

EXAMINATIONS

19 April 2004 (pm)

Advanced Certificate in Derivatives: Further Mathematics, Principles and Practice

Time allowed: Three hours

INSTRUCTIONS TO THE CANDIDATE

1. *Enter all the candidate and examination details as requested on the front of your answer booklet.*
2. *You have 15 minutes at the start of the examination in which to read the questions. You are strongly encouraged to use this time for reading only, but notes may be made. You then have three hours to complete the paper.*
3. *You must not start writing your answers in the booklet until instructed to do so by the supervisor.*
4. *Mark allocations are shown in brackets.*
5. *Attempt all 6 questions, beginning your answer to each question on a separate sheet.*

AT THE END OF THE EXAMINATION

Hand in BOTH your answer booklet, with any additional sheets firmly attached, and this question paper.

*In addition to this paper you should have available Actuarial Tables,
Derivatives Formula Sheet and your own electronic calculator.*

*NOTE: In this examination, you are never required to prove the use of
an arbitrage-free methodology unless clearly stated in the question.*

- 1** (i) Outline the key features of the following options as they might apply to over-the-counter contracts:
- Asian Options
 - Barrier Options
 - Basket Options
 - Bermudan Options
 - Binary (Digital) Options
 - Compound Options
 - Forward Start Options
 - Lookback Options
 - Shout Options
- [9]
- (ii) Set out the main legal risks which might be involved when transacting over-the-counter options. [6]
- [Total 15]

2 An *accrual swap*, or *range swap*, is a fixed-to-floating swap in which, during each payment period of the swap, the fixed-rate receiver accrues the fixed rate only on those days when LIBOR is in a pre-specified range, say between 4% and 5%, otherwise they accrue nothing.

A “*chooser*” *accrual swap* further allows the receiver to specify the mid-point of the range (without altering its width) at the start of each accrual period.

- (i) Suggest a method of valuing the simple accrual swap, and describe its market risk profile. [3]
- (ii) Outline the complications that the introduction of the “chooser” facility adds to the accrual swap. [3]
- (iii) Describe the Monte Carlo (MC) simulation method for valuing complex interest rate options, and discuss the advantages and disadvantages of using it compared with binomial trees. [8]
- (iv) Describe two ways that the MC approach can be refined, from its raw form, to reduce the number of iterations without affecting the accuracy of the end result. [4]
- [Total 18]

- 3 You have been given the following term structures for the years $t = 0$ to 4 of a US dollar swap curve:

<i>Time t in years</i>	<i>Volatility in %</i>	<i>Zero coupon bond prices</i>
0		100.000
1	18%	98.522
2	19%	96.637
3	20%	94.544
4	19%	92.327

Using a Black-Derman-Toy (BDT) model, you have obtained the binomial tree for the annually compounded short rate $r(t)$ for $t = 0$ to 4, using time steps of one year, as follows:

<i>0</i>	<i>1</i>	<i>2</i>	<i>3</i>	<i>4</i>
				5.111
			4.143	
		3.130		3.495
	2.299		2.777	
1.500		2.141		2.390
	1.604		1.862	
		1.464		1.635
			1.248	
				1.118

- (i) (a) Using the tree given, value a four-year 2% cap and 2% floor with annual resets.
- (b) Demonstrate that put-call parity is satisfied for these option prices, and describe how you would calculate the delta with respect to the short rate. [12]

[Note: a four year annual cap or floor has three annual options expiring at the ends of years 1, 2 and 3 respectively. The strike rate may be assumed to have the same day-count convention as $r(t)$.]

- (ii) Explain how your answers might differ from the values which would be obtained from the usual Black (futures) model with the same input term structures. [2]
- (iii) Compare the features of the BDT model with a single-factor Hull-White model of the short rate, and highlight the general strengths and weaknesses of each model. [5]
- [Total 19]

4 As an actuarial consultant from a specialist risk management advisory firm, you are preparing a report on the subsidiary operations of a large UK bank. You have been asked to address system and operational controls for the monitoring and measurement of market risk covering the following three areas:

- (a) Treasury operations in New York and Tokyo, which whilst not as developed as in London nevertheless deal in interest-rate swaps and money-market instruments on a fairly large scale.
- (b) A small trading operation based in Hong Kong, dealing in foreign exchange, equities, bonds and money market instruments from several emerging market countries.
- (c) The insurance division that has a number of complex derivatives on its books, purchased from investment banks, to hedge its portfolio of Guaranteed Equity Bonds.

Set out the major issues you would present in a note for the senior management, including those you would want to investigate further, and indicate what you would expect to recommend as best practice in each area you cover. [15]

5 The one-factor Heath-Jarrow-Morton (HJM) model of the yield curve supposes a generalised risk-neutral process for zero-coupon bond prices $P = P(t, T)$ of:

$$dP = r(t)Pdt + \sigma(t, T, P)dz$$

where t is calendar time (from now), T is maturity time (from now), r is the short rate at time t , σ is the volatility of bond price P , and dz represents standard Brownian motion.

Hence the evolution of bond prices comprises both a drift term and a generalised stochastic term.

- (i) Describe, without supplying all the detailed algebra, how the HJM model as described above can also be expressed in terms of the forward rates $f(t, T)$, and explain why there is no need for an explicit drift rate $\mu(t, T, P)$. [6]
- (ii) Discuss the benefits of using a two-factor HJM model (i.e. with two stochastic components, dz_1 and dz_2) over and above some other popular models of the yield curve. Outline how best the two factors might be chosen, and what considerations would be involved in implementing the model and calibrating it to market prices. [7]
- (iii) The HJM model is a “no-arbitrage” model but not an “equilibrium” model. Outline briefly what you understand by these terms, and the significance of such properties for an interest-rate model. [4]

[Total 17]

- 6** You are the risk manager in charge of a large trading portfolio consisting of international equities and bonds.
- (i) Briefly describe value at risk (VaR) as a risk management tool for your fund. [2]
 - (ii) List the advantages and disadvantages of using the following approaches to VaR:
 - (a) compiling a complete variance-covariance matrix based on historical movements of each sector of the markets in which you trade
 - (b) recalculating the profit and loss on the current portfolio using historical prices
 - (c) running a Monte-Carlo simulation using a time-series model (such as GARCH) to forecast future volatilities [9]
 - (iii) The portfolio also contains a number of forward currency positions, and structured notes with embedded optionality. Explain how you would treat these parts of the portfolio in the VaR framework described above, and why structured notes with certain types of optionality might not fit within it. [5]
- [Total 16]

END OF PAPER