EXAMINATIONS

6 April 2001 (pm)

Advanced Certificate in Derivatives: Further Mathematics, Principles and Practice

Time allowed: Three hours

INSTRUCTIONS TO THE CANDIDATE

1. *You have 15 minutes at the start of the examination in which to read the questions. You are strongly encouraged to use this time for reading only but notes may be made. You then have three hours to complete the paper.*

2. *You must not start writing your answers in the booklet until instructed to do so by the supervisor.*

3. *Write your surname in full, the initials of your other names and your Candidate's Number on the front of the answer booklet.*

4. *Mark allocations are shown in brackets.*

5. *Attempt all 7 questions, beginning your answer to each question on a separate sheet.*

AT THE END OF THE EXAMINATION

Hand in BOTH your answer booklet and this question paper.

In addition to this paper you should have available actuarial tables, derivatives formula sheet and an electronic calculator.

1 BigBank has reached its internal credit limit with one of its most profitable customers, BigLoan. The customer wishes to draw down further borrowings equal to 25% of the bank's internal credit limit. The manager of the bank's credit department has advised senior management that further lending to this client would cause the bank to exceed both its industry and client credit concentration limits.

The senior management of the bank have suggested that there must be a way around this problem which will allow the bank to continue to lend to its most profitable customer. Further, senior management believe that to turn down the loan would seriously damage the relationship with the client.

(i) Describe how a plain vanilla credit default swap with a "AAA-rated" financial institution, CRisk, might solve the bank's problem. [3]

(ii) Suggest a simple approximation to pricing a plain vanilla credit default swap like the one above and give a brief explanation of why it works. [2]

(iii) Discuss the weaknesses in the approach in (ii) above. [4]

(iv) Describe briefly the ways in which credit default swaps are settled and comment on the implications of the different approaches to settlement. [3]
[Total 12]

2 (i) State the risks faced by the writer of an unhedged option where the gamma of the option is large and negative and the delta is zero. [2]

(ii) Derive an expression for the gamma and the vega of a European call option on a stock index that pays a continuous dividend at rate q. You may find it useful to use the following expression

$$d_1^2 - d_2^2 = 2\ln\left(\frac{S}{X}\right) + 2\,(r-q)\,(T-t)$$ [8]

(iii) Determine or show algebraically the circumstances in which it is possible to make a position in an OTC European call option on a stock index both gamma neutral and vega neutral by adding an exchange-traded European option on the index to the portfolio.

You may assume that (with the usual notation) r, σ and q are constant and common to both options. [7]
[Total 17]

3 You are an actuarial consultant from a specialist risk management advisory firm, on assignment to a major UK bank. The bank has asked you to assess its current risk management approach to some of its subsidiary operations.

You have been asked to report on systems and operational controls for the measurement and monitoring of market risk in the following parts of the bank:

- The treasury operations in New York and Tokyo which, though not as developed as in London, are nevertheless sizeable and deal in interest-rate swaps and money-market instruments on a fairly large scale.

- The insurance division in London, which has a number of complex derivatives on its books, purchased from investment banks, to hedge its portfolio of Guaranteed Equity Bonds.

Draft brief notes as preparation for a report to the senior management, illustrating the major issues which you would want to investigate, and indicate what you would expect to recommend as best practice in each area you cover.

[15]

4 (i) Show, by general reasoning, that it is never optimal to exercise early an American call option on a stock that does not pay dividends. [5]

(ii) An exotic option provides that the strike price of an American call option on a stock that does not pay dividends grows at a continuously compounded rate h per annum.

Show that, when $0 < h < r$, it is never optimal to exercise the American call option early where r is the continuously compounded risk-free rate of interest. [12]

[Total 17]

5 (i) Write down, in terms of the forward swap rate, the formula for the value of a *European swaption* exerciseable in T years, where the owner has the right on exercise to pay, in exchange for floating rate funds, annual fixed rate funds of X% per annum for n further years. Define any terms you use. [4]

(ii) Calculate the value of a €10,000,000 European payer swaption exercisable after 2 years into a 3-year annual LIBOR swap paying 6.5% fixed. Use a volatility for the forward swap of 12% per annum and the following annual term structure:

Term	*Discount factor*
0	1.000
1	0.942
2	0.886
3	0.830
4	0.779
5	0.731

[6]

(iii) In what ways are volatilities on swaptions linked to those on bond options of equivalent maturity? [2]

[Total 12]

6 (i) Describe the method of Monte Carlo (MC) simulation applied to a bond (or swap) yield curve developing under the influence of a stochastic process. [3]

(ii) Discuss the advantages and disadvantages of using this approach to value interest rate options, compared with others such as binomial/trinomial trees. [6]

(iii) Describe three ways that the MC approach can be refined, from its raw form, to reduce the number of iterations without affecting the accuracy of the end result. [6]

[Total 15]

7 The one-factor Heath-Jarrow-Morton (HJM) model, or series of models, of the yield curve supposes a generalised risk-neutral process for zero-coupon bond prices $P = P(t, T)$ of:

$$dP = rPdt + \sigma \, dz$$

where:

t = calendar time from now
T = maturity time from now
r = $r(t)$ is the short rate at time t, and
σ = $\sigma(t, T, P)$ is the volatility of bond price P,

i.e. the evolution of bond prices comprises both a drift term and a (generalised) stochastic term. As usual, the final term dz represents standard Brownian motion.

(i) Describe, without giving all the detailed algebra, how the HJM model as described above can also be expressed in terms of the instantaneous forward rates $f(t, T)$, and explain why there is no need for an explicit drift rate $\mu(t, T, P)$. [5]

(ii) Discuss the benefits of using a two-factor HJM model (i.e. with two stochastic components, dz_1 and dz_2) over and above some other popular models of the yield curve. Outline how best the two factors might be chosen, and what considerations would be involved in implementing the model and calibrating it to market prices. [7]
[Total 12]