EXAMINATIONS

11April 2002 (pm)

Advanced Certificate in Derivatives: Further Mathematics, Principles and Practice

Time allowed: Three hours

INSTRUCTIONS TO THE CANDIDATE

1. *Enter all the candidate and examination details as requested on the front of your answer booklet.*

2. *You have 15 minutes at the start of the examination in which to read the questions. You are strongly encouraged to use this time for reading only, but notes may be made. You then have three hours to complete the paper.*

3. *You must not start writing your answers in the booklet until instructed to do so by the supervisor.*

4. *Mark allocations are shown in brackets.*

5. *Attempt all 5 questions, beginning your answer to each question on a separate sheet.*

Graph paper is not required for this paper.

AT THE END OF THE EXAMINATION

Hand in BOTH your answer booklet, with any additional sheets firmly attached, and this question paper.

In addition to this paper you should have available Actuarial Tables, Derivatives Formula Sheet and your own electronic calculator.

1 A life insurance company is considering buying various tranches of swaptions from an investment bank to help hedge its liabilities relating to non-profit pension savings contracts (deferred cash sums). These provide the policyholder with the option to effect an annuity on guaranteed minimum terms at retirement.

Each swaption contract gives the insurance company an option which, if exercised on a specified date, allows it to swap the coupons of a floating rate bond of a specified term for the coupons of a fixed rate bond of the same term. The effective initial date of each swap coincides with the exercise date of the swaption. An option premium is payable to the investment bank at the commencement date of each contract.

The parameters of the swaptions (option exercise dates, strikes, nominals) are such that the swaption cash flows are expected to match quite closely the cash flows in respect of the guaranteed annuities.

The investment bank has agreed that it is willing periodically to quote "close-out" prices on up to £5 million nominal of underlying bonds. The total volume of bonds involved in the transactions is £100 million nominal.

The insurance company is considering whether these derivatives will be admissible under the asset valuation regulations. It is therefore considering the seven conditions for admissibility as set out in DTI Prudential Guidance Note 1995/3.

(i) Explain the following requirements in the PGN 1995/3, and discuss whether the derivatives described above satisfy them:

(a) "covered" [4]
(b) "in connection with" [4]
(c) capable of being readily closed out [4]
(d) admissible underlying assets [5]

(ii) List the three other requirements that must be satisfied in order that the derivatives can be treated as admissible assets. [2]

[Total 19]

2 The Kingdom of Merovence has one international bond issue, which is denominated in sterling and has exactly one year to maturity. It bears a coupon of 8% paid annually and is trading actively at a price of 95. A coupon has just been paid.

You are given the following market information and details:

- All interest rate curves are constant over the one year period.
- One year sterling Government bonds stand at a yield of 6% per annum.
- The sterling swap curve (annual fixed to semi-annual floating) stands at 6.5% per annum.
- The Examiners Guild Agency recently awarded the Merovence bond a credit rating of QQQ.

- Over the last 20 years, the Examiners Guild Agency estimates that QQQ grade entities defaulted at a rate of 8% in the year period following that rating being given.

You are also given the following product definitions (as they apply to this question):

- **Asset swap**: A synthetic floating rate instrument comprised of a specific bond and an attached swap which together make the package equivalent to a par-value floating rate note paying a margin over LIBOR.

- **Credit default swap**: An instrument which on a specified event of default pays a cash amount equal to the difference between par plus outstanding payments and the price of the reference asset at default.

- **Credit default option**: An instrument which on a specified event of default pays a pre-determined fixed amount.

You may assume that default can only occur on payment due dates. You should also ignore tax considerations.

(i) Draw a simple tree to show potential outcomes, and develop an equation of value as at today. Define any terms used. [3]

(ii) The Merovence bond is combined with a one year interest rate swap to create an asset swap. Calculate the equivalent par semi-annual floating rate coupon of the package. [4]

(iii) Calculate the risk-neutral value of a one year default swap on the Merovence bond. [4]

(iv) Explain why it is not possible to calculate the risk neutral value of a one year credit default option which pays out 20% in the event of default. [3]

(v) Estimate a value for the credit default option from the information provided. [3]

(vi) Discuss the issues arising when attempting to price the following instruments (calculations are not required):

(a) a six month credit default swap
(b) a two year credit default swap

In both cases the default trigger event is the failure of any Merovence government financial obligation. [3]

[Total 20]

3 You are a risk management consultant to a small investment bank based in a major European city. The bank has daily value-at-risk (DVaR) and stress-test reporting for its main trading areas (swaps, bonds and foreign exchange), but there is no market risk reporting for the smaller trading areas (credit and equity products), and DVaR numbers are not yet used as a decision-making tool. Other non-market risk assessment is minimal.

The management is concerned that they have fallen behind other similar banks in developing their overall risk framework. You have been asked to prepare a proposal for enlarging the bank's risk management capability into a full Enterprise Risk Management (ERM) framework. This is potentially a very big project, so as part of your preparation for the report, you are setting out to write a few summary notes.

(i) Define the concept of ERM, and list the major categories of risk that you would include within it. State the advantages of ERM for the bank. [6]

(ii) Describe the principal projects that will be required as the bank moves towards ERM, and the obstacles that might be encountered in implementation. (You should cover the management controls, decision-making processes, system and analytical development, but without going into specific details.) [10]

(iii) Describe the potential use of market and credit risk measurement as a tool for performance evaluation. [5]

In your notes, state clearly any assumptions you are making about the structure of the bank and its trading environment, which might need further clarification.

[Total 21]

4 (i) The money market interest rate curve is currently flat at 7% per annum for all maturities (an unusual occurrence).

(a) Calculate the value of a forward rate agreement (FRA) contract in the size of USD 1 billion bought at a strike of 8%. The contract is a "30–33" FRA, that is, it expires in 2½ years with an underlying term of 3 months. [2]

(N.B. the bought side of an FRA profits if interest rates increase.)

(b) Calculate the value of a futures contract of the same size and due dates as (a). (You may ignore initial margin.) [1]

(c) Describe the general circumstances in which the value of a futures contract would be the same as that of an FRA. [2]

(ii) A trade is undertaken:

Sell USD 11.84 billion "30–33" FRAs @ 7%
Sell USD 10 billion "30–33" futures @ 93.00

(a) Compute the profit or loss on the trade in the cases where the interest rate curve changes instantaneously to 5% flat or 9% flat. [2]

(b) Give reasons why the trade might have been undertaken, and explain the relative sizes of the two "legs". [2]

(iii) Consider two random variables s_1 and s_2, which follow Wiener processes given by:

$$ds_i = \mu_i s_i\, dt + \sigma_i s_i\, dz_i \quad \text{for } i = 1, 2$$

such that $\mathbf{E}(dz_1 dz_2) = \rho_{12} dt$.

(a) Using the generalised version of Ito's lemma, find the process followed by the product $s_1 s_2$. [4]

(b) Using in addition the following data, calculate the risk neutral expected price differential between the contracts traded in (ii) above:

σ_1 [3 month FRA] = 25%
σ_2 [2½ year zero coupon rate] = 20%
$\rho_{12} = 0.8915$ [5]

(iv) Comment briefly on your answer to (ii) (a) in the context of the result in (iii) (b). [1]

[Total 19]

5 Assume the short rate of interest r follows the log-normal stochastic process:

$$dr = \mu(r,t)rdt + \sigma(r,t)rdz$$

where z is a standard Brownian motion.

A traded asset $V = V(r, t, T)$, which depends only on this short rate r and its evolution over time, satisfies the following parabolic partial differential equation (PDE) in the " real world":

$$\frac{\partial V}{\partial t} + r\left(\mu - \lambda(r,t)\sigma\right)\frac{\partial V}{\partial r} + \frac{1}{2}\sigma^2 r^2 \frac{\partial^2 V}{\partial r^2} = rV$$

where t = time since outset, T = time to maturity, μ and σ are respectively the drift and volatility of the short rate, and $\lambda(r, t)$ is the "market price of risk".

(i) Explain what is meant by the "real world" and the "market price of risk" λ, in this context. Demonstrate (either by argument, or by algebra) that λ cannot be a function of any specific traded asset V. [6]

(ii) The "finite difference" (FD) method can be used to obtain an approximate numerical solution to the above PDE.

Describe how you would go about setting up this valuation in the case where V is a 1-year American call option on a 3-year par bond, allowing specifically for initial and boundary conditions. If necessary in your answer you may assume that your chosen model is any suitable one-factor model of the yield curve. [10]

(iii) The binomial lattice valuation method is a widely-used alternative to the FD method.

Explain briefly how the construction of these two methods in fact achieves the same result, and comment on the suitability of each method for the valuation of interest rate options. [5]

[Total 21]