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ALLOWING FOR ASSET, LIABILITY AND BUSINESS RISK IN THE VALUATION OF A LIFE OFFICE

BY S. J. B. MEHTA, M.A., F.I.A.

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ABSTRACT

The theme of this paper is that life office valuation can be based on an analysis of the characteristics of the individual cash flows which form the net cash flow of an office. These cash flows may be compared with the income and gains derived from traded securities such as equities and gilts in order to assess their values. Standard asset pricing models of modern portfolio theory enable one to allow for lapse, mortality and other risks.

The methodology outlined in the paper is illustrated by reference to the very different types of revenue generated by non-profit, unit-linked and with-profits products. It is also used, in the valuation of future new business, to allow for the risks that expected sales volumes and profitability levels will not be achieved.

The paper provides a rationale for the choice of discount rates used in the valuation of a life office and facilitates the comparison of life office and alternative investment opportunities.

KEYWORDS

Appraisal Value; Life Office; Modern Portfolio Theory; Valuation

1. INTRODUCTION

1.1.1 This paper explores how an examination of asset, liability and business risk can assist in valuing the policyholders' and shareholders' interests in a life office. Investment in a life office is one of many possible investment choices open to investors. The proposition set forth in this paper is that an investor can examine the individual cash flows which make up the aggregate net revenue of the office and can consider the returns available on traded securities in order to obtain an assessment of comparative value. For this purpose, some individual items of life office income and expenditure can be compared directly with future cash flows from quoted securities. Other insurance cash flows have more complex characteristics and need to be valued using general asset pricing models such as those developed in modern financial theory.

1.1.2 It is possible, having derived the value of individual cash flow components, to determine an equivalent single discount rate. If required, separate rates can be assessed for net worth, in-force value and the value of future new business. It is interesting to assess these equivalent discount rates in order to understand the overall riskiness of the life office.

1.1.3 The paper focuses on general methodology rather than on the particular purpose for which the valuation is used. The reader is referred to Bangert (1973) for an outline of some of the reasons for life office acquisition and merger, for a

discussion of the principles of valuation, as well as for the need for pragmatism. Burrows & Whitehead (1987) provides a practical guide to life office appraisal, and Sherris (1987) outlines how the Capital Asset Pricing Model can be used to value individual cash flows.

1.1.4 Sections 1.2, 1.3 and 1.4 provide a brief introduction to asset valuation, life office valuation, and the characteristics of life office profit. The detailed approach suggested in the paper is described in Section 2. Section 3 discusses how investment return assumptions can help to provide a benchmark against which to assess discount rates for use in a valuation. Sections 4, 5 and 6 illustrate the methodology by exploring the special features of non-profit, unit-linked and with-profits business. Section 7 examines the effect of sales and lapse risk on the value of future new business sales. Additional considerations required to value a whole company are discussed in Section 8.

1.1.5 The author believes that the approach set out in this paper has not been comprehensively applied to life office valuation in the past and should be considered for use in the future.

1.2 *Valuation of Assets and Liabilities*

1.2.1 An asset is a resource from which future economic or other benefits are expected to flow.

1.2.2 The value of an asset can be assessed by reference to:

- prices at which transactions in the asset have taken place, or
- values of other assets, having regard to their expected returns and risks.

Similar observations apply to the valuation of a liability, to the valuation of an interest in an asset and to the valuation of an asset which consists of a combination of various assets and liabilities, such as a life office. Where a comparative reference value is not available, it may be possible to assign individual values to the assets and liabilities which make up the combined asset.

1.3 *Life Office Valuation*

1.3.1 In recognition of the different characteristics of the shareholders' interests in net worth, in-force business, and future new business, a separate valuation of each of these components of value is performed in many life office appraisals. For example:

- Net worth may be assessed as the market value of the 'other than long-term' fund assets less liabilities.
- Future profits arising from the long-term fund assets and liabilities may be discounted using an interest rate which has some regard to the returns available on traded securities.
- The value of profits arising from all future new business ('existing structure value') may be determined by applying a multiple to the value of one year's sales. The multiple used is likely to be influenced, for example, by values

implied in transactions involving other companies, having regard to differences in new business growth and other prospects.

1.3.2 Selection of a single rate of discount to value all future cash flows, including those from future new business, is an equally valid approach, but it suffers from two main drawbacks:

(i) The choice of the appropriate discount rate is more difficult. The rate selected will be affected by the relative proportions of net worth, in-force business and future new business for each company being valued.
(ii) The values derived for the individual components of value may not be valid. For example, existing structure value may be overstated and in-force value understated, compared to an open market valuation.

1.3.3 Before considering the subject of life office valuation in detail, it is useful to examine the nature of life office cash flows and profit more closely.

1.4 *Life Assurance Business Profit Characteristics*

1.4.1 A major difference between the business of life assurance and that of many other businesses is that cash flows arising from business sold in the year emerge over many years rather than in the year of sale.

1.4.2 Positive cash flows arise primarily from returns on the invested assets, from future premiums expected to be paid by policyholders and from assets to be acquired out of future positive net cash flows. The future premiums can be viewed as a special type of asset, namely as a set of (typically) fixed monetary payments subject to decrement arising from lapse or from death of the policyholder.

1.4.3 Set against the positive cash flows are negative cash flows payable to agents and employees, suppliers, policyholders and the Inland Revenue. The special characteristics of unit-linked and with-profits business result in some degree of alignment between the interests of policyholders and shareholders, so that some shareholder cash flows are proportionate to certain policyholder cash flows.

1.4.4 There are, however, similarities between profit for a life assurance business and for a manufacturing concern. For the latter, economic profit can be defined as:

- achieved investment return on net worth, including realised and unrealised gains, plus
- profit earned in the year, that is income less associated costs in respect of sales in the year, less
- overhead costs.

1.4.5 For life assurance business, sales in the year generate future cash flows. The overall profit generated in the year may be expressed as:

- achieved investment return on net worth plus achieved return on the value of expected future cash flows from business sold in previous years, plus

- the value of cash flows arising from sales in the year (earned profit), less
- overhead costs.

1.4.6 The comparison is not taken further in this paper, and for this reason the terms net worth, income, costs and overhead costs as applicable to a manufacturing company have not been defined. The similarities end here, because net worth for a manufacturing company is not the same as net worth for a life company. Also, as seen above, cash flows derived from the in-force business have different characteristics from the returns on net worth.

1.4.7 The three components of life assurance business overall profit may have very different risk characteristics. For example, overhead costs could be relatively stable whereas value added from sales in the year could vary significantly from year to year.

2. VALUATION METHODOLOGY

2.1 *Introduction*

2.1.1 Individual life office cash flows exhibit generally simpler risk/return characteristics than the aggregate net cash flows of an office. Section 2.2 discusses how 'simple' cash flows can be valued by reference to the future cash flow characteristics of quoted securities. Some of the cash flows are more complex or interact with each other. Modern financial theory has developed a number of ways in which the risk and return of complex assets can be valued. These are outlined in Sections 2.3 and 2.4 and described in more detail in Appendix A. The relationship between the equivalent overall discount rate required to value net cash flows and the assets and liabilities of the office is noted in Section 2.5. The effects of establishing statutory reserves, and of holding surplus or solvency capital, are considered in Sections 5 to 8.

2.2 *Simple Cash Flows*

2.2.1 For a non-profit product, an office expects to receive a set of fixed payments in the future (premiums), to incur some expenses and to pay a fixed sum on maturity of the policy. The effects of policy lapse, mortality and other risks, as well as taxation, should not be ignored. They create an additional layer of complication which can be considered separately.

2.2.2 The future premiums resemble the coupon payments and maturity proceeds of gilt-edged securities, and a gilt portfolio could be constructed to replicate closely the pattern of expected future premium receipts. The lack of a full complement of gilt maturity dates does not present a major difficulty in the valuation, because the yield curve is reasonably smooth. Since lapses and mortality are being ignored at this stage, shareholders could be expected to be largely indifferent as to whether future profits and liabilities are funded from premium receipts or gilt coupon and maturity payments. The yield curve can, in principle, be analysed to assess the implicit value of a payment at any future date.

2.2.3 There is an alternative way of approaching the same valuation problem.

Many investors in the bond and equity markets are able to borrow, or to reduce the proportion of their portfolio invested in gilts. These investors can rearrange their portfolios in order to offset the effect of the future premium receipts. The value of these receipts can thus be assessed by reference to yields available on government bonds and the cost of borrowing.

2.2.4 Similar considerations apply in valuing policy maturity proceeds and expenses incurred by the office. In regard to expenses, it should be noted that fixed-interest government bond receipts are subject to inflation risk and, arguably, this risk is reflected in the level of long-term bond yields (see Appendix B.2). For this reason, to the extent that expenses are real rather than nominal monetary payments, they can be valued using an interest rate net of the premium for inflation risk.

2.2.5 Expected asset cash flows include income as well as proceeds arising from sale of the asset in the future. In the author's view, future asset cash flows need to be valued by adjusting market values to reflect the difference between life fund taxation and tax implicit in the market valuation of traded securities. Use of unadjusted market values could be inappropriate. For example, if a 100% tax rate were applied to life fund asset returns, a nil value, not a market value, could be attributed to these returns.

2.2.6 The approach adopted is to assess investment return assumptions, that is internal rates of return equating future expected asset cash flows with market values (Appendix B). These returns are assessed both for a gross investor and a net corporate investor, in Section 3. The projected asset cash flows, net of allowance for life office taxation, are then valued using risk adjusted discount rates consistent with the underlying investment return assumptions. Effectively the assets are valued at market prices, and an adjustment is made in respect of the value of life fund taxation payable in addition to that assumed for direct investment by a gross or net investor.

2.3 *Capital Asset Pricing Model*

2.3.1 The methods outlined in Section 2.2 can be used to value future cash flows, allowing for the effect of lapses, mortality and other risks on the expected level of future cash flows. Different considerations are required to adjust for the effect of these risks on the variability of future cash flows.

2.3.2 One approach, suggested by Sherris, to estimating the additional discount rate required to allow for risk, is to use a standard asset pricing model such as the Capital Asset Pricing Model (CAPM) described in Appendix A. Under CAPM, risk is divided into non-diversifiable and diversifiable components. The diversifiable component is that part of risk which, in the limit, does not contribute to the overall riskiness of a well diversified portfolio of assets (as the number n of assets held increases, the contribution to total risk increases less rapidly than $1/n$). The non-diversifiable component of risk, 'systematic' risk, relates to returns which are correlated with a broadly based index of asset prices. According to CAPM, if unsystematic risk were 'priced' (that is contributed

towards higher expected return from an asset), portfolio investors could purchase securities with high unsystematic risk and achieve higher expected total portfolio returns for only a small addition to the variability of the total portfolio return. The price of unsystematic risk would fall as demand for securities carrying this risk increased. Similar arguments could be developed for individual cash flows.

2.3.3 If CAPM is true, the required return on any asset is a linear combination of the risk free rate and the expected return on the market index, weighted according to the asset's systematic risk factor, beta. Any cash flow which is not correlated with the market index has a beta of zero and is valued using the risk free rate; other cash flows can be valued using a discount rate dependent on their systematic risk. In practice, modifications to CAPM, and the Arbitrage Pricing Theory also described in Appendix A, allow for other systematic risk factors and not just variability in market returns. If it is assumed that mortality rates and real expense levels are not correlated with the market index and other 'priced' risk factors discussed in Appendix A.2 and A.3, mortality and expenses risks are entirely unsystematic and are not associated with an increase in required returns. As seen in Sections 7 and 8, unsystematic risk may result in an increase in the cost of trading and in the risk of corporate failure. These effects need to be factored into the valuation process, even if no addition to the discount rate is made. The concluding section of this paper comments further on the use of a risk free rate to value cash flows subject to only unsystematic risk.

2.3.4 Section 5.5 describes how CAPM can be used to estimate the additional discount rate required to allow for lapse risk.

2.3.5 Use of an asset pricing model such as CAPM is one of the very few ways in which discount rates required to compensate for risks borne can be assessed. The lack of any simple and realistic alternative framework is one reason why the author would advocate greater use of these models in life office appraisal valuations. Some of the limitations of these models disappear when used to assess asset risks, rather than liability and business risks, if life office portfolio investments are reasonably well diversified.

2.4 *Option Pricing Theory*

2.4.1 Some insurance cash flows accrue to shareholders only in the event of a suitably favourable or unfavourable asset return. For example, an office which has a mismatch of assets against guaranteed liabilities may only be able to continue writing new business if it achieves a sufficiently high return on its asset portfolio. The shareholders benefit from value attributed to future new business if asset returns exceed a minimum level. This example is a complicated form of a call option on an asset, a security which provides the right to purchase the asset at a prespecified (exercise) price at or before an exercise date.

2.4.2 Options cannot be valued accurately using standard asset pricing models because of the skewness of the return distribution of option prices. Black & Scholes (see Appendix A.4) showed that if the returns from the underlying asset

have a normal or lognormal distribution, the returns from an option can be replicated instantaneously by purchasing an interest in the underlying asset and borrowing a proportion of the exercise price. From this finding they developed a general option pricing equation, solutions of which provide estimates of option values.

2.4.3 In practice, insurance company option pricing problems are typically too complex to model and solve accurately. Valuation entails developing simplified examples to capture the essential factors which determine the discount rate, and then applying stochastic modelling techniques to incorporate the detailed asset return and liability cash flow interactions.

2.5 *Equation of Value*

2.5.1 As noted in the Introduction, it is instructive to assess the equivalent overall discount rate required to value net cash flows, based on the assessed values of the assets and liabilities. This can be determined from the assumption that the value of the shareholders' interest E is equal to the value of the assets less the value of the liabilities. If the assets have a value of A at time 0 and provide an overall return of r, and if the liabilities are worth L based on a discount rate of g, the discount rate e required to value the shareholders' interest can be found from the equations of value at time 0 and time 1:

$$A - L = E,$$

and

$$A\,(1+r) - L\,(1+g) = E\,(1+e).$$

Solving:

$$rA - gL = eE,$$

or

$$e = r + (r-g)L/E.$$

This equation, applied in the context of a firm financed partly by equity E and partly by debt L, is a standard result of modern financial theory. Its extension to insurance business follows, given the analogy which can be drawn between policyholders' guaranteed liabilities and corporate debt.

2.5.2 In the absence of any significant likelihood of default, the discount rate required to value net cash flows increases linearly with asset risk and with the ratio of policyholders' liabilities to shareholders' equity. The equation remains valid in the presence of corporate default risk, but g and L are then dependent on the ratio of policyholders' liabilities to equity. This is because an increase in shareholders' equity reduces the probability that variability of asset or liability cash flows will result in insufficient assets to meet the liabilities in full. The effect of default risk and whole company valuation is considered further in Section 8.

2.6 *Summary*

2.6.1 In this paper it is suggested that a life office valuation should start from a consideration of the nature of the individual cash flows which make up the

expected future net cash flows of the office. These cash flows are generally simpler than net cash flows, and the values of traded securities such as bonds and equities form a suitable starting point for comparative valuation. Values need to be adjusted for risks incurred by the life office. In this regard, modern portfolio theory suggests that suitable allowance can be made by dividing risk into its systematic and unsystematic components. Some insurance cash flows are contingent on the outcome of other asset returns. Option pricing theory can be used to assess the systematic risk of these contingent cash flows.

3. INVESTMENT RETURNS AND DISCOUNT RATES

3.1 *Introduction*

3.1.1 Discount rates used in the valuation of insurance company assets and liabilities need to be consistent with the assumptions made regarding future investment returns. Section 3.2 summarises the investment return assumptions used in this paper and Sections 3.3 and 3.4 consider the effect of taxation on the level of market prices and on investor risks and returns.

3.2 *Investment Return Assumptions*

3.2.1 Some of the analyses of historic and other data that can be made in order to set investment return assumptions are described in Appendix B. The examples in this paper assume the following:

	Long-Term Investment Return Assumptions % p.a.
Inflation	5
Real short-term interest rates (three-month Treasury bills)	3
Real long-term interest rates	4
Equity risk premium (relative to short-term interest rates)	6
Short-term interest rates	8
Index-linked gilt returns	8
Long-term interest rates	$9\frac{1}{4}$
Equity returns	$14\frac{1}{2}$

A further assumption is that the standard deviation of equity returns is 15%.

3.2.2 A multiplicative model is assumed, so that, for example, the equity return assumption is $(1{\cdot}05 \times 1{\cdot}03 \times 1{\cdot}06 - 1) \times 100$. It is also assumed that the term premium, the difference between expected future short-term interest rates and long-term interest rates, arises from inflation risk. Since index-linked gilts are not subject to inflation risk, the return on index-linked gilts is assumed to be the same as the expected future level of short-term interest rates.

3.2.3 The assumed level of equity return, $14\frac{1}{2}$% p.a., is based on an 8% p.a. level of Treasury bill yield and a 6% p.a. level of equity risk premium. A standard approach in actuarial literature is to compare historic equity returns with

inflation, in order to assess the prospects for future real equity returns. In contrast, the normal practice in modern financial empirical and theoretical research is to measure equity returns relative to Treasury bill yields rather than to inflation rates. This is because it is believed that investors compare equity risk and return with the risks and returns of alternative investment opportunities. Thus, equity risk results in investors requiring higher returns from equities than from Treasury bills irrespective of the level of real Treasury bill yields. The practice results in a higher estimate of total equity return than if inflation rates are used as a base and the measurement period includes a period of low real Treasury bill yields, such as in the period 1940 to 1980. For example, consider a period of measurement when real Treasury bill returns averaged 0% p.a. and real equity returns were 6% p.a. If the expected future Treasury bill real yield is 6% p.a., the traditional approach would result in an assessment of expected future equity returns of 6% p.a., the same as the Treasury bill yield. There would be no reward for risk. Modern financial theory would suggest use of a 12% p.a. real equity return assumption, based on a 6% p.a. level of equity risk premium. This level of equity risk premium is typical of the results of empirical research into a number of equity markets across the world. A full review of the arguments is beyond the scope of this paper.

3.2.4 It should be noted that the choice of equity risk premium is important, because of the effect it has on the discount rates selected and on the resulting asset and liability valuations.

3.3 *Taxation*

3.3.1 The demand and supply curves for any asset, and consequently its market price, are likely to be influenced by taxation. In terms of the U.K. bond markets, interest receipts are taxed in the hands of certain investors, and the issuer of a bond generally obtains tax relief on the interest payments. The market price of an interest coupon of £100 may fall short of £100 if there is insufficient demand from gross investors or lenders for whom the coupon is worth £100. This would also be true if there is a sufficient supply from borrowers who obtain tax relief (or tax revenue, for the government), or from sales made by tax payers. Similar observations hold in relation to dividends from equities and the extent to which the attaching tax credits carry value. Since many investors have a choice of whether to invest in bonds or in equities, there is likely to be some linkage between the effects of taxation on bond and equity prices. One factor which suggests that the effect of tax on market prices is limited is the increasing dominance of gross investors, such as pension funds and also banks and multinational portfolio investors, who pay tax only on the difference between asset returns and the amounts credited to depositors or clients. Other factors are the reduced level of government borrowing requirement, the restricted availability of personal tax relief on interest payments, the increased opportunities for tax-free saving and investment and the interdependence of U.K. and overseas markets.

3.3.2 Examination of the yields available on zero coupon eurobonds and of the prices paid for preference shares also suggests that market prices may be closer to the discounted value of gross rather than net returns. On the other hand, prices of very low coupon gilts clearly do exhibit a significant tax effect. Arguably, markets are segmented, with equities and bonds in the main incorporating value requirements of gross investors, but with certain tax favoured investment categories reflecting the tax position of net investors. Overall, however, there is relatively little firm empirical evidence on the degree to which the tax position of net or gross investors is factored into market prices. It is useful to develop both pre-tax and after-tax investment return expectations in the absence of conclusive evidence that one or other is closer to the assumption set underlying observed asset prices.

3.4 *After-Tax Risk and Return*

3.4.1 The following table shows the approximate after-tax returns available to a net corporate investor based on the gross returns specified in Section 3.2. It is assumed that the rate of corporation tax is 33%, basic rate tax is 25% and that corporation tax is applied to equity capital gains net of indexation relief. For example, the net return on equities, assuming a 5% p.a. gross dividend yield (3·75% net), 4% p.a. real growth (2·7% net), and 5% p.a. inflation, is $1{\cdot}0375 \times 1{\cdot}027 \times 1{\cdot}05 - 1 = 12\%$ p.a.

	Asset Returns and Risks for a Net Corporate Investor	
	Gross Return %p.a.	Net Return % p.a.
Treasury bills	8	$5\frac{1}{2}$
Index-linked gilts	8	7
Long-term gilts	$9\frac{1}{4}$	$6\frac{1}{4}$
Equities	$14\frac{1}{2}$	12
Standard deviation of equity returns	15	10

3.4.2 It is interesting to note that the reduction in expected net equity return for a net investor is accompanied by lower equity volatility, since a fall in equity prices will be accompanied by a reduction in any liability to tax on capital gains. For example, consider an investment which provides a return of $+1$ if the result of one toss of a coin is a 'head' and -1 if the outcome is a 'tail'. For a gross investor, the mean return is $0{\cdot}5 \times ((1) + (-1))$, that is 0. The standard deviation is the square root of $0{\cdot}5 \times ((1)^2 + (-1)^2)$, that is 1. For a net investor paying 30% tax, the mean return is $0{\cdot}5 \times ((1) \times 0{\cdot}7 + (-1) \times 0{\cdot}7) = 0$ assuming immediate tax relief for losses. The standard deviation is the square root of $0{\cdot}5 \times ((1 \times 0{\cdot}7)^2 + (-1 \times 0{\cdot}7)^2)$, that is 0·7.

3.4.3 The CAPM (see Appendix A.2) market line is a straight line in a risk/return plane. It passes through the point of nil risk and a risk free return, and also

the point of 'equity' risk and an equity risk return. Equity risk is defined here as the annualised standard deviation of equity returns.

3.4.4 For a gross investor, this risk free return is usually taken to be the return on Treasury bills. For a net investor, Treasury bills and gilts are tax-inefficient investments and it can be argued that the (higher) net return on index-linked gilts represents a better measure of risk free return opportunities. With these assumptions, the gross and net CAPM market lines based on a geometric model become:

CAPM Risk/Return Market Line

	Risk Free Rate % p.a.	Excess Return on Equities % p.a.	Volatility of Equity Returns % p.a.
Gross investor	8	6	15
Net investor	7	$4\frac{1}{2}$	10

3.4.5 These CAPM market lines can be used to assess the discount rates required to value insurance company cash flows attributable to shareholders (or to the long-term fund), according to the degree of systematic risk borne. If the gross investor CAPM line is used, the returns valued should be grossed up to allow for the recoverability of tax credits on distributions made by the insurance company. If the net investor line is used, allowance should be made not just for taxes borne by the insurance company, but also for the reduction in return and risk arising from taxation of gains on the investors' holding in the company. The effect of investor tax and life assurance taxation on the assessment of discount rates is considered further in Appendix C.

3.5 *Summary*

3.5.1 This Section discusses the degree to which tax is factored into market prices, and the effect of tax on investment risk and return. Some of the theoretical arguments suggest that markets reflect the tax position of gross investors and borrowers. In view of the lack of conclusive empirical evidence, the sets of investment return opportunities available to both gross and net investors are considered. These sets of returns provide a framework for assessing discount rates required to value more complex assets and liabilities. They are needed as CAPM input parameters, where a CAPM approach to valuation is used.

4. NON-PROFIT PRODUCTS

4.1 *Introduction*

4.1.1 For non-profit products, policyholder liabilities can be likened to debt. Asset risk is borne primarily by shareholders. The liabilities result in a 'gearing-up' of asset risk and this increases the discount rate required to value net cash flows.

4.1.2 The valuation of liabilities and of assets is discussed in Sections 4.2 and 4.3. Section 4.4 notes that option pricing theory can be used to value policyholders' rights to guaranteed cash values, and to value corporate choice as to the interest rates credited to policies. Although accurate modelling of these options is likely to be difficult, the result of applying a simple model, described in Appendix D and summarised in Section 4.5, shows that the options can be valued approximately by valuing stochastic net cash flows, using a discount rate which depends on the difference between short and longer-dated bond yields.

4.2 *Valuing Non-Profit Liabilities*

4.2.1 A starting point for determining the discount rate required to value non-profit liabilities is to use the interest rate on government bonds of similar term, as discussed in Section 2.2. Where lapses are a significant characteristic of the business, a stochastic model which allows for the interaction of interest rates and lapse rates is desirable. The result needs to be adjusted to reflect riskiness introduced by any options granted to policyholders or shareholders because the expected cost, not the systematic risk, is allowed for in the stochastic modelling process. Option pricing theory is needed to value the systematic risk.

4.3 *Asset Risk*

4.3.1 The assets backing non-profit liabilities are typically fixed-interest or floating-rate securities.

4.3.2 Corporate securities offer interest margins over government stock of similar type and term for some or all of the following reasons:

(i) Expected default costs.
(ii) Systematic default risk. Because defaults tend to be high during periods when, for example, equity markets are falling, default costs have 'systematic' risk.
(iii) Transaction costs.
(iv) Taxation differences.
(v) Premium for illiquidity.

If the last three factors are assessed to have a significant impact on asset yield, it may be appropriate to use a discounted cash flow rather than market value approach to asset valuation. The discount rate selected needs to allow for systematic default risk generated by investment in corporate bonds. This risk can be estimated using correlation analysis between the interest margin over government stock and the stock market index, as discussed in Appendix A.

4.3.3 The existence of fixed policyholders' liabilities results in a gearing-up of asset risk borne by shareholders. Consider an example where shareholders' equity, E, is 7% of policyholders' liabilities, L, and the total assets are invested in corporate rather than government bonds yielding an additional 0·2% p.a. of interest r, in respect of systematic default risk. This risk increases the discount rate required to value future net cash flows by about 3% p.a. (0·2 + 0·2/·07, based on the equation derived in Section 2.5).

4.4 *Policyholder Put and Company Call Options*

4.4.1 Lapse risk arises from:

- Variations in lapse rates. A method for dealing with systematic lapse risk is outlined in Section 5.5.
- An option against the office, for guaranteed cash value products, if interest rates increase sufficiently to offset the effect of any surrender value penalty (the policyholders' put option).

4.4.2 For certain types of products, the company has an option to reduce the interest rates credited to policies if interest rates fall. This is equivalent to a call option on the underlying asset, with exercise price equal to the value of a bond purchased at the time of exercise. In the event of an interest rate fall, the company can reduce the credited rate and sell the original bond investment for a profit. If interest rates rise, the credited rate can be maintained and the original bond held to maturity.

4.5 *Option Valuation Results*

4.5.1 Both put and call option features arise for interest sensitive single premium deferred annuity (SPDA) products commonly sold in the U.S.A. Appendix D summarises the results of applying option pricing theory to value the put and call option features for an SPDA, with certain simplified liability and asset characteristics, for a variety of input parameters and sensitivities. Application of the methodology to typical U.K. products is straightforward.

4.5.2 In the examples given, the call option has a high level of systematic risk, so that stochastic cash flows arising from the option need to be valued using a high discount rate. The put option has low systematic risk and is valued using a low discount rate. An examination of the sensitivity results presented in the Appendix shows that a key factor determining the level of discount rates required to value the options is the term premium (the difference between the expected return on the initial bond investment over the exercise period and the return on a bond with term to maturity equal to the exercise period). Other factors, such as the volatility of bond yields, the credited rate spread and the option term, have a relatively smaller impact on the level of discount rate.

4.5.3 Based on the parameters used in the examples, the systematic risk of the put and call options can be allowed for approximately (without the need for exact modelling of the option features), by reducing the discount rates used to value the stochastic liability cash flows by approximately 0·05% p.a. for the policyholder put option, and 0·25% p.a. for the shareholder call option, in total 0·3% p.a. In other words, the total value of the liabilities, allowing for the Black–Scholes values of the put and call options, can be determined approximately by assessing the mean discounted value of the stochastic liability cash flows. The discount rate is the yield on government bonds, with term equal to the liability term, reduced by 0·3% p.a. Systematic risk introduced by a term premium of 1% p.a., as in the numerical examples, gives rise to the need for a 0·3% p.a. adjustment to the discount rate.

4.5.4 The examples illustrate that, although exact modelling of option terms may be impractical, construction of a simplified model representing the main option features can assist in the choice of discount rate used to value liability payments.

4.6 *Summary*

4.6.1 Use of government bond yields of appropriate term is suggested for valuing non-profit product premium and liability payments. An adjustment may be required to allow for systematic lapse risk. Expenses can be valued using expected future short-term interest rates. Stochastic cash flow modelling, combined with approximate option pricing techniques, is required to value guaranteed cash value and interest rate options. Assets can be valued using discounted cash flow techniques if, for example, liquidity risk is borne by policyholders.

4.6.2 The following table illustrates the equivalent overall discount rate required to value net cash flows, based on the examples of asset and liability risk premium levels discussed in this Section. By 'risk premium' is meant the additional yield, required because of systematic risk, over and above the yield on a government bond with term equal to the average term of the liabilities.

Source of Risk	Value	Risk Premium % p.a.	Effect on Overall Discount Rate % p.a.
SPDA liability with interest rate put and call options	(90)	(0·3)	1·8 (90/15 × 0·3)
Assets invested in fixed interest bonds	100	0·2	1·3 (100/15 × 0·2)
Surplus invested in equities, say	5	5·0	1·7 (5/15 × 5·0)
Total	15	–	4·8

4.6.3 In the above table, if the government bond yield is 9% p.a., the overall discount rate required to value net cash flows allowing for surplus is 13·8% p.a. (9% + 4·8%). The rate required to value net cash flows before allowing for surplus is 13·7% p.a. (9% + 90/10 × 0·3% + 100/10 × 0·2%). It can be seen that, since the values of the assets and liabilities are both substantially larger than shareholders' equity, a relatively small addition to the level of asset or liability risk results in a significant increase in the overall discount rate required to value the shareholders' interests.

5. UNIT-LINKED POLICIES

5.1 *Introduction*

5.1.1 Profits from unit-linked business arise from a variety of sources. Some cash flow streams are relatively certain, and can be valued using a discount rate close to the yield on a government bond. Unit fee income is proportional to the market value of the linked asset portfolio, and can be valued using a discount rate which corresponds to the asset return rate. These cash flow characteristics are

considered in Sections 5.2 and 5.3. Section 5.4 examines in brief the effect of surplus.

5.1.2 Variability in policy lapse rates creates additional risk and is considered separately, in Section 5.5. Section 5.6 describes how the equivalent aggregate discount rate required to value net cash flows can be found.

5.2 *Product Characteristics*

5.2.1 Premiums payable under unit-linked policies are divided between amounts invested for the benefit of policyholders, accumulated in the unit fund, and amounts which accrue for the benefit of shareholders. Shareholders also benefit from unit fees and other charges deducted from the unit funds. These items of income are required to pay for policyholder benefits not covered by the unit funds, for expenses, commissions and taxation, and to provide a margin of profit.

5.2.2 The shareholders have a relatively limited direct interest in the unit fund, since the full unit fund is usually paid out to policyholders on maturity of the policy. In order to recoup initial costs, the amount paid out on policy surrender in the first few policy years is sometimes less than the accumulated unit fund, giving rise to a profit when a policy lapses. For other types of policy, initial costs are recouped by a reduction in the allocation rate in the first few policy years. The full accumulated unit fund is then typically paid out on surrender.

5.2.3 In the U.K., pensions business profits are subject to tax, whilst the tax base for life business is investment income and chargeable gains less relievable expenses. The effect of tax for life unit-linked policies is mitigated, because companies levy a 'tax' deduction from the unit funds based on the amount of investment income and gains. For many offices, the net effect of tax less tax charged to the policyholders' funds is that the net cost to shareholders can be expressed as tax on profits less tax relief on expenses.

5.2.4 For some offices relievable expenses exceed taxable income, and payment of tax is deferred. The tax deduction from the unit fund may then exceed the effective rate of tax borne on investment income and gains. This leads to a 'tax' profit, proportional to the amount of life business investment income and gains, which offsets the effect of deferral of tax relief on acquisition costs.

5.2.5 Finally, shareholders have a direct interest in non-unit reserves to the extent that such reserves are set conservatively.

5.2.6 Viewed in this way, shareholders' profits arising from a unit-linked policy can be assessed as the sum of:

(i) premiums not allocated to the unit funds,
(ii) expense charges less expenses,
(iii) mortality charges less costs,
(iv) lapse profits,
(v) unit fee income,
(vi) tax deducted from the unit funds less tax payable, and

(vii) surplus from income and gains on assets backing non-unit reserves, and releases of these reserves.

5.2.7 If it is assumed that tax payable is a proportion e of investment income and chargeable gains less expenses and p of profit, each of the first five items can be appropriately scaled to allow for tax:

- the premium, mortality, lapse and unit fee net cash flows can be multiplied by $(1-p)$, and
- the expense cash flow can be valued net of tax relief and profits tax.

A separate adjustment is required to allow for spreading of tax relief on acquisition costs.

5.2.8 For a typical company, there is little risk of insolvency (the Appointed Actuary will insist on an adequate level of reserves). Because of this, the full net liability to policyholders will be paid and all the items can be valued separately. Risk of insolvency is considered in Section 8.

5.2.9 The value of the shareholders' interest in non-unit reserves is considered in Section 5.4.

5.3 *Cash Flow Risk Characteristics*

5.3.1 Ignoring lapses (these are considered in Section 5.5), premiums not allocated to the unit funds are an almost certain income stream. Expenses and mortality costs are also broadly uncorrelated with the main risk factors identified in Appendix A. Diversification and arbitrage arguments outlined in Section 2 suggest that company specific or unsystematic risk will not be priced, that is, such risks will not be reflected in the required valuation discount rate. Since the premium and other cash flows emerge over a period, it would be consistent to value them using government bond yields of equivalent term. To the extent that it is believed that real expenses are not subject to inflation risk, and that the term premium is related to this risk, expenses can be valued using expected future short-term yields rather than a long-term bond yield.

5.3.2 Unit fees vary in line with the market value of the linked asset portfolio. If the linked asset portfolio is considered to be a series of individual funds, one for each unit allocation, discounting at a bond rate would be appropriate prior to the unit allocation, and at a risk rate from the point of allocation to receipt of the fees. The risk rate will correspond to the riskiness of the linked asset portfolio. A similar argument applies to any tax 'profit' arising from unit fund deductions in excess of tax borne on unit income and gains.

5.3.3 Tax deductions from a linked life fund are usually not dissimilar to the tax rates suffered by a net corporate investor, and consequently the risk rate which can be used to value future unit fees is similar to the net asset return (both have the same after tax risk characteristics). The effect of tax on the discount rate is considered further in Appendix C.

5.3.4 These discount rates need to be adjusted to allow for lapse risk as discussed in Section 5.5.

5.4 *Non-Unit Reserves and Surplus*

5.4.1 Shareholder value arises in respect of non-unit reserves and surplus, because the underlying assets which back these quantities generate income and gains and may eventually become distributable. Since the main liability risks are considered separately, cash flows arising from assets backing non-unit reserves and surplus can be discounted at the asset return rate, adjusted to allow for the difference between the tax position of the fund and tax implicit in the capital market line.

5.5 *Lapse Risk*

5.5.1 An increase in policy lapses in a period gives rise to reductions in the value of future premiums not allocated to unit funds, expenses, mortality payments and unit fees. The increase in lapse rate in the period will also have an effect on any lapse profits in the year, arising from payments on policy lapse to the extent that these are less than the amount of the unit funds.

5.5.2 There appears to be some correlation between adverse lapse experience and factors such as economic downturns, stock market falls and interest rate rises. In the absence of such correlation, lapse risk would be entirely 'unsystematic', that is, unrelated to the APT and CAPM risk factors, and arbitrage arguments could be developed to suggest that it should not be priced. Any agency and other costs (see Sections 7 and 8) associated with a poor lapse experience would then need to be allowed for purely by use of stochastic modelling techniques, rather than in the use of a higher discount rate.

5.5.3 Based on historic lapse data for a number of life offices obtained by the author, a very approximate estimate of the link between regular premium policy lapse rates and stock market returns is for a market decline of 15% to be associated with an increase in the level of lapse rates of 3% (in policy year one) and 1% (for policy years three and over). Expressed differently, the correlation between lapse rates and market returns, multiplied by the ratio of the standard deviation of lapse rates to the standard deviation of market returns, is 3/15 in policy year one and 1/15 for policy years three and over.

5.5.4 CAPM can be used to provide an estimate of the excess return required in respect of lapse risk. From Appendix A.2, the additional discount rate required to value a cash flow stream subject to lapse risk can be assessed as:

- the equity risk premium (say 6% p.a.), times
- the correlation between the market index and the value of the cash flow stream, times
- the standard deviation of changes in the value of the cash flow stream, divided by
- the standard deviation of market returns (say 15% p.a.).

5.5.5 The risk return required to allow for lapse risk in the valuation of the premium, mortality, and unit fee cash flows can then be gauged. If a 1% increase in the lapse rate were to reduce the value of these future cash flows by 1%, the

additional premium would be 6% × 1/15 = 0·4% p.a. A lower additional risk rate is needed for life business, 0·3% p.a., because of the effect of tax. The premium is not required when valuing overhead maintenance costs, since these are generally not affected by policy lapse.

5.5.6 The effect of systematic risk on the value of lapse profits can be much greater. The following table illustrates the effect for a policy where the penalty on lapse is expressed as a proportion of the unit fund. It allows for a 1% increase in lapse rates, and a 15% market fall, at the beginning of policy year three.

	Prior to Market Fall			After Market Fall		
Policy Year	Lapse Rate %	Lapse Penalty £	Profit £	Lapse Rate %	Lapse Penalty £	Profit £
3	10	1000	100	11	850	93
4	10	500	45	10	425	38
5	10	500	40	10	425	34
6+	10	0	0	10	0	0
Present value at 12% p.a.			172			154

A 10% reduction in the value of future lapse profits (in the example from 172 to 154) resulting from a 15% market fall, would increase the discount rate needed to value future lapse profits by 4% p.a. (6% × 10/15) for pension policies and 3% p.a. for life policies.

5.5.7 The effect of lapses in policy year one can be allowed for in existing structure value and this is considered in Section 7.

5.5.8 The analysis outlined in this section suggests that risk introduced by variability in ultimate lapse rates is normally small relative to equity risk. This conclusion is consistent with the author's experience that variations in embedded values arising on account of ultimate lapse experience are typically much smaller than variations in stock market levels.

5.6 *Equation of Value*

5.6.1 The value of the shareholders' interest in a unit-linked office is equal to the value of the office's total assets, less the value of net liabilities to policyholders, $E = A - L$, as in Section 2.5. However, the main policyholder liabilities are determined by reference to the value of the assets held in the unit funds. For this reason, it is useful to restate the equation of value by deducting the value of the unit fund assets from both A and L. Shareholders' equity E, is then the value of non-unit fund reserve assets plus the value of future cash flows less the value of net cash flows into the unit funds. The equation of value becomes:

$$E = S + V_1 + V_2 + \ldots$$

where: S is equal to non-unit reserves and surplus, and
V_i are the values of individual items of cash flow i, measured net of movements into the unit funds.

5.6.2 The equivalent aggregate discount rate used to value net cash flows is the internal rate of return at which the value of these cash flows is equal to E. It can be determined approximately as the weighted average of the discount rates used to value the individual cash flows, with weights equal to the values of these cash flows. This approximation is valid only if the individual cash flow distributions are reasonably similar to each other.

5.7 *Summary*

5.7.1 The following table illustrates the discount rates that could be used to value individual cash flows based on the observations made in this Section and the net and gross discount rates set out in Appendix C. It assumes that the Treasury bill rate is 8% p.a. and that unit funds are invested entirely in equities yielding an expected return of $14\frac{1}{2}$% p.a. The level of additional return required in respect of lapse risk, as outlined in Section 5.5, has been rounded to $\frac{1}{4}$% p.a. for life policies and $\frac{1}{2}$% p.a. for pension policies. It has been assumed that expense charges have the same variability characteristics as expenses; a fixed charge subject to inflation risk would be treated in the same way as premium receipts. No account has been taken of the additional lapse risk in the initial period of a policy.

Valuation of Unit-Linked Business
Cash Flows: Discount Rates

	Net Investor		Gross Investor	
	Life Policies % p.a.	Pension Policies % p.a.	Life Policies % p.a.	Pension Policies % p.a.
Premiums not allocated to unit funds	$8\frac{1}{4}$	$8\frac{3}{4}$	$9\frac{1}{4}$	$9\frac{3}{4}$
Expense charges less expenses	$7\frac{1}{4}$	$7\frac{1}{2}$	$8\frac{1}{4}$	$8\frac{1}{2}$
Overhead costs	7	7	8	8
Mortality charges less costs	$8\frac{1}{4}$	$8\frac{3}{4}$	$9\frac{1}{4}$	$9\frac{3}{4}$
Lapse profits	11	$12\frac{1}{4}$	12	$13\frac{1}{4}$
Unit fee income:				
Prior to allocation	$8\frac{1}{4}$	$8\frac{3}{4}$	$9\frac{1}{4}$	$9\frac{3}{4}$
After allocation	$12\frac{1}{4}$	14	$13\frac{1}{4}$	15

As discussed in Section 3.4, the gross investor cash flows to be discounted should include an allowance for tax credits on distributed profits.

5.7.2 The equivalent overall discount rate required to value net cash flows will vary according to product design and company experience. If surrender charges are not dependent on the size of the unit fund and if all future unit fees are funded (that is, anticipated at the point of sale), an aggregate discount rate close to a government bond yield could be justified to value the in-force business of a large unit-linked office. If overhead costs are large and cannot be fully recouped from expense charges, and if future surrender profits and unit fees are substantial, an overall discount rate close to the expected return on equities could be justified.

5.7.3 As an example, consider an office which sells an equal volume of life and

pensions policies. The products are designed to recoup initial costs by having a reduced allocation rate in the first two policy years. The bid/offer spread meets the cost of paying renewal commission and a part of maintenance expenses. The annual management charge provides the main source of ongoing profit. The in-force value to a net investor for a large office with total annualised premium income of 1000 could be broken down as follows:

Value attributed to	Discount Rate % p.a.	Value
Future premiums not allocated to unit funds	$8\frac{1}{2}$	300
Expenses	$7\frac{1}{4}$	(200)
Mortality charges less costs	$8\frac{1}{2}$	0
Lapse profits	$11\frac{1}{2}$	0
Unit fee income	11*	200
Net cash flows	11	300

* In between the $8\frac{1}{2}$% p.a. rate in respect of the period prior to allocation to the unit fund and the 13% p.a. rate for the period after allocation.

5.7.4 In the example, the equivalent overall discount rate required to value net cash flows has been calculated as a weighted average rate. In practice, an internal rate of return should be used and this will differ from the weighted average rate.

6. WITH-PROFITS BUSINESS

6.1 *Introduction*

6.1.1 For a with-profits office, the value of the long-term fund assets less the net guaranteed obligations of the office can be compared to shareholders' equity in a non-profit office. The equity is shared between policyholders and shareholders, and is available for ultimate distribution in the form of bonus additions and transfers to the profit and loss account. The overall discount rate required to value equity is considered in Section 6.2. For a U.K. with-profits office, shareholders' transfers are normally linked to the cost of bonus declarations. Section 6.3 observes that the discount rate required to value transfers is the same as that required to value bonuses during the period prior to bonus declaration.

6.1.2 The following further observations can also be made:

(i) The amount of estate required to support bonus policy can be assessed.
(ii) The level of mathematical reserves has a second order effect on shareholder value.
(iii) The distribution of assets needed to approximately correspond to liability and bonus payments can be derived.
(iv) The value of assets not required to support the in-force business, free capital, varies according to the uses to which this capital is put.

These observations are discussed in Sections 6.4 to 6.9.

6.1.3 Asset shares can be developed by accumulating the component cash flows separately, using returns from appropriately matched assets. The assessment of asset shares using risk rates of return rather than the earned rate is not considered further in this paper.

6.2 *Equity in a With-Profits Office*

6.2.1 Defining equity E as the value of the long-term assets less the guaranteed component of net liabilities L, the discount rate e required to value bonus payments during the period prior to bonus declaration, and to value shareholder transfers, can be found from the equation derived in Section 2.5:

$$e = r + (r - g)\, L/E,$$

where: r is the earned rate on the long-term fund assets, and
g is the aggregate rate of discount required to value the guaranteed component of net liabilities (non-bonus payment-related cash flows net of premium receipts).

6.2.2 For some offices equity is substantial and is used to support new business, or to smooth bonuses by holding back surplus in order to mitigate the effect of asset value fluctuations. For these offices a closer analysis of the discount rate needed to value bonuses is required.

6.3.1 *Assessment of Discount Rates*

6.3.1.1 Use of close to a long-term government bond yield is suggested for valuing future premiums, sums assured and bonuses after declaration. This is because the main risk affecting these cash flows is lapse risk, and lapse risk after the initial period is estimated to be small (see Section 5.5).

6.3.1.2 The correlation between real expense levels and other economic factors may be small, so that their systematic risk is low. If it is believed that the term premium is related to inflation risk, expenses could be valued using expected future short-term bond yields, that is long-term government bond yields net of the term premium.

6.3.1.3 Cash value payments and bonuses prior to declaration should also be discounted using rates which reflect the variability of these payments. It is useful to construct a simplified model and to introduce complexities gradually, as in the following paragraphs.

6.3.2 *Mutual Office*

6.3.2.1 Consider a mutual office in which the guaranteed element of liability (for sums assured and declared bonuses, less future premiums) and also expenses have been fully reassured (or are matched against appropriate investments and excluded from further consideration). No new business is being written and reversionary and terminal bonus payments will be set to exhaust the fund. Bonuses once declared are reassured.

6.3.2.2 The variability ('risk') of bonus declarations is similar to the variability

of the fund's assets: any reduction in asset values will result in an equal expected reduction in aggregate bonus payments and any increase in asset values will be fully reflected in increased bonuses.

6.3.2.3 The office can choose, for any individual policy or policy type and for any individual year of bonus declaration, to depart from a matching of bonuses with asset returns. However, it cannot do so once all years and all policies are taken into account.

6.3.3 *Proprietary Fund*

6.3.3.1 Consider next a proprietary with-profits fund which in other respects is similar to the fund described above. Transfers to profit and loss are a proportion of the cost of bonuses and the two together will be set to exhaust the fund.

6.3.3.2 As for the mutual fund, bonus payments would need to be reduced in the event of any reduction in value of the asset portfolio. To the extent that asset appreciation is also reflected in the bonus payments, the variability of bonus declarations and shareholder transfers is equal to that of the asset return. In practice offices smooth bonus declarations, and this is considered next.

6.3.4 *Symmetric Bonus Smoothing Policy*

6.3.4.1 Bonus smoothing policy is described here as symmetric if the change in bonus levels due to a fall in asset prices relative to expectations is equal and opposite to the change arising from an increase in asset prices. An office with a large estate can choose the degree to which it smooths bonus declarations. The choice may depend on marketing or other considerations, not just financial ones. At one extreme bonus declarations can be fixed in advance. Shareholders' transfers would also be fixed and could be valued using a government bond yield. Use of a discount rate between the bond rate and the asset return rate could be justified if an office maintains a bonus smoothing policy, under which a change in asset prices results in a less than proportionate change in bonuses.

6.3.4.2 At the opposite extreme, as in Sections 6.3.2 and 6.3.3, an office may have no surplus of assets beyond the amount required to meet future expected bonus payments. If, as before, the guaranteed liabilities are matched by government bonds, shareholders' transfers and bonuses prior to declaration can be valued using a discount rate equal to the expected return on the assets backing these payments. The effect of taxation is considered in Appendix C.

6.3.4.3 More generally, the degree of smoothing of bonuses will depend on the variability of the returns on the underlying assets and on whether any surplus is available to mitigate asset return variability. The discount rate required to value shareholders' transfers can be determined by assessing the proportion of gilt-edged investment in a notional portfolio, such that the aggregate variability in the return on the notional asset portfolio is equal to the variability of bonus declarations.

6.3.4.4 Offices are likely to smooth reversionary bonus rates to a greater extent than terminal bonuses. Consequently, a discount rate closer to the gilt rate than

to the equity return rate may be appropriate to value future reversionary bonuses.

6.3.4.5 A substantially higher link is likely between terminal bonus rates and equity returns, requiring use of a discount rate closer to the equity return rate to value terminal bonus payments. Assessment of the risk and discount rate for terminal bonus payments needs to be consistent with the overall variability of policy maturity proceeds. The latter is likely to depend on the variability of policy asset shares. Because terminal bonuses can be viewed as balancing items required to track asset shares to a greater or lesser extent, the discount rate required to value them may be high unless a heavy smoothing policy is adopted.

6.3.5 *Asymmetric Bonus Smoothing Policy*

6.3.5.1 An alternative bonus smoothing policy is for bonus declarations to follow asset price increases, but to be maintained if asset prices decline. For example, terminal bonuses could be set such that the proceeds on maturity of a policy are a proportion, say 90%, of the asset share, subject to a minimum maturity payment equal to the sum assured and accumulated reversionary bonuses. Reversionary bonus rates could be fixed. For an office which adopts this bonus smoothing policy the value of shareholders' transfers can be assessed using option pricing theory. The terminal bonuses are given by:

$$\text{Max } (0.9A - G, 0), \text{ or}$$
$$0{\cdot}9 \times \text{Max } (A - G/0.9, 0),$$

where: A is the (variable) asset share on maturity of the policy, and
G is the sum assured and accumulated reversionary bonuses.

6.3.5.2 The expression is that for a call option on an asset which provides the asset return in excess of an exercise price, if asset returns exceed the exercise price. The cost of the bonus declaration can then be assessed as the value of a call option on the asset portfolio, plus the reversionary bonus payments discounted using a government bond yield. Option pricing theory enables an approximate value of the call option to be assessed.

6.3.5.3 In practice, an office's bonus policy is likely to be complex, and complex option pricing formulae, as well as simulation techniques, are likely to be needed.

6.4 *Required Surplus*

6.4.1 Required surplus is defined here as the value of assets required to meet future payments to existing policyholders and shareholders, and expenses, less the amount of mathematical reserves. Future payments to policyholders include expected future bonuses and may be assessed, for example, by projecting policy asset shares. In other words, required surplus is the amount of estate needed to support expected future bonus or other payments. The estate is defined as the excess of the value of assets over the mathematical reserves.

6.4.2 The amount of required surplus can be determined by discounting future

cash flows at the relevant risk rates discussed in Section 6.3. There is an element of circularity in the calculation of required surplus, because the risk rates themselves may depend on the amount of available surplus (as discussed in Section 6.3.4). The amount of required surplus depends on the extent to which an office adopts a policy of smoothing bonus rates. Higher surplus is required, and a lower discount rate is used to value future bonuses, if policyholders and shareholders are to be insulated from the effects of a decline in asset prices.

6.4.3 This methodology generally results in a higher estimate of required surplus than traditional methods, because a lower rate of discount (closer to the gilt rate than to the earned rate) is used to value future expenses, sums assured and reversionary bonus payments. This has a greater impact than the use of the lower rate to value future premium receipts. This is because the value of future expenses, sums assured and reversionary bonus payments generally exceeds the value of future premiums.

6.5 *Statutory Valuation Basis*

6.5.1 Assets backing both required surplus and the mathematical reserves are required to meet future payments to existing policyholders and shareholders. For this reason the valuation methodology disregards the strength of the reserving basis, except insofar as this affects the total of required surplus and reserves. The valuation basis is also used to determine the amount transferred to the profit and loss account. The appraisal valuation needs to reflect expected future levels of the valuation basis, so that the amounts of transfer can be projected. It may also need to reflect systematic risk, if future changes in the valuation basis are assessed to be correlated with common economic factors.

6.6 *Asset Matching*

6.6.1 An office may have a bonus smoothing policy which results in terminal bonuses and reversionary bonuses prior to declaration exhibiting variability characteristics equal to some factor times that of equities. Other cash flows may have a relatively low variability. The proportion of the asset portfolio which needs to be invested in gilts in order to match approximately the variability characteristics of an office's net liabilities can be found.

6.6.2 If the estate falls short of required surplus, values should be based on bonus smoothing policy, and bonuses limited to reflect the amount of the estate and the office's long-run target asset mix. The discount rate used to value bonus cash flows would be increased to reflect the restriction on bonus smoothing.

6.7 *Free Capital*

6.7.1 Free capital is defined here as the value of assets less the amount of mathematical reserves and required surplus.

6.7.2 Free capital can be used for some or all of the following:

(i) reducing investment mismatching risk,
(ii) subsidising future new business through bonus rate or bonus smoothing policy, and

(iii) maintaining a 'free asset' ratio higher than needed for (i) and (ii) in order to promote new business sales.

6.7.3 It should be noted here that these uses are not necessarily residual to bonus policy. For example, an office may prefer to limit bonus smoothing than to reduce new business subsidies. For this reason it is necessary to quantify the amount of capital required for each potential use. The total needs to be compared with the amount of available estate in order to avoid double counting.

6.7.4 The first of the uses of free capital is considered next.

6.8 *Free Capital Required for Asset/Liability Mismatching*

6.8.1 Mismatching gives rise to risk of 'failure', that is the possibility that asset values will fall below the level required in order that new business can be sold. Free capital held to reduce the risk of failure has value to shareholders, because it reduces the risk that existing structure value will be lost. Risk of failure is discussed further in Section 8.

6.8.2 Offices may use equity and property investments to back guaranteed liabilities. The variability of bonus payments may differ from the variability of returns on the backing assets. These policies result in:

(i) higher expected bonus declarations arising from higher expected returns on the investments,
(ii) a corresponding increase in the riskiness of bonus declarations because of the riskiness of equity/property investment returns,
(iii) tax benefits in respect of life policies, since indexation relief favours equity compared with bond investment, and
(iv) a risk of failure and a need to hold free capital to reduce this risk.

The higher expected maturity proceeds may also result in increased new business sales.

6.8.3 The first two factors result in an increase both in expected shareholders' transfers and in their variability. These offset each other, and consequently guaranteed liabilities can be valued using fixed-interest gilt yields, irrespective of the office's actual investment policy. For the same reasons, bonuses can be valued using discount rates depending on bonus smoothing policy rather than on investment policy.

6.8.4 The effect of indexation relief can be allowed for by valuing the guaranteed liabilities at a rate higher than the fixed-interest yield. An addition of $1\frac{1}{4}\%$ p.a. (basic rate tax multiplied by the expected future rate of inflation) can be made to value life business guaranteed liabilities which are backed by equities, property or index-linked gilts.

6.8.5 A mismatching strategy results in a gain to shareholders if higher expected maturity proceeds facilitate new business sales. On the other hand, from the shareholders' viewpoint, risk of failure is undesirable, because shareholders could lose their interest in future new business and, perhaps, a part of their interest in the in-force business. These two factors result in a need to assign

shareholder value to free capital held to offset risk of failure. The cost of failure, and the value of reducing it, can be assessed using option pricing techniques. The value generated by enhanced new business sales can, in principle, be calculated directly.

6.9 *Free Capital Required to Support Sales Distribution Capability*

6.9.1 Besides free capital needed for mismatching purposes, a part of future cash flows may be set aside to enhance the office's sales distribution capability.

6.9.2 To the extent that these cash flows are not distributed, they do not directly generate value for shareholders or policyholders. A separate value can, however, be assigned to these cash flows for embedded value purposes, for example by considering the opportunity cost of alternative uses.

6.9.3 The discount rate required to value future new business sales prior to the point of sale can be reduced for an office with capital which is used to support the sales distribution capability and to which no separate value is assigned. This is because uncertainty as to whether future expected sales will, in fact, be achieved is reduced if the office is able, if necessary, to increase new business subsidies. This is likely to be an inefficient use of capital from the point of view of shareholders, since they only benefit if circumstances prove adverse. If the discount rate is not reduced, then it would be appropriate to assign a (discounted) value to this capital when calculating an appraisal value.

6.10 *Summary*

6.10.1 To conclude this Section, the overall discount rate used to value terminal bonuses and reversionary bonuses prior to declaration should be consistent with the amount of available estate. A high degree of bonus smoothing policy will require a higher proportion of gilt-edged investment, a lower discount rate to value bonuses and hence a greater utilisation of the estate. For example, consider an office with a mix of 50% life and 50% pensions business and with the following characteristics:

Value attributed to	Discount Rate % p.a.	Value
Equity assets	13	8,000
Gilt assets	8	2,000
Future premiums	$8\frac{1}{2}$	3,000
Future expenses	$7\frac{1}{2}$	(200)
Sum assured related liability payments	$8\frac{1}{2}$	(3,000)
Bonus declared to date	$8\frac{1}{2}$	(1,800)
Equity	12·9	8,000
Future bonuses and shareholder transfers	12·9	8,000

6.10.2 The discount rates shown in the table are the average of the net investor required returns for investments in the life and pension funds shown in Appendix C.4. A $\frac{1}{4}$% p.a. addition has been made to the discount rates shown for the premium, expense and liability cash flows, to allow for lapse risk. The results

were rounded to the nearer $\frac{1}{2}$% p.a. The discount rates used to value the expense cash flows were based on short-term rather than long-term bond returns.

6.10.3 The discount rate required to value equity is the internal rate of return at which net cash flows are valued at 8,000. In the table, for the sake of simplicity, the weighted average discount rate has been calculated, although it is recognised that this may not equal the internal rate of return.

6.10.4 The office is approximately matched in that the value of the guaranteed obligations and expenses is equal to the value of the gilt portfolio and future premiums. With a 100% equity portfolio, the weighted average discount rate required to value bonus declarations and shareholders' transfers would increase to 14·1% p.a. Expected future bonuses and their volatility would increase. If the office had assets worth 5,000 invested entirely in equities (not, as in the table above, assets of 10,000 invested partly in gilts), the weighted average discount rate would become 16% p.a.; bonus levels would need to be reduced and the office would not be able to maintain its bonus smoothing policy.

6.10.5 If the assets are worth 20,000, leading to equity of 18,000, the office can smooth bonuses and use a part of the estate to support new business. The weighted average discount required to value the whole equity, assuming a 100% equity investment policy, is $13\frac{1}{2}$% p.a. The increase in available equity enables the office to adopt a greater degree of bonus smoothing. If bonuses are smoothed so that the level of risk is in between that of gilts and equities, a discount rate of $10\frac{1}{2}$% p.a. would be appropriate (that is, half-way between the discount rate required for gilts, 8% p.a., and for equities, 13% p.a.). The value of future bonuses and transfers is increased to, say, 10,000. The remaining equity, 8,000, could be valued using a discount rate of $(18{,}000 \times 13\frac{1}{2}\% - 10{,}000 \times 10\frac{1}{2}\%)/8{,}000 = 17\frac{1}{4}\%$ p.a. This rate of discount would need to be factored in when assessing value attributed to alternative uses of the estate.

6.10.6 As noted in Section 6.8, an option pricing valuation method may be needed to value future bonuses, if the amount of estate is relatively small and if the assets and liabilities are not matched. Terminal bonuses are likely to have different risk characteristics from reversionary bonuses, and can be valued using a higher discount rate or using option pricing techniques, as suggested in Section 6.3.

6.10.7 Although Sections 6.7, 6.8 and 6.9 refer primarily to with-profits offices, they also have relevance to non-profit and unit-linked offices which use free capital to support new business activities or to offset risk arising from mismatching assets and liabilities.

7. EXISTING STRUCTURE VALUE

7.1 *Introduction*

7.1.1 Existing structure value (also known as 'goodwill') is value attributed to future new business. Future sales are uncertain and a different rate of discount is required to value profits in respect of the period prior to the point of sale than from the date of sale.

7.1.2 New business sales and lapse risks result in a need to apply a higher discount rate to value future gross profit margins up to the point of sale than to value overhead costs and future initial expenses. Lapse risk arises because initial expenses may not be recouped if a policy lapses. The effect of both sales and lapse risk is examined in Sections 7.2 and 7.3. For a with-profits office, the existence of an estate can substantially reduce lapse and sales risk borne by shareholders. Sections 7.4 and 7.5 review the special considerations required for with-profits offices distributing business through a direct salesforce and through brokers, respectively.

7.2 *New Business Risk*

7.2.1 Future earned profits (Section 1.4) are subject to a variety of 'risks'. A number of such risks impinge on expected levels of earned profit, and can be allowed for in assessing the level of future growth. Other risks can be divided between unsystematic risks, those unrelated to common market factors, and systematic risks. Both types of risk increase costs. This is because policyholders, agents, staff and suppliers may require better terms from an office which exhibits volatile results and which could cease to trade. For the reasons stated in Section 2, unsystematic risks may not give rise to an increase in required discount rates.

7.2.2 Systematic risk arises because sales may fall and lapse rates may increase in an economic downturn. These two effects are compounded by the existence of overhead costs and initial expenses, as illustrated below:

	Value Prior to Market Fall	Value After Market Fall	
		Reduced Sales	Increased First-Year Lapse Rates
Earned profit gross of initial expenses	100	90	87
Marginal initial expenses	(60)	(54)	(54)
Acquisition overhead costs	(10)	(10)	(10)
Earned profit	30	26	23

7.2.3 Section 6.8 considered some of the effects of holding free capital to support the sales distribution capability of an office.

7.3 *Non-Profit Business*

7.3.1 For a non-profit or unit-linked office, the gearing introduced by overhead costs and first-year lapse risk can be allowed for by developing separate existing structure multiples for earned profit gross of initial expenses, marginal initial expenses, and acquisition overhead expenses. Estimates of the required multiples can be derived using CAPM, based on an examination of historic new business sales and first-year lapse rates relative to stock market returns or other economic factors. For example, a decline of 10% in new business sales (relative to prior expectations) and an increase of 3% in first-year lapse rates (see Section 5.5)

might be associated with an equity market return of 15% below expectations. With these assumptions, the additional discount rates, based on an equity risk premium of $4\frac{1}{2}$% p.a. for a net investor and 6% p.a. for a gross investor are:

	Additional Discount Rates	
	Net Investor % p.a.	Gross Investor % p.a.
New business sales risk	$10 \times 4\frac{1}{2}/15 = 3{\cdot}0$	$10 \times 6/15 = 4{\cdot}0$
Lapse risk	$3 \times 4\frac{1}{2}/15 = 0{\cdot}9$	$3 \times 6/15 = 1{\cdot}2$

7.3.2 In practice, overhead costs are also subject to systematic risk, since in a severe economic downturn an office may cease to transact new business. For the purpose of illustration, it is assumed that the level of this risk is one-third of the level of new business sales risk, resulting in a 1% p.a. and $1\frac{1}{4}$% p.a. additional discount rate for the net and gross investor assumptions, respectively.

7.3.3 The overall discount rates, assuming a risk free rate of 7% p.a. for a net investor and 8% p.a. for a gross investor are:

	Risk Discount Rates	
	Net Investor % p.a.	Gross Investor % p.a.
Earned profit gross of inital expenses	$7+3+0{\cdot}9=11$, say	$8+4+1{\cdot}2=13\frac{1}{4}$, say
Marginal initial expenses	$7+3=10$	$8+4=12$
Acquisition overhead costs	$7+1=8$	$8+1\frac{1}{4}=9\frac{1}{4}$

7.3.4 The overall multiples required for an office where sales margins and expenses are increasing with inflation at 5% p.a. are illustrated in the following table:

		Derivation of Existing Structure Multiples	
	Value £m	Net Investor	Gross Investor
Earned profit gross of initial expenses	100	$1/(11-5)=16{\cdot}7\times$	$1/(13.25-5)=12{\cdot}1\times$
Marginal initial expenses	(60)	$1/(10-5)=20{\cdot}0\times$	$1/(12-5)=14{\cdot}3\times$
Acquisition overhead costs	(10)	$1/(8-5)=33.3\times$	$1/(9.25-5)=23.5\times$
Earned profit and existing structure value	30	£137m	£117m
Overall multiple	—	$5\times$	$4\times$

7.3.5 The effect of adjusting for systematic risk in first-year lapse rates and in new business sales is to reduce dramatically estimated existing structure values (in the example, to £137m for the net investor assumption and £117m for the gross investor assumption).

7.3.6 The low values shown in the table result from high initial expenses relative to the value of gross earned profit. An increase in gross earned profit to £110m with no increase in expenses, for example, would increase existing structure value to between £238m and £304m. The lower the responsiveness of

new business sales to a market decline, the higher the calculated level of existing structure multiples.

7.3.7 For a large efficient company, acquisition overhead costs would be spread over a larger number of policies, and higher multiples apply. For a less efficient company, overhead costs may form a larger proportion of total costs, but possibly these costs should be discounted at a rate which reflects higher systematic risk (the office might be more likely to be closed to new business in a severe recession), rather than the riskless rate shown in the table. The 'small company' effect (Appendix A.2) could be allowed for by increasing the gap between the discount rates used to assess the multiples for gross earned profit, initial expenses and overhead costs.

7.3.8 Existing structure multiples are also increased if a portion of initial expenses are clawed back on policy lapse (such expenses should be deducted from profit and not included in initial expenses in deriving the multiples), or through the use of persistency bonuses, volume overrides and share option schemes. On the other hand, it should be noted that there may be a significant delay between a decline in sales and management action to reduce costs, resulting in a higher effective level of overhead expenses than a strict analysis might suggest.

7.4 *With-Profits Office—Direct Salesforce*

7.4.1 For with-profits offices, somewhat different considerations apply in the assessment of existing structure multiples. Assume that new business sales depend largely on salesforce size and productivity rather than on bonus subsidy.

7.4.2 The gearing from overhead costs and first-year lapse risk, noted in Section 7.2, needs to be allowed for primarily in assessing the cost to the estate of writing new business, rather than in assessing the value of profit and loss transfers. Because of this, the existing structure multiples are relatively high. Against this, the assessed cost of writing new business is also higher.

7.4.3 For assessing the value of shareholders' transfers a multiple of 12 times (gross investor) and 17 times (net investor) could be appropriate, based on the following analysis and on the new business sales and lapse risk levels assumed in Section 7.3.

	Net Investor % p.a.	Gross Investor % p.a.
Risk free rate	7.00	8.00
New business sales risk	3.00	4.00
First-year lapse risk	0.90	1.20
New business growth	(5.00)	(5.00)
Net discount rate	5.90	8.20
Multiple	17×	12×

7.4.4 In practice, the lower existing structure multiple appropriate for the gross investor is often found to be offset by higher in-force and new business values. These higher values arise because the effect of grossing-up cash flows to

allow for the tax credit on distributions to shareholders may outweigh the effect of using a higher discount rate.

7.4.5 For some offices, new business sales may require support from the estate. In practice, there is a possibility that sales would reduce if the new business subsidy is eliminated, particularly if the subsidy is cut before similar action is taken by other offices. A lower multiple is appropriate for that part of total sales which is dependent on the estate in this way, reflecting investment risk of the underlying assets and the expected period of support.

7.5 *With-Profits Office: Broker Distribution*

7.5.1 New business sales subsidy is likely to be a significant portion of the estate, since offices are under pressure to maintain bonus levels.

7.5.2 Whilst the subsidy continues, a high level of sales may be achieved. Sales arising as a result of the subsidy are subject to sales and first-year lapse risk. They are also subject to asset risk, since a reduction in the amount of estate available to subsidise new business will reduce the period during which the subsidy continues. The discount rate applied to values arising from the subsidy should reflect the impact of all three risks.

7.5.3 At some point the subsidy may cease. If this occurs before similar action is taken by the office's competitors, new business volume is likely to substantially reduce (or the office may close to new business), and bonus rates will reduce. These features should be incorporated in any projection of expected future new business. Alternatively, the derivation of existing structure value can be based on the projected ultimate level of sales, and a reduced existing structure multiple applied in respect of the temporary additional volumes and higher bonuses.

7.5.4 For the last offices to eliminate the subsidy, new business sales may continue at an undiminished level, because there will presumably always be some demand for with-profits products. For these offices, the existing structure value should reflect the impact of the eventual decline in bonus rates.

7.5.5 In practice, an office needs a relatively strong estate in order to pursue a policy of subsidising new business. The interaction of bonus policy, new business subsidy, and estate size is a complex subject which is not considered further in this paper.

7.6 *Summary*

7.6.1 This section shows how a consideration of systematic risk leads to the assessment of separate multiples for gross earned profit, acquisition costs and overhead expenses, rather than a single multiple as is currently common in life office appraisal valuations. The special considerations needed for with-profits offices are investigated.

7.6.2 It should be noted that in any appraisal, the value placed on existing structure is likely to involve considerable exercise of judgement. This is because existing structure multiples are sensitive to small changes in assumptions, and

because of the need to interpret transactions and other data. It should also be noted that further research is required into the magnitude of systematic sales and lapse risks.

8. WHOLE COMPANY VALUATION

8.1 *Introduction*

8.1.1 An office with an insufficient amount of capital may be unable to make the best use of its other resources. On the other hand, too much capital may lead to inefficiencies.

8.1.2 Section 8.2 introduces the concept of target surplus, whilst Section 8.3 shows how option pricing theory can be used to assess the optimal level of surplus or estate required to ensure that an office will be able to continue writing new business.

8.1.3 The market valuations of quoted life offices, supported by analyses of transactions in insurance companies, can be used to validate the component approach to life office valuation. Such analysis is useful, because transaction prices provide objective evidence as to the value placed on insurance companies by investors. Section 8.4 briefly considers the uses and limitations of beta analysis in the process of validation.

8.2 *Target Surplus*

8.2.1 The level of assets in relation to the liabilities of a life office has a number of implications for company valuation.

8.2.2 Surplus can be defined as the excess of the value of assets over reserves calculated on a statutory minimum basis. Target surplus can be defined as management's long-term desired level of surplus, expressed, for example, in relation to statutory minimum reserves or solvency margin requirements. The value of liabilities (for a with-profits office, net guaranteed liabilities) can be defined as the value of policyholder and other liabilities discounted at appropriate risk-adjusted rates of return as derived in previous Sections.

8.2.3 Both target surplus and reserves in excess of the value of liabilities are similar to the extent that neither will be distributed. Nevertheless, it may be appropriate to operate a target surplus policy based on statutory reserves rather than value of liabilities, because the former may be a better guide to minimum surplus requirements set by regulators, and capital needs resulting from agency costs. Agency costs arise because a more risky company, with a lower level of surplus, may find it more difficult and costly to attract staff, policyholders and intermediaries and to arrange reinsurance or financing. These costs, as well as inefficiencies which may arise if capital exceeds an optimal level (perhaps assessed as suggested in Section 8.3), can be allowed for directly, in the valuation of net worth, in-force business and existing structure.

8.2.4 The ratio of target surplus to reserves is in some ways analogous to an equity/debt ratio, since policyholder liabilities have a number of similarities to debt. Operating a target surplus policy is then similar to targeting a particular

level of debt to equity. Of course, capital market theory would suggest valuing debt at market value and disregarding regulatory valuation rules so that the surplus/reserve ratio needs to be interpreted with care.

8.3 *Risk of Failure*

Failure here is defined as the event that asset values fall below minimum regulatory standards, resulting in a company being unable to continue writing new business and in shareholders losing existing structure value. An increase in surplus reduces the risk of failure. Insolvency is a special case of failure, and arises when an office is unable to meet its guaranteed obligations in full. The risk of insolvency for U.K. life offices is generally small, and so can normally be ignored in the valuation process.

8.3.2 Firstly, consider only existing business. All liabilities are payable at time 1. Maximum shareholder value at time 1 is given by:

$$\text{Max } i\ (A_i - L_i, 0),$$

where: i is the set of alternative management strategies at time 0,
A_i is the (variable) asset return at time 1,
L_i is the amount payable to policyholders at time 1,
and it is understood that the mean of a stochastic process is being assessed.

8.3.3 Shareholder value in this example is a call option on the underlying asset A_i, with exercise price L_i. Shareholder value is maximised by choosing an asset with high return variability and by minimising the amount of asset utilised against the specified level of policyholders' liability (that is, minimising target surplus). For the shareholder gains by the full amount of asset appreciation, whereas policyholders meet the cost of any excess asset depreciation (in the event of bankruptcy).

8.3.4 Introducing new business, assume that the shareholder will lose the value of future new business, B, if asset values fall below a minimum level, R_i. Shareholder value is given by:

$$\text{Max } i\ (A_i - L_i, 0) + (\text{if } A_i > R_i)\ B.$$

8.3.5 Consider an example where the policyholders' liability L_i is 950 and is equal to R_i. The value of the asset is 1,000 and the potential loss of existing structure value, B, is 100. The asset consists of the payoff from a gamble which provides 1,100 if a 'head' is thrown, and 900 on a 'tail'. Time 1 is the time it takes to throw the coin. Shareholder value is $0{\cdot}5 \times 250 + 0{\cdot}5 \times (0) = 125$.

8.3.6 If the policyholders' liability is reduced to 900 and surplus is increased from 50 to 100, shareholder value is increased to $0{\cdot}5 \times 300 + 0{\cdot}5 \times 100 = 200$. Injecting 50 of surplus adds 75 of value. Further increases in surplus do not create additional shareholder value (beyond the amount injected).

8.3.7 Optimal target surplus is an increasing function of risk of failure and of agency costs. A higher level of surplus reduces the probability that assets will fall

below the level required to continue writing new business, and hence preserves existing structure value.

8.3.8 If the office matches its liabilities, or has sufficient surplus to ensure that its assets will always exceed the minimum required level, there is no risk of failure. Shareholder value can be assessed by taking the difference between the market values of the assets and the liabilities and adding in existing structure value.

8.3.9 If the risk of failure is increased, an option pricing approach is needed to value the shareholder's call option, as seen above.

8.4 *Insurance Company Beta Ratios*

8.4.1 Under the Capital Asset Pricing Model (Appendix A), the beta of an asset determines the discount rate required to value the asset. The quoted U.K. life offices have beta ratios averaging approximately one (see, for example, the beta ratios published quarterly by London Business School Financial Services). The ratios are influenced by gearing introduced by holding company debt, so that the beta for a typical ungeared U.K. quoted life office could be approximately 0.9. This ratio is the weighted average of beta ratios applicable to net worth, in-force business and existing structure value. The following table illustrates the relationship:

Component	Illustrative Proportion of Total Value	Beta Ratio	Net Investor Discount Rate % p.a.	Gross Investor Discount Rate % p.a.
Net worth	0.1	1.0	12	14½
In-force value	0.6	0.8	11	13
Existing structure value	0.3	1.1	12½	15
Total	1.0	0.9	11½	14

8.4.2 The existing structure discount rate is a composite of the rate required to value business in-force (that is, from the point of sale) and new business up to the point of sale. If these have an equal weight, the discount rate required to value new business up to the point of sale is 17% p.a. (gross investor) or 14% p.a. (net investor). This corresponds to a multiple of 8 times for the gross investor and 11 times for the net investor assumption, for an office with expected future long-term growth equal to 5% p.a. In practice, as discussed in previous Sections, beta ratios for the individual components of value could differ significantly from the ratios shown in the table.

8.4.3 Observed insurance company beta ratios provide one possible means to assess discount rates. These discount rates may be a useful check on the results of the analyses suggested in Sections 4 to 7. However, beta ratios suffer from considerable measurement error, and also only indirectly take account of the risks to which companies are exposed. A close analysis of the risk/return characteristics of the individual cash flows which make up the overall return to

shareholders facilitates understanding of the sensitivity of company values to changes in company structure or circumstances. The approach to valuing individual assets and liabilities by comparing their risk characteristics with those of traded securities, and the use of option pricing arbitrage relationships where options exist, should enable the derivation of more reliable discount rates than those derived from beta analysis.

8.5 *Summary*

8.5.1 As seen in previous Sections, different types of insurance business and different risk factors can cause substantial variations in the amount of risk borne by shareholders and in discount rates which would compensate for this risk. A particularly important risk factor is the level of surplus held by the office in relation to the amount of its liabilities. If surplus is sufficiently high the office is not likely to fail, and full allowance can be given to the existing structure in an assessment of the total appraisal value.

8.5.2 Analysis of the beta ratios of quoted companies can provide a guide to the overall levels of discount rate required to value in-force business and existing structure.

9. CONCLUSION

9.1 The approach developed in this paper provides a practical framework for assessing the value of a life office. Value is assessed in the context of the risk and return trade-off implicit in the market valuation of securities traded on the principal securities exchanges.

9.2 The method proposed is to examine each life office cash flow separately and, where possible, to draw a comparison between these cash flows and the cash flows expected from traded securities such as conventional and index-linked gilts and equities. For example, premium, expense and guaranteed liability payments are compared with gilt coupon and maturity payments; unit fee income and bonus payments are compared with the returns from the underlying life office assets. The cash flows can then be valued using discount rates based on the investment return assumptions selected for each traded security. In general, the value of future net cash flows is assumed to equal the sum of the values of the individual cash flows.

9.3 In this paper, discount rates are presented on both a net and a gross investor basis. There is relatively little firm empirical evidence as to the extent to which taxation of investment returns affects the level of market prices, although, in the author's view, the theoretical arguments favour use of a gross basis.

9.4 Use of general arbitrage arguments, and of asset pricing models such as CAPM and APT, is suggested to allow for mortality, lapse and other risks. Risks are divided into systematic and unsystematic components, the former being those related to common economic factors such as stock market returns, industrial production, personal consumption, inflation and the term premium. Modern portfolio theory suggests that cash flows subject to systematic risk may

need to be valued using discount rates in excess of a risk free rate, whereas no such excess is required to value cash flows subject only to unsystematic risk. For this reason an addition to the discount rate is required to allow for lapse risk, but no addition is made for mortality risk, since this is assumed to be broadly uncorrelated with the principal economic factors. This risk does, however, adversely affect the value of a company, as seen in Sections 7 and 8.

9.5 Valuation of risky cash flows which are subject to only unsystematic risk using a risk free rate seems counterintuitive. It may be felt that, not withstanding diversification and arbitrage arguments to the contrary, potential investors in an imperfect market will, in practice, require a risk return for bearing unsystematic risk. The same argument could, however, also be applied to suggest that an investor may require more than a market return from any asset. Just as potential investors will seek to achieve a transaction price below a 'fair' price, a seller will strive for a higher price. The eventual outcome will depend on the balance of supply and demand rather than necessarily on assessed value.

9.6 Section 7 examines risk in relation to the valuation of future new business. Gross profits from new business sales are subject to sales risk, with a high systematic risk component, whereas overhead costs have low systematic risk. Similarly, first-year lapses affect gross profits, but not acquisition expenses. Because of this, separate valuation of gross profit margins, marginal initial expenses and acquisition overhead costs is suggested for the determination of existing structure value.

9.7 For some offices, surplus levels may be insufficient to ensure that regulatory approval for continued new business sales will be forthcoming under all realistic scenarios. This may be true, even though the risk of insolvency is negligible. Existing structure value is retained in circumstances when the value of an office's assets exceeds the value of its liabilities by a sufficient margin, in much the same way as a call option on an asset provides a payoff in the event that the asset price exceeds the exercise price.

9.8 For other offices, there is little likelihood that existing structure value is at risk, and an additive rather than an option pricing approach can be used to assess company value. The total value of the company is then the sum of its in-force and existing structure values.

9.9 The author has had the opportunity to consider the relationship of transaction and market prices with appraisal values for many offices over a number of years. In his view, application of the techniques outlined in the paper, with suitable exercise of judgement, is likely to provide realistic estimates of value. The methodologies explored represent a refinement of existing techniques widely used in appraisal value calculations, and result in a closer degree of alignment with empirical and theoretical research work in the field of finance undertaken in the last 40 years.

9.10 Modern financial theory provides many insights into the relationship between price, risk and return. A few of these insights have been used in developing the valuation approaches explored in this paper. However, the theory

has much wider application, for example in product development and pricing, bonus philosophy, investment policy, profitability assessment, profit recognition and life office financial control. The author looks forward to the start of a more extensive debate within the profession on the uses and limitations of modern financial theory.

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APPENDIX A

ASSET PRICING MODELS

A.1 *Introduction*

A.1.1 The last few decades have seen a number of breakthroughs into the understanding of how risk affects asset prices. In 1952 Markowitz noted that investors are concerned primarily with the return on their whole portfolio of assets, and that the risk of any individual asset is of concern only to the extent that it contributes to the risk of that portfolio. Since Markowitz' original pioneering work several asset pricing theories have emerged. The Capital Asset Pricing Model, Arbitrage Pricing Theory, and Option Pricing Theory stand out, and are the most widely used. These theories enable the market price of an asset to be estimated once the cash flow distribution and risk characteristics of the asset have been assessed. A brief description of each theory follows.

A.2 *Capital Asset Pricing Model*

A.2.1 Sharpe, Lintner and Mossin developed the Capital Asset Pricing Model (CAPM) in 1964 and 1965, based on the observation that many institutional portfolios are well diversified and that the risk of an individual asset can be assessed by measuring its contribution to the standard deviation of returns on the market portfolio ('systematic' or 'non-diversifiable' risk). CAPM states that the required return on an asset depends solely on its systematic risk and equals:

$$r(f)+b\,(\check{r}(m)-r(f))$$

where: $r(f)$ is the return on a risk free asset,
$\check{r}(m)$ is the expected return on some broadly based index of asset prices (the 'market' index), and
b is the asset's systematic risk factor, beta.

Beta is equal to:

- the correlation coefficient between returns on the asset and the market index, times
- the standard deviation of returns on the asset, divided by
- the standard deviation of returns on the market index.

A.2.2 The beta of the market index is, by definition, 1.0, and the beta of an asset measures the responsiveness of the asset price to changes in the market index. An asset which is expected to rise or fall in price by 5% when the market index rises or falls by 10% has a beta of 0.5.

A.2.3 CAPM draws considerable strength from two empirical observations:

- Individual share prices tend to rise or fall in tandem (that is, systematic risk is large).
- The returns from a relatively small reasonably diversified portfolio of shares

are likely to be close to that of the market index (95% of unsystematic risk is likely to have been diversified away in a portfolio of 40 shares chosen at random).

A.2.4 The empirical evidence for a strict and stable linear relationship between expected return and systematic risk relative to a single market index is mixed. For example, many studies, particularly in the U.S.A., have found that returns on small companies exceed the CAPM equilibrium levels. There is rather more evidence for a modified form of CAPM, the two-factor or zero beta model, under which the risk-free rate is replaced by the return on a minimum variance portfolio uncorrelated with the market index. The zero beta version can be derived using more realistic assumptions than those required for the single index model. Another criticism levied at CAPM is that not just variance, but also skewness of returns, contribute towards investor risk/return preferences, and that some degree of skewness is exhibited in typical asset return distributions. These criticisms have led to a number of alternative versions of CAPM including multi-factor models, to allow for industry specific risk and the effects of tax, and models which seek to correct for possible misspecification of the market index (the consumption based CAPM attributable to Breedon).

A.2.5 In the author's view, CAPM provides a useful benchmark from which to assess expected returns on an asset.

A.3 *Arbitrage Pricing Theory*

A.3.1 Ross presented the Arbitrage Pricing Theory (APT) in 1976. Under APT, expected returns on an asset depend linearly on the responsiveness of the asset's return to one or more of an unspecified number of common factors. Examples of factors which have been suggested include the market index, industrial production, inflation and the differentials between the yields on short- and long-term government bonds (the term premium) and on low grade and high grade corporate bonds.

A.3.2 APT is more general than CAPM, in that asset prices may depend on a variety of factors. Under the conditions of riskless arbitrage used to derive the APT model, shares which are priced at a discount to their expected price will be bought to replace shares or combinations of shares with similar risk characteristics not priced at a discount.

A.3.3 The evidence in favour of APT is stronger than for CAPM. However, there is only a limited volume of data on the number, size and type of the APT factors and risk premia. For this reason, examples in this paper are based on CAPM rather than APT.

A.4 *Option Pricing Theory*

A.4.1 In 1973 Black & Scholes published their paper, on 'The Pricing of Options and Corporate Liabilities'.

A.4.2 A call option on an asset is a security consisting of the right to acquire the asset for a pre-specified price (the exercise price) at (sometimes on or before)

the option expiry date. A put option provides the right to sell the asset at the exercise price.

A.4.3 Black & Scholes showed that, if the asset return is normally or lognormally distributed, it is possible to replicate the return distribution of a call option by buying shares in the asset and borrowing a proportion of the exercise price. The value of the option is given by the value of the shares bought less the amount of borrowing. The proportions of shares and borrowing need to be continuously rebalanced during the life of the option in order to maintain instantaneous replication of the option return distribution. Black & Scholes noted that opportunities for riskless arbitrage limit departures of the price of a traded option from its theoretical price.

A.4.4 The expected value of an option can be assessed using a stochastic model of the asset return distribution. However, the Black–Scholes (B–S) valuation approach is required in order to allow for the option's systematic risk.

A.4.5 Option pricing techniques have been applied in the valuation of many different securities including shares, fixed-interest securities and convertible bonds. For example, ownership of the equity in a firm can be considered to be a call option on the firm, with exercise price equal to the amount of debt financing. It can be used for everyday capital budgeting decisions where a positive value is achieved only if costs are recouped, and in the evaluation of natural resource investments.

A.4.6 The B–S theory is more robust than CAPM and APT, in that the B–S estimated value of a contingent claim on an asset depends on the price of the asset and on the construction of a portfolio which matches the return distribution of the option. Estimated values are dependent on the expected variability in the asset return, not on the expected return. Finally, it should be noted that estimated option prices are known to be reasonably accurate (relative to observed transaction prices) for short-term contracts. The range of comparisons conducted for long-term contracts is more limited, although the results to date appear encouraging (see, for example, the valuation of equity notes in the article by Courtadon & Merrick in *The Revolution in Corporate Finance* edited by Chew & Stern).

APPENDIX B

INVESTMENT RETURN ASSUMPTIONS

B.1 *Introduction*

B.1.1 The methodology explored in this paper requires that discount rates selected to value future cash flows be consistent with investment returns available on equally risky investments. The level of available investment return therefore plays an important part in the valuation process.

B.1.2 This Appendix sets out some of the analyses of historic and other data that can be made in order to obtain an informed assessment of future asset returns.

B.2 *Real Short-Term Interest Rates*

B.2.1 Pre-tax real returns on three-month Treasury bills in the U.K., measured by taking the geometric average of Treasury bill returns divided by the consumer price inflation index, have been as follows:

- in the 116 years between 1824 and 1939: over 3% p.a.,
- in the 41 years between 1940 and 1980: minus 0.9% p.a., and
- in the 10 years between 1981 and 1990: 4.7% p.a.

B.2.2 Data on interest rates in the U.K. since 1970 can be found in the CSO publication *Financial Statistics*. The publication *The British Economy Key Statistics 1900–1970* (London and Cambridge Economic Service) provides data since 1900. Pepper, in a lecture to the Faculty of Actuaries on 20 February 1984, 'The Long-Term Future of Interest Rates Both Real and Nominal', provides a useful discussion, as well as the information on real rates since 1824.

B.2.3 Factors which may have impinged on the level of real yields in the period between 1940 and 1980 include the Second World War, exchange controls, the operation of interest rate policy, credit restrictions and the effect of severe inflation from about 1970. Since 1980 the absence of exchange controls has led to an expectation that real returns in the U.K. will match those in other countries, subject to differences arising from purchasing power or other deviations in currency rates. The average real return on short-term instruments for the leading industrialised nations in the period 1979 to 1990 was 3% p.a.

B.2.4 If, as the author believes, investors and issuers focus on real rather than nominal yields, analysis of the structure of interest rates can provide input into anticipated future real yields:

- Short-term bond yields can be compared with available inflation forecasts.
- Long-term bond yields net of a term premium adjustment (to obtain an equivalent short-term bond yield), can be compared with inflation forecasts derived from survey data, historic trend analyses or from an economic model. The term premium is considered in Section B.3.

- Analysis of the index-linked gilt yield curve may provide a best estimate, particularly if it is believed that these gilts do not incorporate a term premium. This might be true, for example, if the existence of a term premium is related to inflation risk.

B.2.5 The results of an analysis conducted along these lines are likely to depend on the relative weight given to the level of real yields in the period 1940 to 1980 compared with the other evidence. In the author's view, bond market prices at the time of writing are broadly consistent with short-term real Treasury bill yields of approximately 5% p.a. for the next few years, declining to perhaps 3% or $3\frac{1}{2}$% p.a. in the longer term.

B.3 *Term Premium*

B.3.1 The term premium, defined here as the difference between expected future short-term interest rates and long-term interest rates, measures the additional return expected from investing in long-term bonds. In practice, measurement of the term premium is usually based on the difference between current short- and long-term rates. As noted by Pepper, the term premium tends to be high during periods when real short-term interest rates are low and vice versa. Thus, in the U.K., there has been only a small term premium during the 1980s, in contrast to the high premium experienced in the fifty-year period to 1979 (1·9% p.a.). Similar results have been experienced in the U.S.A. This conclusion needs to be qualified, because empirical and theoretical investigations into the term structure of interest rates have not yet reached a consensus.

B.3.2 In the context of a real yield on Treasury bills of say 3% p.a., a term premium of 1% p.a. would result in a real yield on long-term government bonds of 4% p.a.

B.4 *Long-Term Inflation Assumptions*

B.4.1 Long-term inflationary expectations can be assessed by deducting from long-term bond yields the assumed real long-term bond yield. As defined above, the real long-term bond yield is the sum of the forecast real short-term bond yield and the term premium. It has been assumed that government bond yields reflect the requirements and expectations of gross investors (see Section 3.3). Because of this, no adjustment for tax is needed when comparing yields and inflation levels.

B.4.2 The examples used in this paper assume that long-term inflation expectations amount to about 5% p.a. and that the real yield on long-term government bonds is 4% p.a.

B.5 *Equity Returns*

B.5.1.1 A guide to prospective returns on the equity market can be found from:

- an assessment of corporate growth prospects,
- a consideration of return on capital employed,

– an analysis of historic returns, and
– consensus forecasts.

These are considered in turn.

B.5.2 *Real Economic Growth*

B.5.2.1 It can be argued that corporate sector growth will bear some relation to growth in the economy as a whole. An assumption as to future growth in the economy, say 1% p.a., can then be used to assess prospects for future profitability and equity returns. Analysis of these, consistent with growth prospects, could be made as in the following example.

B.5.2.2 Consider a firm which is expected to grow in real terms at 1% p.a., and to pay a gross dividend of 5% p.a. Capital employed is 1500 at the start of the year, made up of 500 of debt and 1000 of equity taken at market value. These levels of debt to equity ratio and dividend yield are similar to the average for large companies quoted on the U.K. Stock Exchange. Inflation is assumed to equal 5% p.a., and bond and equity yields are assumed to remain constant over time. Tax is assumed to be paid on real earnings, and the effect of capital or other tax allowances is ignored. In order to have grown at a 1% p.a. real rate, the company value at the end of the year needs to be 1,590, an increase of 90 made up of 75 (inflationary growth) and 15 (retained earnings). The level of inflation adjusted pre-tax profit required to achieve this growth is 124 (8% of capital employed), as shown below:

Inflation adjusted profit pre-tax and interest	124
Interest (9% of 500)	(45)
Tax (33% of 124 – 45)	(26)
Equity earnings	53
Net dividend (5% × 1000, less tax)	38
Retained earnings	15

B.5.2.3 The 6% increase in total company value, to 1,590, has a geared effect on the value of equity. Debt is still worth 500 at the end of the year, and consequently equity has increased to 1090 (+9%). The equity return in this example is 14%, made up of a 5% dividend yield and 9% growth. If the company does not issue further debt, its debt/equity ratio declines over time, resulting in a declining level of equity risk and return.

B.5.2.4 If the company maintains a target debt/equity ratio of one-half by reducing its equity base and issuing debt, its equity return will be a constant 14% p.a. The equity return derived depends on the level of debt/equity ratio as well as on the profitability, interest rate, inflation, dividend and tax assumptions. A' company which has a lower (higher) level of target debt/equity ratio is likely to achieve a lower (higher) rate of return on equity, corresponding to a reduced (increased) overall level of equity risk.

B.5.3 *Return on Capital Employed*

B.5.3.1 The average inflation adjusted pre-tax return on industrial capital employed (measured at replacement cost) in the period 1963 to 1990 has been approximately 8% p.a. (source: *Bank of England Quarterly Bulletins*). This equates to a 14% p.a. equity return, using the gearing level and other assumptions stated in the previous example, and assuming that replacement cost is equal to the market value of the total debt and equity capital base.

B.5.3.2 The assessment of prospective returns on capital can be supported by consultation with investment analysts or experts specialising in the various sectors of the equity market.

B.5.4 *Historic Returns*

B.5.4.1 The geometric average risk return on equities, derived by dividing equity returns by the return on Treasury bills, during the 72-year period between 1919 and 1990 was 6.3% p.a. The arithmetic mean excess return was about 2% p.a. higher (source: the annual equity-gilt study by BZW). In the author's view, the geometric mean provides the better measure of the equity risk premium. If Treasury bills have a real yield of 3% p.a. and inflation is 5% p.a., the total equity return based on a 6% p.a. equity risk premium would be $14\frac{1}{2}$% p.a. ($1{\cdot}06 \times 1{\cdot}03 \times 1{\cdot}05$).

B.5.5 *Consensus Forecasts*

B.5.5.1 An alternative approach to assessing future return prospects is to use estimates prepared by equity market analysts and fund managers. These estimates tend to be lower than the results produced from the analyses suggested above. For example, at the time of writing, typical fund manager forecast long-term returns are:

Inflation	– 5% p.a. to 6% p.a.
Real yield on Treasury bills	– 0% p.a. to 3% p.a.
Real yield on long-term bonds	– 2% p.a. to 4% p.a.
Risk premium on equities (relative to Treasury bills)	– 4% p.a. to 6% p.a.

B.6 *Volatility of Investment Returns*

B.6.1 For option pricing and stochastic modelling purposes, estimates of the volatility of asset yields and returns are required.

B.6.2 The annualised standard deviation of stock market returns during the last 70 years has a 'normal' level of approximately 15% p.a. which, however, varies significantly from time to time. For example, volatility was much higher at times during 1974 and 1987 (so that estimates of volatility based on shorter measurement periods including these years are often higher than 15% p.a.). The distribution of equity returns has a 'tail' which is fatter than lognormal, resulting in extreme returns occurring more frequently than for a lognormal distribution.

B.6.3 There is some evidence that share prices regress towards levels based on

long-run average corporate earnings, that is, there is some 'normal' level of earnings yield. If return is measured as the logarithm of the ratio of the value of a share portfolio at the end and beginning of a period, the standard deviation of eight-year returns, for example, could be 0·8 or 0·9 times the standard deviation that would be observed if there were no mean reversion. In other words, mean reversion reduces long-term volatility and risk. The Institute of Actuaries Maturity Guarantees Working Party model incorporated a higher level of volatility of one-year returns, reducing to approximately 15% p.a. for returns measured over an eight-year period. In the model, shares were assumed to regress towards an average dividend yield rather than towards an earnings yield.

B.6.4 One method which can be used to measure the volatility of bond yields is to assess the standard deviation of a series of the natural logarithm of yield rates. If the standard deviation is sY, where Y is the expected yield, the one standard deviation range of yields in one year's time is Ye^{-s} to Ye^{s}. Yields typically exhibit some degree of mean reversion. Thus, the one standard deviation range of the expected yield in n years' time will be less than $Ye^{-s\sqrt{n}}$ to $Ye^{s\sqrt{n}}$. For example, the reduction over a 20-year period might be one-third. Under typical investment market conditions, the parameter s could be 0·25 for Treasury bills and 0·15 for ten-year bonds. In some periods, volatility is much higher than these levels.

B.6.5 In the author's view, allowance for a mean reversion effect should be considered in all long-term option pricing and stochastic modelling work.

APPENDIX C

DISCOUNT RATES AND TAXATION

C.1 *Introduction*

C.1.1 Life fund taxation may reduce the variability of returns to shareholders and consequently may affect the discount rates required to value the shareholders' interest in a life office.

C.2 *Capital Market Line*

C.2.1 As discussed in Section 3.4, net and gross capital market lines can be developed to represent the investment opportunity set for net and gross investors:

	Net Investor % p.a.	Gross Investor % p.a.
Risk free rate	7	8
Equity return	12	$14\frac{1}{2}$
Variability of equity return	10	15
Equity risk premium	$4\frac{1}{2}$	6

The risk free rate for a net investor has been assumed to equal the return available on index-linked gilts.

C.2.2 As noted in Section 3.3, it is not clear which of these two lines more closely represents the risk/return profile implicit in the principal securities markets.

C.3 *Effect of Taxation on Asset and Shareholder Return Variability*

C.3.1 Consider the effect of a doubling of asset prices on terminal bonus payments in a with-profits fund. The tax rate on income and (indexed) gains is assumed to equal 25%, and additional tax of 6% is payable on NCI profit (grossed-up shareholders' transfers). Shareholders' transfers are assumed to equal one-ninth of bonus payments.

An asset worth 1000 can be used to pay a bonus of B where:

$$1000 = B + \tfrac{1}{9} \times B + \tfrac{1}{9} \times B \times 0{\cdot}06/0{\cdot}75,$$

so that:

$$B = 893.$$

A doubling of the asset price results in an ability to pay a bonus of B where:

$$2000 - 250\ (\text{tax}) = B + \tfrac{1}{9} \times B + \tfrac{1}{9} \times B \times 0{\cdot}06/0{\cdot}75,$$

so that:

$$B = 1563.$$

The result of the doubling in price is to increase bonus payments, and hence shareholders' transfers, by 75%.

C.3.2 Similar arguments can be developed to assess the variability of shareholders' profits relative to the variability of asset returns according to the type of fund and tax position of the life office:

	Ratio of Variability of Shareholders' Profits to Asset Returns		
	Net Life Fund %	Gross Life Fund %	Pension Fund %
Tax on income and gains	25	8	0
With-profits fund/bonuses	75	92	100
Unit-linked fund/unit fees	75	75	100
Non-profit fund/surplus	75	92	100

The term 'life fund' is taken to mean the fund backing basic life assurance and general annuity business liabilities. The term 'pension fund' means the fund backing the pension business liabilities. In practice separate funds are not maintained, but an approximate apportionment can be made and the distinction is useful when considering the effect of tax.

C.3.3 In the table, the term 'gross life fund' refers to a life fund for which investment income and chargeable gains less relievable expenses is less than NCI profit. The effective tax rate on income and gains, shown in the table as 8%, can vary between 0% and 25%, depending on the period over which payment of tax is deferred. The variability of unit fee income is shown as 75% of the variability of asset returns, because it is assumed that basic rate tax is to be deducted from unit income and gains within the linked funds.

C.4 *Net Investor Discount Rates*

C.4.1 For a typical net corporate investor, the risk/return profile of a direct investment is fairly similar to the risk/return profile of the same investment held via a life fund. A difference arises because corporation tax for the net corporate investor is at a higher rate than for policyholders' income in the life fund. For both the investor and the fund, tax is paid on income and real gains. As seen above, tax reduces the riskiness of the returns for both.

C.4.2 In order to value pension fund gross asset returns using the return requirements of a net investor, it is necessary to use the net capital market line. Since the investment return in a pension fund is exempt from tax, the volatility of

returns in a pension fund is 4/3 times the volatility in a life fund (assuming basic rate tax equal to 25%). Based on a risk free rate of 7% p.a. and an equity risk premium of $4\frac{1}{2}$% p.a., the return required by a net investor on an equity investment held in a pension fund is $1{\cdot}07 \times (1 + \frac{4}{3} \times 0{\cdot}045) - 1 = 13\frac{1}{2}$% p.a.

C.4.3 The following table summarises the position for other types of asset, based on the same risk free rate and equity risk premium assumptions. The net asset returns shown reflect the tax position of the life fund, whereas the required returns reflect the tax position of a net corporate investor.

	Asset Return and Net Investor Required Return Assumptions			
	Life Fund		Pension Fund	
	Net Asset Return % p.a.	Required Return % p.a.	Asset Return % p.a.	Required Return % p.a.
Short-term bonds	6	7	8	7
Index-linked gilts	$7\frac{1}{4}$	7	8	7
Long-term bonds	7	8	$9\frac{1}{4}$	$8\frac{1}{4}$
Equities	$12\frac{1}{4}$	12	$14\frac{1}{2}$	$13\frac{1}{2}$

C.4.4 The adverse tax treatment of fixed-interest securities in the life fund results in net returns being less than those required by a net investor whose investment opportunity set includes index-linked gilts, property and other favourably taxed investments. The extent of the disadvantage is approximately equal to the effect of indexation relief (basic rate tax, 25%, multiplied by the assumed level of inflation, 5% p.a.). For the net investor, returns available via investment through the tax-free pension fund exceed the returns available on alternative equally risky direct investments, even after allowance is made for the dampening effect on risk caused by taxation.

C.4.5 Overall, tax effects result in net investor discount rates being:

- about 0·25% p.a. lower than the net asset yield, for equity and property assets held in the life fund,
- about 1% p.a. higher than the net asset yield, for fixed-interest assets held in the life fund, and
- about 1% p.a. lower than the asset yield, for assets held in the pension fund.

The discount rates derived above are those appropriate for net investors or for a market dominated by net investors.

C.4.6 It is interesting to note that double taxation of real gains (firstly within the *I-E* computation and again in the hands of a corporate net investor when he sells his insurance company shares) results in a substantial Inland Revenue participation in the risk and return of the shareholders' interest in the long-term fund assets.

C.5 *Gross Investor Discount Rates*

C.5.1 The methodology used to determine net investor discount rates can also be used to derive gross investor rates. Discount rates required to value returns to a gross investor are as follows:

Asset Return and Gross Investor Required Return Assumptions

	Life Fund		Pension Fund	
	Net Asset Return % p.a.	Required Return % p.a.	Asset Return % p.a.	Required Return % p.a.
Short-term bonds	6	8	8	8
Index-linked gilts	7¼	8	8	8
Long-term bonds	7	9	9¼	9¼
Equities	12¼	13	14½	14½

The adverse tax treatment of fixed-interest securities in the life fund results in net returns being less than returns required by a gross investor.

C.5.2 Overall, tax effects result in gross investor discount rates assets being:

- about 0·75% p.a. higher than the net asset yield, for equity and property assets held in the life fund,
- about 2% p.a. higher than the net asset yield, for fixed-interest assets held in the life fund, and
- the same as the asset yield, for assets held in the pension fund.

C.5.3 The discount rates derived above are those appropriate for gross investors or for a market dominated by gross investors. They need to be applied to cash flows inclusive of an allowance for the tax credit which accompanies distributions to investors. With basic rate tax at 25%, shareholder profits calculated net of insurance company tax can be multiplied by 4/3 in order to assess the return to a gross investor.

APPENDIX D

PUT AND CALL OPTION FEATURES OF A SINGLE PREMIUM DEFERRED ANNUITY

D.1 *Introduction*

D.1.1 The examples in this Appendix are based on a single premium deferred annuity ('SPDA') product, and the policyholder put and company call options described in Section 4.4.

D.2 *Policy Characteristics*

D.2.1 Key SPDA product and experience characteristics typically include:

(i) A high premium (resulting in sensitivity to interest rate considerations).
(ii) Accumulation of the initial premium with credited interest, which can be a function of various factors including the initial yield on the portfolio, competitor (new money) rates and surrender rates.
(iii) After the first year, credited rates might equal the lesser of the portfolio yield minus 1·5% p.a. and 0·5% p.a. above the competitor rate. The second of these two terms gives rise to the company call option. The office has an option to reduce credited rates further, subject to a minimum guarantee of say 4% p.a., but the value of this option may be limited if existing structure value is to be maintained.
(iv) Surrender penalties in the first six years (for example, 6% of the accumulated premium declining to 1%).
(v) Low surrender rates in the first six years, followed by a mass lapse on expiry of the surrender value penalty period, and by high surrenders thereafter.
(vi) High surrender rates if the credited rate falls significantly below the competitor rate. This feature is the policyholder put option. High surrender rates are also associated with companies which experience bad publicity (for example, arising from a decline in surplus or in asset values).

D.3 *Simplified Model*

D.3.1 The SPDA has an initial face value FV of 100 and initial term N equal to six years. The credited rate, $C1$, is $CS1$ (1% p.a.) below the yield $Y1$ on a zero coupon N year government bond. At duration T years, the policyholder has an option to surrender his policy for the accumulated value times a surrender value factor SV, and to switch into a new product with term $(N-T)$. The company has an option to reduce the credited rate. The new credited rate at time T, $C2$, is $CS2$ below the yield at time T on a zero coupon $(N-T)$ year bond. For the company call option a yield margin M is also credited and included in $C2$.

D.3.2 A proportion W of policyholders take advantage of the option to switch

if it is to their advantage to do so. The company is assumed always to take advantage of its option.

D.3.3 The company has surplus E in excess of the value of the N year bond (S), at the outset of the policy.

D.4 *Yield Curve Characteristics*

D.4.1 At the outset of the policy, the expected annual return from Treasury bills (that is, exclusive of any term premium) is $R1$ over the N year period and $R2$ during the first T years. The total return on the N year bond is $Y1$ including a term premium of $P1 (R1 + P1 = Y1)$.

D.4.2 The total return on the T year bond is $Y2$ including a term premium of $P2 (R2 + P2 = Y2)$. The expected term premium in T year's time on an $(N-T)$ year bond is $P3$. The expected yield on the $(N-T)$ year bond in T years is $Y3$, where:

$$(1 + Y3 - P3)^{N-T} = (1 + R1)^N / (1 + R2)^T.$$

D.4.3 The standard deviation factor of $(N-T)$ year bond yields is V (see Appendix B.6), so that the one standard deviation range of the yield in T years' time is:

$$V1 = Y3 \times \exp(\pm V \times \sqrt{T})$$

and the standard deviation of bond price in T years is approximately:

$$Z = Y3 \times V \times \sqrt{T} \times (N - T).$$

The expected return on the N year bond over T years, $Y4$, is given by:

$$(1 + Y4)^T = (1 + Y1)^N / (1 + Y3)^{N-T}.$$

D.5 *Option Statistics*

D.5.1 The initial value of the bond matching the expected maturity proceeds is:

$$S = FV \times ((1 + C1)/(1 + Y1))^N.$$

The option exercise price in T years' time is:

$$K = FV \times SV \times (1 + C1)^T \times ((1 + C2)/(1 + Y3))^{N-T}$$

where SV is set to 1 if the value of the company call option is to be assessed.

D.5.2 The value of the put option granted to policyholders, P, is W times the Black–Scholes put option price evaluated with input parameters S, Z, K, T and $Y2$.

The Black–Scholes put option price based on these parameters is:

$$SN(-d_1)-K^1 N(-d_2)$$

where:

$d_1=(\ln(S/K^1)+Z^2/2)/Z$,
$d_2=f_1-Z$,
$K^1=K\exp(-TY2)$, and
$N(x)$ is the probability that a normally distributed variable with mean zero and variance 1 is less than x.

The expected value of the put option at time T, P^1, is W times the Black–Scholes put option price evaluated with input parameters $S\times(1+Y4)^T$, Z, $K\times(1+Y2)^T$, T and $Y2$. Similar input parameters are used to assess the value and expected value of the call option. The Black–Scholes value of the call option is $SN(d_1)-K^1N(d_2)$.

D.5.3 The discount rate required to value the put and call options is given by:

$$(1+D)^T=P^1/P \text{ and } C^1/C.$$

The overall discount rate, I, required to value future maturity and surrender cash flows is solved by iteration.
The value of the policyholders' liability, L, is given by $(S+P)$ and $(S-C)$.
The value of the shareholders' interest, MV, is $(E-P)$ and $(E+C)$.

D.6 *Model Results*

D.6.1 The put option results shown in Tables D.1 and D.2 include:

(i) the value of a bond (S) required to meet the maturity proceeds assuming no surrender option,
(ii) the surrender value (K) at the time of option expiry,
(iii) the value of the put option given to policyholders at policy inception, and expected value at the time of option expiry (P and P^1),
(iv) the discount rate (D) consistent with the option pricing value of the surrender payments,
(v) the overall discount rate (I) required to value policyholder cash flows,
(vi) the value of liabilities to policyholders ($L=S+P$, that is, the value of the bond plus the value of the put option) compared with stochastic payments discounted using government bond yields ($L1$), and
(vii) shareholder value assessed on an option pricing basis ($MV=E-P$, where E is surplus measured by reference to the bond price S and P is the cost of the put option) compared with the value of net cash flows discounted at, say, 12% p.a.

D.6.2 The call option results are similar in format. The value of policyholder liabilities is the value of the bond less the value of the company call option ($L=S-C$), and shareholder value is equal to surplus plus the value of the call ($MV=E+C$).

D.6.3 The calculated value of policyholders' liabilities (L), allowing for the put option, is up to 0·4% higher than the value that would be calculated by multiplying stochastic cash flows by discount factors derived from an analysis of government bond prices ($L1$). This is because systematic risk inherent in the put

Table D.1 POLICYHOLDER PUT OPTION

Input		Examples											
		1	2	3	4	5	6	7	8	9	10	11	12
Bond Term	N	6·000	6·000	6·000	6·000	6·000	6·000	6·000	6·000	6·000	6·000	6·000	6·000
Option Term	T	3·000	3·000	3·000	3·000	3·000	3·000	3·000	3·000	3·000	5·000	1·000	1·000
Risk Free Rate N yrs	$R1$	0·080	0·080	0·080	0·080	0·080	0·080	0·080	0·080	0·080	0·080	0·080	0·080
Risk Free Rate T yrs	$R2$	0·080	0·080	0·080	0·080	0·080	0·080	0·080	0·080	0·080	0·080	0·080	0·080
Term Premium N yrs	$P1$	0·010	0·010	0·010	0·010	0·010	0·010	0·010	0·010	0·010	0·010	0·010	0·010
Term Premium T yrs	$P2$	0·005	0·010	0·005	0·010	0·005	0·005	0·005	0·005	0·005	0·005	0·005	0·005
Term Premium $N-T$ yr Bond in T yrs	$P3$	0·005	0·010	0·010	0·005	0·005	0·005	0·005	0·005	0·005	0·005	0·010	0·010
Volatility Factor ($N-T$ yr Bond Yield)	V	0·150	0·300	0·150	0·150	0·300	0·150	0·150	0·150	0·150	0·150	0·150	0·150
Credited Rate Spread to N yr Bond	$CS1$	0·010	0·010	0·010	0·010	0·010	0·005	0·010	0·010	0·010	0·010	0·010	0·010
Credited Rate Spread to $N-T$ yr Bond in T yrs	$CS2$	0·010	0·005	0·010	0·010	0·010	0·010	0·005	0·010	0·010	0·010	0·010	0·010
Surrender Factor	SV	0·970	0·970	0·970	0·970	0·970	0·970	0·970	0·980	0·970	0·990	1·000	0·950
Initial Face Value	FV	100·000	100·000	100·000	100·000	100·000	100·000	100·000	100·000	100·000	100·000	100·000	100·000
Withdrawal Rate	W	0·300	0·600	0·300	0·300	0·300	0·300	0·300	0·300	0·600	0·300	0·300	0·300
Surplus (excess over the asset value S)	E	5·000	5·000	5·000	5·000	5·000	5·000	5·000	5·000	5·000	5·000	5·000	5·000
Calculation													
N yr Bond Yield	$Y1$	0·090	0·090	0·090	0·090	0·090	0·090	0·090	0·090	0·090	0·090	0·090	0·090
N yr Credited Rate	$C1$	0·080	0·080	0·080	0·080	0·080	0·085	0·080	0·080	0·080	0·080	0·080	0·080
T yr Bond Yield	$Y2$	0·085	0·090	0·085	0·090	0·085	0·085	0·085	0·085	0·085	0·085	0·085	0·085
$N-T$ yr Bond in T yrs: Yield	$Y3$	0·085	0·090	0·090	0·085	0·085	0·085	0·085	0·085	0·085	0·085	0·090	0·090
Return on N yr Bond over T yrs	$Y4$	0·095	0·090	0·090	0·095	0·095	0·095	0·095	0·095	0·095	0·091	0·090	0·090
$N-T$ yr Bond in T yrs: Credited Rate	$C2$	0·075	0·085	0·080	0·075	0·075	0·075	0·080	0·075	0·075	0·075	0·080	0·080
$N-T$ yr Bond Price Volatility at T	Z	0·066	0·140	0·070	0·066	0·133	0·066	0·066	0·066	0·066	0·029	0·068	0·068
N yr Bond Asset Value	S	94·620	94·620	94·620	94·620	94·620	94·279	94·620	94·620	94·620	94·620	94·620	94·620
Exercise Price	K	118·845	120·518	118·860	118·845	118·845	120·503	120·511	120·070	118·845	144·123	103·136	97·979
Value of Put Option	P	0·421	2·397	0·462	0·293	1·126	0·310	0·572	0·529	0·842	0·265	0·778	0·248
Value of Put Option at T	P'	0·276	3·105	0·439	0·267	1·083	0·191	0·398	0·362	0·552	0·082	0·776	0·239
Discount Rate to Value Surrender Payments	D	−13·11%	9·00%	−1·72%	−3·02%	−1·32%	−14·83%	−11·43%	−11·87%	−13·11%	−20·91%	−0·22%	−3·42%
Overall Discount Rate for Cash Flows	I	8·96%	9·00%	8·98%	8·98%	8·94%	8·97%	8·95%	8·95%	8·92%	8·96%	8·99%	8·99%
Total Value of Liability to Policyholders	L	95·041	97·018	95·082	94·913	95·747	97·589	95·192	95·149	95·462	94·885	95·398	94·868
Policyholder Liability Valued Using Bond Yield	$L1$	94·836	97·018	94·964	94·826	95·468	97·429	94·931	94·904	95·052	94·675	95·335	94·841
Value of Shareholders' Equity	MV	4·579	2·603	4·538	4·707	3·874	4·690	4·428	4·471	4·158	4·735	4·222	4·752
Net Cash Flows Discounted at 12% p.a.		4·070	2·211	3·960	4·076	3·548	4·125	3·991	4·014	3·891	4·203	3·644	4·062

Table D.2 COMPANY CALL OPTION

Input		Examples											
		1	2	3	4	5	6	7	8	9	10	11	12
Bond Term	N	6·000	6·000	6·000	6·000	6·000	6·000	6·000	6·000	5·000	6·000	6·000	6·000
Option Term	T	3·000	3·000	3·000	3·000	3·000	3·000	3·000	3·000	3·000	5·000	1·000	1·000
Risk Free Rate N yrs	$R1$	0·080	0·080	0·080	0·080	0·080	0·080	0·080	0·080	0·080	0·080	0·080	0·080
Risk Free Rate T yrs	$R2$	0·080	0·080	0·080	0·080	0·080	0·080	0·080	0·080	0·080	0·080	0·080	0·080
Term Premium N yrs	$P1$	0·010	0·010	0·010	0·010	0·010	0·010	0·010	0·010	0·010	0·010	0·010	0·010
Term Premium T yrs	$P2$	0·005	0·010	0·005	0·010	0·005	0·005	0·005	0·005	0·005	0·005	0·005	0·005
Term Premium $N-T$ yr Bond in T yrs	$P3$	0·005	0·010	0·010	0·005	0·005	0·005	0·005	0·005	0·005	0·005	0·010	0·010
Volatility Factor ($N-T$ yr Bond Yield)	V	0·150	0·300	0·150	0·150	0·300	0·150	0·150	0·200	0·150	0·150	0·150	0·300
Credited Rate Spread to N yr Bond	$CS1$	0·010	0·010	0·010	0·010	0·010	0·005	0·010	0·000	0·010	0·010	0·010	0·010
Credited Rate Spread to $N-T$ yr Bond in T yrs	$CS2$	0·010	0·005	0·010	0·010	0·010	0·010	0·005	0·000	0·010	0·010	0·010	0·010
Margin over Competitor Yield (if exercised)	M	0·005	0·005	0·005	0·005	0·005	0·005	0·005	0·000	0·005	0·005	0·005	0·005
Initial Face Value	FV	100·000	100·000	100·000	100·000	100·000	100·000	100·000	100·000	100·000	100·000	100·000	100·000
Surplus (excess over the asset value S)	E	5·000	5·000	5·000	5·000	5·000	5·000	5·000	5·000	5·000	5·000	5·000	5·000
Calculation													
N yr Bond Yield	$Y1$	0·090	0·090	0·090	0·090	0·090	0·090	0·090	0·090	0·090	0·090	0·090	0·090
N yr Credited Rate	$C1$	0·080	0·080	0·080	0·080	0·080	0·085	0·080	0·090	0·080	0·080	0·080	0·080
T yr Bond Yield	$Y2$	0·085	0·090	0·085	0·090	0·085	0·085	0·085	0·085	0·085	0·085	0·085	0·085
$N-T$ yr Bond in T yrs: Yield	$Y3$	0·085	0·090	0·090	0·085	0·085	0·085	0·085	0·085	0·085	0·085	0·090	0·090
Return on N yr Bond over T yrs	$Y4$	0·095	0·090	0·090	0·095	0·095	0·095	0·095	0·095	0·093	0·091	0·090	0·090
$N-T$ yr Bond in T yrs: Credited Rate	$C2$	0·080	0·090	0·085	0·080	0·080	0·080	0·085	0·085	0·080	0·080	0·085	0·085
$N-T$ yr Bond Price Volatility at T	Z	0·066	0·140	0·070	0·066	0·133	0·066	0·066	0·088	0·044	0·029	0·068	0·135
N yr Bond Asset Value	S	94·620	94·620	94·620	94·620	94·620	94·279	94·620	100·000	95·496	94·620	94·620	94·620
Exercise Price	K	124·238	125·971	124·246	124·238	124·238	125·971	125·971	129·503	124·813	146·256	105·546	105·546
Value of Call Option	C	1·774	4·576	1·915	2·382	4·238	2·397	1·314	3·337	1·147	0·653	1·576	4·058
Value of Call Option at T	C'	3·927	5·927	3·202	4·010	7·153	5·069	3·054	6·287	2·877	3·176	1·891	4·619
Discount Rate to Value the Call Option	D	30·33%	9·00%	18·69%	18·96%	19·06%	28·35%	32·46%	23·50%	35·86%	37·20%	19·98%	13·84%
Overall Discount Rate for Cash Flows	I	8·76%	9·00%	8·89%	8·86%	8·75%	8·71%	8·80%	8·72%	8·75%	8·73%	8·97%	8·97%
Total Value of Liability to Policyholders	L	92·846	90·044	92·705	92·238	90·382	94·882	93·306	96·663	94·349	93·967	93·044	90·563
Policyholder Liability Valued Using Bond Yield	$L1$	91·545	90·044	92·113	91·524	89·020	93·311	92·229	95·078	93·244	92·508	92·877	90·363
Value of Shareholders' Equity	MV	6·774	9·576	6·915	7·382	9·238	7·397	6·314	8·337	6·147	5·653	6·576	9·058
Net Cash Flows Discounted at 12% p.a.		6·790	8·137	6·349	6·843	8·877	7·528	6·225	8·317	6·287	5·994	5·722	7·849

option results in a reduction in the discount rate required to value the policyholder liabilities, and in a consequent increase in the value of these liabilities. There is no such increase if the expected return on the N-year bond ($Y4$) over the T-year period equals the return on a T-year bond ($Y3$), that is, there is no term premium.

D.6.4 The calculated value of policyholders' liabilities (L), allowing for the call option, is up to $1\frac{1}{2}\%$ higher than the value that would be calculated by applying government bond discount factors to the stochastic cash flows. The call option has a relatively high systematic risk. This high risk reduces the value of the option compared with the expected cost discounted at the bond rate. The value of the policyholders' liabilities is correspondingly higher than the level derived from a stochastic model discounted using a government bond yield.

D.6.5 In practice, the expected future lapse rate, W, may depend on the term structure of interest rates and, therefore, may also carry systematic risk. Further research is required to model this feature.

ABSTRACT OF THE DISCUSSION

Mr S. J. B. Mehta (introducing the paper): A large number of papers have been written on subjects related to life office appraisal values. Why should a further paper be of interest to the profession? There are, in my view, three main areas where the paper which is before you fills some gaps:

(1) Over the years appraisal value techniques have diverged from traditional actuarial methods such as those of the gross premium and bonus reserve valuations: the former applies a risk discount rate to net cash flows; the latter approaches place values on each of the individual life office cash flows. It is useful and interesting to bring these approaches closer into line.
(2) I perceive a gap of communication between actuaries and capital market researchers. Although the subject of modern portfolio theory has been introduced to the profession in a number of papers, the theory is conspicuous by its absence in everyday actuarial work. Likewise, the many tools developed by actuaries are largely unused outside our profession. Perhaps an inter-mingling of techniques is called for.
(3) The paper introduces a general methodology of comparing life office cash flows with returns from traded securities, in order to assist in the selection of discount rates for use in the valuation of a life office. In many appraisal valuations hitherto, discount rates have been selected on the basis of judgement and experience rather than by application of the techniques of asset pricing theory. Tackling the selection problem using an alternative approach may be beneficial.

Under the methodology proposed, the same value is assessed for a given cash flow whether it is attributable to policyholders or to shareholders, to a mutual office or to a proprietary company. Higher rates of discount are derived for risky cash flows than for less risky cash flows.

Use of this value-based, rather than profit-based, system highlights corporate strengths and weaknesses. For example, appraisal values determined for a non-profit company investing in high risk corporate bonds, for a unit linked office which recoups expenses primarily from unit fee income, for a with-profits office which is heavily mismatched, or for any company with high overhead costs, are reduced compared with values assessed using existing methods. The techniques suggested can be used not just in assessing the value of the policyholders' or shareholders' interests in a life office; they can also be used in refining existing approaches to a number of subjects, such as the development of policy asset shares, product pricing, demutatlisation and asset liability matching.

Mr N. J. Dumbreck (opening the discussion): In the introduction to their paper entitled, 'The Determination of Life Office Appraisal Values' (*J.I.A.* **114,** 411), Burrows & Whitehead commented that an entire paper could be devoted to the subject of determining risk discount rates. It is 5 years since that paper was presented, and while there have been further written contributions, this is the first opportunity we have had for a debate focusing on this and related topics.

The author takes as his starting point the capital asset pricing model and aims to quantify the systematic, or market-related risks inherent in long-term insurance business, by looking at individual cash flows and comparing them with cash flows from traded securities. I agree that examination of the nature and variability of individual cash flows must be the right way to get a better understanding of the appropriate levels of risk to be allowed for in life company appraisal values.

In reviewing the paper, it is appropriate to consider the validity of the CAPM itself. The model is widely used, particularly in the United States of America, but empirical evidence suggests that it does not work well in practice. The assumptions underlying the model are very strict, and as a recent paper by Cummins (*ASTIN Bulletin,* **20,** 125) points out, nearly every one of them is violated in the real world. For example, the model assumes that all investors have the same economic view of the world, are subject to the same tax rates and have no bias as between upside and downside risks. The author acknowledges these shortcomings, but argues that the model should be used in the absence of a better alternative. I would have preferred to have seen more critical evaluation of the suitability of the capital asset pricing model in the paper.

Section 3 introduces the investment return assumptions which form the basis of the examples. I

have difficulty with one of the assumptions, and that is the assumed level of return on equities. The author questions the standard approach in actuarial literature of comparing historic equity returns with inflation, and advocates the method used in modern financial research of adding a risk premium to Treasury bill returns. It is interesting to compare the results of the two approaches. In their paper 'A Realistic Approach to Pension Funding' (*J.I.A.* **119**, 229), Thornton & Wilson used the traditional approach, and concluded that, if inflation is 5% p.a., equities may be expected to achieve a return of around 11%- 11½% p.a. This result was based on an average dividend yield of 5% p.a. and an average real dividend growth of 1% p.a. Other 'traditional' papers have produced similar results. The paper assumes a 14½% overall equity return, and at current dividend yields this implies average real dividend yield growth of 4% p.a., or thereabouts. Sustained dividend growth at this level seems to me to be unattainable unless either the economy grows remarkably quickly or else the providers of loan finance to companies are very unambitious. My verdict is a victory for traditional actuarial methods over modern financial theory, although perhaps the problem is with the assumptions, and particularly the assumed 6% risk premium for equities. The paper indicates that this is the average risk premium observed over the last 72 years, but much has changed over that period. Many large investors, pension funds and with-profits life funds now regard equities as the most natural investment relative to their liabilities, and are not necessarily looking for a large risk premium. I suggest that the risk premium is more likely to be of the order of 3%, and this would bring the results of the two approaches closely into line.

The methodology for valuing in-force business is developed in Sections 4-6. Some of the parallels between life office and comparable asset cash flows are more convincing than others. For example, it would be very difficult to assess accurately the increase in lapse rates associated with a given fall in market values, and, although both may be consequences of an economic downturn, the market fall may well precede the rise in lapse rates by a year or two. For with-profits business, it is argued that the discount rate should be dependent on bonus smoothing policy, rather than on the volatility of the underlying returns. This may be appropriate for an office which has a well established smoothing policy, but for a large number of offices there is evidence that this policy has not yet settled down. I would not wish to give undue weight to current bonus smoothing policy in an appraisal value calculation.

The examples in the paper are interesting. There is evidence that discount rates for published appraisal values have been reducing in recent years from the once common 15% p.a., but they are still typically higher than the assumed net return on equities. The examples imply that net investor discount rates should be below the net return on equities for most types of business, a conclusion which appears to be consistent with the relatively low level of beta for quoted life offices. In numerical terms, the effect of this conclusion is to some extent masked in the paper by the use of a very high assumed return on equities. For a unit-linked plan with low lapse profits and funded management charges, it is suggested that for a net investor profits should be discounted at a rate close to the net gilt yield. The consequences of accepting these conclusions are fairly significant.

In Section 7.3 I like the approach of applying different multiples to marginal and overhead initial expenses, but was slightly alarmed at the sensitivity of the overall multiple to small variations in the individual components.

One consequence of looking at individual cash flows rather than statutory surpluses, is that the choice of statutory valuation basis becomes largely irrelevant to the appraisal value calculation, particularly for non-profit and unit-linked business. No additional return is required, simply because profits are not immediately distributable. A corollary is that there should be no locking-in adjustment in respect of that part of the net worth of the office held to meet the minimum solvency margin. I have no difficulty in accepting this, but I suspect that others will. Another interesting conclusion is that discount rates should differ as between life and pensions business, because of the cushioning effect of capital gains tax and the advantageous tax treatment of pension fund assets.

The paper does not put forward any specific methods for allowing for unsystematic risks, but it is clear from § 9.5 that the author is not comfortable that such risks can be ignored entirely. Neither am I, and I now give two examples to explain why:

(1) Consider two offices which are identical in all respects but one, which sell term assurance business. One office guarantees that the premium rates will remain unchanged, but the other has

the right to increase rates for existing business if mortality experience worsens. If the offices are valued with no allowance for unsystematic risks, then the two values will be identical; but common sense suggests that the office which does not offer guarantees is worth more.

(2) Consider the determination of goodwill multiples. Received wisdom is that the main factor to be taken into account is the security of the distribution system. The approach described in the paper recognises the difference between broker distribution and a direct sales force for with-profits business, because the former is more likely to be affected by market movements which make the office uncompetitive. However, it would not necessarily produce different goodwill multiples for, on the one hand, a bank-owned life office which has very secure distribution via its parent company branches, and for, on the other hand, an office selling via down-market tied agents. This lack of differentiation seems wrong.

In considering unsystematic risk, it is also important to bear in mind the purpose for which the appraisal value is being prepared. In evaluating a life company for the purchase of a controlling interest, for example, the assumption that unsystematic risk can be diversified away appears to be invalid.

Practical application of the methods advocated in the paper is likely to be complicated, and the difficulties of communicating the approach to users of the appraisal valuation would be considerable. One conclusion is that, even using the additional techniques which the author has developed, different investors and their advisers are likely to come up with very different values for the shares of an insurance company. Investors will differ in their tax circumstances and in their economic assumptions. In addition, considerable judgement is required in applying the methods described for valuing the in-force business and goodwill. The concept of a wholly objective expert valuation, which all investors should accept as representing the true value of their shares, seems unattainable.

The author has taken us half a step forward by suggesting a more scientific method of determining discount rates and goodwill multiples, but some aspects of the proposed method and the underlying theory require further investigation before a major breakthrough can be claimed.

Mr M. H. D. Kemp: A considerable number of financial practitioners in other industries treat the insights provided by modern portfolio theory almost as 'received wisdom'. I think that a greater airing of these techniques within actuarial practice, as provided by this paper, is to be applauded.

The results of any analysis along the lines suggested in the paper depend crucially on the assumptions used, particularly the assumed rates of return. I have certain comments on some of the author's assumptions, particularly the one relating to the future return on equities, although I would add that the choice of such assumptions is a matter for professional judgement.

The author's approach, like the CAPM, seems to involve coming up with an assumed risk free rate of return and then adding a risk premium dependent on the asset category concerned. In § 3.4.4 the author states that for a gross investor the risk free return is usually taken to be the return on Treasury bills. Sadly, although this assumption is often made, it is not necessarily correct. The risk free return may not be the return on Treasury bills. What is 'risk free' depends on what is risky to the investor concerned. For long-term investors, such as pension funds and life offices, who probably dominate the equity market in the United Kingdom, it is more usual, at least in the actuarial profession, to assume that 'risk free' assets are ones that in some sense match the liabilities, assets like conventional or index-linked gilts.

The use of Treasury bills by the original protagonists of the CAPM has, unfortunately, a knock-on effect on the assumptions made for the long-term returns on equities. In Appendix B.5.4 the author notes that equities have out-performed Treasury bills by about 6% p.a. between 1919 and 1990, during a period when Treasury bills gave what the author seems to believe is an artificially low 0% p.a. real return. In the future he expects Treasury bills to yield a more normal 3% p.a. return, and so has come up with an overall long-term assumed rate of return on equities of about 9% p.a.—that is, 6% plus 3%. Is it not equally plausible to assume that the aberration is not in the 0%, but in the 6% p.a. out-performance that equities have given over cash, because cash performed poorly relative to whatever the true risk free asset was? Hence, is it not equally plausible to assume equities will continue to offer a 6% real return as the opener has suggested?

That alternative viewpoint seems to be widespread. My firm recently carried out a survey of about a

dozen leading fund managers. The consensus forecast in Appendix B just about fits into the findings of our survey. However, the manager who thought that the real yields on Treasury bills were at the high end of his 0%–3% p.a. range also thought that the risk premium on equities was at the low end of the 4%–6% p.a. range that the author also suggested. Given the particularly favourable equity performance during the 1980s, I would adopt a more cautious rate than the author for what seems to be a key assumption in the appraisal valuation technique.

Mr N. A. M. Franklin, F.F.A: The author's worked examples are particularly helpful, even though the results may be based on a number of questionable assumptions—for example, on systematic lapse risk. They provide, nevertheless, a real insight into the author's approach, as well as a basis for further development.

The key to the paper seems to be the equation of value in § 2,5,1. The discount rate applied to future cash flows is that appropriate to allocated assets, plus an element that reflects the difference between asset and liability risk, multiplied by the financial gearing or 'debt/equity' ratio. This gearing element can be very significant, as is demonstrated for a U.S. single premium deferred annuity contract in Section 4.6. I wonder how many U.S. managements were conscious of the impact of junk bonds on the risk discount rate when they priced their SPDA contracts? There are strong parallels between such contracts and the unitised with-profits life bonds that have been issued in recent years in the U.K. (with equities taking the place of junk bonds). It would be interesting to apply the author's techniques to these contracts.

The section on with-profits business is the least developed, perhaps reflecting the complexity of the business. However, the basic idea, as expressed in Section 6.2, is intuitively very appealing. The examples show that the discount rate to apply to future bonuses (and associated profit and loss transfers) is very sensitive to the size of the free estate and the investment policy; that is, the more the risk discount rate is increased the smaller the free estate (that is the higher the gearing) and the higher the equity proportion of total assets. It is difficult to argue with such a conclusion.

Mr A. D. Smith: This paper attempts to introduce the methods of modern financial economics into actuarial work. I believe that such techniques will become more important in a world where actuaries often attempt to adopt a 'market value' approach to life office valuation. Inevitably there is a gap between the ideal assumptions underlying the theory and the real world. Some 'fixes' must be made in order to explain observed effects in terms of factors outside the scope of the financial models employed. I do not think that some of the 'fixes' used by the author are wholly appropriate.

In § 4.3.1 the author mentions some reasons why corporate securities offer interest margins over government stocks of similar type and term. Some of these reasons relate to the fact that the market is not totally frictionless. Transaction costs and a premium for illiquidity are mentioned as significant factors. If corporate securities are considered illiquid, the liabilities of a life office must be even more so. It would be interesting to see what adjustment, if any, should be made to reflect this illiquidity.

The two theories of option pricing and the CAPM are not consistent with each other, and I give here an example to illustrate this. It is necessarily oversimplified, but the principles carry over to the more complex real world. Consider two cases:

(1) This is a unit-linked endowment product, with a term of 10 years and a maturity guarantee of £10,000. Suppose that the expected value of the maturity payment is actually £15,000. Because of the equity participation, the appropriate net discount rate will be close to the assumed net return on equity, that is 12% in § 3.4.1. Applying CAPM, the discount rate might be around 11%, depending on the distribution of returns assumed. The net present value of the liabilities, ignoring mortality, expenses and the need to set up reserves, may then be calculated as £5,283.

(2) This is a non-profit endowment, with a term of 10 years and a maturity value of £10,000. The appropriate net discount rate would then be determined by the net return on long-term gilts, that is 6¼% in § 3.4.1. The net present value of the liability, again ignoring mortality, expenses and the need to set up reserves, will now be £5,454.

The critical point to note from these cases is that the unit-linked policy incurs a payment at least as large as the conventional one, while the value assigned to the liabilities is smaller. This is in

contradiction to the option pricing approach, where larger payments always mean larger present values. The discrepancy between the models arises because of the differing assumptions. The Black and Scholes option pricing formula relies, not so much on the distribution of the asset price on maturity, but on the path taken to get there. In particular, continuous trading is assumed possible between now and the maturity date. In contrast, intermediate trading actually invalidates CAPM, where it is assumed that a chosen asset allocation is maintained for each investor during the period under consideration. The author's suggestion in § 2.4.2, that skewness is the criterion for abandoning CAPM, is misleading; instead liquidity should point to the use of a Black and Scholes approach.

Mr A. J. Sanders: Assessing a life office appraisal value is an inexact science, in which judgement inevitably plays a major role. Replacement of judgement with objective determination is a laudable aim, but it may not always be possible, successful or even desirable. The use of asset pricing models in determining the discount rates to be used to value future cash flows does shed further light on some of the issues. However, the models have their limitations; the evidence in their favour and their ability to predict returns accurately is questionable. The different types of risk for which allowance is being made need to be kept firmly in mind. There is a danger of using complex methods to take account of fairly minor risks, whilst relying heavily on commercial judgement for the major elements of risk.

Models such as the CAPM are applied mainly to shares quoted on stock exchanges, for which there is a reasonably efficient market. The model assumes that risk is assessed having regard to past share price movements and other information in the public domain. In the U.K. there are only about a dozen quoted life or composite insurers, and most U.K. life operations are not quoted. Some of the information available for the appraisal valuation may not be in the public domain. In these circumstances the validity of asset pricing models is questionable.

The major risks in a life operation often involve the distribution channel, such as salesforce or intermediary loyalty, and also future lapse and expense experience. Thus, the amount of risk diversifiable under CAPM is significant. The argument in § 2.3.2, that diversifiable risk does not require a higher rate of discount, depends upon there being an efficient market. This may, perhaps, have some validity in assessing the trading price of a quoted share with a substantial number of shareholders, but these circumstances do not apply in most appraisal value exercises. The argument that a higher return is not required for risks that could be diversified in a large, efficient market does not sound very convincing if applied to a small company with only one shareholder—where there is no question of the risks being arbitraged.

I am also uncomfortable with the equation of value in § 2.5.1, which derives the shareholders' required rate of return from the overall return on assets and the discount rate for the liabilities. The concept appears sensible in the circumstances that, when an investor borrows in order to make an investment, his required rate of return depends upon the rate of interest he is paying on his debt. However, the application to the shareholders' interest in a life operation seems more tenuous. The analogy that is drawn between policyholders' guaranteed liabilities and corporate debt does not seem justified. Shareholders are not borrowing at the rate of interest used for discounting the liabilities. The consequent result, quoted in § 4.3.3, that the rate of shareholders' discount depends upon the ratio of shareholders' equity to policyholders' liabilities, has surprising implications. In that example, an additional yield of 0·2% p.a. on corporate bonds resulted in an increase in the shareholders' discount rate of about 3% p.a. If there had been more equity capital, then the increase in discount rate could be reduced or eliminated. The portfolio, therefore, would be worth more to an acquirer with a large amount of capital than one with a small amount of capital. This appears a curious result. It suggests that the purpose of the appraisal valuation needs to be kept firmly in mind when considering the applicability of these methods. In particular, the value to a controller of a life operation, who can influence the degree of risk taken, may be different from the value to a minority shareholder.

In the application of the CAPM, identifying the diversifiable and undiversifiable components of the risk can be both difficult and subjective. To take the example of lapse experience in § 5.5.3, the author gives an approximate estimate of the link between regular premium lapse rates and stock market returns. A market decline of 15% is associated with an increase in lapse rates of 3% in year 1 and 1% in years 3 and over. That may be a reasonable estimate, but there could be many other different estimates. The problem here is that lapse rates depend upon many other factors not necessarily

correlated with investment movements—tax rates, sales force turnover, etc. Irrespective of how these risks are allowed for, it is difficult to assess and measure the undiversifiable element reliably. The lapse risk may well not be symmetric; for example, potential losses from bad experience may be much greater than gains from good experience. Moreover, conditions change rapidly, and it is likely that correlations that appear to have applied in the past will not necessarily be appropriate in the future. In these circumstances, it has to be asked whether the application of the theory really adds any additional insight into the assessment of an appraisal value.

Mr P. G. Scott: At the present time there are a number of substantial changes affecting insurance offices, not only from the changing investment scene, but also from the changing liability profile, where the use of some of the techniques outlined in the paper can lead to greater understanding.

On assumptions, a number of speakers have suggested that the equity returns look rather high. I am also in that camp. Total returns from equities in the range of 10%-11% p.a. are nearer the central assumption that we should be using, rather than 14½%. However, we must be careful to understand the purpose for the appraisal. If we are trying to create an appraisal value to encourage shareholders to pay more for a business, then perhaps the level of returns we are looking at are justified as one of the scenarios.

There are two types of capital needed to support with-profits bonds. There is the usual new business strain, where we look at expense loadings versus actual expenses, and we find—because with-profits bonds are front-end loaded products—that there is very little capital strain. Then there is the mismatching reserve per unit of single premium written. Using the techniques described in the paper, it is found that the amount of capital needed to invest the funds in the broad range of assets in a with-profits office is substantial. I believe that the key benefit of the paper will come from a better understanding of the interactions between assets and liabilities.

Mr D. M. Pike, F.F.A: The practical difficulties facing any actuary wishing to use the methods advocated in the paper should not be underestimated, not least the presentational ones. In most situations it is much more critical for the actuary to be perceived to have experience and credibility rather than theory.

When actuaries calculate appraisal values, they are normally considering the economic value, or a range of economic values, of the business. The papers by Bangert (*J.I.A.* **99,** 131) and by Burrows & Whitehead (*J.I.A.* **114,** 411) discuss how the price at which a purchase is transacted may differ from the appraisal value. As I understand asset pricing models, they have been developed for the purpose of assessing the price at which investors might buy shares for investment purposes as part of a diversified portfolio. The author does not discuss the purpose of the valuation, so it is not clear what resulting value is given by use of the methods described. Although an attempt is made to distinguish net and gross investors, the author does not recognise any other features which might differentiate purchasers of a life office, for instance skills or resources which could be employed to add value; or a foreign currency. For these reasons, I do not feel the method can ever be more than a benchmark for risk discount rates, and even for that we must clarify what the results represent.

The origin of the idea of using option pricing theory in life office appraisals lies in the U.S.A., and the discussion of non-profit products in Section 4 and Appendix D is related to single premium deferred annuities (SPDA), which are U.S. products. The method is more readily appropriate to the nature of life business in the U.S.A., where there are guaranteed surrender values on the liability side and mainly fixed-interest investments on the assets side.

Given that asset pricing theories have been developed in relation to traded securities like equities and gilts, I am not comfortable with extending their application to the individual elements into which the cash flows from unit-linked policies are broken down in § 5.2.6. For instance, lapse risk (Section 5.5) depends on:

(1) 'a very approximate estimate' that a market decline of 15% is associated with an increase in lapse rates of 3% in year 1 and 1% in years 3 and over, whereas I would be surprised if the relationship is linear,
(2) an equity risk premium of 6% which has already been commented on, and
(3) a standard deviation of market returns of, say, 15%.

A further assumption is made that a 1% increase in the lapse rate reduces the value by 1%. Presumably this can be evaluated from two runs of a projection, but does it not depend on the discount rate which one is trying to determine?

There are ideas worth exploring in the treatments of with-profits business, goodwill and the whole company in Sections 6, 7 and 8. I note in §8.4.1 that quoted U.K. life offices have a beta of approximately one. In an article in the *Journal of Risk and Insurance* (Risk/Return Relationship for Life—Health, Property—Liability and Diversified Insurers, **58**, 322), Hoyt & Trieschmann analysed the returns on quoted U.S. life and health insurers for the period 1973–87 using CAPM, and estimated beta to be 0·52. The U.S. quoted life and health insurers are not particularly representative, even of that market, since most of the large U.S. life insurers are either mutual, or part of a multi-line group, or a conglomerate. Nevertheless, the different results for beta seem interesting. I suggest two reasons:

(1) U.S. insurers invest their assets largely in fixed-interest rather than equities, and so are less correlated to the equity index.
(2) Goodwill for U.S. insurers is commonly a much smaller proportion of the total value than for U.K. insurers. The same article show betas for property and casualty (P & C) and multi-line insurers of 0·95 and 1·08, respectively.

Mr D. J. Keeler: As appraisal value techniques have become accepted within the financial community, it is the relationships between the risk discount rate, traditionally applied to net cash flows, and the assumed long-term investment returns on assets which generate the most discussion. Whilst this is commonly the province of actuaries, financial advisers such as merchant banks are normally consulted before an appropriate risk discount rate is established. In a number of cases experts in modern financial theory have been consulted, but there have been problems in applying the advice received—particularly owing to the impact of life office taxation. It is against this background that the author has laid out the groundwork, which may be developed in due course into the bridge between modern financial theory and actuarial work.

The author derives the value of individual cash flow components, and then calculates an equivalent discount rate which could have been applied to net cash flows. The examples given in §§ 5.7.3 and 6.10.1 illustrate equivalent discount rates broadly consistent with those used in the traditional approach. However, in the with-profits business example, changes in asset mix and in the size of the estate have a significant effect on the discount rate. Similarly, in the unit-linked example, changes in product design or in the product itself (such as to a single premium bond) can have a material effect on the discount rate. This is also consistent with the traditional approach, which would not necessarily apply the same discount rate to all companies—there is, after all, no substitute for judgement. The advantage of the approach suggested by the author is in attempting to quantify an appropriate difference in discount rates for two companies.

I would like to see further work on the valuation of free capital. I agree with the observation in § 6.1.2 that the value of free capital varies according to the uses to which it is put. However, I believe that strict application of the financial theory contained in the paper suggests that free capital should be taken at face value. In the light of statements made in § 6.9.3 regarding the inefficient use of capital, and the illustration in § 6.10.5 which suggested that this part of the estate earned 17¼% p.a., I find it difficult to accept that free capital held within the estate of a with-profits company should be taken at face value. My reservations come from a belief that the financial theory applied to this free capital requires a more perfect knowledge of the market than exists, or a greater freedom of action than regulation permits. Consideration of the opportunity cost of alternative uses, as suggested in § 6.9.2, may be appropriate according to the purpose of the valuation, but this would be difficult if certification were required that the valuation had been carried out in a manner consistent with the company's current management record.

Mr M. J. Hesketh (a visitor): I am here to represent the consumers of embedded value and appraisal value calculations. These are the shareholders and the institutional fund managers who invest in proprietary life companies.

The stock market has been, for a long time, very cynical about embedded values. The number that

they have always had difficulty in understanding is this mysterious discount rate. This paper provides a rigorous framework on which a judgement can be made as to the applicability of the numbers that are given to the market. We have not yet reached a consensus as to whether this methodology leads to exactly what we are looking for, but I feel confident that, in the fullness of time, there will be an agreed basis on which figures can be rigorously calculated. What will then be needed is a simplified version of what has been agreed, in order that it can be understood by a wider audience.

Stock market people find it difficult to understand the difference between appraisal value and embedded value, which in the paper is called 'existing structure value', and which in commercial and industrial companies would be called goodwill. In simplified terms, what is required to understand goodwill is to see how many years of future profit a purchaser is being required to pay now. Conventional wisdom says that, for life companies, distribution is the key. The most desirable form of distribution in current market perception is 'bancassurance', which all companies would like to have, but very few will achieve, because there are not enough banks. There is also greater understanding of the strength and the desirability of home service distribution. In the undesirable category are those distributors who tackle cold customers. How did that work in practice? In the mid 1980s huge premiums over embedded values were paid for almost any kind of distribution. That was because people did not fully understand how the market would develop in the post-Financial Services Act era.

Today, premiums of any kind cannot be guaranteed. It is an unpleasant fact that many life companies are trading at possibly very significant discounts, even to embedded value, so the market is not particularly interested in attributing any goodwill at all. If one result of this debate is that the life industry finds a way to bring the stock market to an understanding of the mysteries of life assurance, then it would have been a giant step forward.

Dr C. D. Pickup: The approach in the paper of simplifying a highly complex business into the most basic elements of its cash flows, and examining the valuation of each of these cash flows in turn on the basis of its own risk characteristics, gives a good understanding of the real risks underlying the business, and attempts to quantify the extent of such risks and their impact on value. In particular, the techniques highlight the extent to which the net profits are 'geared for any particular type of product or company, and this enables a more rational decision to be made as to the overall risk discount rate to apply to the net cash flows.

The methodology in the paper has not been widely used for valuing life companies, and I suspect that there will be considerable debate and additional research before these techniques become widely accepted and commonly used. We need to be convinced of the validity of the theory in general, and also of its applicability at the micro level. We must also be able to apply the methodology correctly in a practical situation.

CAPM has been with us since the mid-1960s, and, notwithstanding the 'mixed empirical evidence' referred to by the author, it is fairly widely accepted. It is certainly consistent with our intuitive notion of the trade-off between risk and return. However, the theory of CAPM was developed in the context of the performance of traded securities relative to a market index. An intellectual leap of faith must be made to believe that the theory can be applied to the component cash flows which are not tradeable securities, and cannot, in most cases, be disentangled from each other. An example is the application of CAPM in § 5.5.4 to estimate the excess return required in respect of lapse risk. If there is little correlation between the market index and lapse rates, as appears to be the case, then we are asked to discount future premiums at close to a risk-free rate. Given the choice of investing in a gilt or in a stream of future life premiums on the same terms, I know which I would prefer. The theory assumes that all other (unsystematic) risks are diversifiable and should not be priced, but it is hard to believe that this applies in practice.

The complexity of the approach is another factor which may limit its use. There are likely to be many complex interactions in a company, for example between lapses and other cash flows such as expenses, and those interactions may be very difficult to allow for. The author has used simple examples to illustrate the methodology, and, whilst these are extremely helpful in understanding the approach, some of the more complex areas are only touched upon. A typical office provides many types of policyholder options, and the author acknowledges that these are often too complex to model and solve accurately.

As with all methods of valuing life companies, a considerable degree of judgement is required in estimating certain parameters. It is interesting to note the sensitivity of the results to the assumptions used. For example, in the valuation of goodwill in Section 7.3, a small change to the allowance for lapses changes the results significantly. If the discount rate to be applied to gross profit is increased from 11% to 12% p.a. then the corresponding multiple for a net investor decreases from 16·7 to 14·3. The total goodwill then decreases from £137m to --£100m. That said, the assessment of existing structure value is one of the most subjective aspects in the appraisal of a life office, and perhaps this paper helps to explain it.

It is also clear that different products can have very different risk profiles, so that as well as using a much wider range of discount rates to value different products in different companies, it is highly dangerous to generalise the results obtained from any one example. The example of a regular premium unit-linked contract shown in § 5.7.3 implies a discount rate of around 11% p.a. for net cash flows. If we consider instead a portfolio of single premium bonds, then applying exactly the same methodology and similar underlying assumptions results in a discount rate for the net cash flows of close to 16% p.a.

However, all this runs the risk of sounding unduly negative. Much of what interests me in my daily work is the confrontation of my intuition with the results of careful analysis. The resolution of that confrontation frequently leads to a fuller understanding.

Professor A. D. Wilkie: I was pleased to hear that this paper was going to contain a discussion of the application of the CAPM and other concepts from financial economics to actuarial valuations. This subject requires discussion, but I am a little disappointed in the result. The author has given us a wealth of detail, but I do not think he has satisfactorily discussed the fundamental principles.

The fundamental point is about adjusting for risk. If the amount of cash flow at the end of one period is a random variable, then one way of calculating the present market value of that random variable is to discount the mean at a special discount rate, which we call a risk-adjusted rate. The theory of the CAPM suggests that the adjustment should be proportional to the beta coefficient between the random variable being discounted and the random variable representing return on all risky assets in the economy.

The author quotes Sherris (*J.I.A.* **114,** 581), who suggests that the concept of a risk adjusted discount rate over one period can be applied over several periods. Sherris is using the results of Fama, whose paper (*Journal of Financial Economics*, **5,** 3) the author also refers to. Fama, however, seems to assume that risks are independent over successive periods. He also uses net cash flows. However, I do not think that Fama's model is satisfactory for dealing with another type of risk that I will call timing risk. For a simple example of timing risk, assume that at the end of year 1 a claim is payable which will either be £0 or £1 million, with equal probabilities. At the end of year 2 another claim is payable, again of amount £0 or £1 million, with equal probabilities. The total of claims may be £0, £1 million or £2 million, with appropriate probabilities. Contrast this with the situation where £1 million is certainly payable, but at the end of year 1 or year 2, with equal probabilities. The total claim is £1 million with certainty, but there is uncertainty about the date of payment. Looking at the years individually, the situation seems to be the same as in the first case, but this ignores the dependence between years. The second case is much less risky than the first.

It is easy to elaborate the example to bring in correlations with the market, either with different beta coefficients in different years or with correlation between market returns in successive years. Fama's model could easily bring in varying beta coefficients to allow for timing risk, but this elaborates the model enormously.

The assumptions on which the basic CAPM is based are many, and are wholly unrealistic. Papers have been written discussing how the strict conditions of the basic CAPM can be relaxed while still preserving its general result, but the author has not really discussed these.

The assumptions made in deriving the basic CAPM include the following:

(1) All investors and all assets are enclosed within a single market; there is no external market.
(2) There are no taxes or transaction costs; or, possibly, all investors are subject to the same taxes. The author's discussion of values to a gross and to a net investor is counter to this assumption.

(3) All investors have the same sort of liabilities, measured in a single currency. In some papers they are real assets, but the concept of different intermediary investors with different portfolios of liabilities is not part of the basic CAPM. It is, therefore, difficult to see how it should be applied when the main investors in the stock market are intermediaries, and the primary investments, insurance policies and pensions rights, are not themselves traded.

(4) All investors have the same views about the joint distribution of returns on assets: for example, that they use the same means, variances, co-variances and beta factors. Investors only differ in having different utility functions, though all are considered to be risk-averse. They, therefore, end up with the same portfolio of risky assets. This contradicts the possibility that one investor might assess a company differently from another or consider that by taking control of that company he might be able to change its fortunes. The basic CAPM does not say much about investors with different opinions, nor about purchase for control.

(5) Investors invest over one single time period. It is not difficult to extend it, as Fama has done, to sucessive time periods, provided that returns in successive periods are all independent. Interest rates and the premium for risk in successive periods may differ, but in a pre-determined rather than a random way. We may need one yield curve for interest rates, another for the risk premium, another for the standard deviation of risky assets, and further ones for all the beta coefficients which can vary with time.

(6) All assets and liabilities are traded, and the CAPM is a description of how equilibrium market values are determined. A high interest rate for discounting a relatively risky asset means the same high rate for discounting an equally risky liability. This is counter-intuitive to actuaries, who would want to reduce the discount rate for risky liabilities. I am not entirely clear which way the author expects discount rates for liabilities to be adjusted. This seems an area worth further research.

I have referred to the basic CAPM. The conditions for it that I have mentioned are sufficient, but may not be necessary. It may be possible to relax many of them and still have a usable model. The author has not done this, nor has he quoted others who have. Much more research needs to be done.

Mr S. P. Taylor-Gooby: As with any new venture into uncharted territory, this paper will, no doubt, meet with resistance. There are those who misunderstand the theory, and those who see it as an attack on their own science, which they prefer to believe is foolproof. It would be easy in a work of this size and scope to pick out bits and to attack them, but that misses the point. The paper draws our attention to many useful insights where established actuarial valuation theory falls down; for example, in ignoring systematic investment risk. To take an example of a company with a portfolio of guaranteed bonds investing in gilts; traditional actuarial theory would increase the value of that company if it sold all the gilts and invested the proceeds in equities. That is an absurd result.

In identifying systematic option risks, we have seen a number of U.S. companies offering guaranteed liabilities, investing in high risk securities, and keeping the extra profits. Traditional valuations of these companies have often resulted in high values, where the companies are, in fact, worthless. There is also the effect of gearing on risk. I have seen many valuations of companies which have high levels of debt, either in a holding company or through reassurance, which are not reflected in the valuation. Another insight is in the way taxation affects investment risk. By adopting an appropriate level of gearing, a net life fund investing in equities can replicate the returns offered by a gross pension fund.

On the other hand, the theory is not yet sufficiently complete that we should throw away traditional methods. Instead, we should learn from all the tools at our disposal, and apply them intelligently where appropriate. There are many areas where further research is necessary, particularly on the effect of transaction costs, illiquidity premiums and taxation differences, each of which is very small in itself, but which can combine to have a large effect on the difference between two large numbers.

The worst mistake any of us can make, however, is to believe that we have a theory that provides all the answers and to apply it unthinkingly. Occasionally, for example, we see appraisals where discounted future income techniques imply infinite asset values, or enormous goodwill valuations that do not make sense. Even the most carefully researched and intellectually sound valuation theory

will produce results which are rubbish if the basic actuarial assumptions used are not sound. In the end this can only discredit the profession and all of our techniques.

Mr M. McIvor (closing the discussion): The financial world is developing very quickly, and whereas, some years ago, the actuarial profession led in that regard, there are now many others who can offer tools and techniques which are of value to us. It is our duty, both to ourselves and to our profession, to make sure that we use the best techniques available.

This paper encourages us to do just that. However, we could go further, as there is still a hint in it that life companies are special, through the relationships within them between activities this year and those that will occur in later years. I would argue that most companies in industry have relationships which go on between this year's activities and those for many years to come. We should make every endeavour to understand more of the relationships and the risk management techniques which are employed by auto companies, drug companies and even construction companies, as a means of enabling us to manage ours even better.

Considering valuation methodology, there are three areas on which I will comment: the cash flow components that have been addressed in the paper; CAPM; and the application of CAPM. I like the approach of breaking down a life company into cash flow components. The way in which it is done in the paper is clearly recognised as valuable, and several speakers have offered ways in which it can be developed further. Most importantly it differentiates the components by size, duration, certainty and quality.

The paper points to the need to look at options and to evaluate them. These options may vary substantially. There are options for management, for shareholders and for policyholders. We have to be clear which ones we are specifically addressing. To value the cash flow components using option pricing theory, we have to take account of a point raised by several speakers: that these components are not part of a well defined or traded market. Also, the size and frequency of transactions are not that apparent. Both of these elements have an impact on the discount rate which should be applied. There are also other factors which need evaluation when setting the discount rate. When we look at the various components and their valuation, we should not simply add them to establish a value for the company. We have seen the stock markets use this process over several decades, and each time the market concludes that it does not work.

Professor Wilkie has provided some comments on CAPM, as have others. CAPM is built on strict conditions. They are important, and there is a great debate as to the strength of the theory in practice. However, it provides a framework, and as has been pointed out, improved tools may lead to a better understanding of the valuation process. I believe that CAPM provides an effective structure for development, but whether we should make it the sole arbiter is a different question.

Within asset pricing models a key component in the valuation is volatility risk. Actuaries have had the difficulty of deciding the appropriate interest rate for many years. Switching our allegiance to volatility would not make the world any easier to manage. In addition, when reference is made to testing these models in practice, that testing has been in liquid markets, and several speakers have brought to our attention the fact that these components are not present in liquid markets. We cannot trade the elements.

At several points in the paper the author refers to small risks, or risks that can be ignored. The professionals in the options and future markets use these models day in and day out, and are very respectful of small risks, which can multiply very rapidly. Additionally, market prices are only meaningful while markets remain liquid. In trying to use option pricing theory on these cash flows within life companies, we are not able to manage the risks as market participants may.

Mr Pike and other speakers drew attention to the question of whether these valuation techniques are being used for shareholders or for corporate acquisitions. There are two distinct groups of questions here:

(1) A single shareholder is buying in on the basis that he can reassess his position, and he does not have to project forward all the elements in his valuation for ever.
(2) A corporate acquisition is different. If the work of this paper can be implemented satisfactorily, then certainly the approach used will give further insight. If the potential acquirer understands

the elements of the overall price, he has a better opportunity to see whether the profile of values meshes with his own initial position and his objectives. Mr Sanders developed these issues further.

Turning to investment returns and discount rates, it is noteworthy how important tax is and how sensitive the relationships in this part of the paper are. Great care is required, and Mr Hesketh has already referred to how quickly value can change in a market. Another risk is that our predictions of returns are not regularly questioned. The whole balance of returns within the economy goes through significant changes, and we need to be aware of this. It is simply not good enough to look over a 70- or 25-year period, or any other period, and to come to a single conclusion. We must recognise what the economic developments during those periods were, and make reasonable judgements.

In § 5.2.8, it is claimed that with a typical unit-linked company there is little risk of insolvency. This may be a bold statement in these changing times. Indeed, low risk is referred to in many parts of the paper. If there is low risk, we may redefine or improve our expected distribution of outcomes, and we have further opportunity to refine our option valuation process, but ultimately we shall still be relying on the integrity of our model.

In the paper it is suggested that the different unit-linked portfolios should be valued at different rates of interest. Whilst the theory is not unreasonable, the distribution of units is strongly influenced by the policyholder. Here is yet another option which needs to be addressed.

Several speakers have commented on the importance of the assumptions with respect to existing structure multiples, and the importance of small changes in those assumptions. We need to know the full range of multiples which could be applied. How strongly are we committed to each of the underlying assumptions? How great a financial commitment should we make to the results? We can then ask: do we wish to accept the risks involved, or do we wish to mitigate them?

Turning to the whole company valuation, Mr Hesketh suggested that we may be moving forward to a rigorous framework. Whilst such a prospect is welcome, my experience of stock markets suggests that market valuations are difficult to use to validate a particular theory, because they can vary dramatically. Furthermore, I noted the phrase, 'ignoring some element of risk'. We ignore any element of risk at our peril.

In § 8.4.3, the difficulties of measuring beta are acknowledged, and this has led the author to propose a whole variety of techniques so that we may take the next step.

In § 9.3, the question of tax surfaces. This is a difficult issue for life offices and investment practitioners. At the short-term end of the market, operators are taxed on profits, and it is sensible to start from a gross basis. The next step is less obvious. Certainty of your own tax basis and objectives is vital. If, for market purposes, you need to use another basis, you know that your level of risk is significant, and you should manage accordingly.

In § 9.5, the author suggests that there may be some difficulty, as an investor may require more than a market return from any asset. I do not see this as a problem. Indeed, an investor is likely to be making an investment for that very reason. Surely what the pricing models are saying is that, although the investments are made expecting a higher return, the outcome must be levelled out. The 'requirement' in this statement is key, but it is only key in the process of asset selection in terms of outcome.

The President (Mr H. H. Scurfield): I begin by mentioning some practical points. I am pleased that the question of equity returns has been raised, because the paper suggests that they might be of the order of 14%–15% in the future. We are going through a time of change, and we should not lead ourselves into a false sense of security by expecting that size of return. I am also pleased to see, in § 6.3.2.3, a reminder that we can pay out an excess of asset shares for some policies some of the time, but not for all policies all the time, and, in § 6.8.5, that the risk of failure is undesirable. However, the author only talks about the risk of failure from the shareholder's point of view, and I wondered whether there were not some other people to be concerned as well, such as policyholders.

In § 6.7.2 there is reference to the subsidising of future new business. I do not like that expression. I think that the author is referring to new business strain. Other Europeans are rightly worried when there is talk about subsidising new business, thinking that we intend to flood the market in a particular country.

The issues behind this paper have been at the core of actuarial thinking throughout our 144 years of existence, and the subject matter is of considerable practical significance at this time. Many actuaries involved in the practicalities of bonus policy and business risk within life assurance will not have considered using asset pricing models. The value of this paper lies in doing just that. I am sorry not to have heard from more practitioners. For many this paper may have seemed removed from the day-to-day practicalities, but we are a learned profession, and it is entirely right and proper that we should be attempting to cross new boundaries, and in particular, to bring techniques from one part of our work to bear upon another. It is all the more relevant to be doing so at this time, when with-profits life assurance is going through such a time of change and re-examination, and there seems to be an economic discontinuity. Gone are the high and relatively stable returns of the 1980s. We have moved into times of lower returns, and perhaps, even more importantly, higher risk.

We are grateful to the author, Shyam Mehta, for his paper. The discussion serves to demonstrate the interest there has been, and we thank him very much.

Mr S. J. B. Mehta (replying): The philosophy presented in the paper is that, just as different rates of discount are implied in the market valuation of gilts and equities, separate rates should be used to value the various types of payments made by an office. Use of a bond yield is suggested for the contractual obligations of an office; straightforward techniques are presented for determining the discount rates needed to value other cash flows.

WRITTEN CONTRIBUTION

The author subsequently wrote: First, I should like to clear up a misunderstanding. Contrary to the impression given by a number of speakers, the paper does not rely primarily on CAPM. In essence it is suggested that asset valuation should be based on market prices and that liabilities should be valued using appropriately selected risk rates, rather than a discounted cash flow approach with a single risk discount rate.

Guaranteed payments to policyholders result in shareholder risk in very much the same way as borrowing does. Corporate investment in equities likewise results in shareholder risk and can be valued using an equity risk rate of return. Pricing models assist where more complex risks are borne. They typically compare the degree of risk borne with equity or other risks of traded or other assets, and in the paper I suggest that CAPM is just one of a number of models which could be applied. Perhaps these observations, together with those in Section 2, address the concerns of Dr Pickup and Mr Pike, who question whether asset pricing models can be applied to value individual cash flows. I believe it to be unproductive to look at the minutiae of the conditions under which these models apply and that it is better to take instead a pragmatic view. There are many papers which discuss the detailed assumptions on which asset pricing models are based, and I do not agree with Professor Wilkie that one more would have been particularly useful. The individual risks and cash flows need to be examined, and a judgement or analysis made as to whether values are likely to vary by 20% in any year—equity level risk—or whether a much more modest level of variation is to be expected. For example, use of CAPM may be a reasonable solution, even if it is believed that CAPM is not an exact model. At the very least it provides an order of magnitude estimate of the cost of risk borne, for instance in relation to lapses where hitherto no estimate has been available. It would be useful to hear from those speakers who questioned the use of CAPM what alternatives they would suggest. The actuary is, of course, free to use another model, but should recognise that in any valuation some implicit or explicit assumption as to the price of risk is required.

Mr Smith suggests that there is a discrepancy between the Black and Scholes option pricing model and CAPM, and that the latter is invalid as it does not allow for the possibility of continuous trading. There are, in fact, several versions of CAPM, including ones which assume continuous trading. In the example given, the skew distribution of returns for the unit-linked product over discrete time is equivalent to an instantaneously symmetric distribution which varies over time. Application of CAPM would require discounting stochastic payments under the policy, allowing for the maturity guarantee, and using a time varying discount rate. The two models would then give the same result.

Several speakers have questioned my assumption of a 6% p.a. return on equities in excess of Treasury bill and index-linked gilt returns. Use of a lower return assumption is not always cautious, and 'caution' may be different for a purchaser and for a seller. It should also be noted that the methodology suggested in the paper does not depend on any particular level of assumption. Appendix B sets out some of the logic and analyses which can be used to assess returns. For example, the real dividend growth rate assumption needs to be viewed in the context of the effects of gearing outlined in § B.5.2.2.

Mr Kemp appears to agree with me that cash has performed poorly over a part of the period 1919–90. He postulates that there is a 'true' risk-free asset which may have performed better, in support of his view that the excess return on equities is less than 6% p.a. It would be interesting to know what this alternative asset is.

One aspect of the analysis needs further discussion. The geometric average of historic excess returns on equities is 6·3% p.a., as noted in § B.5.4; this is an estimator of the mode of the underlying distribution. Equity returns are often represented by a lognormal distribution, a distribution which is characterised by a significant degree of positive skewness. For the levels of variability experienced in the past, the mean expected return exceeds the mode by approximately 2% p.a. On further reflection, I would withdraw the statement in § B.5.4.1 that the geometric mean provides the better measure of the equity risk premium, and would suggest that the analysis should also have regard to the arithmetic mean, approximately 8% p.a. I am not suggesting that this is the only factor to be taken into account, and I hope that the Appendix will prove useful in assisting actuaries to come to their own views on an appropriate assumption.

The opener argues that unsystematic risks should not be ignored. I fully agree. In his example, a stochastic projection of results for the office which guarantees premium rates would result in a lower valuation than for the office which exercises its rights to increase rates if mortality experience worsens. I have a few further thoughts about the appropriate discount rate required to value unsystematic risks. Although a risk-free rate is used for the original form of the CAPM, a number of empirical studies of the zero beta version of the CAPM have found that in the U.S.A. unsystematic risks are priced to return approximately 1% p.a. higher than the risk-free rate. The insights and arguments of modern portfolio theory do apply, but perhaps not exactly. Mr Sanders referred to the example of a small company with only one shareholder. For such a company, a stochastic projection is likely to show that unsystematic risks do, indeed, have a very significant effect on value. I maintain that it is still appropriate to use a lower discount rate to value unsystematic than systematic risks. The sole shareholder is faced with a risk/return opportunity set in the traded securities markets, and these markets provide a benchmark against which the comparative value of his company can be judged. An investor who is particularly averse to illiquidity or to unsystematic rather than systematic risk would presumably take steps to reduce these, perhaps by diversifying. In § 9.5 I argue that investors may require more, and sellers less, than a market return on the transaction price. The requirements of both sides need to be borne in mind in assessing a value.

I now comment on two other points made by Mr Sanders. The analogy between guaranteed liabilities and corporate debt appears to me even closer if the debt is in zero coupon form. The equation of value does not imply that a portfolio is worth more to a capital rich acquirer. The discount rate used to value the acquisition including the newly injected capital reduces if this capital has a lower level of risk than the acquisition; the discount rate required to value the acquisition itself is unchanged. I agree with Mr Franklin and Mr Taylor-Gooby that many useful insights can be gained from the equation of value.

A number of speakers commented on the valuation of existing structure. It was noted that there is a wide range of possible existing structure multiples, depending, for example, on the method of sales distribution. Mr Hesketh highlighted how risks and perceptions of risk have changed radically over time. The methodology developed in Section 7 illustrates the sensitivity of multiples to a variety of factors. For example, the economics of a bank-based distribution system differs from that of a direct sales force system, and this results in a corresponding difference in risk characteristics. I hope the techniques suggested will prove valuable and will provide insight into the effect of risk on multiples required for companies using different distribution channels and which have differing circumstances. I would not wish to suggest that the exercise of judgement is to be superseded.

Mr Keeler refers to the question of whether free capital is worth its face value. I suggest that the value depends on the use to which this capital is put. At one extreme, an office with a large estate may derive little or no benefit from free capital if it is not likely to be distributed or used. For another office, existing structure value may depend crucially on the availability of free capital. One possible approach would be to value free capital as the difference between appraisal value with and without this capital. Another approach would be to value it at its face value and to assign any difference to existing structure or in-force value. I agree with Mr Keeler that inefficiencies in the use of capital should be reflected in its valuation.

I take the closer's point that 'small' risks may turn out to be more important than one might imagine. I hope that a careful analysis of the various risk factors will at least draw attention to possible areas of difficulty.

I thank the President for his kind observations and also for expressing his concern that a balance of realism needs to be struck when considering risk and return. I submit that use of a market value based approach to valuing both assets and liabilities could reduce the difficulty of communicating value calculations to non-actuaries, although the evaluation of existing structure will, of course, retain a significant subjective element. In response to the point raised by Mr Pike, the resulting value for a shareholder is an estimate of the value of future returns discounted at risk rate(s) based on the risk/return tradeoff implicit in the market prices of traded securities. Overall value to any particular purchaser or seller, or value assessed for a particular purpose, may need to be adjusted to reflect individual risk/return preferences and elements of value contributed or given up.