The Connection between Population Structure and Bond Yields

Phase 3 Final Report

April 2020

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AUTHORS Doug Andrews
Steve Bonnar
Jaideep Oberoi
Aniketh Pittea

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Canadian Institute of Actuaries
Institute and Faculty of Actuaries

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Douglas Andrews and Stephen Bonnar
University of Waterloo
Jaideep Oberoi & Aniketh Pittea
University of Kent[1]

This report provides a high-level review of the literature that addresses the relationship between bond prices/yields and demographic variables, a deep dive into one of those papers (Favero et al., 2016), and empirical analysis that starts with the methodology of that paper and extends the analysis to look at the stability of the relationship over time, to longer-dated bonds, and to the sovereign bonds of various countries. We find that the results of Favero et al. hold in many (but not all) circumstances. An increase in the ratio of the "middle-aged" to the "young" population is associated with a reduction in bond yields at various points on the yield curve from three months to 30 years. An increase in the ratio of the "middle-aged" to the "old" population is associated with an increase in those bond yields.

When we examine shorter sub-periods within the data, these relationships are not nearly so strong. Generally, they still hold for the sub-period 1960–1974. In some instances, particularly for Canada, they hold for the sub-period 1990–2015. However, the turmoil in the bond (and other) markets that occurred during the significant run-up and then run-down in inflation and bond yields during the sub-period 1975–1989 resulted in unstable relationships between bond yields and demographic ratios. This should not be a surprising result in that the demographic ratios are very-slow-moving series, whereas bond yields (and inflation) moved very rapidly during this period.

Finally, we note that the relationship is not at all stable when looking at the circumstances of the UK, Germany, and Japan. It may be that the smaller (or absence of a) "baby boom" in these countries precludes us from finding this relationship.

[1] Funding for this work has been received from the following organizations: the Canadian Institute of Actuaries, the Institute and Faculty of Actuaries, the Society of Actuaries, the Social Sciences and Humanities Research Council, the University of Kent, and the University of Waterloo. Additionally, we acknowledge helpful comments received from the review group established by the three actuarial organizations. This article is a sub-project of a larger project entitled Population Aging, Implications for Asset Values, and Impact for Pension Plans: An International Study. In addition to the authors, the project team for the larger project includes researchers Lori Curtis and Kathleen Rybczynski, University of Waterloo; Mark Zhou, Canada Mortgage and Housing Corporation; Miguel Leon-Ledesma and Pradip Tapadar, University of Kent; and contributor Tony Wirjanto, University of Waterloo. The full project team thanks all who have made this project possible. Excellent research support for this paper has been provided by Giovanna Apicella, University of St. Gallen; and Lina Wang, University of Waterloo.

1. Introduction

This report forms part of a larger research agenda that began in 2014 and was funded, in part, through June 2019. The main researchers are in Canada and the United Kingdom, with funders in Canada, the UK, and the US. The larger research agenda seeks to identify and quantify the impact of changes in population structure on asset values over long time periods, and illustrate the impact of results of the research by applying it to large pension plans in Canada, the UK, and the US.

This report documents the work that the authors have done for the Society of Actuaries in the third phase of the research funded by that organization. Specifically, this third phase comprises the following work:

- Perform a two-part literature review. The first part is a literature review with a specific focus on papers providing a quantitative approach to how demographic structure affects interest rate levels and the term structure of bonds.
- Develop a model based on the insights in the literature that uses the data available, including the projections from our models, to quantify how demographic factors influence interest rates.
- We expect that fuzzy mathematics may be useful in the development of this model. The second part of the literature review identifies papers presenting fuzzy mathematical approaches that may have actuarial applications.

The empirical analysis starts with the methodology of Favero et al. (2016) and extends the analysis to look at the relationship to other demographic variables, the stability of such relationships over time, to longer-dated bonds, and to sovereign bonds of various countries. Favero et al. include the ratio of the “middle-aged” to the “young” population in their analysis. We find that the results of Favero et al. hold in many (but not all) circumstances. An increase in the ratio of the “middle-aged” to the “young” population is associated with a reduction in bond yields at various points on the yield curve from three months to 30 years. An increase in the ratio of the “middle-aged” to the “old” population (i.e., pensioners) is associated with an increase in those bond yields.

The economic rationale for these relationships is an expectation that the “old” population may have an increased demand for bonds to finance their retirement. If true, this increased demand would push down bond yields when the “old” population gets larger relative to the “middle-aged” population, and vice versa. This is consistent with the empirical results that show that an increase in the ratio of the “middle-aged” to the “old” population (i.e, relatively fewer pensioners) is associated with an increase in bond yields.

Similarly, there is an expectation that the “young” will want to borrow in order to smooth consumption over time. Thus, an increase in the proportion of the “young” relative to the “middle-aged” population would push up bond yields, and vice versa. Again, this is consistent with the empirical results that an increase in the ratio of the “middle-aged” to the “young” population (i.e., fewer young people) is associated with a decrease in bond yields.

When we examine shorter sub-periods within the data, these relationships are not nearly so strong. Generally, they still hold for the sub-period 1960–1974. In some instances, particularly for Canada, they hold for the sub-period 1990–2015. However, the turmoil in the bond (and other) markets that occurred during the significant run-up and then run-down in inflation and bond yields during the sub-period 1975–1989 resulted in unstable relationships between bond yields and demographic ratios. This should not be a surprising result in that the demographic ratios are very-slow-moving series, whereas bond yields (and inflation) moved very rapidly during this period.

Finally, we note that the relationship is not at all stable when looking at the circumstances of the UK, Germany, and Japan. It may be that the smaller (or absence of a) “baby boom” in these countries precludes us from finding this relationship.

The balance of this report is structured as follows:

- Section 2 provides a high-level summary of some of the papers that draw a connection between demographic factors and bond returns[2].
- Section 3 provides a detailed summary of one of those papers: Favero et al. (2016).
- Section 4 describes the data that we use for the analysis.
- Summary statistics are provided in Section 5.
- Section 6 contains our analysis that shows the results of our replication of the methodology in Favero et al. for the US and Canada.
- Section 7 contains additional analysis using the methodology of Favero et al. on the longer end of the yield curve, examining both coupon and zero-coupon bond yields.
- Section 8 concludes the text.
- Additional analysis for other countries (UK, Germany, and Japan) is contained in Appendix A.
- Appendix B contains a survey of the academic literature that addresses bond market characteristics using fuzzy mathematics.

2. Bond Markets and Demographic Factors

The notion that there should be a demographic influence on bond returns is often based on the life-cycle savings hypothesis proposed by Modigliani and Brumberg (1954) in the early 1950s. In short, this hypothesis suggests that lifetime utility is maximized when people borrow when young, invest for retirement when middle-aged, and live off their investments once they are retired.

[2] A more detailed literature review covering a range of asset classes can be found at www.soa.org/Files/resources/research-report/2018/lit-review-popl-aging-asset-values-impact-pension.pdf. Note that this detailed literature review cites a working-paper version of Favero et al. (2016). For the purposes of the descriptions in the two literature reviews, these two papers by Favero and his colleagues are the same.

Arnott and Chaves (2012) use a variety of control variables in their regressions. One common theme emerges from all their analyses: large populations of retirees (65-plus) seem to erode the performance of financial markets as well as economic growth. This effect is less pronounced for bonds than equities, likely because they are sold later in retirement than stocks, based on widespread financial advice, and because they provide a less risky, more stable income in retirement than stocks.

Favero et al. (2016) develop a simple theoretical model of the yield curve. They consider the ratio of the middle-aged (40–49) to the young (20–29) population in the US as the relevant demographic variable to determine the persistent component of interest rates. Their model predicts (and the empirical analysis shows) a negative correlation between their demographic variable and bond yields.

Roy et al. (2012) include both demographic and economic variables in their regressions. The result from regressing 10-year government bond yields on the Yuppie/Nerd ratio (the ratio of the number of 20- to 34-year-olds to the number of 40- to 54-year-olds) and inflation provides a good regression fit. Bond yields tend to go up when the Yuppie/Nerd ratio rises, and vice versa.

3. Detailed Summary of Favero et al. (2016)

Favero et al. (2016) has been chosen as the paper for a deeper dive into the relationship between demographic variables and bond yields because the authors both create a theoretical model for bond yields and conduct empirical analysis supporting that theory. They start their paper by noting that short-term bond yields[3] exhibit significant persistence[4], while the economic factors that central banks use to administer short-term rates (such as inflation and the output gap) have much lower levels of persistence[5].

Favero et al. (2016) state that: "[r]ecent evidence shows that the behavior of interest rates is consistent with the decomposition of spot rates as the sum of two processes: (i) a very persistent long-term expected value and (ii) a mean-reverting component". Most modelling of interest rates assumes that the long-term expected value is constant. What Favero and his colleagues contribute to the literature is to model the long-term expected value as being affected by a demographic variable. Specifically, the demographic variable that they use is the ratio of the middle-aged (40–49) to young (20–29) population, MY.

In their simple framework, the yield to maturity of the one-period bond, $y_t^{(1)}$, is determined by the action of the monetary policy maker, and all the other yields on n-period (zero-coupon) bonds can be expressed as the sum of average expected future short rates[6]. That is the

[3] Favero et al. (2016) use three-month rates for their analysis.

[4] Autocorrelations of US three-month T-Bill yields at lags of one, two, and three periods are in excess of 0.85.

[5] Autocorrelations of US inflation and the output gap at lags of one, two, and three periods range from -0.03 to 0.25.

[6] The term $E_t[y_{t+i}^{(n)}|I_t]$ is the current period expectation of future bond yields based on all the information known at time t (i.e., I_t).

expectations hypothesis (EH) and the term premium, $rpy_t^{(n)}$.[7]

$$y_t^{(n)} = \frac{1}{n}\sum_{i=0}^{n-1} E_t[y_{t+i}^{(1)}|I_t] + rpy_t^{(n)} \quad (1)$$

$$y_t^{(1)} = y_t^* + \beta(E_t[\pi_{t+k}] - \pi^*) + \gamma E_t[x_{t+q}] + u_{1,t+1} \quad (2)$$

Favero et al. (2016) then describe the process as follows:

> In setting the policy rates, the Fed reacts to variables at different frequencies. At the high frequency, the policy maker reacts to cyclical swings reflected in the output/unemployment gap, x_{t+q}; that is, transitory discrepancies of output from its potential level, and in deviation of inflation, π_{t+k}, from the implicit target (π^*) of the monetary authority. Monetary policy shocks, $u_{1,t+1}$, also happen. As monetary policy impacts macroeconomic variables with lags, the relevant variables to determine the current policy rate are k-period ahead expected inflation and q-period ahead expected output gap. However, cyclical swings are not all that matter to set policy rates. We posit that the monetary policy maker also takes into account the equilibrium level of interest rates, y_t^*, (which is determined by the sum of a time-varying real interest rate target and the inflation target π^*) according to the slowly evolving changes in the economy that take place at a generational frequency, that is, those spanning several decades. We relate this to the age structure of population, MY_t, as it determines the savings behavior of the middle-aged and young population.

So, the innovation that Favero et al. (2016) introduce is to model the equilibrium interest rate as follows: $y_t^* \equiv \rho_0 + \rho_1 MY_t$, which they refer to as the demographically adjusted affine term structure model (ATSM). To reduce to the traditional ATSM model, ρ_1 would be set to zero.

In their empirical work[8], Favero et al. (2016) find that their additional parameter δ_2 is highly significant with the expected negative sign. They also conduct tests of the forecasting effectiveness of the demographically adjusted ATSM and find that it dominates the traditional ATSM.

While Favero et al. (2016) is one of many papers in this literature, it is notable for containing both a theoretical framework and empirical testing of that framework. In addition, they accomplish this with a very parsimonious specification.

4. Data and Methodology

Historical data for five countries (the US, Canada, the UK, Germany, and Japan) for the period 1960–2015 were collected. Data on monthly Consumer Price Index (CPI) were obtained from the Federal Reserve Bank of St. Louis database. Yearly population data were obtained from the United Nations Population Division. We then determined two demographic ratios: $M\ to\ Y$ and $M\ to\ O$. $M\ to\ Y$ is the ratio of the "middle-aged" population (age 40–49) to the "young" population (age 20–29). $M\ to\ O$ is the ratio of the "middle-aged" population to the "old"

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[7] Favero et al. (2016) use the following notation: $p_t^{(n)}$ is the log price of an n-year discount rate bond at time t. The continuously compounded spot rate is then $y_t^{(n)} \equiv -\frac{1}{n}p_t^{(n)}$.

[8] Their empirical formulation is slightly different from the theoretical structure described above and utilizes the parameter δ_2 rather than ρ_1.

population (age 60–69). These annual data were disaggregated to a monthly frequency using the Denton–Cholette method as described in Dagum and Cholette (2006).

Three-month (r_3) T-Bill yields and 10-year (r_{10}) and 20-year (r_{20}) coupon bond yields (all at monthly frequency) were obtained from the Federal Reserve Bank of St. Louis database.

Ten-year ($r_{10}{}'$) and 20-year ($r_{20}{}'$) zero-coupon yields are determined from r_3, r_{10}, and r_{20} by assuming that the one-year forward rates between three months and 10 years (and for periods in excess of 10 years) are all the same. Then, $r_{10}^{'}$ can be calculated by solving the following equation:

$$1 = \frac{r_{10}/2}{(1+r_3)(1+r_{10}')^{.25}} + \frac{r_{10}/2}{(1+r_3)(1+r_{10}')^{.75}} + \cdots + \frac{r_{10}/2}{(1+r_3)(1+r_{10}')^{9.25}} + \frac{1+r_{10}/2}{(1+r_3)(1+r_{10}')^{9.75}} \quad (3)$$

Knowing r_3, r_{20}, and $r_{10}^{'}$, we can solve for $r_{20}^{'}$ by solving the equation:

$$1 = \sum_{i=1}^{20} \frac{r_{20}/2}{(1+r_3)(1+r_{10}')^{(i/2-.25)}} + \sum_{j=1}^{20} \frac{r_{20}/2}{(1+r_3)(1+r_{10}')^{9.75}(1+r_{20}')^{(j-1)/2}} + \frac{1+r_{20}/2}{(1+r_3)*(1+r_{10}')^{9.75}(1+r_{20}')^{10}} \quad (4)$$

Finally, after obtaining $r_{10}^{'}$ and $r_{20}^{'}$, the five-year zero-coupon yield ($r_5^{'}$) can be calculated as follows:

$$r_5^{'} = \left[(1+r_3)(1+r_{10}^{'})^{4.75}\right]^{\frac{1}{5}} - 1 \quad (5)$$

And the 30-year zero-coupon yield ($r_{30}^{'}$) can be calculated as follows:

$$r_{30}^{'} = \left[(1+r_3)(1+r_{10}^{'})^{9.75}(1+r_{20}^{'})^{20}\right]^{\frac{1}{30}} - 1 \quad (6)$$

5. Summary Statistics

This section and those that follow in the body of this report focus on the US and Canada. Appendix A broadens the analysis to include the other three countries (UK, Germany, and Japan). Figures 1 and 2 show the trend of the $M\ to\ Y$ and $M\ to\ O$ ratios over time.

Figure 1. $M\ to\ Y$ **Ratio over Time**

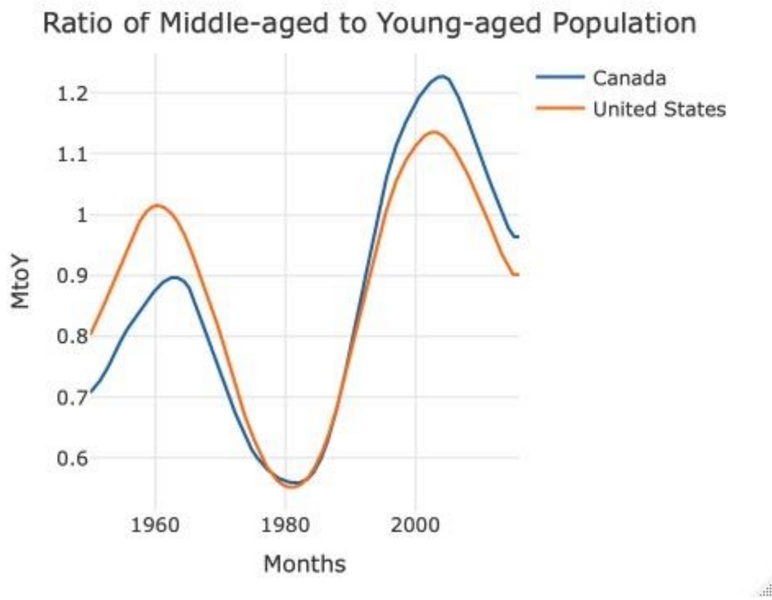
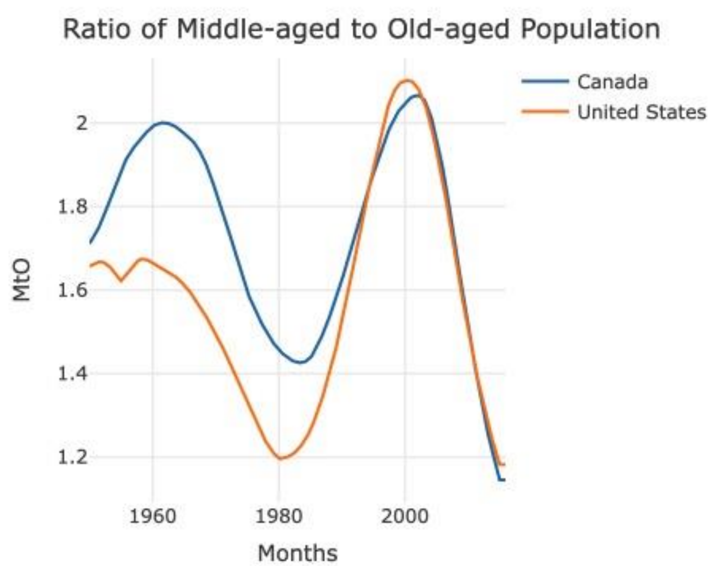


Figure 2. $M\ to\ O$ **Ratio over Time**

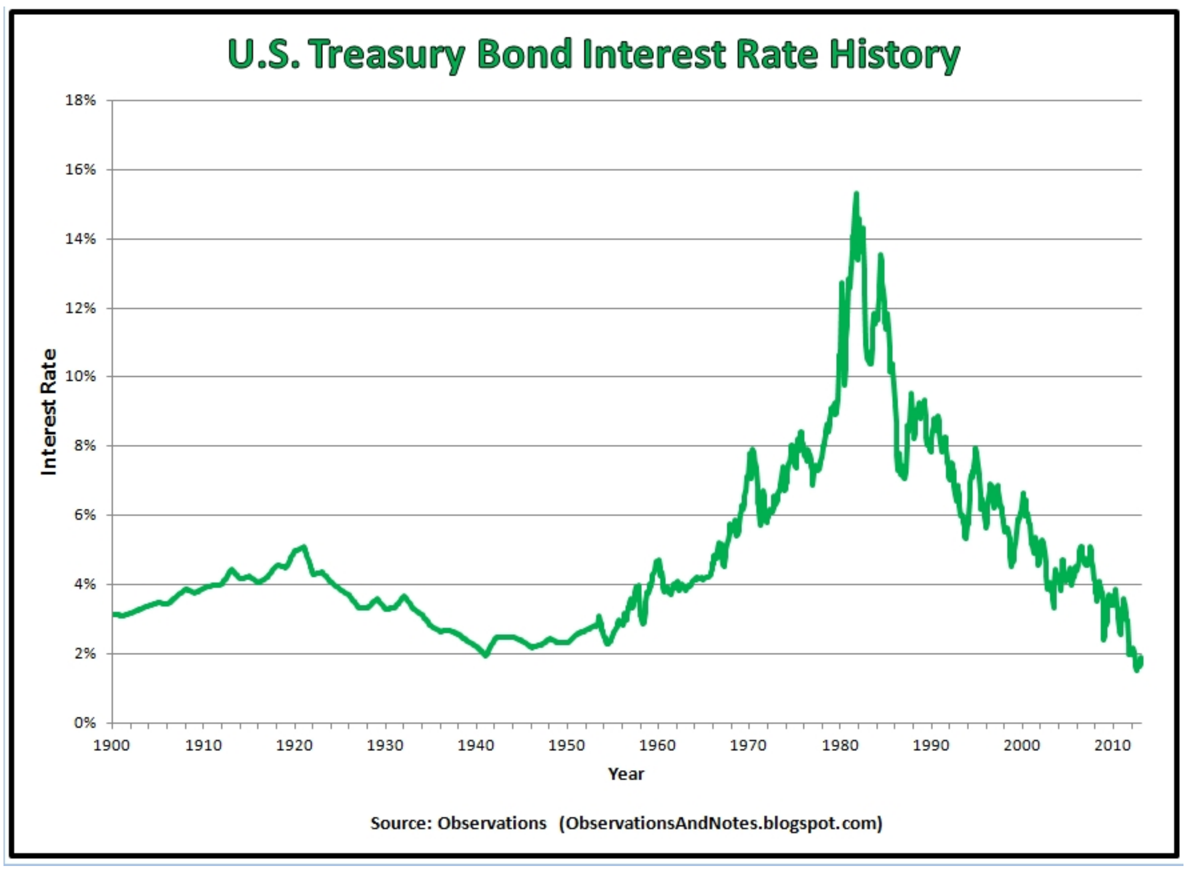


The trend in these ratios is similar between Canada and the US. The trend in the other countries is different, as can be seen in Appendix A. The trend in the North American ratios is largely driven by the influence of the baby boom generation. The ratio $M\ to\ Y$ rises until the mid-1960s, when the baby boom generation starts to turn 20. The ratio subsequently falls until the mid-1980s, when the baby boomers start to turn 40. The subsequent rise lasts until the mid-2000s, when the baby boomers start to turn 60.

The influence of the baby boom generation on the $M\ to\ O$ ratio naturally starts later. Until the mid-1980s there is a general downward trend in this ratio, reflecting the relatively small number of births that occurred during the Depression and Second World War years. Then the ratio starts to move upward as the baby boomers begin to turn 40. It then falls, starting in the mid-2000s, as the baby boomers begin to turn 60.

Unlike the demographic ratio curves that exhibit multiple peaks, bond yields since 1960 have had a single secular peak in the early 1980s, as shown in Figure 3. The secular rise and fall of bond yields was largely driven by the secular rise and fall in actual inflation and inflation expectations. This strongly suggests that in order to detect any relationship between the demographic ratios and bond yields, it is necessary to control in some way for inflation. It would be best to control for inflation expectations, but these are not observable. As a result, we control for price inflation. Of note, the significant rise and fall in bond yields, actual inflation, and inflation expectations during the 1970s and 1980s was a one-off occurrence in the period since 1900. Figure 3 shows that this episode was the exception, not the norm.

Figure 3. US Bond Yields



6. Analysis Using Methodology of Favero et al. (2016)

Initially, we examine whether the relationship found in Favero et al. (2016) holds in a slightly different time period[9] and for Canada as well as the US. Column 3 of Table 1 sets out results that are directly comparable to those shown in Favero et al. As can be seen, the coefficient on the variable $M\ to\ Y$ is negative and significant, consistent with the results found in Favero et al. As we extend the analysis to Canada (columns 1 and 2), and add the variable $M\ to\ O$ (columns 2 and 4) we see that the coefficients on the variable MY are all negative and significant. The coefficients on the variable $M\ to\ O$ are all positive and significant[10].

Table 1. Replication of Favero

	Dependent Variable: Three-Month Yield			
	Canada (1)	Canada (2)	US (3)	US (4)
CPI	0.018 (0.020)	0.165*** (0.013)	−0.010 (0.012)	−0.003 (0.004)
M to Y	−14.112*** (1.677)	−36.392*** (2.003)	−10.579*** (3.144)	−22.493*** (2.186)
M to O		16.514*** (1.052)		8.753*** (0.952)
Constant	17.730*** (.0443)	1.250 (1.313)	15.791*** (1.368)	11.584*** (1.374)

Note: *p<0.1; **p<0.05; ***p<0.01

As Figure 3 illustrates, the environment for yields varied dramatically over the period 1960–2015. In order to assess the impact of the varying environment on our results, we examine regressions that used data from three different time periods. Table 2 summarizes the results of those regressions. What we can see from the table is that the signs of the coefficients are as expected for the periods 1960–1974, and 1990–2015 (though the coefficients are not significant for the US regression for the time period 1960–1974). However, the signs are reversed for the tumultuous period starting after the first oil shock and ending with the close of the 1980s. We suspect that there are other factors influencing bond yields in this period than price inflation and the demographic variables.

[9] The data in Favero et al. (2016) run from 1961Q3 to 2013Q4. The data in this paper run from 1960Q1 to 2015Q4.
[10] The figures in brackets in all of the tables (other than in heading rows) are the standard errors, adjusted for both heteroskedasticity and autocorrelation.

Table 2. Analysis of Different Time Periods

	Dependent Variable: Three-Month Yield					
	Canada 1960–1974 (1)	Canada 1975–1989 (2)	Canada 1990–2015 (3)	US 1960–1974 (4)	US 1975–1989 (5)	US 1990–2015 (6)
CPI	3.090*** (0.473)	−0.442 (0.360)	0.107 (0.072)	0.740*** (0.102)	0.008 (0.033)	−0.010 (0.019)
M to Y	−14.011** (6.902)	141.875 (102.252)	−32.335*** (6.797)	−37.957 (37.397)	716.352*** (161.339)	−14.476** (6.120)
M to O	59.402*** (11.276)	−130.659* (74.493)	14.185*** (3.749)	73.496 (49.167)	−506.108*** (105.966)	7.314*** (2.520)
Constant	−140.918*** (25.699)	137.716** (64.499)	5.605 (5.519)	−101.067** (46.380)	222.599*** (40.883)	7.035*** (1.976)

Note: *$p<0.1$; **$p<0.05$; ***$p<0.01$

7. Extension of Empirical Analysis to Long Yields

7.1 Analysis of Coupon Bonds

Favero et al. (2016) build up a yield curve by using the EH and term premia. Our analysis directly observes 10-year and 20-year coupon bond yields, and indirectly calculates five-, 10-, 20-, and 30-year zero-coupon bond yields. Table 3 shows the results for regressions on the 10-year and 20-year coupon bond yields for the full period of data, 1960–2015. As with the regressions for three-month yields, the coefficients on the variable $M\ to\ Y$ are negative and significant, while the coefficients on the variable $M\ to\ O$ are positive and significant.

Table 3. Analysis of Long-Term Coupon Bonds

	Dependent Variable: Par Bond Yield			
	Canada 10-Year (1)	Canada 20-Year (2)	US 10-Year (3)	US 20-Year (4)
CPI	0.161*** (0.014)	0.166*** (0.015)	0.007** (0.003)	0.010*** (0.003)
M to Y	−33.455*** (2.245)	−33.590*** (2.543)	−22.796*** (2.468)	−17.606*** (2.432)
M to O	16.248*** (1.070)	16.263*** (1.143)	8.655*** (0.905)	8.509*** (1.100)
Constant	0.087 (1.160)	0.070 (1.151)	11.628*** (1.486)	6.088*** (1.263)

Note: *$p<0.1$; **$p<0.05$; ***$p<0.01$

Similarly to the analysis for three-month yields, we also examine regressions for coupon bonds during three different time periods. The results of these regressions are summarized in Table 4 for Canada and Table 5 for the US.

The relationships for the Canada bonds are qualitatively similar to those for three-month T-Bills. The coefficient on $M\ to\ Y$ is negative and significant in the first and third periods; the coefficient on $M\ to\ O$ is positive and significant in the first period, but not significant in the third period. It appears as if in the second period there is too much “noise” going on to detect any “signal” from the demographic variables.

The situation for US bonds is quite different. The coefficients are only significant for the 10-year bond in the first period, and they have the expected signs. None of the coefficients in the other time periods and for the 20-year bond are significant. This result suggests that the relationship between bond yields and demographic variables is not stable over time.

Table 4. Analysis of Different Time Periods

	Dependent Variable: Canadian Par Bond Yield					
	10-Year 1960–1974 (1)	10-Year 1975–1989 (2)	10-Year 1990–2015 (3)	20-Year 1960–1974 (4)	20-Year 1975–89 (5)	20-Year 1990–2015 (6)
CPI	0.761*** (0.219)	0.063 (0.228)	−0.073 (0.088)	0.774*** (0.210)	0.078 (0.203)	−0.044 (0.100)
M to Y	−18.407 *** (4.984)	−12.710 (64.974)	−13.858 * (7.818)	−18.400 *** (5.017)	−14.560 (57.125)	−16.358 * (8.767)
M to O	19.726 *** (7.329)	−14.166 (46.702)	5.598 (4.049)	19.921*** (7.186)	−12.546 (41.444)	6.735 (4.568)
Constant	−27.517 ** (13.707)	36.975 (39.941)	16.360 *** (5.797)	−28.063 ** (13.239)	35.198 (36.019)	15.060** (6.697)

Note: *$p<0.1$; **$p<0.05$; ***$p<0.01$

Table 5. Analysis of Different Time Periods

	Dependent Variable: US Par Bond Yield					
	10-Year 1960–1974 (1)	10-Year 1975–1989 (2)	10-Year 1990–2015 (3)	20-Year 1960–1974 (4)	20-Year 1975–1989 (5)	20-Year 1990–2015 (6)
CPI	0.444*** (0.093)	0.059** (0.028)	−0.062*** (0.009)	−0.017 (0.143)	0.044*** (0.007)	−0.062*** (0.009)
M to Y	−62.153*** (13.491)	100.449 (147.525)	2.821 (3.759)	−19.262 (14.038)	6.360 (39.649)	2.821 (3.759)
M to O	89.693*** (21.366)	−89.685 (97.341)	−1.667 (1.554)	14.958 (24.765)	−3.109 (25.639)	−1.667 (1.554)
Constant	−94.709*** (24.479)	58.514 (37.649)	16.354*** (1.380)	−1.496 (31.186)	3.527 (9.397)	16.354*** (1.380)

Note: *p<0.1; **p<0.05; ***p<0.01

7.2 Analysis of Zero-Coupon Bonds

We have calculated the yield on zero-coupon bonds of five-, 10-, 20-, and 30-year terms using the assumption that the one-year forward rates between one and 10 years are all the same. We also assume that the one-year forward rates between 10 and 20 years are all the same, and that the one-year forward rates at terms in excess of 20 years are the same as the one-year forward rates between 10 and 20 years. The details of our methodology are provided in Section 4.

Table 6 shows the results of the regressions for Canada bonds, while Table 7 shows the results of the regressions for US bonds. As with the regressions for the coupon bond yields for the full time period, all of the coefficients on the demographic variables are significant. All of the coefficients on $M\ to\ Y$ are negative, and all of the coefficients on $M\ to\ O$ are positive.

We have also looked at regressions for the zero-coupon bonds by sub-periods, which we have not presented. Qualitatively, the results are similar to those for coupon bonds analyzed by sub-period.

Table 6. Analysis of Long-Term Zero-Coupon Bonds

	Dependent Variable: Canada Zero-Coupon Bond Yield			
	Five-Year (1)	10-Year (2)	20-Year (3)	30-Year (4)
CPI	0.161*** (0.013)	0.160*** (0.015)	0.167*** (0.016)	0.165*** (0.015)
M to Y	−33.760*** (2.164)	−33.114*** (2.460)	−33.496*** (2.715)	−33.476*** (2.427)
M to O	16.325*** (1.039)	16.285*** (1.148)	16.362*** (1.243)	16.342*** (1.120)
Constant	0.122 (1.154)	−0.164 (1.153)	−0.156 (1.190)	−0.111 (1.149)

Note: *$p<0.1$; **$p<0.05$; ***$p<0.01$

Table 7. Analysis of Long-Term Zero-Coupon Bonds

	Dependent Variable: US Zero-Coupon Bond Yield			
	Five-Year (1)	10-Year (2)	20-Year (3)	30-Year (4)
CPI	0.007* (0.004)	0.009** (0.004)	0.011*** (0.003)	0.010*** (0.003)
M to Y	−23.757*** (2.423)	−22.973*** (2.622)	−17.235*** (2.677)	−19.236*** (2.228)
M to O	8.851*** (0.970)	8.718*** (0.906)	8.547*** (1.195)	8.621*** (0.991)
Constant	12.235*** (1.438)	11.630*** (1.580)	5.580*** (1.357)	7.667*** (1.217)

Note: *$p<0.1$; **$p<0.05$; ***$p<0.01$

8. Conclusion

As mentioned in the introduction, the empirical analysis starts with the methodology of Favero et al. (2016) and extends the analysis to look at the stability of the relationship over time, to longer-dated bonds, and to the sovereign bonds of various countries. Our general conclusion from the empirical analysis is that the results of Favero et al. hold in many (but not all) circumstances. An increase in the demographic ratio $M\ to\ Y$ is associated with a reduction in bond yields at various points on the yield curve from three months to 30 years. An increase in the demographic ratio $M\ to\ O$ is associated with an increase in those bond yields.

When we examine shorter sub-periods within the data, these relationships are not nearly so strong. Generally, they still hold for the sub-period 1960–1974. In some instances, particularly for Canada, they hold for the sub-period 1990–2015. However, the turmoil in the bond (and other) markets that occurred during the significant run-up and then run-down in inflation and bond yields during the sub-period 1975–1989 resulted in unstable relationships between bond yields and demographic ratios. This should not be a surprising result in that the demographic ratios are very-slow-moving series, whereas bond yields (and inflation) moved very rapidly during this period.

Finally, we note that the relationship is not quite as stable when looking at the circumstances of the UK, Germany, and Japan. It may be that the smaller (or absence of a) baby boom in these countries precludes us from finding a stable relationship.

Practitioners should be interested in the future impact of population changes on bond yields. Over the next 45 to 50 years, the baseline population projections of both Statistics Canada and the US Census Bureau project that the M to Y ratio will increase and the M to O ratio will decrease. If we assume that the regression coefficients for the full time period set out in Table 3 will also hold for the future, then we can expect that 10- and 20-year bond yields in Canada will be lower by roughly 40 basis points (0.40%) relative to what they would have been under a stationary population structure. US 10-year bond yields will be lower by 40 basis points, and 20-year bond yields will be lower by 30 basis points. In developing best estimate assumptions for bond yields or returns, actuaries should consider the influence of future changes in population structure.

Appendices

A. Extension of Empirical Analysis to Other Countries

Figures 4 and 5 expand on Figures 1 and 2 by including the time trend of $M\ to\ Y$ and $M\ to\ O$ for the UK, Germany, and Japan. Because of the smaller (or non-existent) impact of the baby boom in these other countries, the "up/down" pattern that was exhibited for Canada and the US is not present. This may mute the impact of these demographic ratios on sovereign bond yields in these countries.

Figure 4. $M\ to\ Y$ Ratio over Time

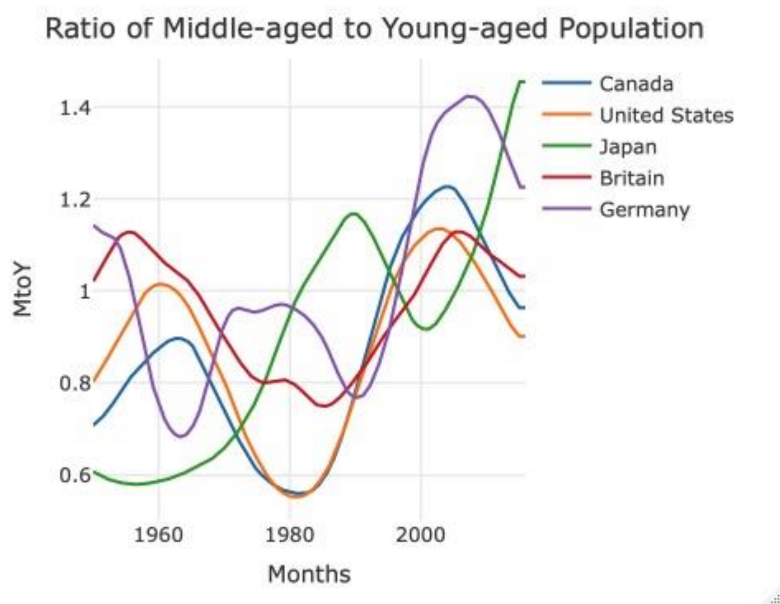


Figure 5. $M\ to\ O$ Ratio over Time

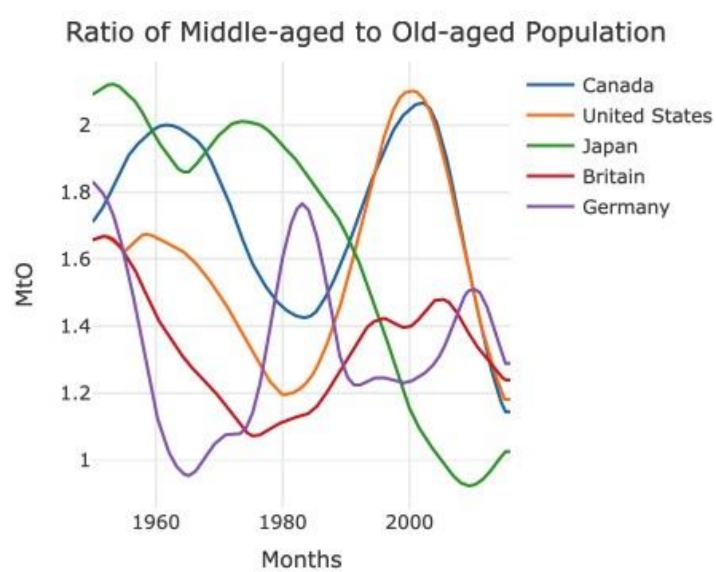


Table 8 shows the results of the regression of three-month bill yields against CPI, $M\ to\ Y$, and $M\ to\ O$ for the various countries. The additional countries do not show as strong an association between yields and the demographic ratios. The UK, which had a “baby blip”, shows less significance of the coefficients. The German coefficient on $M\ to\ Y$ is not significant, and neither of the coefficients for Japan are significant. Of note, the signs of the coefficients are consistent with their North American counterparts.

Tables 9, 10, and 11 show the results of regressions for longer bonds for the UK, Germany, and Japan, respectively. For the UK, the coefficient on $M\ to\ Y$ is negative and significant in all of the regressions, while the coefficient on $M\ to\ O$ is not significant in any of the regressions. The regressions for the German bond yields show that the coefficient on $M\ to\ Y$ is negative and significant in all of the regressions, and the coefficient on $M\ to\ O$ is positive and significant in all of the regressions. Finally, for Japan, the coefficient on $M\ to\ Y$ is negative and significant for the zero-coupon bond regressions, and the coefficient on $M\ to\ O$ is positive and significant in all of the regressions.

Table 8. Comparison of Three-Month Yields

	Dependent Variable: Three-Month Yield				
	Canada (1)	US (2)	UK (3)	Germany (4)	Japan (5)
CPI	0.165*** (0.013)	−0.003 (0.004)	−0.025* (0.013)	−0.047** (0.018)	−0.124 (0.087)
M to Y	−36.392 *** (2.003)	−22.493 *** (2.186)	−29.525 *** (3.811)	−3.430 (2.283)	−1.724 (2.425)
M to O	16.514*** (1.052)	8.753*** (0.952)	9.949* (5.669)	3.860*** (1.272)	1.378 (1.341)
Constant	1.250 (1.313)	11.584*** (1.374)	22.759*** (5.333)	6.232* (3.340)	13.254 (12.932)

Note: *p<0.1; **p<0.05; ***p<0.01

Table 9. Analysis for the UK

	Dependent Variable: UK Bond Yield					
	10-Year Coupon (1)	20-Year Coupon (2)	Five-Year Zero (3)	10-Year Zero (4)	20-Year Zero (5)	30-Year Zero (6)
CPI	−0.022* (0.013)	−0.025* (0.014)	−0.023 (0.014)	−0.022* (0.013)	−0.025* (0.015)	−0.024* (0.014)
M to Y	−23.318*** (3.046)	−22.587*** (2.892)	−23.864*** (2.882)	−22.448*** (3.433)	−21.964*** (3.184)	−22.362*** (3.153)
M to O	1.491 (5.604)	0.303 (5.822)	2.077 (5.551)	0.088 (5.959)	−0.857 (6.239)	−0.208 (6.068)
Constant	28.748*** (5.509)	29.829*** (5.803)	28.455*** (5.498)	29.909*** (5.784)	30.891*** (6.174)	30.318*** (5.991)

Note: *p < 0.1; **p < 0.05; ***p < 0.01

Table 10. Analysis for Germany

	Dependent Variable: German Bond Yield					
	10-Year Coupon (1)	20-Year Coupon (2)	Five-Year Zero (3)	10-Year Zero (4)	20-Year Zero (5)	30-Year Zero (6)
CPI	−0.056*** (0.007)	−0.054*** (0.007)	−0.058*** (0.009)	−0.058*** (0.010)	−0.055*** (0.009)	−0.056*** (0.008)
M to Y	−2.890*** (0.627)	−2.905*** (0.588)	−3.622*** (1.022)	−2.824*** (0.920)	−2.876*** (0.740)	−2.992*** (0.758)
M to O	3.098*** (0.435)	2.955*** (0.402)	4.781*** (0.800)	2.989*** (0.672)	2.874*** (0.524)	3.210*** (0.550)
Constant	8.397*** (1.406)	8.477*** (1.327)	7.284*** (2.182)	8.747*** (1.866)	8.713*** (1.581)	8.481*** (1.644)

Note: *$p<0.1$; **$p<0.05$; ***$p<0.01$

Table 11. Analysis for Japan

	Dependent Variable: Japanese Bond Yield					
	10-Year Coupon (1)	20-Year Coupon (2)	Five-Year Zero (3)	10-Year Zero (4)	20-Year Zero (5)	30-Year Zero (6)
CPI	−0.161 ** (0.073)	−0.156 ** (0.068)	−0.192 *** (0.037)	−0.131 ** (0.051)	−0.137 ** (0.063)	−0.145 *** (0.052)
M to Y	1.313 (0.841)	−0.434 (0.946)	−5.264 *** (0.316)	−5.459 *** (0.369)	−4.929 *** (0.465)	−5.074 *** (0.405)
M to O	4.253 *** (0.790)	5.805 *** (0.796)	7.862 *** (0.401)	7.806 *** (0.412)	7.785 *** (0.505)	7.801 *** (0.425)
Constant	10.086 (8.007)	11.097 (7.624)	20.394 *** (3.959)	14.989 *** (5.102)	15.298 ** (6.340)	16.096 *** (5.215)

Note: *$p<0.1$; **$p<0.05$; ***$p<0.01$

B. Literature Review of Bond Analysis Using Fuzzy Mathematics

For this literature review, we look at papers published since 2000 that connect fuzzy mathematics with bond markets. Our focus is on sovereign bonds, but this literature is so small that we expand the scope to look at any type of bond. In total, we found seven papers that satisfy our criteria. We supplement the review with one paper published in 1998, because it has many citations in other papers. We start the review with the abstract from this paper, and continue chronologically with the other abstracts.

Ramaswamy (1998) "illustrates the use of fuzzy decision theory to structure portfolios by considering the case of a fund manager who is allowed to hold only government bonds and plain vanilla options on them. [The author assumes] that the fund manager holds a portfolio of US Treasuries consisting of the current two-, five-, and 10-year bonds. Using the current yield to maturity of these bonds and the yield of the one-year T-Bill, the [author constructs a] linearly interpolated par yield curve. The approach is such that a given target rate of return is achieved for an assumed market scenario. If the assumed market scenario turns out to be incorrect, the portfolio is guaranteed to secure a given minimum rate of return. The methodology is useful in the management of assets against given liabilities or in forming structured portfolios that guarantee a minimum rate of return."

Michalopoulos et al. (2004) "investigate the application of a methodology based on fuzzy-set theory to the selection of an optimal portfolio of Greek government bonds. Investors' goals for the different bond market scenarios are formulated in fuzzy qualitative terms, while a model of fuzzy mathematical programming is used for the specification of the portfolio that optimally meets the given goals. The reliability of the results obtained with this methodology is checked with the aid of simulations."

Lee and Cheng (2008) examine the opportunities to include high-yield bonds (HYB) in a portfolio of equities and Treasury bonds. "The returns and risks of […] HYB lie between stocks and Treasury bonds. In view of investment opportunities and the rate of return, the advantages of HYB are both lower risks and higher returns. Therefore, HYB can be an important component in portfolios. The purpose of [their] study is to identify critical factors related to the selection of HYB. [The] primary criteria to evaluate HYB selection are obtained from a literature survey and applying the fuzzy Delphi method. Then, a fuzzy analytical hierarchy process is employed to calculate the weights of these criteria, so as to build the fuzzy multi-criteria model of HYB investment." They use three types of criteria:

- bond characteristics, such as liquidity and callability
- financial factors, such as the current ratio, EBIT, total-debt-to-total-assets ratio, asset growth rate, and cash-flow-to-total-debt ratio
- economic environment, such as the real interest rate change and the spread versus Treasuries

"The results indicate that the greatest weight is placed on the dimension of the economic environment. Three critical evaluation criteria related to HYB selection are: (1) spread versus Treasury bonds, (2) bond callability, and (3) a default rate indicator."

Zhang et al. (2010) use the context of the Black–Scholes model and the traditional bond pricing model to discuss the convertible bond fuzzy pricing problem. “The fuzzy pricing formula for convertible bonds and its algorithm are given under the assumption that the risk-free interest rates, stock prices, and stock price volatility are fuzzy numbers. The empirical results of Shanghai International Airport Corporation’s convertible bonds show that the fuzzy model presented in this paper is effective in forecasting the convertible bond’s market price.”

Agliardi and Agliardi (2011) “develop a computational method to implement the effect of imperfect information on the value of defaultable bonds. Fuzzy modelling is adopted and the numerical experiments show that an imprecise value of the underlying asset and/or the barrier triggering default have a material impact on the qualitative shape of the term structure of credit spreads.”

“Based on fuzzy theory and an analytical hierarchy process, [Jiazhong and Min (2013) present] a new credit risk evaluation model for corporate bonds. First, an evaluation indicator system of credit risk of corporate bonds is designed. Second, an analytical hierarchy process is used to determine the level of different indicators, and multistage comprehensive fuzzy evaluation is used to evaluate the credit risk of corporate bonds. Finally, corporate bonds of 10 enterprises are taken as examples to evaluate credit risk and verify the validity and feasibility of the model. The experimental results show that the model does a good job at evaluating the credit risk of different corporate bonds of different enterprises.”

“Catastrophe bonds are financial instruments that enable the transfer of natural catastrophe risk to financial markets. [Nowak and Romaniuk (2017)] is a continuation of [their] earlier research concerning catastrophe bond pricing. They assume the absence of arbitrage and a neutral attitude of investors toward catastrophe risk. The interest rate behaviour is described by the two-factor Vasicek model [(introduced in Hull and White, 1994)]. To illustrate and analyze results, they conduct Monte Carlo simulations using parameters fitted on real data for natural catastrophes. Besides the crisp catastrophe bond pricing formulas, they obtain their fuzzy counterparts, taking into account the uncertainty of the market. Moreover, they propose an automated approach for decision making in a fuzzy environment with relevant examples presenting this method.”

In Nazemi et al. (2017), “fuzzy decision-fusion techniques are applied to predict the loss-given-default of corporate bonds. In their model, they add the principal components derived from more than 100 macroeconomic variables as explanatory variables. However, in order to improve the performance of the model, the Box–Cox transformation of macroeconomic variables is applied prior to loss-given-default modelling. A differential evolution algorithm is used to create an optimized fuzzy rule-based model that fuses the outputs of several base models. They compare the predictions from fuzzy decision fusion techniques with support vector regression techniques, regression trees, and ordinary least squares (OLS) regressions. Their findings show that fuzzy decision fusion techniques increase prediction accuracy of loss-given-default modelling and transformations of macroeconomic factors do not affect the prediction accuracy of fuzzy models.”

As the reader can see, the literature connecting bond markets with fuzzy mathematics is quite small. However, interesting work is being conducted comparing crisp analysis with fuzzy analysis. Consequently, we expect this literature to expand. Additional details of the papers presented in this section are provided in the summary below.

Title	Year	Source	Author(s)	Abstract	Data Utilized
Portfolio Selection Using Fuzzy Decision Theory	1998	Working paper from Bank for International Settlements	Ramaswamy, S.	This paper presents an approach to portfolio selection using fuzzy decision theory. The approach is such that a given target rate of return is achieved for an assumed market scenario. If the assumed market scenario turns out to be incorrect, the portfolio is guaranteed to secure a given minimum rate of return. The methodology is useful in the management of assets against given liabilities or in forming structured portfolios that guarantee a minimum rate of return.	As a numerical example, this paper illustrates the use of fuzzy decision theory to structure portfolios by considering the case of a fund manager who is allowed to hold only government bonds and plain vanilla options on them. It is assumed that the fund manager holds a portfolio of US Treasuries consisting of the current two-, five-, and 10-year bonds. Using the current yield to maturity of these bonds and the yield of the one-year T-Bill, the linearly interpolated par yield curve is constructed.

Using a Fuzzy Sets Approach to Select a Portfolio of Greek Government Bonds	2004	*Fuzzy Economic Review*	Michalopoulos, M., Thomaidis, N.S., Dounias, G.D., Zopounidis, C.	In this paper the authors investigate the application of a methodology based on fuzzy-sets theory to the selection of an optimal portfolio of Greek government bonds. Investors' goals for the different bond market scenarios are formulated in fuzzy qualitative terms, while a model of fuzzy mathematical programming is used for the specification of the portfolio that optimally meets the given goals. The reliability of the results obtained with this methodology is checked with the aid of simulation.	2000–2001, Greece

A Fuzzy AHP Application on Evaluation of High-Yield Bond Investment	2008	*WSEAS Transactions on Information Science and Applications*	Lee, C.-Y., Cheng, J.-H.	The returns and risks of high-yield bonds (HYB) lie between stocks and Treasury bonds. In view of investment opportunities and the rate of return, the advantages of HYB are both lower risks and higher returns. Therefore, HYB has become an important component in portfolios. The purpose of this study is to identify critical factors related to the selection of HYB. Primary criteria to evaluate HYB selection are obtained from the literature survey and applying fuzzy Delphi method (FDM). Then, fuzzy analytic hierarchy process (FAHP) is employed to calculate the weights of these criteria, so as to build the fuzzy multi-criteria model of HYB investment. The results indicate that the greatest weight is placed on the dimension of the economic environment. Three critical evaluation criteria related to HYB selection are: (1) spread versus Treasury, (2) bond callability, and (3) default rate	The analysis is based on the literature review and an experts' questionnaire.

				indicator.	
Fuzzy Pricing Model of Convertible Bonds in China and its Algorithm	2010	*Journal of Systems Engineering*	Zhang, W.-G., Shi, Q.-S., Xiao, W.-L.	On the basis of the Black–Scholes model and the traditional bond pricing model, the convertible bond fuzzy pricing problem is discussed. The fuzzy pricing formula for convertible bonds and its algorithm are given under the assumption that the risk-free interest rates, stock prices, and stock price volatility are fuzzy numbers. The empirical results of Shanghai International Airport Corporation's convertible bonds show that the fuzzy model presented in this paper is effective in forecasting the convertible bond's market price.	Shanghai International Airport Corporation's convertible bonds, China

Bond Pricing under Imprecise Information	2010	*Operational Research*	Agliardi, E., Agliardi, R.	This article develops a computational method to implement the effect of imperfect information on the value of defaultable bonds. Fuzzy modeling is adopted and the numerical experiments show that an imprecise value of the underlying asset and/or the barrier triggering default have material impact on the qualitative shape of the term structure of credit spreads.	The paper provides a qualitative study only. Their model is calibrated to assumed base case values and the analysis is restricted to A- and B-rated firms to study how the model works on investment-grade and high-yield bonds.

Study on Fuzzy Evaluation of Credit Risk of Corporate Bond	2013	*Journal of Digital Information Management*	Jiazhong, O., Min, L.	The credit risk evaluation of corporate bonds is one of the difficult and hot research fields in the related research and plays a key role for corporate financing. Based on the fuzzy theory and analytic hierarchy process, a new credit risk evaluation model of corporate bonds is presented. First an evaluation indicator system of credit risk of corporate bonds is designed through analyzing the characteristics of the evaluation indicator with more details. Second, an analytic hierarchy process is used to determine the level of different indicators, and multistage comprehensive fuzzy evaluation is used to evaluate the credit risk of corporate bonds. Finally, corporate bonds of 10 enterprises are taken for examples to evaluate the credit risk and verify the validity and feasibility of the model. The experimental results show that the model can evaluate the credit risk of different corporate bonds of different enterprises	September 1985 to December 2010, 10 companies (AAA-rated), China

				practically.	
Catastrophe Bond Pricing for the Two-Factor Vasicek Interest Rate Model with Automatized Fuzzy Decision Making	2015	*Soft Computing*	Nowak, P., Romaniuk, M.	Catastrophe bonds are financial instruments, which enable the transfer of natural catastrophe risk to financial markets. This paper is a continuation of the authors' earlier research concerning catastrophe bond pricing. They assume the absence of arbitrage and neutral attitude of investors toward catastrophe risk. The interest rate behavior is described by the two-factor Vasicek model. To illustrate and analyze obtained results, they conduct Monte Carlo simulations using parameters fitted for real data on natural catastrophes. Besides the crisp cat bond pricing formulas, they obtain their fuzzy counterparts, taking into account the uncertainty of the market. Moreover, they propose an automated approach for decision making in a fuzzy environment with relevant examples presenting this method.	The authors consider only example cases.

Fuzzy Decision Fusion Approach for Loss-Given-Default Modeling	2017	*European Journal of Operational Research*	Nazemi, A., Fatemi Pour, F., Heidenreich, K., Fabozzi, F.J.	In this paper, fuzzy decision fusion techniques are applied to predict loss-given-default of corporate bonds. In the authors' model, the principal components derived from more than 100 macroeconomic variables are added as explanatory variables. However, in order to improve the performance of the model, the Box–Cox transformation of macroeconomic variables is applied prior to loss-given-default modelling. A differential evolution algorithm is used to create an optimized fuzzy rule-based model that fuses the outputs of several base models. They compare the predictions from fuzzy decision fusion techniques with support vector regression techniques, regression trees, and OLS regressions. Their findings show that fuzzy decision fusion techniques increase prediction accuracy of loss-given-default modelling and transformations of macroeconomic factors do not affect prediction accuracy of	2002–2012, US

				fuzzy models.	

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