**Faculty of Actuaries** **Institute of Actuaries**

# EXAMINATIONS

April 1999

## Advanced Certificate In Derivatives: Further Mathematics, Principles and Practice

EXAMINERS' REPORT

**1** (i)

Credit line limits

An investment bank could manage its credit exposures to counterparties by setting a maximum credit exposure it will tolerate for each counterparty.

These credit line limits are assessed by the investment bank before it enters into derivatives transactions with counterparties. The limits should take account of the potential exposure that could arise from movements in the underlying variables of the derivative contract.

Credit risk in relation to derivatives is a dynamic concept and so the limits need to reviewed on an ongoing basis by the bank's credit analysts. The frequency of monitoring needs to be appropriate to the volatility of the variables underlying the derivatives contract between the investment bank and the counterparty.

The main problems with credit line limits is that they prevent a bank from lending more than the line limit to the customer even if the terms of the derivative contract are particularly attractive to the bank. A refusal to enter a deal because a customer has reached their credit line limit may damage the customer relationship.

Reducing the underlying exposure by means of collateral

To offset exposure in derivatives transactions counterparties may post collateral. The idea being that a counterparty that has pledged collateral and subsequently defaulted would forfeit that collateral to the non-defaulting counterparty in or towards payment of the defaulted obligation.

The collateral agreement needs to be enforceable.

Cash and government securities are the most common forms of collateral.

In the absence of netting, collateral is posted on a gross basis. Otherwise it is based on the net negative mark-to-market value.

Collateral arrangements may be bilateral or unilateral. Under the former arrangement, there are two-way flows of collateral; the party with the negative mark-to-market value collateralises the exposure of the other party.

Under the latter arrangement, one party is required to deliver collateral to the other party on trades in which it has a negative mark-to-market value. The other party to the transaction is not required to post collateral in any circumstances.

Unilateral collateral arrangements may be used when one of the parties has a lower credit rating than the other.

Collateral arrangements vary to suit the parties. For example, there may be an amount of collateral required up-front, the frequency of collateral calculations may vary from case to case or the obligation to post collateral may be triggered by an event such as a credit downgrade.

The main problems with collateral arrangements is that the two parties need to agree on a valuation model for the contract and agree the rate of interest to be paid on collateral.

Another problem is the additional administration associated with the collateral aspects of the derivative contract.

<u>Contract pay-offs designed to reduce credit risk</u>
The contract pay-off is designed to reduce credit risk.

This approach is sometimes used when a financial institution wishes to buy a derivative contract like an option from a counterparty with a lower credit rating.

An example of such a contract would be where the financial institution referred to above agrees to purchase an option from the counterparty with the lower credit rating but to pay the option premium (adjusted for interest) at expiry of the option rather than up-front.

Another example of such a contract is a range forward contract (a combination of a long forward contract, a long put and a short call designed to have zero cost by choosing the strike prices of the put and the call so that they are equal in value).

The problem with this approach is that the amount of credit risk protection is limited.

<u>Downgrade triggers</u>
Downgrade triggers require a derivatives contract to be closed out when a party to it has suffered a credit downgrade (to some predefined level). There is also a requirement for the downgraded party to pay a cash amount to the other party using a predetermined formula.

The problem with downgrade triggers is that they do not completely eliminate credit risk. If there is a big jump in the credit rating of a counterparty, say from AA to Default, in a very short period of time, the non-defaulting counterparty may still suffer a credit loss.

<u>Credit derivatives</u>
In the context of the question, a credit derivative is a generic term used to describe various swap and option contracts designed to lay off credit risk on loans, in return for either interest payments or payment of a premium.

Credit derivatives can be used to do derivative deals beyond the credit line limit of the counterparty as the derivative dealer is hedging his exposure to the counterparty by means of the credit derivative.

There can be many problems with credit derivatives here is an non-exhaustive list of such problems.

- Default of the credit risk buyer (the insurer as it were) may be highly correlated with that of the reference obligation under the credit derivative. For example, a subsidiary company writing a credit derivative on the debt obligations of its parent.

- The term of a credit derivative contract may be shorter than that of the underlying loan the credit risk of which is being hedged.
- There may be legal uncertainty regarding the enforceability of credit derivatives contracts in some jurisdictions.
- There may be a mis-match between the reference asset in the credit derivative contract and the underlying asset being hedged.
- The payoff from the credit derivative may be a fixed monetary amount whereas the loss suffered by the protection buyer may be variable up to the par value of the bond.

(ii)

'AAA' rated subsidiaries (sometimes referred to as special purpose vehicles in this solution) may have some or all of the following characteristics in order to gain a 'AAA' rating despite the fact that their parent company does not have a 'AAA' rating.

<u>Capitalisation</u>
Credit rating agencies require the capital of a 'AAA' rated entity to withstand the most extreme financial stress scenarios involving severe market movements. Thus many special purpose vehicles (SPVs) have a large cushion of excess capital.

<u>Operating guidelines</u>
The SPVs may operate within strict guidelines to achieve diversification of credit risk throughout their portfolio.

Such guidelines may put a percentage of capital limit or a monetary limit on the exposure to any one counterparty (including the parent) or may limit total exposure to counterparties in certain rating categories, countries or industries to a percentage of capital or a fixed monetary amount.

<u>Bankruptcy procedures</u>
The 'AAA' rated subsidiary must be legally separate from the parent in that the parent company may guarantee the subsidiary but the subsidiary must not guarantee the debts of the parent. In the case of bankruptcy or insolvency of the parent it is vital that the SPV is not combined with it.

<u>Transaction matching</u>
Transaction matching tries to ensure that the SPV does not at any time have any open or unhedged positions.

The usual way of achieving this result is to require the SPV to enter into a mirror transaction with its parent for every transaction it enters into with a third party and for the parent to collateralise the SPV's exposure it.

Sometimes the parent may be required to collateralise any excess exposures of the SPV.

In addition, the SPV may assign to its parent any transaction with a counterparty that has caused it to breach its operating guidelines.

Such a breach may arise from the downgrading of one of the SPV's counterparties.

(iii)

The underlying asset being hedged is unquoted whereas under the terms of the credit derivative a publicly quoted bond of the issuer (the reference asset) must fail to meet its interest or capital repayments.

If the unquoted bond ranks below all the publicly quoted bonds, it is possible to have a default on the unquoted bond without a having a default on the publicly quoted bonds.

Thus there is an asset mis-match between the reference asset and the underlying asset.

The investment bank might be heavily exposed to the pharmaceutical industry so that an industry-wide default crisis may cause the bank which wrote the credit derivative to default at the same time as the pharmaceutical bond.

The term of the credit derivative contract may be shorter than that of the underlying bond.

## 2

(i)

<u>General comments</u>
Can only buy options, not sell them, as per the comment at the start of the question. *It is common for a Treasury to want to avoid the risk of selling options.*

Want to avoid buying outright options as volatility is high.

*Parts (a) to (c) are looking for a description of the broadly correct type of option:*

(a)
1 year Asian-style Put (sell \$, buy Yen on exercise) to give average strike.

Slightly cheaper than straight Puts on \$ because average life of option is in fact less than 1 year.

(b)
Barrier option i.e. 6 month knock-out Put. Option cancels if barrier is reached.

Much cheaper than straight Put on \$ because could be cancelled.

Best (for this situation) to make strike in-the-money with barrier out-of-the-money well in the “never reach” zone.

For example, Reverse Knock-Out \$ Put with strike at ¥115 (say) and barrier just below ¥100 (say ¥95), so that option cancels if ¥95 touched.

*Alternative:*
If bank was bearish on $ without a specific level, could use a normal Knock-Out Put with strike at Y109 and cancellation above Y120, so benefit if $ falls without rallying, or even combine the two into a Double Knock-Out. Depends on strength of view.

(c)
Suggest a form of Accrual Note. Pays out if Yen-$ lies in range.

*At least two versions are possible:*
Range Binary - pays an agreed amount only if Yen-$ between ¥100 and ¥120 on every day up to expiry. Only pays out if Yen-$ lies in range ¥100 to ¥120, otherwise returns nothing and premium is lost.
OR
Range Accrual - accrues for each day that Yen-$ lies between ¥100 and ¥120.

Comment about suitablility - Range Accrual is less aggressive and more common as a "yield enhancement" option, but the question also suggests Range Binary as it is more highly geared.

(ii)

Where the exercise of an option is into a future, there is (usually) no funding effect, so the option holder is never paid to exercise early.

Holder will always be giving up at least a small time premium to exercise early (except very close to expiry, or very long in-the-money) so it will not be optimal.

However, this is not true for currencies (or OTC bonds or shares). It will often be optimal to exercise Call options on high interest-rate currencies and Put options on low interest-rate currencies.

This is because cash can then be deposited for the remainder of the option's life in the high interest-rate currency, and the extra return on that deposit will (more than) pay for the lost time premium.
OR: an alternative way of looking at this is that the interest-rate differential implies that the higher yielding currency should depreciate, so the option on the conversion rate will be worth less later than it is now.

Therefore, such American currency options are worth more than European options because of the extra choice. In the case of the type of Yen-$ options considered in (i), American Calls on US$ would have a higher premium than the equivalent European Calls. Puts would be identical.

## 3

(i)

The tree for short rates is constructed in a bootstrapping fashion by matching the zero coupon bonds at each time step $t$ = 1, 2, 3, 4 etc as valued on the tree with their actual price.

The BDT method uses probabilities of ½ up and down, and it performs the backwards integration very easily.

since if $P_{\mathrm{up}}(t + 1)$ = value of the security at an up node at time t+1, and $P_{\mathrm{down}}(t + 1)$ the equivalent downward node value, then:

$$P(t) = \frac{\frac{1}{2}\left(P_{up}(t+1) + P_{down}(t+1)\right)}{\left(1 + \frac{r(t)}{100}\right)} \qquad (*)$$

for any node (*or equivalent description*).

For a zero coupon bond, final payoffs are always 100 (par), and the tree creates the present value of the final payoff

so e.g. for $t$ = 4:

| | | | | |
|---|---|---|---|---|
| | | | | 100.000 |
| | | | 93.725 | |
| | | 90.149 | | 100.000 |
| | 88.493 | | 95.826 | |
| 87.738 | | 93.477 | | 100.000 |
| | 92.367 | | 97.245 | |
| | | 95.720 | | 100.000 |
| | | | 98.190 | |
| | | | | 100.000 |

The bottom rate of the tree for each $t$ is adjusted to give the exact zero coupon bond price, starting with $t$ = 1 and proceeding to higher values of t after each adjustment.

A Newton-Raphson method is required to solve for the bottom rate.

Other rates are related by the formula:

$$r_j(t) = r_{j-1}(t).\exp(2\sigma(t)\Delta t)$$

going from node $j$-1 to node $j$ up the tree at the same time-step.

Note that the volatility can (and should) be time-dependent, either by using a decay constant ν so that $\sigma(t) = e^{-\nu t}\sigma(0)$ or, as we have done in this question, putting in a specific empirical declining volatility curve.

The question gives Δt = 1 year, i.e. one time step per year, which is a very coarse tree (normally you would use over 100 time steps per year).

*In the above formula and what follows, we have assumed a discounting based on ACT/ACT day-count and r(t) as simple interest rates. If ACT/360 day-count is used, then r(t) in the denominator is multiplied by 365/360, and if r(t) is continuous, then (1 + r(t) /100) is replaced by $e^{-r}$.*
*We have also ignored, for simplicity, leap years and the "modified following business day" convention. A real-life swap trader would include them, with the effect of making years ≠ 365 days.*
*Candidates were not be penalised for including, or adjusting for, any of these effects, as long as they were consistent in their approach.*

(ii)

(a)

<u>Calculate 4-year cap and floor + deltas.</u>
*Marking scheme below may need to be adjusted if candidates make a persistent slip. We assume the strike does not need to be adjusted for day-count, though candidates may do this if consistent.*

The basic discounting process in (*) is used throughout.

Caplet prices: final payoff at node $j$ = max $\{r_j(t) - 3.5, 0\}$

t = 1:

| | |
|---|---|
| | 0.2429 |
| 0.1178 | |
| | 0.0000 |

t = 2:

| | | |
|---|---|---|
| | | 1.5523 |
| | 0.7481 | |
| 0.3629 | | 0.0000 |
| | 0.0000 | |
| | | 0.0000 |

t = 3:

| | | | |
|---|---|---|---|
| | | | 2.9945 |
| | | 1.8138 | |
| | 1.0653 | | 0.8193 |
| 0.6107 | | 0.3967 | |
| | 0.1937 | | 0.0000 |
| | | 0.0000 | |
| | | | 0.0000 |

Floorlet prices: final payoff at node $j$ = max $\{3.5 - r_j(t), 0\}$

t = 1:

| | |
|---|---|
| | 0.0000 |
| 0.5135 | |
| | 1.0584 |

t = 2:

| | | |
|---|---|---|
| | | 0.0000 |
| | 0.1064 | |
| 0.4319 | | 0.2208 |
| | 0.7840 | |
| | | 1.3851 |

t = 3:

| | | | |
|---|---|---|---|
| | | | 0.0000 |
| | | 0.0000 | |
| | 0.1513 | | 0.0000 |
| 0.4118 | | 0.3140 | |
| | 0.6975 | | 0.6486 |
| | | 1.1146 | |
| | | | 1.6270 |

Hence cap price = 0.1178 + 0.3629 + 0.6107 = 1.9105 and floor price = 0.5135 + 0.4319 + 0.4118 = 1.3572. These prices are in % of nominal.

Delta for any caplet or floorlet is $(P_{up} - P_{down})/(r_{up} - r_{down})$ at $t = 0$. Given that $r_{up} - r_{down}$ = 3.752 - 2.416 = 1.336, we get:

Caplet $\Delta$ for $t = 1$: 0.1818 $t = 2$: 0.5600 $t = 3$: 0.6524

Floorlet $\Delta$ for $t = 1$: -0.7922 $t = 2$: -0.5072 $t = 3$: -0.4088

These are all separate options, so we don't add the deltas.
*The numerical values were not required by the examiners; they are given to assist in the understanding of the solution.*

Check put-call parity of prices
If we take any caplet price minus the floorlet price, the final payoff is $r(t)$ - 3.5 at each node.

Hence, since the tree creates the present value of the final payoff, the value at $t = 0$ will be the present value of the forward rate less the present value of the strike, which is put-call parity.

*Numerical examples could have been given instead to show Put-Call parity for full marks.*

(b) Comparison with Black model
Both BDT and Black use a log-normal model so using the caplet volatilities for each time step should give very close answers.

BDT should give exact same answers as Black for caps and floors if properly calibrated and small $\Delta t$ (i.e. $\Delta t \to 0$).

Our BDT tree is very coarse, so won't get very precise answers (though amazingly they are very close). Need a smaller time step to be accurate.

Need to be sure day-count methods are same.

Generally our deltas will be poor due to the large time step.

Black deltas are always w.r.t. the forward rate, not the short rate as we calculated.

BDT deltas can be converted into forward rate deltas by first calculating, then dividing by, the sensitivity of the forward rate to the short rate but this is a messy calculation and needs a small time step.

<u>(c) 2-year bond option</u>
Price the bond at each node at the end of year 2 using the discount process in (*). Remember to allow for coupons due after year 2 but not before (the option will be based on the clean price).

Using these prices, calculate the final payoff at each node of $t = 2$ and value the option in the same way as the caplets above using (*).

The answer will be suspect, as the option life and bond life are too similar.

In reality, the correlation of rates will affect the value of the option (in general, BDT will overvalue). BDT does not have an explicit way of expressing the imperfect correlation of forward rates.

(iii)

<u>Comparison with HW 1-factor model.</u>

Both models can fit all shapes of yield curve perfectly but both are single factor models, so only have limited ability to model simultaneously the yield curve <u>and</u> volatility structure.

Both fail to give explicit imperfect instantaneous correlation of forward rates.

Both achieve a good fit by compromising on the future evolution of the term structure over time with HW generally having a poorer snapshot but better evolution.

HW uses explicit mean reversion whereas BDT this is achieved by a declining volatility.

BDT uses binomial tree to value HW a trinomial because of the extra degrees of freedom.

BDT fits the curve by adjusting the bottom rate whereas HW's trinomial is much more efficient because it avoids areas of low probability, thereby not wasting calculation on unimportant areas of the tree.

BDT is log normal, HW normal which makes BDT easier to calibrate to Black-derived caps and/or swaptions.

Also, theoretically HW could give negative rates, although only with extremely low probability.

In fact, very hard to calibrate HW, as at-the-money options are priced in both normal and log-normal distributions but the differences come away from the current level of rates. Any cap prices will include a range of at-the-money, out-of-the-money and in-the-money caplets.

Need to disentangle these effects, otherwise HW will not have correct balance between volatility and mean-reversion parameters. In some cases, declining volatility as observed in caplet pricing can lead to failure of the trinomial process further out in the tree.

## 4

(i)

<u>Delivery</u>

If a call futures option is exercised, the holder receives a long position in the underlying futures contract plus an amount of cash equal to the current futures price minus the strike price.

If a put futures option is exercised, the holder gets a short position in the underlying futures contract plus an amount of cash equal to the difference between the strike price and the current futures price.

(ii)

<u>Reasons for popularity</u>
Generally speaking, futures options run up lower transaction costs than spot options.

Trading of futures and futures options is arranged in adjacent pits in exchanges and this facilitates arbitrage, hedging and indeed speculation which tend to make the markets more efficient.

When it is cheaper or more convenient to deliver futures contracts on the asset rather than the asset itself, futures options are more attractive to investors than options on the underlying asset. This is true for many commodities and stock indices.

The exercise of a futures option does not generally lead to delivery of the underlying asset - typically the futures contract has a somewhat longer maturity than the option contract.

When investors would find it difficult to come up with the money to buy the underlying asset upon exercise of an option on the underlying, a futures contract (which is essentially cash settled as the future can be closed out immediately) is much more attractive.

(iii) (a)

The price of a stock falls when a dividend is paid; the fall being roughly equal to the amount of the dividend.

Payment of a continuous dividend at a rate $q$ causes the growth rate of the stock to be less than it would otherwise be.

Suppose that with a continuous dividend yield of $q$, a stock price grows from S at time $t$ to $S_T$ at time T.

Then if there were no dividends, it would grow from S at time t to $S_T e^{q(T-t)}$ at time $T$.

[Put another way, in the absence of dividends it would grow from $S\,e^{-q(T-1)}$ at $t$ to $S_T$ at time $T$.]

The same probability distribution applies to the stock price at time $T$ no matter which way one looks at the problem.

So when valuing a European option lasting for a time ($T$ - $t$) on a stock paying a known dividend yield equal to $q$, one can simply reduce the current stock price from $S$ to $S\,e^{-q(T-1)}$ and then value the option as though it paid no dividends.

The standard BS formula is thus altered replacing $S$ with $S\,e^{-q(T-t)}$.

We obtain the price $C$ of a European call option and the price $P$ of a European put option on a stock providing a continuous dividend yield at rate $q$ as:

$$C = Se^{-q(T-t)}N(d_1) - Xe^{-r(T-t)}N(d_2)$$
$$P = Xe^{-r(T-t)}N(-d_2) - Se^{-q(T-t)}N(-d_1)$$

Since $\ln\left(\frac{Se^{-q(T-t)}}{X}\right) = \ln\left(\frac{S}{X}\right) - q(T-t)$

we can write $d_1$ and $d_2$ as:

$$d_1 = \frac{\ln\left(\frac{S}{X}\right) + (r - q + \frac{1}{2}\sigma^2)(T-t)}{\sigma\sqrt{T-t}}$$

$$d_2 = \frac{\ln\left(\frac{S}{X}\right) + (r - q - \tfrac{1}{2}\sigma^2)(T-t)}{\sigma\sqrt{T-t}} = d_1 - \sigma\sqrt{T-t}$$

(b)

Using part (a), with $q = r$ and $S = F$, where F is the futures price:

$$C = e^{-r(T-t)}\left(FN(d_1) - XN(d_2)\right)$$
$$P = e^{-r(T-t)}\left(XN(-d_2) - FN(-d_1)\right)$$

with

$$d_1 = \frac{\ln\left(\frac{F}{X}\right) + \tfrac{1}{2}\sigma^2(T-t)}{\sigma\sqrt{T-t}}$$ and $d_2 = d_1 - \sigma\sqrt{T-t}$ as before.

(iv)

Here $F$ = 19, $X$ = 20, $r$ = 12% = 0.12, $s$ = 20% = 0.2 and $T$ - $t$ = $5/12$ = 0.4167.

Put price = $20N(-d_2)e^{-0.12x0.4167} - 19N(-d_1)e^{-0.21x0.4167}$

where

$$d_1 = \frac{\ln\left(19/20\right) + \tfrac{1}{2}(0.04)(0.4167)}{0.2\sqrt{0.4167}} = -0.3327$$ and $d_2 = d_1 - 0.2\sqrt{0.4167} = -0.4618$

Hence

$$\begin{aligned}\text{Put price} &= e^{-0.12x0.4167}\left[20N(0.4618) - 19N(0.3327)\right] \\ &= e^{-0.12x0.4167}\left[20(0.6778) - 19(0.6303)\right] \\ &= 1.50\end{aligned}$$

**5**

(i)

Daily value at risk (DVaR) for a portfolio is a number which indicates the maximum loss which the portfolio can sustain over a given day for a given confidence level (say 95% or 99% certainty).

Under the variance-covariance framework, DVaR is calculated by taking into account the magnitude of future price movements (variance, or volatility) and the interdependencies of the portfolio constituents (covariance, or correlation).

The sensitivities of each instrument to interest rate changes are collated in terms of fewer, more basic instruments, e.g. 2-year, 5-year and 10-year discount bonds.

Then, assuming a normal distribution of returns, they are combined into a theoretical portfolio using standard statistical theory.

DVaR reports are used for a number of purposes.

- Quantification of risk - either single risk amount for whole firm or broken down into business units.

- Management reporting of risk for regulatory compliance - senior managers must know the risks they are taking. Also independence of risk unit important, so risks cannot be "hidden" from management.

- Individual risk reports allocated to each P&L unit to measure risk-adjusted performance.

- Hedging - enabling expression of risk into a few components which can quickly and easily be hedged if the need arises. Also tests hedge efficiency.

- Stress testing (for large moves) to avoid insolvency in a crisis - actually, variance-covariance DVaR is not so useful for this.

(ii)

(a) Framework and types of instrument

*Below is only one possible implementation..*

Obtain all "positions" for each instrument. A position is a total amount for securities like bonds, which can be aggregated, otherwise is deal-by-deal e.g. for swaps.

Obtain book/desk structure corresponding to P&L units and reporting lines - this is for later decomposition of DVaR.

Choose a limited set of instruments which more-or-less map the entire yield curve. These become the "synthetic securities". These would typically be zero-coupon bonds of length 3 and 6 months, and 1, 2, 3, 5, 7, 10 and 30 years (or similar combinations depending on the type of business).

Obtain price or yield histories for these instruments, possibly from generic series, and calculate the total variance-covariance matrix for all possible combinations. This explains why you need synthetic securities - the matrix would be far too large and have lots of redundant rows if it contained all possible instruments.

One set of instruments is required for each currency traded, although not necessarily a full set for each, and some smaller currencies could be aggregated. The variance-covariance matrix implicitly includes the correlation between currencies themselves.

Express all positions in terms of the synthetic securities. There is no agreed way to do this: for instance, it can be done by linear interpolation, or by matching the sensitivity to rate changes.

Choose a confidence interval for the DVaR and multiply the positions in the synthetic securities by the variance-covariance matrix as per the formulae.

The basic structure allows a mapping of any zero-coupon bond onto the synthetic securities by some sort of interpolation. Hence, if we can express the instrument as a combination of such zeros, we can obtain our VaR.

*Bond and interest-rate futures*
Interest-rate futures are FRAs (forward rate agreements) with cash settlement up-front, so these can be expressed as a loan at expiry for - usually - three months.

Then express this loan as a loan now up to expiry + three months, and a deposit starting now up to expiry. This pair is effectively two zero-coupon bonds.

Bond futures should be considered as positions in the cheapest-to-deliver (CTD) bond at expiry, after adjusting for the pricing factor.

Then express this forward bond position as a bond (see below) with cash payment at the expiry date.

*Fixed rate and floating rate bonds*
Fixed bonds can be considered as a set of zero-coupon bonds representing each cashflow (coupon and final redemption) together with an initial payment to neutralise the currency effect.

Floating-rate notes (FRNs) reset at LIBOR can be considered as a zero-coupon bond on the next reset, as the next payment is fixed and the remaining LIBOR series will always be valued at par.

If the FRN has a margin, this annuity has to be treated like the fixed coupon part of a bond.

*Interest-rate swaps*
These can be decomposed into a fixed-rate bond and a FRN including the margin on the floating side.

(b) Adjustment for incremental VaR

DVaR can be adjusted incrementally since a portfolio of two normally-distributed assets is also normally distributed.

Let $X_1$ be the portfolio value before the new trade, and $X_2$ the value of the new trade itself, where $X_2 = a.\ X_1$ for $a << 1$. Let $X_c$ be the combined portfolio, and $\sigma_k$ the respective volatilities of the $X_k$.

Then $(1+a)^2\sigma_c^2 = \sigma_1^2 + a^2\sigma_2^2 + 2a\rho_{12}\sigma_1\sigma_2$, and since $a$ is known to be small, the middle term can be ignored, so $(1+a)\sigma_c \approx \sqrt{\sigma_1^2 + 2a\rho_{12}\sigma_1\sigma_2} = \sigma_1\sqrt{1+2a\beta_{12}}$ where $\beta_{12}$ is the new asset's portfolio beta ( = covariance / new variance).

Expanding the square root term for small $a$ (ignoring terms in $a^2$), we get $(1+a)\sigma_c \approx \sigma_1(1+a\beta_{12})$.

So, since DVaR is proportional to the variance of a portfolio times its value, we have shown that combined portfolio has approximately extra DVaR of $\Delta$DVaR = $a.\beta_{12}.(\mathrm{DVaR}_{old})$. Hence we have defined IVaR = incremental DVaR in a simple formula.

Alternative is to recalculate entire portfolio at regular intervals throughout the day, which could be very computer intensive and slow if the portfolio is large.

## 6

GN1 requires appointed actuaries to consider how the company's assets have been valued in determining the mathematical reserves taking into account the degree to which assets and liabilities are matched.

The derivatives in question are over-the-counter (OTC) derivatives and are assets available to back the liabilities to policyholders.

The actuary would need to consider whether the value placed on these contracts adequately reflects their realisable values. In the light of the death benefit, provision may need to be made for any shortfall between realisable value and the estimated cost of paying death claims.

The actuary should also consider the extent to which the guarantee can be matched using the new zero-coupon bonds available in the UK market and the implications of this for the value of the five OTC derivative contracts.

The actuary would need to consider the element of judgement involved in the models and input parameters used to price such derivatives. It is also important to consider the risk that the model proves inaccurate or does not capture some fundamental aspect of the market.

The actuary would need to consider whether the model is reasonable in the context of historical experience especially in relation to price movements in the FTSE 100.

The actuary should be satisfied that the hedging techniques operated by the investment banks are adequate particularly if markets jump suddenly near the end of the life of the single premium bonds.

The actuary would need to have regard to the terms on which a part of the derivative contracts can be closed out prior to maturity to match lapses in the underlying liabilities and the frequency of such close outs.

The actuary needs to be aware that realisable value of the derivative contracts can change in value very quickly.

He or she should not rely on valuations struck even a day ago in deciding surrender values. These should be based on the next available price matching the amount paid out to the realisable value of the part of the derivative closed out. This may involve surrenders waiting until the end of a calendar month or quarter before surrender can take place.

Consistency between the valuation of assets and liabilities is paramount in writing this type of business.

Checks should be carried out on the aggregation of counterparty risk and market exposure in order to ensure compliance with admissibility limits.

This is particularly important in the context of counterparty exposure following large market movements. The actuary should ensure that there are adequate controls to assess this type of exposure on a timely basis and that there are adequate free assets to cover any likely admissibility restrictions arising from such market movements.

The actuary would need to consider the ability of the company to meet its liabilities if there were a change in the tax treatment of the derivatives contracts in question.

Uncovered derivatives positions may arise from the failure to close out some part of the derivative contracts when payments have been made in respect of lapses. Such exposures must be carefully monitored as they are uncovered derivative positions.

The impact of the derivative contracts on the overall position of the company must be considered by the appointed actuary.

The actuary must consider the effect that derivative positions may have on the income or redemption yield of the portfolio as this yield may be used to help the actuary to choose the valuation rate of interest.

The credit risk of the derivative contracts must also be considered in the context of establishing the valuation rate of interest.

The actuary should ensure continuous monitoring of the derivative positions to fulfil his or her responsibilities under paragraph 4.1 of GN1.

When preparing a financial condition report, the actuary should have regard to the derivative contracts in his portfolio.

The actuary may need to consider the adequacy of controls in the context of the use of derivatives. In particular, that the use is consistent with the objectives and policies laid down by the board for the use of derivatives.

If controls are inadequate, the actuary may need to set aside additional reserves while they are being put right.