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Analysis of Bonus Systems in Motor Insurance

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TABLE OF CONTENTS

	page
Introduction	1
1. Description of the bonus system to be analyzed	1
2. Definition of states and transition rules of a bonus system	3
3. Estimation of parameters in the model	6
4. Statistical properties of the estimators	8
5. The model as an aid in tariffing	9
6. Case study	10
Conclusion	13
Table 1-3 : B-values	
Appendix 1 : Basic data	
Appendix 2 : Parameter estimates and smoothed probabilities	
Appendix 3 : Portfolio movements (case 1)	

Introduction

A continuous time Markov Chain model is proposed for the analysis of bonus systems in motor insurance which as a particular merit takes explicitly into account the effect of cancellation of policies. Most analysis of bonus systems treat these as "closed" systems while in reality they are very "open" systems with a considerable change of the portfolio within one year. Relating to Danish experiences there is a yearly number of cancellations (excluding lapses i.e. failure to renew) corresponding to 15-20 % of the portfolios and a level of new business of approximately the same size making the real changes of the portfolios rather small.

The model is being used for tariffing purposes in deriving the overall level of the loss ratio - or rather the denominator part of it - by estimating expected total premium income for the whole portfolio or part of it given the values of certain external parameters. These are the number of new policies issued and trends in the claims frequency and the rate of cancellation. Special attention will be given to the analysis of data from a portfolio comprising about 150.000 policies which is one third of the whole motor portfolio of the Baltica Insurance Company.

In this analysis stress will be put on the macro-aspect of the bonus system i.e. on the behaviour of the portfolio as a whole and not on the behaviour of the individual policy. As will be evident from the theoretical set-up this macro-view of the bonus system is also the core of the model.

1. Description of the bonus system to be analyzed.

The bonus system for which a case study has been done is the one presently used for the motor portfolio in the Baltica Insurance Company for third party and own damage coverage. The bonus system consists of 11 classes, 0,1,...,10. New policies are normally placed in class 4 but can - depending on their past claims experience - be placed in any of the ten other classes. The transition rules are:

- in case of a claimfree insurance period the policy is transferred one step forward in the bonus system ($i \rightarrow i+1$) with the exception of class 4 (as far as new business is concerned) where the policy in this case remains for two years

- for each claim in the insurance period the policy is transferred two steps backward in the bonus system ($i \rightarrow i-2$, $i \rightarrow i-4$ etc.).

Below are shown in Diagram 1 and 2 the classes of the bonus system and the transition rules for the case of claimfree experience and the case of exactly one claim in the insurance period, respectively. For reasons that will become apparent when we look at the actual data we divide the classes into two subsystems: one for new business and one for the portfolio (excluding new business). Furthermore, we introduce an interim diary class 4.1 corresponding to the above transition rule for class 4. Thereby we operate with 23 classes which will be referred to as

$$\{ C_i, i=1, \dots, I \}$$

with $I=23$ for the Baltica case.

1. Claimfree in the insurance period

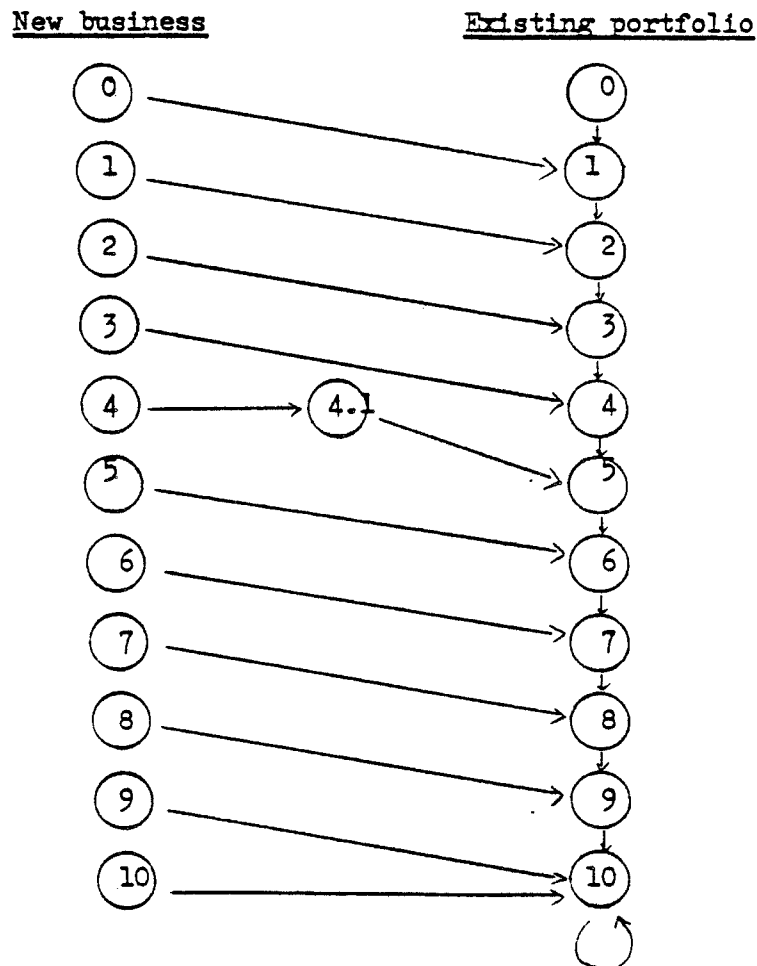


Diagram 1

2. Exactly one claim in the insurance period

New business

Existing portfolio

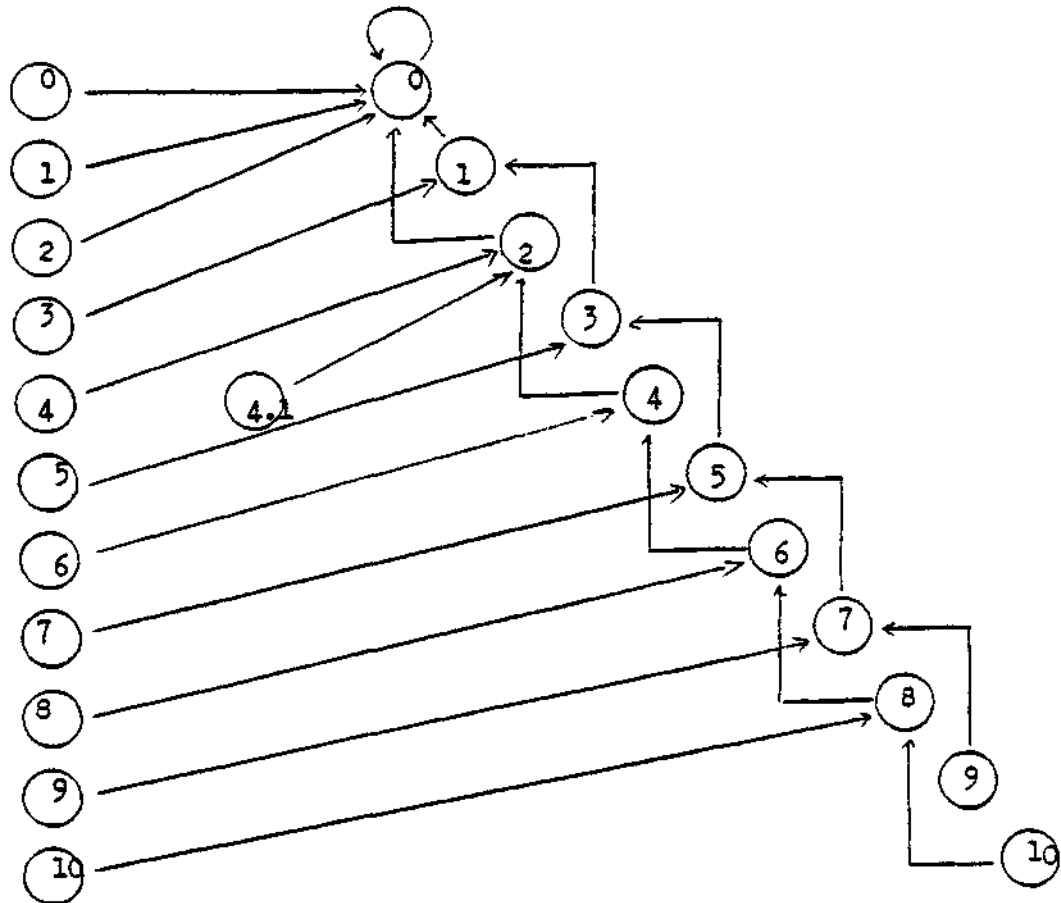


Diagram 2

2. Definition of states and transitions rules of a bonus system.

The movement of a policy in a bonus system can be described by means of

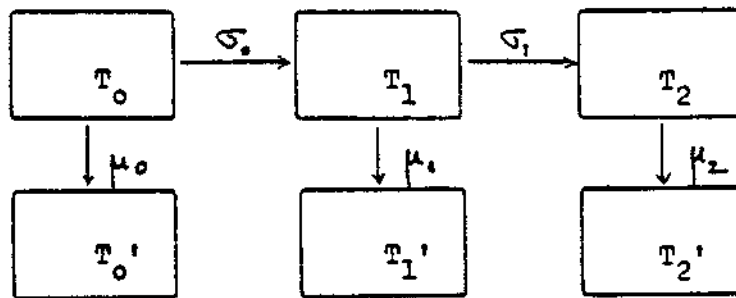
- a) the events taking place during the insurance period
(= period between renewals), and
- b) the transition rules applied at each time of renewal.

a. Events taking place during the insurance period.

At the beginning of an insurance period a given policy is by definition claimfree in the bonus class to which it was transferred at the time of renewal. By the end of the insurance period this policy can be in one of the following six states:

- T_0 : claimfree
 T_0' : cancelled as claimfree
 T_1 : has incurred exactly one claim
 T_1' : cancelled after having incurred exactly one claim
 T_2 : has incurred two or more claims
 T_2' : cancelled after having incurred two or more claims.

The possible direct transitions are given in the following diagram:



The model will be described as a time-homogeneous Markov Chain.

The transition intensities are:

1. claim-rate for policies with exactly k claims: σ_k ($k=0,1$)
2. cancellation rate for policies with exactly k claims: μ_k ($k=0,1$)
3. cancellation rate for policies with more than one claim: μ_2

The intensities depend on the bonus class considered and may e.g. vary from calendar year to calendar year.

Let $\alpha_k = \sigma_k + \mu_k$ ($k=0,1$) and $\alpha_2 = \mu_2$. Then the transition probabilities are given by

$$p_{00}(z) = \exp[-\alpha_0 z]$$

$$p_{00'}(z) = \frac{\mu_0}{\alpha_0} (1 - \exp[-\alpha_0 z])$$

$$p_{0k}(z) = \int_0^z p_{0,k-1}(u) \sigma_{k-1} \exp[-\alpha_k(z-u)] du \quad (k=1,2)$$

$$p_{0k'}(z) = \int_0^z p_{0,k-1}(u) \mu_k du \quad (k'=1,2')$$

or - if we assume that $\mu_2 = \mu_1$ -

$$\phi_{01}(z) = \frac{\sigma_0}{\alpha_1 - \alpha_0} (\exp[-\alpha_0 z] - \exp[-\alpha_1 z])$$

$$\phi_{02}(z) = \frac{\sigma_0 \sigma_1}{\alpha_1 - \alpha_0} \left(\frac{\exp[-\alpha_0 z] - \exp[-\alpha_1 z]}{\alpha_1 - \alpha_0} - z \exp[-\alpha_1 z] \right)$$

$$\phi_{01'}(z) = \frac{\sigma_0 \mu_1}{\alpha_0 - \alpha_1} \left(1 - \frac{1}{\alpha_1} \exp[-\alpha_1 z] + \frac{1}{\alpha_0} \exp[-\alpha_0 z] \right)$$

$$\begin{aligned} \phi_{02'}(z) = & \frac{\sigma_0 \sigma_1 \mu_1}{\alpha_1 - \alpha_0} \left(\frac{1}{\alpha_0(\alpha_1 - \alpha_0)} (1 - \exp[-\alpha_0 z]) - \right. \\ & \left. - (1 - \exp[-\alpha_1 z]) \left(\frac{1}{\alpha_1(\alpha_1 - \alpha_0)} + \frac{1}{\alpha_1} + \frac{1}{\alpha_1^2} \right) \right) \end{aligned}$$

If one gets slightly different expressions.

Let $A = \{1, 2\}$ and $B = \{1', 2'\}$. Then

$$\phi_{0A}(z) = \sigma_0 \frac{\exp[-\mu_1 z] - \exp[-\alpha_0 z]}{\alpha_0 - \mu_1}$$

$$\phi_{0B}(z) = \frac{\sigma_0 \mu_1}{\alpha_0 - \mu_1} \left(\frac{1 - \exp[-\mu_1 z]}{\mu_1} - \frac{1 - \exp[-\alpha_0 z]}{\alpha_0} \right)$$

b. Transition rules

We will assume that the bonus system has been so designed that for each bonusclass is given a set of transition rules that only depend on the bonusclass considered and the state T_k ($k=0,1,2$), where the policy is situated at the end of the insurance period. Let

$$X_k(C_i) \quad , \quad k=0,1,2$$

denote the transition rules corresponding to T_k ($k=0,1,2$), i.e. a policy which at the end of the insurance period in bonus class C_i is situated in state T_k is transferred to bonusclass $X_k(C_i)$ and placed in state T_0 in this bonusclass. Typically one has

$$X_0(C_i) = C_{i+1}$$

and

$$X_k(C_i) = C_{i-ak} \quad \text{for } k=1,2 \text{ and some } a.$$

c. Definition of underlying stochastic process.

The state space of the process described in the preceding paragraphs is

$$\left\{ \pi_k(C_i), k=0,1,2,0',1',2', i=1,\dots,I \right\}$$

Let $X(t)$ denote the sample path of the process. We will assume that the process is a time-continuous Markov Chain with transition probabilities as given on p. 5.

3. Estimation of parameters in the model.

a. Derivation of the likelihood function.

I. Continuous observation over a fixed period (complete sampling scheme).

Suppose we observe $\{X(t)\}$ continuously over say a calendar year period (ref. Diagram 3 below). Consider a given bonus class and let n denote policy No. n . Define

$$\begin{aligned} N_n^k &= \text{transition frequency of } k \rightarrow k+1 \text{ during} \\ &\quad \text{the period} \quad (k=0,1) \\ M_n^k &= \text{transition frequency of } k \rightarrow k' \text{ during} \\ &\quad \text{the period} \quad (k=0,1,2) \\ v_n^k &= \text{the observed time for which policy No. } n \\ &\quad \text{has occupied state } k \quad (k=0,1,2). \end{aligned}$$

Then the likelihood function is

$$L = \prod_{k=0}^2 \exp(-\mu_k v_n^k) \prod_{k=0}^2 \mu_k^{M_n^k} \prod_{k=0}^2 \frac{1}{v_n^k} N_n^k$$

with $v^k = \sum_n v_n^k$, $M^k = \sum_n M_n^k$, and $N^k = \sum_n N_n^k$.

Maximum likelihood estimators are the usual occurrence/exposure rates

$$\begin{aligned} \hat{G}_k &= \frac{N^k}{v^k} & k=0,1 \\ \hat{\mu}_k &= \frac{M^k}{v^k} & k=0,1,2 \end{aligned}$$

II. Observation at fixed points (incomplete sampling scheme).

Suppose we observe $\{X(t)\}$ for policy No. n only at each renewal date, i.e. the end of each insurance period. Consider a given bonus class and define

$$M_n^k = \begin{cases} 1 & \text{if policy No. } n \text{ is in state } k \text{ at the end} \\ & \text{of the insurance period} \\ 0 & \text{otherwise} \end{cases}$$

($k=0,1,2,0',1',2'$). Then the likelihood function is

$$L = \prod_n \prod_k p_{ok}^{M_n^k}$$

with $p_{ok} = p_{ok}(1)$ assuming the length of the insurance period is 1. The maximum likelihood estimators of the transition probabilities p_{ok} are given by

$$\hat{p}_{ok} = M^k / M$$

with $M^k = \sum_n M_n^k$ and $M = \sum_k M^k = N$, the number of observed policies.

From this one gets the m.l.e. of the parameters and by solving the equations

$$\exp[-\alpha_0] = M^0 / M$$

$$\sum_{k=0}^2 \alpha_k (1 - \exp[-\alpha_k]) = M^{0'} / M$$

etc.etc. Most of the equations will have to be solved by some iteration procedure. By using the approximation $1 - \exp(-x) \approx x$ for small values of x one easily gets suitable initial values for the iteration procedure (e.g. Newton-Raphson). The case of unequal insurance periods can be handled too. In that case one will have to work with $p_{ok}(z_n)$ with z_n being the exposure of policy No. n . The equations involved become a bit more complicated but can easily be solved by a simple algorithm similar to the one indicated above.

The observations in a given bonus class in 1962 using Lexis diagram

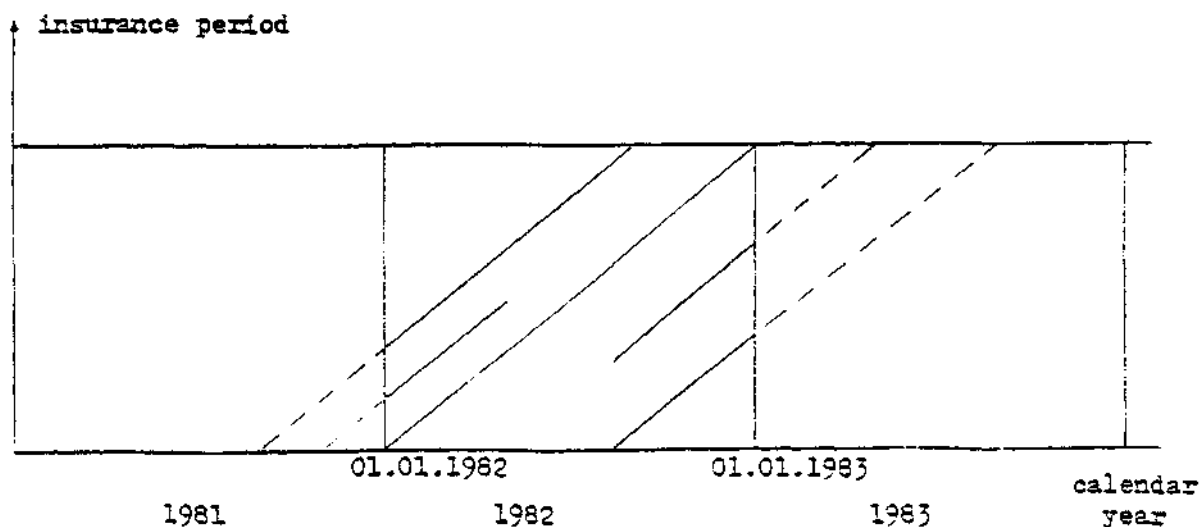


Diagram 3

4. Statistical properties of the estimators.

Let $\hat{\theta}_N$ denote the vector of maximum likelihood estimators $(\hat{\sigma}_k, \hat{\mu}_k)$. Then it is well-known that asymptotically $\hat{\theta}_N$ is normally distributed

$$\hat{\theta}_N \underset{\sim}{\sim} N(\theta_0, \frac{1}{N} \zeta(\theta_0)^{-1})$$

where $\zeta(\theta_0)$ is the Fisher information matrix given by

$$\zeta(\theta) = - \left\{ E_{\theta} \frac{\partial^2 L}{\partial \theta_i \partial \theta_j} \right\}_{i,j=0,1,0',1',2'}$$

$$(\theta_0 = \sigma_0, \theta_1 = \sigma_1, \theta_{0'} = \mu_0, \theta_{1'} = \mu_1, \theta_2 = \mu_2, \theta_{2'} = \mu_2).$$

In the complete sampling scheme all covariances vanish while in the incomplete sampling scheme one uses the fact that the M^k 's are Bernoulli s.v. with parameters p_{ok} in deriving mean values (and in this case the estimators are not asymptotically independent).

If you look at the asymptotic relative efficiency you will in most cases find the lack of efficiency in estimating the parameters using the incomplete sampling scheme is insignificant.

For the complete sampling scheme the following expressions are obtained for the determination of variance terms

$$\begin{aligned} -E \frac{\partial^2 L}{\partial \theta_k^2} &= \frac{1}{\theta_k^2} E N^k = \frac{1}{\theta_k^2} \int_0^1 p_{ok}(t) \theta_k dt \quad \text{for } k=0,1 \\ -E \frac{\partial^2 L}{\partial \theta_{k'}^2} &= \frac{1}{\theta_{k'}^2} E M^{k'} = \frac{1}{\theta_{k'}^2} \int_0^1 p_{ok}(t) \theta_{k'} dt \quad \text{for } k=0,1,2 \end{aligned}$$

i.e. the information matrix is diagonal with the following non-zero entries

$$\begin{aligned} &\frac{1}{\sigma_0^2} \int_0^1 p_{00}(t) \sigma_0 dt \\ &\frac{1}{\sigma_1^2} \int_0^1 p_{01}(t) \sigma_1 dt \\ &\frac{1}{\mu_0^2} \int_0^1 p_{00}(t) \mu_0 dt \\ &\frac{1}{\mu_1^2} \int_0^1 p_{01}(t) \mu_1 dt \\ &\frac{1}{\mu_2^2} \int_0^1 p_{02}(t) \mu_2 dt = \frac{1}{\mu_1^2} \int_0^1 p_{02}(t) \mu_1 dt \quad (\text{as } \mu_2 = \mu_1 \\ &\quad \text{according to assumptions}). \end{aligned}$$

Statistical inference could be based upon the asymptotic distribution of the parameters, e.g. by using linear normal models and treating the estimators as if they were distributed exactly as normal s.v. with known variance-covariance matrix.

It is not the intention to go into detail about these matters but only mention that management - in order to assess the efficiency of the bonus system - is interested in testing

- if there are significant differences among the σ_k 's of the different classes, e.g. hypotheses of the type $\sigma_0(1) > \sigma_0(2) > \dots > \sigma_0(i) > \dots > \sigma_0(I)$ i referring to bonus class C_i
- if there are significant differences between $\mu_0(i)$ and $\mu_i(i)$ for $i=1, \dots, I$.

5. The model as an aid in tariffing.

a. B-values

As the premium in motor insurance in most of the risk categories is linked to the claims frequency experience of the individual policyholder, for tariffing purposes it is necessary to know more than the level and structure of the risk premium in order to access the development of the loss ratio for a future period of say 3-5 years. Focus is put on the loss ratio as this indirect measure of profitability is the one used and understood by management.

In order to access future expected premium income you will have to know the expected claims frequency, the expected level of new business, and the expected cancellation rate during the period of interest.

With the bonus system is given a bonus scale $\overline{\pi}_i$, $i=1, \dots, I$ such that the one-period premium is $\overline{\pi}_i \cdot P$ for all policies in bonus class C_i . Here P is a common tariff-premium for the risk group considered. The bonus scale used at present in Baltica is

C_i	0	1	2	3	4	5	6	7	8	9	10
$\overline{\pi}_i$	1.113	.9646	.8162	.742	.6678	.5936	.5194	.4452	.371	.2968	.2226

Let N_i denote the number of policies in bonus class C_i and define B as

$$B = \sum_i P \overline{\pi}_i N_i / \sum_i P N_i .$$

The relative measure B is a convenient way of expressing the changes in the level of the premium income during the period of interest.

$B \cdot P \cdot \sum_i N_i$ is the expected premium income.

b. Estimation of expected total premium income

Estimation of expected total premium income is done simply by applying the probabilities $p_{ok}(1)$'s and $p_{ok}'(1)$'s to the number of policies in each of the bonus classes at the beginning of the year to obtain the similar number of policies in each of the bonus classes at the end of the year. The process is continued in an obvious manner for each of the years in the period of interest.

You get

year 0 : portfolio at beginning of year 0
 + new business in year 0
 - cancellations in year 0
 = portfolio at end of year 0
year 1 : = portfolio at beginning of year 1
 + new business in year 1
 - cancellations in year 1
 = portfolio at end of year 1
year 2 : = portfolio at beginning of year 2
 etc.etc.

Weighing the number of policies (or rather policyyears) in each of the bonus classes with the bonus scale you find the following B-values:

Portfolio at beginning of the year, excluding new business	: B_0
do	, including new business : $B_{0,n}$
Portfolio at end of the year, excluding cancellations	: B_1
do	, including cancellations : $B_{1,c}$
New business	: B_n
Cancellations	: B_c

where primarily $B_{1,c}$ is the one of importance in estimating the level of premium income.

6. Case study

Below are shown examples based on the data for 1980 (in a complete observational scheme) from a motor portfolio of the Baltica Insurance Company comprising 150.000 policies to illustrate the performance of the model.

We are looking at the biggest single risk category, namely privately owned cars having an insurance coverage including a 300 Dkr deductible. In Appendix 1 the basic data are shown and in Appendix 2 the estimates of the parameters in the model. In order to achieve monotonicity among the parameters of the model the estimates, i.e. claim frequencies and cancellation rates, have been smoothed. The monotonicity requirements - which for most bonus classes are fully justified by the data - consist mainly in the following relations

- the claims frequency in bonus class C_1 decreases with i
- the cancellation rate in bonus class C_1 decreases with i .

The model was implemented on our APL-system.

Taking the 31.12.1981 portfolio as starting point we are interested in evaluating the portfolio development during the five-year period 1982-1986 by means of the B-values. We will do that for different sets of assumptions regarding amount of new business and level of claims frequencies and cancellation rates during that period. In one of the cases we will show the portfolio movements for each bonus class. For all other cases we will focus on the B-values alone.

- CASE 1 Claims frequencies as in 1980 for all the years 1982-1986
 Cancellation rates as in 1980 for all the years 1982-1986
 Amount of new business as in 1980 for all the years 1982-1986
 - see Appendix 3 for the detailed portfolio movements.
- CASE 2 Amount of new business in 1982 is 50% higher than in 1980.
 Otherwise the same assumptions as in case 1.
- CASE 3 Amount of new business in 1982 is 50% lower than in 1980.
 Otherwise the same assumptions as in case 1.
- CASE 4 Amount of new business in 1982 is 10% higher, in 1983 20%
 higher, in 1984 30% higher, in 1985 40% higher and in 1986
 50% higher than in 1980.
 Otherwise the same assumptions as in case 1.
- CASE 5 Amount of new business in 1982 is 10% lower, in 1983 20%
 lower, in 1984 30% lower, in 1985 40% lower and in 1986 50%
 lower than in 1980.
 Otherwise the same assumptions as in case 1.
- CASE 6 Claims frequencies are for each of the years 1982-1986 10%
 higher than in 1980.
 Otherwise the same assumptions as in case 1.
- CASE 7 As in case 6 but with 25% higher claims frequencies.
- CASE 8 As in case 6 but with 100% higher claims frequencies.
- CASE 9 As in case 6 but with 25% lower claims frequencies.

- CASE 10 Cancellation rates are for each of the years 1982-1986 10% higher than in 1980.
Otherwise the same assumptions as in case 1.
- CASE 11 As in case 10 but with 25% higher cancellation rates.
- CASE 12 Both claims frequencies and cancellation rates are 25% higher in bonus class 10.
Otherwise the same assumptions as in case 1.
- CASE 13 As in case 12 but with 25% lower claims frequencies and cancellation rates.

In the tables 1-3 are shown the B-values and the relative differences between the B-values.

It is surprising that the B-values exhibit only rather small deviations even in cases where you alter the assumptions drastically as regards new business, claim frequencies and cancellation rates.

If we e.g. look at case 6 and 7 we note that an overall increase in the claim frequencies and thereby in the risk premiums by 10% and 25% respectively is followed by an overall premium increase of only 1-2% as compared with the "normal" situation (case 1). This shows that the bonus system as a sort of self-adjusting instrument or mechanism is of little or no value at all. The reason is in our view that the level of claim frequency is just simply too small to justify bonus systems as the ones in use by most companies. Statistically the bonus system is justified in the sense that claim frequencies are significantly different from bonus class to bonus class. The question is whether this is so in an economic sense of the concept: significance, as seen from a company point of view. Due to competition, however, it is hardly possible to imagine motor insurance without bonus systems.

What is worth noting is in almost all the cases the tendency in the course of time of a decrease in the B-values which is due to the fact that more and more of the policies end up in class 10 and stay there. This general tendency is of course most dangerous if account of it has not been made in the tariff premium calculation.

Conclusion

I have hoped to show that the proposed model is a suitable and practical instrument for gauging different strategies as to new business, cancellations, bonus scales etc.etc. relating to the bonus system of motor insurance. I have stressed the evaluation of the total premium income for a forecasting period of five years through the use of the B-values.

It is possible to work not only on mean values but to consider the model as a "true" stochastic model, e.g. by doing simulation assuming that the parameters as maximum likelihood estimators follow a normal distribution. I am not sure, however, what would be the benefits from such an approach as I am convinced that changes in the level of new business, claim frequencies, and cancellation rates as a consequence of company policies or general changes in society (oil crises etc.) exhibit a far greater effect on the parameters of the model than do the purely stochastic variations in the parameters when dealing with portfolios of the size presented in the present case study.

Table 1 **B-values (in %)**

Case	B _o					B _n all years					B _o				
	<u>1982</u>	<u>1983</u>	<u>1984</u>	<u>1985</u>	<u>1986</u>	<u>1982</u>	<u>1983</u>	<u>1984</u>	<u>1985</u>	<u>1986</u>	<u>1982</u>	<u>1983</u>	<u>1984</u>	<u>1985</u>	<u>1986</u>
1	31.311	29.754	28.810	28.323	28.103	49.427	46.678	44.894	43.862	43.436	43.287				
2	31.311	30.222	29.087	28.470	28.169	49.427	48.018	45.540	44.187	43.529	43.249				
3	31.311	29.359	28.668	28.338	28.212	49.427	45.083	44.445	43.843	43.705	43.718				
4	31.311	29.948	29.222	28.967	28.986	49.427	47.028	45.959	45.650	45.898	46.351				
5	31.311	29.757	28.684	27.978	27.483	49.427	46.435	44.242	42.426	40.991	39.649				
6	31.311	29.853	28.958	28.491	28.281	49.427	46.738	45.147	44.195	43.809	43.685				
7	31.311	30.004	29.184	28.748	28.556	49.427	46.817	45.517	44.685	44.359	44.274				
8	31.311	30.790	30.364	30.124	30.056	49.427	47.075	47.215	46.937	46.926	47.058				
9	31.311	29.513	28.449	27.918	27.674	49.427	46.503	44.244	43.007	42.484	42.273				
10	31.311	29.568	28.541	28.028	27.807	49.427	46.419	44.353	43.234	42.818	42.722				
11	31.311	29.304	28.171	27.630	27.415	49.427	46.049	43.596	42.384	42.008	42.003				
12	31.311	29.969	29.165	28.766	28.625	49.427	44.561	42.983	42.129	41.858	41.896				
13	31.311	29.535	28.449	27.872	27.575	49.427	49.193	47.334	46.212	45.654	45.315				

Table 2

B-values (in %)

Case	$B_{o,n}$					$B_{l,o}$				
	<u>1982</u>	<u>1983</u>	<u>1984</u>	<u>1985</u>	<u>1986</u>	<u>1982</u>	<u>1983</u>	<u>1984</u>	<u>1985</u>	<u>1986</u>
1	32.845	31.506	30.721	30.344	30.203	32.015	30.871	30.266	30.001	29.852
2	33.519	31.882	30.925	30.434	30.223	32.737	31.225	30.448	30.074	29.860
3	32.112	31.205	30.649	30.415	30.361	31.353	30.692	30.300	30.166	30.097
4	32.984	31.985	31.560	31.539	31.741	32.290	31.467	31.198	31.272	31.456
5	32.703	31.195	30.099	29.315	28.701	31.988	30.648	29.717	29.050	28.437
6	32.845	31.600	30.863	30.508	30.381	32.140	31.067	30.491	30.242	30.105
7	32.845	31.742	31.079	30.758	30.654	32.327	31.365	30.834	30.610	30.494
8	32.845	32.480	32.201	32.087	32.124	32.247	32.885	32.626	32.573	32.592
9	32.845	31.277	30.373	29.946	29.775	31.702	30.386	29.717	29.419	29.242
10	32.845	31.356	30.517	30.137	30.014	32.004	30.723	30.074	29.809	29.681
11	32.845	31.148	30.245	29.870	29.872	31.992	30.520	29.818	29.563	29.474
12	32.845	31.725	31.094	30.827	30.788	32.104	31.133	30.647	30.485	30.423
13	32.845	31.283	30.341	29.854	29.613	31.916	30.594	29.869	29.501	29.265

Table 3 Relative B-values (in %)

Case	$B_{1,0} : \frac{1982 = 100.0}{}$					$B_{1,0}$	<u>case 1 = 100.0</u>				
	<u>1982</u>	<u>1983</u>	<u>1984</u>	<u>1985</u>	<u>1986</u>		<u>1982</u>	<u>1983</u>	<u>1984</u>	<u>1985</u>	<u>1986</u>
1	100.0	96.4	94.5	93.7	93.2	100.0	100.0	100.0	100.0	100.0	100.0
2	100.0	95.4	93.0	91.9	91.2	102.3	101.1	100.6	100.2	100.2	100.0
3	100.0	97.9	96.6	96.2	96.0	97.9	99.4	100.1	100.5	100.5	100.8
4	100.0	97.5	96.6	96.8	97.4	100.9	101.9	103.1	104.2	104.2	105.4
5	100.0	95.8	92.9	90.8	88.9	99.9	99.3	98.2	96.8	96.8	95.3
6	100.0	96.7	94.9	94.1	93.7	100.4	100.6	100.7	100.8	100.8	100.8
7	100.0	97.0	95.4	94.7	94.3	101.0	101.6	101.9	102.0	102.0	102.2
8	100.0	98.9	98.1	98.0	98.0	103.8	106.5	107.8	108.6	108.6	109.2
9	100.0	95.8	93.7	992.8	92.2	99.0	98.4	98.2	98.1	98.1	98.0
10	100.0	96.0	94.0	93.1	92.7	100.0	99.5	99.4	99.4	99.4	99.4
11	100.0	95.4	93.2	92.4	92.1	99.9	98.9	98.5	98.5	98.5	98.7
12	100.0	97.0	95.5	95.0	94.8	100.3	100.8	101.3	101.6	101.6	101.9
13	100.0	95.9	93.6	92.4	91.7	99.7	99.1	98.7	98.3	98.3	98.0

NUMBER OF POLICY YEARS

1980

Bonus class	Existing Portfolio			New business		
	before the first claim	after the first claim	total	before the first claim	after the first claim	total
0	12	0	12	6	1	7
1	1443	178	1621	56	8	64
2	2744	216	2960	81	13	94
3	2174	113	2287	19	1	20
4	2295	115	2410	7210	733	7943
4.1	4709	288	4997	0	0	0
5	1937	48	1985	123	4	127
6	5875	238	6113	557	35	592
7	8424	299	8723	750	20	770
8	11676	337	12013	798	29	827
9	21429	577	22006	4527	125	4652
10	74051	1248	75299	1144	24	1168
TOTAL	136769	3657	140426	15271	993	16264

NUMBER OF CLAIMS

1980

Bonus class	Existing portfolio			New business		
	before the first claim	after the first claim	total	before the first claim	after the first claim	total
0	5	0	5	2	0	2
1	615	126	741	32	10	42
2	822	121	943	45	6	51
3	418	67	485	4	2	6
4	370	36	406	2904	508	3412
4.1	1048	127	1175	0	0	0
5	309	14	323	22	2	24
6	840	71	911	120	25	145
7	1025	88	1113	111	5	116
8	1117	69	1186	97	11	108
9	1857	117	1974	461	24	485
10	4471	162	4633	131	2	133
TOTAL	12897	998	13895	3929	595	4524

NUMBER OF CANCELLATIONS

1980

Bonus class	Existing portfolio			New business		
	before the first claim	after the first claim	total	before the first claim	after the first claim	total
0	5	8	13	1	3	4
1	762	281	1043	32	15	47
2	1198	386	1584	27	22	49
3	705	129	834	11	0	11
4	520	118	638	3276	1874	5150
4.1	1559	289	1848	0	0	0
5	499	88	587	37	8	45
6	1230	213	1443	133	52	185
7	1220	195	1415	126	42	168
8	1211	194	1405	116	33	149
9	1864	253	2117	555	108	663
10	5206	475	5681	101	47	148
TOTAL	15979	2629	18608	4415 1.346	2206	6621

CLAIM FREQUENCIES

1980

Bonus class	Existing portfolio			New business		
	before the first claim	after the first claim	total	before the first claim	after the first claim	total
0	417	0	417	333	0	286
1	426	708	457	571	1250	656
2	300	560	319	556	462	543
3	192	593	212	211	2000	300
4	161	313	168	403	693	430
4.1	223	441	235	0	0	0
5	160	292	163	179	500	189
6	143	298	149	215	714	245
7	122	294	128	148	250	151
8	96	205	99	122	379	131
9	87	203	90	102	192	104
10	60	130	62	115	83	114
TOTAL	94	273	99	257	599	278

CANCELLATION RATES

1980

Bonus class	Existing portfolio			New business		
	before the first claim	after the first claim	total	before the first claim	after the first claim	total
		—				
0	417	0	1083	167	3000	571
1	528	1579	643	571	1875	734
2	437	1787	535	333	1692	521
3	324	1142	365	579	2000	650
4	227	1026	265	454	2557	648
4.1	331	1003	370	0	0	0
5	258	1833	296	301	2000	354
6	209	895	236	239	1486	313
7	145	652	162	168	2100	218
8	104	576	117	145	1138	180
9	87	438	96	123	864	143
10	70-	381	75	88	1958	127
TOTAL	117	719	133	289	2222	407

CLAIM PROBABILITIES

1980

Bonus class	Existing portfolio			New business		
	before the first claim	after the first claim	total	before the first claim	after the first claim	total
0	435	283	0	607	74	0
1	385	91	31	319	79	50
2	479	71	18	411	130	27
3	597	66	19	454	29	27
4	679	71	10	424	65	17
4.1	575	85	18	1000	0	0
5	659	50	6	619	48	9
6	703	68	10	635	65	21
7	766	68	10	729	46	4
8	819	59	6	766	53	9
9	841	58	6	799	55	5
10	877	44	3	816	43	1
TOTAL	810	53	7	579	59	14

CANCELLATION PROBABILITIES

1980

Bonus class	Existing portfolio			New business		
	before the first claim	after the first claim	total	before the first claim	after the first claim	total
0	283	0	283	131	188	319
1	340	153	493	341	211	552
2	309	123	432	221	211	432
3	253	65	318	400	90	490
4	188	53	241	305	188	493
4.1	254	68	322	0	0	0
5	211	74	285	239	85	324
6	176	43	219	192	88	280
7	127	29	157	144	77	221
8	94	21	115	128	45	172
9	80	16	95	110	31	141
10	66	10	76	80	60	140
TOTAL	105	25	130	223	126	349

SMOOTHED CLAIM PROBABILITIES

1980

Bonus class	Existing portfolio			New business		
	before the first claim	after the first claim	after the second claim	before the first claim	after the first claim	after the second claim
0	390	84	35	336	52	16
1	391	90	33	356	54	18
2	485	80	23	377	58	18
3	598	68	15	399	63	18
4	676	68	10	423	68	18
4.1	573	85	19	1000	0	0
5	654	69	12	641	47	10
6	701	69	10	677	50	10
7	767	66	8	715	52	9
8	817	58	6	755	55	7
9	841	58	5	798	56	5
10	878	44	3	843	56	3
TOTAL	810	53	6	581	61	13

SMOOTHED CANCELLATION PROBABILITIES

1980

Bonus class	Existing portfolio			New business		
	before the first claim	after the first claim	total	before the first claim	after the first claim	total
0	324	177	501	325	271	596
1	328	158	486	321	251	572
2	305	108	412	316	230	546
3	250	70	320	311	208	519
4	193	53	246	306	186	491
4.1	256	67	323	0	0	0
5	214	51	265	207	94	301
6	177	42	219	184	79	263
7	126	33	159	159	64	224
8	96	23	119	133	50	183
9	80	17	97	106	35	141
10	66	9	75	77	22	99
TOTAL	105	26	131	219	126	345

PORTFOLIO MOVEMENTS IN 1982

Bonus class	Number of policies as at												
	01.01. 1982	31.12.1982											
		0	1	2	3	4	5	6	7	8	9	10	
0	926	110	352										
1	518	64		203									
2	2468	253			1198								
3	2255	33	152			1349							
4	1916	20		130			1296						
4.1	3411	65		289			1956						
5	6970		83		480			4559					
6	2068			22		143			1450				
7	7528				64		495			5773			
8	12804					79		744			10457		
9	11566						57		667			9724	
10	100600							280		4453		88284	
0	86	6	29										
1	13	1		5									
2	218	17			82								
3	17	0	1			7							
4	7026	127		476		2972							
5	334		3		16			214					
6	382			4		19			259				
7	503				4		26			360			
8	685					5		37			517		
9	579						3		32			462	
10	4315							11		241		3638	
TOTAL	167188	695	620	1126	1843	4574	3832	5847	2408	10827	10975	102105	

APPENDIX 3

PORTFOLIO MOVEMENTS IN 1983

Bonus class	Number of policies as at											
	01.01. 1983	31.12.1983										
		0	1	2	3	4	5	6	7	8	9	10
0	695	83	264									
1	620	76		243								
2	1126	115			547							
3	1843	27	124			1103						
4	1602	16		108			1083					
4.1	2972	57		251			1704					
5	3832		45		264			2507				
6	5847			61		404			4100			
7	2408				20		158			1847		
8	10827					67		629			8842	
9	10975						54		633			9227
10	102105							284		4520		89605
0	86	6	29									
1	13	1		5								
2	218	17			82							
3	17	0	1			7						
4	7026	127		476		2972						
5	334		3		16			214				
6	382			4		19			259			
7	503				4		26			360		
8	685					5		37			517	
9	579						3		32			462
10	4315							11		241		3636
TOTAL	159011	525	468	1147	933	4577	3029	3684	5024	6967	9360	102929

PORTFOLIO MOVEMENTS IN 1984

Bonus class	Number of policies at at											
	01.01. 1984	31.12.1984										
		0	1	2	3	4	5	6	7	8	9	10
0	525	62	200									
1	468	57		183								
2	1147	118			557							
3	933	14	63			558						
4	1604	16		108			1085					
4.1	2972	57		251			1704					
5	3029		36		209			1981				
6	3684			38		254			2583			
7	5024				42		330			3853		
8	6967					43		405			5690	
9	9360						46		540			780
10	102929							287		4556		9031
0	86	6	29									
1	13	1		5								
2	218	17			82							
3	17	0	1			7						
4	7026	127		476		2972						
5	334		3		16			214				
6	382			4		19			259			
7	503				4		26			360		
8	685					5		37			517	
9	579						3		32			
10	4315							11		241		30
TOTAL	152800	475	332	1065	910	3859	3195	2936	3414	9010	6207	102

PORTFOLIO MOVEMENTS IN 1985

Bonus class	Number of policies as at												
	01.01. 1985	31.12.1985											
		0	1	2	3	4	5	6	7	8	9	10	
0	475	56	180										
1	332	41		130									
2	1065	109			517								
3	910	13	61			544							
4	886	9		60			599						
4.1	2972	57		251			1704						
5	3195		38		220			2090					
6	2936			31		203			2059				
7	3414				29		224			2618			
8	9010					56		524			7358		
9	6207						30		358			5219	
10	102295							285		4528		89771	
0	86	6	29										
1	13	1		5									
2	218	17			82								
3	17	0	1			7							
4	7026	127		476		2972							
5	334		3		16			214					
6	382			4		19			259				
7	503				4		26			360			
8	685					5		37			517		
9	579						3		32			462	
10	4315							11		241		3636	
TOTAL	147855	436	313	956	868	3806	2588	3161	2708	7747	7876	99087	

PORTFOLIO MOVEMENTS IN 1986

Bonus class	Number of policies as at												
	01.01. 1986	31.12.1986	0	1	2	3	4	5	6	7	8	9	10
0	436	52	166										
1	313	38		123									
2	956	98			464								
3	868	13	59			519							
4	834	9		56			564						
4.1	2972	57		251			1704						
5	2588		31		178			1693					
6	3161			33		218			2217				
7	2708				23		178			2077			
8	7747					48		450			6327		
9	7876						39		454			6622	
10	99087								276		4386		86956
0	86	6	29										
1	13	1		5									
2	218	17			82								
3	17	0	1			7							
4	7026	127		476		2972							
5	334		3		16			214					
6	382			4		19			259				
7	503				4		26			360			
8	685					5		37			517		
9	579						3		32			482	
10	4315								11		241		3636
TOTAL	143704	417	289	947	767	3789	2514	2682	2962	7063	6844	97675	