

EXAMINATIONS

10 April 2000 (am)

Advanced Certificate in Derivatives: Further Mathematics, Principles and Practice

Time allowed: Three hours

INSTRUCTIONS TO THE CANDIDATE

1. *You have 15 minutes at the start of the examination in which to read the questions. You are strongly encouraged to use this time for reading only but notes may be made. You then have three hours to complete the paper.*
2. *You must not start writing your answers in the booklet until instructed to do so by the supervisor.*
3. *Write your surname in full, the initials of your other names and your Candidate's Number on the front of the answer booklet.*
4. *Mark allocations are shown in brackets.*
5. *Attempt all 6 questions, beginning your answer to each question on a separate sheet.*

AT THE END OF THE EXAMINATION

Hand in BOTH your answer booklet and this question paper.

In addition to this paper you should have available actuarial tables, derivatives formula sheet and an electronic calculator.

1 A pension scheme awards deferred annuities to members who leave service before retirement. The deferred annuities fall under the Social Security Act (1985) and must be increased over the period from leaving service to the members' normal retirement dates in line with increases in the Retail Prices Index (RPI) but subject to a maximum of 5% per annum compound. While the rate of inflation as measured by the RPI may be negative from one period to another the amount of the annuity on the vesting date cannot be less than that awarded on leaving service.

- (i) Consider a member who has just left the pension scheme and has been awarded a deferred annuity of £1 per annum, deferred for n years. Let $Q(t)$ denote the value of the RPI at time t with the current value being $Q(0) = 1$. Let $R(n) = 1.05^n$.
- (a) Write down an expression for the annualised annuity benefit re-valued after n years.
- (b) Explain how the benefit can be viewed as combining two European options and describe these options. [6]
- (ii) Let δ denote the uniform force of interest that can be earned on fixed money investments and let η denote the uniform force of interest earned on investments linked to the RPI.

Suppose that the RPI follows the following diffusion process:

$$d(\log Q(t)) = \mu(t, Q(t)) dt + \sigma dz$$

where

$\mu(t, Q(t))$ is the mean rate of change

σ is the standard deviation (a constant) of the diffusion process

Z is a unit Normal (Brownian motion) process

- (a) Show that the value of the deferred annuity at time t , V_t , is given by:

$$V_t = Q_0 e^{-\eta T} (1 - \Phi(d_1) - \Phi(f_2)) + A e^{-\delta T} \Phi(f_1) + B e^{-\delta T} \Phi(d_2)$$

where

$$Q_0 = Q(0)$$

$\Phi(\cdot)$ is the standard cumulative Normal distribution function

$$T = n - t$$

$$f_1 = \frac{\log(A e^{-\delta T} / Q_0 e^{-\eta T})}{\sigma \sqrt{T}} + \frac{\sigma \sqrt{T}}{2}$$

$$f_2 = \frac{\log(Ae^{-\delta T} / Q_0 e^{-\eta T})}{\sigma\sqrt{T}} - \frac{\sigma\sqrt{T}}{2}$$

A , lower limit of the annuity (£1 per annum)

B , upper limit of the annuity ($R(n) = 1.05^n$)

$$d_1 = \frac{\log(Q_0 e^{-\eta T} / B e^{-\delta T})}{\sigma\sqrt{T}} + \frac{\sigma\sqrt{T}}{2}$$

$$d_2 = \frac{\log(Q_0 e^{-\eta T} / B e^{-\delta T})}{\sigma\sqrt{T}} - \frac{\sigma\sqrt{T}}{2}$$

- (b) Comment on the suitability of the assumptions used in this model.
- (c) Compare the results that would be obtained using the more traditional and simpler approach of assuming a constant (such as 5%) rate of growth for the RPI.

[14]

[Total 20]

- 2** A UK company needs to convert US \$ 1 million into sterling in slightly over three months time. The current \$/£ spot exchange rate is 1.6080.

An investment bank offers the company a short position in a range forward contract. Under this contract, if the spot price is less than $X_1 = 1.55$ the company sells its dollars at the rate of X_1 . If the spot price is greater than $X_2 = 1.65$ then the company sells its dollars at the rate of X_2 . If the spot rate is between X_1 and X_2 , then the company sells its dollars at the prevailing spot rate.

- (i) Write down an expression for the payoff at expiry from this range forward contract as a function of the spot rate in 90 days time, the minimum and the maximum delivery prices (work in pounds per dollar). Sketch the payoff function. [4]
- (ii) Show that the payoff function is mathematically equivalent to the sum of the payoffs from two European options and identify these two options. [4]

[Total 8]

- 3** The life insurance company where you work is considering using derivatives to hedge liabilities relating to guaranteed annuity options. At a recent investment committee meeting, the admissibility criteria for derivatives was discussed. One of the criteria is that the derivatives must be used for reducing investment risk or for efficient portfolio management purposes.

Summarise the guidance available from Professional Guidance Note 1995/3 on what is meant by “reduction in investment risk” and “efficient portfolio management” and how to assess whether a proposed derivative satisfies these criteria. [18]

- 4** Calculate the price of a three-month European put option on a fifteen-month bond with half yearly coupons of 6%. Use the Hull and White model with parameters $\sigma = 0.015$ and $\alpha = 0.1$. Today, the term structure of interest rates is given by the equation $r(t) = 0.09 + 0.02t$. The strike price of the option is £100 (per £100 nominal). [22]

- 5** A United Kingdom pension fund manager holds a portfolio of US stocks which mirrors the performance of the S&P 500 index. The portfolio is valued at \$100m when the S&P 500 index stands at 1,400. The trustees of the pension scheme have ordered the manager to put in place insurance against a reduction of more than 5% in the value of the portfolio over the next 6 months.

- (i) Calculate the cost of the portfolio insurance if the fund manager buys European put options. [3]
- (ii) Calculate the initial position in risk-free securities the fund manager should take to hedge her exposure dynamically. [3]
- (iii) Discuss the relative merits of the two approaches in (i) and (ii). [2]
- (iv) Calculate the number of contracts the fund manager will need if she decides to provide insurance using 9-month S&P 500 index futures contracts. [3]
- (v) Describe briefly an alternative strategy open to the fund manager involving European call options and show that it leads to the same result as in (i) above. [4]

You may assume the following:

- The dividend yield on the portfolio and the S&P 500 index is 3% per annum.
- The risk free rate of interest is 6% per annum.
- The volatility of the index is 30% per annum.
- Each futures contract on the S&P 500 index is for $250 \times$ the index.

[Total 15]

- 6** You are the consultant specialising in credit risk management with particular emphasis on the credit risk management of derivatives contracts. The board of a derivatives dealer is reviewing its credit risk management practices and has asked you to produce a memorandum on the two main approaches to credit enhancement and credit risk reduction.

Draft the memorandum.

[17]