JIA 116 (1989) 737-743

APPLICATIONS OF MATHEMATICS IN INSURANCE, FINANCE AND ACTUARIAL WORK, 6-7 JULY 1989

This joint seminar was sponsored by the Institute, the Faculty, and The Institute of Mathematics and Its Applications. The day-and-a-half meeting took place at Staple Inn on 6 and 7 July 1989. There were 180 participants with 37 from overseas representing 14 different countries. The seminar was introduced by the President, Mr. R. D. Corley. Seventeen papers were considered. The meeting was chaired by Professor S. Haberman on the first day and by Professor J. McCutcheon and Mr P. N. S. Clark on the second day. Many of the participants attended the dinner at the London Press Centre where they were addressed by Professor H. Ramlau-Hansen on the subject of a Continental approach to actuarial work and by Dr J. Lörper on developments in the European Life Insurance Market.

Papers were distributed in advance of the meeting. Each speaker presented an overview of the subject which was followed by general discussion.

TIME SERIES MODELS FOR INSURANCE CLAIMS

Professor A. Harvey introduced the paper, by Harvey and Fernandes, on Time Series Models for Insurance Claims. He dealt with prediction of the average and total values of insurance claims. The general approach is based on structural time series methodology which is a recent development. Past observations are used to make predictions and explanatory variables can be added. The reduction in road traffic accidents following the introduction of seat belts was used to illustrate the method. It is a dynamic model. The forecasts are maximum likelihood estimates. The method can be extended to deal with multivariate models. It is particularly suitable for restricted data where for example only totals are available. In the discussion the models were compared with the more familiar ARIMA models.

MATCHING

Mr A. J. Wise, a practitioner, set out his views on analysis of matching. Linear algebra provided the framework. The actuarial preliminaries involved comparing assets and liabilities using discounted cash flow techniques. The mathematical preliminaries introduced the rules of vector algebra. The concept of pension fund valuation and matching, the need for asset allocation and interpretation of investment risk were discussed. This leads to the idea of a 'matching portfolio' of assets to match a given portfolio of liabilities. The resulting model depends on five axioms. The first is that a matching portfolio exists. The second is that it is unique. The third is that if absolute matching of cash flows is possible then this is the matching portfolio. The fourth is that the solution is linear in the sense that if the liabilities were scaled up by a factor the

new matching portfolio would be the existing matching portfolio scaled up by the same factor. The fifth and final axiom is the additive property of matching portfolios. If two liability portfolios are combined then the matching portfolio for the combination is the sum of the two individual matching portfolios. The axioms are used to derive a general solution for the matching portfolio. Variations of the model allow for alternative definitions of matching. For example, the criterion could be to maximize the surplus subject to a specified degree of risk. Mismatching was also dealt with.

OPTION PRICING MODELS

Dr D. Blake began with an introduction to basics. This included identifying the holder and the writer of the option, explaining the variation in the value of the option premium over time and the variation in its value over ranges of exercise prices. He presented the value as the sum of the intrinsic value (proceeds if exercised now) and the time value which decays. He looked at the call option in more detail and highlighted the five factors that determine the value. One of these is the exercise price which has a negative effect in that a high price reduces the value. The others are share price, unexpired term, risk free rate of interest and volatility of the share price. These factors affect the option price but positively in that the option price is higher for large values. Two models were introduced. The binomial model and the more complicated Black–Scholes model. Actuarial judgement was suspended for the binomial example. The option price exceeded the expected gain but this device allowed the author to simplify the calculations underlying the stochastic process.

LINEAR MODELS FOR GENERAL INSURANCE PREMIUM RATES

Dr S. Coutts introduced a practical approach to modelling premium rates. He expressed concern on the use of aggregate data which smoothed out the underlying variations in component parts. His talk included a number of ideas from a range of models used. Making use of power available on PCs is important. He also placed the model in the control cycle for determining premium rates for liability and property insurance risks. This begins with the analysis of the General Linear Model (GLIM). This is followed by the marketing management's initial assessment, the establishment of reassurance requirements and the formulation of the Business Plan. The rates are considered by marketing management a second time before being implemented. The next items are control and analysis of surplus from the experience. The results are compared with the business objectives, the database is updated. The updated position is the starting point for the analysis of the general linear model which completes the cycle.

LIFE INSURANCE FOR INSULIN-DEPENDENT DIABETICS

Professor H. Ramlau-Hansen presented the results of a very interesting study of the mortality of insulin-dependent diabetics in Denmark. This involved the application of mathematical and statistical techniques in determining premium rates for non-standard lives. The condition is that of nil or low insulin production. Insulin-dependent diabetes typically affects younger lives and is associated with underweight. Before the discovery of insulin the expectation of life had been one to two years. This contrasts with the much milder condition of non-insulin-dependent diabetes which typically affects older lives and is associated with overweight.

It was established that mortality depended on a total of 11 factors including durational effects. The risks increase with duration from onset of the condition. The model chosen was a three-state-disability model which allowed for 7 of the 11 factors. The investigation was based on records from a hospital in Copenhagen covering some 3000 lives.

CHAIN LADDER AND INTERACTIVE MODELLING

Dr A. Renshaw discussed the prediction of outstanding claims amounts in non-life insurance. It is usual for these estimates to be based on current and past records and almost always take the form of a run-off triangle. Use has been made of the GLIM package with the following aims:

(i) To develop more fully the statistical analogue of the original actuarial chain-ladder technique;
(ii) To investigate the magnitude and nature of predictor instability associated with the technique;
(iii) To suggest a method for improving predictor stability;
(iv) To make the methodology readily available to practioners so that they can make their own judgements on these matters.

Four specific model structures were outlined and a comparison made of the methods used to estimate parameters. The possible methods were Least Squares, Empirical Bayes and Kalman Filter.

A STATE SPACE REPRESENTATION OF THE CHAIN LADDER LINEAR MODEL

Mr R. Verrall also dealt with the chain ladder method for estimating outstanding claims in general insurance business. He demonstrated that Bayesian methods could be applied to the chain ladder linear model. He concentrated on the Kalman Filter and the State Space approach.

Chain ladder, multiplicative model and linear model approaches are identical except in the derivation of parameters. Bayesian methods represented an

improvement on least squares by making better use of the data available. In addition to the central model presented he illustrated alternative possibilities leaving the practitioner with a range of models to consider.

SOME ASPECTS OF THE MODELLING OF PERMANENT HEALTH INSURANCE

Dr H. Waters' paper was concerned with modelling PHI in general and the CMIB multiple state model in particular. He dealt, initially, with the requirements of a model. The purposes are to monitor the experience to provide tables for premium rates and to provide tables for reserving bases. For this the model should be realistic but tractable in that it is capable of producing results. The data must be available. Complex models can be simplified, the reverse is not so. They can also be used to check the approximations and simplifications of a simpler model.

Alternative models were compared. The Manchester-Unity approach involves the use of central sickness rates. The tables are simple to use but rates only vary with age and the statistical properties of the estimates of rates of sickness are not readily determined. The Continuance Table model with disability annuities is useful but has some undesirable features. Claim inception rates do not depend on the policyholder's state of health. Claim termination rates do not distinguish between deaths and recoveries. Some of the approximations involved cannot easily be quantified. He also considered a simplified CMIB model with three states (healthy, sick or dead) where the simplification was at the cost of modelling deferred period experience.

By way of illustration, two technical problems were considered: modelling the IBNR reserve for one year Group PHI business and investigating the statistical properties of sickness rates both demonstrated results from the CMIB model.

AN ACTUARIAL MODEL FOR AIDS

Professor D. Wilkie discussed the paper published in *J.I.A.* **115,** 839. The paper sets out the rationale and the differential equations underlying the model used by the Institute's Working Party on AIDS.

There have been few recoveries from AIDS so this has been excluded from the model. The two main states are clear and HIV positive (or infected), although the established models for spread of infections such as an influenza epidemic were not appropriate. Some aspects of more complex modelling were considered. These included modelling the immune population, heterogeneous sexual activity and infectivity varying with duration since infection. The models require a proliferation of states of health, introduce complicated formulae and are based on data that may not be available.

PROJECTING THE SPREAD OF AIDS INTO THE GENERAL POPULATION

Dr J. Lörper described the model used by a reassurance company to provide answers to practical problems on the spread of AIDS.

The model uses rates of heterosexual contact of high risk groups. There is a complex network of infection. Some groups were of less statistical interest such as haemophiliacs where the risk is controlled and prenatal infection which is not an insurance risk. The models have been refined following overstatement of results for 1987/88.

The results from the model indicate that in the absence of antiselection the AIDS claims will involve a small additional cost compared with the margins available. This optimism contrasts with the generally held views in the United Kingdom and the United States of America.

MODELLING THE OPERATIONS OF A GENERAL INSURANCE COMPANY BY SIMULATION

Mr C. Daykin introduced the paper. The purpose of the model was to provide effective management of uncertainty. For a general insurance company there are four main areas of uncertainty. They are the run-off risk, underwriting risks, inflation and variability in the return on assets. The balance sheet and revenue account, as accounting information, do not deal with this uncertainty. These statements are static, assets and liabilities are not related. There are no valuation rules. There is no attempt to measure the inherent uncertainty.

An alternative approach is desirable if management or statutory authorities want more fully to understand what is going on. The solution is to model the items of future cash flow. The resulting model is dynamic, relates assets to liabilities, renders valuation rules irrelevant and directly models uncertainty.

The run-off or solvency situation can be modelled separately and assumes that the distribution of claims over time is fixed for each class of business. Resulting aggregate claims payments are then randomized.

Wilkie's set of interrelated autoregressive equations are modified to model inflation and changes in asset values and income.

The approach is generalized to provide the management model which is designed to simulate realistic market conditions. This allows both management and supervisor to model the market and the company's position in the market. It takes account of the uncertainty and effect of management decisions on such matters as pricing.

In the discussion three areas of concern were raised. Firstly, the relative importance of oscillation and correlation between rates of inflation and return on assets was questioned. Secondly, the cycle of profits and losses resulting from infrequent changes to premium rates was not dealt with adequately in the model. Finally the model missed 'bursts' of high inflation. Mr Hey argued that the problem was choice of parameters and not choice of models.

USING APL FOR FINANCIAL CALCULATIONS

Mr D. Forfar set out some of the advantages of the computer language APL. Its structure and syntax made it well suited to financial, actuarial, mathematical and statistical calculations. The language deals with scalars, vectors and matrices as single elements. It has a number of very powerful specialist symbols, functions can be called from other functions—and it is highly interactive. Mr Forfar demonstrated the power of the language with short programs to deal with a number of otherwise complex routines. Examples were annuity values using an interest rate that fluctuates randomly and profit testing for a unit-linked policy.

MODELLING A UNIT-LINKED LIFE OFFICE

Mr C. Johnson described the corporate planning system developed by a major unit-linked office. This is a deterministic model—there was no room for uncertainty.

The system comprises a series of interconnected mathematical models. These cover the agency network, new business, staff, expenses, products, 'equity' (model for capital and dividends), assets, Corporation Tax and surplus, embedded values, statutory solvency checks and finally analysis and presentation of results.

Mr Johnson presented a high level pass through the system and its various components. He then presented the agency, products, assets and tax models in more detail.

Obtaining results from the system is an iterative process in that parts of the model may be run several times before the final set of results is produced.

The philosophy in developing the system is one of cost justification. The system should pay for itself. It should highlight differences and the development of solutions to business problems and generally improve management information. In view of this, accuracy and Board confidence are critical features of the system.

MODELLING A WITH-PROFITS LIFE OFFICE

Mr M. Ross had based his work on Professor D. Wilkie's model detailed in *T.F.A.* **39**, 341. He dealt with the problems that actuaries face in setting up a model for a with-profits office.

Mr Ross began with data requirements and went on to consider the interaction and interdependence of bonus policy, investment policy, expansion and solvency considerations. These were presented as points of the actuarial compass. The points made were illustrated by results from a range of model offices.

DRIVING THE PENSION FUND

Professor S. Benjamin introduced the results of his work on applying the classical theory of control systems to pension funding. The approach involved

considering perturbations from a stable system. This leads to an analysis of spike, step, ramp, sine-wave and white noise random variations in the earned rate of interest and an analysis of the consequent effect on pension funding.

THE DISTRIBUTION OF A PERPETUITY

Dr D. Dufresne discussed his work on the distribution of a perpetuity. He set out the convergence conditions and discussed the existence of the present value of random payments discounted by random discount factors. He concluded with a consideration of the applications to risk theory and referred to use of moments of and the distribution of pension funding rates.

CLOSING SESSION

In the closing session Professor S. Benjamin, in paying tribute to the work of the authors, emphasized the need for actuaries and mathematicians to present overviews and expository papers on their respective areas of interest and expertise.

J. E. O'NEILL