AN ACTUARIAL MODEL FOR AIDS

(O.A.R.D., 40)

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1. In this note I describe the mathematical formulation of a model for representing the spread of AIDS in a population, which is designed for actuarial use in dealing with life insurance companies and pension funds. A major requirement of actuaries is that the model should be age-specific, and should take into account normal age-specific mortality as well as the extra sickness and mortality from AIDS.

2. The model has already been used to produce results in the form of projections of AIDS cases and deaths for the United Kingdom population, which have appeared in R. Watson & Sons: 'Preliminary Memorandum on AIDS, July 1987' (Wilkie, 1987) and the Institute of Actuaries AIDS Working Party: 'AIDS Bulletin No. 1' (Daykin *et al.*, 1987a), 'AIDS Bulletin No. 2' (Daykin *et al.*, 1987b) and 'AIDS Bulletin No. 3' (Daykin *et al.*, 1988). These papers all use the same model, though with different parameter assumptions.
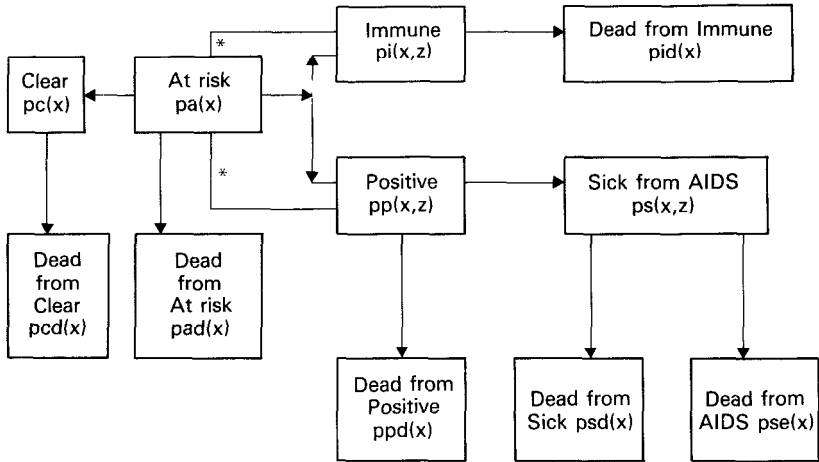
3. In this note I concentrate on explaining the mathematical model, discuss briefly the parameter assumptions, and outline the sort of results that can be produced; I do not present results, which appear in the papers cited above.

4. The model is a Markov stochastic process, applicable to a large population, with time-varying transition intensities between states. I assume that we can deal with a 'cohort' of a single sex and single age independently of other cohorts. I deal only with males at this stage. (The extension to a two-sex model, with various sexualities—male heterosexuals, bisexuals, homosexuals etc.—is more complicated, and has not yet been implemented. Estimating the parameters of such a model at this stage appears to be impossible.)

The model

5. The members of one cohort at age x may be in any of eleven discrete states, which are indicated in Figure 1. Five of these are Live states: Clear, At Risk, Immune, Positive and Sick from AIDS. Six are Dead states, which are kept separate simply to show which Live state someone died in. The dead states are: Dead from Clear, Dead from At Risk, Dead from Immune, Dead from Positive, Dead from Sick (other than from AIDS) and Dead from AIDS. I do not know whether it is practically possible to distinguish the last two categories, and deaths other than from AIDS of those who suffer from AIDS are comparatively trivial.

6. Those in the Clear state are those whose assumed sexual activity is such that they run no risk whatever of becoming infected with AIDS. They form the 'normal' pre-AIDS population for comparative purposes. Those in the At Risk state are treated as exposing themselves to the risk of acquiring AIDS by reason

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*denotes possible infection.

Figure 1. AIDS Model: States and Transitions.

of sexual contact with infected people. Those in the Immune state are assumed to have acquired HIV infection, to be infectious, but to be wholly immune from becoming sick from AIDS or dying from AIDS. Whether such a state can exist I do not know, but its existence has been suggested by Anderson *et al.* (1986), and it is not difficult to implement this feature of the model.

7. Those in the Positive state are HIV Positive, but not yet sick from AIDS; they are infectious and not immune. I assume that it is possible to distinguish between those who are HIV Positive and those who are Sick from AIDS in a discrete way. In reality there are several stages in the transition from HIV infection to Death from AIDS, as described in the Frankfurt study (Brodt *et al.*, 1986), which has been used by Cowell & Hoskins (1987) and by Panjer (1987).

8. I assume that the current age (x) is part of the total status, and that transition intensities can all vary by current age. In addition, since I treat each age cohort (or year of birth cohort) separately, I can assume that each transition intensity varies also by calendar year, so that each cohort has its own set of transition intensities.

9. Duration since entry to the states Immune, Positive and Sick from AIDS are also relevant to the transition intensities. This duration is denoted in each case as z.

10. Conditional on some initial distribution at a starting age x_0, the probability of an individual being in state j (or the proportion of individuals in state j) at age x is denoted by $pj(x)$, where $j = \{c, a, cd, ad, id, pd, sd, se$ (Dead from AIDS)$\}$, as in Figure 1. The corresponding density for those in the Immune,

Positive or Sick from AIDS states at age x and duration z is $pi(x,z)$, $pp(x,z)$ or $ps(x,z)$ respectively.

11. Possible transitions are as shown in Figure 1. Those in any of the Live states may die, and those who are Sick from AIDS may die from AIDS or from causes other than AIDS. Those who are At Risk may change their behaviour and become Clear (there is no representation in the model of transfer from Clear to At Risk). Those who are At Risk may become infected, and at that point are immediately allocated either to the Immune state or to the Positive state, in proportions that may depend on age (and on calendar year, though it seems unlikely that this would actually exercise any influence).

12. Those in the Positive state may become Sick from AIDS, if they do not die first. Infection is possible from the Immunes and Positives, to the At Risk, and how this is represented will be described below.

13. The transition intensity from state j to state k is generally represented by $mjk(x)$, if it depends only on age, x, and by $mjk(x,z)$ if it depends on both age, x, and duration, z.

Differential equations

14. The usual Kolmogorov differential equations describe the transitions between states, with the exception of the transmission of infection. The differential equations are now described.

$$\frac{dpc(x)}{dx} = -mcd(x) \cdot pc(x) + mac(x) \cdot pa(x) \tag{1}$$

Clears are diminished by death and increased by transfers from At Risk to Clear.

$$\frac{dpa(x)}{dx} = -mac(x) \cdot pa(x) - mad(x) \cdot pa(x) - T(x) \tag{2}$$

At Risk are diminished by transfers to Clear and to Dead and by the term $T(x)$, representing transfers to the infectious states. This will be described below.

$$\frac{dpcd(x)}{dx} = +mcd(x) \cdot pc(x) \tag{3}$$

Dead from Clear are increased by deaths among Clears. Similar formulae apply to all the Dead states.

15. We put $w = x - z$, so w is the entry age to the given state. Then, for $z > 0$,

$$\frac{dpi(w+z,z)}{dz} = -mid(x,z) \cdot pi(x,z) \tag{4}$$

Immunes at duration z are diminished only by death.

$$\frac{dpp(w+z,z)}{dz} = -mpd(x,z) \cdot pp(x,z) - mps(x,z) \cdot pp(x,z) \tag{5}$$

Positives at duration z are diminished by death and by transfers to Sick from AIDS.

$$\frac{dps(w + z,z)}{dz} = -msd(x,z) \cdot ps(x,z) - mse(x,z) \cdot ps(x,z) \qquad (6)$$

Those Sick from AIDS at duration z may die from causes other than AIDS or may die from AIDS.

16. The new sick at duration 0 are given by

$$\frac{\partial ps(x,0)}{\partial x} = \int_0^\infty mps(x,z) \cdot pp(x,z)dz \qquad (7)$$

New immunes at duration 0 are given by

$$\frac{\partial pi(x,0)}{\partial x} = fi(x) \cdot T(x) \qquad (8)$$

where $fi(x)$ is the proportion of those newly infected at age x who are assumed to enter the Immune state, and $T(x)$ is the density of newly infected at age x.

New positives at duration 0 are given by

$$\frac{\partial pp(x,0)}{\partial x} = \{1 - fi(x)\} \cdot T(x) \qquad (9)$$

i.e. all those newly infected who do not enter the Immune state.

17. In order to explain $T(x)$, the newly infecteds, we define the total proportions immune and positive at age x as

$$PI(x) = \int_0^\infty pi(x,z)dz \qquad (10)$$

and

$$PP(x) = \int_0^\infty pp(x,z)dz \qquad (11)$$

we define also

$$AI(x) = \int_0^\infty mai(x,z) \cdot pi(x,z)dz \qquad (12)$$

and

$$AP(x) = \int_0^\infty map(x,z) \cdot pp(x,z)dz \qquad (13)$$

Then

$$T(x) = pa(x)\left\{\frac{AI(x) + AP(x)}{pa(x) + PI(x) + PP(x)}\right\} \qquad (14)$$

The intensities of infectivity, $mai(x,z)$ and $map(x,z)$, are a combination of what other authors have treated as two elements: frequency of sexual contact with a new partner, and probability of infection from a new partner. It is assumed that these intensities may vary by the age of the At Risks, perhaps representing varying levels of sexual activity at different ages, and may also vary according to the duration of infection of the infected partner.

18. If there were no immunes, and if the infectivity intensity were a constant m for all ages and durations, then the density of new infections would be

$$T(x) = pa(x) \cdot \left\{\frac{m \cdot PP(x)}{pa(x) + PP(x)}\right\} \qquad (15)$$

which in the absence of any other transitions would lead to the usual logistic growth of the infected proportion in a population.

Numerical solution

19. Although these differential equations appear fearsome, the numerical solution of them is laborious rather than difficult. The technique used is one developed by Waters & Wilkie (1987), and designed to describe sickness with recovery for the purposes of Permanent Health Insurance.

20. An initial age x_0 is chosen. Typically I have used a starting year of 1983, and considered each cohort with its age in that year. Numerical solution of the differential equations is carried out in steps of length h, where h is some convenient fraction of a year, usually $1/4$.

21. Where the probability in a particular state does not depend on duration, approximations of the form

$$\frac{pc(x + h) - pc(x)}{h} = -\{mcd(x) \cdot pc(x) + mcd(x + h) \cdot pc(x + h)\}/2$$
$$+ \{mac(x) \cdot pa(x) + mac(x + h) \cdot pa(x + h)\}/2, \quad (16)$$

which approximates formula (1), are used.

22. Where duration enters into the status, we first subdivide duration into steps also of h, i.e. $(0,h)$, $(h,2h)$, ... The proportion positive at age x in durations $((j-1)h, jh)$ is denoted

$$PPJ(x,j) = \int_{(j-1)h}^{jh} pp(x,z)dz \qquad (17)$$

with similar definitions for the immunes, $PIJ(x,j)$, and the sick, $PSJ(x,j)$. It is only necessary to record durations up to the highest practicable duration, being the

longest duration initially at which Immunes, Positive or Sick are non-zero plus the duration so far $(x - x_0)$.

23. Those in each duration cell among the immunes are carried forward by means of the approximation

$$
\frac{PIJ(x + h, j + 1) - PIJ(x,j)}{h} = -\{mid(x,k) \cdot PIJ(x,j)
$$
$$
+ mid(x + h, k + h) \cdot PIJ(x + h, j + 1)\}/2 \quad (18)
$$

where $k = (j - \cdot 5)h$, the mid point of the relevant interval. Similar but more elaborate expressions apply to $PPJ(x,j)$ and $PSJ(x,j)$.

24. The integrals in formulae (10), (11), (12) and (13) can now be approximated by summations. The approximations used to calculate $pa(x+h)$, $PIJ(x,1)$ and $PPJ(x,1)$ involve a complicated set of non-linear simultaneous equations, which can be solved recursively. All the other approximations can be solved directly.

25. There remains a specification of the initial conditions, which require fractions, adding up to unity, among Clears, At Risks and at least one duration cell of Immunes or Positives. (If there are no infecteds in the initial population none can occur later.) Further duration cells of Immunes, Positives and Sick may also be specified if desired. There is little point in including any Deads initially.

26. The total Living plus total Dead must always equal unity. The total transitions among all states between x and $(x+h)$ must sum to zero. These are useful computational checks.

Transition intensities

27. The model allows a very flexible representation of the transition intensities, varying by age, calendar year and where relevant also by duration in the current state. Our knowledge is far too small at this stage to take advantage of this flexibility. In practice I have used the transition intensities described below.

28. The notation used throughout is as follows. Units are years, except for t.

x is the attained age.
$t = (x - 70)/50$, a convenient transformation of age.
y is the calendar time, measured from end-$1983 = 0$.
d is the duration since entry to the current status.

Intensities that vary by age and calendar year

29. The transition intensities that depend only on age and calendar year are discussed first.

Mortality from Clear

30. In the first set of projections I assumed England and Wales Population Mortality 1983 Males, represented by the formula:

$$
mcd(x) = a_0 + a_1 \cdot t + \exp\{b_0 + b_1 \cdot t + b_2 (2t^2 - 1)\} \quad (19)
$$

where

$$a_0 = - \cdot 000780$$
$$a_1 = - \cdot 001446$$
$$b_0 = -3 \cdot 735111$$
$$b_1 = 4 \cdot 725108$$
$$b_2 = - \cdot 662952$$

This was derived by graduating the available data using the methods described by Forfar, McCutcheon & Wilkie (1988).

31. For making projections for insurance purposes (Bulletin No. 2) I used the C.M.I. experience for 1979–82 for Assured Lives Male Durations 2 and over, which I have graduated by the same formula, with:

$$a_0 = - \cdot 003390$$
$$a_1 = - \cdot 003873$$
$$b_0 = -3 \cdot 351194$$
$$b_1 = 4 \cdot 654752$$
$$b_2 = \cdot 0$$

This is very close to the graduation of the same experience described by the C.M.I. Committee (1988), but without the allowance for duplicates included in the latter graduation.

Mortality from At Risk
32. In all cases I have used the same values as the mortality from Clear.

Transfer from At Risk to Clear
33. I have used a variety of assumptions here, but in each case the intensity varies only by calendar year, and not by age.

34. In projections A, B, C, F and G of Watson's Preliminary Memorandum I assumed that the rate is: zero up to 31.12.86; increasing (in an undefined way) from zero at 31.12.86 to ·1 at 1.1.87; reducing linearly from ·1 at 1.1.87 to zero at 31.12.96; zero thereafter.

35. In Projection D of the Preliminary Memorandum I have assumed zero at all times.

36. In Projection E of the Preliminary Memorandum I have assumed zero up to 31.12.86; increasing to ·05 at 1.1.87; constant ·05 after 1.1.87.

37. In Projections A, C, D and E of Bulletin No. 1 I have assumed zero at all times.

38. In Projections B and F of Bulletin No. 1 I have assumed zero up to 31.12.86; increasing to ·1 at 1.1.87; constant after 1.1.87. This is equivalent, ignoring all other transitions, to reducing the At Risk population by about 40% in five years, and by about 65% in ten years.

Proportion Immune

39. Strictly this is not a transition intensity, but it has the form of one. In my projections so far I have not allowed for Immunes. Anderson *et al.* (1986) make assumptions that 30%, 50% or 100% of those infected are not Immune, so I could use a constant rate of ·7, ·5 or zero. If this proportion occurs naturally I see no reason for it to vary by calendar year, but I suppose it could vary by age. If 'immunity' can be procured by some sort of drug then I suppose this proportion could vary by calendar year. My own opinion is that it is likely to be zero anyway, i.e. that there are no Immunes. Cowell & Hoskins (1987) assume this too.

Intensities that vary also by duration

40. I now consider the transition intensities for which the model permits dependence on duration since entry to the current status as well as dependence on age.

Mortality from Immune

41. I have so far assumed that this does not in fact vary by duration, and have used the same values as the mortality from Clear.

Mortality from Positive

42. I have so far assumed that this also does not in fact vary by duration, and have used the same values as the mortality from Clear.

Mortality from Sick not from AIDS

43. Again I have so far assumed that this does not in fact vary by duration, and have used the same values as the mortality from Clear.

Mortality from Sick from AIDS

44. I have assumed that this does not in fact vary either by duration or by age, and in most projections I have assumed a constant value of ·7. But in projections D and F of Bulletin No. 1 I have assumed that the rate is ·7 up to the end of 1986, and that it then reduces linearly over the five years to the end of 1991 to a value of ·35, remaining constant at ·35 thereafter. A constant rate of ·7, ignoring all other transitions, implies that about half of those sick die within one year, half the survivors within a second year, and so on. The expectation of life of a sick person is about 1·43 years (at all durations of sickness). This is consistent with the published data. A constant rate of ·35 implies a constant expectation of life of about 2·86 years.

Infectivity from Positives to At Risk

45. I have so far assumed that this does not in fact vary by duration, but that it does vary by age. In all the projections of the Preliminary Memorandum I have assumed a constant peak value of ·7 from age 15 to age 50; reducing linearly from ·7 at age 50 to zero at age 70; and zero above age 70. A constant rate of ·7 implies a

'doubling time' in the early stages of the epidemic of about one year, which is consistent with the published data.

46. In projections A, B, D and E of Bulletin No. 1 I have assumed a constant peak value of ·7 from age 25 to age 50; below this range it reduces linearly from ·7 at age 25 to zero at age 15; above this range it reduces linearly from ·7 at age 50 to zero at age 70; below age 15 and above age 70 it is zero.

47. In projections C and F of Bulletin No. 1 I have assumed that the peak rate is ·7 up to the end of 1987, and that it then reduces linearly over the five years to the end of 1992 to a value of ·35, remaining constant at ·35 thereafter. I have assumed that this peak rate applies over the same age range, and that the same linear reductions apply from this peak rate both below and above this peak age range. In fact this means that the intensity is a quartic surface over the lower and higher age bands; for example, for $15 \leq x \leq 25$ and $3 \leq y \leq 8$ (i.e end-1986 to end-1991), we get:

$$map(x,y) = -·007xy + ·091x + ·105y - 1·365,$$

which is a hyperboloid. The reduction in this intensity with time is consistent both with a change of behaviour—a reduced frequency of partner change—and with a spread of infection into groups with a lower frequency of sexual contact.

48. I can implement the reported hypothesis (Anderson, 1988) that the infectivity varies by duration of infection, but I do not know what rates to use. It has been suggested that the rate starts out high, then falls, then rises again as the infected individual developed symptoms. It is not necessary to pay attention to this last point; it is sufficient to assume that the rate rises with duration of positivity.

49. The force of infectivity strictly combines two elements: the infectiousness of infected persons, and the frequency of contact of an At Risk person with infected persons. It is the latter element that justifies the lower values at lower and at higher ages; but I do not know whether the linearity assumptions are reasonable.

Infectivity from Immunes to At Risk

50. I have not yet had to make any assumptions about this. I suppose one could use the same values as for the infectivity from Positives to At Risk. I do not know whether it is suggested that these rates may be different.

Transfer from Positive to Sick

51. This is one intensity about which there is considerable uncertainty, and about which there has been a lot written, without any conclusive results. So far I have assumed that it varies by duration, but not by age or by calendar year. In all the projections for Bulletin No. 1 I have used a Gompertz formula with an upper limit:

$$mps(x,d) = \text{Min } \{\exp(-8·4 + 1·4\ d),\ ·25\}$$

52. In projections A, B, C, D, and E of the Preliminary Memorandum I have used the same function, but in projections F and G of the Preliminary Memorandum I modified it. In projection F I put an upper limit of ·05, and in projection G I put no upper limit.

53. Anderson *et al.* (1986) have used a Weibull formula, as suggested by Lui *et al.* (1986). This is in general:

$$mps(x,d) = c \cdot b \cdot d^{b-1}$$

but they have used $b = 2$ and put $a = 2c$ giving:

$$mps(x,d) = a \cdot d$$

54. In one place Anderson *et al.* use $a = ·237$, but this leads to a very rapid transition to Sick. Later they use $a = ·0628$, which is more comparable with my assumptions.

55. The results for these Weibull models can be compared with my assumptions (see Table 1). It can be seen that my usual assumption (Gompertz with a maximum of ·25) gives a rather longer-tailed distribution of incubation period than either of the Weibull assumptions. If I have no upper limit, I am behind the Weibull with $a = ·0628$ up to duration 6, and then I shoot ahead. With an upper limit of ·05 the distribution is very long-tailed indeed.

56. It is interesting to compare these also with the assumptions of Cowell and Hoskins (1987) and of Panjer (1987), who both based their calculations on data from the Frankfurt Study (Brodt *et al.*, 1986), and with the Weibull distribution of Hyman & Stanley (1988) who use the form

$$mps(x,d) = p \cdot q^{p} \cdot d^{p-1}$$

with $p = 2·4$, $q = ·11$. Their distribution is reasonably close to my Gompertz with a maximum of ·25.

Initial conditions

57. These depend on the purposes of the particular projection. The Preliminary Memorandum and Bulletin No. 1 both give projections for the U.K. male population, and the initial conditions required for this will be described.

Initial proportions

58. The population projections all start from 1983, and initial proportions in each status must be specified for each age in the population of males in 1983, and for each new generation of males, which is assumed to enter at age 15 in each year from 1984 onwards. Initial proportions are needed for:

> Clear
> At Risk
> Positive, subdivided by duration
> Immune, subdivided by duration
> Sick, subdivided by duration

There is no point in including any Dead initially.

Table 1. *Percentage sick by given duration*

Duration (years)	Weibull (a = ·0628)	Weibull (a = ·237)	Gompertz (No max.)	Gompertz (Max. ·25)
1	3·09	11·17	·05	·05
2	11·80	37·75	·25	·25
3	24·62	65·58	1·05	1·05
4	39·49	84·98	4·24	4·24
5	54·39	94·83	16·14	16·14
6	67·71	98·60	51·04	34·69
7	78·53	99·70	94·48	49·13
8	86·60	99·95	100·00	60·39
9	92·14	99·99	100·00	69·15
10	95·67	100·00	100·00	75·97
11	97·76	100·00	100·00	81·29
12	98·91	100·00	100·00	85·43
13	99·50	100·00	100·00	88·65
14	99·79	100·00	100·00	91·16
15	99·91	100·00	100·00	93·16
16	99·97	100·00	100·00	94·64
17	99·99	100·00	100·00	95·82
18	100·00	100·00	100·00	96·75
19	100·00	100·00	100·00	97·47
20	100·00	100·00	100·00	98·03

Duration (years)	Gompertz (Max. ·05)	Cowell & Hoskins	Panjer	Hyman & Stanley
1	·05	·2	1·6	·5
2	·25	2·7	8·1	2·6
3	1·05	12·4	18·8	6·8
4	4·17	23·7	31·3	13·0
5	8·84	33·4	43·7	21·2
6	13·29	41·6	55·0	30·9
7	17·51	48·8	64·6	41·4
8	21·54	55·5	72·6	52·1
9	25·37	61·4	78·9	62·3
10	29·00	66·6	83·9	71·5
11	32·47	71·2	87·8	79·4
12	35·76	75·3	90·8	85·7
13	38·89	78·8	93·1	90·6
14	41·87	81·9	94·8	94·0
15	44·71	84·6	96·1	96·4
16	47·40	86·8	97·1	97·9
17	49·97	88·8	97·8	98·9
18	52·41	90·6	98·4	99·4
19	54·73	92·0	98·8	99·7
20	56·94	93·2	99·1	99·9

59. I have not included Immunes in any of the projections, so their initial proportions are all zero. I also assume zero Sick in 1983 for the initial population, and for each new entrant cohort. The other proportions vary by age.

60. For each projection a peak proportion of males Not Clear (i.e. At Risk plus Positive) is assumed. In projections A, D, E, F and G of the Preliminary Memorandum and in Projections A, B, C and D of Bulletin No. 1 this is 5%. In Projections B of the Preliminary Memorandum it is 10%. In Projection C of the Preliminary Memorandum and in Projections E and F of Bulletin No. 1 it is 2·5%. In each case this peak proportion applies to ages 21 to 50 inclusive. The proportions Clear at these ages are therefore 95%, 90% and 97·5% respectively.

61. When the peak proportion Not Clear is 5% the proportion Not Clear reduces linearly below age 21 to 2% at age 15, and reduces linearly above age 50 to zero at age 70. New entrants are assumed to have 2% Not Clear at age 15.

62. The corresponding figures when the peak proportion is 10% (2·5%) are 4% (1%) at age 15, zero at age 70, and new entrants 4% (1%).

63. The proportion Positive is defined as a proportion of those Not Clear and the proportion At Risk is the residual. When the peak proportion Not Clear is 5% the proportion Positive is ·1% (of the Not Clears—i.e. ·005% of the total at the peak ages) at duration 0 to ·25, reducing by a factor of ·8409 per quarter year (equivalent to halving each year) up to duration 6·5 to 6·75 years. However, if the age at entry minus the duration is less than 15 the proportion Positive is taken as zero. (These assumptions produce smooth results for the peak ages when projecting forward from 1983, but they are not quite consistent for young ages.)

64. The proportion Positive for new entrants is assumed to be ·1% of the Not Clears, i.e. ·002% of the total.

65. The corresponding figures when the peak proportion is 10% (2·5%) are ·05% (·2%) of the Not Clears at duration 0 to ·25, reducing similarly up to duration 6·5 to 6·75. This gives the same proportion of the total (·005%) in the first duration cell for all projections. For new entrants the initial proportion Positive in the first duration cell is ·1% of the Not Clears for all Projections, but this is ·004% (·001%) of the total when the peak proportion Not Clear is 10% (2·5%).

66. The initial proportions Positive have been chosen so as to roughly reproduce the reported numbers of deaths from AIDS in the U.K. for 1984, 1985 and 1986 for each Projection.

Initial population

67. For the population projections an initial population is required. I have taken figures from 'Population Projections 1983–2023' (O.P.C.S., 1985) for the U.K. estimated male population each age in mid-1983 and then used these numbers unadjusted as if they were end-1983 figures. The numbers are given in Table 2.

68. Since it is assumed that those at present above age 70 (and indeed those at age 70) include no At Risk or Positives, these age groups cannot acquire AIDS

and can be excluded from the initial population. However, those initially below age 70 may become Positive below that age (though not above it, since the force of infectivity is reduced to zero above age 70 in all Projections), and may go on to develop AIDS or die from AIDS above age 70.

New entrants

69. The numbers of new male entrants at age 15 for each future year are taken from the same Population Projections. No allowance has been made for the possibility that the existence of AIDS may both change the number of possible parents, and change the amount of sexual activity so as to alter the numbers of babies born in future years. If most of those in the At Risk status are homosexuals, these secondary effects may be small. If the disease spreads widely into the heterosexual population they may need to be taken into account.

70. The numbers of new male entrants at age 15 that have been assumed are given in Table 3.

71. The proportions of new entrants assumed to be At Risk and Positive in the various projections have been discussed above. Unless there is a small 'seed' of Positives, the structure of the model would produce no infection in a new entrant cohort.

Population projections

72. The model, the transition intensities and the initial conditions described above are sufficient to effect a population projection, given also the number of males at each age in the starting year (1983) and the 'new entrants' at age 15 in each future year. The output of any one projection is the number of persons in each state at the end of each future year and the number of transitions between

Table 2. *Estimated numbers of males, U.K. population mid-1983*

Age	Number	Age	Number	Age	Number	Age	Number
15	463,000	30	382,000	45	324,000	60	303,000
16	481,000	31	373,000	46	318,000	61	316,000
17	489,000	32	382,000	47	314,000	62	324,000
18	501,000	33	392,000	48	309,000	63	333,000
19	498,000	34	406,000	49	300,000	64	222,000
20	484,000	35	432,000	50	301,000	65	207,000
21	472,000	36	474,000	51	311,000	66	227,000
22	458,000	37	373,000	52	314,000	67	238,000
23	439,000	38	368,000	53	311,000	68	245,000
24	424,000	39	372,000	54	307,000	69	239,000
25	412,000	40	357,000	55	304,000	70	228,000
26	398,000	41	327,000	56	308,000		
27	391,000	42	303,000	57	311,000		
28	379,000	43	323,000	58	306,000		
29	384,000	44	323,000	59	306,000		

Table 3. *Projected numbers of male 'new entrants' aged 15, U.K.*

Year	Number	Year	Number	Year	Number	Year	Number
1984	458,000	1994	361,000	2004	399,000	2014	404,000
1985	444,000	1995	373,000	2005	406,000	2015	398,000
1986	449,000	1996	371,000	2006	412,000	2016	393,000
1987	421,000	1997	364,000	2007	417,000	2017	388,000
1988	397,000	1998	366,000	2008	420,000	2018	383,000
1989	369,000	1999	366,000	2009	421,000	2019	379,000
1990	356,000	2000	370,000	2010	419,000	2020	375,000
1991	343,000	2001	377,000	2011	417,000	2021	372,000
1992	328,000	2002	384,000	2012	413,000	2022	370,000
1993	333,000	2003	392,000	2013	409,000	2023	370,000

states during each future year, for as far as one cares to run the projection. Interesting figures are the numbers of new cases of AIDS each year and the numbers of deaths from AIDS each year, the numbers Sick from AIDS at the end of each year, the numbers Positive at the end of each year, and the ratios of total mortality of such a population at each age to 'normal' mortality, represented by the mortality of the Clears. These are the sorts of figures that have appeared in Wilkie (1987) and Daykin *et al.* (1987a).

Effect of insurance selection

73. Insurance companies are concerned with the effect of accepting a selected group of the potential population of applicants. In order to reproduce this, I assume first that the model is run from a starting year of 1983 for a number of years to a selection year, say Y, and secondly that the model is run for a number of years after Y keeping track of the proportion in each state at year Y. Thus, for example, Clears at year $Y+t$ who were Clear at time Y are kept separate from Clears at year $Y+t$ who were At Risk at year Y. No new principles are involved; just laborious calculation.

74. It is then possible to consider the subsequent mortality of various groups selected at year Y, for example a group consisting of all Clears and all At Risks, with those Immune, Positive or Sick having been identified and declined for insurance. Other assumptions can readily be made. This is of use for providing guidance for determining premium rates for life assurance contracts and for determining the appropriate provisions (technical reserves) for a life office's portfolio of policies.

Conclusion

75. The model in its present state seems as flexible as is practicable. Indeed many of the transition intensities are permitted to be more flexible than there is knowledge to determine. However, it is hoped that the model can be of practical use, both for its intended purpose of assisting actuarial applications and for providing projections for the total population of any chosen set of assumptions.

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