The Actuarial Profession
making financial sense of the future

VARIABLE ANNUITIES: BRIDGING THE DIVIDE

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Economic Scenario Generators

Models and Calibration

17 September 2010

Agenda

- Trends in risk measurement in the global insurance sector

- Move to more advanced economic models

- Implementing risk management strategies for guarantee risks
  - Case Study

- More on managing market-consistent risks in illiquid markets

- Advanced capital calculation models

Trends in market risk measurement and management in the global insurance sector

Global Context: Market Risk Assessment in Insurance

- Last decade has produced a general global shift away from an ‘actuarial funding’ approach to market risk…
  – Actuarial judgement, margins, long-term, prescription, net premium valuations

- …to more market-based, analytic, economic approach
  – Stochastic models, market-consistent, principle-based => internal models

Global Context: Market Risk Assessment in Insurance

Many examples in regulatory capital assessment:

- In European Union, the Solvency II program and its front-runners:
  - Swiss Solvency Test (2004)
  - UK's Realistic Balance Sheet and ICA (2003/4)
  - South Africa's PGN-110 (2005-7)
  - Canadian regulatory capital for segregated funds business
  - In US, C-3 Phase II for variable annuity products
  - Countries such as Malaysia and Singapore using or considering the use of internal models in regulatory capital
- Similar developments have occurred in financial reporting:
  - MCEV
  - US GAAP

Market-consistency in Life Insurance: An emerging global standard

- These new measures of market risk and cost generally use market-consistent liability valuation as a core element:

- Solvency II Directive, Article 74: "...*calculation of technical provisions shall make use of and be consistent with information provided by the financial markets…*"

- CFO Forum Market Consistent EV Principles:

  Principle 12: *Economic assumptions must be internally consistent and should be determined such that projected cash flows are valued in line with the prices of similar cash flows that are traded on the capital market…*

Implications for Market Risk Management in the Global Insurance Sector (I)

- Sophisticated mark-to-market risk management programs have been put in place by many leading insurance groups over this period:
  - Daily-rebalanced dynamic hedging programs for VA products
  - Substantial static derivative hedges put in place for products such as UK Guaranteed Annuity Options

Implications for Market Risk Management in the Global Insurance Sector (II)

- And has also had major impact on product design:
  - Greater consideration of investment guarantee pricing and market-consistent profitability
  - Creating product structures that can be effectively risk managed
  - e.g. more constraints on policyholders’ underlying asset choices in VA products
  - In some countries, a significant move away from guarantee-intensive business
  - Move to CPPI / volatility controlled products

Implications for stochastic asset models (I)

- Significant spend on enterprise-wide internal models
  - Risk and capital measurement: Economic Capital, Solvency II, etc.
- And on high-performance specialist models for managing complex business
  - Risk and capital management: Dynamic hedging
- Requirement to understand long term nature of risks
  - Risk in hedging program
  - Changes to and capital and return on capital over run-off of business

Implications for stochastic asset models (II)

- Economic scenario generation requirements:
  - Market-consistent modelling and calibration
    - Arbitrage-free; automated calibration capability (stress tests); extrapolation methods
  - Real-world modelling and calibration
    - 1-year projection horizon for VaR approach to capital assessment
    - Integration with market-consistent scenarios (valuation in real-world projections)
    - Real-world model needs to reflect the risk profile of the business
      - Principle-based concept; not one-size-fits-all
      - e.g. no hedging => simple risk exposure (poor equity returns) measured with simple r-w model
      - dynamic hedging =>complex risk exposures (1-day gap risk) need complex r-w models to measure
  - Requirement for nested stochastic models
    - 1 year MCEV VaR capital
    - Projection of capital, hedges etc
    - Requirement for consistency of real world and market consistent models

Move to more advanced economic models

Need for more sophisticated ESG models

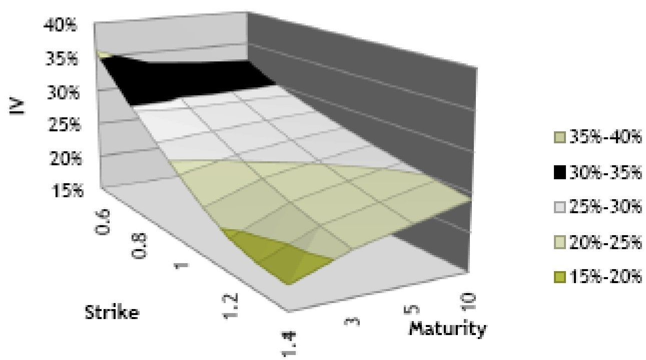
- Require models that capture market prices more accurately
  - Equity implied volatility (market consistent)
  - Interest rate implied volatility (market consistent)

  - Projection of implied volatility (real world)

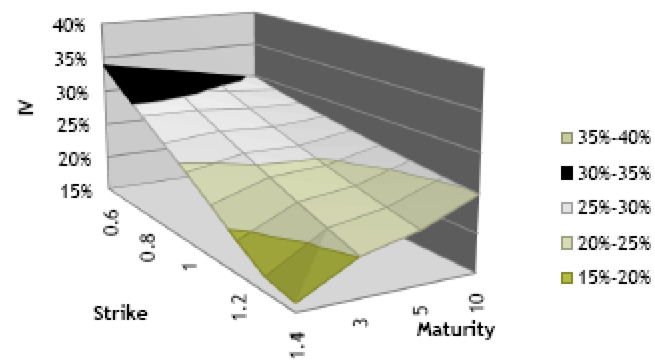
Market consistent models - Equity

- Stochastic Volatility Jump Diffusion model
  - Models change in volatility stochastically
  - Allows for possibility of jumps in equity prices
  - Gives more consistent fit to market implied volatilities
  - Theoretical advantage in pricing complex path-dependent guarantees

Market Implied Volatility Surface

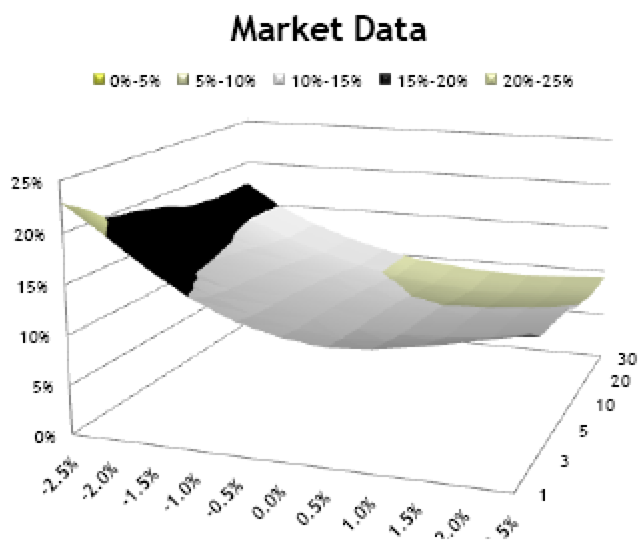


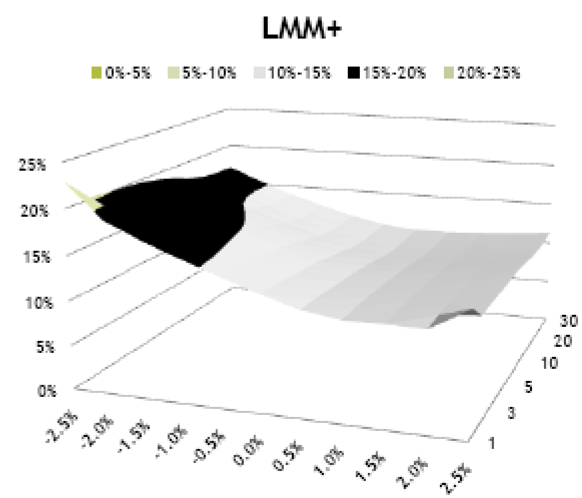
ESG Generated IV Surface from sophisticated model (SVJD)



Market consistent models – Interest rates

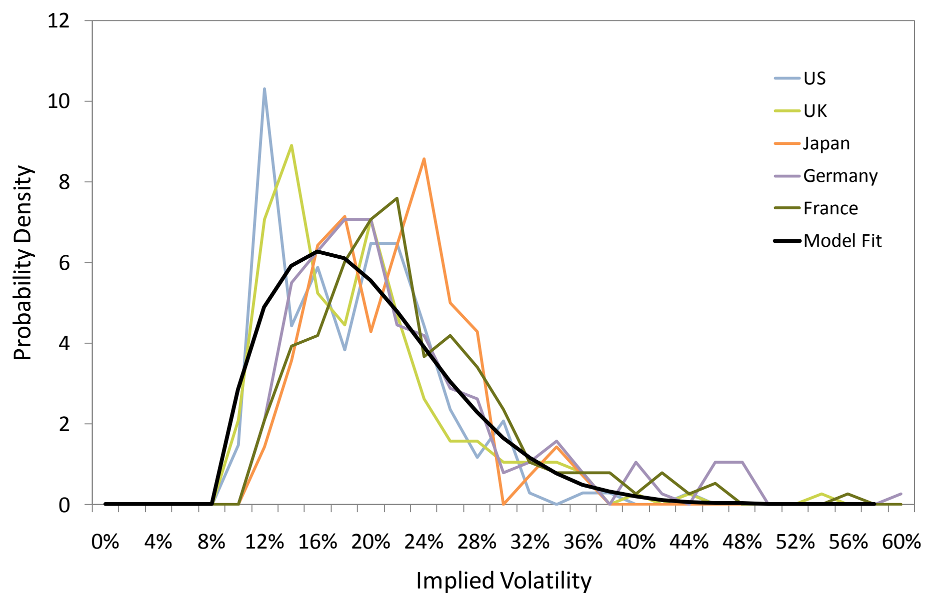
- Stochastic Volatility LMM (LMM+)
  - Models change in volatility stochastically
  - Gives more consistent fit to market implied volatilities by strike, term, tenor
  - Important in pricing complex interest rate sensitive guarantees





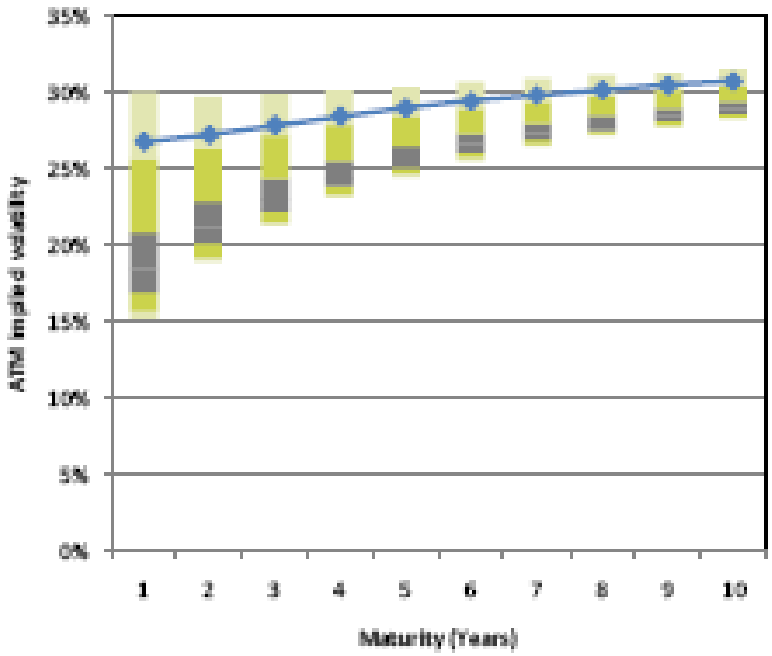
Market consistent models – Projecting volatility

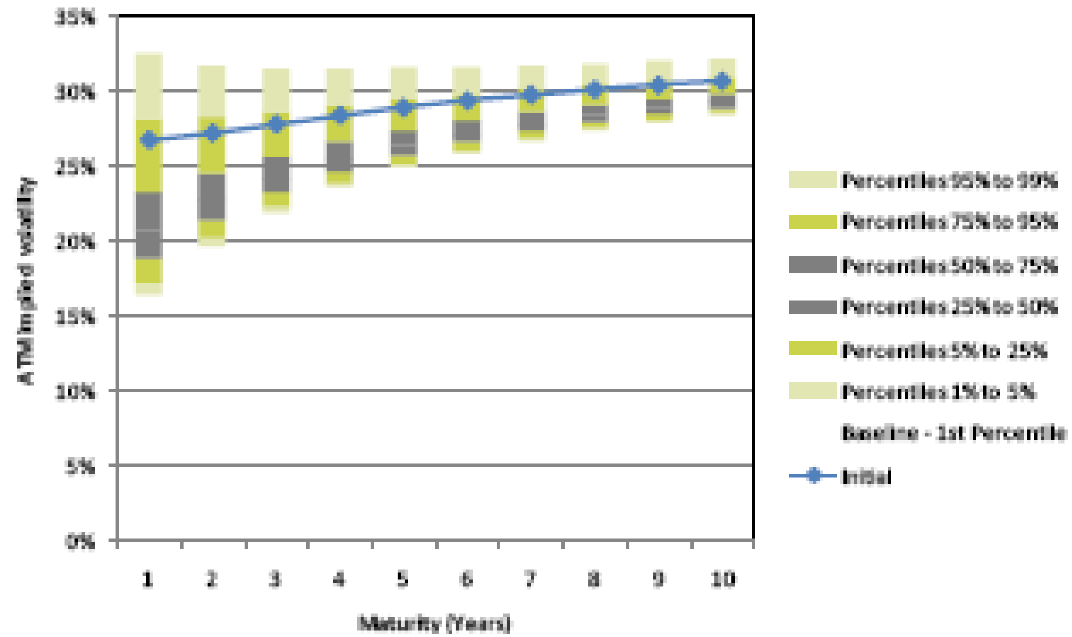
- Can use the same models in real-world mode
  - Allows projection of implied volatilities
  - In a consistent manner to pricing assumptions
  - Important in measuring risks, calculating capital etc



Market consistent models – Projecting volatility

- Projection of full volatility surface





Implementing risk management strategies for guarantee risks

Investment Guarantees: Risk Management Options

- *Do nothing but hold capital*
  - Increasingly uncommon, but may be considered by firms that are not concerned with mark-to-market volatility or its risk-based capital implications.
- *Reinsure*
  - Pass these market risk exposures on via a reinsurance treaty. Used significantly in the past, but reinsurance sector may now have limited appetite for these risks, especially at prices at which the guarantees have been sold.

Investment Guarantees: Risk Management Options (II)

- *Structured OTC hedging solution*
  - Pass the risk on to capital markets through the purchasing of a hedging solution that has been tailored to (permanently) match the characteristics of the liability.
  - Will leave policyholder risks (e.g. lapse behaviour). Like reinsurance, firms may find it difficult to achieve a solution at a cost similar to the product pricing.
- *Dynamic internal hedging*
  - The firm dynamically manages a portfolio of vanilla derivatives in a way that matches the short-term market value sensitivities of the liabilities. This may be, *on average*, a cheaper solution than the structured approach, but it may leave material residual market risks behind.
    - How much cheaper? How much risk?

Dynamic Hedging: The Mechanics

- In a dynamic hedging strategy, a hedge portfolio is held that has similar short-term market risk sensitivities (*greeks*) to the guarantee cost.
- Generally, a stochastic asset-liability model will be required to calculate the *greeks* of the liability
  - Market-consistent economic scenarios
  - Must be quickly and accurately calibrated to market prices
  - Automated daily sensitivity test calibration production
    - i.e. Equity vol up 1%; interest rates down 10bps; etc.
- The hedge portfolio's composition may need to be re-balanced frequently (e.g. daily), to maintain a good match to the guarantee cost's market risk sensitivities.
  - Use simple, easily-traded instruments in hedge portfolio

Market risks left behind by dynamic hedging strategies

- Residual market risks in guarantee delta-hedging program:
  - Market movements occur before hedge portfolio can be re-balanced (gamma risk)
  - The asset path is more volatile than expected over the lifetime of the product (gamma risk)
  - Changes in option-implied volatility result in unhedged guarantee cost increases (vega risk)
  - The underlying instruments used in the hedge portfolio are not perfectly correlated with the underlying assets of the investment product (basis risk)
  - The model used to estimate the Greeks doesn’t provide accurate results (model risk)

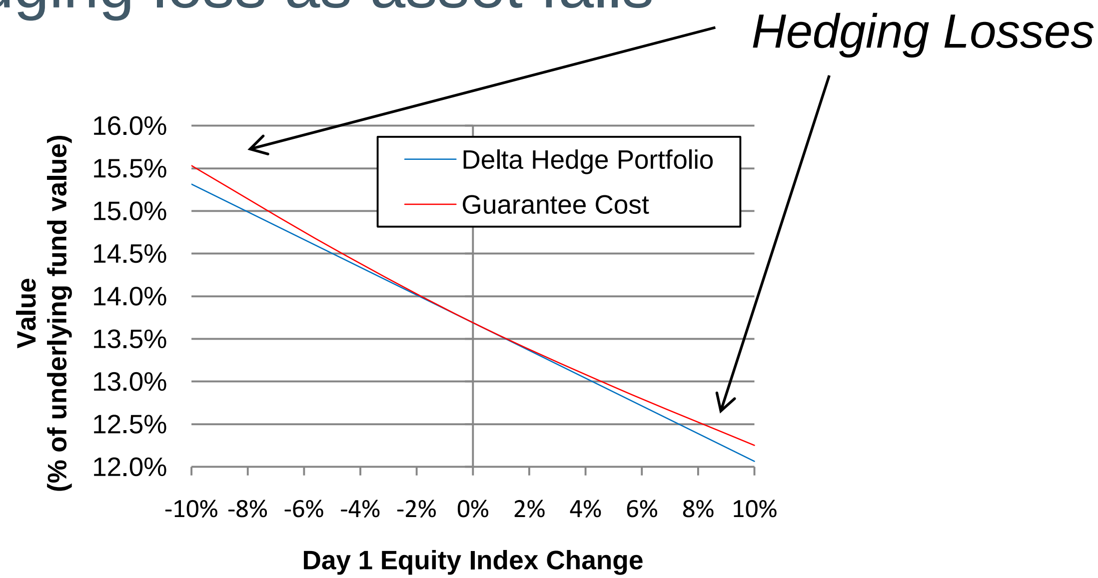
Market risks left behind by dynamic hedging strategies (II)

- Recent market experience has highlighted the size of the loss tail that can result from these ‘second-order’ market risks

- Also operational risks and policyholder behaviour risks

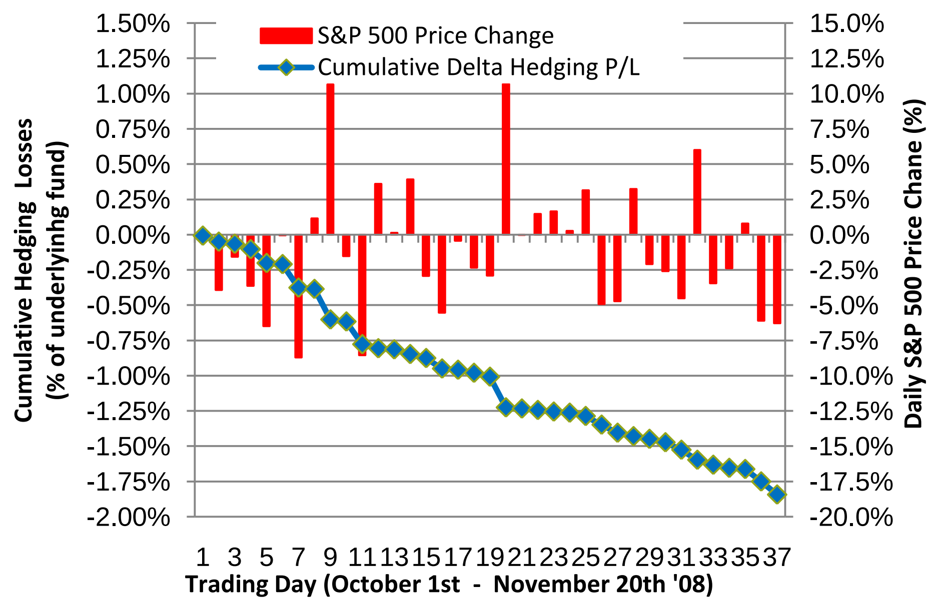
Residual Risks of a Dynamic Hedging Strategy: More on Gamma Risk

- A dynamic hedging strategy will reduce directional sensitivity but can leave exposure to the risk that underlying asset *volatility* is greater than expected
- Note short-term underlying asset *increases* can be as much of a source of hedging loss as asset falls



Residual Risks of a Dynamic Hedging Strategy

- Q4 2008 was a good recent example of the substantial residual gamma risks left behind by a dynamic hedging strategy
- In a B&H case study, a 5% GMWB contract with market-consistent cost of 14% of underlying fund value incurred hedging losses from gamma risk alone of over 2% in less than 2 months, despite daily re-balancing algorithm
  - Before allowing for trading costs, vega risks, etc.



Assessing the gamma risk tail: Modelling needs

- The gamma risk tail will be a function of:
  - How frequently the hedge portfolio can be re-balanced
  - The extent that market volatility may vary over time
  - The likelihood of market ‘gaps’ or jumps (e.g. 10%+ 1-day movements)
  - Expectations for market volatility over the lifetime of the product relative to the option-implied level

Assessing the gamma risk tail: Modelling needs (II)

- A sophisticated real-world daily equity return model will provide a powerful insight into the nature of this risk exposure
  - Daily time-steps
  - Stochastic volatility for underlying assets (expected volatility and its possible variation)
  - Capture gap risk
  - Option-implied volatility modelling (for vega risk)
  - Calibration
  - Risk assessment may use stochastic-on-stochastic => integrated market-consistent modelling

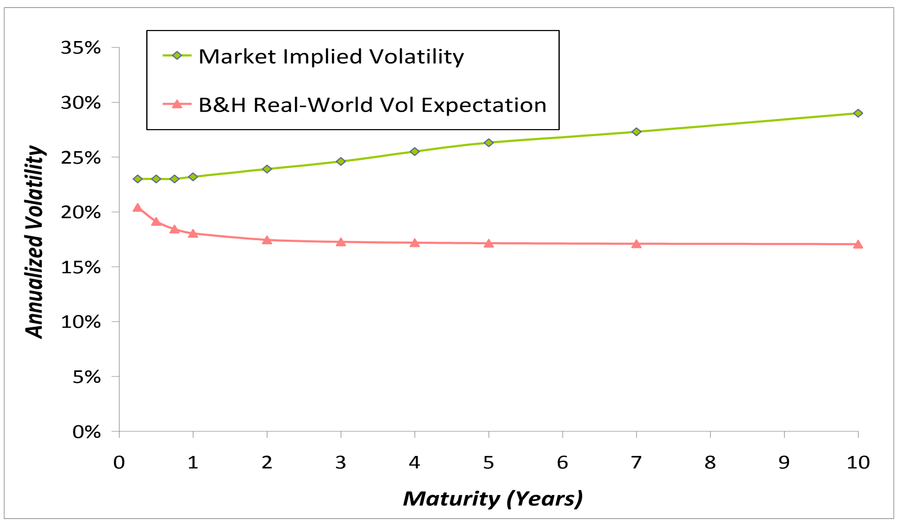
Delta Hedging Case Study

- Money-back guarantee after 10 years
- 10-year option-implied volatility at end-2009 = 29%
- Assume 4% interest rate
- Starting fund value = strike price = 1
- This produces option price of 0.158; option delta = -0.187
- So initial hedge portfolio =
  - -0.187 of equities;
  - (0.158 -0.187) = 0.343 in cash
  - Hedge portfolio = 0.158 = option cost

- Assessing the risk / return in the delta hedge requires:
  - Real-world volatility expectations (i.e. how expensive are option market prices?)
  - Stochastic volatility model to produce variation in realised volatility

Case Study: Option prices and volatility expectations

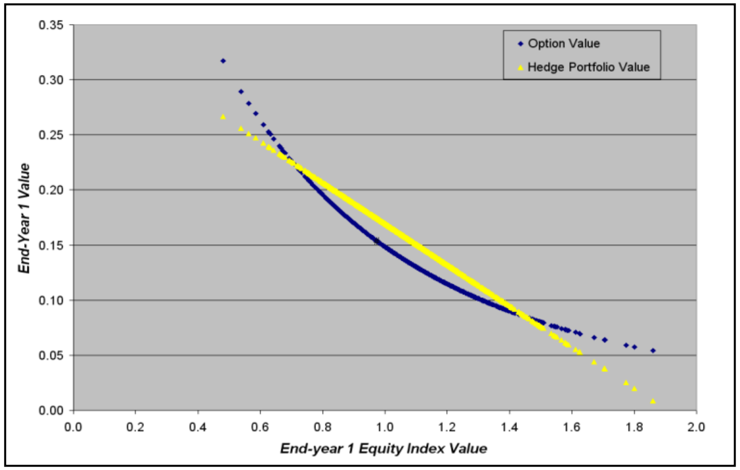
- How do 'real-world' volatility expectations relate to recent option-implied levels?
- This will determine the *expected* hedging profit from a delta hedging strategy
  - This can be considered as the risk premium embedded in option pricing (and hence expected return for manufacturing an option using dynamic hedging
- A stochastic volatility model can then put risk around this expectation



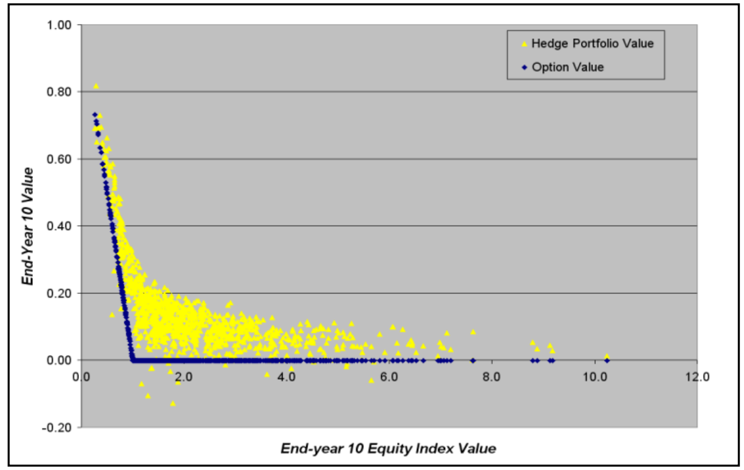
Volatility expectations for S&P 500 at end-2009: Implying substantial risk premiums for delta hedgers

Case Study: Residual Risks and Returns of a Dynamic Hedging Strategy

After 1 year

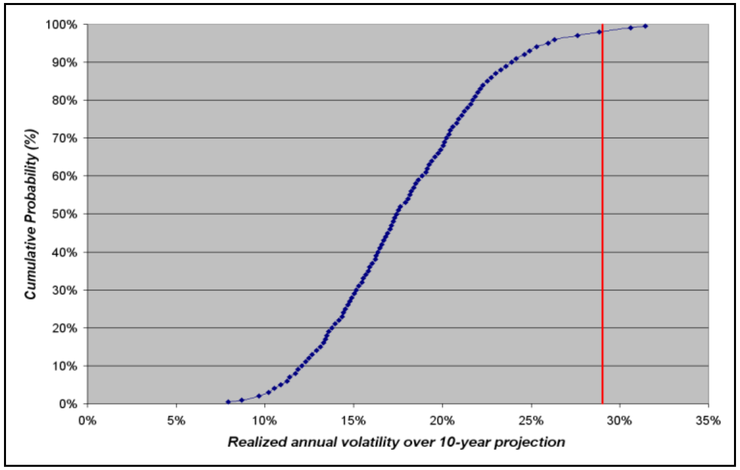


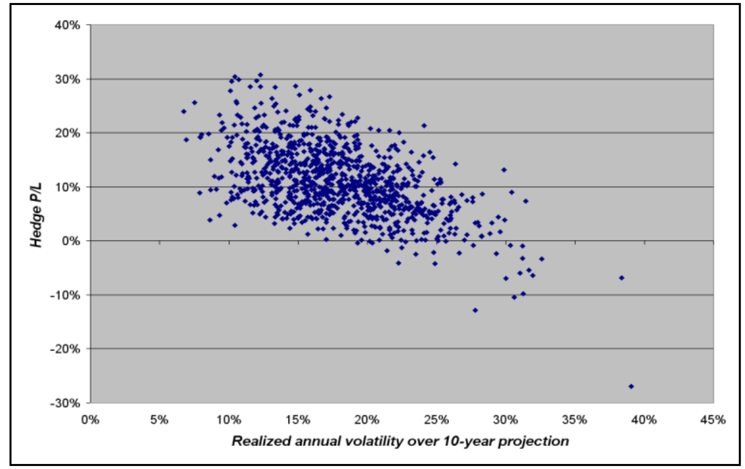
After 10 years



- These modelling assumptions imply a *positive* cumulative hedging profit / loss has a 97% probability (!)
- Driven by assumption that 10-year ‘real-world’ volatility is similar to historical averages, and hence substantially lower than is implied by observed option pricing

Delta Hedging: Behind the results (I)





- Key points:
  – The final hedging P/L is a function of the realized volatility in the projection
  – The real-world volatility calibration does result in some scenarios where 10-year realized vol is higher than the 10-year option-implied vol, but the probability is low (2%)

Delta Hedging: Behind the results (II)

- Key points:
  - Such a model can be used to explore how risk measures and capital requirements vary with different time horizons, confidence levels and volatility expectations
  - Contrasting short term P&L risk with long term cumulative returns
  - A tool for understanding risk drivers and risk premia
  - Highlights the effect of our real world assumptions on profitability
  - The effects not modelled?

More on managing market-consistent risks in very illiquid markets

Market-Consistent Liability Valuation in the Real-World (I)

Consistent with what?

- Risk-free yield curves
  - Treasuries or swaps?
- Equity option-implied volatilities
- Swaption-implied volatilities

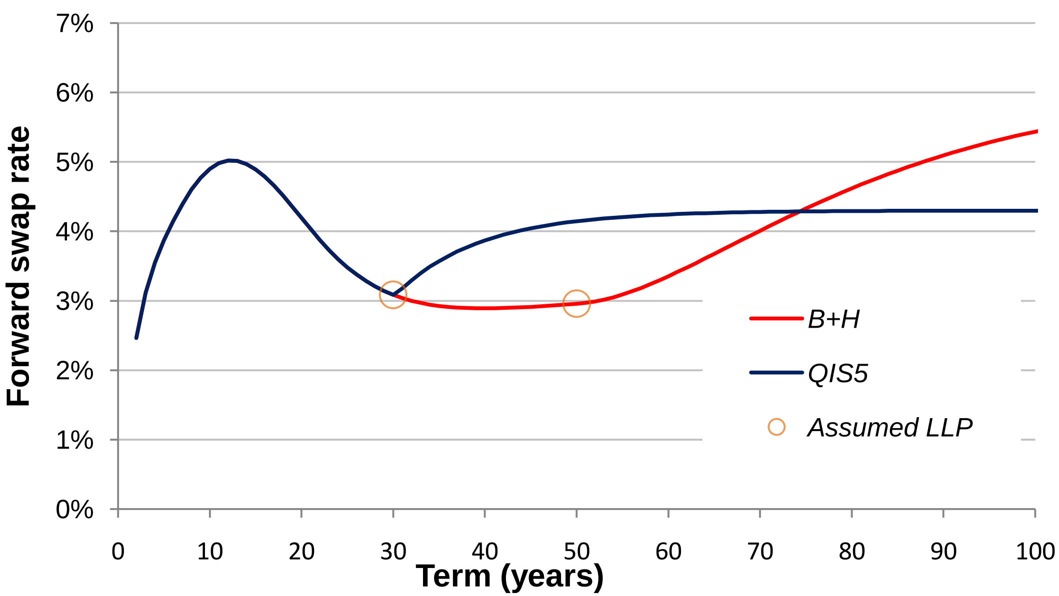
- For all these instruments, we may require prices at maturities for which no transparent price is available…need to extrapolate

Market-Consistent Liability Valuation in the Real-World (II)

- There will also be other model assumptions for which **no** liquid market price may be available
  - e.g. Real estate volatility, some correlations
  - Tend to use real-world assumptions for these parameters (+ risk margin?)
- The lack of relevant liquid market price availability impacts on objectivity of the valuation, and significantly complicates hedging strategies
  - Less clarity on liability valuation characteristics
  - Less assets with which to match these characteristics

Extrapolating Market Prices

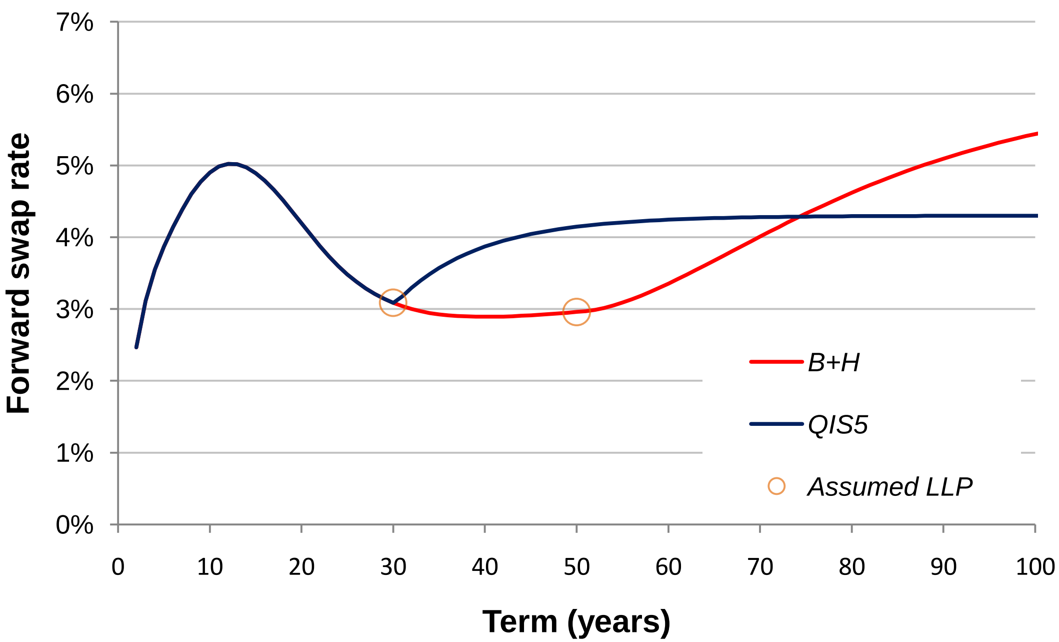
- What is the longest observable market price?
  - e.g. ‘Deep, liquid and transparent markets’ requirements in Solvency II…is longest liquid point of the Euro swap curve 30 years or 50 years?
- What is the ‘unconditional’ ultra-long-term price?
  - e.g. is long-term swap rate 4% or 6%?



Comparison of End-Dec 09 Euro Yield Curve Extrapolations

Extrapolating Market Prices

- How do we interpolate between them?
  - Strength of ‘pull’ will reflect subjective views of the volatility of ultra-long-term price
  - May also put smoothing constraints to avoid discontinuities in yield curves



Comparison of End-Dec 09 Euro Yield Curve Extrapolations

Dynamic Hedging in Very Illiquid Markets

- Even the economies' with the world's most deep and liquid financial markets can create significant liquidity issues for insurance liabilities
  - e.g. very limited equity option availability at maturities of 10 years+
- However, some economies will have very limited access to derivatives, including equity index futures
  - e.g. China
- This can make delta hedging implementation particularly problematic, as the use of index futures would usually be extensive

Dynamic Hedging in Very Illiquid Markets (I)

- Some possible solutions:

1. Use equity index futures contract of an index that is highly correlated with the underlying index
   - This may result in a basis risk loss tail that is highly significant

Dynamic Hedging in Very Illiquid Markets (II)

- Some possible solutions:

2. Net the delta hedging requirement from the underlying asset holding, and dynamically adjust the underlying asset holding to re-balance the hedge portfolio
   - This approach has been used by UK with-profit funds
   - Re-balance underlying holdings as dictated by changing delta

- Issues:
   - Transaction costs of buying and selling the underlying physical index
   - Regulatory capital treatment

Advanced capital calculation methods

Need for nested stochastic simulation

- We use 1000’s of simulations for a single valuation of VA liabilities
- To calculate 1 year MCEV VaR capital we need to do 1000’s of real world simulations and price liabilities within each one
- Many other applications of nested stochastic models in VA business
- ALM models are too slow to perform full nested stochastic calculation
  – Need for acceleration

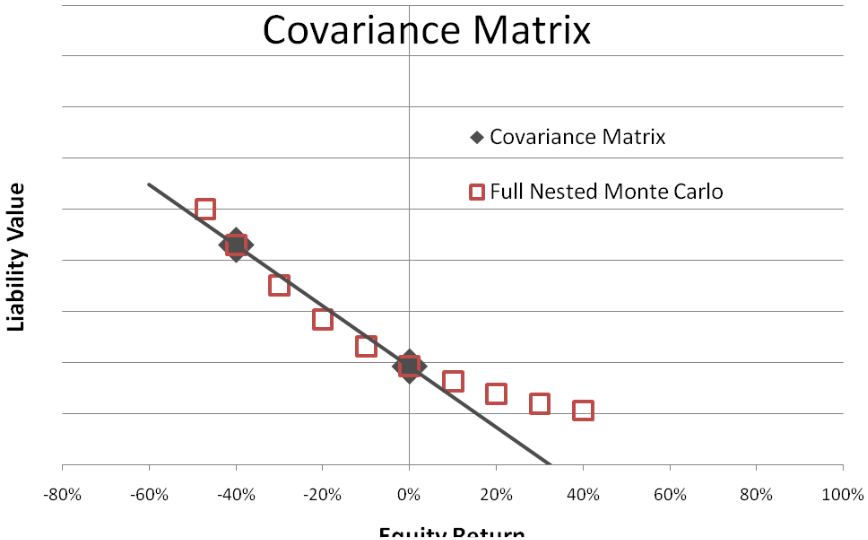
Techniques available

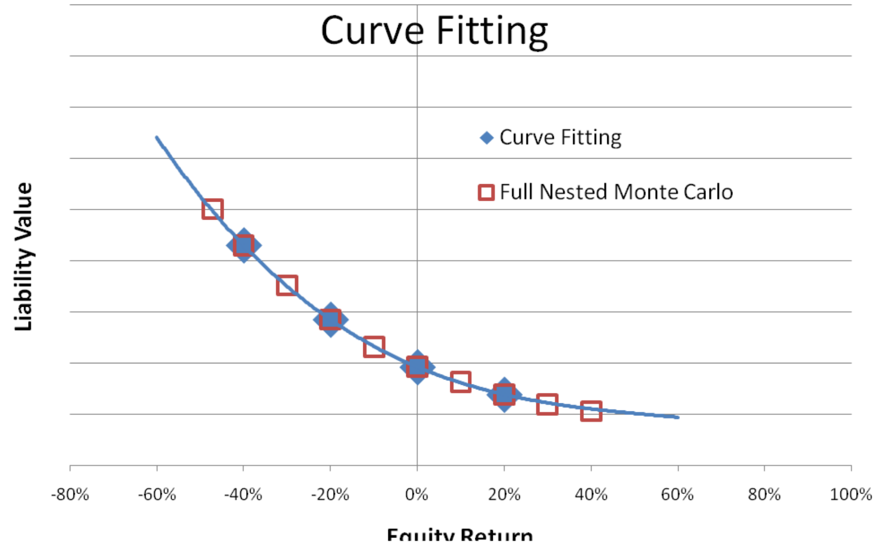
- Covariance Matrix
- Curve fitting
- Replicating portfolios
- Least Squares Monte Carlo
- Policy – Simulation pairing
- Control Variates

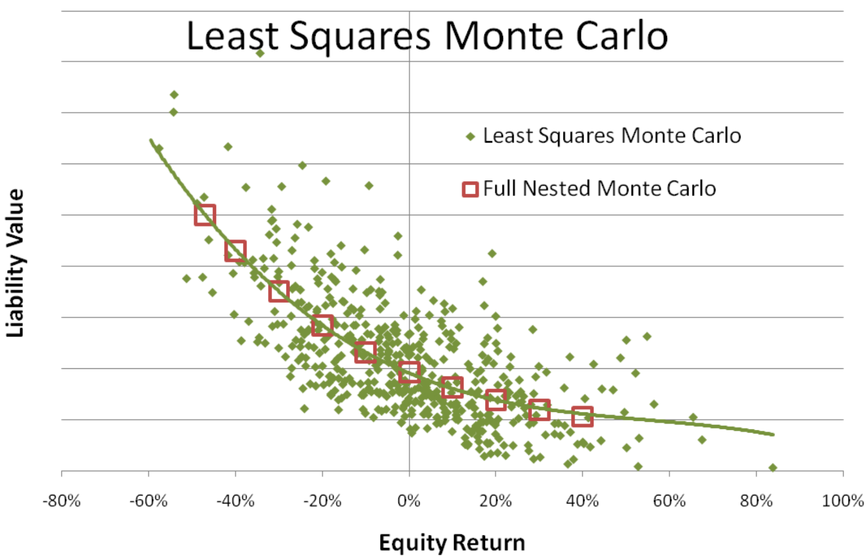
Techniques available

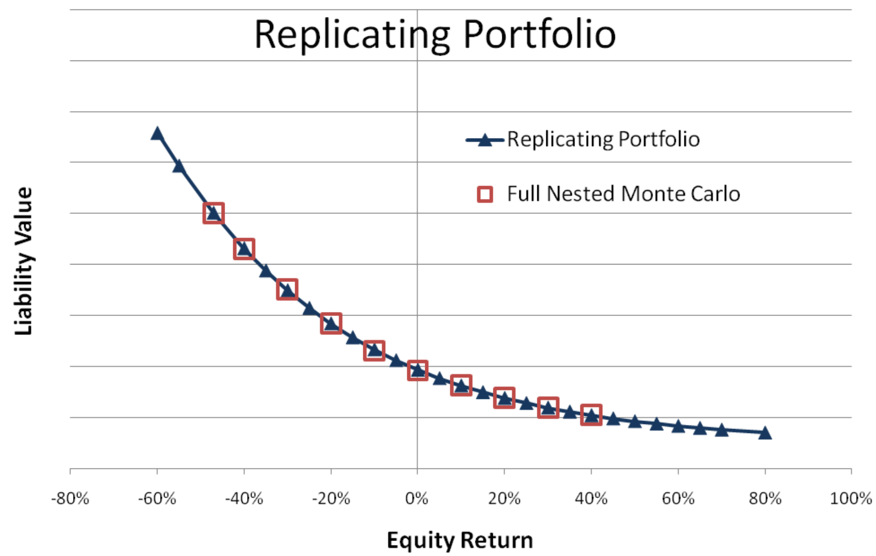
- Covariance Matrix
  - Inaccurate
  - Assumes linearity in risk drivers and in liabilities
- Curve fitting
  - Very time consuming to fit full liability curve in many risk dimensions
- Replicating portfolios
  - Difficult to find RP that behaves in similar way to liabilities
  - Especially in the presence of dynamic rules, complex guarantees

Techniques available



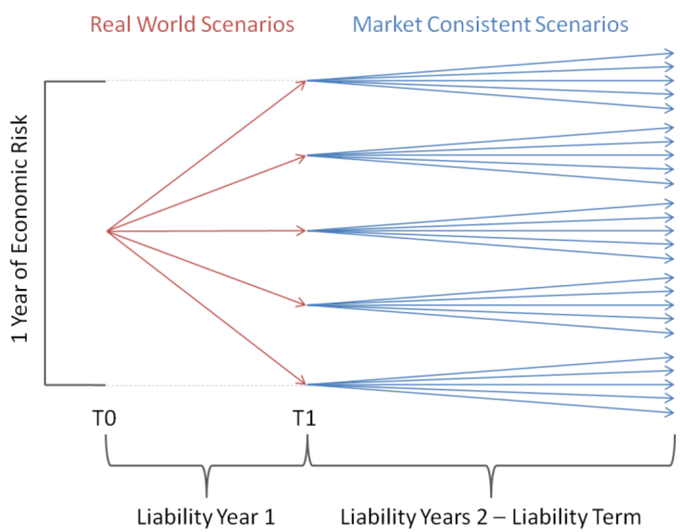


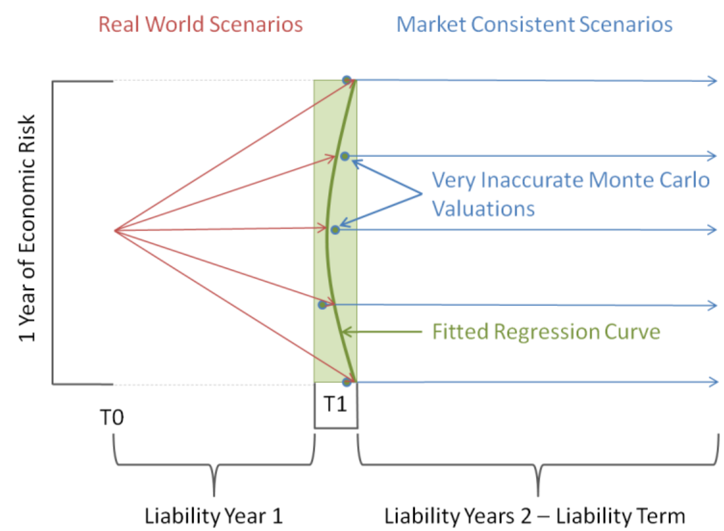




Techniques available

- Least Squares Monte Carlo
  - Instead of full nested Monte Carlo simulation, do only 1 inner scenario



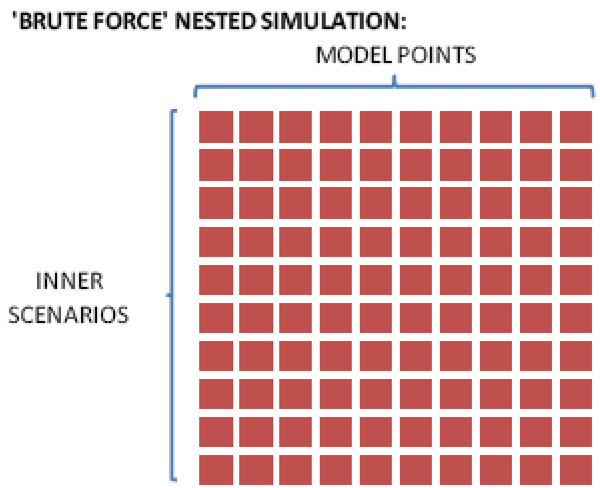


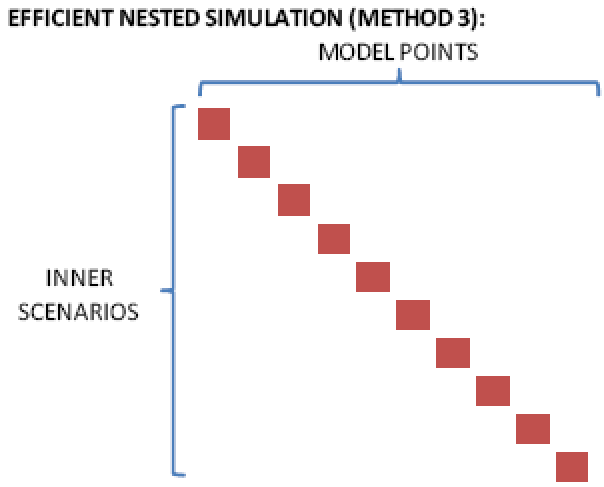
Least Squares Monte Carlo

- Advantages
  - Significant speed advantage over nested stochastic simulation
  - Converges quickly to true liability function
  - Ability to find liability function at future time periods by regressing over inner simulations at future dates
    - Gives ability to do long term capital and hedge efficiency calculations etc
  - More efficient than curve fitting
  - More accurate than replicating portfolios
  - Uses in internal model decisions – management and policyholder behaviour based on MCEV / capital in each simulation

Variance reduction in nested stochastic

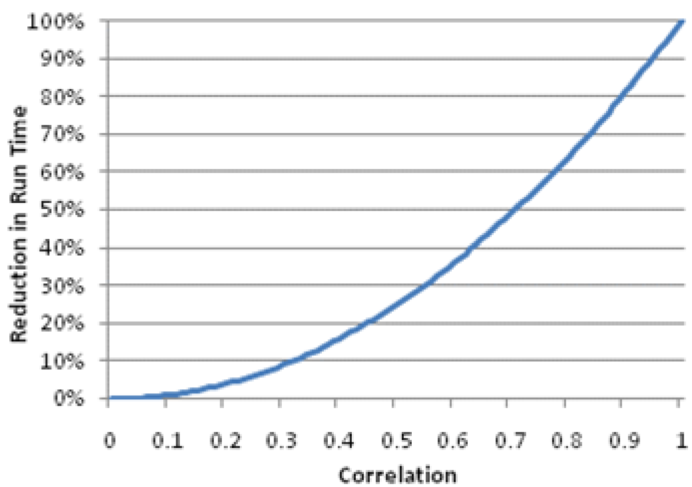
- Policy-Scenario pairing
  - Each policy valued using different market consistent simulations
  - Need more MC sims but same ALM run time
  - Significantly increases accuracy of pricing by “averaging errors” – allowing less ALM sims

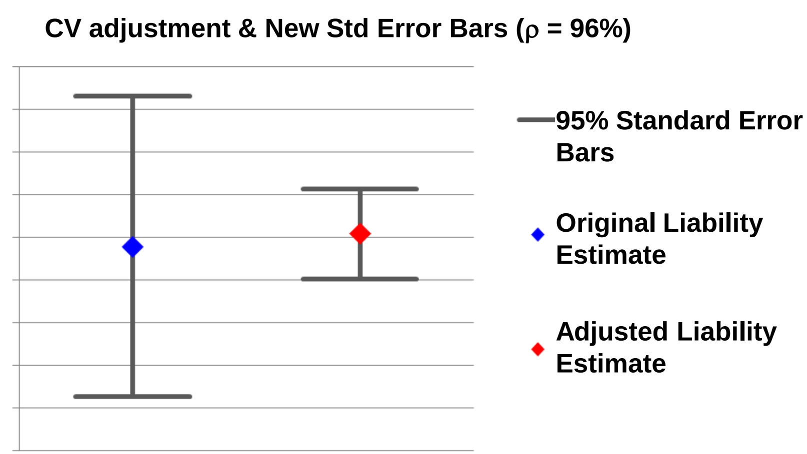




Variance reduction in nested stochastic

- Control Variates
  - For each inner pricing simulation, run fewer simulations
  - Correct for error in pricing by comparing to error in pricing similar assets
  - Depends on degree of “Similarity” measured by correlation





Conclusions

Conclusions

- Previous decade has seen a transformation in how insurance groups' market risks are assessed, reported, capitalised and managed
  - Market-consistent valuation techniques have played a very major role
- Sophisticated market risk management programs have emerged to manage mark-to-market liability valuation volatility
  - Derivatives and dynamic hedging
- Sophisticated risk management creates the need for sophisticated models
  - Real world and market consistent ESGs
  - Nested stochastic programs

Conclusions

- Mark-to-market risk management is particularly complex for insurance liabilities
  - Long-term nature of liabilities and valuation challenges;
  - Limited liquidity of relevant assets
- Recent experience highlights that the measurement of the risks left behind by these strategies is an area that risk actuaries need to further develop
  - Understanding the nature of risks left behind
  - Deploying appropriate models to measure them
  - Actuarial judgement is unavoidable: need for informed and explicit assumptions

Questions or comments?

Expressions of individual views by members of The Actuarial Profession and its staff are encouraged.

The views expressed in this presentation are those of the presenter.