CAMRADATA Analytical Services

Geometry of Finance Momentum Conference 2011

First a bit on CAMRADATA

CAMRADATA:

600 Managers

4,023 products

85 Consultants

Over 300 Pension Scheme Trustees

Over 5,000 online reports run since June 2009

Manager League Tables

Fund Flows Reports

Preliminaries

- Starting points

Transformations

Clockwise rotation as a matrix

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \cos(\theta) & \sin(\theta) \\ -\sin(\theta) & \cos(\theta) \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

Scaling as a matrix

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} A & 0 \\ 0 & B \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

Clockwise rotation and scale as a matrix

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} A & 0 \\ 0 & B \end{bmatrix} \begin{bmatrix} \cos(\theta) & \sin(\theta) \\ -\sin(\theta) & \cos(\theta) \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

Actuarial Maths

Uncorrelated random number to correlated numbers

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \sqrt{\text{covariance matrix}} \times \begin{bmatrix} x \\ y \end{bmatrix}$$

Transforming uncorrelated random number to correlated numbers

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} L1 & 0 \\ L2 & L3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

Our maths is in fact a transformation

Transforming uncorrelated random number to correlated numbers

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \sqrt{EigenValue1} & 0 \\ 0 & \sqrt{EigenValue2} \end{bmatrix} inv\,[EigenVectors] \begin{bmatrix} x \\ y \end{bmatrix}$$

Covariance eigen vectors rotate and covariance eigen values scale
So covariance is a geometric transformation!

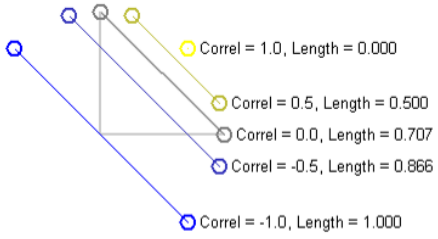
Simple example

0.5 in Y

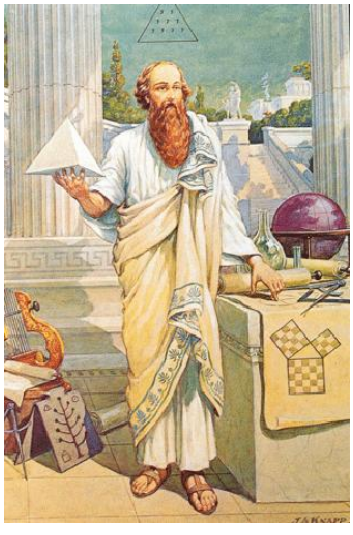
Distance = Risk Between X and Y in uncorrelated space

$Distance = \sqrt{(0.5^2 + 0.5^2)} = 0.7071$

0.5 in X

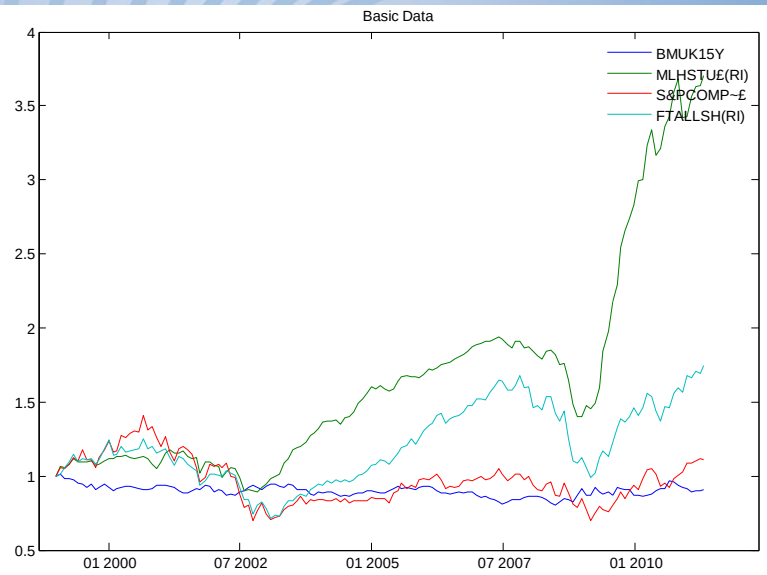


What is the influence of Pythagorean cults?

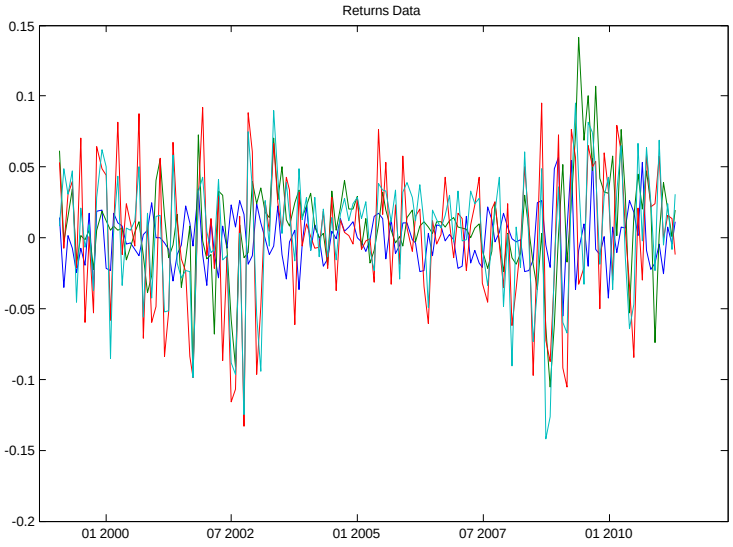


'Establish the triangle and the problem is two-thirds solved': Pythagoras

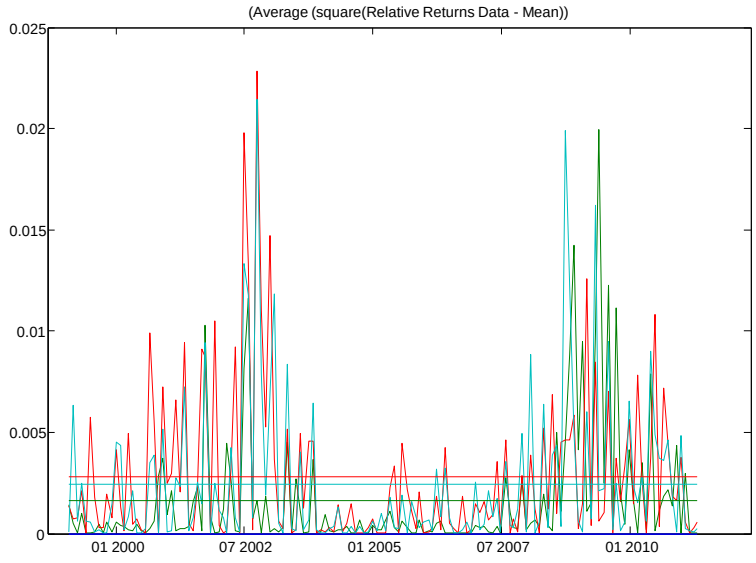
Can we think in triangles?



Normal view



Notion of distance



Link to dimensionality

How many assets do I really hold?

Dimensionality

How many dimensions are there?

[V,D] = eig(corrcoef(mxRets))

nDimensions = sum(min(D(:),1))

Correlation and dimensionality

Example Uncorrelated Assets?

[V,D] = eig(eye(3))

sum(min(D(:),1)) = 3

[V,D] = eig(ones(3))

sum(min(D(:),1)) = 1

Equation for correlation

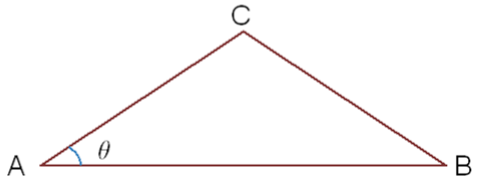
$$cov(X,Y) = \frac{1}{n}\sum(x_i - \overline{x})(y_i - \overline{y})$$

$$\rho_{(x,y)} = \frac{\sigma^2_{(x-y)} - \sigma^2_x - \sigma^2_y}{-2\sigma_x\sigma_y}$$

Cosine Rule

$$\cos(\theta) = \frac{(C-B)^2 - (C-A)^2 - (B-A)^2}{-2(C-A)(B-A)}$$

$$\rho_{(C-A,B-A)} = \frac{\sigma^2_{(C-B)} - \sigma^2_{(C-A)} - \sigma^2_{(B-A)}}{-2\sigma_{(C-A)}\sigma_{(B-A)}}$$



270° in a triangle?

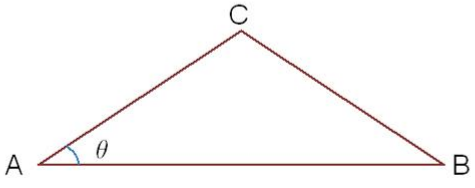
$\sum(\theta_i) = 180°$

$\theta_{(x,y)} = \cos^{-1}(\rho_{(x,y)})$

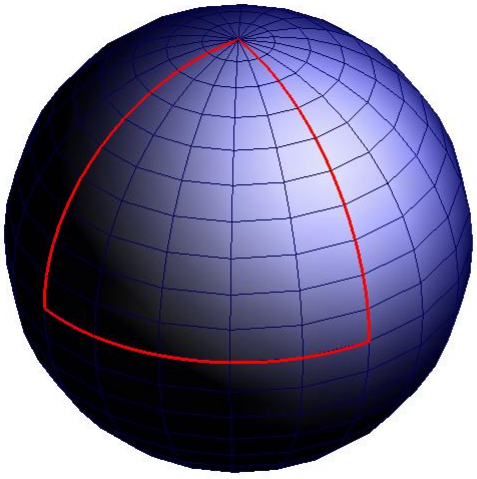
Uncorrelated assets have these angles:

	Asset 1	Asset 2	Asset 3
Assets 1		90	90
Assets 2			90
Assets 3			

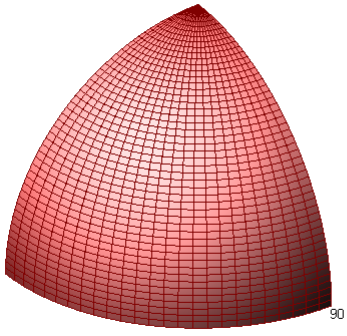
$\sum(\theta_i) = 270°$



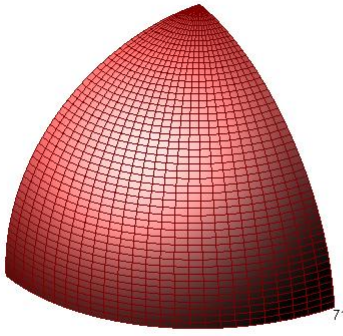
Not a flat triangle, a sphere



Uncorrelated

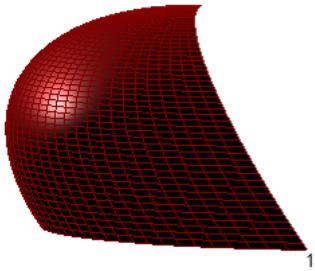


A bit of correlation



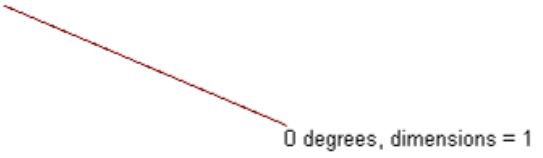
71.9 degrees, dimensions = 2.4

More….

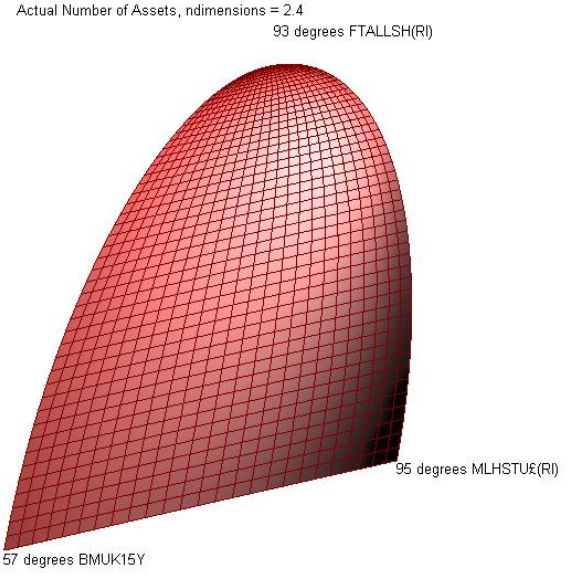


14.1 degrees, dimensions = 1.1

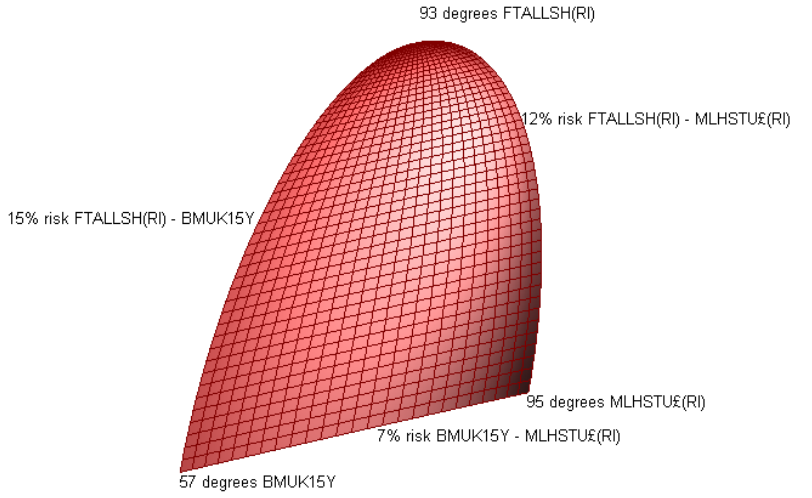
A lot of correlation



Real world visualisation I

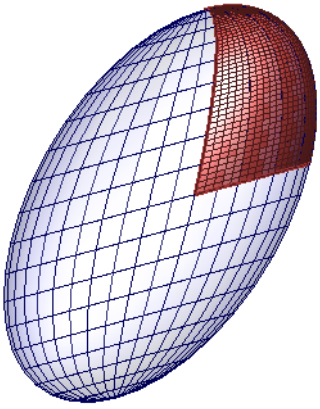


Real world visualisation II



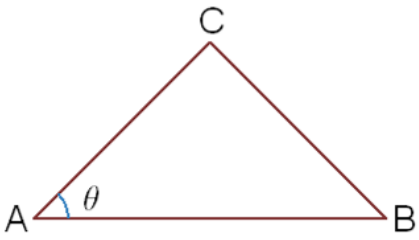
Risk is radius

Only positive?
Radius = risk



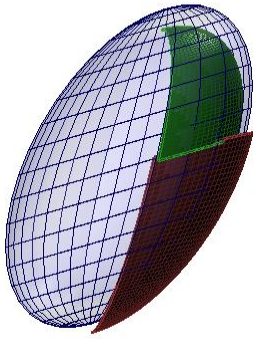
But this is an average triangle

$$\frac{1}{sum(\text{logic criteria})} \sum (x_{\text{logic criteria}} - \overline{x})(y_{\text{logic criteria}} - \overline{y})$$



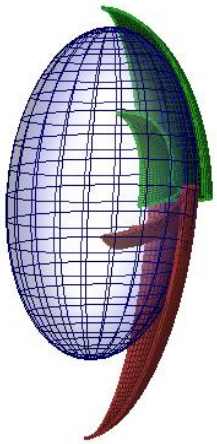
We can disaggregate triangle

Splitting out covariance:
Equity Market Up (Green), N dims = 2.4
Equity Market Down (Red), N dims = 2.5



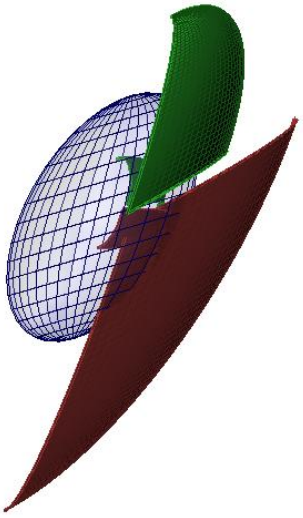
Quartiles of equity movements

Equity Market Q1(Green), N dims = 2.3
Equity Market Q2(Green), N dims = 2.3
Equity Market Q3(Red), N dims = 2.7
Equity Market Q4(Red), N dims = 2.4



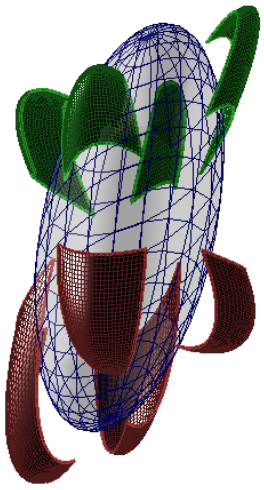
Extremes of equity movement

Equity Market Top 5 prctile N dims = 2.2
Equity Market Bottom 5 prctile N dims = 2.1

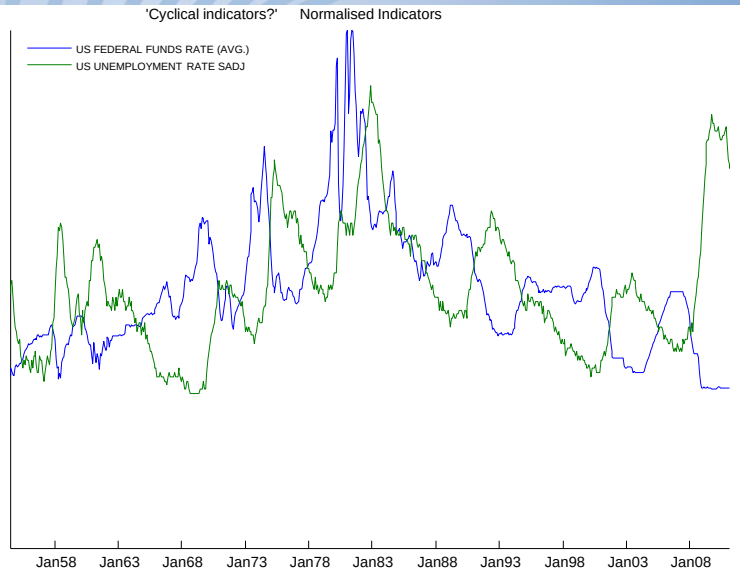


Joint top bottom quartiles

Jointly up/down Equity, High Yield, Government Fixed

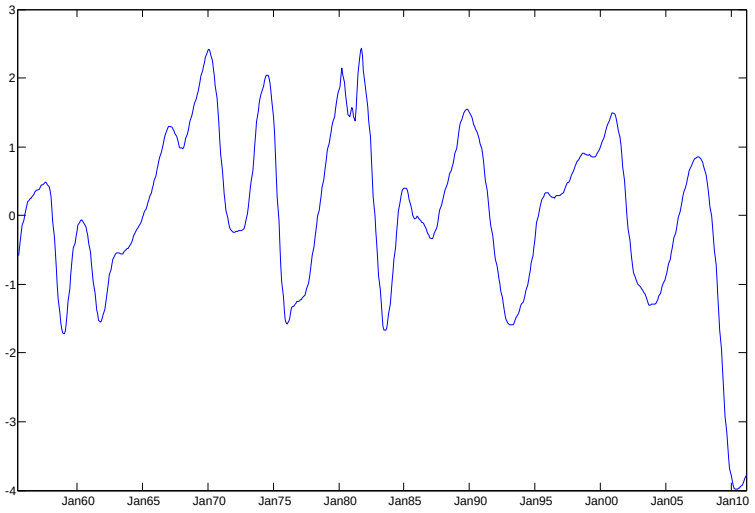


Economic cycle?



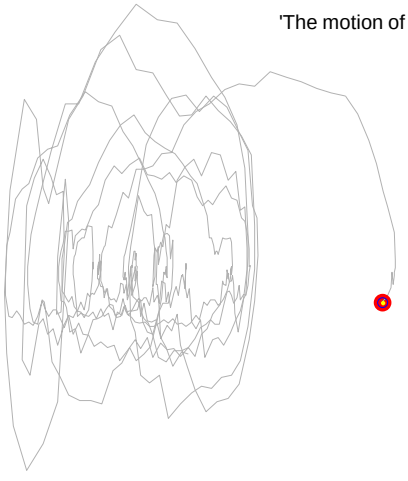
Establish the economic wave

Smoothed federal funds rate-unemployment



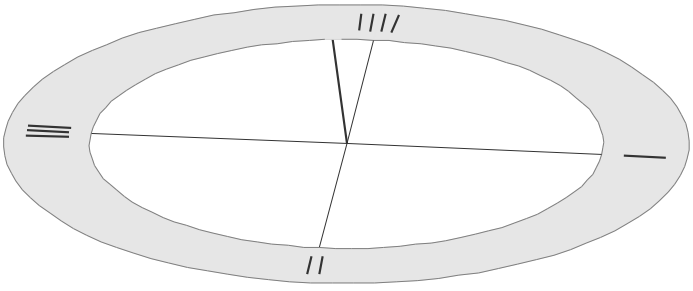
Plot as an economic cycle

'The motion of God is circular': Pythagoras

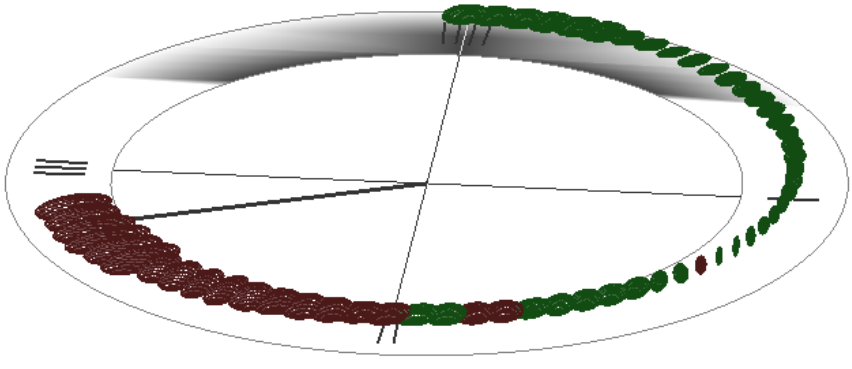


Can we disaggregate covariance by point on clock

Financial Space Time

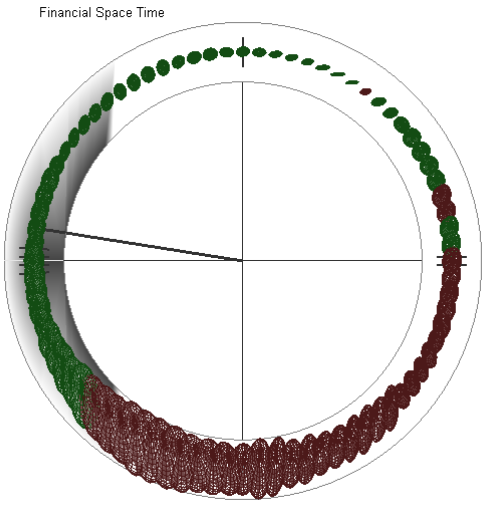


Economic cycle … Data is 97 to present …..moving round the circle



Green = equity up; Red = equity down

97 to June 2011 using this measure



This is a visualisation of financial space-time

- Round Balls = less correlation
- Rugby Balls = more correlation
- Big Balls = more risk