A REVIEW OF PAPERS RELEVANT TO NON-LIFE REINSURANCE (WITH A LIABILITY FOCUS)

by members of the

2006 GIRO "Reinsurance Matters" Working Party

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Introduction

The Reinsurance Matters working party agreed that the Faculty and Institute Reading Guide is somewhat sparse on papers relating to non-life reinsurance. We therefore compiled a list of such papers and reviewed them, preparing this short summary of each in the same format as the official reading guide. We hope that this will prove useful to actuaries looking for reinsurance papers and as such could provide the basis for a reinsurance section in the official guide.

As we are actuaries we couldn't resist a couple of caveats:

- We recognise that whilst quite comprehensive the list of papers we have reviewed is not exhaustive.
- Each review was compiled by a small subset of the working party and as such no single review necessarily reflects the views of the entire group

Key

"Type" must be one of the following three options:

Intro : Introductory and background: assumes little knowledge

Core : Core material – essential knowledge for a practitioner in this area

Spec : *Specialist* – *goes beyond the essential knowledge*

"Tech Level" must be one of the following three options:

Non : Non-technical – a layman can understand this
Stan : Standard – Requires an actuary's technical skills
Bey : Beyond – Goes beyond standard actuarial skills

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#	Yr	Author Keyword	Title of Paper	
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#1	Estimating Pure Premiums By Layer - An	Type: Intro
Finger, R. J.	Approach	Tech Level: Stan
1976		Pages: 19

This early paper presented an approach to the estimation of loss costs by layer of insurance coverage using a LogNormal loss severity model. It lays out the statistical basis for Increased Limits Factors with numerical illustrations that are easy to follow showing the reader how to develop their own ILFs from raw claims data using first principles.

Whilst the subject of this paper is now in universal use (albeit not always using a LogNormal model) the paper is still worth a read by the ILF-novice seeking an explanation of the basics.

#2	On The Theory Of Increased Limits And	Type : Core
Miccolis, R. S.	Excess Of Loss Pricing	Tech Level: Stan
1977	· ·	Pages: 33

This paper offers the reader a great introduction to Increased Limits Factors (ILFs). The first half of the paper presents the mathematical basis for building and using ILFs (as per the 1976 paper by Finger) including a discussion of the impact of trend. The second half of the paper offers both a mathematical basis for and a numerical illustration of how ILFs can incorporate loads for process risk based on the variance of the pure premium.

Whilst this paper is now roughly 30 years old, the techniques described remain valid today and as such this paper is worthwhile reading for those new to reinsurance.

#3	Adjusting size of loss distributions for trend	Type : Core
Rosenberg, S. et al		Tech Level: Stan
1981		Pages: 37

This paper examines methods for adjusting severity distributions for trends, and asserts (using intuition and empirical evidence) that the trend in losses is higher for larger loss sizes (which is put down predominantly to social inflation).

It goes on to show that this difference tails off as claim size increases, and offers ax^b (small b) as a good fit and easy to apply. The effects on ILFs are then demonstrated.

Useful, fundamental stuff (and you don't need to get bogged down in the proofs)

#4	A practical guide to the single parameter	Type : Spec
Philbrick, S.	pareto distribution	Tech Level: Stan
1985		Pages: 40

The paper is a reasonably in-depth analysis of the single parameter pareto distribution. The first section of the paper looks at various loss severity representations before moving on to examining basic properties of the single parameter pareto. A section on parameter estimation follows and the paper concludes with a discussion on the effects of inflation and some worked examples.

Although this is quite long paper, it is readable and has a good balance between text and formulae, and the worked examples at the end are useful. A solid foundation for anyone wishing to learn about the single parameter pareto distribution.

#5	The Mathematics Of Excess Of Loss	Type: Intro
Lee, Y-S.	Coverages And Retrospective Rating – A	Tech Level: Stan
1988	Graphical Approach	Pages: 29

The mathematics of excess of loss coverage and retrospective rating can involve heavy algebra. This paper presents a graphical approach to the theory in order to illustrate and explain many mathematical relations and results. The first half of the paper deals with XoL and as such is straightforward to follow, whereas the second half deals with retrospective rating and therefore assumes a working knowledge of "Table M" and "Table L" as used in the USA when rating workers compensation plans.

The paper offers a quick, easy and useful additional perspective for anyone struggling to conceptualize the mathematics involved.

#6	The effect of trend on excess of loss	Type : Core
Keatinge, CL.	coverages	Tech Level: Stan
1989		Pages: 21

This paper follows on from previous papers, that discuss how (uncapped) excess losses increase proportionately more than the average trend in losses.

It demonstrates that there is a dampening effect from capping excess losses (practical reality of limits for most excess layers), which acts to offset the leveraging effect of the deductible.

By the use of graphs, the paper then demonstrates how this varies when:

- Changing the shape/scale of the loss distribution
- Changing the type of loss distribution
- Allowing the trend to vary by size of loss

#7	Pricing The Impact of Adjustable Features	Type : Core
Bear, R.A and Nemlick,	and Loss Sharing Provisions of Reinsurance	Tech Level: Stan
K.J.	Treaties	
1990		Pages: 64

This paper looks at pricing reinsurance treaties that have specific features operating at the aggregate loss level. (e.g. aggregate deductibles/limits, loss corridors, profit/sliding scale commissions etc).

It considers pricing six examples of specific treaties using two approaches (Lognormal model & Heckman-Meyers collective risk model.) All formulae and assumptions are provided enabling the interested reader to reproduce the results.

Whilst this paper is 'old' in computing power terms and the pricing approaches it considers are not as widely used as they may have been, it may provide a useful learning opportunity for someone wanting to understand these types of features and price them from first principles.

#8	Risk Loads for Insurers	Type : Core
Feldblum, S		Tech Level: Stan
1990		Pages: 36

'Risk Loads for Insurers' is a discussion on the methods available and being used to load expected loss costs for the risk assumed in writing business. The author discusses various approaches including:

- (a) the standard deviation and variance of the loss distribution
- (b) utility functions
- (c) the probability of ruin
- (d) reinsurance cost

The weaknesses of these approaches are commented on before the author goes on to describe his favoured method which is based upon portfolio risk theory.

The author is very black and white in his views on how risk loads should be calculated and the paper is very critical of the methods used by the Insurance Services Office (ISO), which is the major U.S rate making bureau for non-compensation lines of business.

The subsequent review of this paper by Glenn G. Meyers redresses this balance and provides a balanced discussion of all of the approaches commented on by Feldblum including the limitations of applying portfolio theory models to insurance risks.

The key finding and possibly most important part of Feldblum's paper is that by applying the Capital Asset Pricing Model to insurance portfolios it would suggest that that the returns demanded by insurance companies are lower than they should be for the more 'risky' classes of business.

#9	Report of reinsurers security working party	Type: Intro
Hart, D. et al		Tech Level: Stan
1990		Pages: 41

This touches on some key core ideas for anyone considering the issues around RI bad debt and provisions for it. It is very quick to read and digest and even though it is old it remains relevant because of the generality of ideas.

However, we would recommend you also see the 1996 paper "Reinsurance Security" by Sanders et al (#17 in this guide) as this covers much of the same grounds, and then some.

#10	Estimation in the Pareto distribution	Type : Spec
Rytgaard, M.		Tech Level: Adv
1990		Pages: 16

This paper focuses on different methods of estimating the α parameter of a Pareto distribution. Purely theoretical approaches are derived and then contrasted with a more practical approach using market knowledge of the expected α value based on the class of business. The author then credibility weights the two approaches to arrive at a blended answer.

The paper is littered with mathematical formulae but moderates this by providing practical worked examples.

#11	Pricing for credit exposure	Type: Spec
Brown et al		Tech Level: Stan
1992		Pages: 18

This paper explains how to price and design products to allow for credit exposures in general with a focus on bad debt exposures for retrospectively rated policies arising from the possible default of future premium income expected for this type of policy. It takes into account the variability of exposure and creditworthiness of a risk over time.

The level of detail of thought for the methodology in the paper makes it more useful for calculating a small number of large exposures than a large number of small exposures however sections 3 to 6 contain some more generic ideas around credit exposure which may prove useful reading.

#12	Credibility For Treaty Reinsurance Excess	Type : Spec
Patrik, G. & Mashitz, I.	Pricing	Tech Level: Bey
1992		Pages: 52

This paper develops a credibility weighting to apply to experience and exposure rates for excess of loss rating purposes, based upon statistical analysis of loss frequencies assuming a negative binomial distribution. The resulting formulae are intuitively satisfying and the formulation is interesting, although in practice a quick and dirty subjective weighting based on a healthy respect for the uncertainties will often suffice.

Three aspects of this paper are notable even if you do not choose to implement their credibility methods:

- It references some very old papers (1950s and 60s) as the original thinking behind the use of a Negative Binomial frequency distribution
- The tables they publish to illustrate their credibility factors suggest very low confidence in exposure rates and higher confidence in experience rates, which seems contrary to contemporary practice?
- The paper uses an underwriter questionnaire to attempt to extract their views on credibility in a consistent, objective, indexed manner without resorting to direct questions they would likely be unable to answer.

Note that this work was developed further in 2004 in a paper by Mark Cockroft.

#13	How to choose a trend factor	Type : Core
Krakowski, I.		Tech Level: Stan
1994		Pages: 54

The first half of the paper relates to 12 considerations that go into the selection of a trend factor after the maths is understood. In narrative form mostly, the paper guides and provokes more thought into the methodology and assumptions used, to ensure that they are correctly applied as well as chosen e.g. by territory v overall.

The 2nd half of the paper relates to the use of credibility in selecting a trend factor. Although US legislatively focussed, the issues appear to be very transferable, and include a discussion of where the theoretical and practical diverge.

This paper assumes a fair amount of prior knowledge to make it an easy read, but nevertheless provides a useful narrative for practitioners.

#14	Quantifying the uncertainty in claim severity	Type : Spec
Meyers, G.	estimates for an excess layer when using the	Tech Level: Stan
1994	single parameter pareto	Pages: 22

The title of the paper kind of gives it away! The approach considered uses Bayesian Theory, incorporating prior opinions about the distribution of the Pareto parameter. Having first estimated the parameter, q, for the single parameter pareto, the conditional distribution of q* is derived. The author then examines the effect of introducing prior distributions to refine the estimates before moving on considerations of sample size and the choice of prior distributions. The paper closes with by considering how to proceed using a distribution other than the single parameter pareto.

The author concludes by drawing attention to the dangers of relying on model outputs alone and advises the use of prior information (i.e. market knowledge) in pricing negotiations.

#15	Pricing In The London Market: Part 1	Type: Intro
Sanders, D. et al		Tech Level: Non
1995		Pages: 109

This is an important paper for the beginner and comprises two halves.

- The first half of the paper offers a very high level overview of the various techniques in common use for estimating loss costs on reinsurance business. This offers a good introduction to the subject and encompasses a simple explanation of how cat models work, as well as illustrations of experience and exposure rating methods and stochastic modelling. It includes a useful piece on Credibility Theory, with a summary of the findings of the 1992 paper by Patrik and Mashitz and an illustration of how this can be applied in practice.
- The second half of the paper describes how pricing fits within the overall management of an underwriting entity (the control cycle, risk/reward trade and capital allocation, the actuarial role etc) and also presents a relatively lengthy and detailed numerical example and discussion of how Profit Testing techniques could be applied. However, the recent advances in user-friendly DFA software mean that such analysis would usually be handled in a DFA framework nowadays.

The Appendices are a little more technical than the main body of the paper, and introduce a number of mathematical fields that whilst not yet applied to London Market pricing have some features that make them interesting areas for possible further research. These include Neural Networks, option pricing techniques (e.g. Black Scholes) and Extreme Value Theory.

#16	Pricing In The London Market: Part 2	Type: Intro
Sanders, D. et al		Tech Level: Stan
1996		Pages: 34

This sequel to the similarly entitled 1995 paper focuses upon topics in property risk (as opposed to cat) reinsurance pricing. The first half of the paper illustrates a number of key problems that every actuary should heed when applying standard techniques such as curve fitting and using first loss scales. The second half shows how common features like AADs and RPs can be quantified using either Panjer recursion or stochastic modelling. Nowadays simulations have become de rigueur, but the PR example is nevertheless interesting reading.

#17	Reinsurance Security	Type: Intro
Sanders, D. et al		Tech Level: Stan
1996		Pages: 25

This paper outlines the issues that should be considered when assuming recoveries from reinsurers, such as how to make appropriate provision for reinsurance bad debt. The paper discusses the different provisions that will be needed for different types of business, such as latent claims and for different purposes. It provides some methods for analysing bad debts and the problems associated with applying these. Further discussion is provided on bad debt provisioning in other industries, particularly the banking environment, and how these approaches may be used in the insurance industry.

A section is devoted to the role of the reinsurance committee within an insurance company and its monitoring of the developments of a company's reinsurance exposure. Different approaches and possible sources of information are discussed. Special issues for reinsurance security are also outlined, such as the treatment of commutations and the extent to which an insolvent insurer can recover from its reinsurers.

The final section of the paper discusses reports by the Subcommittee on Oversight and Investigations of the Committee on Energy and Commerce of the US House of Representatives (the Dingell Committee) made in the 1990s looking at worldwide insurance solvency regulation.

#18	An integrated pricing and reserving process	Type: Intro
Goldberg, L.	for reinsurers	Tech Level: Non
1997		Pages: 56

The authors describe from a practical perspective the way in which the work done in pricing inwards reinsurance can be used within the reserving process. It assumes that each individual contract underwritten has been analysed and technically rated at outset. This analysis includes assessment of a loss ratio, loss development pattern, payment pattern, distribution of results and an estimated return on equity. It then describes how this analysis can be incorporated into reserving this business either on a contract by contract basis or with data grouped into homogeneous cells. It also describes the feedback loop for renewals.

The paper includes practical approaches to allowing for special contract features in the reserving exercise, for example loss corridors, retrospective rating features etc.

Note that the approaches described in this paper are equally applicable to reserving for outwards reinsurance where this has been analysed at purchase.

#19	Estimating the tails of loss severity	Type : Core
McNeil, A.	distributions using extreme value theory	Tech Level: Stan
1997		Pages: 21

This paper provides a framework for modelling extreme events using parametric or semiparametric distributions and using Extreme Value Theory (EVT). It also explains the key features and results of EVT.

In particular, it discusses the Pickands-Balkema-de Hann Theorem which broadly says losses that exceed high enough thresholds follow the generalised Pareto distribution. The paper also discusses the Fisher-Tippett Theorem which describes the limiting behaviour of appropriately normalised sample maxima as well as a classification for short, medium or heavy tailed distributions.

The analysis is applied to examples of real data for insured losses and explains methods for interpretation of the results observed. In particular, it is related to estimating losses for high layer insurance.

#20	A Simulation Approach in Excess	Type: Core
Papush, D	Reinsurance Pricing	Tech Level: Stan
1997		Pages: 30

This paper systematically describes the steps involved in a stochastic simulation approach to pricing excess of loss reinsurance. It explains the more complicated real-life examples that simpler aggregate loss approaches can't deal with, such as RI layers that "drop down" to provide coverage lower down the programme when the lower layers are exhausted.

The paper uses an example of the data from the reinsurance layers for a large US medical professional organization. The output at different stages is given in good clear appendices.

There is also a useful description of how to estimate the number of simulations required.

#21	Reinsurance Pricing	Type : Core
Chandaria, S. et al		Tech Level: Stan
1998		Pages: 28

The paper provides a good introduction to the practical issues faced by an actuary involved in pricing reinsurance. It makes frequent reference to the 1995 GISG paper 'Pricing in the London Market' which gives an introduction to the three basic pricing methods of, experience rating, exposure rating and frequency/severity analysis. The paper builds on the 1995 paper by discussing the practicalities of these methods, issues faced in the real world and possible solutions. The paper is purely focused on the estimation of the cost of claims of a reinsurance contract and does not comment in detail on the additional steps necessary and techniques used to move from the expected cost of claims to what may be an acceptable rate to be charged.

The appendices include a very useful 'Glossary of loss sensitive contract terms' with worked examples. These cover; Profit Commission, Sliding Scale Commission, Swing Rating, Loss Corridors, Reinstatement Premiums and Aggregate Deductibles.

#22	Reinsurer risk loads from marginal surplus	Type : Core
Kreps, R	requirements	Tech Level: Stan
1998		Pages: 8

This paper looks at reinsurer's pricing of reinsurance looking at the marginal additional capital required to support the new business. The paper puts mathematical formulae on the decision process the reinsurer has to go through to decide whether or not to write the business. Basically the reinsurer has to be paid enough for the additional capital at risk from writing this policy or they should just invest that capital elsewhere.

Although this short paper is full of mathematical formulae it is relatively easy to follow with the results feeling intuitively correct. This paper discusses the practical implementation of this approach but unfortunately lacks worked examples.

#23	Investment Equivalent Reinsurance Pricing	Type : Spec
Kreps, R		Tech Level: Stan
1998		Pages: 31

This paper discusses how to decide on the risk load for a reinsurance contract. This is based on the insurer having assets to cover losses to a certain level, so the risk and return on these assets should be equivalent to other assets. This will depend on the reinsurers attitude to risk and the paper discusses the measures for this that are used.

Simplifying assumptions are made to consider a contract with a single payment at year one and lower limits on the assets required are derived. The use of options is also considered. The paper then goes on to demonstrate that a minimum premium can be derived for high excess layers on this basis.

Single payments at a variable time and multiple payments are considered. As the situations become more complex simulation becomes necessary and a general framework to simulate this is outlined. The benefits of pooling of risks over contracts and over time are also discussed.

The derivation of bounds for reinsurance risk loads may help to verify the risk loads used that are actually used in practice and the simulation discussed can help with modelling more complex situations.

#24	Implementation of PH transforms in	Type: Intro
Wang, G	ratemaking with discussions by Venter	Tech Level: Stan
1998		Pages: 36

Wang presents a method for deciding on contingency provisions in insurance pricing by using proportional hazard (PH) transforms. A PH transform adjusts a probability distribution using a further selected parameter. The value of this parameter will control the amount of margin that is introduced on the original distribution. Simple examples of PH transforms are given and Wang discusses how to use them to price ambiguous risks by viewing the PH transform as an upper confidence limit for the best estimate loss distribution.

The paper then goes on to consider using PH transforms in more complex situations such as pricing excess of loss layers, increased limits ratemaking and pricing of risk portfolios. In addition to details on the technical approach to using PH transforms, the paper relates the method to economic theories around taking decisions under risk, uncertainty and ambiguity.

Venter responds to the paper in a discussion covering a number of areas including the need for consistency between contracts and further how it links to economic theories. Venter also goes on to propose alternative transforms (Esscher) that may work better in certain situations.

The PH transform method provides a relatively straightforward way of loading distributions for risk and could be applied in practice, although selecting a value for the uncertainty parameter is likely to be subjective. Venter's discussion does help to address the difficulties in choosing this parameter, however.

#25	Reinsurance Pricing	Type : Core
Chandaria, S. et al		Tech Level: Stan
1999		Pages: 47

The paper builds upon the 1998 GIRO working party paper 'Reinsurance Pricing, by Chandaria, S et al'. It starts by providing a useful summary of a skeleton process which may be followed when pricing any class of non-proportional reinsurance. The description of the process is then followed by a worked numerical example. The pitfalls that may be faced by following such a pricing process are identified and possible practical solutions are discussed at length, usually aided by numerical worked examples.

Throughout the paper reference is made to commonly used techniques and theorems which may be used to aid the pricing actuary. Since so much of reinsurance pricing involves the approximation of aggregate claims distributions and extreme values, both of these subjects are discussed in detail.

The paper should be of interest to anyone with a basic knowledge of reinsurance pricing who wishes to gain a good understanding of the pricing process and learn about the basic techniques commonly used by pricing actuaries.

#26	Modelling Losses With The Mixed	Type : Spec
Keatinge, C.	Exponential Distribution	Tech Level: Stan
1999		Pages: 45

Finding a parametric model that fits loss data well is sometimes difficult, and the alternative of using an empirical distribution can often be unsatisfactory. Keatinge presents the Mixed Exponential distribution as a compromise between these two approaches. The paper justifies the use of this model, explains how you to go about maximum likelihood fitting of parameters and compares results using Mixed Exponential versus some traditional parametric distributions. It also gives some health warnings about when you shouldn't use this approach, and discusses some possible adjustments to and other uses for the model.

The mixed exponential is an appealing and flexible model, but fitting it to real data is a non-trivial recursive process which could be a hurdle to its application. It also requires some manipulation if you wish to extrapolate beyond your dataset, although it does offer you a lot of control to exercise judgment when fitting this "tail".

#27	Risk load and the default rate of surplus	Type : Spec
Mango, D		Tech Level: Stan
1999		Pages: 47

The aim of this paper is to derive formulae for calculating risk loadings using methodology consistent with the capital markets. Don Mango calculates the risk loads based on the default risk of the supporting capital using capital market techniques. The paper provides a comparison of this technique to other common techniques which in itself provides a useful background of these techniques.

Although this paper is 47 pages long, the main idea and formulae in this paper are covered in the first 8 pages with many worked examples following. This paper also comes with an excel spreadsheet of the 12 worked examples.

#28	Premium Trend Revisited	Type : Core
McCarthy, T.		Tech Level: Stan
2000		Pages: 32

The paper considers trends in claims, when modelling via the adjustment of premiums e.g. rather than via Frequency / Severity / Exposure.

It discusses the need to adjust premiums for changes in 'loss issues' explicitly, and not just exposure rate changes, to avoid misstating rates. For example, this would include allowing for changes in mix of business between territories / business segments with differing underlying trends. There are proofs given, with corresponding real-life practical problems considered.

#29	Heavy-tailed distributions and rating	Type : Core
Beirlant, J. et al.		Tech Level : Stan
2001		Pages: 22

This paper takes us through techniques used in Extreme Value Theory (EVT) for modelling the tail of claim size distributions (e.g. in higher reinsurance XL layers). It deals with the modelling of extreme events using either:

- Non-parametric techniques such as Mean Excess functions and Quantile plotting, or
- Semi-parametric techniques such as Pareto-Type distributions.

It discusses Pareto-type distributions with slowly varying functions and then goes on to show the usefulness of the generalised Burr / Gamma distribution as a fully parametric model for the centre and tail of the distribution together. However, it notes that this needs the use of EVT statistics to best-fit the model (rather than classic techniques).

This is an interesting paper although it is important to remember that you are working in the realms of very little data so there is a limit to how much confidence can be assigned to any inferred model.

#30	Stochastic dominance: A tool for evaluating	Type : Spec
Heyer, D.D.	reinsurance alternatives	Tech Level: Stan
2001		Pages: 23

One of the tasks a reinsurance buyer has to do is to decide which of various alternative reinsurance structures best meets his company's objectives. However, it is surprisingly common for these objectives to be vague, or even non-existent.

This paper explains that a company's objectives, if rational, should meet 3 particular criteria (wealth preference, risk aversion and skewness preference.) These criteria can be used to compare reinsurance strategies, and hence eliminate any stochastically dominated strategies from consideration. The paper does point out that there are other considerations that apply to the decision and that a unique dominant solution might not be practical for other reasons.

The paper concludes with some sample computer code and numerical worked examples. These examples consider the strategies from the perspective of both the reinsured and the reinsurer.

#31	Approximations to the Aggregate Loss	Type : Core
Papush, D. et al	Distribution	Tech Level: Stan
2001		Pages: $6 + Apps$

The paper is concerned with approximating aggregate loss distributions. There are a number of well documented methods of doing this but they usually rely upon being able to approximate separate loss frequency and severity distributions. This paper addresses the question of what type of probability distribution is the most appropriate to use as an approximation to the aggregate loss distribution when it is not practical to obtain separate frequency and severity distributions. In order to test the goodness of fit of different distributions to aggregate claims data, the aggregate claims are first built up from known frequency and severity distributions. This enables the goodness of fit to be tested against a known aggregate loss distribution.

An informative paper, easy to follow and the conclusions provide justification for the general use of a lognormal distribution when frequency and severity data is limited.

#32	Pricing Aggregate and Credit Risk for Risk	Type : Core
Deacon, J.	Sharing Entities	Tech Level: Stan
2002		Pages: 8

This paper discusses the credit and other issues that arise for an insurance company when fronting for a risk sharing entity such as a captive, particularly where the arrangement involves the RSE running a large aggregate retention.

The paper focuses on small enterprises that may typically have low levels of capital. There will be risk that arises if the aggregate attachment point is at a level such that if losses approach or exceed it then the risk sharing entity may not meet all of its liabilities which may be passed to the insurer. This needs to be incorporated into the pricing of a contract. The paper demonstrates this with an example and discusses some of the difficulties allowing for this in the contract pricing.

#33	Tails of Copulas	Type : Spec
Venter, G. et al		Tech Level: Bey
2002		Pages: 46

Copulas allow joint distributions to be separated into the marginal distributions of individual variables and the interdependency of the probabilities. This helps with controlling what parts of the distribution are more strongly associated when using distributions in modelling.

The paper discusses several common copulas and some particularly designed for loss severity distributions. Key properties are discussed and details are given on how to simulate values from distributions with a given copula. This includes some practical methods for visualising copulas and having control over the level of correlation in different parts of the distribution.

The paper also provides methods of assessing appropriate copulas from given data using the properties of the data and considering its descriptive features. Particular applications are considered including joint distributions of loss and loss adjustment expenses and hurricane losses arising in different regions from a given event. Further details on manipulating copulas and their mathematical properties are also provided.

#34	Capital consumption: An alternative	Type : Spec
Mango, D	methodology for pricing reinsurance	Tech Level: Core
2003		Pages: 28

This paper breaks new ground. The paper introduces the concept of consuming capital from a central pool as an alternative to allocating capital to contracts/portfolios. Mango argues that the consumption process in the form of any number of capital calls better describes the process in practice and also gets round the problems of theoretical capital allocation and paying claims above the allocated capital amount. The paper follows on by outlining how, using utility theory, this approach works in practice giving a couple of worked examples. The paper also debates about whether capital allocation makes sense in insurance

This paper is relatively easy to read and understand. It contains little mathematical formulae and gives numerous worked examples to aid understanding. Recommended reading!

#35	Pricing Excess of Loss Treaty with Loss	Type : Spec
Mata, A. et al	Sensitive Features: An Exposure Rating	Tech Level: Stan
2003	Approach	Pages: 46

The authors describe an approach in which a pre-selected expected loss is used to develop an aggregate loss distribution that can incorporate all the characteristics of the treaty, for example swing rated premium or profit commissions. The aggregate loss distribution is based on a severity distribution generated from an exposure rating approach.

The paper includes examples to illustrate the proposed approach.

The paper is aimed at estimating the expected value of loss sensitive features based on the aggregate loss distribution for the layer; it does not cover methods for estimating the expected loss cost to the layer. The approach uses detailed risk information such as original contract deductibles and policy limits.

There is an assumption that the individual losses are mutually independent, and the author recognises that this assumption may not be realistic in practice, and may underestimate the overall risk.

There are two detailed worked examples that show how the approach can be applied in both mono-line and multi-line reinsurance covers.

#36	Bayesian Credibility For XoL Reinsurance	Type: Spec
Cockroft, M.	Rating	Tech Level: Bey
2004		Pages: 21

This paper builds on some previous research (in particular the 1992 Patrik / Mashitz paper) to develop a Bayesian Credibility methodology for weighting experience and exposure rates for an excess of loss reinsurance layer. The paper is very technical containing a lot of statistics; this is not one for the lay reader! As the author states, the proposed methodology offers "significant precision using a macro" but is also "not simple or elegant, and with five separate infinite series summations ... can never be computed exactly".

#37	Rating long-tail excess of loss reinsurance	Type : Core
Cockcroft, M. et al		Tech Level: Stan
2004		Pages: 42

The papers starts off with an outline review of various generic pricing approaches (burning cost, FGU claim projection, exposure and frequency/severity.) It considers the strengths and weaknesses of each under various criteria, including the ability for the methodology and conclusions to be easy to explain to an underwriter.

It then moves on to look specifically at issues related to long-tailed classes and how these can be handled in the various pricing approaches.

It also very briefly covers one of the more practical aspects of pricing which is how to price high layers, blend experience with exposure prices and ensure programme/layer price consistency.

#38	Credit risk	Type : Spec
Bonnard et al		Tech Level: Bey
2005		Pages: 11

This guide to pricing RI credit risk is a set of PowerPoint slides from GIRO 2005.

It evaluates the variability of Return on Capital under traditional ratings and also by allowing for recent rating re-grades and concludes that:

- The two methods considered (traditional ratings and traditional ratings allowing for recent rating re-grades) are not ideal for credit rating
- The purchaser's understanding of uncertainties highly alters the perception of value for money of reinsurance and the extent to which protection is valued

NOTE – There are a few slides with some detailed stats in but they are not essential for an understanding of the message the paper is trying to get across (as detailed above)

#39	D&O reinsurance pricing – A Financial	Type : Core
Alwis, A. et al	Markets Approach	Tech Level: Bey
2005		Pages: 14

This paper sets out an exposure based rating approach for D&O reinsurance using Financial Market pricing techniques instead of relying on traditional ILFs and the assumption that the original business is correctly priced. The authors admit that the paper is a starting point rather than a final solution to a very complex problem.

The paper itself is very readable even for non-D&O practitioners. It begins with a short introduction to the D&O marketplace followed by a look at the limitations of existing experience and exposure rating approaches. They then briefly set out their approach (with most formulae being relegated to an appendix.)

There is, however, little information provided on the exact details of the assumptions used, with the paper merely explaining what factors they considered in setting their assumptions. It would take a fair degree of effort and research for anyone else to implement their approach.

The references are varied including papers on the D&O market and securities class actions as well as more technical financial market papers on debt and correlation modelling.

The authors, in a 2006 paper, have now applied a similar approach to Political Risk reinsurance.

#40	An Improved Method for Experience Rating	Type: Spec
Mata, A. et al	Reinsurance Treaties using Exposure Rating	Tech Level: Stan
2005	Techniques	Pages: 40

This paper sets out ways to use exposure curves to adjust historic claim experience to allow:

- calculation of experience rates for reinsurance layers attaching below the inflated loss reported threshold;
- calculation of trend factors for excess layers, with a method of splitting these between frequency and severity trends within the layer;
- estimation of experience adjustments to allow for changes in limit profile and / or maximum line size.

The methods set out require access to exposure curves which are appropriate to both the class and territory under consideration; the authors note that, other than for US business lines reported to ISO, accurate and contemporary exposure curves may be difficult to obtain.

The methods outlined are described in terms of a single year of origin of data; in practice one would need to apply the calculations to each past year with relevant data and take a weighted average of the results.

The paper includes a worked example to illustrate the methods described.

#41	The modelling of extreme events	Type : Core
Sanders, D.		Tech Level: None
2005		Pages: 39

This paper gives a broad overview of the modelling of extreme events. If it reminds the Extreme Value Theory, it also deals succinctly with various other subjects from Richter scale, Copulas, historical events to asteroid collisions or Chaos and Complexity Theory. This paper raises practical problems and gives a useful general knowledge to all practitioners.

# 42	Stochastic excess of loss pricing within a	Type : Stan
Schirmacher, D. et al	financial framework	Tech Level: Stan
2005		Pages: 54

This paper starts by introducing the Extreme Value Theory for pricing high layer excess of loss reinsurance and derives the Generalised Pareto Distribution formula to calculate the risk premium to XL layers. This first section also describes the QQ plot and how this graph can be used to check whether the empirical distribution is compatible with a known distribution.

The paper then describes fitting GPD severity distributions and a methodology for fitting and a précis of the different frequency distributions. The paper goes on to describe common risk load techniques including VaR, TVaR and Probability of Ruin. Everything is brought together in the IRR pricing model which projects all net cash flows over the term of the policy to derive the office premium from risk premium. There is a running example through the IRR pricing model allowing the reader to follow the thought process.