V-a-R	Quantile	# S.D.'s	V-a-R
99.95	37.521	3.62	6.34	37.663	3.701	6.482	37.447	3.578	6.265	37.268	3.476	6.087
99.9	37.109	3.385	5.928	37.222	3.449	6.041	37.028	3.339	5.847	36.874	3.251	5.693
99.8	36.577	3.081	5.396	36.739	3.174	5.558	36.59	3.089	5.409	36.461	3.015	5.28
99.7	36.371	2.964	5.19	36.464	3.017	5.283	36.323	2.936	5.142	36.208	2.871	5.027
99.6	36.185	2.857	5.004	36.264	2.902	5.083	36.129	2.825	4.948	36.023	2.765	4.842
99.5	36.114	2.817	4.933	36.102	2.81	4.921	35.975	2.737	4.794	35.877	2.681	4.695
99.4	35.846	2.664	4.665	35.967	2.733	4.785	35.846	2.664	4.665	35.754	2.611	4.573
99.3	35.771	2.621	4.59	35.851	2.666	4.67	35.736	2.601	4.555	35.649	2.551	4.468
99.2	35.678	2.568	4.497	35.749	2.608	4.567	35.639	2.546	4.458	35.557	2.499	4.376
99.1	35.523	2.48	4.342	35.659	2.557	4.478	35.553	2.496	4.372	35.474	2.451	4.293
99	35.492	2.462	4.311	35.579	2.511	4.397	35.475	2.452	4.293	35.399	2.409	4.218
98	34.968	2.163	3.787	35.015	2.189	3.834	34.936	2.144	3.755	34.882	2.113	3.701
97	34.596	1.95	3.415	34.653	1.982	3.472	34.599	1.951	3.417	34.557	1.928	3.376
96	34.338	1.803	3.157	34.39	1.832	3.209	34.347	1.808	3.166	34.314	1.789	3.133
95	34.143	1.691	2.962	34.182	1.714	3.001	34.143	1.691	2.962	34.116	1.676	2.935
94	33.964	1.589	2.783	34.01	1.615	2.829	33.971	1.593	2.79	33.949	1.581	2.768
93	33.82	1.507	2.639	33.859	1.529	2.678	33.821	1.507	2.64	33.803	1.497	2.622
92	33.683	1.428	2.501	33.724	1.452	2.543	33.687	1.431	2.506	33.673	1.423	2.491
91	33.578	1.368	2.396	33.601	1.382	2.42	33.565	1.361	2.384	33.554	1.355	2.373
90	33.464	1.304	2.283	33.486	1.316	2.305	33.454	1.298	2.273	33.445	1.293	2.264
89	33.366	1.247	2.185	33.378	1.254	2.197	33.351	1.239	2.17	33.345	1.235	2.163
88	33.256	1.185	2.074	33.277	1.197	2.096	33.254	1.184	2.073	33.25	1.182	2.069
87	33.134	1.115	1.953	33.181	1.142	2	33.164	1.132	1.983	33.162	1.131	1.98
86	33.052	1.068	1.871	33.092	1.091	1.911	33.078	1.083	1.897	33.078	1.083	1.896
85	32.975	1.024	1.794	33.007	1.042	1.826	32.996	1.037	1.815	32.998	1.037	1.817
84	32.888	0.975	1.707	32.926	0.996	1.745	32.919	0.992	1.738	32.921	0.994	1.74
83	32.819	0.935	1.638	32.85	0.953	1.669	32.844	0.95	1.663	32.848	0.952	1.667
82	32.749	0.895	1.568	32.777	0.911	1.596	32.773	0.909	1.592	32.778	0.912	1.597
81	32.681	0.857	1.5	32.707	0.871	1.526	32.704	0.87	1.523	32.71	0.873	1.529
80	32.619	0.821	1.438	32.639	0.833	1.458	32.637	0.832	1.456	32.645	0.836	1.464
79	32.562	0.789	1.381	32.574	0.796	1.393	32.573	0.795	1.392	32.581	0.799	1.4
78	32.485	0.745	1.304	32.511	0.76	1.33	32.51	0.759	1.329	32.519	0.764	1.338
77	32.423	0.709	1.242	32.45	0.725	1.269	32.45	0.724	1.268	32.459	0.73	1.278
76	32.367	0.677	1.186	32.391	0.691	1.21	32.39	0.69	1.209	32.401	0.697	1.22
75	32.308	0.643	1.127	32.333	0.658	1.152	32.332	0.657	1.151	32.344	0.664	1.163
74	32.255	0.613	1.074	32.277	0.626	1.096	32.276	0.625	1.095	32.288	0.632	1.107
73	32.204	0.584	1.023	32.221	0.594	1.04	32.221	0.594	1.04	32.233	0.601	1.052
72	32.149	0.553	0.968	32.167	0.563	0.986	32.166	0.563	0.985	32.18	0.57	0.998
71	32.106	0.528	0.925	32.114	0.533	0.933	32.113	0.532	0.932	32.127	0.54	0.946
70	32.056	0.5	0.875	32.062	0.503	0.88	32.061	0.503	0.88	32.075	0.511	0.894
69	32.004	0.47	0.823	32.01	0.473	0.829	32.01	0.473	0.829	32.024	0.481	0.843
68	31.958	0.443	0.777	31.959	0.444	0.778	31.959	0.444	0.778	31.974	0.453	0.793
67	31.91	0.416	0.729	31.909	0.415	0.728	31.909	0.416	0.728	31.925	0.425	0.744
66	31.85	0.382	0.669	31.859	0.387	0.678	31.86	0.388	0.679	31.876	0.397	0.695
65	31.8	0.354	0.619	31.81	0.359	0.629	31.812	0.36	0.631	31.828	0.369	0.646
64	31.747	0.323	0.566	31.762	0.332	0.581	31.764	0.333	0.583	31.78	0.342	0.599
63	31.701	0.297	0.52	31.714	0.304	0.533	31.716	0.306	0.535	31.733	0.315	0.552
62	31.648	0.267	0.467	31.666	0.277	0.485	31.67	0.279	0.488	31.686	0.288	0.505
61	31.607	0.243	0.426	31.62	0.25	0.439	31.623	0.252	0.442	31.64	0.262	0.459
60	31.558	0.215	0.377	31.574	0.224	0.392	31.577	0.226	0.396	31.594	0.236	0.413
59	31.508	0.187	0.327	31.528	0.198	0.347	31.531	0.2	0.35	31.548	0.21	0.367
58	31.473	0.167	0.292	31.482	0.172	0.301	31.486	0.174	0.305	31.503	0.184	0.322
57	31.43	0.142	0.249	31.437	0.146	0.256	31.441	0.148	0.26	31.458	0.158	0.277
56	31.39	0.119	0.209	31.393	0.121	0.212	31.396	0.123	0.215	31.413	0.133	0.232
55	31.351	0.097	0.17	31.348	0.096	0.167	31.352	0.098	0.171	31.369	0.107	0.188
54	31.313	0.075	0.132	31.304	0.07	0.123	31.308	0.072	0.126	31.324	0.082	0.143
53	31.263	0.047	0.082	31.26	0.045	0.079	31.263	0.047	0.082	31.28	0.057	0.099
52	31.213	0.019	0.032	31.216	0.02	0.035	31.22	0.022	0.038	31.236	0.031	0.055
51	31.174	-0.004	-0.007	31.172	-0.005	-0.009	31.176	-0.003	-0.005	31.192	0.006	0.011
50	31.128	-0.03	-0.053	31.128	-0.03	-0.053	31.132	-0.028	-0.049	31.148	-0.019	-0.033

Mean = 31.181, S.D. = 1.751, Provision = 31.181, 1 Unit = $1,000,000

Dataset: Employers Liability: Forecast

Model: PTF-good1

Scenario 2: Inflation at 7.18%+-0.78% for 6 years then drops to zero.

Reserve Forecast Table

Accident Period vs Development Period

	Cal. Per. Total	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	Reserve	Ultimate
1985	327,998	327,998	2,822,314	4,286,674	5,005,747	4,429,542	2,695,789	1,994,968	1,447,601	902,569	572,356	298,319	167,149	118,820	66,699	37,476	21,076	11,864	2,506	1,315	691	363	0	25,580,529
	363,421	363,421	2,823,353	3,740,983	4,851,694	4,302,127	3,337,231	2,176,450	1,561,371	1,390,771	628,642	87,377	200,919	39,315	23,958	29,482	9,494	11,340	0	3,845	0	-1,245	0	0
1986	3,137,155	314,841	2,709,078	4,114,650	4,804,828	4,251,711	3,082,343	2,282,197	1,389,616	866,408	549,417	307,566	172,337	122,512	68,775	38,644	21,734	4,588	2,405	1,262	663	374	374	23,570,755
	3,165,803	342,451	2,787,749	3,771,543	4,460,001	4,101,358	2,822,690	2,145,689	1,196,979	827,302	537,642	372,893	48,850	-7,546	123,409	51,765	-13,445	215	0	237	600	523	523	523
1987	7,323,749	327,998	2,822,314	4,286,674	5,005,747	5,276,625	3,826,934	2,377,407	1,447,601	902,569	614,748	344,160	192,853	137,106	76,972	43,252	9,126	4,780	2,506	1,315	742	419	1,161	26,164,218
	6,900,955	372,223	2,597,547	4,597,842	4,439,529	4,171,869	4,073,713	2,327,095	1,894,814	807,301	428,835	165,943	100,930	94,769	1,811	88,567	269	0	0	0	1,021	585	1,206	1,206
1988	12,304,967	362,256	3,117,370	4,735,250	6,584,540	6,941,315	4,224,345	2,624,591	1,598,294	1,070,395	729,103	408,206	228,757	162,641	91,313	19,252	10,076	5,278	2,767	1,560	880	497	2,937	32,732,711
	11,547,134	326,349	3,172,571	5,176,011	5,379,296	6,387,889	4,521,772	2,811,563	1,323,411	1,445,179	311,985	562,611	405,192	31,637	434,136	433,892	7,580	2,729	-4,029	2,120	1,212	695	2,637	2,637
1989	16,990,391	351,978	3,028,922	5,478,566	7,621,744	6,744,373	4,104,489	2,550,125	1,667,891	1,117,061	760,927	426,045	238,766	169,766	35,773	18,706	9,790	5,128	2,888	1,628	919	519	5,954	34,489,355
	16,930,957	398,416	3,384,938	6,261,944	7,232,344	6,056,747	4,172,737	2,638,001	1,999,745	1,280,583	518,592	432,369	47,274	15,294	16,795	199	4,548	22,876	3,879	2,213	1,266	725	4,903	4,903
1990	20,097,157	379,738	3,890,728	7,039,194	8,219,945	7,273,483	4,426,533	2,953,994	1,932,258	1,294,265	881,725	493,729	276,725	73,872	38,592	20,180	10,561	5,943	3,347	1,887	1,065	602	12,844	39,103,200
	20,777,343	338,276	3,981,902	7,269,814	7,273,202	6,764,962	4,663,626	3,778,364	1,786,086	2,035,300	966,986	143,383	21,026	-117	20,750	28,458	18,336	7,899	4,497	2,566	1,468	841	10,003	10,003
1991	26,697,014	389,243	3,989,913	6,059,969	7,076,465	6,261,666	4,093,134	2,731,640	1,786,901	1,196,962	815,477	456,656	96,124	63,595	33,224	17,372	9,767	5,497	3,096	1,746	985	557	21,647	33,798,376
	25,151,751	357,601	3,930,777	6,254,476	7,270,856	5,543,963	4,385,023	2,271,060	2,163,980	1,131,551	314,255	-113,095	-18,502	267,189	15,518	2,076	12,863	7,307	4,160	2,374	1,358	779	16,221	16,221
1992	33,503,376	354,477	3,050,078	4,632,526	5,409,584	5,141,467	3,361,048	2,243,179	1,467,447	983,024	669,757	140,906	73,482	48,615	25,398	14,266	8,021	4,514	2,543	1,434	809	425	32,014	27,191,877
	33,007,909	406,312	3,485,198	4,787,544	6,016,700	4,772,276	3,259,134	1,978,110	1,388,877	645,655	305,919	90,952	22,476	12,200	-11,490	18,641	10,565	6,002	3,418	1,951	1,116	595	23,349	23,349
1993	33,263,505	295,203	2,540,061	3,857,901	4,838,880	4,599,349	3,006,808	2,006,857	1,312,915	879,548	225,216	117,345	61,195	40,486	22,722	12,764	7,177	4,039	2,276	1,283	674	354	51,289	24,964,549
	32,793,140	286,900	2,500,351	4,252,026	5,428,687	4,340,034	3,008,771	2,731,563	1,181,851	728,782	371,273	68,942	10,783	3,295	29,496	16,680	9,455	5,373	3,060	1,747	930	496	36,675	36,675
1994	31,473,817	335,836	2,889,680	4,714,267	5,913,229	5,620,882	3,674,814	2,452,831	1,604,758	404,177	256,216	133,496	69,618	49,479	27,770	15,601	8,773	4,938	2,782	1,460	767	403	111,973	27,946,374
	32,018,107	359,335	2,940,485	4,749,103	5,360,802	5,962,373	4,131,260	2,780,882	1,056,343	386,307	226,250	-15,475	-103,263	63,905	36,056	20,392	11,561	6,571	3,743	1,987	1,058	564	78,900	78,900
1995	29,114,365	370,278	3,422,255	5,583,414	7,003,692	6,657,867	4,352,989	2,905,638	715,087	445,629	282,493	147,187	82,457	58,608	32,896	18,482	10,393	5,850	3,067	1,610	845	444	214,653	29,664,117
	29,082,604	272,072	3,351,803	5,088,547	7,048,157	6,656,030	3,895,374	1,595,575	794,846	513,843	47,316	185,902	66,637	75,707	42,721	24,166	13,703	7,789	4,127	2,191	1,166	622	116,174	116,174
1996	29,365,438	373,692	3,453,624	5,634,678	7,067,685	6,718,980	4,392,885	1,103,615	671,648	418,554	265,331	148,512	83,205	59,144	33,199	18,653	10,490	5,495	2,881	1,512	794	417	364,303	32,941,398
	31,282,166	369,338	3,373,646	5,580,846	7,733,248	9,576,139	4,414,667	466,863	777,454	204,599	80,294	119,554	67,238	76,404	43,122	24,397	13,836	7,315	3,876	2,058	1,095	584	169,269	169,269
1997	30,508,236	452,194	4,179,321	6,819,032	8,553,572	8,132,086	2,002,289	1,243,249	756,628	471,512	321,098	179,737	100,705	71,587	40,186	22,580	11,818	6,190	3,246	1,703	895	470	760,216	32,512,542
	27,665,268	339,282	3,732,991	5,227,094	9,228,797	8,435,768	2,888,546	1,135,434	547,640	216,773	258,142	144,724	81,411	92,511	52,220	29,549	15,587	8,240	4,366	2,318	1,234	658	332,390	332,390
1998	32,778,418	457,495	4,228,506	6,899,649	8,655,029	3,100,593	1,885,782	1,170,908	712,602	477,052	324,891	181,872	101,907	72,446	40,671	21,266	11,130	5,830	3,057	1,604	842	443	1,243,012	39,703,619
	32,987,794	278,722	3,494,212	5,806,965	11,247,156	10,692,710	4,906,059	1,673,282	361,501	158,936	261,252	146,499	82,427	93,662	52,878	27,830	14,680	7,761	4,112	2,184	1,162	620	377,154	377,154
1999	36,465,760	1,566,708	14,482,146	23,629,812	11,083,082	9,807,436	5,968,885	3,708,648	2,426,272	1,625,458	1,107,499	620,249	347,698	247,291	129,242	67,606	35,396	18,549	9,729	5,107	2,684	1,411	6,644,190	47,799,907
	34,807,630	261,587	3,279,725	6,885,099	10,432,638	11,101,285	6,269,954	2,925,429	572,672	535,192	888,941	499,279	281,364	320,036	168,220	88,623	46,794	24,763	13,133	6,981	3,718	1,985	1,420,868	1,420,868
2000	49,001,211	1,397,800	12,921,401	7,882,100	9,204,080	8,144,705	4,956,933	3,308,599	2,164,683	1,450,298	988,214	553,478	310,286	205,366	107,331	56,144	29,395	15,404	8,079	4,241	2,229	1,172	9,204,920	42,662,303
	37,162,551	167,544	3,509,353	6,962,743	9,221,279	9,569,081	4,027,383	772,295	511,812	478,261	793,627	445,841	251,302	265,778	139,700	73,598	38,861	20,565	10,907	5,797	3,088	1,648	1,512,375	1,512,375
2001	66,251,939	1,389,714	4,802,771	7,293,884	8,517,209	7,536,891	4,927,636	3,289,244	2,152,151	1,441,989	982,613	550,374	287,131	190,040	99,321	51,954	27,201	14,254	7,476	3,925	2,062	1,085	14,028,458	39,676,043
	38,780,673	158,189	2,663,632	6,601,908	8,424,381	7,799,475	536,710	769,132	510,309	476,553	789,680	443,717	232,548	245,943	129,275	68,105	35,961	19,030	10,093	5,365	2,857	1,525	1,636,505	1,636,505
2002	32,258,341	557,264	4,794,893	7,281,920	8,503,238	8,083,287	5,285,193	3,528,131	2,308,596	1,546,904	1,054,169	549,471	286,660	189,728	99,158	51,869	27,157	14,231	7,464	3,918	2,059	1,083	23,039,079	42,336,220
	36,338,872	155,809	3,402,607	7,405,743	8,332,982	884,791	580,119	827,409	549,591	512,646	847,908	442,990	232,167	245,540	129,063	67,994	35,902	18,999	10,076	5,356	2,853	1,523	2,022,154	2,022,154
2003	36,334,581	401,468	3,454,371	5,246,093	6,580,867	6,256,237	4,090,840	2,731,007	1,787,114	1,197,552	759,452	395,854	206,517	136,685	71,436	37,368	19,565	10,252	5,377	2,823	1,483	780	24,291,210	33,812,510
	38,753,674	476,574	3,196,497	5,848,228	721,461	690,062	454,725	643,081	427,612	398,210	610,856	319,142	167,259	176,894	92,980	48,984	25,865	13,687	7,259	3,858	2,055	1,097	1,780,645	1,780,645
2004	36,625,332	313,423	2,696,799	4,399,712	5,519,476	5,247,523	3,431,468	2,290,956	1,499,245	934,919	592,898	309,040	161,226	106,709	55,770	29,173	15,274	8,004	4,198	2,204	1,158	609	24,609,562	27,825,576
	37,600,773	203,281	3,012,732	482,820	609,733	586,081	388,030	542,265	360,943	310,879	476,890	249,151	130,578	138,099	72,589	38,242	20,192	10,686	5,667	3,012	1,604	856	1,631,864	1,631,864
2005	34,703,794	284,888	2,633,305	4,296,386	5,390,180	5,124,909	3,351,493	2,237,698	1,362,751	849,802	538,919	280,904	146,548	96,994	50,692	26,517	13,883	7,275	3,816	2,003	1,053	554	26,415,683	26,688,881
	32,991,052	273,198	291,649	475,084	602,916	582,197	387,120	532,990	328,082	282,576	433,473	226,468	118,690	125,527	65,980	34,760	18,354	9,713	5,151	2,738	1,458	778	1,687,389	1,687,389
	Total Fitted/Paid		2006	2007	2008	2009	2010	2011	2012	2013	2014	2015	2016	2017	2018	2019	2020	2021	2022	2023	2024	2025	Total Reserve	Total Ultimate
Cal. Per. Total	598,526,543		33,496,540	29,143,446	23,374,930	16,890,292	11,012,169	7,181,639	4,272,538	2,513,123	1,446,903	783,871	431,813	246,042	128,505	67,072	34,417	17,440	8,586	3,941	1,662	554	131,055,481	691,165,059
	560,109,578		1,899,416	1,828,862	1,888,870	1,682,334	1,463,129	1,366,430	1,008,458	767,469	616,515	361,506	227,152	163,213	86,629	46,120	24,521	13,015	6,851	3,508	1,737	778	6,580,520	6,580,520

1 Unit = $1

Inside the table the blues are the observed, the black are means of log-normal distributions and the reds are the corresponding Std Devs of the log-normals.

Given that on the log-scale we have normal distributions in every cell about the trend structure, the corresponding distributions on the dollar scale are log-normals (that are correlated via the parameter estimates).

The predictive log-normal distributions include parameter uncertainty and process variability.

Summaries by Year

Accident Yr Summary

Acc. Yr	Mean Reserve	Mean Ultimate	Standard Dev.	CV Reserve	CV Ultimate
1985	0	25,580,529	0	****	0
1986	374	23,570,755	523	1.4	0
1987	1,161	26,164,218	1,206	1.04	0
1988	2,937	32,732,711	2,637	0.9	0
1989	5,954	34,489,355	4,903	0.82	0
1990	12,844	39,103,200	10,003	0.78	0
1991	21,647	33,798,376	16,221	0.75	0
1992	32,014	27,191,877	23,349	0.73	0
1993	51,289	24,964,549	36,675	0.72	0
1994	111,973	27,946,374	78,900	0.7	0
1995	214,653	29,664,117	116,174	0.54	0
1996	364,303	32,941,398	169,269	0.46	0.01
1997	760,216	32,512,542	332,390	0.44	0.01
1998	1,243,012	39,703,619	377,154	0.3	0.01
1999	6,644,190	47,799,907	1,420,868	0.21	0.03
2000	9,204,920	42,662,303	1,512,375	0.16	0.04
2001	14,028,458	39,676,043	1,636,505	0.12	0.04
2002	23,039,079	42,336,220	2,022,154	0.09	0.05
2003	24,291,210	33,812,510	1,780,645	0.07	0.05
2004	24,609,562	27,825,576	1,631,864	0.07	0.06
2005	26,415,683	26,688,881	1,687,389	0.06	0.06
Total	131,055,481	691,165,059	6,580,520	0.05	0.01

1 Unit = $1

Calendar Yr Summary

Calendar Yr	Mean Reserve	Standard Dev.	CV Reserve
2006	33,496,540	1,899,416	0.06
2007	29,143,446	1,828,862	0.06
2008	23,374,930	1,888,870	0.08
2009	16,890,292	1,682,334	0.1
2010	11,012,169	1,463,129	0.13
2011	7,181,639	1,366,430	0.19
2012	4,272,538	1,008,458	0.24
2013	2,513,123	767,469	0.31
2014	1,446,903	616,515	0.43
2015	783,871	361,506	0.46
2016	431,813	227,152	0.53
2017	246,042	163,213	0.66
2018	128,505	86,629	0.67
2019	67,072	46,120	0.69
2020	34,417	24,521	0.71
2021	17,440	13,015	0.75
2022	8,586	6,851	0.8
2023	3,941	3,508	0.89
2024	1,662	1,737	1.05
2025	554	778	1.41
Total	131,055,481	6,580,520	0.05

1 Unit = $1

In order to find distributions of aggregates by accident year, calendar year and total of all years, we simulate from the predictive log-normal distributions of each cell, incorporating their correlations.

We do this because there is no analytical form for the sum of log-normals.

However, note that the means and Std Devs of the distributions of aggregates are computed via the model using second order moments, including covariances.

The sample quantiles (percentiles) below are based on 10,000 simulations of the forecast triangle.

The Kernel is fitted to the sample frequency plot whereas the log-normal and gamma are just based on the means and std. devs. of the actual forecast distributions.

Note that the distributions are slightly skewed.

Below are the quantiles (percentiles) for the Aggregate Reserve Distribution predicted by the model.

Quantiles and VaR Mean = 131.055, S.D. = 6.581, Provision = 131.055, 1 Unit = $1,000,000

Quantile Statistics and Value at Risk (Acc Year: Total)

%	Sample			Kernel			LogNormal			Gamma		
	Quantile	# S.D.'s	V-a-R	Quantile	# S.D.'s	V-a-R	Quantile	# S.D.'s	V-a-R	Quantile	# S.D.'s	V-a-R
99.995	159.428	4.312	28.372	161.013	4.552	29.957	159.11	4.263	28.054	158.228	4.129	27.173
99.99	157.901	4.08	26.846	159.712	4.355	28.657	157.746	4.056	26.691	156.953	3.935	25.897
99.98	157.823	4.068	26.768	158.143	4.116	27.087	156.337	3.842	25.281	155.63	3.734	24.574
99.97	156.612	3.884	25.557	157.161	3.967	26.106	155.488	3.713	24.433	154.831	3.613	23.776
99.96	155.929	3.78	24.873	156.376	3.848	25.32	154.875	3.62	23.819	154.253	3.525	23.197
99.95	154.8	3.608	23.745	155.73	3.75	24.674	154.392	3.546	23.336	153.797	3.456	22.741
99.94	154.564	3.572	23.508	155.205	3.67	24.149	153.992	3.486	22.937	153.419	3.398	22.364
99.93	154.201	3.517	23.146	154.774	3.604	23.719	153.651	3.434	22.595	153.096	3.349	22.041
99.92	153.971	3.482	22.915	154.424	3.551	23.368	153.352	3.388	22.297	152.814	3.306	21.758
99.91	153.801	3.456	22.745	154.124	3.506	23.068	153.087	3.348	22.032	152.562	3.268	21.507
99.9	153.21	3.367	22.155	153.864	3.466	22.808	152.848	3.312	21.793	152.336	3.234	21.28
99.8	151.838	3.158	20.783	152.276	3.225	21.22	151.231	3.066	20.175	150.799	3	19.743
99.7	151.185	3.059	20.13	151.336	3.082	20.28	150.245	2.916	19.189	149.859	2.857	18.804
99.6	150.313	2.926	19.258	150.614	2.972	19.558	149.525	2.807	18.469	149.172	2.753	18.116
99.5	149.797	2.848	18.742	150.014	2.881	18.959	148.954	2.72	17.899	148.626	2.67	17.57
99.4	149.328	2.777	18.272	149.494	2.802	18.439	148.479	2.648	17.423	148.171	2.601	17.115
99.3	148.859	2.706	17.804	149.035	2.732	17.98	148.071	2.586	17.015	147.779	2.541	16.724
99.2	148.435	2.641	17.379	148.624	2.67	17.568	147.712	2.531	16.656	147.435	2.489	16.379
99.1	148.155	2.598	17.099	148.256	2.614	17.201	147.391	2.482	16.336	147.127	2.442	16.072
99	147.576	2.511	16.521	147.92	2.563	16.864	147.101	2.438	16.046	146.848	2.4	15.793
98	145.333	2.17	14.277	145.675	2.222	14.619	145.103	2.135	14.047	144.922	2.107	13.866
97	144.227	2.002	13.171	144.376	2.024	13.321	143.849	1.944	12.793	143.708	1.923	12.653
96	143.289	1.859	12.234	143.407	1.877	12.351	142.912	1.802	11.857	142.8	1.785	11.745
95	142.514	1.741	11.459	142.587	1.752	11.532	142.155	1.687	11.1	142.064	1.673	11.009
94	141.726	1.621	10.67	141.869	1.643	10.814	141.514	1.589	10.458	141.44	1.578	10.384
93	141.096	1.526	10.04	141.237	1.547	10.182	140.954	1.504	9.898	140.893	1.495	9.838
92	140.536	1.441	9.481	140.682	1.463	9.626	140.454	1.428	9.399	140.406	1.421	9.35
91	139.982	1.356	8.926	140.186	1.387	9.13	140.001	1.359	8.946	139.963	1.354	8.907
90	139.6	1.299	8.545	139.739	1.32	8.683	139.586	1.296	8.53	139.556	1.292	8.501
89	139.137	1.228	8.082	139.33	1.257	8.275	139.201	1.238	8.145	139.179	1.234	8.124
88	138.795	1.176	7.74	138.954	1.2	7.899	138.841	1.183	7.785	138.826	1.181	7.771
87	138.494	1.13	7.438	138.602	1.147	7.547	138.502	1.132	7.447	138.494	1.13	7.439
86	138.193	1.085	7.138	138.271	1.096	7.215	138.182	1.083	7.127	138.18	1.083	7.125
85	137.906	1.041	6.85	137.956	1.049	6.9	137.878	1.037	6.823	137.881	1.037	6.826
84	137.608	0.996	6.552	137.656	1.003	6.6	137.588	0.993	6.532	137.596	0.994	6.54
83	137.279	0.946	6.223	137.369	0.959	6.314	137.31	0.95	6.254	137.322	0.952	6.266
82	136.964	0.898	5.908	137.094	0.918	6.039	137.042	0.91	5.987	137.059	0.912	6.003
81	136.682	0.855	5.626	136.83	0.877	5.774	136.785	0.871	5.729	136.805	0.874	5.749
80	136.44	0.818	5.385	136.575	0.839	5.52	136.536	0.833	5.48	136.559	0.836	5.504
79	136.213	0.784	5.158	136.329	0.801	5.274	136.295	0.796	5.239	136.321	0.8	5.266
78	135.997	0.751	4.942	136.091	0.765	5.036	136.06	0.761	5.005	136.09	0.765	5.035
77	135.809	0.722	4.754	135.86	0.73	4.804	135.833	0.726	4.777	135.865	0.731	4.81
76	135.6	0.691	4.544	135.635	0.696	4.579	135.611	0.692	4.555	135.646	0.698	4.591
75	135.392	0.659	4.337	135.415	0.662	4.359	135.394	0.659	4.339	135.432	0.665	4.376
74	135.172	0.626	4.117	135.199	0.63	4.144	135.183	0.627	4.127	135.223	0.633	4.167
73	134.958	0.593	3.903	134.988	0.598	3.933	134.976	0.596	3.92	135.017	0.602	3.962
72	134.722	0.557	3.666	134.781	0.566	3.726	134.773	0.565	3.717	134.816	0.572	3.761
71	134.52	0.526	3.464	134.577	0.535	3.522	134.574	0.535	3.518	134.619	0.542	3.563
70	134.325	0.497	3.269	134.377	0.505	3.322	134.378	0.505	3.322	134.425	0.512	3.369

TABLE CONTINUES ON NEXT PAGE

Quantiles and VaR Mean = 131.055, S.D. = 6.581, Provision = 131.055, 1 Unit = $1,000,000

Quantile Statistics and Value at Risk (Acc Year: Total)

%	Sample			Kernel			LogNormal			Gamma		
	Quantile	# S.D.'s	V-a-R	Quantile	# S.D.'s	V-a-R	Quantile	# S.D.'s	V-a-R	Quantile	# S.D.'s	V-a-R
69	134.136	0.468	3.081	134.18	0.475	3.125	134.185	0.476	3.13	134.234	0.483	3.178
68	133.936	0.438	2.881	133.986	0.445	2.931	133.996	0.447	2.94	134.046	0.454	2.99
67	133.714	0.404	2.659	133.795	0.416	2.739	133.809	0.418	2.754	133.86	0.426	2.805
66	133.545	0.378	2.49	133.607	0.388	2.551	133.625	0.39	2.569	133.677	0.398	2.622
65	133.378	0.353	2.323	133.421	0.36	2.366	133.443	0.363	2.388	133.496	0.371	2.441
64	133.19	0.324	2.134	133.238	0.332	2.183	133.263	0.336	2.208	133.317	0.344	2.262
63	133.005	0.296	1.949	133.057	0.304	2.001	133.086	0.308	2.03	133.14	0.317	2.085
62	132.847	0.272	1.792	132.877	0.277	1.822	132.91	0.282	1.854	132.965	0.29	1.909
61	132.69	0.248	1.635	132.7	0.25	1.645	132.735	0.255	1.68	132.791	0.264	1.736
60	132.544	0.226	1.488	132.524	0.223	1.469	132.562	0.229	1.507	132.619	0.238	1.563
59	132.366	0.199	1.311	132.35	0.197	1.294	132.391	0.203	1.336	132.448	0.212	1.392
58	132.187	0.172	1.132	132.176	0.17	1.121	132.221	0.177	1.165	132.278	0.186	1.222
57	131.993	0.142	0.938	132.004	0.144	0.949	132.052	0.151	0.996	132.109	0.16	1.053
56	131.796	0.112	0.74	131.833	0.118	0.778	131.884	0.126	0.828	131.941	0.135	0.885
55	131.635	0.088	0.58	131.663	0.092	0.608	131.717	0.1	0.661	131.774	0.109	0.718
54	131.447	0.059	0.391	131.495	0.067	0.439	131.55	0.075	0.495	131.607	0.084	0.552
53	131.296	0.037	0.241	131.327	0.041	0.271	131.385	0.05	0.329	131.441	0.059	0.386
52	131.136	0.012	0.08	131.16	0.016	0.105	131.22	0.025	0.164	131.276	0.033	0.22
51	130.99	-0.01	-0.065	130.995	-0.009	-0.061	131.055	0	-0.001	131.11	0.008	0.055
50	130.836	-0.033	-0.219	130.83	-0.034	-0.226	130.891	-0.025	-0.165	130.945	-0.017	-0.11

Below are quantiles and VaR statistics for the next calendar year 2006

Note that the distributions here are more skewed than for the aggregate reserves.

Quantile Statistics and Value at Risk (Cal. Yr: 2006)

%	Sample			Kernel			LogNormal			Gamma		
	Quantile	# S.D.'s	V-a-R	Quantile	# S.D.'s	V-a-R	Quantile	# S.D.'s	V-a-R	Quantile	# S.D.'s	V-a-R
99.95	40.089	3.471	6.592	40.21	3.535	6.714	40.297	3.581	6.801	40.101	3.477	6.605
99.9	39.58	3.203	6.084	39.814	3.326	6.317	39.843	3.341	6.346	39.674	3.252	6.178
99.8	39.291	3.05	5.794	39.437	3.128	5.941	39.367	3.091	5.871	39.225	3.016	5.729
99.7	39.12	2.96	5.623	39.207	3.007	5.711	39.077	2.938	5.581	38.951	2.872	5.455
99.6	38.984	2.889	5.488	39.029	2.913	5.533	38.866	2.827	5.37	38.75	2.766	5.254
99.5	38.816	2.801	5.32	38.879	2.834	5.383	38.699	2.739	5.202	38.591	2.682	5.095
99.4	38.683	2.731	5.186	38.747	2.764	5.251	38.559	2.665	5.063	38.458	2.612	4.962
99.3	38.567	2.67	5.071	38.629	2.702	5.133	38.44	2.602	4.943	38.344	2.552	4.848
99.2	38.445	2.605	4.949	38.522	2.646	5.026	38.334	2.547	4.838	38.244	2.499	4.747
99.1	38.346	2.553	4.849	38.424	2.594	4.928	38.241	2.498	4.744	38.154	2.452	4.658
99	38.276	2.516	4.78	38.335	2.548	4.839	38.156	2.453	4.659	38.073	2.409	4.576
98	37.607	2.164	4.111	37.697	2.212	4.201	37.571	2.145	4.074	37.512	2.114	4.015
97	37.234	1.967	3.737	37.284	1.994	3.788	37.204	1.952	3.708	37.159	1.928	3.662
96	36.924	1.804	3.427	36.978	1.833	3.482	36.931	1.808	3.434	36.895	1.789	3.398
95	36.672	1.672	3.175	36.739	1.707	3.242	36.71	1.692	3.214	36.681	1.676	3.184
94	36.486	1.574	2.99	36.543	1.604	3.046	36.523	1.593	3.027	36.499	1.581	3.003
93	36.312	1.482	2.815	36.376	1.516	2.879	36.36	1.508	2.864	36.341	1.497	2.844
92	36.182	1.414	2.686	36.229	1.439	2.733	36.215	1.431	2.718	36.199	1.423	2.703
91	36.069	1.355	2.573	36.098	1.369	2.601	36.083	1.362	2.586	36.071	1.355	2.574
90	35.933	1.283	2.436	35.978	1.306	2.481	35.962	1.298	2.465	35.953	1.293	2.456
89	35.824	1.225	2.328	35.866	1.247	2.369	35.85	1.239	2.353	35.843	1.235	2.347
88	35.728	1.175	2.231	35.762	1.193	2.265	35.745	1.184	2.249	35.741	1.182	2.244
87	35.633	1.125	2.137	35.663	1.141	2.167	35.647	1.132	2.15	35.645	1.131	2.148
86	35.539	1.076	2.043	35.57	1.092	2.074	35.554	1.083	2.057	35.553	1.083	2.057
85	35.461	1.034	1.964	35.481	1.045	1.985	35.466	1.037	1.969	35.467	1.037	1.97
84	35.372	0.987	1.875	35.396	1	1.9	35.381	0.992	1.885	35.384	0.994	1.887
83	35.289	0.944	1.793	35.314	0.957	1.818	35.3	0.95	1.804	35.305	0.952	1.808
82	35.205	0.899	1.708	35.236	0.916	1.739	35.223	0.909	1.726	35.228	0.912	1.732
81	35.139	0.865	1.642	35.16	0.876	1.663	35.148	0.87	1.652	35.155	0.873	1.658
80	35.067	0.827	1.57	35.086	0.837	1.59	35.076	0.831	1.579	35.084	0.836	1.587
79	34.989	0.786	1.492	35.015	0.799	1.519	35.006	0.795	1.509	35.015	0.799	1.518
78	34.92	0.749	1.424	34.946	0.763	1.45	34.938	0.759	1.442	34.948	0.764	1.451
77	34.867	0.721	1.37	34.879	0.728	1.382	34.872	0.724	1.376	34.883	0.73	1.386
76	34.804	0.688	1.307	34.814	0.693	1.317	34.808	0.69	1.311	34.819	0.696	1.323
75	34.722	0.645	1.225	34.75	0.66	1.254	34.745	0.657	1.248	34.757	0.664	1.261
74	34.663	0.614	1.166	34.688	0.627	1.192	34.684	0.625	1.187	34.697	0.632	1.2
73	34.603	0.582	1.106	34.627	0.595	1.131	34.624	0.593	1.127	34.637	0.601	1.141
72	34.557	0.558	1.06	34.568	0.564	1.072	34.565	0.562	1.068	34.579	0.57	1.083
71	34.501	0.529	1.004	34.51	0.534	1.013	34.507	0.532	1.011	34.522	0.54	1.026
70	34.438	0.496	0.942	34.453	0.503	0.956	34.451	0.502	0.954	34.466	0.51	0.969
69	34.376	0.463	0.879	34.397	0.474	0.9	34.395	0.473	0.898	34.411	0.481	0.914
68	34.322	0.434	0.825	34.341	0.445	0.845	34.34	0.444	0.844	34.356	0.453	0.86
67	34.274	0.409	0.777	34.287	0.416	0.791	34.286	0.416	0.789	34.303	0.424	0.806
66	34.222	0.382	0.725	34.234	0.388	0.737	34.233	0.388	0.736	34.25	0.397	0.753
65	34.173	0.356	0.676	34.181	0.36	0.684	34.18	0.36	0.684	34.197	0.369	0.701
64	34.125	0.331	0.628	34.129	0.333	0.632	34.128	0.333	0.632	34.146	0.342	0.649
63	34.07	0.302	0.574	34.077	0.306	0.58	34.077	0.305	0.58	34.095	0.315	0.598
62	34.02	0.275	0.523	34.026	0.279	0.529	34.026	0.279	0.529	34.044	0.288	0.547
61	33.963	0.245	0.466	33.975	0.252	0.479	33.975	0.252	0.479	33.994	0.262	0.497
60	33.907	0.216	0.41	33.925	0.226	0.428	33.926	0.226	0.429	33.944	0.236	0.447
59	33.868	0.195	0.371	33.875	0.199	0.379	33.876	0.2	0.379	33.894	0.209	0.398
58	33.827	0.174	0.33	33.826	0.173	0.329	33.827	0.174	0.33	33.845	0.184	0.349
57	33.78	0.149	0.284	33.776	0.147	0.28	33.778	0.148	0.281	33.797	0.158	0.3
56	33.722	0.119	0.225	33.727	0.122	0.231	33.73	0.123	0.233	33.748	0.132	0.251
55	33.678	0.095	0.181	33.679	0.096	0.182	33.681	0.097	0.185	33.7	0.107	0.203
54	33.633	0.072	0.137	33.63	0.07	0.134	33.633	0.072	0.137	33.652	0.082	0.155
53	33.586	0.047	0.089	33.582	0.045	0.085	33.585	0.047	0.089	33.604	0.056	0.107
52	33.531	0.018	0.035	33.533	0.019	0.037	33.538	0.022	0.041	33.556	0.031	0.059
51	33.484	-0.007	-0.013	33.485	-0.006	-0.011	33.49	-0.003	-0.006	33.508	0.006	0.012
50	33.429	-0.035	-0.067	33.437	-0.031	-0.06	33.443	-0.028	-0.054	33.461	-0.019	-0.036

Mean = 33.497, S.D. = 1.899, Provision = 33.497, 1 Unit = $1,000,000

Dataset: Employers Liability: Forecast | Model: PTF-good1

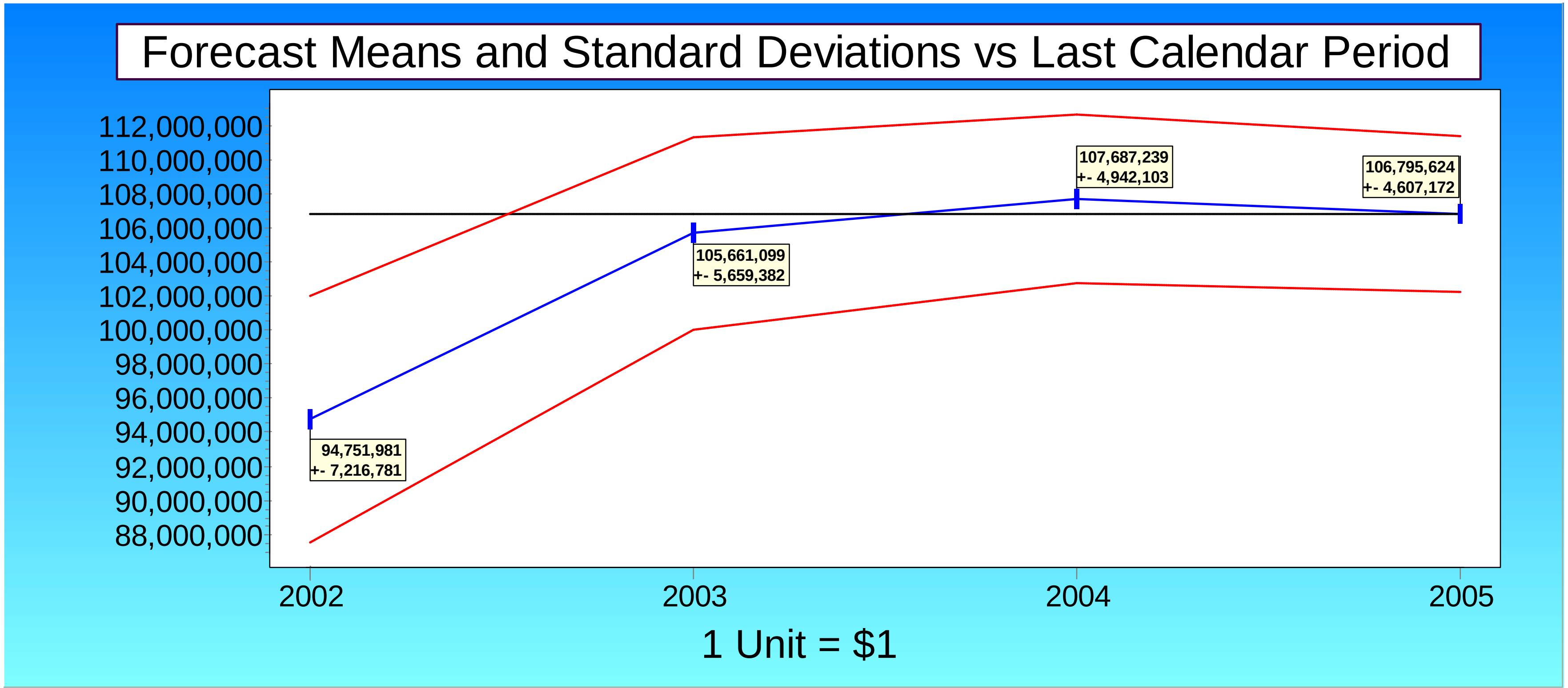
Forecasts from model omitting data from one to three calendar years.

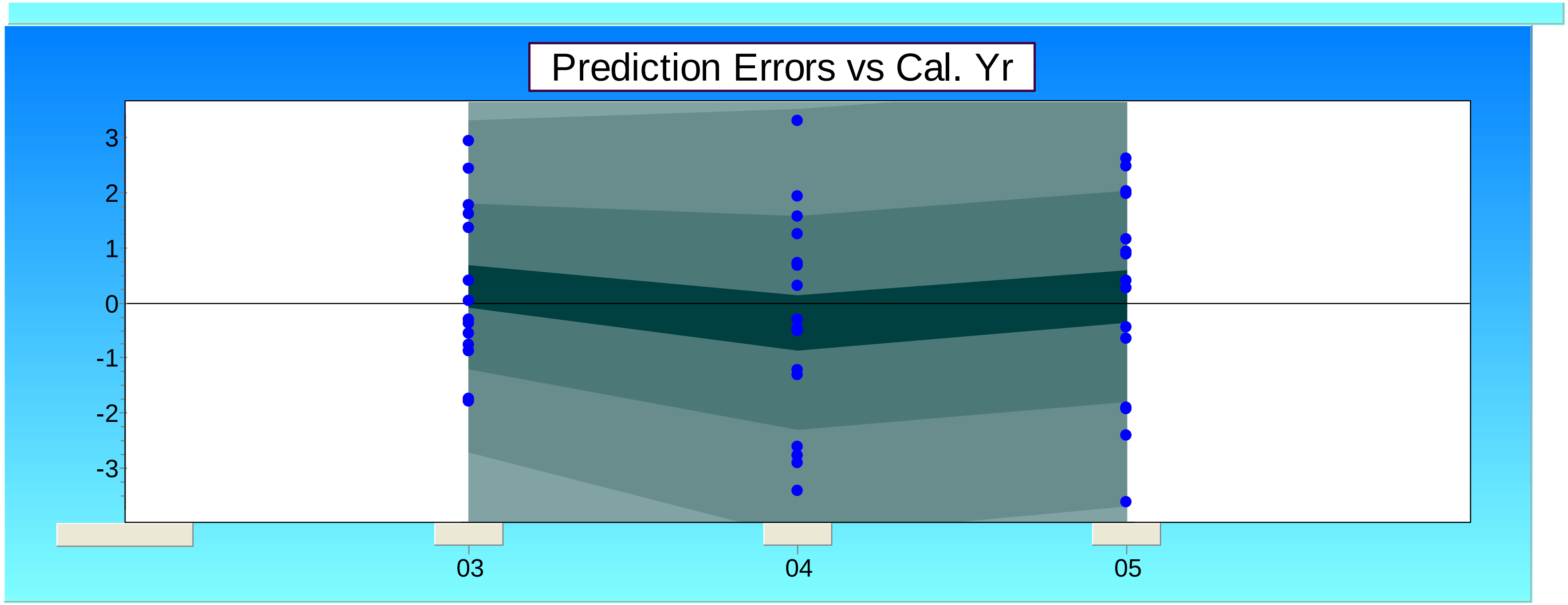
The graph below depicts the how the means and std. devs. of the reserve distributions (beyond 2005) change as years are removed, assuming that Calendar year trend of zero continues.

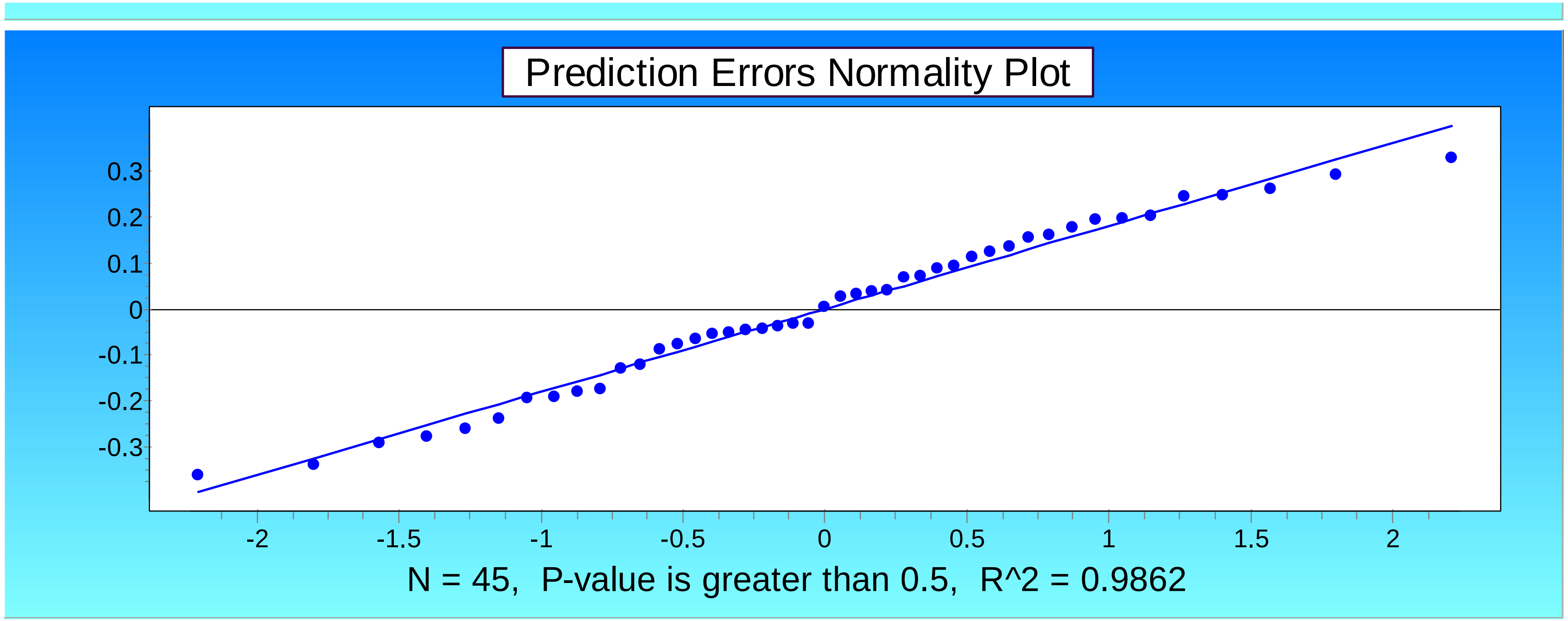
The pair of numbers for any particular year represent the mean and std. dev. of the reserve distribution as predicted at that year's end. For example this would mean that at year end 2003 the data for 2004 and 2005 are not used in estimating the parameters of the model, including process variability.

The prinicipal reasons for stability are that 1. the Calendar year trend since 2002 is zero 2. the decay parameter has been stable for many years and 3. the recent accident years have been flat.

Note that the slightly lower mean value at year end 2002 does not represent significant instability if you also consider the std. devs. of the distributions.


Forecast Means and Standard Deviations vs Last Calendar Period
112,000,000
110,000,000
108,000,000
106,000,000
104,000,000
102,000,000
100,000,000
98,000,000
96,000,000
94,000,000
92,000,000
90,000,000
88,000,000
94,751,981
+- 7,216,781
105,661,099
+- 5,659,382
107,687,239
+- 4,942,103
106,795,624
+- 4,607,172
2002
2003
2004
2005
1 Unit = $1


Prediction Errors vs Cal. Yr
3
2
1
0
-1
-2
-3
03
04
05


Prediction Errors Normality Plot
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
-2
-1.5
-1
-0.5
0
0.5
1
1.5
2
N = 45, P-value is greater than 0.5, R^2 = 0.9862

Appendix F3.5.5

Quantiles after removing two years then forecasting using scenario 2 (all quantiles are stable compared with the Quantile results obtained on 4 Forecast Sc.2)

Quantile Statistics and Value at Risk (Acc Year: Total)												
%	Sample			Kernel			LogNormal			Gamma		
	Quantile	# S.D.'s	V-a-R	Quantile	# S.D.'s	V-a-R	Quantile	# S.D.'s	V-a-R	Quantile	# S.D.'s	V-a-R
99.995	159.34	3.844	29.615	160.836	4.038	31.111	163.128	4.335	33.404	161.881	4.173	32.156
99.99	158.399	3.721	28.674	159.985	3.927	30.26	161.476	4.121	31.752	160.355	3.975	30.63
99.98	157.578	3.615	27.853	159.025	3.803	29.3	159.771	3.9	30.046	158.774	3.77	29.049
99.97	157.464	3.6	27.739	158.408	3.723	28.683	158.747	3.767	29.022	157.821	3.646	28.096
99.96	157.101	3.553	27.376	157.934	3.661	28.209	158.006	3.67	28.281	157.131	3.557	27.406
99.95	157.056	3.547	27.331	157.539	3.61	27.814	157.423	3.595	27.698	156.586	3.486	26.862
99.94	156.937	3.532	27.212	157.192	3.565	27.468	156.942	3.532	27.217	156.136	3.428	26.411
99.93	156.921	3.53	27.196	156.881	3.525	27.157	156.531	3.479	26.806	155.751	3.378	26.026
99.92	156.032	3.414	26.307	156.598	3.488	26.873	156.171	3.432	26.446	155.414	3.334	25.69
99.91	155.991	3.409	26.266	156.335	3.454	26.61	155.852	3.391	26.127	155.115	3.295	25.39
99.9	155.864	3.392	26.139	156.089	3.422	26.364	155.564	3.354	25.839	154.845	3.26	25.12
99.8	153.791	3.123	24.066	154.261	3.184	24.536	153.619	3.101	23.894	153.015	3.023	23.29
99.7	152.622	2.972	22.897	153.067	3.029	23.342	152.435	2.947	22.71	151.897	2.878	22.172
99.6	151.91	2.879	22.185	152.191	2.916	22.466	151.572	2.835	21.847	151.079	2.771	21.354
99.5	151.118	2.777	21.393	151.51	2.827	21.785	150.888	2.747	21.163	150.43	2.687	20.705
99.4	150.385	2.681	20.66	150.962	2.756	21.237	150.319	2.673	20.594	149.89	2.617	20.165
99.3	149.963	2.627	20.238	150.503	2.697	20.778	149.83	2.609	20.105	149.425	2.557	19.7
99.2	149.744	2.598	20.019	150.099	2.644	20.374	149.401	2.554	19.676	149.016	2.504	19.291
99.1	149.367	2.549	19.642	149.74	2.598	20.015	149.018	2.504	19.293	148.651	2.456	18.926
99	149.089	2.513	19.364	149.413	2.555	19.688	148.671	2.459	18.946	148.32	2.413	18.595
98	146.744	2.209	17.019	146.967	2.238	17.242	146.285	2.149	16.56	146.036	2.117	16.311
97	145.092	1.994	15.367	145.341	2.027	15.616	144.791	1.955	15.066	144.599	1.93	14.874
96	143.919	1.842	14.194	144.164	1.874	14.439	143.677	1.811	13.952	143.524	1.791	13.799
95	143.086	1.734	13.361	143.229	1.753	13.504	142.777	1.694	13.053	142.653	1.678	12.928
94	142.253	1.626	12.528	142.441	1.65	12.716	142.016	1.595	12.291	141.915	1.582	12.19
93	141.521	1.531	11.796	141.759	1.562	12.034	141.352	1.509	11.627	141.27	1.498	11.545
92	140.928	1.454	11.203	141.155	1.483	11.43	140.759	1.432	11.034	140.694	1.424	10.969
91	140.431	1.389	10.706	140.612	1.413	10.887	140.223	1.362	10.498	140.172	1.356	10.447
90	140.004	1.334	10.279	140.113	1.348	10.388	139.731	1.299	10.006	139.692	1.294	9.967
89	139.513	1.27	9.788	139.647	1.288	9.922	139.275	1.239	9.55	139.247	1.236	9.522
88	139.147	1.223	9.422	139.205	1.23	9.48	138.849	1.184	9.124	138.831	1.182	9.106
87	138.656	1.159	8.931	138.783	1.176	9.058	138.449	1.132	8.724	138.44	1.131	8.715
86	138.353	1.12	8.628	138.379	1.123	8.654	138.071	1.083	8.346	138.07	1.083	8.345
85	137.874	1.058	8.149	137.99	1.073	8.265	137.711	1.037	7.987	137.717	1.037	7.992
84	137.517	1.011	7.792	137.615	1.024	7.891	137.369	0.992	7.644	137.381	0.994	7.656
83	137.15	0.964	7.425	137.255	0.977	7.53	137.04	0.949	7.315	137.059	0.952	7.334
82	136.811	0.92	7.086	136.908	0.932	7.183	136.725	0.908	7	136.749	0.912	7.024
81	136.476	0.876	6.752	136.575	0.889	6.85	136.421	0.869	6.696	136.45	0.873	6.725
80	136.16	0.835	6.436	136.254	0.847	6.529	136.127	0.831	6.402	136.161	0.835	6.436
79	135.788	0.787	6.063	135.945	0.807	6.22	135.843	0.794	6.118	135.881	0.799	6.156
78	135.491	0.748	5.766	135.648	0.769	5.923	135.567	0.758	5.842	135.609	0.764	5.884
77	135.239	0.716	5.514	135.361	0.731	5.636	135.299	0.723	5.574	135.345	0.729	5.62

TABLE CONTINUES ON NEXT PAGE

Quantile Statistics and Value at Risk (Acc Year: Total)

%	Sample			Kernel			LogNormal			Gamma		
	Quantile	# S.D.'s	V-a-R	Quantile	# S.D.'s	V-a-R	Quantile	# S.D.'s	V-a-R	Quantile	# S.D.'s	V-a-R
76	134.973	0.681	5.248	135.084	0.696	5.36	135.038	0.69	5.313	135.087	0.696	5.362
75	134.714	0.648	4.989	134.817	0.661	5.092	134.783	0.656	5.058	134.835	0.663	5.111
74	134.488	0.618	4.763	134.558	0.627	4.833	134.534	0.624	4.809	134.589	0.631	4.864
73	134.218	0.583	4.493	134.306	0.595	4.581	134.29	0.592	4.565	134.348	0.6	4.623
72	133.965	0.55	4.24	134.061	0.563	4.336	134.051	0.561	4.326	134.112	0.569	4.387
71	133.716	0.518	3.991	133.822	0.532	4.097	133.817	0.531	4.092	133.88	0.539	4.155
70	133.516	0.492	3.791	133.588	0.501	3.863	133.587	0.501	3.862	133.652	0.51	3.927
69	133.313	0.466	3.588	133.359	0.472	3.634	133.361	0.472	3.636	133.428	0.481	3.703
68	133.072	0.434	3.347	133.134	0.443	3.41	133.138	0.443	3.413	133.207	0.452	3.482
67	132.851	0.406	3.126	132.913	0.414	3.188	132.918	0.414	3.193	132.989	0.424	3.264
66	132.646	0.379	2.921	132.695	0.386	2.97	132.702	0.386	2.977	132.774	0.396	3.049
65	132.439	0.352	2.714	132.48	0.358	2.756	132.488	0.359	2.763	132.562	0.368	2.837
64	132.206	0.322	2.481	132.268	0.33	2.543	132.277	0.331	2.552	132.352	0.341	2.627
63	131.999	0.295	2.274	132.059	0.303	2.334	132.069	0.304	2.344	132.144	0.314	2.42
62	131.848	0.276	2.123	131.851	0.276	2.126	131.862	0.277	2.137	131.939	0.287	2.214
61	131.644	0.249	1.919	131.645	0.249	1.92	131.658	0.251	1.933	131.735	0.261	2.01
60	131.435	0.222	1.71	131.441	0.223	1.716	131.455	0.225	1.73	131.533	0.235	1.808
59	131.239	0.196	1.514	131.238	0.196	1.513	131.254	0.198	1.529	131.332	0.209	1.607
58	131.02	0.168	1.295	131.036	0.17	1.311	131.055	0.173	1.33	131.133	0.183	1.408
57	130.849	0.146	1.124	130.836	0.144	1.111	130.857	0.147	1.132	130.935	0.157	1.21
56	130.608	0.115	0.883	130.637	0.118	0.912	130.66	0.121	0.935	130.738	0.132	1.013
55	130.422	0.09	0.697	130.439	0.093	0.714	130.464	0.096	0.739	130.542	0.106	0.817
54	130.226	0.065	0.501	130.242	0.067	0.517	130.269	0.071	0.544	130.347	0.081	0.622
53	130.021	0.038	0.296	130.046	0.042	0.321	130.075	0.045	0.35	130.153	0.056	0.428
52	129.829	0.014	0.105	129.851	0.016	0.126	129.882	0.02	0.157	129.959	0.03	0.234
51	129.656	-0.009	-0.069	129.657	-0.009	-0.068	129.689	-0.005	-0.036	129.766	0.005	0.041
50	129.438	-0.037	-0.287	129.463	-0.034	-0.262	129.497	-0.03	-0.228	129.572	-0.02	-0.153
49	129.228	-0.064	-0.497	129.27	-0.059	-0.455	129.305	-0.055	-0.42	129.379	-0.045	-0.345
48	129.053	-0.087	-0.672	129.078	-0.084	-0.647	129.113	-0.079	-0.612	129.187	-0.07	-0.538
47	128.905	-0.106	-0.82	128.886	-0.109	-0.839	128.921	-0.104	-0.804	128.994	-0.095	-0.731
46	128.707	-0.132	-1.018	128.694	-0.134	-1.031	128.729	-0.129	-0.996	128.801	-0.12	-0.924
45	128.492	-0.16	-1.233	128.503	-0.159	-1.222	128.537	-0.154	-1.188	128.607	-0.145	-1.118
44	128.297	-0.185	-1.428	128.311	-0.183	-1.414	128.344	-0.179	-1.381	128.413	-0.17	-1.311
43	128.133	-0.207	-1.592	128.119	-0.208	-1.606	128.151	-0.204	-1.574	128.219	-0.195	-1.506
42	127.961	-0.229	-1.764	127.927	-0.233	-1.798	127.957	-0.229	-1.768	128.024	-0.221	-1.701
41	127.769	-0.254	-1.956	127.734	-0.258	-1.991	127.763	-0.255	-1.962	127.828	-0.246	-1.897
40	127.563	-0.281	-2.162	127.54	-0.284	-2.185	127.567	-0.28	-2.157	127.631	-0.272	-2.093
39	127.398	-0.302	-2.327	127.344	-0.309	-2.381	127.371	-0.305	-2.354	127.434	-0.297	-2.291
38	127.236	-0.323	-2.489	127.147	-0.335	-2.578	127.174	-0.331	-2.551	127.234	-0.323	-2.49
37	127.029	-0.35	-2.695	126.948	-0.36	-2.777	126.975	-0.357	-2.75	127.034	-0.349	-2.691
36	126.813	-0.378	-2.912	126.748	-0.386	-2.977	126.774	-0.383	-2.95	126.832	-0.375	-2.893
35	126.582	-0.408	-3.143	126.545	-0.413	-3.18	126.573	-0.409	-3.152	126.628	-0.402	-3.097
34	126.382	-0.434	-3.343	126.34	-0.439	-3.385	126.369	-0.436	-3.356	126.422	-0.429	-3.302
33	126.205	-0.457	-3.52	126.132	-0.466	-3.592	126.163	-0.462	-3.562	126.215	-0.456	-3.51
32	125.992	-0.484	-3.733	125.922	-0.494	-3.803	125.955	-0.489	-3.77	126.005	-0.483	-3.72
31	125.752	-0.516	-3.973	125.709	-0.521	-4.016	125.745	-0.517	-3.98	125.792	-0.51	-3.933
30	125.547	-0.542	-4.178	125.492	-0.549	-4.233	125.532	-0.544	-4.193	125.576	-0.538	-4.148

TABLE CONTINUES ON NEXT PAGE

Appendix F3.5.7

Quantile Statistics and Value at Risk (Acc Year: Total)

%	Sample			Kernel			LogNormal			Gamma		
	Quantile	# S.D.'s	V-a-R	Quantile	# S.D.'s	V-a-R	Quantile	# S.D.'s	V-a-R	Quantile	# S.D.'s	V-a-R
29	125.334	-0.57	-4.391	125.272	-0.578	-4.453	125.316	-0.572	-4.409	125.358	-0.567	-4.367
28	125.101	-0.6	-4.623	125.048	-0.607	-4.677	125.097	-0.601	-4.628	125.136	-0.596	-4.589
27	124.898	-0.626	-4.827	124.82	-0.637	-4.905	124.875	-0.63	-4.85	124.911	-0.625	-4.814
26	124.684	-0.654	-5.041	124.588	-0.667	-5.137	124.648	-0.659	-5.077	124.682	-0.655	-5.043
25	124.446	-0.685	-5.279	124.35	-0.698	-5.375	124.418	-0.689	-5.307	124.448	-0.685	-5.277
24	124.205	-0.716	-5.52	124.107	-0.729	-5.618	124.183	-0.719	-5.542	124.21	-0.716	-5.515
23	123.953	-0.749	-5.772	123.858	-0.761	-5.867	123.943	-0.75	-5.782	123.966	-0.747	-5.758
22	123.699	-0.782	-6.026	123.603	-0.795	-6.122	123.698	-0.782	-6.027	123.717	-0.78	-6.008
21	123.414	-0.819	-6.311	123.341	-0.829	-6.384	123.447	-0.815	-6.278	123.462	-0.813	-6.263
20	123.162	-0.852	-6.563	123.071	-0.864	-6.654	123.189	-0.848	-6.536	123.2	-0.847	-6.525
19	122.848	-0.893	-6.877	122.794	-0.9	-6.931	122.924	-0.883	-6.801	122.93	-0.882	-6.795
18	122.593	-0.926	-7.132	122.508	-0.937	-7.217	122.651	-0.918	-7.074	122.652	-0.918	-7.073
17	122.326	-0.96	-7.399	122.211	-0.975	-7.513	122.368	-0.955	-7.356	122.364	-0.955	-7.361
16	122.006	-1.002	-7.719	121.905	-1.015	-7.82	122.076	-0.993	-7.649	122.065	-0.994	-7.659
15	121.686	-1.043	-8.039	121.588	-1.056	-8.137	121.772	-1.032	-7.953	121.755	-1.034	-7.97
14	121.355	-1.086	-8.37	121.256	-1.099	-8.469	121.455	-1.073	-8.27	121.431	-1.076	-8.294
13	120.977	-1.135	-8.748	120.91	-1.144	-8.815	121.123	-1.116	-8.602	121.092	-1.12	-8.633
12	120.692	-1.172	-9.033	120.545	-1.191	-9.179	120.774	-1.162	-8.951	120.735	-1.167	-8.99
11	120.316	-1.221	-9.409	120.163	-1.241	-9.562	120.405	-1.21	-9.32	120.356	-1.216	-9.368
10	119.864	-1.28	-9.86	119.757	-1.294	-9.968	120.012	-1.261	-9.713	119.954	-1.268	-9.771
9	119.424	-1.337	-10.301	119.325	-1.35	-10.4	119.591	-1.315	-10.134	119.521	-1.324	-10.204
8	118.969	-1.396	-10.755	118.864	-1.41	-10.861	119.135	-1.374	-10.59	119.053	-1.385	-10.672
7	118.51	-1.456	-11.215	118.367	-1.474	-11.358	118.636	-1.439	-11.089	118.539	-1.452	-11.186
6	117.958	-1.527	-11.767	117.82	-1.545	-11.905	118.081	-1.511	-11.644	117.967	-1.526	-11.758
5	117.442	-1.594	-12.283	117.206	-1.625	-12.519	117.451	-1.593	-12.274	117.317	-1.61	-12.408
4	116.726	-1.687	-12.999	116.481	-1.719	-13.244	116.716	-1.688	-13.009	116.556	-1.709	-13.169
3	115.782	-1.81	-13.943	115.573	-1.837	-14.152	115.818	-1.805	-13.907	115.625	-1.83	-14.1
2	114.53	-1.972	-15.195	114.365	-1.993	-15.36	114.635	-1.958	-15.09	114.396	-1.989	-15.329
1	112.741	-2.204	-16.984	112.582	-2.225	-17.143	112.796	-2.197	-16.929	112.476	-2.239	-17.249

Mean = 129.725, S.D. = 7.705, Provision = 129.725, 1 Unit = $1,000,000

The real paid loss array is regarded as a sample path from the fitted model depicted in sheet 2 under Model Displays.

Below are the residual graphs for a model that has only one parameter in each direction for each of four triangles, the real data and three simulated triangles from the fitted model.

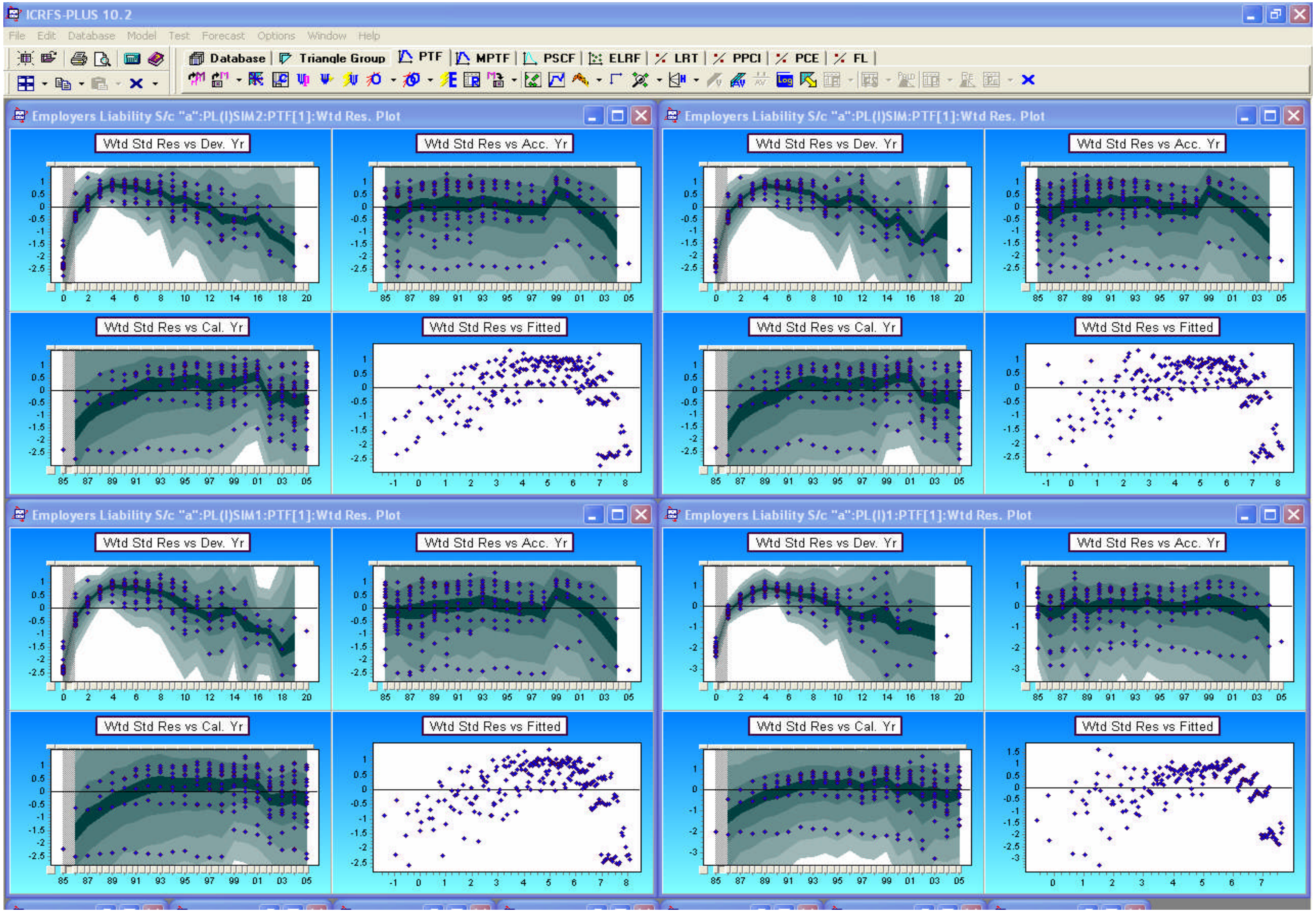
It is almost impossible to distinguish in patterns between the real data and the three simulated triangles.

This demonstrates that the features in the model, fitted to the real data correspond to the features in the data.

Moreover the forecast distributions for the simulated triangles and the real data are statistically the same.

	Mean Reserve	SD Reserve
Real data	106,795,624	4,607,172
Sim 1	103,514,833	4,431,096
Sim 2	105,497,719	4,222,907
Sim 3	105,890,574	4,324,997

These features in the real data cannot be replicated by any link-ratio method or any derivative thereof, period.


ICRFS-PLUS 10.2
Employers Liability S/c "a":PL(I)SIM2:PTF[1]:Wtd Res. Plot
Employers Liability S/c "a":PL(I)SIM:PTF[1]:Wtd Res. Plot
Employers Liability S/c "a":PL(I)SIM1:PTF[1]:Wtd Res. Plot
Employers Liability S/c "a":PL(I)1:PTF[1]:Wtd Res. Plot
Wtd Std Res vs Dev. Yr
Wtd Std Res vs Acc. Yr
Wtd Std Res vs Cal. Yr
Wtd Std Res vs Fitted

Dataset: Employers Liability: Comparison of models for PL(I) and CRE(I)

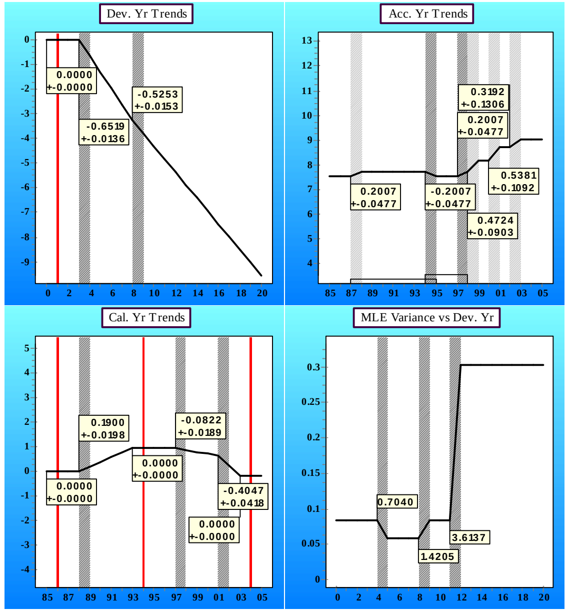
Model: MPTF-optimal1

Final Correlations Final Weighted Residual Correlations Between Datasets
6 iterations were executed

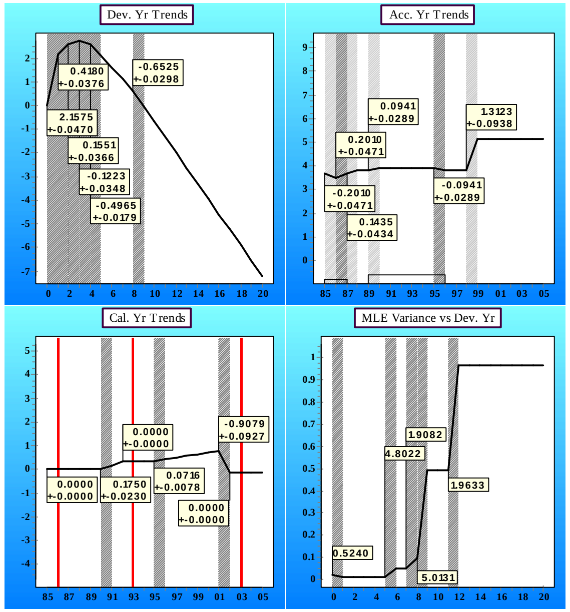
	PL(I)1	CRE(I)1
PL(I)1	1	0.20364
CRE(I)1	0.20364	1

CRE = Case Reserve Estimates
PL = Paid Losses

CRE Model



PL Model



The following comments are pertinent:

1. Along the calendar periods there some similarity in trend patterns, for example a drop from 2001, an increase around 1990, and both are zero from around 2003. Quite often the PL lag the CRE.

However the PL are growing from 95-01 and the CRE are not - they are decreasing.

2. Along the Accident years the total increase from 97-03 is about the same in both loss development arrays

3. The process correlation between the two loss development arrays is 0.203. This means that if a random outcome in a cell in one array is above its trend line the probability that it is above the trend line in the other array is no longer 0.5

4. There is no overall indication that the PL have been increasing at a higher rate than the CRE.

Summary of Appendix F4

Here we use the ELRF modelling framework to model the cumulative Paid Losses.

We show that the features in the data cannot be captured by any link-ratio method.

We only illustrate this with volume-weighted averages, but all our arguments apply to any average link-ratios.

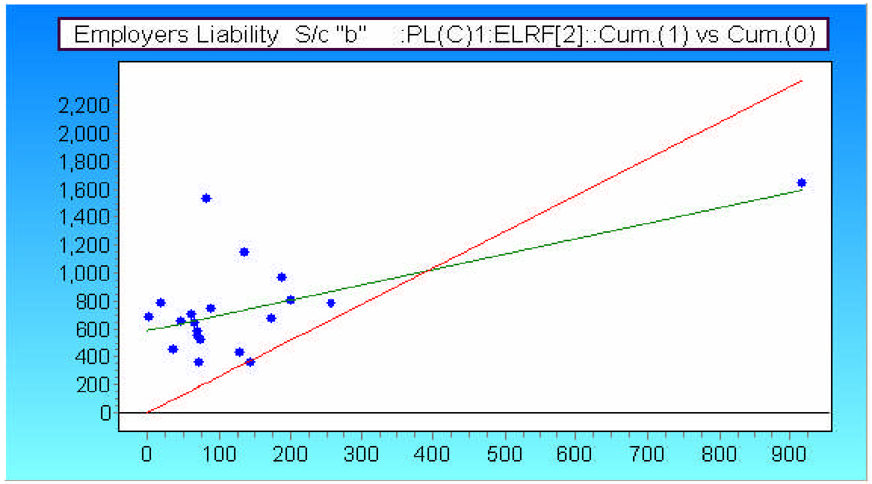
The accident years were adjusted by the exposures below.

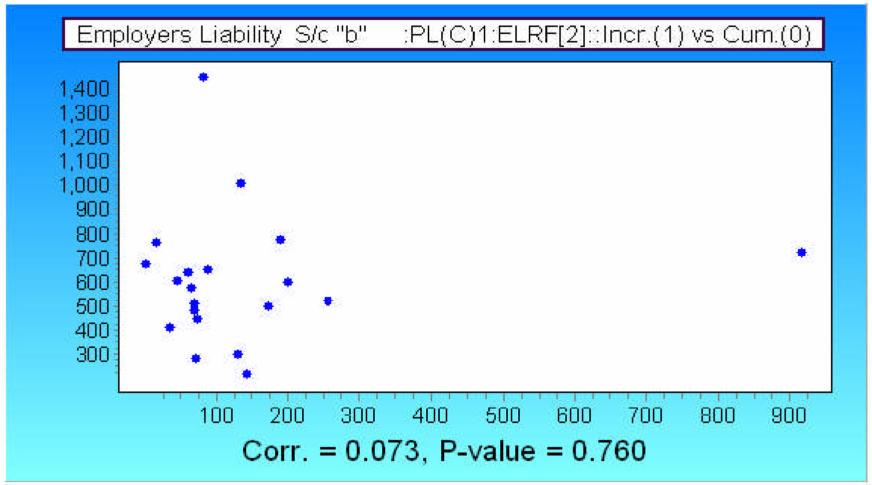
Appendix F4 is uses data from Employers' Liability subclass "b".

Subclass of Employers Liability

Accident Period	Exposures
1985	446
1986	512
1987	883
1988	1171
1989	1381
1990	962
1991	651
1992	612
1993	443
1994	325
1995	286
1996	336
1997	503
1998	301
1999	240
2000	152
2001	113
2002	130
2003	92
2004	88
2005	66

Note that for any of the models in the ELRF framework the modeller cannot choose nor has control over the salient assumptions in the projections. This is further discussed in Uncertainty Working Group Test Template.doc



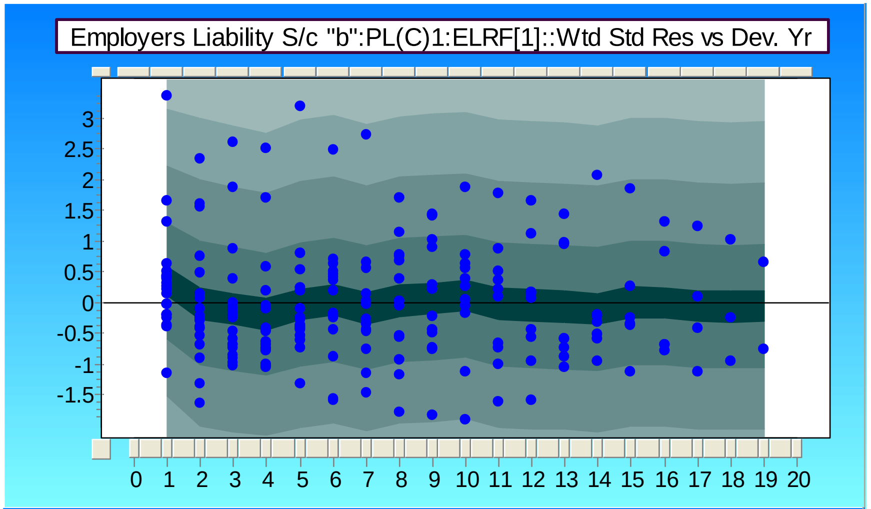


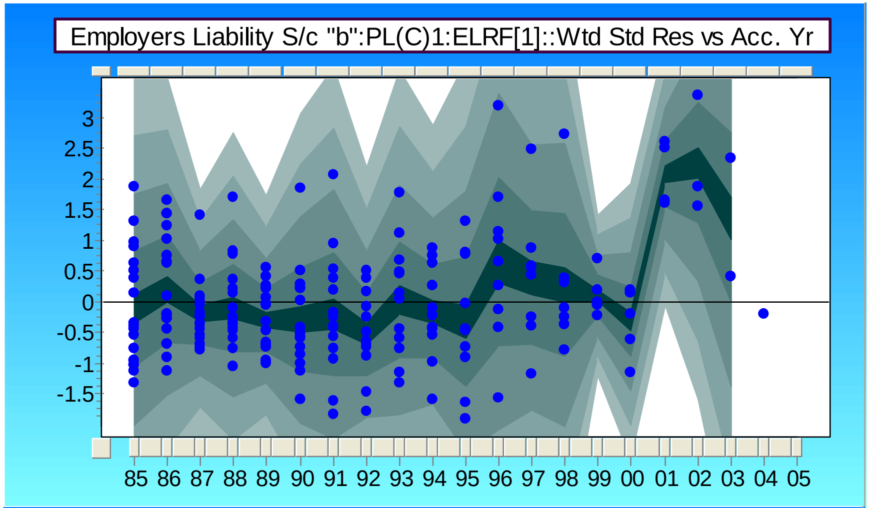
An average ratio is a regression through the origin, yet for these data it is obvious that an intercept is needed.

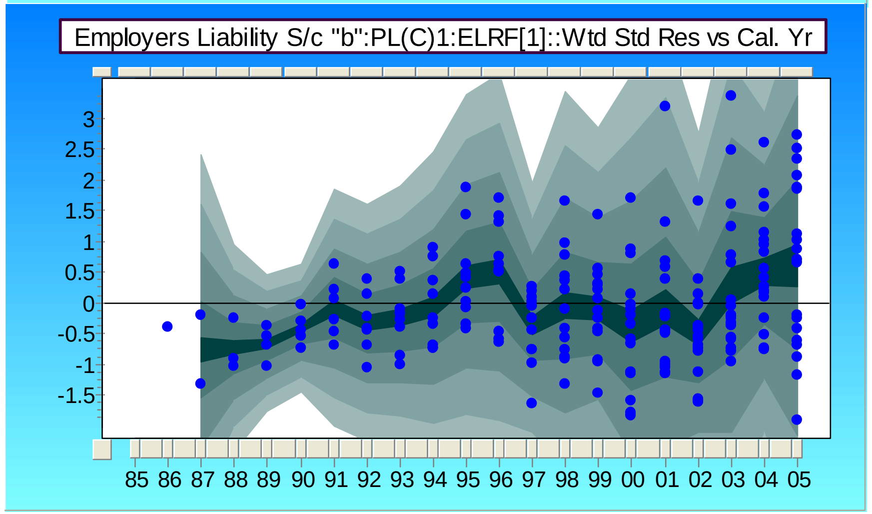
Incrementals in Dev Yr.1 are not correlated to the cumulatives in Dev. Yr. zero, so any link ratio from zero to one has no predictive power.

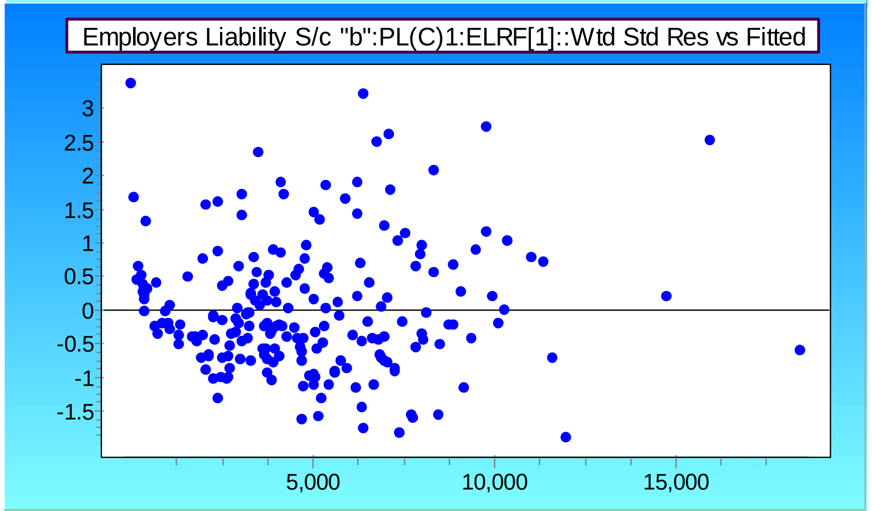
The weighted residuals below represent the residuals of fitting volume-weighted averages (as regression estimators).

Note that the trend structure in the data is not captured. Moreover we showed in the previous sheet that for some periods link ratios have no predictive power whatsoever.

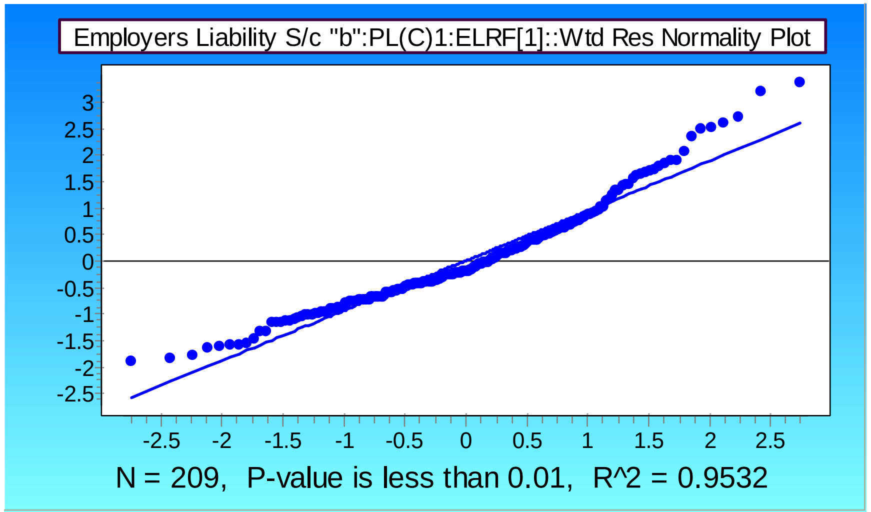








Residuals are not normally distributed.



Employers' Liability:Forecast:Reserve Forecast Summaries:Accident Periods

Accident Yr Summary					
Acc. Yr	Mean		Standard Dev.	CV	
	Reserve	Ultimate		Reserve	Ultimate
1985	0	2,681,470	0	****	0
1986	207,382	4,260,652	53,256	0.26	0.01
1987	377,929	4,064,324	118,813	0.31	0.03
1988	739,274	5,747,407	197,551	0.27	0.03
1989	1,010,346	6,619,931	243,124	0.24	0.04
1990	1,132,616	6,379,035	217,777	0.19	0.03
1991	1,403,619	7,005,605	203,861	0.15	0.03
1992	1,294,431	5,716,867	202,417	0.16	0.04
1993	1,114,836	4,532,711	160,100	0.14	0.04
1994	1,156,373	4,271,824	151,558	0.13	0.04
1995	1,410,553	4,730,236	164,061	0.12	0.03
1996	1,710,512	5,241,271	208,552	0.12	0.04
1997	2,438,467	6,956,118	306,944	0.13	0.04
1998	2,127,977	5,372,060	234,747	0.11	0.04
1999	2,350,370	5,136,188	273,875	0.12	0.05
2000	2,969,212	5,713,587	292,897	0.1	0.05
2001	3,305,219	5,417,264	308,274	0.09	0.06
2002	3,198,321	4,320,671	450,185	0.14	0.1
2003	3,430,801	3,963,033	742,660	0.22	0.19
2004	1,551,995	1,622,149	699,885	0.45	0.43
2005	1,410,124	1,421,962	1,980,809	1.4	1.39
Total	34,340,358	101,174,366	2,989,449	0.09	0.03

1 Unit = $1
CV of forecast for last accident yr is 140.47 %
Model may be inappropriate

Note that the CVs increase as we go down the accident years.

This is because the model has not related the numbers in the triangle in any meaningful way to each other.

Typically for a parsimonious model that capture the features of the data the CVs will decrease.

The latest accident years have more cells to be forecast and therefore the CV of the aggregate should decrease rather than increase.

All the above answers are meaningless because the features of the model have nothing to do with features in the data.

Summary of Appendix F5

This appendix provides the forecasted reserve distribution and quantiles for the aggregate of the two subclasses of business Employers' Liability business.

The aggregate distribution of two lines is obtained by designing a composite model for the two lines which includes correlations between the lines of business (the data are not aggregated).

There are two types of correlations between two lines of business; process correlation and parameter correlation. Process correlation is the correlation between two sets of residuals. When you run the individual PTF models in MPTF for the two lines of business, the process correlation is 22%.

This means that if in one line of business an observation in a particular cell is above the trend line, then the likelihood that it is above the trend line in the other line of business is greater than 50%. Process correlation induces parameter correlation because composite model estimates parameters in one line of business also using the data in the other line of business.

Process correlation and parameter correlation induce correlations between all pairs of accident years, calendar years, and aggregates in the two lines of business. The correlation between the reserves is 7.7%. As a result of this low correlation, the required capital for the two lines of business at the 95th percentile, say, is almost the same as if they were independent.

The forecast scenario used included continuing calendar trends as follows.

For Employers Liability subclass "a"

2005-2011	8.39% +- 0.76%	These assumptions are discussed in Appendix F5.3.
2011-2025	0% +- 0%	

For Employers Liability subclass "b"

2005-2006	24% +- 2.7%	These assumptions are discussed in Appendix F5.4.
2006-2025	0% +- 0%	

The forecasts for the reserve distribution for the subclasses are shown below.

	Mean of Reserve Dsn.	Std. Dv. Of Reserve Dsn.	CoV
Subclass "a"	**131,366,189**	**6,202,006**	0.04721159
Subclass "b"	**37,154,492**	**3,429,684**	0.09230873

It is important to recognise that all forecast distributions are conditional on an explicit set of assumptions that are easily interpretable. If any of these assumptions are varied then the resulting predictive distributions will be different.

Appendix F5.1	Quantiles and V@R for the aggregate reserve for both LOBs.
Appendix F5.2	Process correlation between the two LOBs
Appendices F5.3 and F5.4	Model displays for the two LOBs.
Appendix F5.5	Forecast table for the aggregate of the two LOBs. Means and SDs of distributions are displayed for each cell and for aggregates across cells.
Appendix F 5.6	Accident year summaries of the aggregate of the two LOBs and for each LOB
Appendix F5.7	Calendar year summaries of the aggregate of the two LOBs and for each LOB
Appendix F5.8	Capital allocation by LOB based on a covariance formula.

Employers Liability:Composite DS:MPTF[optimal-1]:PALD:Acc. Yr:Total:Quantiles & VAR

Quantile Statistics and Value at Risk (Acc Year: Total)

Mean = 168.521, S.D. = 7.314, Provision = 168.521, 1 Unit = $1,000,000

%	Sample			Kernel			LogNormal			Gamma		
	Quantile	# S.D.'s	V-a-R	Quantile	# S.D.'s	V-a-R	Quantile	# S.D.'s	V-a-R	Quantile	# S.D.'s	V-a-R
99.995	196.92	3.883	28.399	198.44	4.091	29.92	199.318	4.211	30.797	198.485	4.097	29.964
99.99	195.655	3.71	27.134	197.454	3.956	28.934	197.84	4.009	29.32	197.089	3.906	28.569
99.98	195.51	3.69	26.99	196.346	3.804	27.825	196.311	3.8	27.79	195.641	3.708	27.12
99.97	195.25	3.654	26.729	195.65	3.709	27.129	195.39	3.674	26.869	194.766	3.588	26.246
99.96	194.611	3.567	26.091	195.119	3.636	26.598	194.723	3.582	26.202	194.132	3.502	25.612
99.95	194.141	3.503	25.62	194.676	3.576	26.155	194.198	3.511	25.677	193.633	3.433	25.112
99.94	193.931	3.474	25.41	194.291	3.523	25.77	193.763	3.451	25.242	193.219	3.377	24.698
99.93	193.69	3.441	25.169	193.949	3.477	25.428	193.392	3.4	24.871	192.865	3.328	24.344
99.92	193.11	3.362	24.589	193.644	3.435	25.124	193.067	3.356	24.546	192.555	3.286	24.034
99.91	192.773	3.316	24.252	193.369	3.397	24.848	192.778	3.317	24.258	192.279	3.248	23.759
99.9	192.635	3.297	24.114	193.118	3.363	24.597	192.518	3.281	23.997	192.031	3.214	23.51
99.8	190.948	3.066	22.428	191.421	3.131	22.901	190.755	3.04	22.235	190.344	2.984	21.823
99.7	190.165	2.959	21.644	190.372	2.987	21.851	189.68	2.893	21.159	189.312	2.843	20.791
99.6	189.156	2.821	20.635	189.596	2.881	21.075	188.894	2.785	20.373	188.557	2.739	20.036
99.5	188.629	2.749	20.108	188.982	2.797	20.461	188.27	2.7	19.75	187.957	2.657	19.436
99.4	188.236	2.696	19.715	188.472	2.728	19.951	187.751	2.629	19.23	187.456	2.589	18.935
99.3	187.689	2.621	19.169	188.032	2.668	19.511	187.305	2.568	18.784	187.026	2.53	18.505
99.2	187.344	2.574	18.823	187.646	2.615	19.125	186.912	2.515	18.392	186.647	2.478	18.126
99.1	187.101	2.54	18.581	187.296	2.567	18.775	186.562	2.467	18.041	186.308	2.432	17.788
99	186.728	2.489	18.207	186.979	2.524	18.458	186.244	2.423	17.723	186.001	2.39	17.481
98	184.469	2.18	15.948	184.667	2.208	16.146	184.054	2.124	15.534	183.88	2.1	15.36
97	182.91	1.967	14.39	183.08	1.99	14.559	182.679	1.936	14.158	182.543	1.917	14.022
96	181.586	1.786	13.065	181.865	1.824	13.345	181.65	1.795	13.129	181.541	1.78	13.021
95	180.689	1.664	12.168	180.915	1.695	12.394	180.818	1.681	12.297	180.729	1.669	12.208
94	179.837	1.547	11.316	180.14	1.589	11.619	180.112	1.585	11.592	180.04	1.575	11.519
93	179.269	1.469	10.748	179.486	1.499	10.965	179.496	1.501	10.975	179.437	1.492	10.916
92	178.74	1.397	10.219	178.916	1.421	10.395	178.946	1.425	10.425	178.898	1.419	10.377
91	178.227	1.327	9.707	178.408	1.352	9.887	178.447	1.357	9.926	178.409	1.352	9.888
90	177.775	1.265	9.254	177.947	1.289	9.427	177.989	1.295	9.468	177.96	1.29	9.439
89	177.395	1.213	8.875	177.524	1.231	9.003	177.564	1.236	9.044	177.543	1.233	9.022
88	176.97	1.155	8.449	177.132	1.177	8.611	177.167	1.182	8.647	177.152	1.18	8.632
87	176.665	1.113	8.144	176.764	1.127	8.244	176.794	1.131	8.273	176.785	1.13	8.265
86	176.278	1.061	7.758	176.418	1.08	7.897	176.441	1.083	7.92	176.438	1.082	7.917
85	175.97	1.019	7.45	176.088	1.035	7.568	176.105	1.037	7.584	176.107	1.037	7.586
84	175.673	0.978	7.153	175.775	0.992	7.255	175.784	0.993	7.264	175.791	0.994	7.27
83	175.376	0.937	6.855	175.475	0.951	6.954	175.477	0.951	6.956	175.488	0.953	6.967
82	175.059	0.894	6.538	175.185	0.911	6.664	175.182	0.911	6.661	175.197	0.913	6.676
81	174.83	0.863	6.309	174.905	0.873	6.384	174.897	0.872	6.376	174.915	0.874	6.395
80	174.555	0.825	6.034	174.633	0.836	6.112	174.622	0.834	6.101	174.644	0.837	6.123
79	174.275	0.787	5.754	174.369	0.8	5.848	174.355	0.798	5.834	174.38	0.801	5.859
78	174.035	0.754	5.515	174.112	0.764	5.591	174.096	0.762	5.575	174.124	0.766	5.603
77	173.775	0.718	5.254	173.86	0.73	5.339	173.844	0.728	5.323	173.875	0.732	5.354
76	173.576	0.691	5.055	173.614	0.696	5.093	173.599	0.694	5.078	173.632	0.699	5.111
75	173.362	0.662	4.841	173.372	0.663	4.852	173.359	0.662	4.838	173.395	0.666	4.874
74	173.128	0.63	4.607	173.136	0.631	4.615	173.125	0.629	4.604	173.163	0.635	4.642
73	172.849	0.592	4.329	172.903	0.599	4.382	172.896	0.598	4.375	172.935	0.604	4.415
72	172.594	0.557	4.074	172.675	0.568	4.154	172.671	0.567	4.15	172.712	0.573	4.192
71	172.377	0.527	3.856	172.451	0.537	3.931	172.45	0.537	3.929	172.493	0.543	3.973
70	172.149	0.496	3.628	172.232	0.507	3.711	172.233	0.508	3.713	172.278	0.514	3.757
69	171.915	0.464	3.394	172.016	0.478	3.495	172.02	0.478	3.499	172.066	0.485	3.546
68	171.717	0.437	3.196	171.804	0.449	3.283	171.81	0.45	3.289	171.858	0.456	3.337
67	171.519	0.41	2.998	171.596	0.42	3.075	171.603	0.421	3.082	171.652	0.428	3.131
66	171.305	0.381	2.785	171.391	0.392	2.87	171.399	0.393	2.878	171.449	0.4	2.928
65	171.132	0.357	2.611	171.19	0.365	2.669	171.197	0.366	2.676	171.248	0.373	2.727
64	170.927	0.329	2.407	170.991	0.338	2.47	170.997	0.339	2.477	171.049	0.346	2.529
63	170.762	0.306	2.241	170.795	0.311	2.275	170.8	0.312	2.28	170.853	0.319	2.332
62	170.577	0.281	2.056	170.602	0.285	2.082	170.605	0.285	2.084	170.658	0.292	2.138
61	170.398	0.257	1.878	170.412	0.259	1.891	170.412	0.259	1.891	170.465	0.266	1.945
60	170.213	0.231	1.692	170.223	0.233	1.702	170.22	0.232	1.699	170.274	0.24	1.753
59	169.979	0.199	1.459	170.035	0.207	1.515	170.029	0.206	1.509	170.084	0.214	1.563

TABLE CONTINUES ON NEXT PAGE

Mean = 168.521, S.D. = 7.314, Provision = 168.521, 1 Unit = $1,000,000

%	Sample			Kernel			LogNormal			Gamma		
	Quantile	# S.D.'s	V-a-R	Quantile	# S.D.'s	V-a-R	Quantile	# S.D.'s	V-a-R	Quantile	# S.D.'s	V-a-R
58	169.792	0.174	1.271	169.85	0.182	1.329	169.84	0.18	1.32	169.895	0.188	1.375
57	169.63	0.152	1.109	169.666	0.157	1.145	169.653	0.155	1.132	169.708	0.162	1.187
56	169.463	0.129	0.942	169.483	0.132	0.962	169.466	0.129	0.945	169.521	0.137	1
55	169.277	0.103	0.756	169.3	0.107	0.78	169.28	0.104	0.76	169.335	0.111	0.815
54	169.09	0.078	0.569	169.119	0.082	0.598	169.096	0.079	0.575	169.15	0.086	0.63
53	168.932	0.056	0.411	168.937	0.057	0.416	168.911	0.053	0.391	168.966	0.061	0.445
52	168.776	0.035	0.255	168.756	0.032	0.235	168.728	0.028	0.207	168.782	0.036	0.261
51	168.609	0.012	0.089	168.574	0.007	0.053	168.545	0.003	0.024	168.598	0.011	0.078
50	168.434	-0.012	-0.086	168.392	-0.018	-0.129	168.362	-0.022	-0.159	168.415	-0.014	-0.106
49	168.238	-0.039	-0.283	168.209	-0.043	-0.311	168.18	-0.047	-0.341	168.232	-0.04	-0.289
48	168.071	-0.061	-0.449	168.026	-0.068	-0.495	167.997	-0.072	-0.523	168.048	-0.065	-0.472
47	167.924	-0.082	-0.596	167.842	-0.093	-0.679	167.815	-0.097	-0.706	167.865	-0.09	-0.656
46	167.721	-0.109	-0.8	167.656	-0.118	-0.864	167.632	-0.122	-0.889	167.682	-0.115	-0.839
45	167.508	-0.138	-1.013	167.47	-0.144	-1.051	167.449	-0.147	-1.072	167.498	-0.14	-1.023
44	167.307	-0.166	-1.214	167.282	-0.169	-1.239	167.265	-0.172	-1.255	167.313	-0.165	-1.207
43	167.123	-0.191	-1.398	167.093	-0.195	-1.427	167.081	-0.197	-1.439	167.129	-0.19	-1.392
42	166.928	-0.218	-1.592	166.903	-0.221	-1.617	166.897	-0.222	-1.624	166.943	-0.216	-1.578
41	166.721	-0.246	-1.8	166.712	-0.247	-1.808	166.711	-0.247	-1.809	166.757	-0.241	-1.764
40	166.556	-0.269	-1.965	166.52	-0.273	-2	166.525	-0.273	-1.996	166.569	-0.267	-1.951
39	166.338	-0.298	-2.183	166.327	-0.3	-2.194	166.337	-0.298	-2.183	166.381	-0.293	-2.14
38	166.099	-0.331	-2.422	166.133	-0.326	-2.388	166.149	-0.324	-2.372	166.191	-0.318	-2.329
37	165.945	-0.352	-2.576	165.937	-0.353	-2.584	165.959	-0.35	-2.562	166	-0.345	-2.521
36	165.78	-0.375	-2.74	165.74	-0.38	-2.78	165.767	-0.376	-2.753	165.808	-0.371	-2.713
35	165.589	-0.401	-2.931	165.542	-0.407	-2.978	165.574	-0.403	-2.946	165.613	-0.397	-2.907
34	165.385	-0.429	-3.136	165.343	-0.435	-3.178	165.38	-0.429	-3.141	165.417	-0.424	-3.104
33	165.188	-0.456	-3.333	165.141	-0.462	-3.38	165.183	-0.456	-3.338	165.219	-0.451	-3.302
32	165.008	-0.48	-3.512	164.937	-0.49	-3.583	164.984	-0.484	-3.537	165.018	-0.479	-3.502
31	164.813	-0.507	-3.708	164.732	-0.518	-3.789	164.782	-0.511	-3.738	164.815	-0.507	-3.705
30	164.565	-0.541	-3.956	164.524	-0.546	-3.997	164.578	-0.539	-3.943	164.61	-0.535	-3.911
29	164.328	-0.573	-4.192	164.313	-0.575	-4.208	164.371	-0.567	-4.149	164.401	-0.563	-4.12
28	164.132	-0.6	-4.389	164.1	-0.604	-4.421	164.161	-0.596	-4.36	164.189	-0.592	-4.331
27	163.92	-0.629	-4.601	163.884	-0.634	-4.637	163.948	-0.625	-4.573	163.974	-0.622	-4.547
26	163.697	-0.66	-4.824	163.665	-0.664	-4.856	163.731	-0.655	-4.79	163.755	-0.652	-4.766
25	163.475	-0.69	-5.045	163.442	-0.694	-5.079	163.509	-0.685	-5.011	163.531	-0.682	-4.989
24	163.293	-0.715	-5.228	163.216	-0.725	-5.305	163.284	-0.716	-5.237	163.303	-0.713	-5.217
23	163.062	-0.746	-5.458	162.986	-0.757	-5.535	163.053	-0.748	-5.468	163.07	-0.745	-5.45
22	162.806	-0.781	-5.714	162.751	-0.789	-5.769	162.817	-0.78	-5.703	162.832	-0.778	-5.689
21	162.592	-0.811	-5.929	162.512	-0.821	-6.008	162.575	-0.813	-5.945	162.587	-0.811	-5.933
20	162.358	-0.843	-6.163	162.268	-0.855	-6.253	162.327	-0.847	-6.194	162.336	-0.846	-6.185
19	162.077	-0.881	-6.444	162.018	-0.889	-6.503	162.072	-0.882	-6.449	162.077	-0.881	-6.443
18	161.818	-0.916	-6.703	161.761	-0.924	-6.76	161.808	-0.918	-6.712	161.811	-0.917	-6.71
17	161.584	-0.948	-6.936	161.497	-0.96	-7.024	161.536	-0.955	-6.985	161.535	-0.955	-6.986
16	161.313	-0.985	-7.208	161.225	-0.998	-7.296	161.254	-0.994	-7.267	161.248	-0.994	-7.273
15	161.066	-1.019	-7.455	160.942	-1.036	-7.579	160.96	-1.034	-7.561	160.95	-1.035	-7.57
14	160.78	-1.058	-7.741	160.648	-1.076	-7.873	160.654	-1.076	-7.867	160.639	-1.078	-7.882
13	160.423	-1.107	-8.098	160.341	-1.118	-8.179	160.332	-1.119	-8.188	160.313	-1.122	-8.208
12	160.116	-1.149	-8.405	160.017	-1.163	-8.504	159.995	-1.166	-8.526	159.969	-1.169	-8.551
11	159.833	-1.188	-8.687	159.674	-1.209	-8.846	159.637	-1.215	-8.884	159.606	-1.219	-8.915
10	159.457	-1.239	-9.064	159.304	-1.26	-9.217	159.256	-1.267	-9.265	159.218	-1.272	-9.303
9	159.103	-1.288	-9.417	158.901	-1.315	-9.619	158.847	-1.323	-9.673	158.801	-1.329	-9.719
8	158.696	-1.343	-9.824	158.455	-1.376	-10.066	158.405	-1.383	-10.116	158.35	-1.391	-10.171
7	158.169	-1.415	-10.351	157.953	-1.445	-10.568	157.919	-1.449	-10.602	157.854	-1.458	-10.667
6	157.498	-1.507	-11.022	157.382	-1.523	-11.138	157.379	-1.523	-11.142	157.301	-1.534	-11.219
5	156.853	-1.595	-11.667	156.733	-1.612	-11.788	156.765	-1.607	-11.756	156.673	-1.62	-11.848
4	156.136	-1.693	-12.385	155.986	-1.714	-12.534	156.046	-1.706	-12.474	155.937	-1.72	-12.584
3	155.289	-1.809	-13.232	155.087	-1.837	-13.433	155.168	-1.826	-13.353	155.035	-1.844	-13.485
2	154.183	-1.96	-14.338	153.861	-2.004	-14.66	154.008	-1.984	-14.513	153.842	-2.007	-14.679
1	151.941	-2.267	-16.579	151.874	-2.276	-16.647	152.197	-2.232	-16.323	151.974	-2.262	-16.547

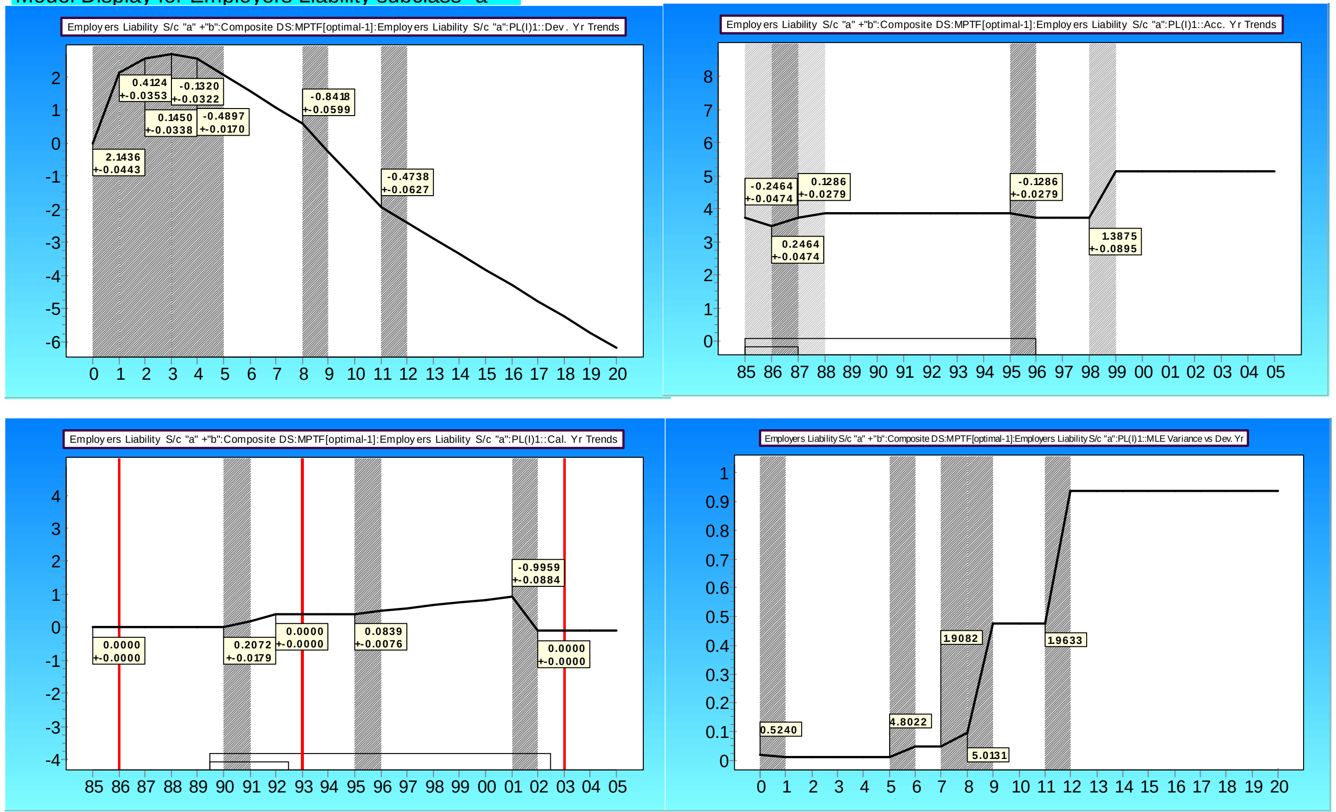
Mean = 168.521, S.D. = 7.314, Provision = 168.521, 1 Unit = $1,000,000

Employers Liability:Composite DS:MPTF[optimal-1]:Weighted Residual Covariances Between Datasets:Final Correlations

Final Weighted Residual Correlations Between Datasets		
	Employers Liability subclass "a":PL(I)1	Employers Liability subclass "b":PL(I)1
Employers Liability subclass "a":PL(I)1	1	0.224567
Employers Liability subclass "b":PL(I)1	0.224567	1
5 iterations were executed		

The process correlation between the two datasets as measured by this model is 22%.

Model Display for Employers Liability subclass "a"



The above model display shows the trends in the three directions along with process variance for the Employers Liability UC 0110 data after incoporating process correlation.

Model comments:

Process variance is high particularly after development period 11.

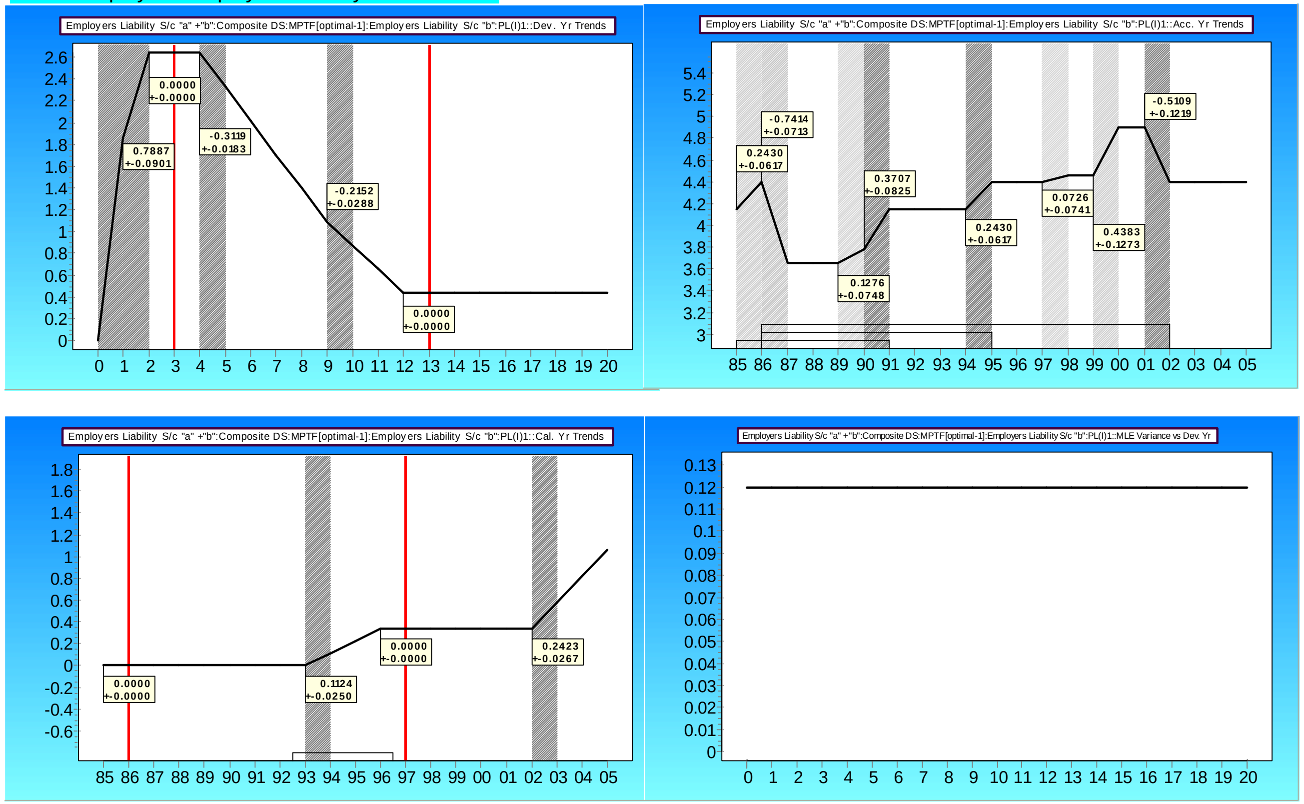
The development trends show that the company is still paying a significant amount even at development period 6.

There have been a number of changes in accident levels, but the level has been stable since 1999.

The calendar direction is fairly volatile with a number of calendar year trends. A conservative forecast would incoporate a positive calendar trend (eg: 8%+-0.78%) in the near future.

In respect of future calendar year trends we have adopted a consertive view and assumed that the future calendar trend reverts to the 8.39%+_ 0.76% for six years (2005-2011) before reverting to zero. This assumption is critical to the forecast distributions.

Model Display for Employers Liability subclass "b"



The model display above shows the trends in the three directions along with process variance for the Employers Liability UC 0012 data after incoporating process correlation.

Model comments:

There are no changes in process variance by development period. Initially, process variance is higher than for UC0110.

The development trends show that the company is still paying a significant amount even at development period 5.

There have been a number of changes in accident levels. The current level has been stable since 2002.

The calendar direction is fairly volatile with a number of calendar year trends.

In respect of future calendar year trends we have adopted a consertive view and assume that the recent large positive calendar trend of 24% +- 2.87% continues for at one more year before reverting to zero. This assumption is critical to the forecast distributions.

Employers Liability: Composite DS:MPTF[optimal-1]: Forecast:Aggregate:Reserve Forecast Table

Accident Period vs Development Period

	Cal. Per. Total	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	Reserve	Ultimate
1985	371,807	371,807	3,092,226	4,802,587	5,486,039	4,860,184	3,028,502	2,317,309	1,743,011	1,110,873	616,037	313,270	186,522	154,874	125,926	106,498	93,409	84,555	69,794	86,027	107,834	136,389	0	28,262,000
	478,653	478,653	3,054,306	4,004,796	5,044,041	4,498,646	3,513,731	2,382,079	1,731,117	1,526,103	724,896	257,662	294,723	91,136	99,462	60,147	67,813	112,023	40,585	65,787	85,019	129,272	0	0
1986	3,449,799	357,574	2,942,008	4,636,973	5,263,752	4,689,467	3,521,368	2,692,002	1,690,003	1,106,639	636,888	369,998	223,892	185,036	156,162	136,783	123,726	102,204	125,964	157,876	199,651	253,802	253,802	27,877,452
	3,485,668	431,362	3,040,670	4,150,149	4,838,383	4,541,616	3,144,698	2,361,280	1,399,143	1,045,055	701,428	525,607	163,379	178,214	244,901	149,780	61,427	82,034	136,531	199,783	228,210	94,140	94,140	94,140
1987	8,122,527	377,931	3,131,300	4,888,345	5,571,796	5,966,720	4,487,202	2,844,860	1,800,190	1,172,113	711,454	385,059	222,201	184,457	150,234	127,263	89,821	107,147	132,018	165,527	209,637	209,037	418,674	30,268,126
	7,533,589	488,123	2,859,152	5,117,642	4,826,260	4,573,057	4,382,958	2,558,125	2,087,463	991,822	617,930	278,437	236,606	202,842	61,510	180,778	85,971	69,729	101,876	129,172	77,841	77,688	113,942	113,942
1988	13,674,163	419,850	3,462,326	5,440,002	7,449,753	7,978,614	4,969,044	3,201,795	2,046,253	1,441,388	862,979	472,775	276,794	229,470	188,992	123,730	144,949	176,851	220,621	278,765	277,731	277,021	833,516	38,571,423
	12,548,899	495,556	3,423,021	5,871,152	6,045,053	6,772,348	4,998,324	3,225,429	1,593,862	1,887,727	382,823	816,403	583,644	189,532	494,119	523,950	130,285	192,648	112,030	103,428	103,155	102,976	191,389	191,389
1989	18,892,537	417,931	3,427,902	6,489,938	8,893,858	7,892,357	4,999,557	3,251,600	2,249,332	1,556,327	936,279	519,228	308,331	255,272	153,287	175,488	211,379	261,934	329,943	328,344	327,251	326,501	1,312,040	41,405,025
	18,376,437	498,745	3,770,019	7,335,137	7,977,961	6,608,136	4,617,670	3,170,227	2,305,151	1,587,376	708,373	705,909	176,879	79,590	104,940	137,114	166,351	143,409	122,300	121,862	121,580	121,393	269,010	269,010
1990	22,557,262	399,592	3,988,842	7,508,594	8,581,660	7,673,107	4,846,897	3,375,477	2,281,396	1,573,737	935,262	502,502	287,382	145,449	153,944	176,785	213,385	265,442	262,937	261,230	260,063	259,262	1,308,935	45,645,710
	22,930,560	407,783	4,437,986	8,116,963	7,855,717	7,400,485	5,323,151	4,302,043	2,218,437	2,288,046	1,159,598	201,559	127,923	16,912	116,900	144,581	218,692	99,329	98,528	98,033	97,717	97,511	256,899	256,899
1991	30,153,425	419,494	4,201,532	6,538,480	7,535,288	6,770,916	4,629,556	3,180,147	2,150,602	1,484,205	883,566	476,854	171,186	162,782	180,486	213,456	262,901	259,445	257,100	255,502	254,409	253,659	1,543,015	40,921,730
	28,319,915	401,905	4,302,360	6,945,790	8,156,845	6,378,916	5,151,438	2,771,520	2,478,978	1,376,124	349,872	65,680	68,456	392,145	186,054	352,631	97,016	95,768	95,018	94,553	94,256	94,062	270,193	270,193
1992	37,899,325	394,513	3,261,224	5,175,392	5,958,794	5,799,656	3,917,589	2,694,284	1,824,917	1,261,044	754,440	231,113	182,025	177,106	205,232	250,210	245,961	243,088	241,138	239,809	238,901	238,162	1,697,268	33,279,566
	36,090,149	445,163	3,873,583	5,376,808	6,930,624	5,504,153	3,828,273	2,250,251	1,526,529	769,774	414,761	287,097	140,743	166,297	68,243	92,441	90,745	89,754	89,154	88,779	88,538	88,350	281,898	281,898
1993	37,212,539	324,038	2,709,774	4,295,191	5,327,322	5,104,779	3,440,070	2,361,211	1,595,053	1,099,858	303,451	204,788	164,315	160,655	189,261	183,546	179,697	177,095	175,329	174,126	173,153	172,538	1,424,746	29,755,882
	36,085,684	320,763	2,695,707	4,874,905	5,969,137	4,834,813	3,263,004	3,062,041	1,315,776	967,392	456,236	194,398	205,731	171,232	71,103	68,042	66,341	65,362	64,778	64,418	64,149	63,989	226,269	226,269
1994	35,176,920	359,142	3,015,018	5,095,314	6,294,645	6,019,655	4,039,691	2,763,058	1,857,621	515,638	320,125	207,283	160,595	158,361	147,822	140,751	135,989	132,770	130,584	128,827	127,720	127,020	1,229,846	32,179,698
	35,705,302	374,943	3,134,302	5,369,278	5,775,133	6,563,261	4,511,286	2,911,676	1,275,786	553,108	319,403	117,463	44,214	67,390	58,359	53,384	50,643	49,108	48,222	47,602	47,260	47,063	190,665	190,665
1995	32,706,667	399,726	3,625,022	6,076,233	7,514,616	7,185,281	4,820,196	3,295,943	878,795	609,108	390,946	267,632	221,045	180,198	167,562	159,083	153,373	149,512	146,426	144,488	143,268	142,496	1,607,451	34,376,597
	33,652,902	296,004	3,763,575	5,485,848	7,513,465	7,211,298	4,415,095	1,860,767	1,092,444	776,126	136,386	218,139	80,723	77,955	66,809	60,637	57,233	55,327	54,047	53,368	52,992	52,775	238,892	238,892
1996	33,215,762	404,720	3,619,707	6,088,764	7,508,161	7,183,143	4,824,141	1,324,922	909,371	657,344	452,657	351,337	251,039	204,368	191,896	183,527	177,890	173,410	170,608	168,850	167,742	167,042	2,207,711	38,315,565
	36,084,135	393,373	3,543,228	5,895,788	8,170,497	10,375,471	5,192,045	624,556	1,158,156	502,229	252,511	132,994	90,697	84,860	74,676	69,098	66,027	64,015	62,974	62,409	62,090	61,903	317,569	317,569
1997	34,164,465	506,838	4,505,027	7,614,497	9,353,865	8,956,143	2,462,754	1,730,480	1,246,743	948,449	724,780	499,791	363,452	295,467	280,185	269,931	261,827	256,779	253,623	251,643	250,395	249,605	3,957,478	40,227,455
	31,318,779	358,375	3,938,368	5,627,544	10,156,293	9,389,914	3,403,399	1,884,588	1,068,636	442,859	290,618	184,803	130,370	118,146	106,819	100,663	96,724	94,732	93,676	93,090	92,752	92,551	568,520	568,520
The above table shows	36,425,028	496,056	4,467,052	7,476,163	9,258,248	3,377,814	2,244,086	1,559,867	1,109,268	869,312	562,343	366,965	255,456	208,424	192,769	180,466	172,833	168,079	165,107	163,241	162,066	161,323	3,628,383	45,333,073
	36,536,449	306,229	3,689,136	6,253,143	11,995,665	11,231,031	5,317,770	1,955,620	956,095	237,470	252,127	145,941	94,880	92,986	78,751	69,599	65,143	62,938	61,805	61,196	60,854	60,656	527,345	527,345
1999	40,402,067	1,738,650	15,959,383	26,325,986	11,449,881	10,208,268	6,433,608	4,182,506	2,918,997	2,014,392	1,198,587	645,814	370,597	307,854	239,782	197,689	171,559	155,277	145,091	138,695	134,662	132,110	8,771,108	52,712,643
	38,438,626	307,131	3,464,725	7,324,949	11,062,092	11,742,461	6,739,436	3,300,742	628,979	582,395	775,116	379,199	192,815	241,502	157,640	108,366	80,241	64,880	56,848	52,748	50,649	49,548	1,346,629	1,346,629
2000	54,289,240	1,573,041	14,432,361	8,342,251	9,694,352	8,714,323	5,564,105	3,949,155	2,673,431	1,846,671	1,102,643	599,445	347,714	275,376	218,844	183,888	162,188	148,666	140,207	134,895	131,546	129,427	12,044,097	48,245,854
	41,166,092	307,195	3,617,963	7,511,621	9,917,515	10,347,010	4,500,453	845,522	577,022	532,010	703,287	345,578	177,266	203,660	134,941	95,165	72,990	61,207	55,178	52,130	50,567	49,740	1,552,288	1,552,288
2001	73,924,017	1,575,232	4,993,416	7,728,899	8,982,986	8,079,884	5,562,002	3,814,101	2,573,225	1,772,609	1,046,762	552,559	293,418	231,961	179,648	147,300	127,220	114,707	106,879	101,963	98,865	96,904	16,820,122	44,579,752
	43,321,240	160,458	2,748,885	6,929,566	9,208,320	8,712,400	621,597	825,509	561,761	523,203	701,134	340,997	158,885	185,195	120,667	82,704	60,991	49,107	42,887	39,715	38,095	37,248	1,677,408	1,677,408
2002	35,321,453	585,947	4,967,294	7,666,375	8,905,292	8,635,328	5,792,865	3,961,021	2,661,936	1,827,928	1,065,588	506,859	256,836	202,461	150,234	117,939	97,892	85,400	77,585	72,677	69,583	67,626	25,649,757	46,069,249
	40,049,723	156,405	3,489,409	7,738,537	9,035,141	933,223	629,112	872,262	592,027	556,348	754,644	334,747	152,640	181,105	115,638	76,347	53,041	39,599	32,167	28,227	26,188	25,131	2,023,136	2,023,136
2003	40,180,074	424,976	3,601,831	5,588,352	7,032,127	6,721,987	4,506,889	3,080,310	2,068,756	1,419,879	766,118	363,898	184,019	145,042	107,416	84,150	69,707	60,707	55,077	51,542	49,313	47,902	26,814,841	36,868,372
	43,407,825	489,198	3,288,602	6,275,731	757,692	729,152	493,040	683,205	463,952	435,942	543,522	241,025	109,829	130,384	83,199	54,862	38,035	28,311	22,923	20,060	18,577	17,808	1,810,063	1,810,063
2004	41,283,942	337,986	2,862,648	4,864,978	6,015,218	5,752,190	3,859,583	2,639,606	1,774,303	1,129,303	613,351	296,372	153,558	121,215	91,841	73,677	62,402	55,376	50,981	48,220	46,480	45,379	27,694,034	30,980,201
	42,771,658	221,080	3,065,087	530,751	654,347	632,685	429,961	585,851	398,401	342,746	425,764	189,540	87,143	102,693	66,082	44,269	31,509	24,305	20,417	18,395	17,355	16,815	1,704,135	1,704,135
2005	40,206,282	308,042	2,825,509	4,715,393	5,852,836	5,593,098	3,747,330	2,559,669	1,588,190	1,008,490	544,324	258,771	131,020	103,277	76,577	60,067	49,818	43,432	39,437	36,928	35,346	34,345	29,303,857	29,588,893
	38,641,302	285,036	306,514	512,653	640,198	620,866	422,490	576,701	359,125	309,455	385,754	171,094	77,996	92,561	59,087	38,991	27,067	20,184	16,375	14,354	13,308	12,765	1,747,421	1,747,421
	Total Fitted/Paid		2006	2007	2008	2009	2010	2011	2012	2013	2014	2015	2016	2017	2018	2019	2020	2021	2022	2023	2024	2025	Total Reserve	Total Ultimate
Cal. Per. Total	669,629,300		40,152,894	35,155,581	28,479,904	21,014,185	14,187,514	9,614,941	6,101,316	3,936,035	2,623,111	1,899,358	1,479,245	1,182,439	843,120	626,381	461,240	312,441	204,596	131,311	80,725	34,345	168,520,681	795,464,268
	626,943,587		2,134,235	2,020,694	1,980,152	1,720,152	1,458,652	1,314,889	939,590	706,590	558,450	349,163	262,725	219,099	155,839	117,543	90,634	62,804	43,397	30,576	22,150	12,765	7,314,170	7,314,170

1 Unit = $1

The above table shows the forecasted reserve distribution for each cell (mean is in black, standard deviation in red) along with the data (mean in black, observed in blue).

The aggregate total reserve has a distribution with a mean of 168.5 Million and a standard deviation of 7.3 Million)

Note from the quantiles table that there is skewness in all the distributions.

Employers Liability: Composite DS: MPTF[optimal-1]: Forecast:
Aggregate: Reserve Forecast Summaries:Accident Periods

Aggregate of both LOBs

Accident Yr Summary

Acc. Yr	Mean Reserve	Mean Ultimate	Standard Dev.	CV Reserve	CV Ultimate
1985	0	28,262,000	0	****	0
1986	253,802	27,877,452	94,140	0.37	0
1987	418,674	30,268,126	113,942	0.27	0
1988	833,516	38,571,423	191,389	0.23	0
1989	1,312,040	41,405,025	269,010	0.21	0.01
1990	1,308,935	45,645,710	256,899	0.2	0.01
1991	1,543,015	40,921,730	270,193	0.18	0.01
1992	1,697,268	33,279,566	281,898	0.17	0.01
1993	1,424,746	29,755,882	226,269	0.16	0.01
1994	1,229,846	32,179,698	190,665	0.16	0.01
1995	1,607,451	34,376,597	238,892	0.15	0.01
1996	2,207,711	38,315,565	317,569	0.14	0.01
1997	3,957,478	40,227,455	568,520	0.14	0.01
1998	3,628,383	45,333,073	527,345	0.15	0.01
1999	8,771,108	52,712,643	1,346,629	0.15	0.03
2000	12,044,097	48,245,854	1,552,288	0.13	0.03
2001	16,820,122	44,579,752	1,677,408	0.1	0.04
2002	25,649,757	46,069,249	2,023,136	0.08	0.04
2003	26,814,841	36,868,372	1,810,063	0.07	0.05
2004	27,694,034	30,980,201	1,704,135	0.06	0.06
2005	29,303,857	29,588,893	1,747,421	0.06	0.06
Total	#########	#########	7,314,170	0.04	0.01

1 Unit = $1

LOB UC 0110

Accident Yr Summary

Acc. Yr	Mean Reserve	Mean Ultimate	Standard Dev.	CV Reserve	CV Ultimate
1985	0	25,580,529	0	****	0
1986	1,155	23,571,536	1,714	1.48	0
1987	3,341	26,166,398	3,637	1.09	0
1988	7,319	32,737,092	6,679	0.91	0
1989	12,890	34,496,291	10,382	0.81	0
1990	21,775	39,112,131	15,915	0.73	0
1991	31,346	33,808,075	21,187	0.68	0
1992	39,308	27,199,170	25,008	0.64	0
1993	53,175	24,966,436	32,382	0.61	0
1994	97,835	27,932,237	58,018	0.59	0
1995	162,340	29,611,803	79,820	0.49	0
1996	254,902	32,831,997	111,835	0.44	0
1997	559,913	32,312,238	243,491	0.43	0.01
1998	1,021,125	39,481,731	293,548	0.29	0.01
1999	6,238,208	47,393,925	1,238,792	0.2	0.03
2000	8,939,135	42,396,518	1,380,636	0.15	0.03
2001	13,892,202	39,539,787	1,524,646	0.11	0.04
2002	23,055,053	42,352,195	1,924,838	0.08	0.05
2003	24,566,249	34,087,548	1,726,268	0.07	0.05
2004	25,148,795	28,364,809	1,605,288	0.06	0.06
2005	27,260,125	27,533,323	1,680,000	0.06	0.06
Total	131,366,189	691,475,767	6,202,006	0.05	0.01

1 Unit = $1

LOB UC 0112

Accident Yr Summary

Acc. Yr	Mean Reserve	Mean Ultimate	Standard Dev.	CV Reserve	CV Ultimate
1985	0	2,681,470	0	****	0
1986	252,647	4,305,917	93,868	0.37	0.02
1987	415,333	4,101,728	113,365	0.27	0.03
1988	826,197	5,834,331	190,344	0.23	0.03
1989	1,299,150	6,908,735	267,402	0.21	0.04
1990	1,287,160	6,533,579	254,360	0.2	0.04
1991	1,511,669	7,113,656	266,610	0.18	0.04
1992	1,657,960	6,080,396	277,661	0.17	0.05
1993	1,371,571	4,789,446	220,105	0.16	0.05
1994	1,132,010	4,247,461	175,254	0.15	0.04
1995	1,445,111	4,764,794	216,004	0.15	0.05
1996	1,952,809	5,483,568	284,060	0.15	0.05
1997	3,397,566	7,915,217	485,516	0.14	0.06
1998	2,607,259	5,851,342	403,170	0.15	0.07
1999	2,532,900	5,318,719	393,933	0.16	0.07
2000	3,104,962	5,849,336	547,751	0.18	0.09
2001	2,927,921	5,039,965	520,202	0.18	0.1
2002	2,594,704	3,717,054	401,527	0.15	0.11
2003	2,248,592	2,780,823	345,912	0.15	0.12
2004	2,545,238	2,615,392	385,782	0.15	0.15
2005	2,043,732	2,055,570	302,516	0.15	0.15
Total	37,154,492	103,988,500	3,429,684	0.09	0.03

1 Unit = $1

The above table shows the forecasted reserve distribution for each cell (mean is in black, standard deviation in red) along with the data (mean in black, observed in blue).
The aggregate total reserve has a distribution with a mean of 168.5 Million and a standard deviation of 7.3 Million)

Employers Liability: Composite DS: MPTF[optimal-1]:
Forecast:Aggregate:Reserve Forecast Summaries:Calendar Periods

Aggregate of both LOBs

Calendar Yr Summary			
Calendar Yr	Mean Reserve	Standard Dev.	CV Reserve
2006	40,152,894	2,134,235	0.05
2007	35,155,581	2,020,694	0.06
2008	28,479,904	1,980,152	0.07
2009	21,014,185	1,720,152	0.08
2010	14,187,514	1,458,652	0.1
2011	9,614,941	1,314,889	0.14
2012	6,101,316	939,590	0.15
2013	3,936,035	706,590	0.18
2014	2,623,111	558,450	0.21
2015	1,899,358	349,163	0.18
2016	1,479,245	262,725	0.18
2017	1,182,439	219,099	0.19
2018	843,120	155,839	0.18
2019	626,381	117,543	0.19
2020	461,240	90,634	0.2
2021	312,441	62,804	0.2
2022	204,596	43,397	0.21
2023	131,311	30,576	0.23
2024	80,725	22,150	0.27
2025	34,345	12,765	0.37
Total	168,520,681	7,314,170	0.04
1 Unit = $1			

LOB Subclass

Calendar Yr Summary			
Calendar Yr	Mean Reserve	Standard Dev.	CV Reserve
2006	33,519,508	1,848,318	0.06
2007	29,508,011	1,802,724	0.06
2008	23,768,077	1,824,205	0.08
2009	17,179,088	1,601,101	0.09
2010	11,142,452	1,370,170	0.12
2011	7,132,762	1,248,581	0.18
2012	4,068,435	879,103	0.22
2013	2,259,762	650,795	0.29
2014	1,186,083	504,771	0.43
2015	624,074	276,022	0.44
2016	365,224	178,727	0.49
2017	242,027	137,920	0.57
2018	150,914	88,522	0.59
2019	94,180	57,741	0.61
2020	56,618	37,342	0.66
2021	33,482	24,071	0.72
2022	19,058	15,215	0.8
2023	9,921	9,075	0.91
2024	4,734	5,197	1.1
2025	1,777	2,658	1.5
Total	131,366,189	6,202,006	0.05
1 Unit = $1			

LOB Subclass

Calendar Yr Summary			
Calendar Yr	Mean Reserve	Standard Dev.	CV Reserve
2006	6,633,386	831,656	0.13
2007	5,647,570	700,405	0.12
2008	4,711,827	584,537	0.12
2009	3,835,097	479,536	0.13
2010	3,045,061	386,608	0.13
2011	2,482,179	323,867	0.13
2012	2,032,880	273,862	0.13
2013	1,676,274	234,483	0.14
2014	1,437,028	211,034	0.15
2015	1,275,283	197,173	0.15
2016	1,114,021	181,158	0.16
2017	940,412	161,632	0.17
2018	692,206	121,301	0.18
2019	532,201	97,096	0.18
2020	404,622	78,949	0.2
2021	278,959	55,320	0.2
2022	185,538	38,751	0.21
2023	121,389	28,007	0.23
2024	75,992	20,828	0.27
2025	32,568	12,100	0.37
Total	37,154,492	3,429,684	0.09
1 Unit = $1			

Employers Liability: Composite DS: MPTF[optimal-1]: Forecast:Aggregate:Reserve Forecast Summaries:Correlations and SDs:By Datasets:Totals:Correlations

Reserve Forecast Correlations Between Datasets (Totals)		
	Employers Liability subclass a	Employers Liability subclass b
Employers Liability subclass a	1	0.076853
Employers Liability subclass b	0.076853	1

The reserve forecast correlation of 7.7% shows a good diversification level between these two lines.
That is, the risk capital required at the 95th percentile, for example is not significantly higher than were

Capital Allocation (Totals)	
	%
Employers Liability subclass a	74.96
Employers Liability subclass b	25.04

The capital allocation formula is based on the variances of the distributions for each LOB and the covariance between them.

Summary of Appendix F6

This workbook provides the forecasted reserve distribution and quantiles for the Gross and Net data of the Marine Cargo Line of Business.

Estimates of the Net of reinsurance and Gross reserve distribution for the Marine Cargo Line of business is obtained by designing a composite model for the two lines which includes correlations between the Net and Gross data (the data are not aggregated).

The process correlation between the Net and Gross data is very high: 90%. The correlation between reserve distributions is also very high at 85%.

The trend structure is approximately the same in each triangle (see model displays). The process variance is higher for the Net data than the Gross data!

The forecast scenario used included continuing calendar trends as follows.
For both Net and Gross data
2005 - 2022 2.48% +- 1.72%

The forecasts for the reserve distribution for the Gross and Net data are shown below.

	Reserve Mean	Reserve Standard Deviation	Coefficient of Variation
Gross	**2,267**	**634**	0.27966476
Net	**2,144**	**613**	0.28591418

The coefficient of variation of the reserves for the Net data is slightly higher than the coefficient of variation for the Gross data. This indicates that the Reinsurance program for the cedant is not optimal in terms of ceding risk. The reason for this phenomenon is that the process variability for the Net data is higher than for the Gross data. See the Model Displays.

Marine Cargo Gross Data: Quantiles & VAR

Mean = 2.267, S.D. = 0.634, Provision = 2.267, 1 Unit = $1,000

Quantile Statistics and Value at Risk (Acc Year: Total)

%	Sample			Kernel			LogNormal			Gamma		
	Quantile	# S.D.'s	V-a-R	Quantile	# S.D.'s	V-a-R	Quantile	# S.D.'s	V-a-R	Quantile	# S.D.'s	V-a-R
99.995	6.643	6.903	4.376	6.883	7.281	4.615	6.349	6.439	4.082	5.601	5.259	3.333
99.99	6.299	6.36	4.031	6.556	6.767	4.289	6.057	5.979	3.79	5.408	4.955	3.141
99.98	5.972	5.845	3.705	6.22	6.236	3.953	5.767	5.521	3.5	5.211	4.645	2.944
99.97	5.894	5.722	3.627	6.053	5.972	3.786	5.598	5.255	3.331	5.094	4.46	2.827
99.96	5.854	5.659	3.587	5.933	5.783	3.666	5.478	5.066	3.211	5.011	4.328	2.743
99.95	5.774	5.532	3.506	5.833	5.626	3.566	5.385	4.919	3.118	4.945	4.224	2.678
99.94	5.735	5.471	3.468	5.743	5.484	3.476	5.31	4.8	3.043	4.891	4.139	2.624
99.93	5.733	5.468	3.466	5.658	5.35	3.391	5.246	4.699	2.978	4.845	4.066	2.578
99.92	5.552	5.182	3.285	5.578	5.223	3.311	5.19	4.611	2.923	4.805	4.003	2.537
99.91	5.423	4.978	3.155	5.502	5.102	3.234	5.141	4.534	2.874	4.769	3.947	2.502
99.9	5.341	4.85	3.074	5.431	4.991	3.164	5.098	4.465	2.83	4.737	3.897	2.47
99.8	4.915	4.177	2.648	5.005	4.319	2.738	4.81	4.011	2.542	4.523	3.559	2.256
99.7	4.756	3.926	2.489	4.791	3.981	2.523	4.641	3.744	2.373	4.395	3.357	2.128
99.6	4.575	3.641	2.308	4.648	3.756	2.381	4.52	3.555	2.253	4.302	3.21	2.035
99.5	4.496	3.516	2.228	4.542	3.589	2.275	4.427	3.407	2.16	4.229	3.095	1.962
99.4	4.418	3.393	2.151	4.457	3.454	2.189	4.35	3.286	2.083	4.169	3	1.902
99.3	4.348	3.283	2.081	4.384	3.34	2.117	4.285	3.184	2.018	4.117	2.918	1.85
99.2	4.279	3.174	2.012	4.321	3.24	2.054	4.229	3.094	1.961	4.072	2.847	1.805
99.1	4.239	3.11	1.971	4.265	3.151	1.997	4.179	3.016	1.912	4.032	2.784	1.765
99	4.206	3.058	1.939	4.214	3.072	1.947	4.134	2.945	1.867	3.996	2.727	1.729
98	3.854	2.504	1.587	3.877	2.54	1.61	3.836	2.475	1.569	3.75	2.34	1.483
97	3.653	2.185	1.385	3.68	2.229	1.413	3.658	2.195	1.391	3.6	2.102	1.332
96	3.518	1.973	1.25	3.539	2.007	1.272	3.53	1.992	1.263	3.489	1.927	1.222
95	3.399	1.786	1.132	3.43	1.835	1.163	3.429	1.833	1.162	3.4	1.788	1.133
94	3.315	1.653	1.048	3.342	1.695	1.074	3.345	1.701	1.078	3.326	1.671	1.059
93	3.242	1.538	0.975	3.268	1.578	1	3.274	1.588	1.006	3.262	1.57	0.995
92	3.186	1.449	0.919	3.203	1.477	0.936	3.211	1.488	0.943	3.206	1.48	0.938
91	3.133	1.366	0.866	3.146	1.386	0.879	3.154	1.4	0.887	3.154	1.4	0.887
90	3.083	1.287	0.816	3.094	1.304	0.826	3.104	1.319	0.836	3.108	1.326	0.841
89	3.04	1.219	0.773	3.045	1.228	0.778	3.057	1.246	0.79	3.065	1.259	0.798
88	2.992	1.143	0.725	3	1.157	0.733	3.014	1.178	0.747	3.025	1.196	0.758
87	2.939	1.06	0.672	2.958	1.091	0.691	2.974	1.115	0.707	2.988	1.137	0.721
86	2.907	1.009	0.64	2.919	1.028	0.652	2.937	1.056	0.67	2.953	1.082	0.686
85	2.868	0.947	0.6	2.882	0.971	0.615	2.902	1.001	0.634	2.92	1.03	0.653
84	2.833	0.893	0.566	2.848	0.916	0.581	2.868	0.948	0.601	2.889	0.981	0.622
83	2.804	0.847	0.537	2.815	0.865	0.548	2.837	0.899	0.57	2.859	0.933	0.592
82	2.772	0.796	0.505	2.785	0.817	0.518	2.807	0.851	0.539	2.83	0.888	0.563
81	2.739	0.744	0.471	2.756	0.771	0.488	2.778	0.806	0.511	2.803	0.845	0.536
80	2.717	0.71	0.45	2.728	0.727	0.461	2.75	0.762	0.483	2.776	0.803	0.509
79	2.693	0.672	0.426	2.702	0.685	0.434	2.724	0.721	0.457	2.751	0.763	0.484
78	2.667	0.631	0.4	2.676	0.645	0.409	2.699	0.68	0.431	2.726	0.724	0.459
77	2.643	0.593	0.376	2.652	0.607	0.385	2.674	0.642	0.407	2.702	0.686	0.435
76	2.618	0.553	0.351	2.628	0.57	0.361	2.65	0.604	0.383	2.679	0.65	0.412
75	2.597	0.52	0.33	2.606	0.534	0.339	2.627	0.568	0.36	2.657	0.614	0.389
74	2.577	0.489	0.31	2.584	0.499	0.317	2.605	0.532	0.338	2.635	0.58	0.368
73	2.559	0.461	0.292	2.562	0.466	0.295	2.583	0.498	0.316	2.613	0.546	0.346
72	2.537	0.425	0.27	2.542	0.433	0.274	2.562	0.465	0.295	2.593	0.513	0.325
71	2.513	0.388	0.246	2.521	0.401	0.254	2.541	0.432	0.274	2.572	0.481	0.305
70	2.495	0.359	0.227	2.501	0.37	0.234	2.521	0.4	0.254	2.552	0.45	0.285
69	2.479	0.335	0.212	2.482	0.339	0.215	2.501	0.369	0.234	2.533	0.419	0.265
68	2.459	0.302	0.191	2.463	0.309	0.196	2.482	0.339	0.215	2.513	0.388	0.246
67	2.44	0.272	0.173	2.445	0.28	0.177	2.463	0.309	0.196	2.495	0.359	0.227
66	2.423	0.245	0.155	2.426	0.251	0.159	2.445	0.28	0.178	2.476	0.329	0.209
65	2.407	0.221	0.14	2.409	0.223	0.141	2.427	0.252	0.159	2.458	0.301	0.191
64	2.389	0.192	0.122	2.391	0.195	0.124	2.409	0.223	0.142	2.44	0.272	0.173
63	2.373	0.167	0.106	2.374	0.168	0.107	2.391	0.196	0.124	2.422	0.244	0.155
62	2.354	0.138	0.087	2.357	0.141	0.09	2.374	0.169	0.107	2.405	0.217	0.137

TABLE CONTINUED ON THE NEXT PAGE

Quantile Statistics and Value at Risk (Acc Year: Total)

%	Sample			Kernel			LogNormal			Gamma		
	Quantile	# S.D.'s	V-a-R	Quantile	# S.D.'s	V-a-R	Quantile	# S.D.'s	V-a-R	Quantile	# S.D.'s	V-a-R
61	2.335	0.108	0.068	2.34	0.115	0.073	2.357	0.142	0.09	2.387	0.19	0.12
60	2.318	0.08	0.051	2.324	0.089	0.056	2.34	0.115	0.073	2.37	0.163	0.103
59	2.302	0.054	0.034	2.307	0.064	0.04	2.324	0.089	0.057	2.354	0.136	0.086
58	2.287	0.031	0.019	2.292	0.038	0.024	2.308	0.064	0.04	2.337	0.11	0.07
57	2.273	0.009	0.005	2.276	0.014	0.009	2.292	0.038	0.024	2.32	0.084	0.053
56	2.258	-0.015	-0.009	2.26	-0.011	-0.007	2.276	0.013	0.008	2.304	0.058	0.037
55	2.245	-0.036	-0.023	2.245	-0.035	-0.022	2.26	-0.012	-0.007	2.288	0.032	0.021
54	2.227	-0.064	-0.04	2.23	-0.059	-0.038	2.244	-0.036	-0.023	2.272	0.007	0.004
53	2.215	-0.083	-0.052	2.215	-0.083	-0.053	2.229	-0.06	-0.038	2.256	-0.018	-0.011
52	2.2	-0.106	-0.067	2.2	-0.106	-0.067	2.214	-0.085	-0.054	2.24	-0.043	-0.027
51	2.185	-0.13	-0.083	2.185	-0.13	-0.082	2.198	-0.108	-0.069	2.224	-0.068	-0.043
50	2.171	-0.152	-0.097	2.17	-0.153	-0.097	2.183	-0.132	-0.084	2.208	-0.093	-0.059
49	2.158	-0.172	-0.109	2.156	-0.176	-0.111	2.169	-0.156	-0.099	2.193	-0.117	-0.074
48	2.143	-0.196	-0.124	2.142	-0.198	-0.126	2.154	-0.179	-0.113	2.177	-0.142	-0.09
47	2.128	-0.22	-0.139	2.127	-0.221	-0.14	2.139	-0.202	-0.128	2.162	-0.166	-0.105
46	2.113	-0.243	-0.154	2.113	-0.243	-0.154	2.124	-0.225	-0.143	2.146	-0.191	-0.121
45	2.099	-0.266	-0.169	2.099	-0.266	-0.168	2.11	-0.249	-0.158	2.131	-0.215	-0.136
44	2.086	-0.286	-0.181	2.085	-0.288	-0.182	2.095	-0.272	-0.172	2.116	-0.239	-0.152
43	2.072	-0.309	-0.196	2.071	-0.31	-0.196	2.081	-0.294	-0.187	2.1	-0.264	-0.167
42	2.056	-0.333	-0.211	2.057	-0.332	-0.21	2.066	-0.317	-0.201	2.085	-0.288	-0.182
41	2.042	-0.355	-0.225	2.043	-0.354	-0.224	2.052	-0.34	-0.216	2.069	-0.312	-0.198
40	2.03	-0.375	-0.237	2.029	-0.376	-0.238	2.037	-0.363	-0.23	2.054	-0.336	-0.213
39	2.017	-0.395	-0.25	2.015	-0.397	-0.252	2.023	-0.386	-0.245	2.039	-0.361	-0.229
38	2.004	-0.416	-0.263	2.002	-0.419	-0.266	2.008	-0.409	-0.259	2.023	-0.385	-0.244
37	1.991	-0.436	-0.276	1.988	-0.441	-0.279	1.994	-0.431	-0.273	2.008	-0.409	-0.259
36	1.977	-0.457	-0.29	1.974	-0.462	-0.293	1.979	-0.454	-0.288	1.992	-0.434	-0.275
35	1.963	-0.48	-0.304	1.961	-0.484	-0.307	1.965	-0.477	-0.303	1.977	-0.459	-0.291
34	1.946	-0.506	-0.321	1.947	-0.505	-0.32	1.95	-0.5	-0.317	1.961	-0.483	-0.306
33	1.934	-0.525	-0.333	1.933	-0.527	-0.334	1.935	-0.523	-0.332	1.945	-0.508	-0.322
32	1.922	-0.545	-0.345	1.92	-0.549	-0.348	1.921	-0.547	-0.346	1.929	-0.533	-0.338
31	1.909	-0.565	-0.358	1.906	-0.57	-0.361	1.906	-0.57	-0.361	1.913	-0.558	-0.354
30	1.896	-0.586	-0.371	1.892	-0.592	-0.375	1.891	-0.593	-0.376	1.897	-0.584	-0.37
29	1.882	-0.607	-0.385	1.878	-0.614	-0.389	1.876	-0.617	-0.391	1.881	-0.61	-0.386
28	1.868	-0.631	-0.4	1.864	-0.636	-0.403	1.861	-0.641	-0.406	1.864	-0.636	-0.403
27	1.855	-0.651	-0.412	1.85	-0.658	-0.417	1.846	-0.665	-0.421	1.848	-0.662	-0.419
26	1.842	-0.671	-0.425	1.836	-0.681	-0.432	1.83	-0.689	-0.437	1.831	-0.688	-0.436
25	1.827	-0.695	-0.44	1.821	-0.704	-0.446	1.815	-0.714	-0.452	1.814	-0.715	-0.453
24	1.813	-0.717	-0.455	1.806	-0.727	-0.461	1.799	-0.739	-0.468	1.796	-0.743	-0.471
23	1.799	-0.739	-0.469	1.791	-0.751	-0.476	1.783	-0.764	-0.484	1.779	-0.77	-0.488
22	1.783	-0.764	-0.484	1.776	-0.775	-0.491	1.767	-0.79	-0.5	1.761	-0.799	-0.506
21	1.768	-0.787	-0.499	1.761	-0.799	-0.507	1.75	-0.816	-0.517	1.743	-0.828	-0.525
20	1.753	-0.812	-0.515	1.745	-0.825	-0.523	1.733	-0.842	-0.534	1.724	-0.857	-0.543
19	1.737	-0.837	-0.53	1.728	-0.85	-0.539	1.716	-0.869	-0.551	1.705	-0.887	-0.562
18	1.718	-0.866	-0.549	1.711	-0.877	-0.556	1.699	-0.897	-0.569	1.685	-0.918	-0.582
17	1.7	-0.894	-0.567	1.694	-0.904	-0.573	1.681	-0.925	-0.587	1.665	-0.95	-0.602
16	1.682	-0.924	-0.585	1.676	-0.932	-0.591	1.662	-0.955	-0.605	1.645	-0.982	-0.623
15	1.665	-0.951	-0.603	1.658	-0.962	-0.61	1.643	-0.985	-0.624	1.623	-1.016	-0.644
14	1.644	-0.983	-0.623	1.639	-0.992	-0.629	1.623	-1.016	-0.644	1.601	-1.051	-0.666
13	1.627	-1.01	-0.64	1.619	-1.023	-0.649	1.603	-1.048	-0.664	1.578	-1.087	-0.689
12	1.609	-1.038	-0.658	1.598	-1.056	-0.669	1.582	-1.081	-0.685	1.554	-1.125	-0.713
11	1.585	-1.076	-0.682	1.576	-1.09	-0.691	1.56	-1.116	-0.708	1.529	-1.165	-0.738
10	1.56	-1.115	-0.707	1.554	-1.126	-0.714	1.536	-1.153	-0.731	1.502	-1.207	-0.765
9	1.537	-1.152	-0.73	1.53	-1.163	-0.737	1.511	-1.192	-0.756	1.474	-1.251	-0.793
8	1.513	-1.189	-0.754	1.504	-1.203	-0.763	1.485	-1.234	-0.782	1.444	-1.299	-0.824
7	1.485	-1.235	-0.783	1.477	-1.246	-0.79	1.456	-1.279	-0.811	1.411	-1.351	-0.856
6	1.456	-1.279	-0.811	1.448	-1.293	-0.819	1.425	-1.328	-0.842	1.375	-1.408	-0.892
5	1.425	-1.328	-0.842	1.415	-1.344	-0.852	1.39	-1.383	-0.877	1.335	-1.471	-0.933
4	1.388	-1.388	-0.88	1.378	-1.404	-0.89	1.351	-1.446	-0.917	1.288	-1.544	-0.979
3	1.347	-1.451	-0.92	1.332	-1.475	-0.935	1.303	-1.521	-0.964	1.233	-1.632	-1.034
2	1.28	-1.558	-0.987	1.274	-1.567	-0.993	1.243	-1.616	-1.024	1.162	-1.744	-1.105
1	1.205	-1.676	-1.062	1.192	-1.696	-1.075	1.153	-1.757	-1.114	1.056	-1.911	-1.211

Mean = 2.267, S.D. = 0.634, Provision = 2.267, 1 Unit = $1,000

Marine Cargo Net Data: Quantiles & VAR												
Mean = 2.144, S.D. = 0.613, Provision = 2.144, 1 Unit = $1,000												
Quantile Statistics and Value at Risk (Acc Year: Total)												
%	Sample			Kernel			LogNormal			Gamma		
	Quantile	# S.D.'s	V-a-R	Quantile	# S.D.'s	V-a-R	Quantile	# S.D.'s	V-a-R	Quantile	# S.D.'s	V-a-R
99.995	7.015	7.952	4.871	7.257	8.347	5.113	6.131	6.508	3.987	5.384	5.29	3.24
99.99	6.959	7.859	4.815	7.096	8.083	4.952	5.843	6.039	3.699	5.196	4.983	3.052
99.98	6.949	7.843	4.805	6.836	7.659	4.692	5.558	5.573	3.414	5.005	4.669	2.86
99.97	6.477	7.073	4.333	6.545	7.185	4.401	5.391	5.301	3.247	4.89	4.483	2.746
99.96	5.853	6.054	3.709	6.147	6.535	4.003	5.274	5.109	3.13	4.809	4.35	2.664
99.95	5.788	5.948	3.644	5.88	6.099	3.736	5.183	4.96	3.039	4.744	4.245	2.6
99.94	5.699	5.803	3.555	5.712	5.825	3.568	5.108	4.839	2.964	4.692	4.159	2.548
99.93	5.484	5.451	3.339	5.587	5.621	3.443	5.045	4.736	2.901	4.647	4.086	2.503
99.92	5.311	5.17	3.167	5.492	5.465	3.348	4.991	4.647	2.847	4.608	4.022	2.464
99.91	5.224	5.028	3.08	5.417	5.342	3.273	4.943	4.569	2.799	4.573	3.965	2.429
99.9	5.221	5.023	3.077	5.356	5.244	3.212	4.9	4.499	2.756	4.542	3.914	2.398
99.8	4.998	4.659	2.854	5.016	4.688	2.872	4.617	4.037	2.473	4.334	3.574	2.189
99.7	4.795	4.327	2.651	4.8	4.335	2.656	4.452	3.767	2.308	4.208	3.37	2.064
99.6	4.588	3.989	2.443	4.629	4.057	2.485	4.334	3.575	2.19	4.118	3.222	1.974
99.5	4.445	3.756	2.301	4.497	3.84	2.352	4.242	3.425	2.098	4.047	3.106	1.903
99.4	4.354	3.608	2.21	4.39	3.666	2.246	4.167	3.303	2.023	3.988	3.01	1.844
99.3	4.28	3.487	2.136	4.3	3.519	2.156	4.104	3.199	1.96	3.938	2.928	1.794
99.2	4.205	3.364	2.061	4.222	3.392	2.078	4.049	3.109	1.905	3.894	2.857	1.75
99.1	4.122	3.229	1.978	4.153	3.28	2.009	4	3.029	1.856	3.855	2.793	1.711
99	4.034	3.085	1.89	4.093	3.181	1.949	3.956	2.958	1.812	3.82	2.735	1.676
98	3.71	2.556	1.566	3.731	2.591	1.587	3.665	2.483	1.521	3.581	2.346	1.437
97	3.501	2.215	1.357	3.529	2.261	1.385	3.492	2.2	1.348	3.434	2.107	1.29
96	3.36	1.986	1.216	3.387	2.029	1.243	3.367	1.996	1.223	3.327	1.931	1.183
95	3.255	1.813	1.111	3.277	1.85	1.133	3.269	1.836	1.124	3.241	1.791	1.097
94	3.169	1.674	1.025	3.187	1.703	1.043	3.187	1.703	1.043	3.169	1.673	1.025
93	3.095	1.552	0.951	3.111	1.578	0.967	3.117	1.589	0.973	3.107	1.571	0.963
92	3.026	1.44	0.882	3.044	1.469	0.9	3.056	1.489	0.912	3.052	1.482	0.908
91	2.971	1.349	0.827	2.985	1.373	0.841	3.001	1.4	0.857	3.002	1.401	0.858
90	2.915	1.259	0.771	2.933	1.287	0.789	2.952	1.319	0.808	2.957	1.327	0.813
89	2.87	1.185	0.726	2.885	1.21	0.741	2.907	1.245	0.763	2.915	1.259	0.771
88	2.827	1.115	0.683	2.842	1.139	0.698	2.865	1.177	0.721	2.877	1.196	0.733
87	2.785	1.046	0.641	2.802	1.073	0.658	2.826	1.114	0.682	2.841	1.137	0.697
86	2.75	0.99	0.606	2.765	1.013	0.621	2.79	1.055	0.646	2.807	1.082	0.663
85	2.719	0.939	0.575	2.73	0.957	0.586	2.756	0.999	0.612	2.775	1.03	0.631
84	2.685	0.883	0.541	2.698	0.904	0.554	2.724	0.946	0.58	2.744	0.98	0.6
83	2.652	0.829	0.508	2.667	0.854	0.523	2.693	0.896	0.549	2.715	0.933	0.571
82	2.625	0.784	0.481	2.639	0.808	0.495	2.664	0.849	0.52	2.688	0.887	0.544
81	2.6	0.744	0.456	2.612	0.763	0.468	2.636	0.803	0.492	2.661	0.844	0.517
80	2.575	0.703	0.431	2.586	0.721	0.442	2.61	0.76	0.466	2.635	0.802	0.491
79	2.547	0.658	0.403	2.561	0.681	0.417	2.584	0.718	0.44	2.611	0.762	0.467
78	2.527	0.625	0.383	2.537	0.642	0.393	2.559	0.678	0.415	2.587	0.723	0.443
77	2.504	0.588	0.36	2.515	0.605	0.371	2.535	0.639	0.391	2.564	0.685	0.42
76	2.487	0.56	0.343	2.493	0.569	0.349	2.512	0.601	0.368	2.541	0.648	0.397
75	2.466	0.525	0.322	2.471	0.534	0.327	2.49	0.565	0.346	2.52	0.613	0.375
74	2.445	0.491	0.301	2.45	0.5	0.306	2.468	0.53	0.324	2.498	0.578	0.354
73	2.424	0.458	0.28	2.43	0.467	0.286	2.447	0.495	0.303	2.478	0.544	0.334
72	2.406	0.428	0.262	2.41	0.435	0.266	2.427	0.462	0.283	2.457	0.512	0.313
71	2.389	0.4	0.245	2.391	0.403	0.247	2.407	0.429	0.263	2.438	0.479	0.294
70	2.37	0.369	0.226	2.372	0.372	0.228	2.388	0.397	0.243	2.418	0.448	0.274
69	2.351	0.338	0.207	2.354	0.342	0.209	2.368	0.366	0.224	2.399	0.417	0.255
68	2.334	0.31	0.19	2.335	0.312	0.191	2.35	0.336	0.206	2.381	0.386	0.237
67	2.313	0.276	0.169	2.317	0.283	0.173	2.332	0.306	0.188	2.363	0.357	0.219
66	2.297	0.25	0.153	2.3	0.254	0.156	2.314	0.277	0.17	2.345	0.327	0.201
65	2.278	0.219	0.134	2.282	0.226	0.138	2.296	0.248	0.152	2.327	0.299	0.183
64	2.26	0.189	0.116	2.265	0.198	0.121	2.279	0.22	0.135	2.31	0.27	0.166
63	2.245	0.165	0.101	2.249	0.171	0.105	2.262	0.193	0.118	2.293	0.242	0.148
62	2.229	0.138	0.085	2.232	0.144	0.088	2.245	0.166	0.101	2.276	0.215	0.132

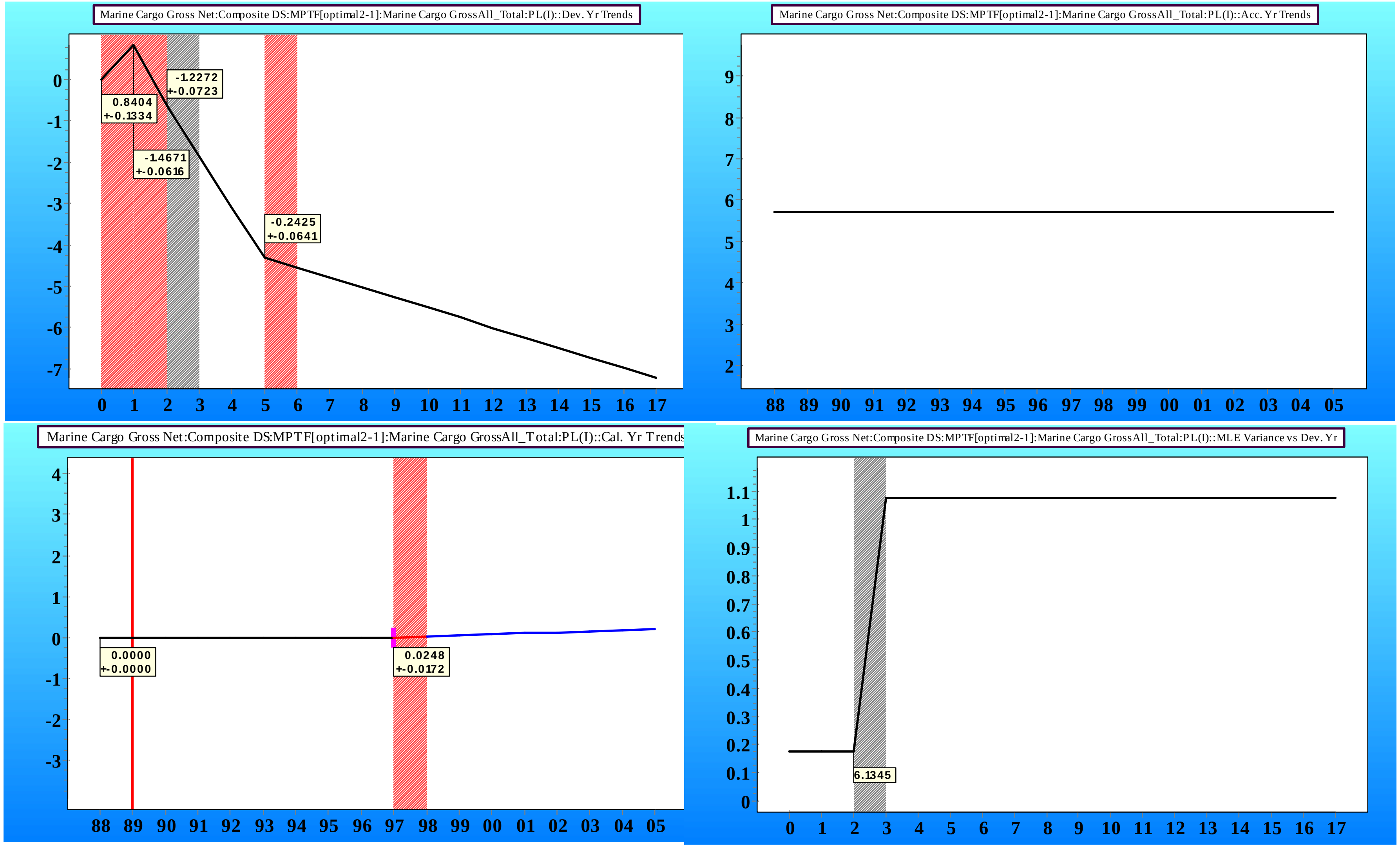
TABLE CONTINUED ON THE NEXT PAGE

Quantile Statistics and Value at Risk (Acc Year: Total)

%	Sample			Kernel			LogNormal			Gamma		
	Quantile	# S.D.'s	V-a-R	Quantile	# S.D.'s	V-a-R	Quantile	# S.D.'s	V-a-R	Quantile	# S.D.'s	V-a-R
61	2.212	0.112	0.068	2.216	0.117	0.072	2.229	0.139	0.085	2.259	0.188	0.115
60	2.197	0.086	0.053	2.2	0.091	0.056	2.213	0.112	0.069	2.242	0.161	0.098
59	2.183	0.063	0.039	2.184	0.066	0.04	2.197	0.086	0.053	2.226	0.134	0.082
58	2.166	0.036	0.022	2.169	0.04	0.025	2.181	0.061	0.037	2.21	0.108	0.066
57	2.152	0.013	0.008	2.153	0.015	0.009	2.166	0.035	0.022	2.194	0.082	0.05
56	2.137	-0.012	-0.007	2.138	-0.009	-0.006	2.15	0.01	0.006	2.178	0.056	0.034
55	2.121	-0.037	-0.023	2.123	-0.034	-0.021	2.135	-0.014	-0.009	2.163	0.03	0.019
54	2.107	-0.06	-0.037	2.108	-0.058	-0.036	2.12	-0.039	-0.024	2.147	0.005	0.003
53	2.092	-0.085	-0.052	2.094	-0.082	-0.05	2.105	-0.063	-0.039	2.132	-0.02	-0.012
52	2.079	-0.106	-0.065	2.079	-0.106	-0.065	2.091	-0.087	-0.053	2.116	-0.045	-0.028
51	2.066	-0.127	-0.078	2.065	-0.129	-0.079	2.076	-0.111	-0.068	2.101	-0.07	-0.043
50	2.052	-0.15	-0.092	2.051	-0.153	-0.094	2.062	-0.135	-0.082	2.086	-0.095	-0.058
49	2.039	-0.172	-0.105	2.036	-0.176	-0.108	2.047	-0.158	-0.097	2.071	-0.119	-0.073
48	2.023	-0.197	-0.121	2.022	-0.199	-0.122	2.033	-0.181	-0.111	2.056	-0.144	-0.088
47	2.009	-0.221	-0.136	2.008	-0.222	-0.136	2.019	-0.205	-0.125	2.041	-0.168	-0.103
46	1.994	-0.246	-0.151	1.994	-0.244	-0.15	2.004	-0.228	-0.14	2.026	-0.193	-0.118
45	1.978	-0.271	-0.166	1.981	-0.267	-0.163	1.99	-0.251	-0.154	2.011	-0.217	-0.133
44	1.967	-0.289	-0.177	1.967	-0.289	-0.177	1.976	-0.274	-0.168	1.996	-0.241	-0.148
43	1.951	-0.315	-0.193	1.954	-0.311	-0.19	1.962	-0.297	-0.182	1.981	-0.265	-0.163
42	1.938	-0.336	-0.206	1.94	-0.333	-0.204	1.948	-0.319	-0.196	1.967	-0.29	-0.177
41	1.925	-0.358	-0.219	1.927	-0.355	-0.217	1.934	-0.342	-0.21	1.952	-0.314	-0.192
40	1.911	-0.38	-0.233	1.914	-0.376	-0.23	1.921	-0.365	-0.224	1.937	-0.338	-0.207
39	1.899	-0.399	-0.245	1.9	-0.398	-0.244	1.907	-0.388	-0.237	1.922	-0.362	-0.222
38	1.888	-0.418	-0.256	1.887	-0.419	-0.257	1.893	-0.41	-0.251	1.907	-0.387	-0.237
37	1.875	-0.439	-0.269	1.874	-0.44	-0.27	1.879	-0.433	-0.265	1.892	-0.411	-0.252
36	1.862	-0.46	-0.282	1.861	-0.462	-0.283	1.865	-0.456	-0.279	1.877	-0.435	-0.267
35	1.85	-0.48	-0.294	1.848	-0.483	-0.296	1.851	-0.479	-0.293	1.862	-0.46	-0.282
34	1.838	-0.499	-0.306	1.835	-0.504	-0.309	1.837	-0.502	-0.307	1.847	-0.485	-0.297
33	1.826	-0.52	-0.318	1.822	-0.525	-0.322	1.823	-0.524	-0.321	1.832	-0.509	-0.312
32	1.813	-0.54	-0.331	1.809	-0.546	-0.335	1.809	-0.548	-0.335	1.817	-0.534	-0.327
31	1.801	-0.561	-0.343	1.796	-0.568	-0.348	1.794	-0.571	-0.35	1.801	-0.56	-0.343
30	1.789	-0.58	-0.356	1.783	-0.589	-0.361	1.78	-0.594	-0.364	1.786	-0.585	-0.358
29	1.774	-0.604	-0.37	1.77	-0.611	-0.374	1.766	-0.618	-0.378	1.77	-0.611	-0.374
28	1.761	-0.626	-0.383	1.757	-0.633	-0.387	1.751	-0.641	-0.393	1.754	-0.636	-0.39
27	1.748	-0.646	-0.396	1.743	-0.655	-0.401	1.737	-0.665	-0.408	1.738	-0.663	-0.406
26	1.734	-0.67	-0.41	1.729	-0.677	-0.415	1.722	-0.689	-0.422	1.722	-0.689	-0.422
25	1.72	-0.692	-0.424	1.716	-0.699	-0.428	1.707	-0.714	-0.437	1.705	-0.716	-0.439
24	1.707	-0.713	-0.437	1.702	-0.722	-0.442	1.692	-0.739	-0.452	1.689	-0.743	-0.455
23	1.694	-0.735	-0.45	1.688	-0.745	-0.456	1.676	-0.764	-0.468	1.672	-0.771	-0.472
22	1.68	-0.757	-0.464	1.673	-0.768	-0.471	1.661	-0.789	-0.483	1.655	-0.799	-0.49
21	1.664	-0.784	-0.48	1.659	-0.792	-0.485	1.645	-0.815	-0.499	1.637	-0.828	-0.507
20	1.649	-0.807	-0.495	1.644	-0.817	-0.5	1.629	-0.841	-0.515	1.619	-0.857	-0.525
19	1.635	-0.831	-0.509	1.629	-0.841	-0.515	1.612	-0.868	-0.532	1.601	-0.887	-0.543
18	1.618	-0.859	-0.526	1.613	-0.866	-0.531	1.595	-0.896	-0.549	1.582	-0.918	-0.562
17	1.604	-0.882	-0.54	1.598	-0.892	-0.546	1.578	-0.924	-0.566	1.563	-0.949	-0.582
16	1.588	-0.908	-0.556	1.581	-0.919	-0.563	1.56	-0.953	-0.584	1.543	-0.982	-0.601
15	1.569	-0.938	-0.575	1.565	-0.946	-0.579	1.542	-0.983	-0.602	1.522	-1.015	-0.622
14	1.554	-0.963	-0.59	1.548	-0.974	-0.596	1.523	-1.013	-0.621	1.501	-1.05	-0.643
13	1.539	-0.988	-0.605	1.53	-1.003	-0.614	1.504	-1.045	-0.64	1.479	-1.086	-0.665
12	1.522	-1.015	-0.622	1.511	-1.033	-0.633	1.483	-1.079	-0.661	1.456	-1.124	-0.688
11	1.499	-1.052	-0.645	1.492	-1.065	-0.652	1.462	-1.113	-0.682	1.431	-1.163	-0.713
10	1.478	-1.088	-0.666	1.471	-1.098	-0.673	1.44	-1.15	-0.704	1.406	-1.205	-0.738
9	1.459	-1.118	-0.685	1.45	-1.134	-0.694	1.416	-1.189	-0.728	1.379	-1.249	-0.765
8	1.437	-1.155	-0.707	1.426	-1.172	-0.718	1.391	-1.23	-0.753	1.35	-1.297	-0.794
7	1.408	-1.201	-0.736	1.401	-1.213	-0.743	1.363	-1.274	-0.781	1.318	-1.348	-0.826
6	1.382	-1.244	-0.762	1.374	-1.257	-0.77	1.334	-1.323	-0.811	1.284	-1.404	-0.86
5	1.355	-1.289	-0.79	1.343	-1.308	-0.801	1.3	-1.377	-0.844	1.245	-1.467	-0.899
4	1.317	-1.349	-0.827	1.308	-1.365	-0.836	1.262	-1.439	-0.882	1.201	-1.54	-0.943
3	1.277	-1.416	-0.867	1.266	-1.434	-0.878	1.217	-1.513	-0.927	1.148	-1.626	-0.996
2	1.223	-1.503	-0.921	1.211	-1.523	-0.933	1.16	-1.607	-0.985	1.08	-1.737	-1.064
1	1.146	-1.63	-0.998	1.131	-1.653	-1.013	1.074	-1.746	-1.07	0.979	-1.902	-1.165

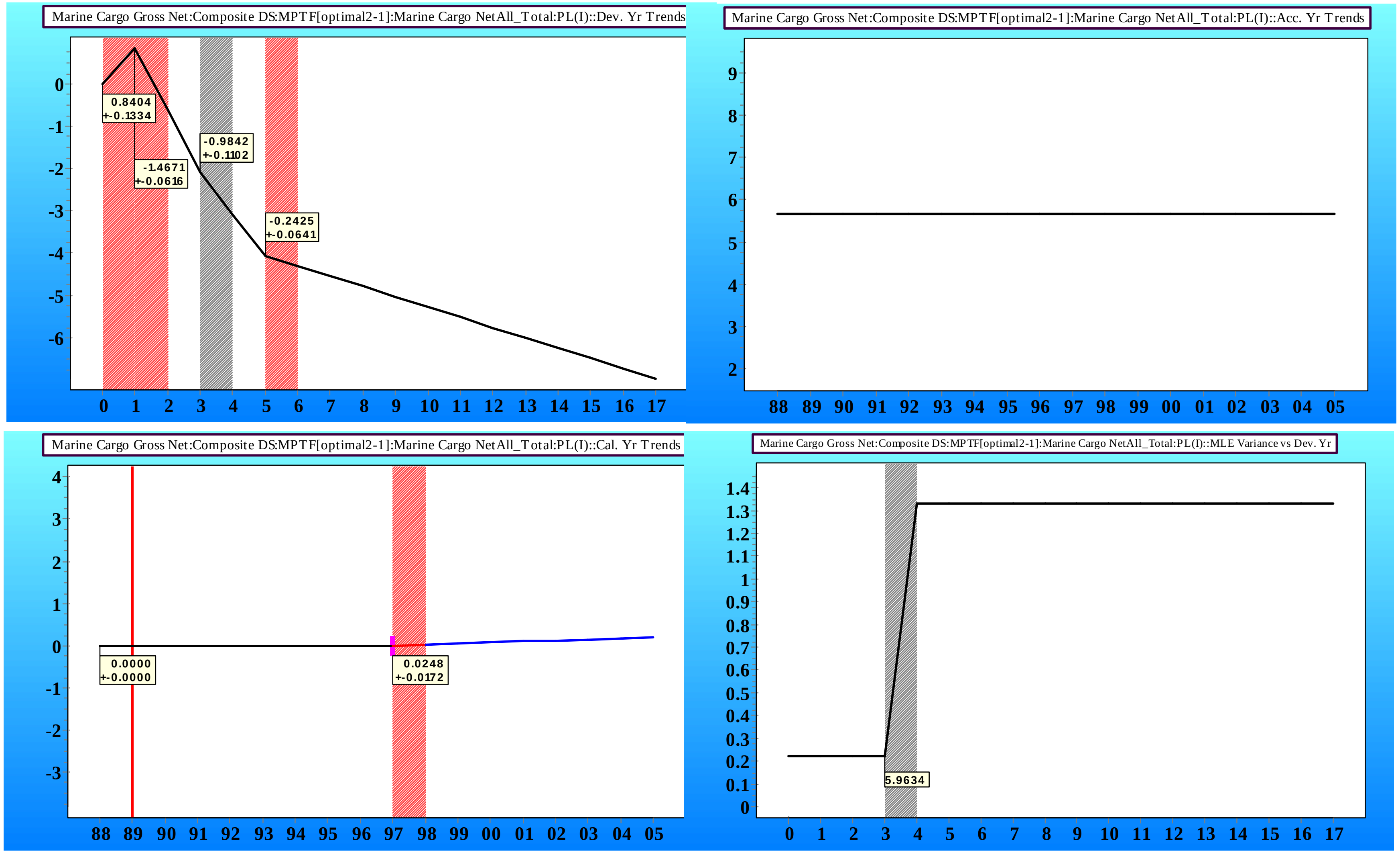
Mean = 2.144, S.D. = 0.613, Provision = 2.144, 1 Unit = $1,000

Marine Cargo Gross Data: Model Displays



The model for the Gross reserves is shown above. To be conservative, the 2.48% +- 1.7% calendar trend was retained even though it is not statistically significant. Process variance is very high from development year 3 onward.

Marine Cargo Net Data: Model Displays



The model for the Net reserves is shown above. To be conservative, the 2.48% +- 1.7% calendar trend was retained even though it is not statistically significant. Process variance is very high from development year 3 onward and is higher than for the Gross data!

Marine Cargo Gross Net:Composite DS:MPTF[optimal2-1]:Weighted Residual Covariances Between Datasets:Final Correlations

Final Weighted Residual Correlations Between Datasets		
	Marine Cargo GrossAll_Total:PL(I)	Marine Cargo NetAll_Total:PL(I)
Marine Cargo GrossAll_Total:PL(I)	1	0.902865
Marine Cargo NetAll_Total:PL(I)	0.902865	1
4 iterations were executed		

The process correlation (90%) between Gross and Net data is very high (as we would expect).

Marine Cargo Gross Data: Forecast

Marine Cargo Gross Net:Composite DS:MPTF[optimal2-1]:Forecast:Marine Cargo GrossAll_Total:PL(I):Reserve Forecast Table

Accident Period vs Development Period

	Cal. Per. Total	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	Reserve	Ultimate
1988	338	338	783	180	83	24	7	6	4	3	3	2	2	1	1	1	1	1	1	0	1,326
	359	359	750	172	27	25	2	2	0	2	-11	0	1	-3	0	0	0	0	0	0	0
1989	1,121	338	783	180	83	24	7	6	4	3	3	2	2	2	1	1	1	1	1	1	1,538
	1,075	325	1,026	180	45	-5	2	-6	0	1	1	1	-29	1	1	1	-6	0	1	1	1
1990	1,301	338	783	180	83	24	7	6	4	4	3	2	2	2	1	1	1	1	1	1	1,550
	1,619	422	864	276	12	-6	-13	-2	1	-5	0	0	0	0	0	0	0	1	1	2	2
1991	1,384	338	783	180	83	24	7	6	5	4	3	2	2	2	1	1	1	1	1	2	1,544
	1,425	354	850	252	47	42	2	-11	5	0	0	0	0	0	0	0	1	1	1	3	3
1992	1,408	338	783	180	83	24	7	6	5	4	3	2	2	2	1	1	1	1	1	3	1,154
	1,470	275	652	170	22	-2	0	36	-3	1	0	0	0	0	0	2	2	1	1	3	3
1993	1,415	338	783	180	83	24	7	6	5	4	3	3	2	2	1	1	1	1	1	5	1,318
	1,198	285	829	127	36	38	2	5	-7	2	0	0	0	-4	2	2	2	1	1	4	4
1994	1,421	338	783	180	83	25	8	6	5	4	3	3	2	2	1	1	1	1	1	7	1,337
	1,342	297	647	190	236	32	7	-38	25	-54	11	-21	0	3	2	2	2	1	1	5	5
1995	1,425	338	783	180	85	26	8	6	5	4	3	3	2	2	1	1	1	1	1	9	1,266
	1,060	242	681	194	74	58	-2	28	0	-2	-15	0	3	3	2	2	2	1	1	7	7
1996	1,429	338	783	184	87	26	8	6	5	4	3	3	2	2	1	1	1	1	1	12	1,166
	1,093	187	710	197	23	20	5	12	-4	2	3	4	3	3	2	2	2	2	1	8	8
1997	1,432	338	802	189	89	27	8	7	5	4	3	3	2	2	2	1	1	1	1	16	1,144
	1,310	152	908	90	1	-6	23	23	-10	-53	5	4	3	3	2	2	2	2	1	10	10
1998	1,469	346	822	194	92	28	8	7	5	4	4	3	2	2	2	1	1	1	1	21	951
	1,429	180	668	88	2	0	-16	8	1	6	5	4	3	3	2	2	2	2	1	13	13
1999	1,507	355	842	199	94	28	9	7	6	4	4	3	2	2	2	1	1	1	1	27	1,689
	1,284	433	1,136	96	-1	3	-1	-5	8	6	5	4	4	3	3	2	2	2	1	15	15
2000	1,547	364	864	204	96	29	9	7	6	5	4	3	2	2	2	1	1	1	1	35	1,332
	1,560	393	708	222	35	-44	-16	10	8	7	5	4	4	3	3	2	2	2	2	19	19
2001	1,588	373	886	209	99	30	9	7	6	5	4	3	3	2	2	1	1	1	1	45	1,431
	1,348	488	788	63	31	16	13	11	8	7	6	5	4	3	3	2	2	2	2	24	24
2002	1,629	383	909	215	102	31	9	8	6	5	4	3	3	2	2	1	1	1	1	77	1,666
	1,263	271	1,041	210	68	44	14	11	9	7	6	5	4	3	3	2	2	2	2	52	52
2003	1,673	393	933	220	104	32	10	8	6	5	4	3	3	2	2	1	1	1	1	183	1,855
	1,534	381	1,231	60	147	45	14	11	9	7	6	5	4	3	3	2	2	2	2	158	158
2004	1,718	403	958	226	107	32	10	8	6	5	4	3	3	2	2	2	1	1	1	415	1,947
	2,039	653	879	104	152	47	15	12	9	7	6	5	4	4	3	3	2	2	2	198	198
2005	1,764	414	984	233	110	33	10	8	7	5	4	3	3	2	2	2	1	1	1	1,410	1,948
	1,486	538	460	108	156	48	15	12	10	8	6	5	4	4	3	3	2	2	2	521	521
	Total Fitted/Paid		2006	2007	2008	2009	2010	2011	2012	2013	2014	2015	2016	2017	2018	2019	2020	2021	2022	Total Reserve	Total Ultimate
Cal. Per. Total	25,568		1,387	413	186	78	46	36	29	23	18	14	11	9	7	5	3	2	1	2,267	26,162
	23,895		510	199	167	55	26	21	18	15	13	11	9	8	6	5	4	3	2	634	634

1 Unit = $1

The above table shows the forecasted reserve distribution for each cell (mean is in black, standard deviation in red) along with the data (mean in black, observed in blue).

Accident Yr Summary							
Acc. Yr	Mean			Standard Dev.		CV	
	Reserve	Ultimate				Reserve	Ultimate
1988	0	1,326		0		****	0
1989	1	1,538		1		1.84	0
1990	1	1,550		2		1.35	0
1991	2	1,544		3		1.13	0
1992	3	1,154		3		0.99	0
1993	5	1,318		4		0.89	0
1994	7	1,337		5		0.81	0
1995	9	1,266		7		0.75	0.01
1996	12	1,166		8		0.69	0.01
1997	16	1,144		10		0.65	0.01
1998	21	951		13		0.61	0.01
1999	27	1,689		15		0.58	0.01
2000	35	1,332		19		0.56	0.01
2001	45	1,431		24		0.55	0.02
2002	77	1,666		52		0.67	0.03
2003	183	1,855		158		0.86	0.09
2004	415	1,947		198		0.48	0.1
2005	1,410	1,948		521		0.37	0.27
Total	2,267	26,162		634		0.28	0.02

1 Unit = $1

Marine Cargo Net Data: Forecast

Marine Cargo Gross Net:Composite DS:MPTF[optimal2-2]:Forecast:Marine Cargo NetAll_Total:PL(I):Reserve Forecast Table

Accident Period vs Development Period

	Cal. Per. Total	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	Reserve	Ultimate
1988	318	318	737	169	39	26	10	8	6	5	4	3	2	2	2	1	1	1	1	0	1,125
	297	297	628	155	25	25	2	2	2	2	-11	0	1	-3	0	0	0	0	0	0	0
1989	1,055	318	737	169	39	26	10	8	6	5	4	3	2	2	2	1	1	1	1	1	1,255
	910	282	788	176	43	-6	2	-7	0	1	1	1	-30	8	1	1	-6	0	2	2	2
1990	1,224	318	737	169	39	26	10	8	6	5	4	3	3	2	2	1	1	1	1	2	1,354
	1,325	382	755	233	7	-6	-13	-2	0	-5	0	0	0	0	2	0	0	2	2	3	3
1991	1,263	318	737	169	39	26	10	8	6	5	4	3	3	2	2	1	1	1	1	3	1,283
	1,243	288	723	242	-4	32	0	-6	5	0	0	0	0	0	0	0	2	2	2	4	4
1992	1,289	318	737	169	39	26	10	8	6	5	4	3	3	2	2	1	1	1	1	5	1,025
	1,262	237	572	159	19	2	0	36	-3	1	-3	0	0	0	0	3	2	2	2	5	5
1993	1,299	318	737	169	39	26	10	8	6	5	4	3	3	2	2	2	1	1	1	7	1,149
	1,061	245	705	120	38	37	4	5	-11	2	0	0	0	-4	3	3	2	2	2	7	7
1994	1,306	318	737	169	39	26	10	8	7	5	4	3	3	2	2	2	1	1	1	9	1,187
	1,063	205	588	222	71	32	7	-8	27	6	45	-19	0	4	3	3	3	2	2	8	8
1995	1,312	318	737	169	40	27	10	8	7	5	4	4	3	2	2	2	1	1	1	12	1,132
	959	218	618	163	72	56	-3	27	-18	-19	13	-7	5	4	4	3	3	2	2	10	10
1996	1,317	318	737	174	41	28	11	9	7	6	4	4	3	2	2	2	1	1	1	16	1,029
	1,047	167	612	175	14	15	25	12	-33	23	3	6	5	4	4	3	3	2	2	13	13
1997	1,320	318	755	178	42	28	11	9	7	6	5	4	3	2	2	2	1	1	1	21	882
	1,016	148	712	51	-5	-1	8	11	-9	-53	8	6	5	5	4	3	3	2	2	16	16
1998	1,356	326	774	182	43	29	11	9	7	6	5	4	3	3	2	2	1	1	1	28	932
	1,206	175	667	69	0	0	-16	8	1	10	8	7	6	5	4	3	3	3	2	19	19
1999	1,392	334	793	187	44	30	12	9	7	6	5	4	3	3	2	2	1	1	1	36	1,504
	1,234	433	967	73	-1	3	-1	-5	13	10	8	7	6	5	4	3	3	3	2	24	24
2000	1,429	342	813	192	45	30	12	10	8	6	5	4	3	3	2	2	2	1	1	46	1,262
	1,385	393	629	219	35	-44	-16	16	13	11	9	7	6	5	4	4	3	3	2	30	30
2001	1,467	351	834	197	47	31	12	10	8	6	5	4	3	3	2	2	2	1	1	60	1,438
	1,273	486	784	61	31	16	21	17	14	11	9	7	6	5	4	4	3	3	2	38	38
2002	1,506	360	856	202	48	32	13	10	8	6	5	4	3	3	2	2	2	1	1	93	1,679
	1,284	271	1,038	210	68	55	22	17	14	11	9	8	6	5	4	4	3	3	3	69	69
2003	1,547	370	879	207	49	33	13	10	8	7	5	4	4	3	2	2	2	1	1	145	1,815
	1,508	381	1,230	60	26	56	23	18	14	12	9	8	6	5	5	4	3	3	3	77	77
2004	1,588	380	902	213	50	34	13	11	9	7	6	5	4	3	2	2	2	1	1	362	1,893
	2,087	652	878	111	27	58	23	19	15	12	10	8	7	6	5	4	4	3	3	142	142
2005	1,632	390	927	219	52	35	14	11	9	7	6	5	4	3	3	2	2	1	1	1,299	1,837
	1,478	538	489	115	28	60	24	19	15	12	10	8	7	6	5	4	4	3	3	525	525
	Total Fitted/Paid		2006	2007	2008	2009	2010	2011	2012	2013	2014	2015	2016	2017	2018	2019	2020	2021	2022	Total Reserve	Total Ultimate
Cal. Per. Total	23,620		1,276	358	143	94	61	49	39	31	24	19	15	12	9	6	4	3	1	2,144	23,782
	21,638		518	141	77	73	41	33	27	23	19	16	14	11	9	8	6	4	3	613	613

1 Unit = $1

The above table shows the forecasted reserve distribution for each cell (mean is in black, standard deviation in red) along with the data (mean in black, observed in blue).

Accident Yr Summary

Acc. Yr	Mean			Standard Dev.		CV	
	Reserve	Ultimate				Reserve	Ultimate
1988	0	1,125		0		****	0
1989	1	1,255		2		2.15	0
1990	2	1,354		3		1.56	0
1991	3	1,283		4		1.3	0
1992	5	1,025		5		1.13	0.01
1993	7	1,149		7		1.02	0.01
1994	9	1,187		8		0.93	0.01
1995	12	1,132		10		0.85	0.01
1996	16	1,029		13		0.79	0.01
1997	21	882		16		0.75	0.02
1998	28	932		19		0.71	0.02
1999	36	1,504		24		0.68	0.02
2000	46	1,262		30		0.65	0.02
2001	60	1,438		38		0.64	0.03
2002	93	1,679		69		0.73	0.04
2003	145	1,815		77		0.53	0.04
2004	362	1,893		142		0.39	0.07
2005	1,299	1,837		525		0.4	0.29
Total	2,144	23,782		613		0.29	0.03

1 Unit = $1