Adjusting for Bias in Mortality Forecasts

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Mortality & Longevity Conference

Edinburgh

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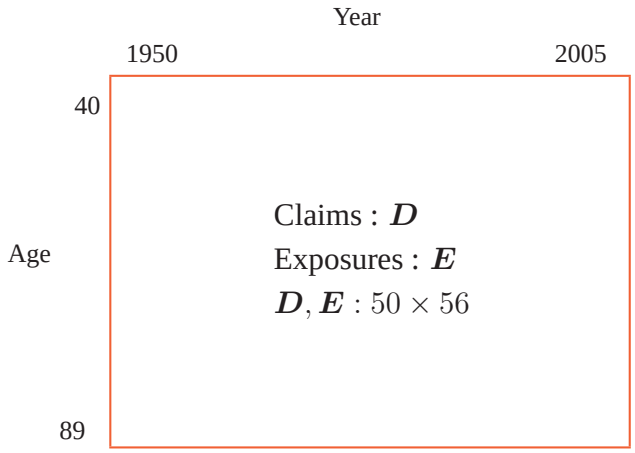
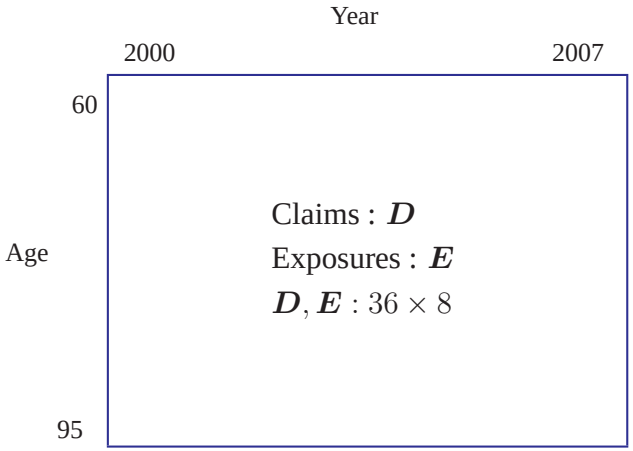
CMI male assured lives data



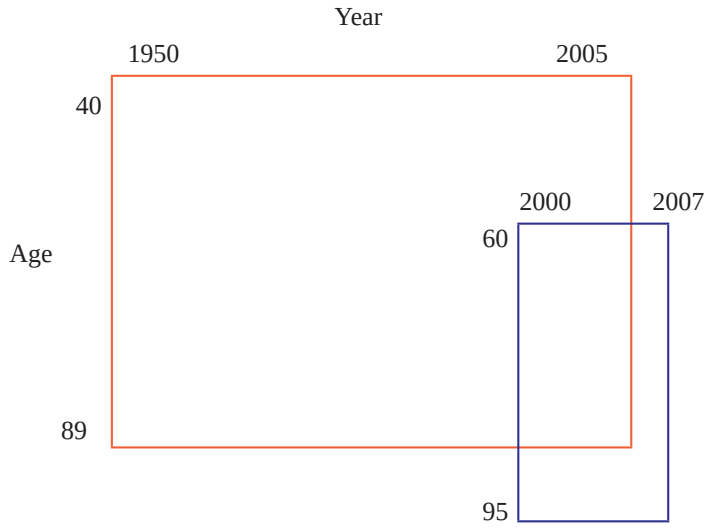
Company male pensioner data

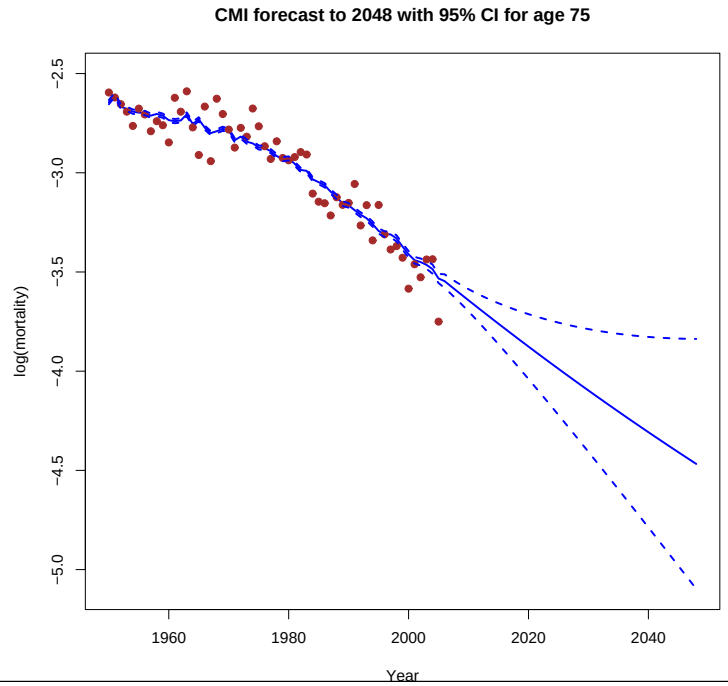


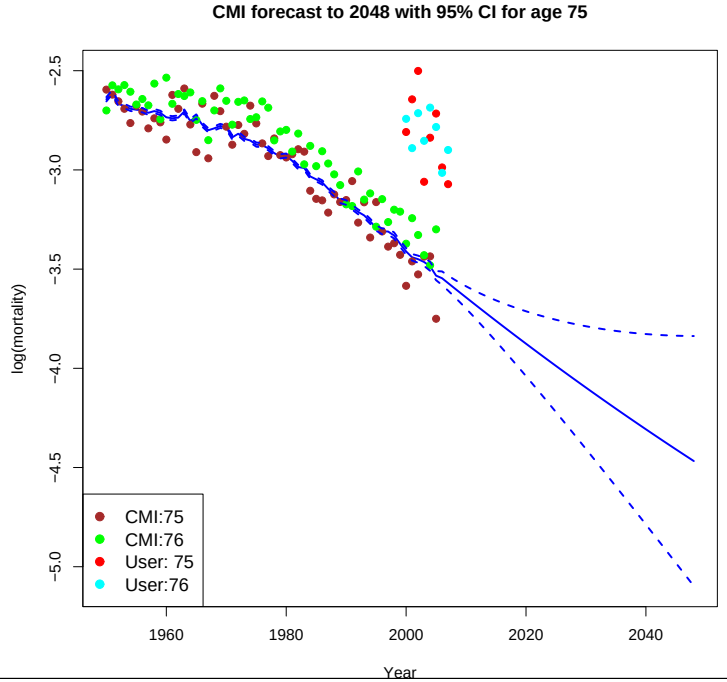
Data splitting

- Social class: two levels
- Pension size: two levels
  - ⋆ Level 00: high status, large pension
  - ⋆ Level 01: high status, small pension
  - ⋆ Level 10: low status, large pension
  - ⋆ Level 11: low status, small pension

CMI & company data

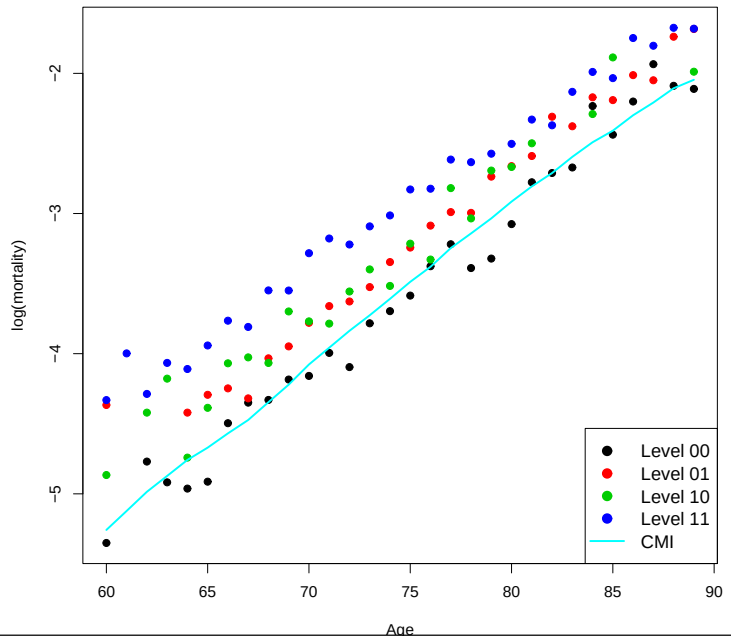


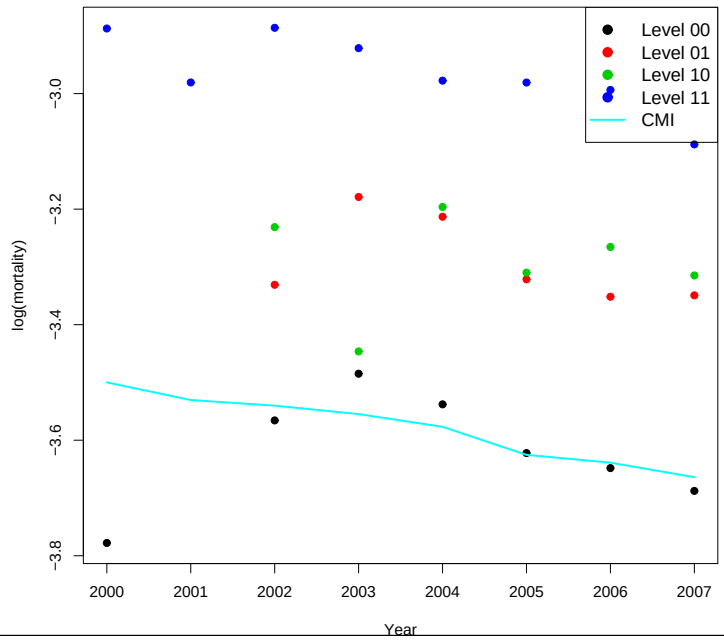




Conclusions

- Forecast with CMI data possible.
- CMI forecast biased (basis risk) for company data.
- Company data insufficient for stand-alone forecast.





The Piggyback Model

Gaps between CMI and user forecasts are

- Constant in time
- Linear function of age

The Piggyback Assumption

- Very strong.
- Doing nothing is also an assumption!

Plan of Talk

- Forecasting with the CMI data
  - ⋆ CMI output sheets
- Estimating the gap functions
  - ⋆ user output sheets

The Lee-Carter Model

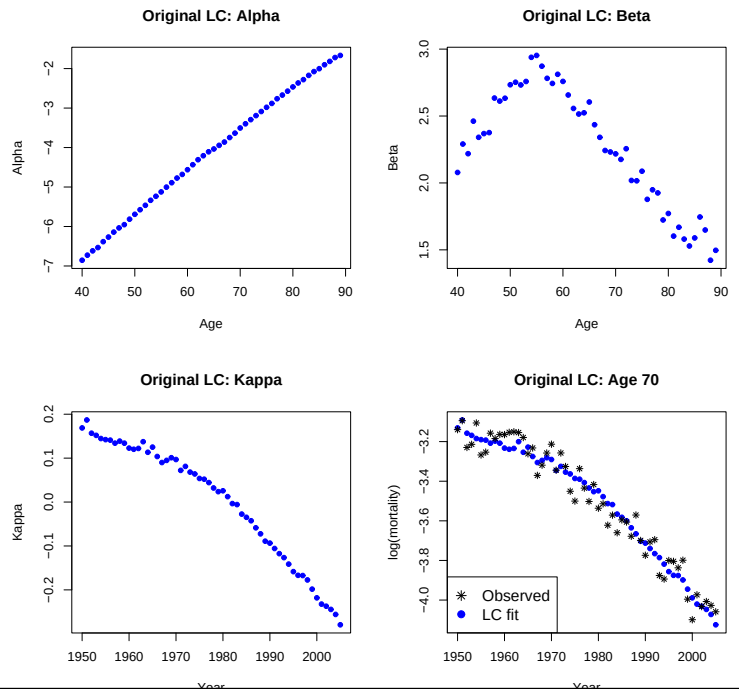
Lee-Carter (1992)

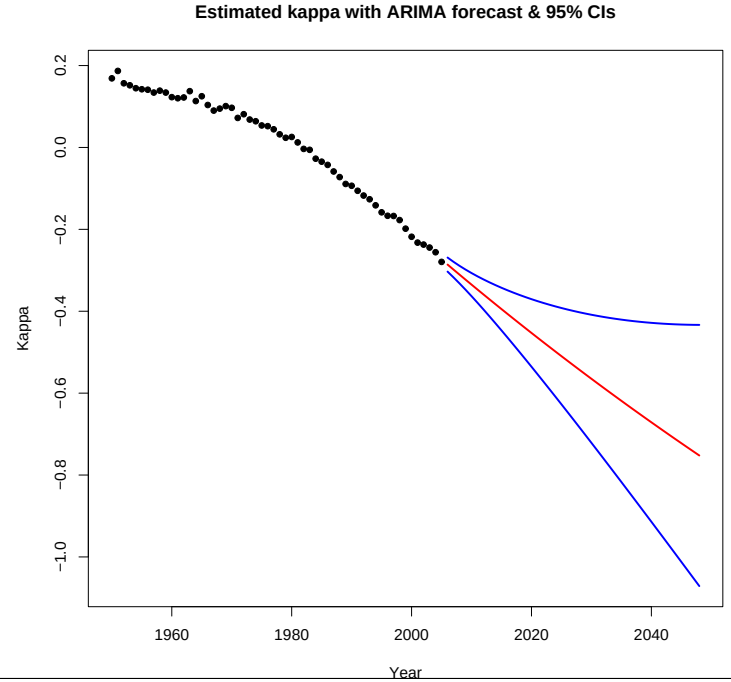
$$\log \mu_{ij} = \alpha_i + \beta_i \kappa_j,\ i = 1, \ldots, n_a,\ j = 1, \ldots, n_y,$$

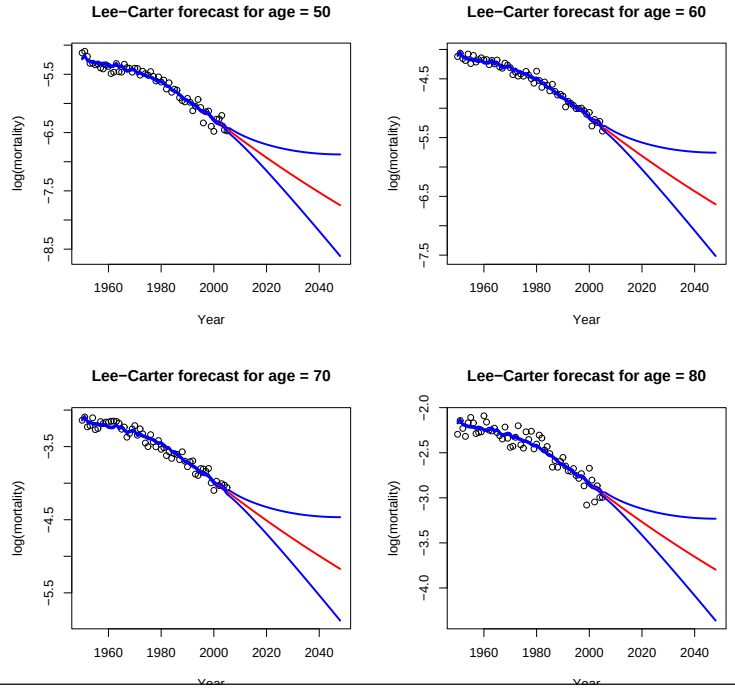
$$\sum \kappa_j = 0,\ \sum \kappa_j^2 = 1.$$

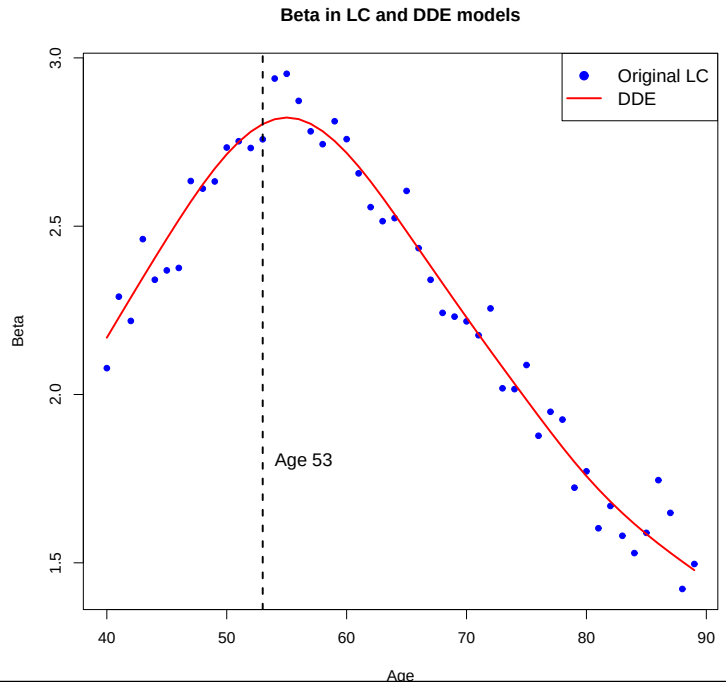
$$D_{ij} \sim \mathcal{P}(E_{ij}\mu_{ij})$$

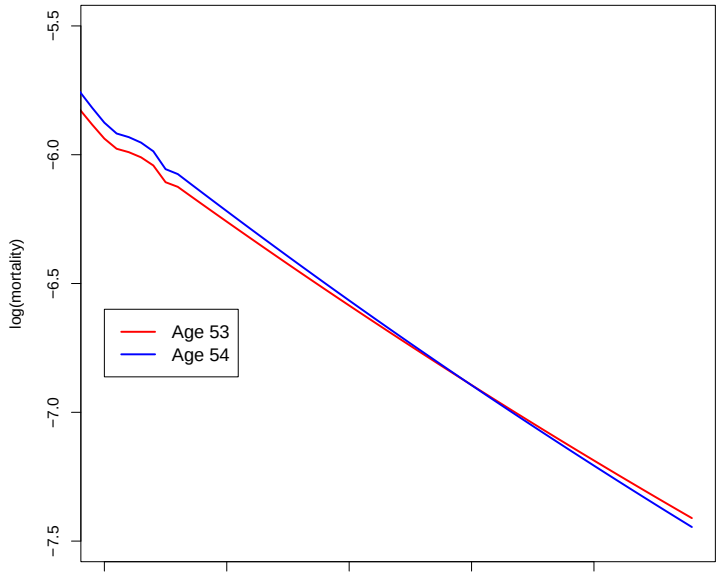
Estimation: Maximum likelihood

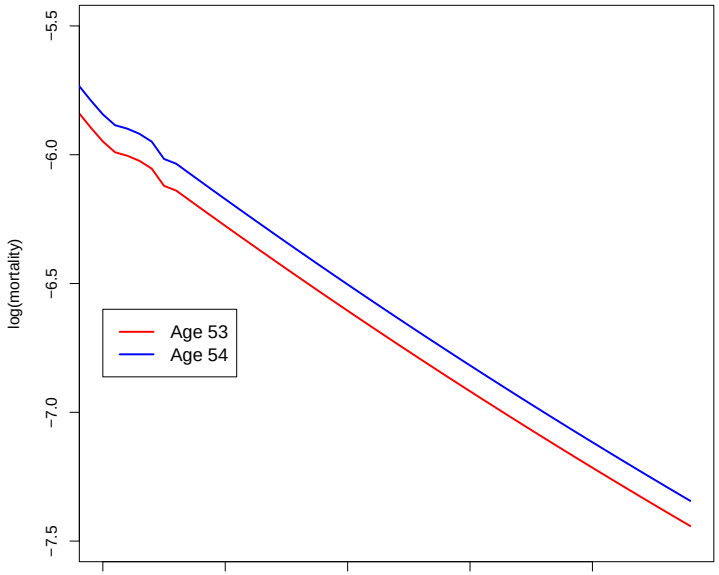





Lee−Carter forecast for age = 50
Lee−Carter forecast for age = 60
Lee−Carter forecast for age = 70
Lee−Carter forecast for age = 80
log(mortality)
Year
1960
1980
2000
2020
2040


Beta in LC and DDE models
Original LC
DDE
Age 53
Beta
Age
40
50
60
70
80
90


Lee−Carter model: age 53 and age 54 forecasts
Age 53
Age 54
log(mortality)
Year
2000
2010
2020
2030
2040


DDE model: age 53 and age 54 forecasts
Age 53
Age 54
log(mortality)
Year
2000
2010
2020
2030
2040

The Delwarde-Denuit-Eilers Model

Delwarde-Denuit-Eilers (2007)

$$\log \mu_{ij} = \alpha_i + \beta_i \kappa_j,\ i = 1, \ldots, n_a,\ j = 1, \ldots, n_y,$$

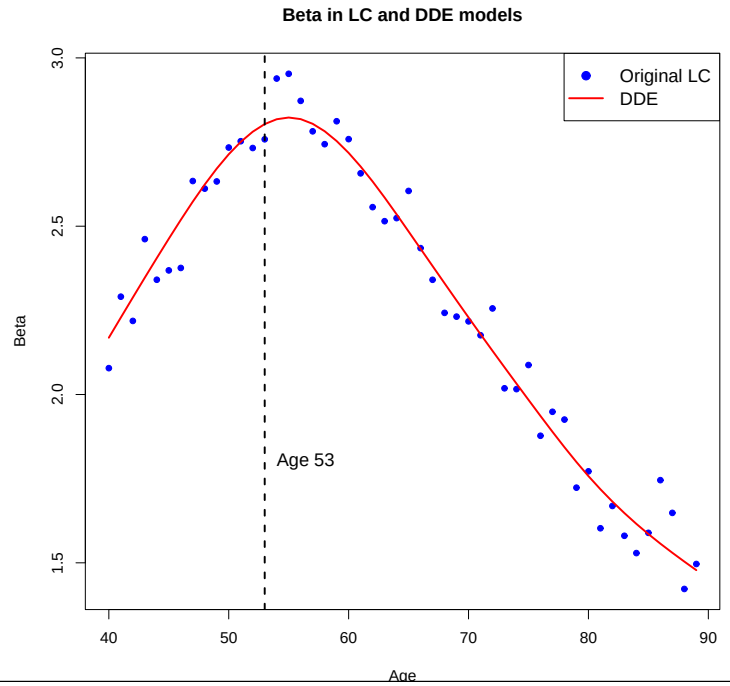
$$\sum \kappa_j = 0,\ \sum \kappa_j^2 = 1.$$

$$\beta_i = \sum_1^c B_j(x_i)\theta_j$$

$\{B_1(x), \ldots, B_c(x)\}$ is B-spline basis

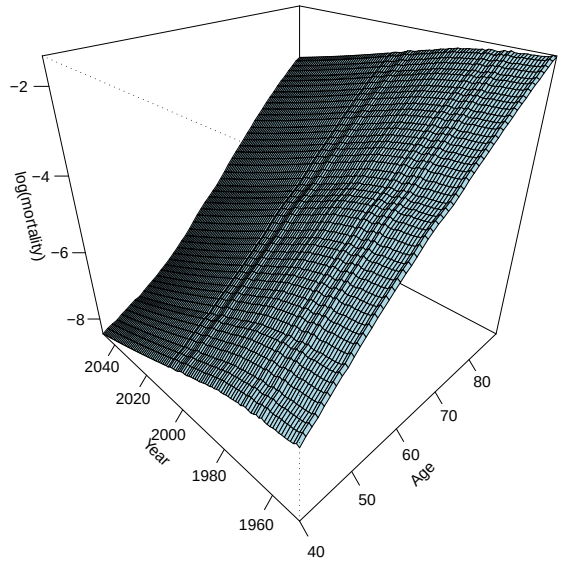
$$D_{ij} \sim \mathcal{P}(E_{ij}\mu_{ij})$$

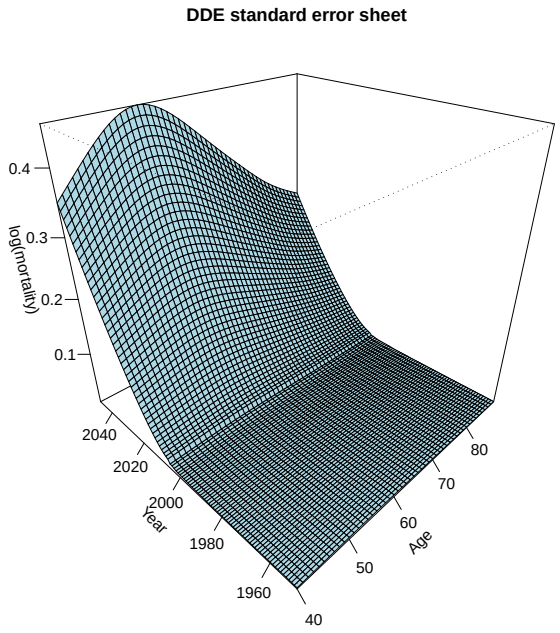
Estimation: Maximum penalized likelihood (P-splines)

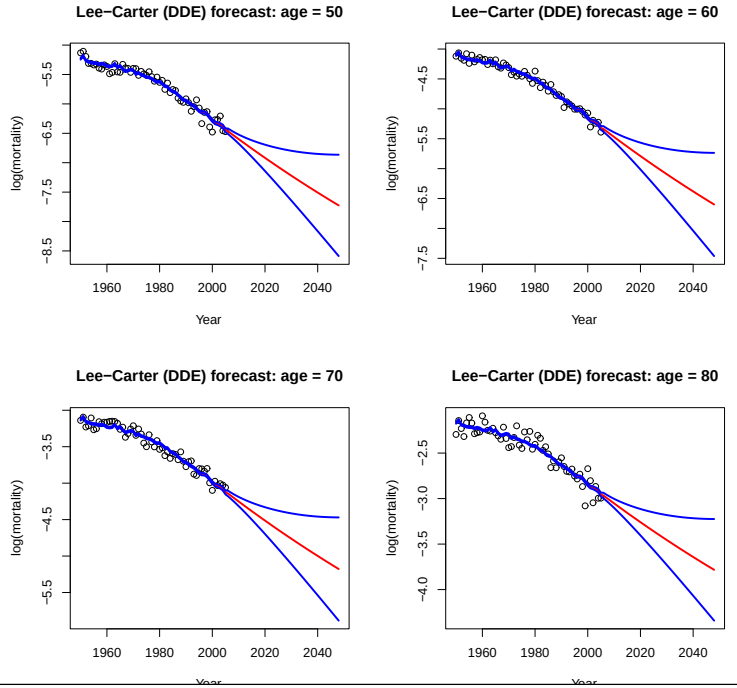


Between the Sheets

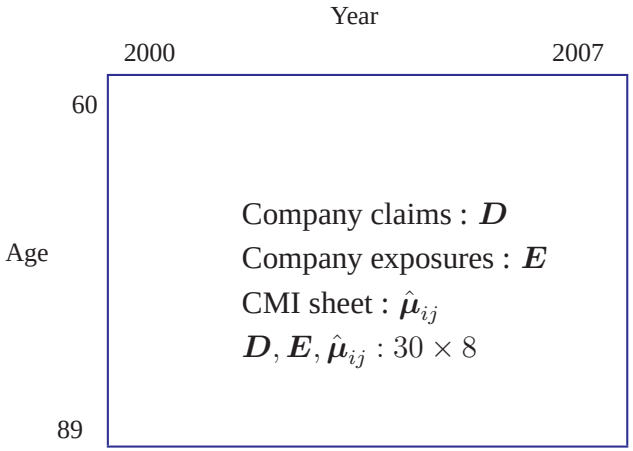
- Mean sheet - fitted and forecast surface by age and year
- Standard error sheet - SE surface by age and year







Company data & CMI sheet (trimmed)



Piggyback model

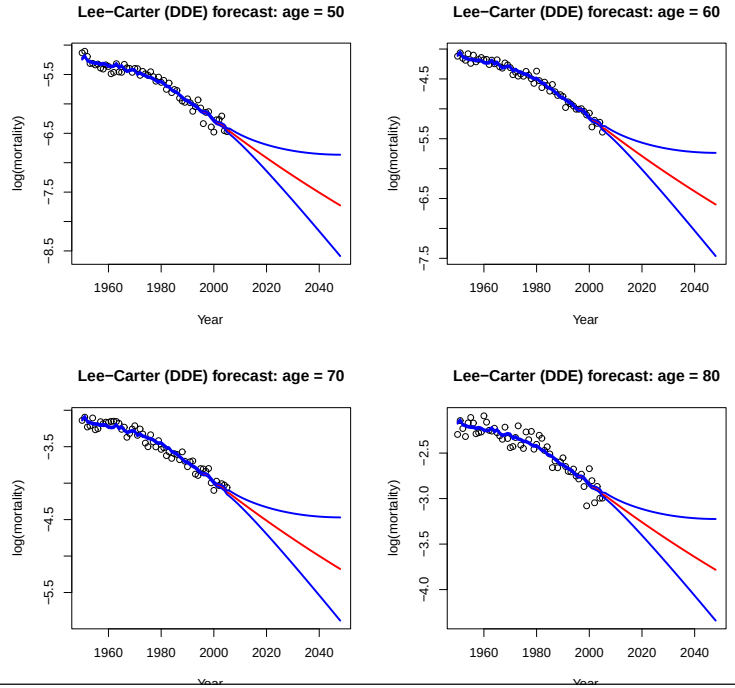
$$D_{ij} \sim \mathcal{P}(E_{ij}\mu_{ij})$$

$$60 \le i \le 89,\ 2000 \le j \le 2007$$

$$\log \mu_{ij} = \hat{\alpha}_i + \hat{\beta}_i \hat{\kappa}_j + a_0 + a_1 x_i$$

In R,

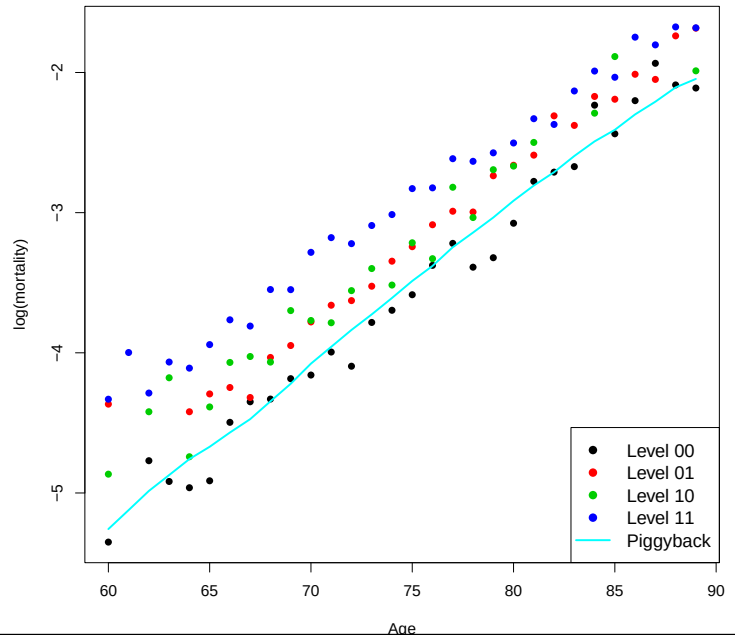
```
glm(Claims ~ X + offset(log(Exposure)) +
      offset(CMI.Sheet), family = poisson)
```

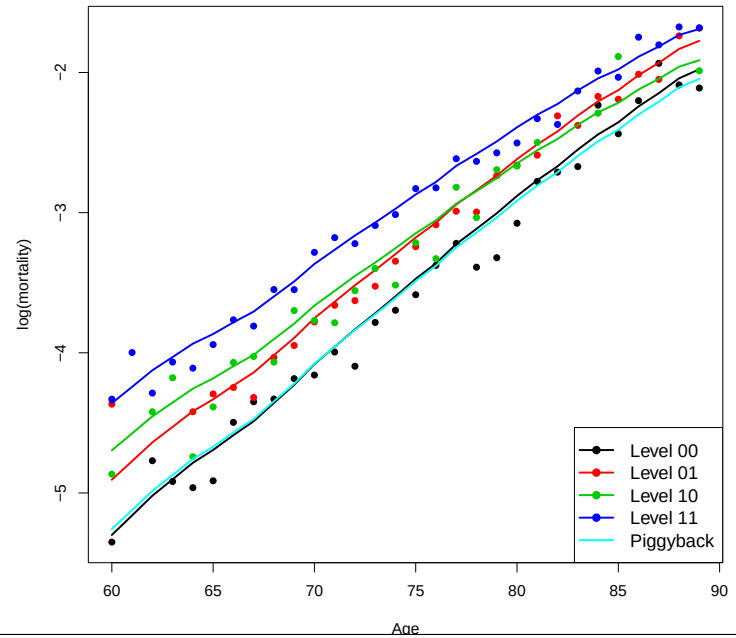


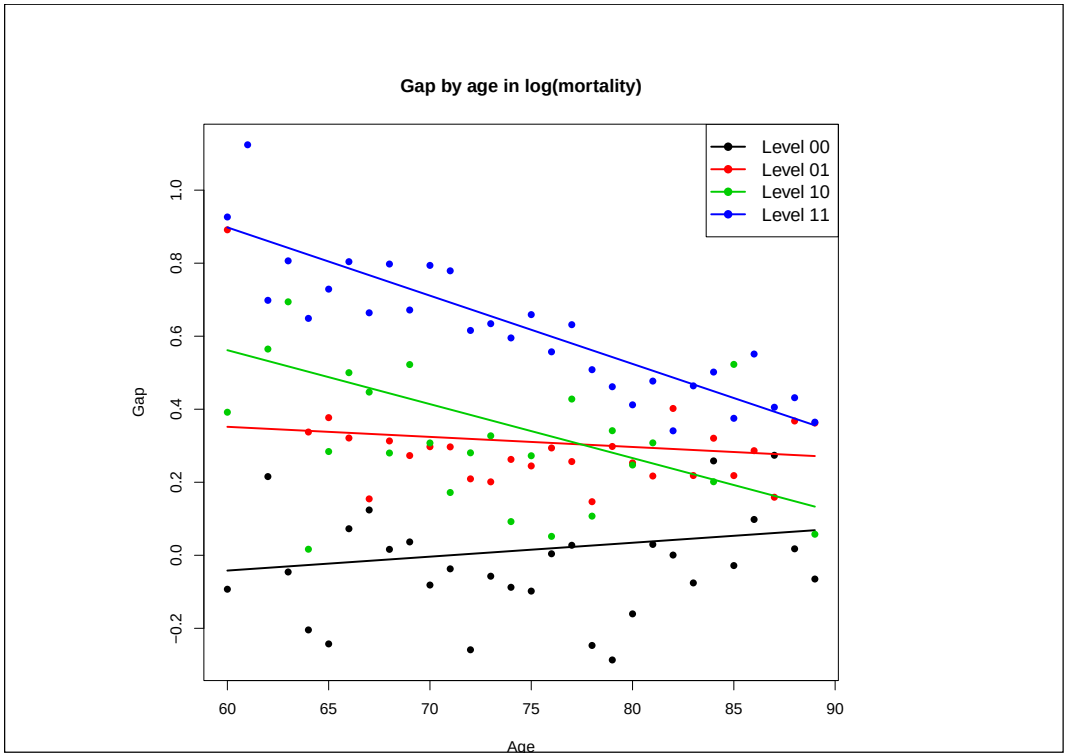
Piggyback forecast

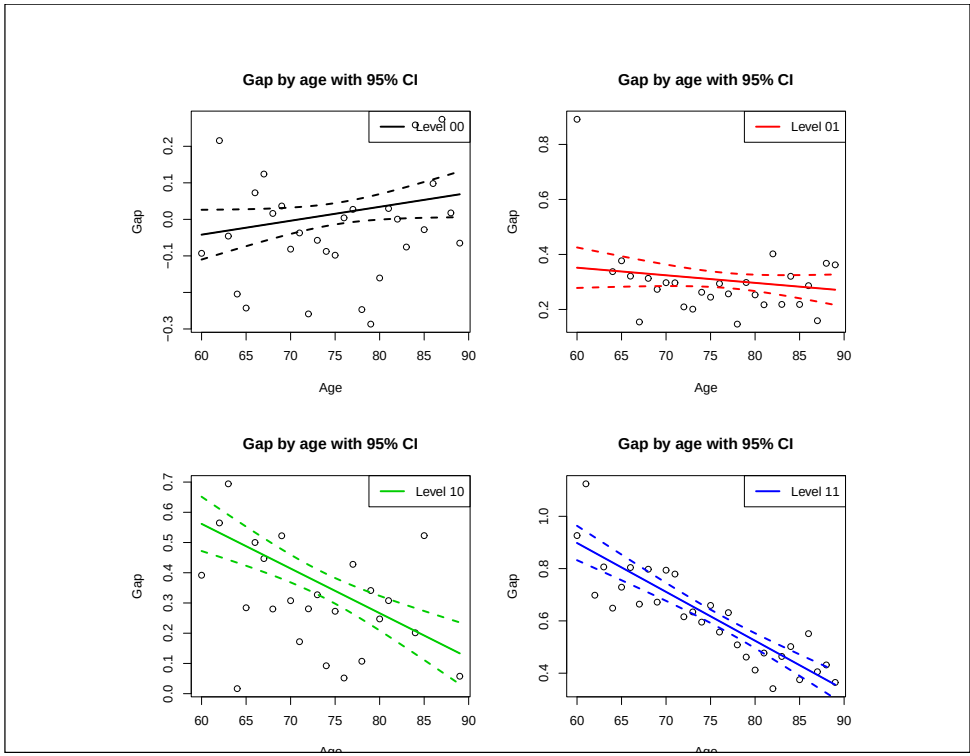
$$\log \hat{\mu}_{ij} = \hat{\alpha}_i + \hat{\beta}_i \hat{\kappa}_j + \hat{a}_0 + \hat{a}_1 x_i$$

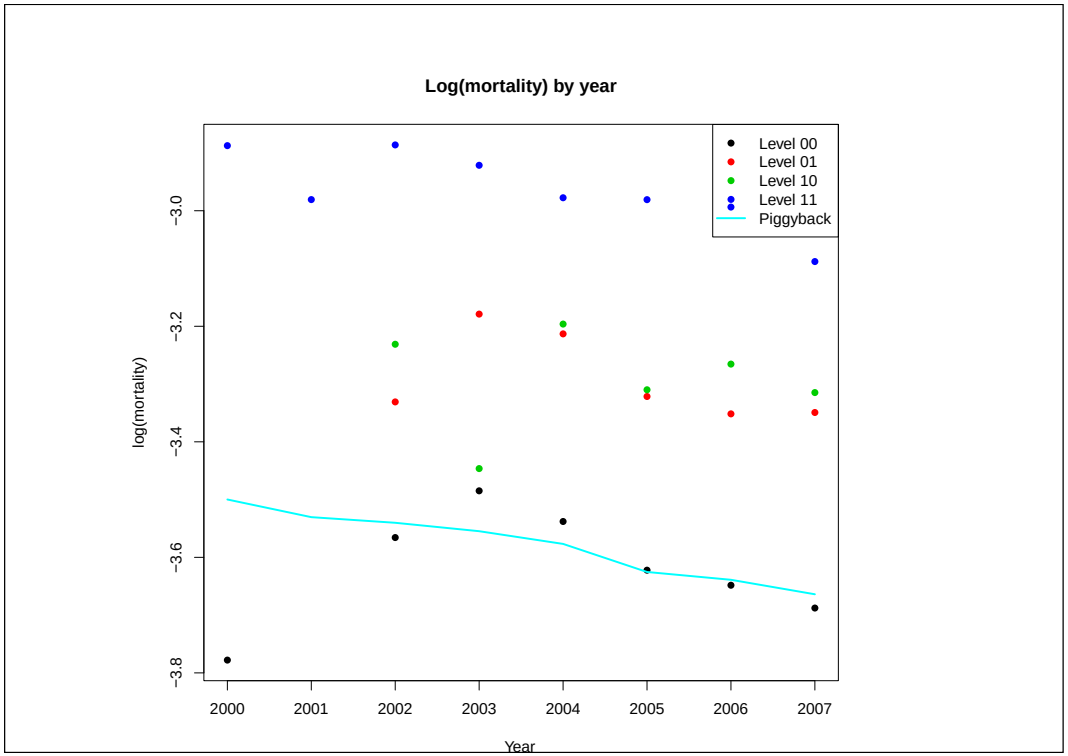
$$60 \leq i \leq 89,\ 2000 \leq j \leq 2048$$

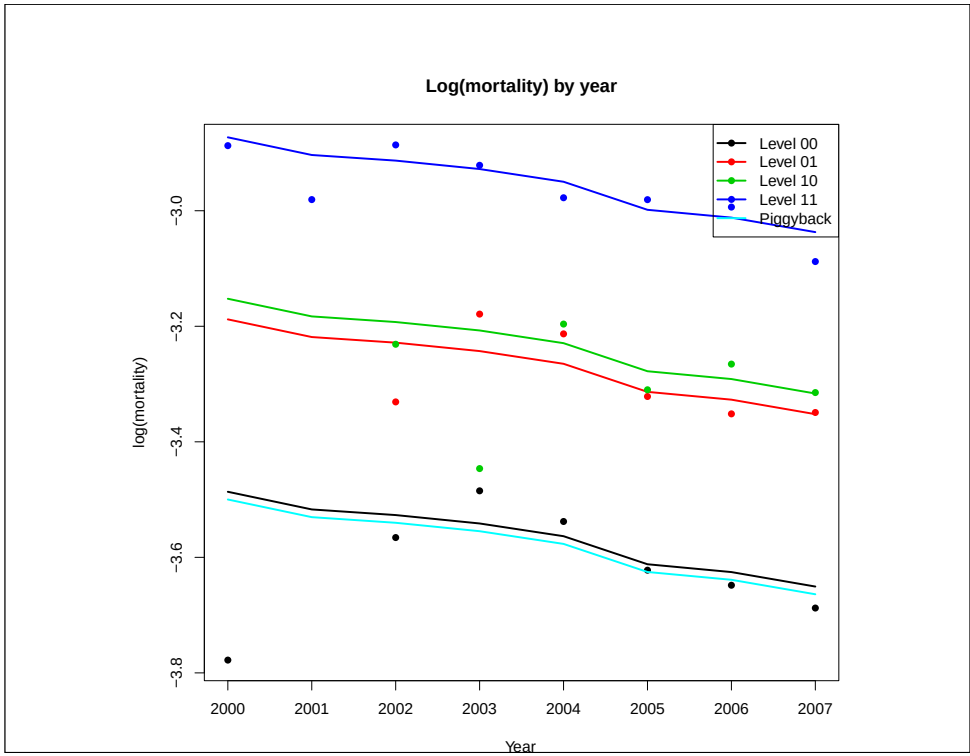


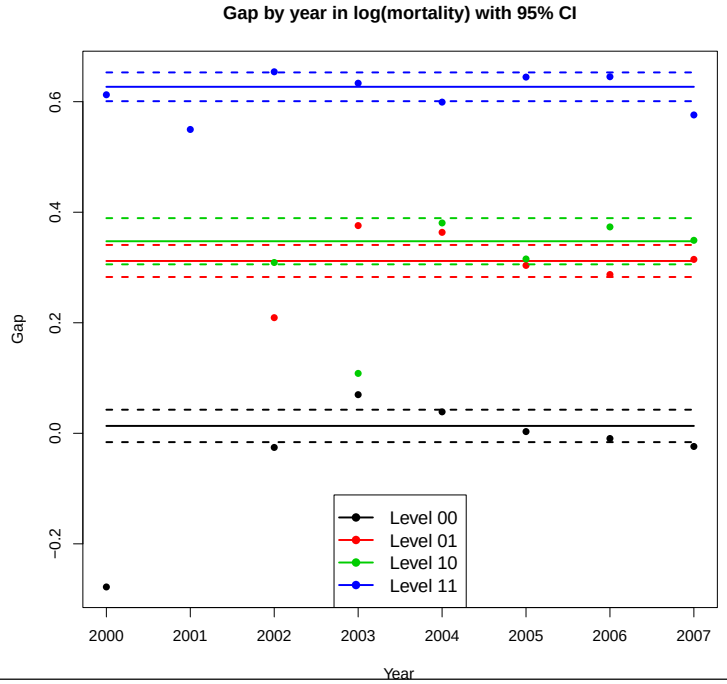


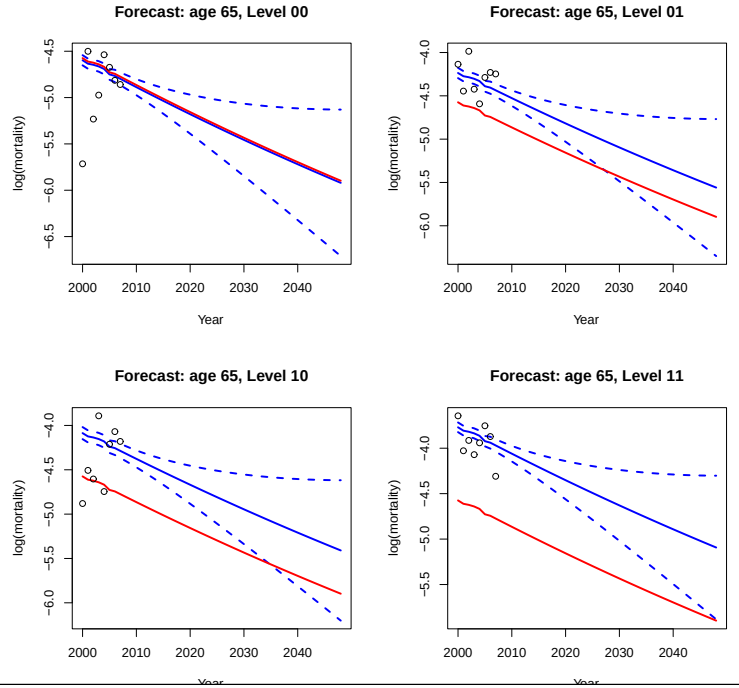

Gap by age in log(mortality)
Level 00
Level 01
Level 10
Level 11
Gap
Age

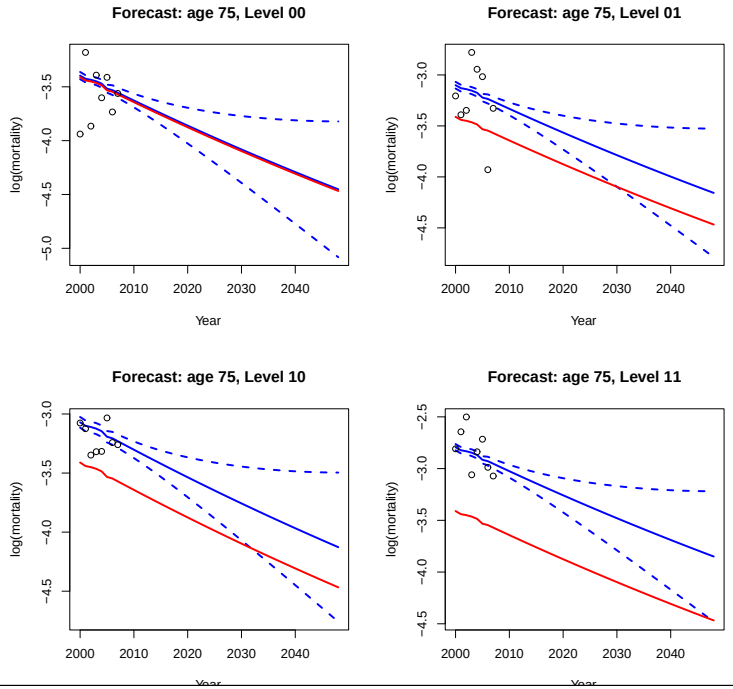

Gap by age with 95% CI
Level 00
Level 01
Level 10
Level 11
Gap
Age


Log(mortality) by year
Level 00
Level 01
Level 10
Level 11
Piggyback
log(mortality)
Year


Log(mortality) by year
Level 00
Level 01
Level 10
Level 11
Piggyback
log(mortality)
Year







Conclusions

Piggyback models allow the actuary

- To adjust forecasts based on standard data sets (CMI, UK population, etc) for basis risk.
- To make forecasts with CIs with very limited data.

Input requirements

- Mean and standard error sheets from CMI, UK,... forecast.
- Company data by age and year split by relevant factors
  - ⋆ social class, postcode, etc.

Output format

- Company mean and standard error sheets split by input factors
  - ⋆ suitable for actuarial tasks: stress testing, valuation, pricing, reserving, etc.

References

- Delwarde, Denuit & Eilers (2007) Statistical Modelling, 7, 29-48.
- Lee & Carter (1992) Journal of the American Statistical Association, 87, 659-671.
- IDC's web page: talks and papers
  `www.ma.hw.ac.uk/~iain/`