The Actuarial Profession
making financial sense of the future

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Jakub Borowicz

In a Capital Model Parameter Uncertainty Diversifies Away ?

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Introduction (1/2)

- Recently a lot of consideration was given to the phenomenon of parameter uncertainty (PU).
- The problem was brought up both by practitioners and regulators.

"(…) and therefore some prudence is expected to be included in the estimate in order to cover model and parameter uncertainties"

[Solvency II QIS3 Technical Specifications]

"(…) parameters themselves may not be fixed and might follow their own distribution. Sophisticated ICAs will therefore include some allowance for parameter uncertainty. "

[Lloyds 2006 ICA Guidance and Instructions]

Introduction (2/2)

- Most capital models include PU through the use of Bootstrapping Techniques (and Bayesian) to
  - When parameterising or modelling Reserving Risk
  - less often capital models introduce parameter uncertainty to Underwriting Risk and other risk types.
- It is commonly accepted that PU brings more variability to the risk distributions, and thus has a tendency to increase in the capital requirements especially when parameters are estimated from small samples.
- In consequence it may not be desirable to "experiment" with this phenomenon, and simply state it as your model limitation ?

Goals

- What really matters is to capture the whole company risk profile, as correctly as possible, then apply a prescribed risk measure and risk tolerance and hope that a resulting capital requirement is sensible
- Practitioners know that it all comes down to diversification or dependencies, and to use of judgement to which the capital models are most sensitive
- In this presentation we will try to answer the question:
  - Does the parameter uncertainty diversifies away when estimating capital requirements and under what assumptions.
  - What is the impact of dependencies on the statement above.
  - How to "use" parameter uncertainty to help us make better founded expert judgements

What is Parameter Uncertainty

- Let's generate two Poisson samples with $\lambda = 4$:

								Average
3	1	5	5	2	3	8	7	4.25
5	6	5	8	5	0	7	2	4.75

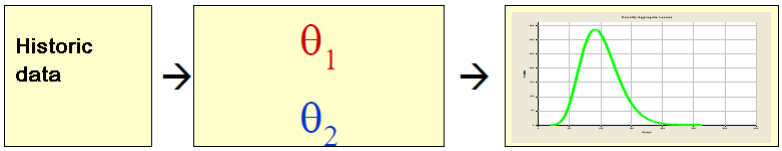
- In the first case parameter estimation would give λ_{MLE}= 4.25 and in the second λ_{MLE} = 4.75
- Given the finite sample we can't estimate the true parameter

Methods of Assessing Parameter Uncertainty

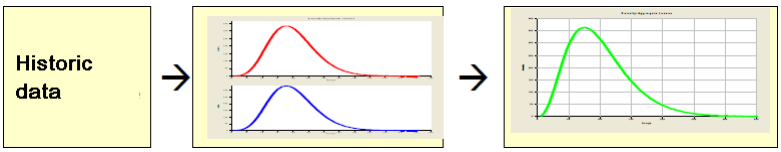
- "classical" statistical methods such an assumption on the asymptotic normality of maximum likelihood estimates
- Bayesian methods (involving sophisticated sampling algorithm and MCMC)
- Bootstrapping
  - Parametric (fitting parameters)
  - non-parametric bootstrapping (sampling with replacement)

Effect of PU:

- Without PU



- Including PU



What do we mean by using PU in a model ?

- Estimate parameter of a single distribution that “we don’t know” and include it as distribution of parameters of the original distribution to generate a posterior one. We would repeat this for all the distributions in the model to come up with the new risk profile.
- Or anwser a question:What is the distribution of my mean, percentile ?
- Use bootstrapping embeded in the reserve risk method to come up with parameters for each origin period and then use LogNormal ?
- What is the distribution of the price of reinsurance contract?
- What is the distribution of the capital requirement (not the capital requirement with the new posterior distributions)

Parametric Bootstrapping used differently (1/4)

First, let’s consider a simple model:

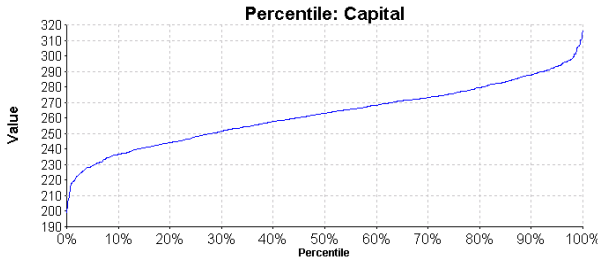
- ***m*** identical lines of business with losses distributed using the same loss distribution ***D***
- All “correlated” using normal dependency with correlation matrix ***R***
- We have ***n*** historical data points for each LoB:
  - to estimate parameters of a LogNormal distribution
  - and correlation matrix between ***R***
- We calculate capital using VaR(99) on the sum of losses from all LoBs and we come up with ***C***

Parametric Bootstrapping used differently (2/4)

- Let's generate a sample to which we will be estimating the parameters and in consequence Capital C.
  - Distribution of claims : Exponential(2)
  - Number of Lines of Business : 40
  - Number of Years : 25
  - Correlation (40x40) between all pair of LoBs : 50%
- Theoretically for this example the capital requirement VaR(99) on the aggregate loss should be equal to **262**.
- We use the method of moments for fitting distribution parameters and empirical correlation to fit the correlation matrix (allowing for different correlation parameters).
- Depending on the random sample we can get **250, 260, 232,** etc…

Parametric Bootstrapping used differently (3/4)

- The graph below shows the result of this estimation for 10 000 different pre generated samples:



- Depending on how "lucky" we were with the original sample we can get closer or further away form the true theoretical value of 262

Parametric Bootstrapping used differently (4/4)

In the previous example:

- we have treated the **capital** as a random variable.
- We have used a parametric bootstrapping to obtain pseudo multivariate samples.
- To each sample we have fitted distribution of parameters and correlations
- For each set of parameters we have computed the capital requirement, this gave us a distribution of that requirement.
- Now we can compute any statistic on that distribution, i.e. CoV(Capital Requirement) = 20%

A "classical" PU analysis would consider

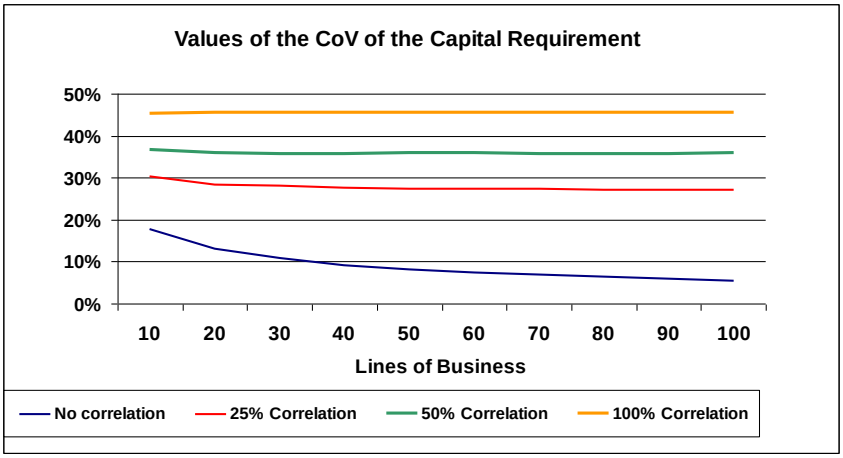
- parameters of individual distributions and their correlation as a random variable
- And the capital requirement would be calculated only once (yielding a higher figure)

Does an increasing number and size of LoBs decrease error in estimation of the capital ? (1/2)

- The previous example was very computational intensive, it involved calculating capital requirement of 10 000 capital models ! (Igloo Enterprise)
- Let's now create 10 000 capital models for each permutation of :
  - Number of years : {5,10,25,50}
  - Number of LoBs : {20,40, …,480,500}
  - Correlation between LoBs : {0,25%,50%,100%}
- Now we are really asking for trouble…
- … we need 4 * 25 * 4 * 10000 capital models with 10000 simulations each, that is 1 bilion simulations for each of the 40 PCs I had.
- The Solvency II stopped for the moment ☺

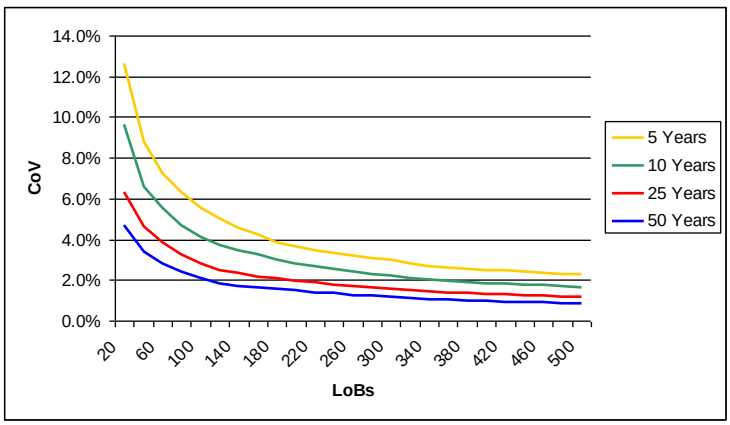
Does an increasing number and size of LoBs decrease error in estimation of the capital ? (1/2)

- Answer: for this model Yes if there is no correlation



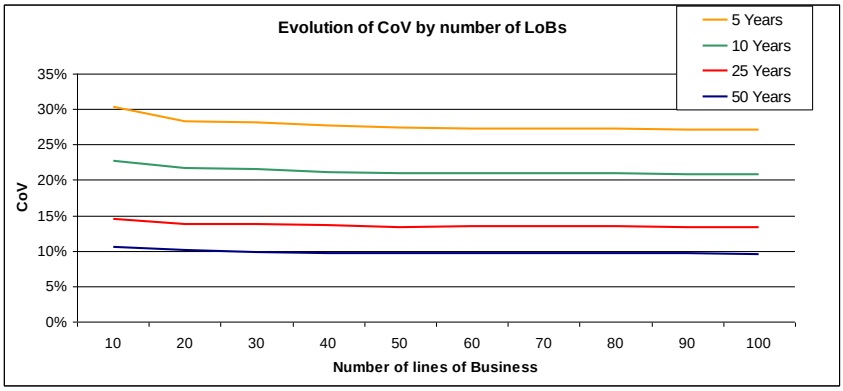
When LoBs are independent

- With different number of years, we obtain the following results for the CoV of the capital:



When LoBs are dependent (correl = 25%)

- With different number of years, we obtain the following results for the CoV of the capital:



- Also we note that both graphs show that there is more volatility in the error of capital if we have smaller datasets

Theory backing the 100% and 0% Correl

- When the correlation is 100%, the LoBs are said to be comonotonic, under this assumption, we have to following result for the Value-at-Risk (Capital) (see D.VINCKE .2003) :

$$VaR_\alpha\left(\sum_i LoB_i\right) = \sum_i VaR_\alpha(LoB_i)$$

- We can further demonstrate that CoV stays constant :

$$CoV\left(VaR_\alpha\left(\sum_i LoB_i\right)\right) = CoV\left(VaR_\alpha\left(LoB_1\right)\right)$$

- For 0% correlation CoV tends to 0, this proof is bit more difficult involving Central Limit Theorem.

Theory backing the Correl $\in$ (0%,100%)

- The behavior of a sum of dependant risks (but not comonotonic) is complex and it is continuous subject of actuarial research.
- For a set of identically distributed dependant random variables there are results of Mario V. Wutrich, combined with further study of P.Barbe, A.L.Fougere & C.Genest showing that:

$$VaR_\alpha\left(\sum_i X_i\right) = q_{\alpha/\Delta} \times VaR_\alpha(X_1)$$

- This interesting result shows that the behaviour of the aggregate capital can be explained by the behaviour of one line of business and a constant factor q. The behaviour of the constant q is complex could explain our interesting results
- But why doing Maths if we have use Monte Carlo ?

Intermediate Conclusions

- When estimating parameters we are not always under or over estimating
- If we don't get the correlation right – we will have an error in estimating capital requirement
  - no mater how many marginal parameters we are estimating,
  - no mater how many lines of business we split business into
- This is consistent when we change:
  - Distributions (LogNormal, Normal etc…)
  - Dependency structures (Normal, Gumbel)
- Limitations: We are not considering Systemic Risk, Inflation etc… It's a very simple model

Correlation, Size & Granularity & Systemic Risk

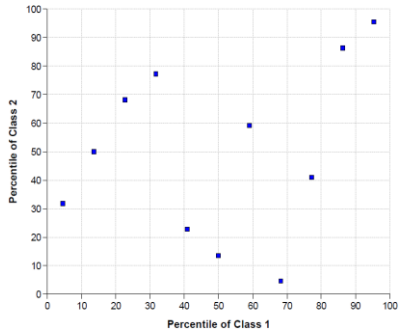
- The more we make the portfolio granular (introducing more lines of business to parameterise):
  - the same estimation error we are making when estimating the capital requirement
  - Are we getting the same capital requirement in practice though ?
  - No because we are making more judgements
- So maybe Correlation can be regarded as a Systemic risk ?

How to defend judgement when estimating Dependency ?

- Benchmarks, back working from what we believe is right etc…
- Practical Method 1:
  1. Regroup the business to different “buckets”
  2. calculate correlations & risk parameters and overall risk profile
  3. Compare the aggregate risk profile of each different “bucketing”
- Practical Method 2:
  - Let PU work for your advantage…

How to defend judgement when estimating Dependency ?

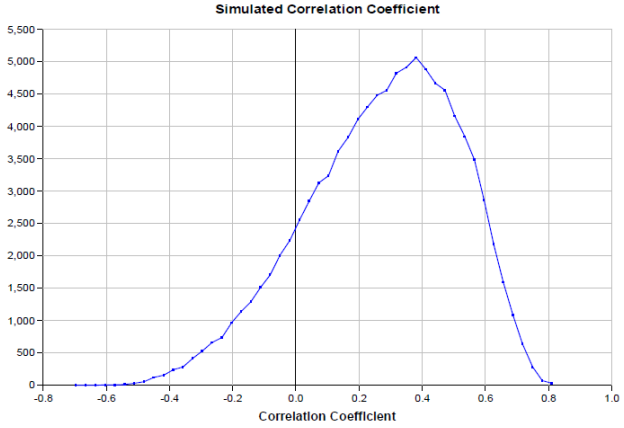
- Let's consider the following Scatter plot of losses from 2 LoBs (for 11 years):



- Calibration of Dependency gives: 38%.

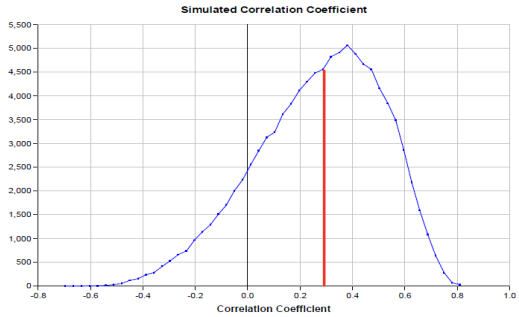
How to defend judgement when estimating Dependency ?

- Let's simulate 10000 samples with 38% Correlation and estimate correlation from each simulated sample :



How to defend judgement when estimating Dependency ?

- When we then overlay our judgement…

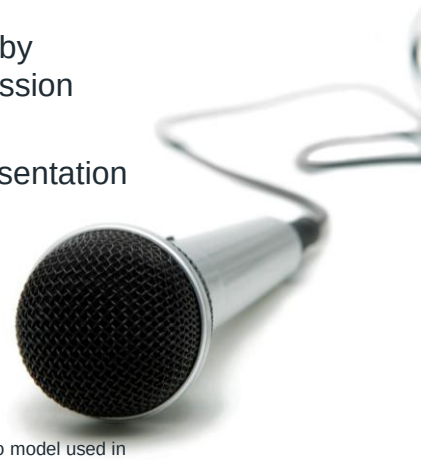


- We have a justification that this "falls" in the range of possible correlation outcomes
- it's not much but "ticks a part of a box"

Questions or comments?

Expressions of individual views by members of The Actuarial Profession and its staff are encouraged.

The views expressed in this presentation are those of the presenter.



Thanks to Paul Carricano for help in preparing the Igloo model used in this example and for background research