Faculty of Actuaries **Institute of Actuaries**

ADVANCED CERTIFICATE IN DERIVATIVES: FURTHER MATHEMATICS, PRINCIPLES AND PRACTICE

Examination Paper

April 1999

Time allowed: Three hours

INSTRUCTIONS TO THE CANDIDATE

1. *You have 15 minutes at the start of the examination in which to read the questions. You are strongly encouraged to use this time for reading only but notes may be made. You then have three hours to complete the paper.*

2. *You must not start writing your answers in the booklet until instructed to do so by the supervisor.*

3. *Write your surname in full, the initials of your other names and your Candidate's Number on the front of the answer booklet.*

4. *Mark allocations are shown in brackets.*

5. *Attempt all 6 questions, beginning your answer to each question on a separate sheet.*

AT THE END OF THE EXAMINATION

Hand in BOTH your answer booklet and this question paper.

In addition to this paper you should have available actuarial tables, derivatives formulae sheet and an electronic calculator.

ACiD/A99

1 (i) Outline three methods by which an investment bank can reduce the risk of default in a derivatives contract and comment on any problems associated with each method. [9]

(ii) Some financial institutions which do not have a "AAA" rating from a credit rating agency have set up special operating subsidiaries with a "AAA" credit rating for the purpose of trading derivatives.

Describe the characteristics of such subsidiaries which allow them to achieve a "AAA" credit rating despite the fact that the parent company does not have a "AAA" credit rating. [5]

(iii) A life assurance company holds an unquoted bond issued by a multinational pharmaceutical company.

To hedge against the risk of default, the investment manager has bought a credit derivative from a bank. Under the terms of the credit derivative, the bank will take delivery of the bond and pay the life insurance company the par value of the bond in the event that the pharmaceutical company fails to meet its interest or capital repayment obligations on any of its publicly quoted bonds.

Describe briefly any potential weaknesses inherent in this particular credit-risk hedge. [2]

[Total 16]

2 (i) You are the treasurer of a Japanese bank. Your bank is trying to find ways to improve the low returns now available from Yen deposits, and wishes to purchase exotic currency options to enhance the return by expressing views on future movements in both the Yen-$ spot rate itself (currently ¥112) and the volatility of that rate. After a recent sharp fall in the Yen-$ spot rate due to US dollar weakness, option implied volatility is at an historic high.

Describe with reasons, a possible type of exotic option which you could purchase and which would be best suited to each of the three following scenarios:

(a) The bank is going to receive a stream of US$ payments which must be converted to Yen. You want to guarantee a minimum average exchange rate to Yen throughout the next 12 months at the lowest cost.

(b) You expect that the US dollar will continue to weaken over the next six months, but that most of the move has now happened and the last "leg" towards ¥100 will be resisted by the authorities.

(c) You think that the US$ has now "found a level" and will trade in the range ¥100 to ¥120 over the next three months. Further, you wish to "gear up" this view, as you do not think there is any likelihood of the rate breaching either level in that time.

[Calculations and expiry diagrams are not required.] [6]

(ii) One of your colleagues who trades options on bond futures has suggested that there is no difference between using European-style and American-style currency options. Explain briefly why he might be mistaken.

[2]
[Total 8]

3 (i) You are using the Black-Derman-Toy (BDT) model of the discrete short-term rate *r* to value interest-rate swaps and options.

Under BDT, the process for *R*, the continuous-time equivalent of *r*, is given by the formula:

$$d(\ln R) = \left(\theta(t) + \frac{\partial \ln \sigma(t)}{\partial t} \ln R \right) dt + \sigma(t) dz$$

where $\sigma(t)$ is the short-rate volatility and $z(t)$ is a standard Brownian motion.

Describe briefly how you would use the term structures of zero coupon bond prices and caplet volatilities to derive the BDT rate tree for a single currency.

[4]

(ii) You have been given the following term structures for the years $t = 0$ to 4 of a Euro-zone swap curve:

Time t in years	*Zero coupon bond prices*	*Caplet volatility in %*
0	100.000	
1	97.023	22.0
2	94.124	22.5
3	91.008	21.5
4	87.738	20.0

You have then obtained the binomial tree for the annual short rate $r(t)$ in %, for $t = 0$ to 3 and time steps of one year, as follows:

0	*1*	*2*	*3*
			6.695
		5.132	
	3.752		4.355
3.069		3.272	
	2.416		2.833
		2.086	
			1.843

(a) Calculate, using the tree, the value of a four-year 3.5% cap and floor with annual resets and explain how you would calculate their deltas with respect to the short rate. Demonstrate that Put-Call parity is satisfied for the option prices.

(b) Explain how your answers might differ from the values which would be obtained from the usual Black futures model with the same input term structures.

(c) Describe *without calculation* how the BDT model above might be used value a European two-year option on a four-year bond, and discuss briefly how accurate the answer might be.

[Note: a four year annual cap or floor has three annual options expiring at the ends of years 1, 2 and 3 respectively. The strike rate may be assumed to have the same day-count convention as $r(t)$.]

[12]

(iii) Compare the features of the BDT model above with the single-factor Hull-White model of the short rate, and list the strengths and weaknesses of each model. [8]

[Total 24]

4 (i) Describe briefly what must be delivered upon exercise of exchange traded put and call options on futures contracts. [4]

(ii) Discuss the reasons for the popularity of options on futures contracts. [4]

(iii) (a) Demonstrate how the Black-Scholes formulae for European options on non-dividend paying stocks can be extended to apply to European options on stocks paying a continuous dividend at a known dividend yield.

(b) Develop formulae for European put and call options on a futures contract, assuming that the contract can be treated as a security with a continuous dividend yield equal to the risk-free rate of interest. [8]

(iii) Calculate the price of a 5-month European put option on a futures contract when the futures price is £19, the risk-free rate of interest is 12% per annum, the strike price is £20 and the volatility of the futures price is 20% per annum. [2]

[Total 18]

5 (i) Describe the concepts behind the variance-covariance approach to Value at Risk (VaR) and explain the uses of daily VaR reports for managing market risk in a large, multi-currency fixed-income trading department. [6]

(ii) (a) Outline how you would set up a daily VaR-based risk management reporting structure for the fixed income department, and in particular how you would cope with the following types of instrument:

- bond and interest-rate futures
- fixed rate and floating rate bonds
- interest-rate swaps

(b) Describe briefly how in practice you would adjust the VaR values intraday to allow for new trades as they take place. [12]

[Total 18]

6 A life insurance company has just written £700 million of 5-year single premium bonds over the six week period ending on 31st March 1999 (the "closing date"). The policyholders invested an average of £25,000 and were guaranteed a return equal to the greater of:

- 95% of their original investment and
- the amount of their original investment multiplied by the ratio of the FTSE 100 index five years from the closing date to the FTSE 100 index on the closing date.

Early surrender values are linked to the value of the underlying derivative contracts but the amount payable on death is equal to the amount of the original single premium investment. The liability to policyholders is backed by five different derivative contracts each with a different investment bank.

You have just taken over the post of appointed actuary of the company and have sufficient expertise to assess the value of the derivative contracts independently of the investment banks. You have been asked by the board of the company to assess whether suitable reserves or provisions exist for this block of single premium business.

Discuss the issues that you would consider in responding to the board's request.

[16]