Institute and Faculty of Actuaries

Mortality models: comparison and application in old-age populations of selected economies

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Agenda

- Purpose
- Data
- Models
- Methodology
- Results
- Conclusion

Purpose

Bring together mortality models from different regions:

“UK – Europe” and “Asia Pacific – North America”

- Lee-Carter (LC)
- Renshaw-Haberman (RH)
- Cairns-Blake-Dowd (CBD) [first version]
- Booth-Maindonald-Smith (BMS)
- Hyndman-Ullah (HU)

Assess efficacy in old-age male and female populations of developed and emerging economies across Europe and Asia

- United Kingdom (developed, Europe)
- Poland (emerging, Europe)
- Japan (developed, Asia)
- Taiwan (emerging, Asia)

Purpose

Project mortality rates stochastically and quantify financial impact of mortality uncertainty

- Get distribution of outcomes, consider best estimate, 70% and 95% confidence intervals

Understand trends across selected populations

- Differences in forecasted rates for developed and emerging economies

Data

- Human Mortality Database (www.mortality.org)
- Male and female
- Age group 65-69, 75-79, 85-89 and 65-89
- Varying histories across the four economies
  - UK: 1947 – 2009 (63 years)
  - Poland: 1958 – 2009 (52 years)
  - Japan: 1947 – 2009 (63 years)
  - Taiwan: 1970 – 2009 (40 years)

Models

- Five models considered are extrapolative in nature
- Allow stochastic projections to be made
- Typically involve terms related to age x, period t and cohort c

1. Lee-Carter (LC)

   $$\text{Log } m(t,x) = A_x^{(1)} + A_x^{(2)}P_t^{(2)} + E(t,x)$$

2. Renshaw-Haberman (RH)

   $$\text{Log } m(t,x) = A_x^{(1)} + A_x^{(2)}P_t^{(2)} + A_x^{(3)}C_c^{(3)} + E(t,x)$$

Models

3. Cairns-Blake-Dowd (CBD) [first version]

   $\text{Logit } q(t,x) = P_t^{(1)} + (x - \bar{x})P_t^{(2)} + E(t,x)$

4. Booth-Maindonald-Smith (BMS)

   $\text{Log } m(t,x) = A_x^{(0)} + \sum_{i=1}^{n} A_x^{(i)} P_t^{(i)} + E(t,x)$

5. Hyndman-Ullah (HU)

   $\text{Log } m(t,x) = \mu(x) + \sum_{i=1}^{n} B(t,i)D(x,i) + \sigma(t,x)E(t,x) + E_2(t,x)$

Methodology

- For each population, performed 3 types of analyses
  - A: In-sample testing (over full period of data available, compared model-fitted mortality rates to actual for each age group)
  - B: Out-of-sample testing (fitted model over subset of full period, compared forecasted mortality rates to actual over remaining 20 years for each age group)
  - C: Out-of-sample testing for cohort aged 67 in 1990 (fitted model over subset of full period, compared forecasted rates to actual over remaining 20 years to 2009)
- Metrics used
  - A: Which model maximised the Bayes' Information Criterion, generated the smallest residuals and absolute residuals (used 5% p-value)
  - B: Which model generated the smallest residuals and absolute residuals
  - C: Graphical comparison of forecasted to actual cohort mortality rates

Methodology

- In selecting the best model for each of the eight populations, more weight placed on model's forecasting than its fitting ability
- Model selection criteria (in descending order of importance)
  - Analysis B: closeness of forecasted to actual rates for all age groups from 1990 to 2009
  - Analysis C: closeness of forecasted to actual rates for cohort aged 67 from 1990 to 2009
  - Analysis A: closeness of fitted to actual rates for all age groups over full period of data available
- Application of selected model
  - Fit model over full period of data
  - Make stochastic mortality projections over 20 years for cohort aged 67 in 2010
  - Analyse distribution of mortality projections
  - Financial impact quantified via price of theoretical 20-year level annuity

Selection process – UK Males

- Analysis A: BIC = Log-likelihood – ½ x number of parameters x log(number of observations)

Model	LC	RH	CBD	BMS	HU
BIC	-14266	-10374	-12973	-4939	-12978

- BMS model only uses data periods meeting its assumption of linearity of mortality improvement (only 21 of 63 available years), so emerges with artificially good BIC measure
- Once it is excluded, the RH model fares best under the BIC

Selection process – UK Males

- Analysis B: out-of-sample testing based on calibration from 1947 to 1989, forecast over 1990 to 2009
- Residuals shown in scientific notation (x10^{-4}), p-values greater than 5% in brackets

Model	65-89	65-69	75-79	85-89
LC	131	67	146	155
RH	-4 (26%)	47	16	-109
CBD	146	71	139	239
BMS	100	32	93	169
HU	154	72	151	242

- RH model generated lowest residuals, and is associated with p-value greater than 5%

Selection process – UK Males

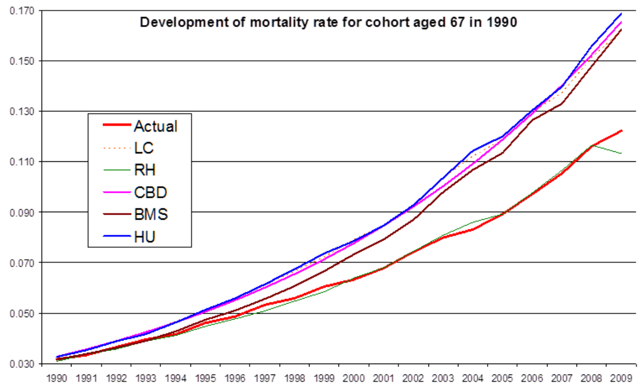
- Analysis B: out-of-sample testing based on calibration from 1947 to 1989, forecast over 1990 to 2009
- Absolute residuals shown in scientific notation (x10^{-4})

Model	65-89	65-69	75-79	85-89
LC	134	67	146	167
RH	54	49	32	116
CBD	146	71	139	239
BMS	101	33	93	173
HU	154	72	151	243

- When absolute residuals are considered, again the performance of the models are similar

Selection process – UK Males

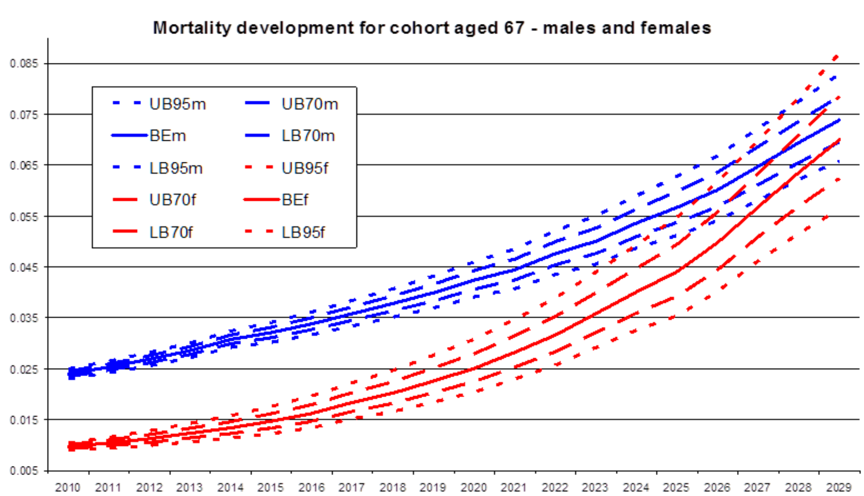
- Analysis C: out-of-sample testing for cohort aged 67, based on calibration from 1947 to 1989, forecast over 1990 to 2009



- RH model performed very well up to 2009. Underestimate in 2009 largely due to $A_{86}^{(2)}P_{2009}^{(2)}$ term, with $P_{2009}^{(2)}$ expected to be low and $A_{86}^{(2)}$ to be high (greater sensitivity to period effect)

Results – UK

- Stochastic mortality projections for UK males (RH model) and females (BMS model)

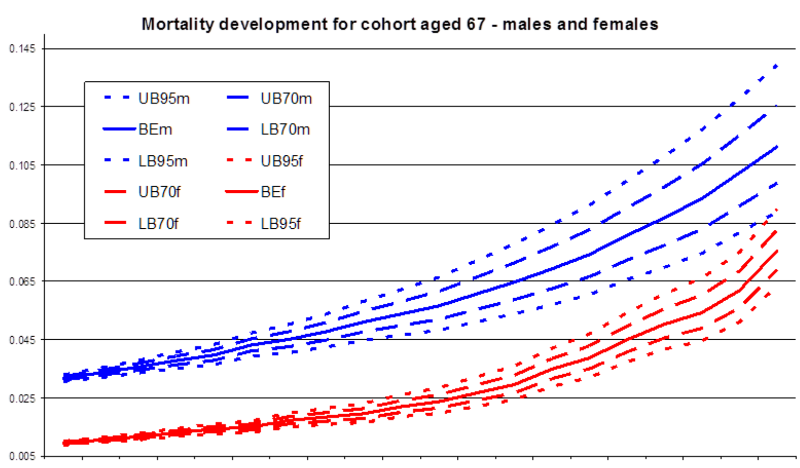


Results – UK

- Gap between male and female mortality expected to narrow, with males experiencing greater mortality improvements over time
- Consistent with research showing UK males to be smoking less, taking up less hazardous occupations
- More uncertainty around female rates (wider confidence intervals), possibly down to model selection (BMS model has more parameters than RH model)
- Confidence bands not perfectly symmetrical (wider in higher mortality part): log mortality effect
- Financial impact of mortality uncertainty not large (price variability of 2% for males and 3% for females)

Results – Poland

- Stochastic mortality projections for Polish males (BMS model) and females (RH model)

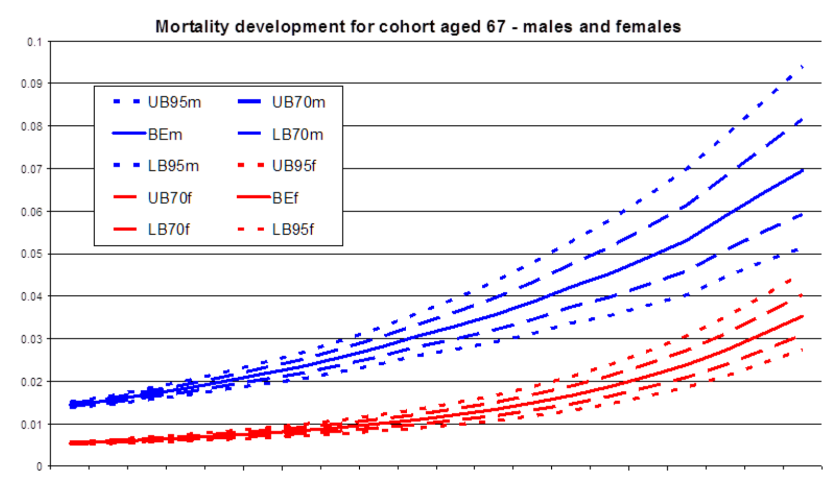


Results – Poland

- Unlike the UK, gap between male and female mortality not expected to narrow noticeably
- Uncertainty around male mortality wider than around female
  - Historical experience
  - Model chosen
- Less smoothness in projected mortality rates
  - Historically, mortality rates only declined after 1990. Before then, in some years the rates actually increased. More variability in the history compared to the UK.

Results – Japan

- Stochastic mortality projections for Japanese males and females (BMS model)

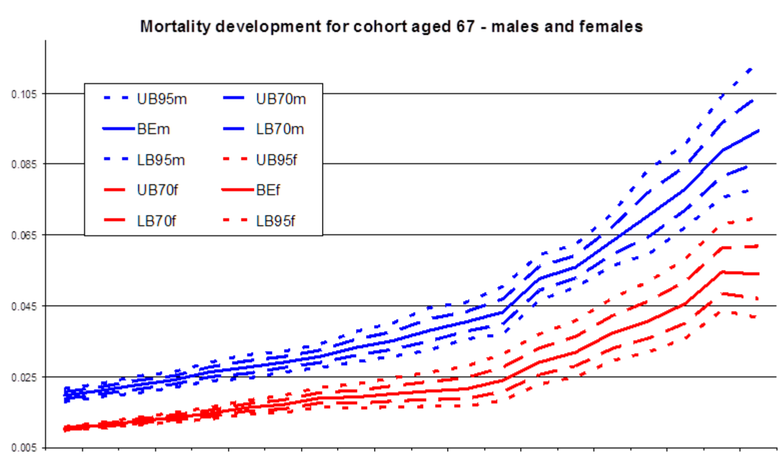


Results – Japan

- Unlike the UK and similar to Poland, gap between male and female mortality also not expected to narrow
- Uncertainty around male mortality also wider than around female
  - Historically saw more variability in male rates
  - Smoking rates amongst males remain much higher (34% vs 11% in 2012)

Results – Taiwan

- Stochastic mortality projections for Taiwanese males (HU model) and females (BMS model)



Results – Taiwan

- Unlike the UK and similar to Poland and Japan, gap between male and female mortality also not expected to narrow
- More jagged rates being forecasted
  - Historical mortality experience not always smooth
  - Only after the late 1970s (with fast economic growth) did rates decline steadily
  - Fewer years of data available
- Uncertainty around male mortality also wider than around female
  - Historically saw more variability in male rates
  - HU model has more parameters than BMS

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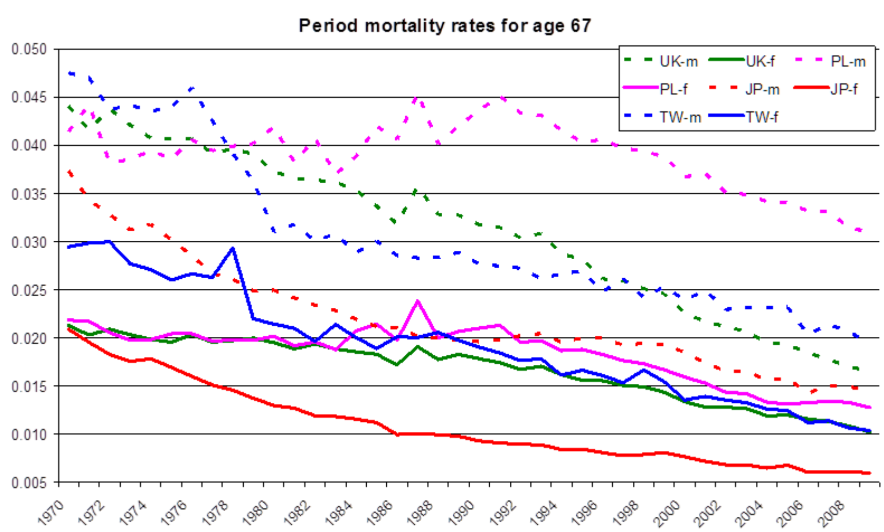
Historical mortality rates



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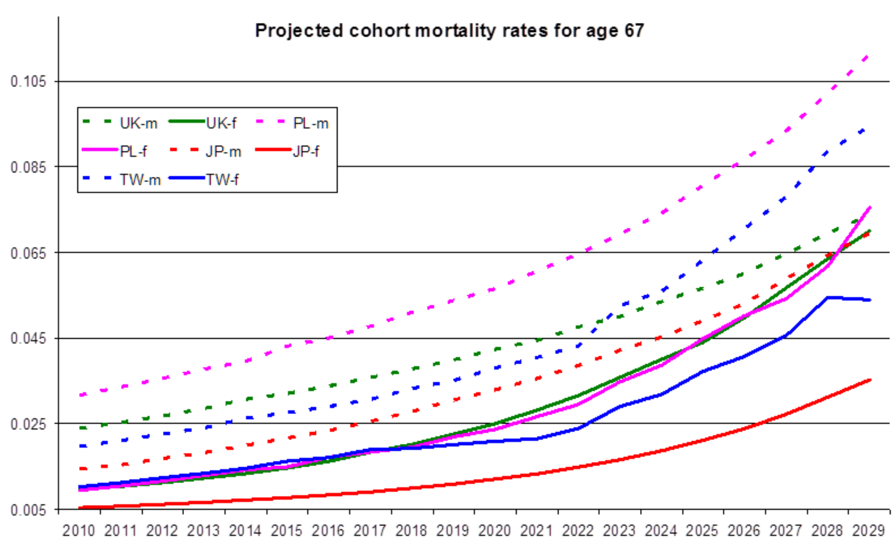
Selected Models

- Selected models for eight populations

Model	Male	Female
UK	RH	BMS
Poland	BMS	RH
Japan	BMS	BMS
Taiwan	HU	BMS

- BMS model did best in 5 of the 8 populations, and reasonably well in other populations too
- Where the BMS model did well, the HU model also did reasonably well, for example in emerging economies with more complex mortality patterns requiring more parameters
- RH model did well where a clear cohort effect exists

Best estimate projected rates



Theoretical annuity prices

- Theoretical prices of 20-year level annuity, at interest rate of 1.5% for a 67-year old

Annuity Prices	95% LB	70% LB	BE	70% UB	95% UB
UK males	12.6	12.5	12.3	12.2	12.1
Poland males	11.6	11.4	11.1	10.9	10.6
Japan males	13.9	13.7	13.5	13.2	13.0
Taiwan males	13.1	12.9	12.7	12.5	12.3
UK females	14.7	14.5	14.3	14.0	13.8
Poland females	14.6	14.5	14.3	14.1	13.9
Japan females	15.8	15.7	15.6	15.4	15.3
Taiwan females	14.8	14.6	14.4	14.2	14.0

LB = Lower Bound; BE = Best Estimate; UB = Upper Bound

- A higher price suggests a longer life expectancy in old age, as interest rate used is the same for all populations

Financial impact of longevity uncertainty

- Variability in annuity prices relative to best estimate

% Price difference	95% LB	70% LB	70% UB	95% UB
UK males	2.1%	1.1%	-1.1%	-2.1%
Poland males	4.4%	2.4%	-2.4%	-4.6%
Japan males	3.3%	1.8%	-1.9%	-3.6%
Taiwan males	3.1%	1.7%	-1.7%	-3.4%
UK females	2.9%	1.6%	-1.7%	-3.3%
Poland females	2.5%	1.3%	-1.4%	-2.8%
Japan females	1.5%	0.8%	-0.9%	-1.8%
Taiwan females	2.6%	1.4%	-1.5%	-3.0%

LB = Lower Bound; BE = Best Estimate; UB = Upper Bound

- A greater variability means more uncertainty in mortality outcomes
- If interest rate is lowered, extent of differential in annuity prices due to mortality uncertainty widens

Conclusion

- Male and female mortality rates expected to converge in the UK
- BMS model worked particularly well in female and Asian populations
- Females in selected emerging economies expected to outlive males in developed ones
- Less mortality uncertainty expected for
  - Females than males (2.5% vs 3.5% in price difference)
  - Developed than emerging economies (3% vs 3.5%)
  - Asian than European economies (just below 3% vs just above 3%)
- Populations with longer life expectancies more sensitive to changes in interest rates, particularly to further decreases in rates

Room for further research

- Room for further research:
  - Extend analysis to other economies based on this framework
  - Consider later versions of the CBD model

Questions Comments

Expressions of individual views by members of the Institute and Faculty of Actuaries and its staff are encouraged.

The views expressed in this presentation are those of the presenter.