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Setting Long Term Interest Rate Assumptions

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19 June 2019

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Introduction

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Long Term Interest Rate Forecasting

Setting long term interest rate assumptions is very challenging

- What do we even mean by long term? (should 30 years be different from 50 or 100 years?)
- Many forecasting methodologies at our disposal (different results)
- To what extent can we use history as a guide?
- Not enough data to adequately backtest long term targets (can we extrapolate the efficacy of shorter term back tests?)
- So much uncertainty (economic, social, geo-political etc.)

Once targets are set can they be incorporated into a parsimonious stochastic modelling framework?

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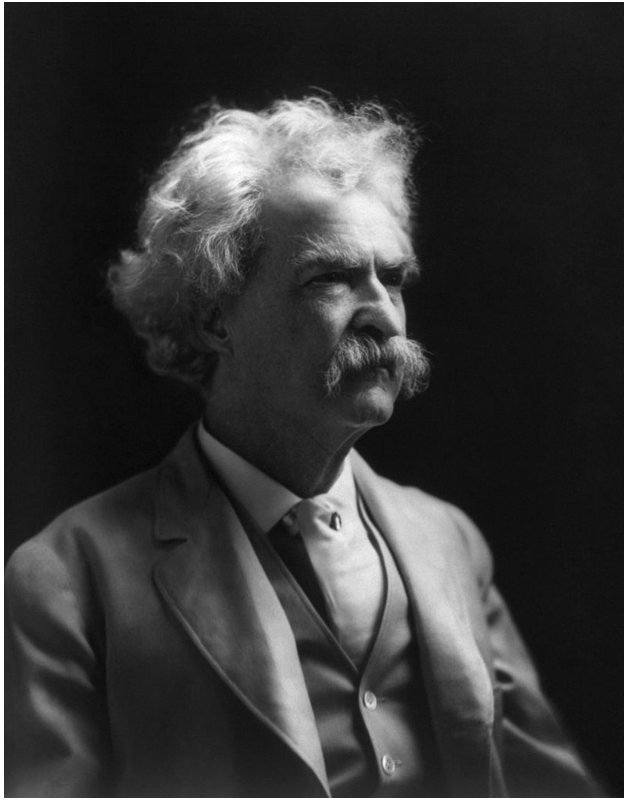
Long Term Interest Rate Forecasting



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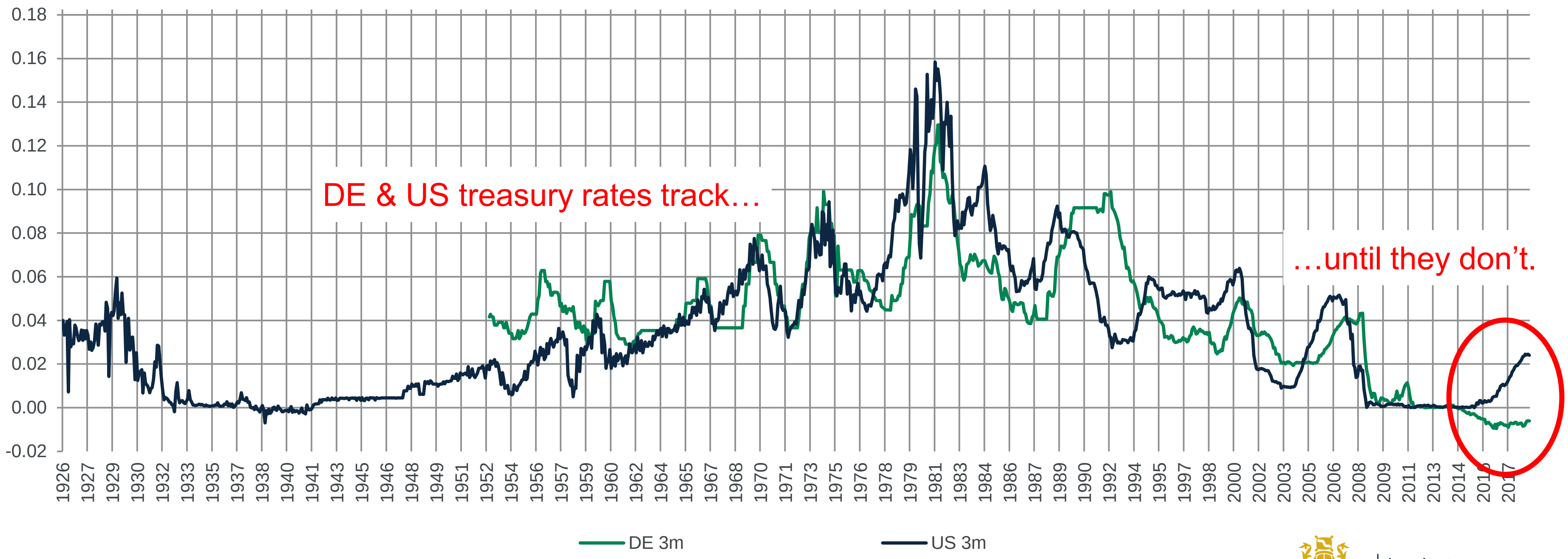
German and US 3-month Treasury Rates



Source: Conning Inc./Bloomberg

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Long Term Interest Rate Forecasting

We will compare two possible methodologies

- Use forward curve metric
- Econometric forecasts

We will consider specifically the forecasts of UK Gilt Yields

- 2018 forecast start date
- Consider a 5 and 30 year forecasting time horizon

Finally we consider whether it is possible to implement such forecasts in a parsimonious stochastic interest rate model

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Forecasting Methods

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Forward Rate

Forward rates are interest rates that can be locked in today for an investment in a future time period.

Let's denote F(t, T, S) the simply compounded forward interest rate prevailing at time t for the expiry T > t, and maturity S > T .

$$F(t, T, S) = \frac{1}{S-T}\left(\frac{P(t,T)}{P(t,S)} - 1\right)$$

where P(t, T) is the T–maturity zero coupon.

Equivalently the continuously compounded forward rate F(t, T, S) is targeted

$$F(t, T, S) = \frac{1}{S-T} Log\left(\frac{P(t,T)}{P(t,S)}\right)$$

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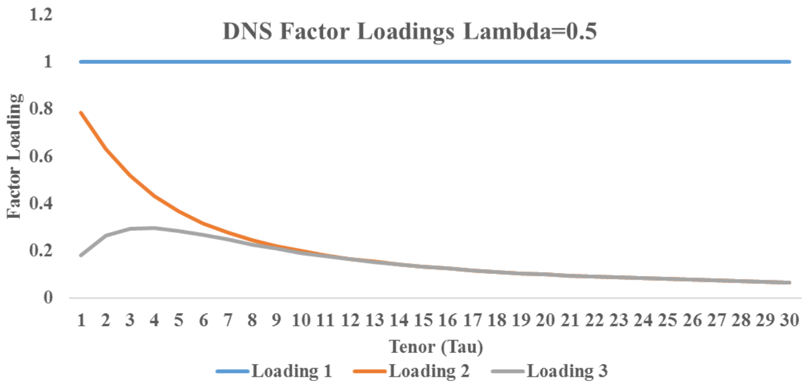
Dynamic Nelson Siegel (Basic Idea)

- Dynamic Nelson Siegel Model (DNS) is a popular framework for analysing and forecasting interest rates
  - Backed by a large body of research (e.g. Diebold and Li 2005/2006)
  - Outperforms other methods on data from multiple economies
  - Parsimonious, intuitive, relatively simple to estimate
- Three factor model
- Fix λ and fit β's to historical yield curves (OLS)
- For example with Gilt yields.....

$$y_t(\tau) = \beta_{1,t} + \beta_{2,t}\left[\frac{1-\exp(-\lambda_t\tau)}{\lambda_t\tau}\right] + \beta_{3,t}\left[\frac{1-\exp(-\lambda_t\tau)}{\lambda_t\tau} - \exp(-\lambda_t\tau)\right] + \varepsilon_t.$$

Level | Slope | Curvature



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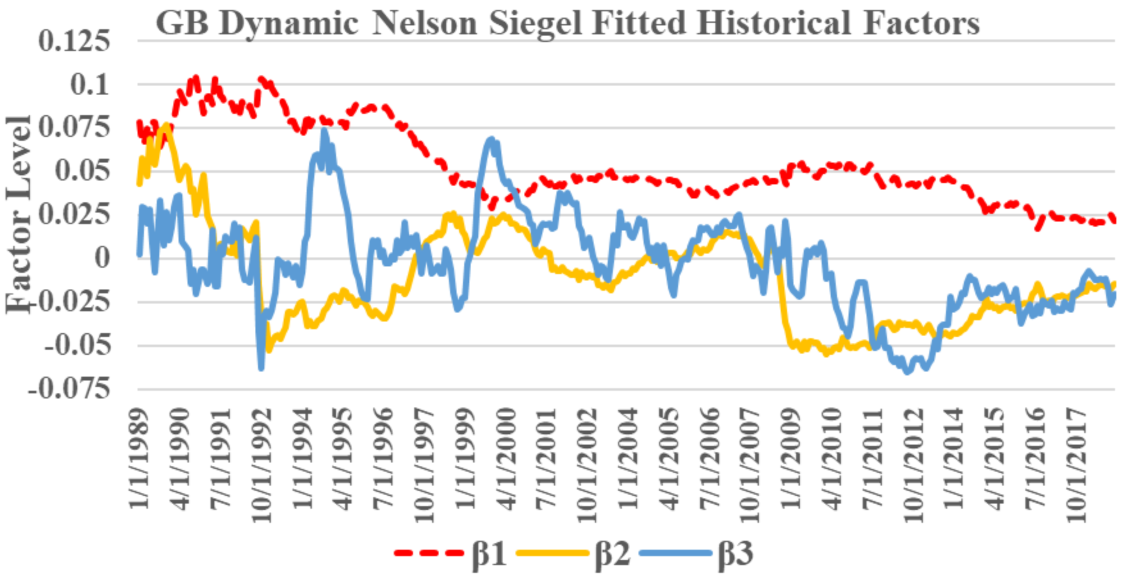
Dynamic Nelson Siegel (Basic Idea)

- Factors β are dynamic
- B1,t closely follows the yield levels as expected
- “Shape” factor movements track term structure movements
- Build ARIMA model to forecast future yields curves



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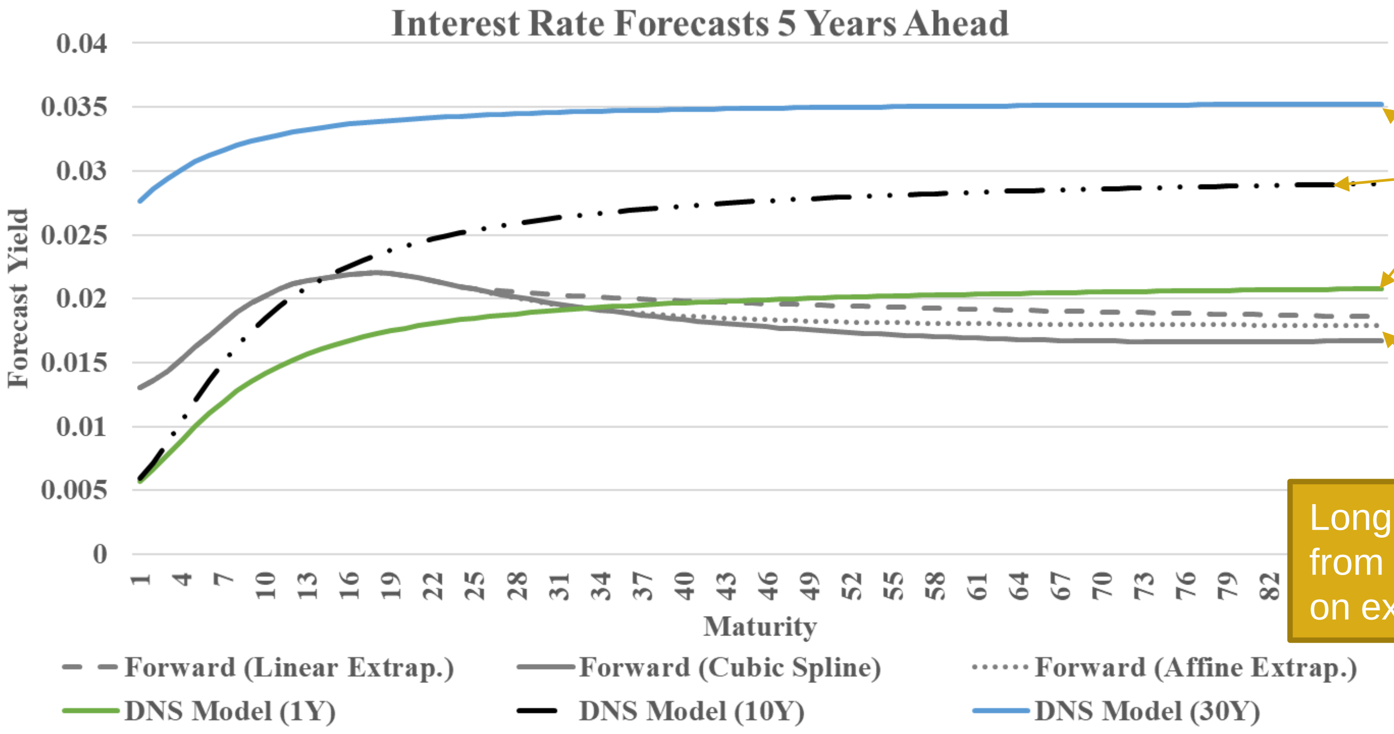
Results and Conclusion

2023 Forecasts – Medium Term



DNS Forecasts dependent on estimation time window used (1,10 or 30 years)

Long maturity forecasts from forwards dependent on extrapolation technique

Source: Conning Inc.

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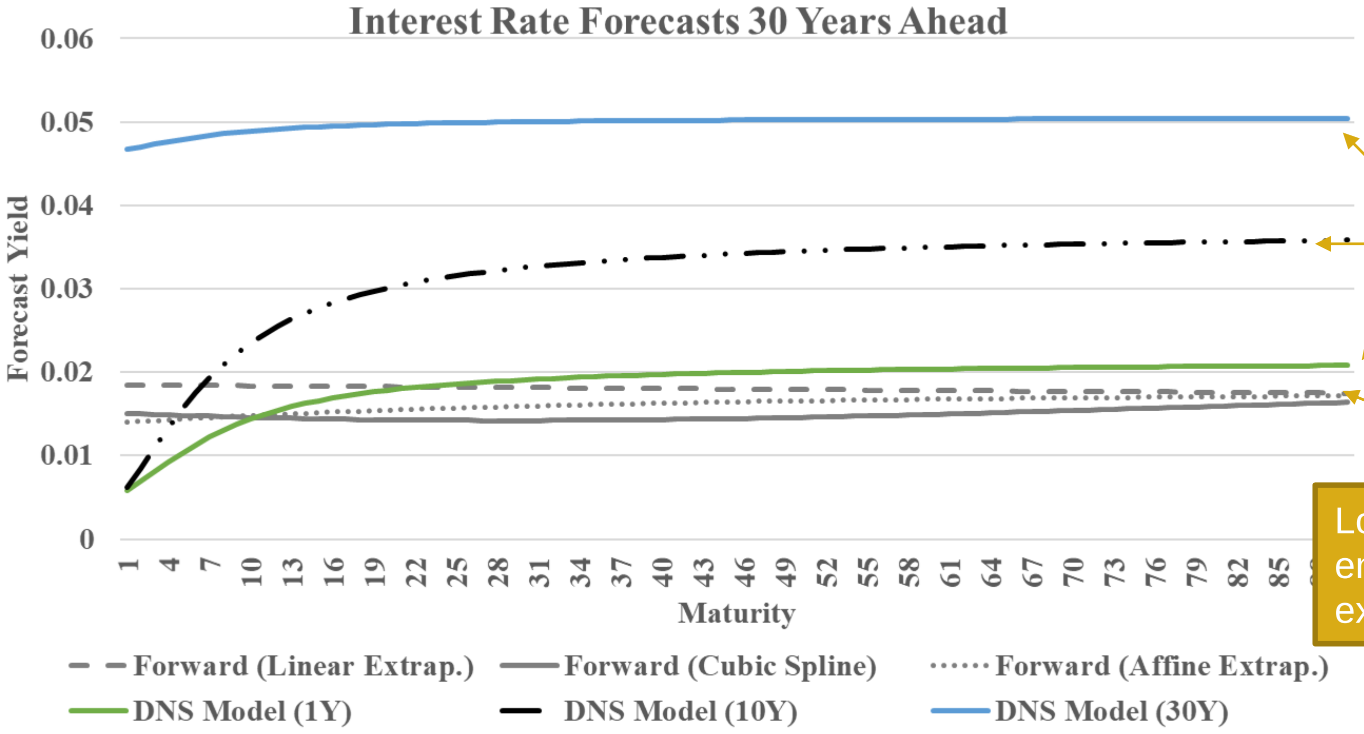
2048 Forecasts – Long Term



DNS Forecast window dependent but shapes are reasonable. But which is most appropriate?

Long term forward forecasts entirely dependent on extrapolation technique!!!!

Source: Conning Inc.

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Practical Considerations - Calibrating to Forecasts

Is it possible to incorporate these forecasts into a stochastic simulation?

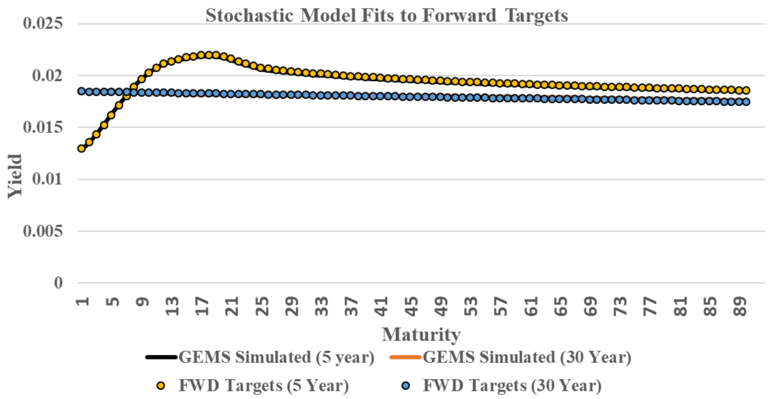
Use an extended 3 Factor CIR model

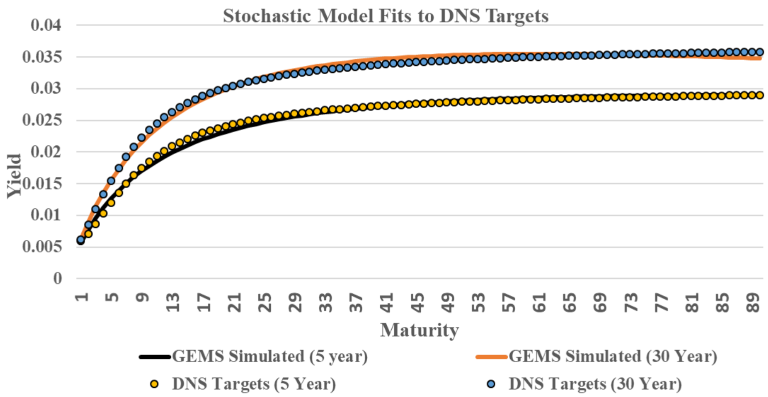
$$dx_i(t) = [\vartheta_i - \kappa_i x_i(t)]\, dt + \sigma_i \sqrt{x_i(t)} dW_i(t)$$

$$\mathbb{E}_0^Q \left[\exp\left(- \int_t^T r(\tau) d\tau \right) \right] = e^{\left(-\int_t^T l(s)ds + \vec{A}(\tau) + \vec{B}(\tau)\cdot \vec{x}(t)\right)}$$

Requirements;

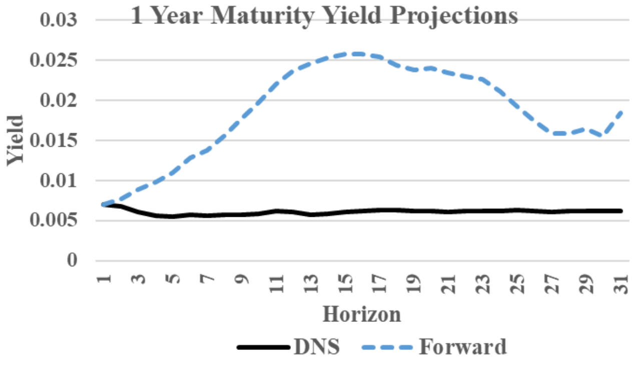
- Must fit the initial curve to 120 years
- Fit DNS or FWD targets at the 5 and 30 year horizon simultaneously
- Must remain arbitrage free

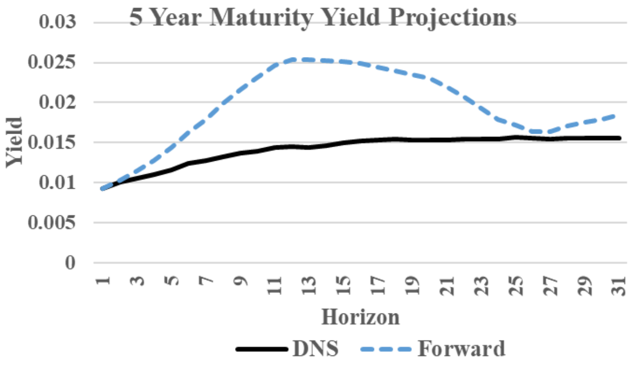


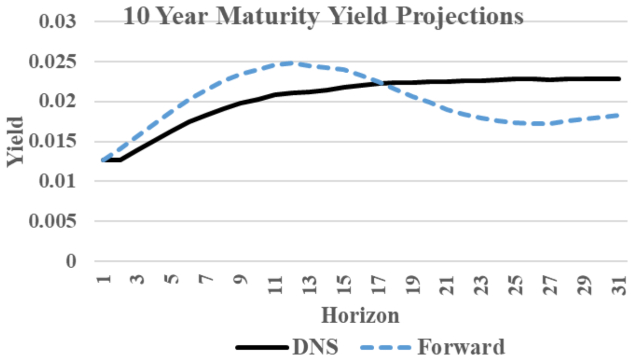


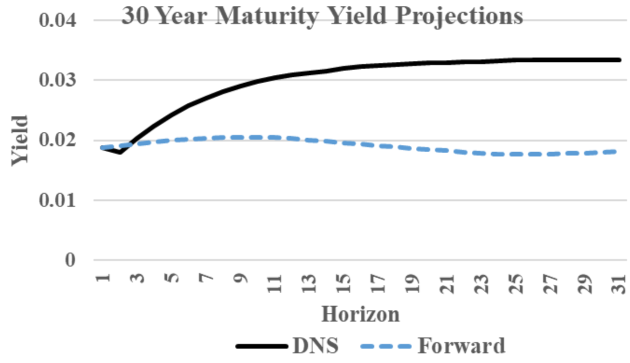
Source: Conning Inc. GEMS ESG

Practical Considerations - Calibrating to Forecasts









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Summary and Conclusions

There is no right or wrong view of future interest rates in the long term

More important that we have a robust, automatable, repeatable, explainable, justifiable approach

Forward Curve

- Fulfils many of the requirements
- Mostly suitable for short and medium term forecasting
- Unclear how to apply it to longer term forecasting

Econometric forecasting with DNS model

- Is a valuable tool for setting long term interest rate assumptions
- Data window to use is the only judgment required

A combined approach is possible using the forward curve for short term forecasts and econometric modelling for the longer term.

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