EXAMINATIONS

7 April 2003 (pm)

Advanced Certificate in Derivatives: Further Mathematics, Principles and Practice

Time allowed: Three hours

INSTRUCTIONS TO THE CANDIDATE

1. *Enter all the candidate and examination details as requested on the front of your answer booklet.*

2. *You have 15 minutes at the start of the examination in which to read the questions. You are strongly encouraged to use this time for reading only, but notes may be made. You then have three hours to complete the paper.*

3. *You must not start writing your answers in the booklet until instructed to do so by the supervisor.*

4. *Mark allocations are shown in brackets.*

5. *Attempt all 6 questions, beginning your answer to each question on a separate sheet.*

AT THE END OF THE EXAMINATION

Hand in BOTH your answer booklet, with any additional sheets firmly attached, and this question paper.

In addition to this paper you should have available Actuarial Tables, Derivatives Formula Sheet and your own electronic calculator.

NOTE: In this examination, you are never required to prove the use of an arbitrage-free methodology unless clearly stated in the question.

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1 You are a non-executive Board member of a fund management organisation. The organisation uses significant leverage to manage client funds and measures performance in absolute terms (i.e. against a zero benchmark). Client portfolios consist of a global mix of bonds, equities, futures, vanilla and exotic options and money-market instruments.

Executives have just finished a presentation to the Board and are now inviting discussion. In the presentation it was proposed that the organisation implement a variance/covariance Value at Risk (VaR) model as the single measure of risk, operating on a 95% confidence interval and a 1-day horizon. Risk limits would then be set against the VaR measure. The rationale for the proposal was that only this approach could show the maximum loss potential.

(i) Set out fully the positive and negative aspects of the proposal. [14]

(ii) State clearly, with reasons, whether or not you would support the proposal as made. [3]

(Note: You may ignore considerations of cost and return on investment relating to the proposal.) [Total 17]

2 You are considering option strategies based on a non-dividend paying equity, priced at 105. Implied volatility is 20% per annum, and financing rates are 6% per annum.

(Note: Precise numerical calculations are **not** required for either parts (i) or (ii).)

(i) For each of the following strategies, draw profit/loss diagrams against equity price at three times: now, 2 months from now and three months from now. Assume that the options were dealt at fair prices today.

(a) Buy European style three month put option, strike 100.

(b) Buy American style three month put option, strike 100.

(c) Buy six month European style call option strike 110, sell three month American style call option strike 110. [8]

(ii) For each of the following strategies, draw the Gamma profiles against equity price at three times: now, two months from now and **nearly** three months from now.

(a) Buy 1 three month knock-out European Call option strike 120, knock-out at 90.

(b) Buy 1 three month European style Call option strike 100, sell 2 three month European style Call options strike 110. [6]

[Total 14]

3 You are asked to consider two corporate bonds:

Bond name	*Maturity*	*Annual coupon*	*Clean price*	*Gross redemption yield per annum*
X	1 year	6%	99.50	6.53%
Y	2 year	5%	98.00	6.09%

Money market rates, which can be taken as risk-free, are 4.75% compounded annually for all maturities.

(i) Calculate the equivalent risk free bond prices for each corporate bond. [2]

(ii) Calculate the single premium risk neutral price of a 1 year Credit Default Swap (CDS) on bond X, assuming default can only take place on final maturity. [3]

(iii) Calculate the single premium risk neutral price of a 2 year CDS on bond Y assuming default can only take place on final maturity. [2]

(iv) If default can take place on any payment date, demonstrate with reasons why a risk-neutral hedging strategy cannot be generated for a 2 year CDS on bond Y. [6]

(Hint: use a suitable binomial tree for parts (ii), (iii) and (iv).)

For the remainder of this question, you may assume that bond X and bond Y are from the same issuer under (virtually) identical documentation, with full cross-default protection.

(v) Find a formula for p_{01}, the risk neutral probability of default of the issuer, in terms of the post default recovery rate, and show three illustrative values. In particular, explain what happens to p_{01} if the post default recovery rate is 100% of the corporate bond cashflows. [3]

(vi) Discuss reasonable assumptions for the value of the post default recovery rate, and use it to estimate the value of a single premium continuously defaultable CDS on bond Y. [3]

(vii) Briefly discuss how to set up an optimal hedging strategy for a purchased CDS as in (vi). [2]

(Note: The buyer of a CDS will receive, on a default trigger event, an amount equal to: (principal + payments outstanding) – (post trigger bond value).

A continuously defaultable CDS will pay this amount on default whenever it should occur. You may ignore taxes and accrued interest.) [Total 21]

4 You are the Appointed Actuary of a life assurance company that makes extensive use of over-the-counter derivatives to manage its risks relating to investment guarantees which it has written. You have been asked to conduct a review of the adequacy of management controls for the derivative portfolio in the context of your role as Appointed Actuary.

(i) Summarise the guidance available to you from GN25 to assist you with this task. [5]

(ii) Describe how in practice such a review might proceed. [10]

[Total 15]

5 The current zero coupon yield curve in a given fixed income market is as follows (all rates assume annual compounding):

Term	*Rate % p.a.*
1 year	3.25
2 year	3.35
3 year	3.65
4 year	3.95
5 year	4.20
6 year	4.35
7 year	4.50

(i) Calculate the continuously compounded 5-year zero coupon rate. [1]

(ii) Calculate the value of a zero coupon bond of maturity 3½ years. [1]

(iii) Calculate the fixed leg coupon of a 5-year par value annual to annual interest rate swap. [2]

(iv) Calculate the fixed leg coupon of a forward-starting 5-year par value annual to annual interest rate swap commencing in two years time. [3]

(v) A constant maturity swap (CMS) is a swap where each payment on the floating leg is reset to be equal to the fixed leg coupon of a par value interest rate swap of the given maturity that is being traded in the market at each reset date.

(a) Calculate the fixed leg coupon of an annual to annual par value 1-year CMS which is based on the 5-year swap rate.

(b) Explain, without giving detailed algebra, how you would value a 2-year CMS based on the 5-year swap rate. What further information would you require?

[5]

(Note: You may use a simplified day-count basis, and ignore taxes.) [Total 12]

6 In your bank's Treasury division, vanilla caps, floors and swaptions are currently valued using the Black model. You have been asked to propose a (single currency) term structure yield curve model for pricing and hedging exotic interest-rate swaps and options.

(i) Discuss why you might need a full yield curve model, and describe the features which would be desirable in your model. [4]

(ii) Comment on the suitability (or otherwise) of the following in the context of a full yield curve model:

(a) one-factor equilibrium models
(b) no arbitrage models

Define all the terms you use. [5]

The Hull-White (HW) one-factor model for valuing interest-rate swaps and options projects the short-term rate r according to the formula:

$$dr = a(t)\big(b(t) - r\big)dt + \sigma(t)dz$$

where $\sigma(t)$ is the short-rate volatility and $z(t)$ is a standard Brownian motion.

(iii) Describe how you would construct a trinomial rate tree for this model, calibrated to the term structures of zero coupon bond prices and caplet volatilities, when $a(t) = a$ and $\sigma(t) = \sigma$ are constants for all t. [8]

(Note: You may wish to use some algebra, but you do not need to derive the branching equations.)

(iv) Comment on how far the model in (iii) fulfils the desirable features you listed in (i). [4]

[Total 21]