# R E P O R T

# O F

# W O R K I N G   G R O U P

# S T U D Y I N G   A P P L I C A T I O N S

# O F   R E I D   P A P E R

C O N T E N T S

# SECTION 1

## INTRODUCTION

The format of this report results from the way in which the Study Group tackled the question of investigating further aspects of the approach to estimating outstanding claims described in the Reid paper. While most of the Group was concerned with examining specific aspects arising from the practical application of this method, one member working in parallel devised a fundamentally different approach to the same problem, and this is described in detail in P.J. Cooper's paper which will be available at the Seminar.

Section 2 of this report refers to a number of specific developments of the method which have arisen since the time of writing the original paper. It may be worth emphasising here that a development of very considerable significance is that of the 'interpolation method' which has been found in a number of practical examples to produce very close approximations to those based upon fitting the mathematical model of the M surface. Since it is possible in principle to automate the entire process of claim reserving with the help of this interpolation process (subject, of course, to the provision of a specific basis), with the availability of data processing capacity in the industry the applicability of the method is substantially increased.

Section 3 of the paper summarises the basic principles of the original method as outlined in the paper.

Section 4 is concerned with the application of the model from a large portfolio to a very small portfolio in a different class of business. It is to be expected, therefore, that the fit of the model will be less than perfect. It is interesting to see to what extent the parameters of the model are capable of absorbing the salient features of the experience on the small account and modifying that of the large account to match. It emerges that the experience on the large account is sufficiently distinct that it is not possible to force a close match. The implication is that a 'mother' account more closely related to the 'infant' should be considered. A further possibility which is discussed is that of using the data from the small account itself through the interpolation method to provide the model for assessing reserves, and the fit of this may be compared with the previous approaches. Clearly if an account of this size were being evaluated in isolation the concept of security margins outlined in the original paper would be of considerable importance.

Section 5 discusses the results of applying the model from one company's Private Car account to the evaluation of a second company's Private Car account, in a rather similar manner to that of Section 4. However, on this occasion because of the identity of types of business considered the fit of the model to experience is close, although differences in rate of settlement are indicated. The results of this approach are compared with those of the Chain Ladder in a variety of forms.

The paper comprising Section 6 is interesting in setting the model used in the original paper in its context as a special case of a much more general class of models. This paper attempts to find constraints which will establish the uniqueness of such a model and thus its estimability in terms of the conventional log-linear formulation. The author concludes that 'with the usual run-off there will always be models which cannot be estimated; in particular those involving an interaction between origin year and development year'.

## SECTION 2

## ESTIMATION OF OUTSTANDING CLAIM RESERVES

### Background

In Dr. Reid's paper (JIA 1978 Vol.105 page 211) an original method of estimating outstanding claim reserves was described. The Study Group has been concerned with considering this method and its applications.

It may be helpful to briefly re-state the underlying philosophy and an outline of principles behind the method without going into the algebra.

### Philosophy

The actuary is faced with a general insurance account for which he has to estimate the reserves needed for the outstanding claims and, perhaps, to certify their adequacy.

Given that:-

(a) his statement will of necessity imply some assessment of the future, and

(b) claim settlement is to some extent 'random'

a method is required which will incorporate explicit assumptions regarding future values of exogenous variables, such as inflation, and which will quantify in respect of the random variation the confidence which may be attached to the valuation. The basis adopted and degree of confidence required may depend upon the purpose for which the valuation is required, for example whether for rate making or solvency assessment.

The method of approach should use historic data and involve no subjective judgement in a particular case other than in choosing the valuation basis. The valuation basis itself should be capable of comparison with previous experience.

### Approach

The approach is to use a simple probabilistic model with an assumed basis. The similarity to the actuarial theory of life assurance is obvious.

Information on at least one 'base' year's claims is required. For this base year the run-off needs to be sufficiently complete so that reliable estimates can be made of the few claims still outstanding.

For years more recent than the base year, claims data is required which may be used to update the base year data.

Each claim is regarded as being described by:-

(i) A year of origin (or notification)

(ii) An amount being the total of all payments on that claim

(iii) A closure datewhich may be defined as the date of first settlement, or some other convention if more convenient.

Continued.......

The cohort of claims arising from a particular year of origin develops through 'development time' as progressively more claims are settled. Because of the correlation of amount with the time interval between occurrence and settlement ('settlement time') and also variations in the rate of claim settlement, it is necessary to consider the joint distribution of amount and settlement time for each year's cohort of claims. It is also necessary to allow for the differing proportions of claims settled at no cost in each origin year's cohort of claims.

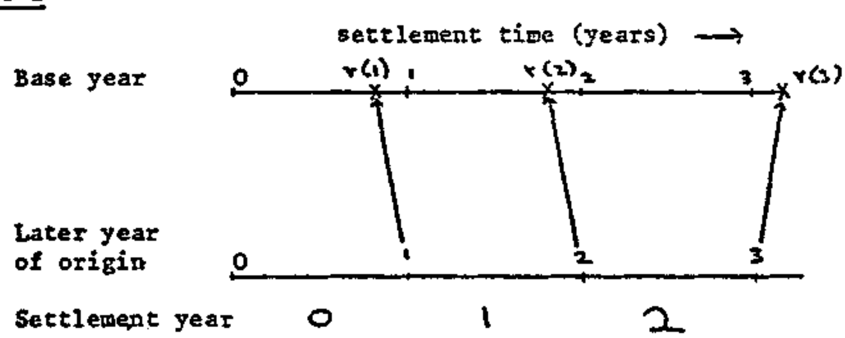
So, the first step is to derive a model representing the base year's cohort of claims by amount and settlement time, treating nil claims separately.

The second step is to modify this model for each subsequent cohort of claims so that it represents the experience so far developed for each cohort. The modifications that may be made to the base year model are:-

(a) different proportions of nil claims
(b) transformed time scale reflecting changed settlement rates
(c) changes to the amount scale reflecting inflation in claim cost whether monetary, social or from any source.

The effect of these modifications is that settlements for a later year of origin take place in a settlement time which is an appropriately stretched or contracted version of the base year's settlement time. Thus claims settled in years of development 1, 2, 3 etc of a later year of origin may be related to claims settled in successive time intervals of the base year, say intervals 0 to r(1), r(1) to r(2), r(2) to r(3) etc (see Figure 1).

Figure 1



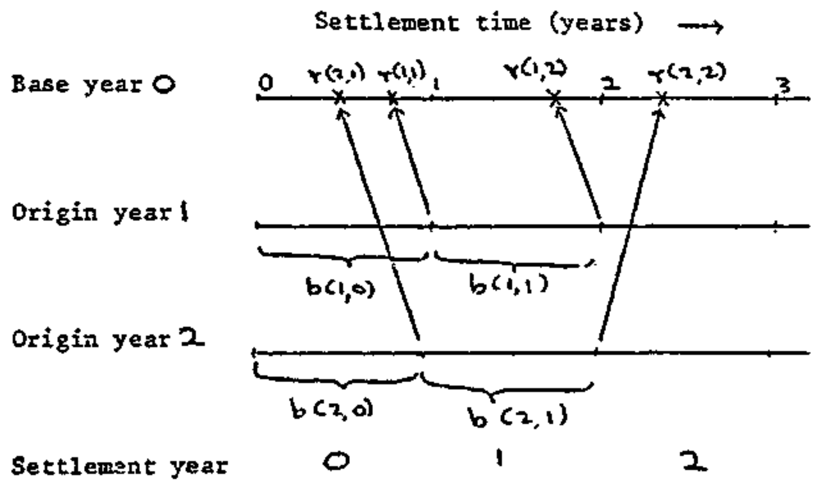

In the situation depicted above, at the end of settlement year 0 the later year of origin has experienced a slower rate of settlement than the base year, so r(1) falls before year 1. At the end of settlement year 1 there is a similar position. At the end of settlement year 2, the later year of origin now has a faster settlement rate, so r(3) falls after the end of 3 years.

The example in Figure 1 shows only variations in settlement rate. There is also the varying level of overall claim cost at different points of time. A certain group of claims is defined by origin year i and settlement year j.

Continued........

To this group will correspond a period r(ij) to r(i,j + 1) of the base year settlement time axis, and the distribution by size of base year claims whose settlement times fall in this period. The quantity b(ij) is the scale transformation to bring this distribution as closely as possible into conformity with the distribution by size of the group of claims being considered. Since each b relates to a different section of the development experience of origin year i's claims, and since also the sections into which successive origin years' experience are divided would not normally be similar, it is not possible to meaningfully compare any one value of b with any other. The position is represented in Figure 2:-

Figure 2



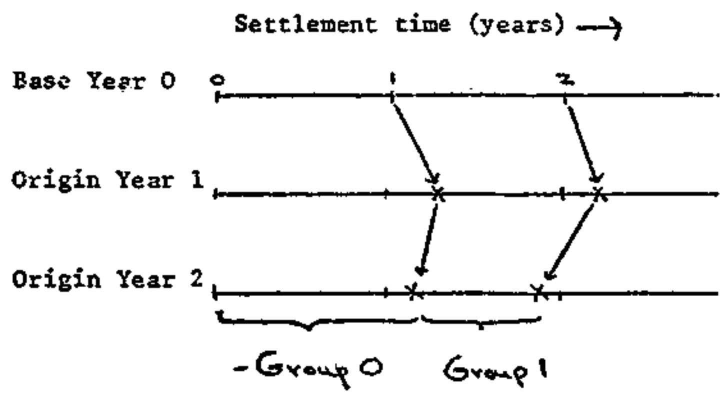

In the situation shown above, origin year 1 has at the end of settlement years 0 and 1 a slower rate of settlement than the base year so the equivalent points on the base year time settlement axis, r (1,1) and r (1,2) fall before years 1 and 2 respectively. The cost of claims from origin year settled in settlement year 0 is increased by a factor b(1,0) over those claims from the base year settled in the period 0 to r (1,1). The cost of claims from origin year 1 settled in settlement year 1 is increased by a factor b(1,1) over those claims from the base year settled in the period r (1,1) to r (1,2). The cost of claims from origin year 2 settled in settlement year 1 is increased by a factor b(2,0) over those claims from the base year settled in the period 0 to r (2,1). The cost of claims from origin year 2 settled in settlement year 2 is increased by a factor b(2,1) over those claims from the base year settled in the period r (2,1) to r (2,2).

The third step is to extrapolate using the modified model to estimate the claim experience for those parts of the development of incomplete years of origin that still lie in the future. Because the rates of settlement vary from cohort to cohort, claims settled in a fixed settlement time interval are not comparable with those claims settled in a similar interval from another cohort.

Continued......

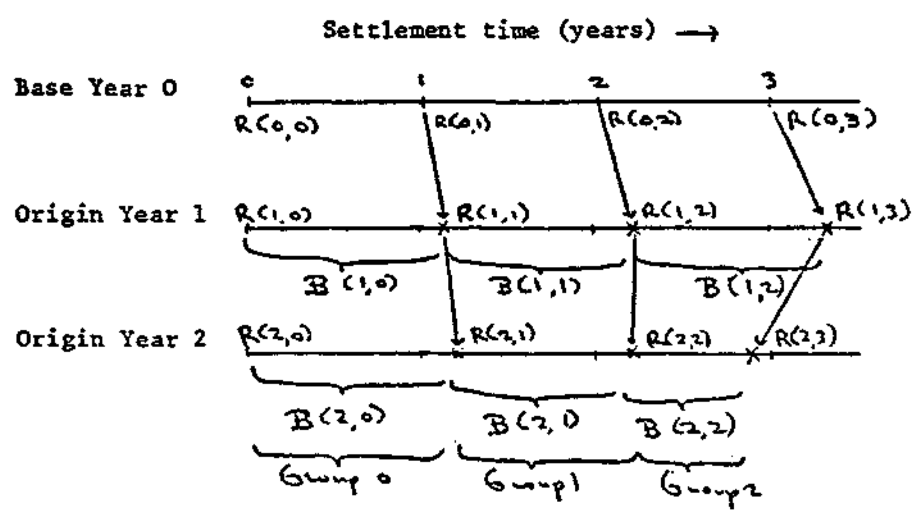
Thus, it is necessary to define notional groups of claims so that the corresponding groups from different cohorts are comparable.. This is achieved by defining the group boundaries in terms of the proportion of a given cohort of claims that have been settled at that settlement time. Thus, the first group will be defined by the claims settled in the first development year of the base year, and the representatives of this group for later development years are those claims settled in the initial development period for each origin year until such time as the proportion of claims settled for that origin year reaches the proportion settled in the first development year of the base year. Later groups of claims are defined similarly in terms of each origin year's cumulative proportions settled. The definition of claim groups in this way allows the comparison of one origin year with another taking into account both changes in claim settlement rate and claim cost, and permits the extrapolation beyond the present time. Figure 3 demonstrates this definition.

Figure 3




We can define a new set of quantities called R's to measure rate of settlement, another set called B's to measure cost levels relative to corresponding values for the base year and these are demonstrated in Figure 4.

Figure 4




The calculation of reserves is now a routine matter.

# S E C T I O N 3

## RECENT DEVELOPMENTS

### 1. Interpolating the M Surface

In the original paper the attempt was made to develop a mathematical form for the M surface which represents the bivariate distribution of base year claims by amount and time to settlement. It was clear that an alternative approach to this part of the problem could have been based upon interpolation methods but it was felt that difficulties might arise in using such methods because of the extreme skewness of the data particularly in the claim size direction.

Having developed such formulae, however, it became a question of considerable interest to see to what extent the results of using these formulae in order to construct estimates of reserves for outstanding claims were affected by substituting an interpolative approach. To test these, a simple method of linear interpolation was used in a number of examples in which the number of data points had been slightly augmented in those areas where it was felt sparseness might be a problem. The results of this exercise established that in practical terms in the examples considered the differences in reserves resulting were insignificant.

This result is of potentially very considerable significance inasmuch as it would now appear possible to automate totally the reserving process described in the original paper. Furthermore, if several complete base years are available for a particular account, it would be a simple matter to calculate the reserves in respect of a group of outstanding claims on the basis of the pattern of each of these base years and to compare the estimated reserves resulting from them. Similarly if it were available, another company's data might be used as a basis for valuation (but see comments in chapter 1 of this report).

### 2. The Inclusion of Information from Case Estimates

In some accounts, particularly where the number of claims is relatively small it may be considered by the actuary that the claim pattern of settled claims is not sufficiently well-established to provide a basis for estimating outstanding claims. In such a case he may feel that other types of information should be aggregated with the settlement pattern in order to produce a firmer estimate. Such information might include available case estimates which in the case of a well-established company it might be felt contained relevant information.

In this event the possibility would arise of attempting to extract information from these case estimates and to combine it with that derived from the settlement pattern. One way of doing this would be to form a very broad classification of case estimates, to introduce a probability structure representing the pattern movement of case estimates through time, and on the basis of this structure to augment the likelihood function upon which reserves are based in the original approach.

The possibilities here are several and are described in more detail in a forthcoming paper: it has not yet been possible to test the structure proposed here against actual data, but clearly this would be regarded as an area for future investigation.

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## 3. Variable Inflation

The original model allowed for inflation through the parameters b and B. One possible extension of the model would be to consider the possibility that inflation may vary with size of claim and one way of doing this would be to allow b to be replaced by $b_1 + b_2 \log x$ where x represents claim size. This possibility has been examined and has been found useful on occasions other than those where purely a steady but differential inflation rate was applying. Thus, it can be particularly helpful in representing the change in distribution occurring where for some specific reason a particular sub-group of claims is revalued.

An alternative view of this process derives from the possibility of testing the statistical significance of the parameter $b_2$, and in consequence deriving a statistical test of the goodness-of-fit of the model to the data. As often occurs in an actuarial context, statistical criteria have to be tempered with actuarial reality here and the commercial significance of very small but nevertheless perhaps statistically significant differences must be assessed pragmatically.

Yet one further possibility is that of adding further terms in powers of log x to the inflation rate proposed above and in this way effectively distorting the original surface to some other surface which might, for example, be used to fit data from a different account altogether.

## SECTION 4

## The application of the model to a small portfolio

In considering the application of D.H.R.'s model to small insurance portfolios a number of problems arise, irrespective of whether the portfolio is small because the company being considered is small or because the portfolio is a small part of a large company's total business. A small portfolio is probably one where there are under 1,000 claims a year and may be as few as 100, however the business can still be long tail with up to 15 years if not longer possibly being required before almost all the claims relating to any one year of origin have been settled. D.H.R. gave an example in section 7.5 of his Institute paper of the fitting of the model directly to a small account. On that occasion the direct approach appears to have been satisfactory. However that might not always be the case. Some of the problems which might occur, together with possible solutions are set out below.

(1) Because there are relatively so few claims in any one year, the shape of the run off will fluctuate from origin year to origin year much more than it would with a large portfolio, making it unwise to take any particular year as the base year. One method of dealing with this would be presumably to combine several origin years. Possible methods of doing this are as follows:

  (i) Estimate M surfaces for each "base" year separately and then average the coefficients obtained to produce the final M surface.

  (ii) Combine the "base" years, by increasing the claims from the earlier years by suitable rates of inflation to produce a composite base year, and then fit a M surface to this.

Cont'd.....

(iii) Add the "base" years together without any inflation adjustment and fit the M surface to this composite base year. The "base" years are then included as separate years in the fitting of the rest of the model.

Unfortunately lack of sufficient data for a suitable portfolio has so far meant that it has not been possible to investigate which of these approaches might in practice be best.

(2) Another method of dealing with the problem of constructing a model for a small portfolio is to take a M surface which has been found appropriate for a much larger portfolio of a similar class of business. This M surface is then applied without any adjustment to the small portfolio, the actual fit being achieved by the estimation of the b's and r's from the small portfolio's data. This approach was successfully illustrated in section 7.7 of D.H.R.'s Institute paper. A further example of this approach is given below, where it is also illustrated how by permitting a relaxation on the fitting of the b's an improved fit can be obtained. In this example the small portfolio was a commercial vehicle comprehensive account, with about 400 claims reported per annum and the M surface was taken from the private car comprehensive account of the same company. Three different cases were considered.

(i) The M surface for the large account was derived by the method outlined in D.H.R.'s Institute paper. The M surface used was as follows:

| | Term | Coefficient | |
|---|---|---|---|
| Non-zero | $x$ | - 0.63500 | 00 |
| | $x^2$ | 0.13240 | 01 |
| | $x^4$ | - 0.37301 | - 01 |
| | $t$ | 0.10917 | 01 |
| | $xt$ | 0.43541 | 00 |
| | $x^2t$ | - 0.60653 | 00 |
| | $x^4t$ | - 0.26142 | - 01 |
| | $xt^2$ | - 0.55224 | - 01 |
| | $x^2t^2$ | 0.71649 | - 01 |
| | $x^4t^2$ | - 0.33646 | - 02 |
| | $t^2e^{-t}$ | - 0.26825 | 01 |
| | $t^{2.3}e^{-t}$ | 0.42790 | 01 |
| | $t\ \ e^{-6t}$ | 0.13357 | 01 |
| | $x^4e^{-4x}e^{-2t}$ | 0.17927 | 02 |
| | $x^2e^{-4x}e^{-2t}$ | 0.14153 | 02 |
| | $x^{10.5}e^{-5x}t^6e^{-9t}$ | 0.59583 | 05 |
| | $x^{12}e^{-8x}t^6e^{-9t}$ | - 0.17857 | 07 |
| | $x^{12}e^{-7x}t^4e^{-t}$ | 0.12887 | 02 |
| | $x^9e^{-4x}t^8e^{-3t}$ | - 0.18687 | 01 |
| | $x^9e^{-3x}t^5e^{-3t}$ | 0.54616 | 00 |
| Zero | $t$ | 0.20851 | 01 |
| | $t^2e^{-3t}$ | - 0.10275 | 02 |

The rest of the model was then fitted to the small portfolio by the usual method.

(ii) The M surface for the large account was fitted by the interpolation method. The rest of the model was then fitted to the small portfolio by the usual method.

Cont'd....

(iii) As for (i) above but after the $r_{jk}$ and $b_{jk}$ had been estimated by the mathematical model, in an attempt to improve the fit of means and claim size distributions the $b_{jk}$ were allowed to vary with claim size, taking the form

$$b = b_1 + b_2 (\log_{10} (X) - 1)$$

where X is actual claim size.

In the calculations the base year was taken as 1973 and we looked at the 4 years of notification, 1975-78 (there was only one claim outstanding for 1974 at 31.12.78). The resultant $r_{jk}$ and $p_j$ values were:

| tification Year | Model | $r_{jk}$ | | | | $p_j$ |
|---|---|---|---|---|---|---|
| | | k: | | | | |
| | | 1 | 2 | 3 | 4 | |
| 1975 | (i) & (iii) | 1.04 | 1.84 | 3.31 | - | 0.290 |
| | (ii) | 1.05 | 1.89 | 3.70 | - | 0.300 |
| 1976 | (i) & (iii) | 0.99 | 2.00 | 2.55 | - | 0.312 |
| | (ii) | 0.99 | 2.04 | 2.66 | - | 0.311 |
| 1977 | (i) & (iii) | 0.95 | 1.88 | - | - | 0.297 |
| | (ii) | 0.93 | 1.89 | - | - | 0.296 |
| 1978 | (i) & (iii) | 1.00 | - | - | - | 0.299 |
| | (ii) | 0.99 | - | - | - | 0.300 |

(The lack of a fourth r for 1975 is due to the fact no claims for 1975 were settled in 1978)

A comparison of actual and modelled numbers of claims settled in each development year is shown in the following tables:

Cont'd....

Non-zero Settlements

| Notification Year | Model | Development Year | | | |
|---|---|---|---|---|---|
| | | 1 | 2 | 3 | 4 |
| 1975 | Actual | 220 | 85 | 16 | 0 |
| | (i) & (iii) | 217 | 89 | 19 | 0 |
| | (ii) | 216 | 87 | 18 | 0 |
| 1976 | Actual | 180 | 82 | 9 | |
| | (i) & (iii) | 172 | 91 | 8 | |
| | (ii) | 174 | 90 | 7 | |
| 1977 | Actual | 220 | 120 | | |
| | (i) & (iii) | 214 | 125 | | |
| | (ii) | 214 | 125 | | |
| 1978 | Actual | 259 | | | |
| | (i) & (iii) | 259 | | | |
| | (ii) | 259 | | | |

Zero Settlements

| Notification Year | Model | Development Year | | | |
|---|---|---|---|---|---|
| | | 1 | 2 | 3 | 4 |
| 1975 | Actual | 73 | 52 | 9 | 0 |
| | (i) & (iii) | 78 | 49 | 7 | 0 |
| | (ii) | 82 | 50 | 7 | 0 |
| 1976 | Actual | 60 | 62 | 2 | |
| | (i) & (iii) | 66 | 55 | 3 | |
| | (ii) | 67 | 54 | 3 | |
| 1977 | Actual | 70 | 75 | | |
| | (i) & (iii) | 74 | 71 | | |
| | (ii) | 76 | 69 | | |
| 1978 | Actual | 94 | | | |
| | (i) & (iii) | 94 | | | |
| | (ii) | 94 | | | |

Cont'd....

The $b_{jk}$ obtained were:

| Year | Model | Development Year 1 | 2 | 3 |
|---|---|---|---|---|
| 1975 | (i) & (iii) | 1.58 | 1.34 | n.a. |
| | (ii) | 1.64 | 1.34 | n.a. |
| 1976 | (i) & (iii) | 1.85 | 2.68 | |
| | (ii) | .1.82 | 2.50 | |
| 1977 | (i) & (iii) | 2.64 | | |
| | (ii) | 2.54 | | |

For model (iii) the $b_{jk1}$ and $b_{jk2}$ then obtained were:

| Year | Development Year 1 | | 2 | |
|---|---|---|---|---|
| | $b_1$ | $b_2$ | $b_1$ | $b_2$ |
| 1975 | 0.82 | 0.47 | 0.54 | 0.47 |
| 1976 | 1.11 | 0.53 | 1.73 | 0.37 |
| 1977 | 1.11 | 0.88 | | |

The comparison of means is:

| Year | Model | Development Year 1 | 2 |
|---|---|---|---|
| 1975 | Actual | 305 | 613 |
| | (i) | 267 | 506 |
| | (ii) | 283 | 501 |
| | (iii) | 261 | 521 |
| 1976 | Actual | 396 | 728 |
| | (i) | 320 | 856 |
| | (ii) | 313 | 811 |
| | (iii) | 333 | 771 |
| 1977 | Actual | 427 | |
| | (i) | 440 | |
| | (ii) | 410 | |
| | (iii) | 403 | |

Cont'd....

It is also possible to compare actual and modelled distributions by size of settled claims for each notification year/development year. This is done in the following table:

| Notification Year | Development Year | Model | 10 | 25 | 100 | 300 | 500 | 1,000 | 2,000 | 5,000 | 10,000 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | Number of claims settled at an amount greater than £: | | | | | | | | |
| 1975 | 1 | Actual | 85 | 74 | 47 | 21 | 13 | 7 | 1 | | |
| | | (i) | 85 | 82 | 53 | 25 | 12 | 3 | 0 | | |
| | | (ii) | 85 | 75 | 52 | 25 | 13 | 4 | 1 | | |
| | | (iii) | 85 | 74 | 49 | 24 | 13 | 4 | 1 | | |
| | 2 | Actual | 16 | 14 | 9 | 5 | 4 | 4 | 2 | | |
| | | (i) | 16 | 16 | 11 | 6 | 4 | 2 | 1 | | |
| | | (ii) | 16 | 14 | 10 | 6 | 4 | 2 | 1 | | |
| | | (iii) | 16 | 14 | 10 | 6 | 4 | 2 | 1 | | |
| 1976 | 1 | Actual | 82 | 76 | 40 | 21 | 17 | 8 | 5 | | |
| | | (i) | 82 | 81 | 53 | 28 | 15 | 5 | 1 | | |
| | | (ii) | 82 | 73 | 52 | 26 | 15 | 5 | 1 | | |
| | | (iii) | 82 | 76 | 52 | 29 | 17 | 6 | 2 | | |
| | 2 | Actual | 9 | 9 | 7 | 5 | 5 | 2 | 1 | | |
| | | (i) | 9 | 9 | 8 | 5 | 4 | 2 | 1 | | |
| | | (ii) | 9 | 8 | 7 | 5 | 4 | 2 | 1 | | |
| | | (iii) | 9 | 9 | 7 | 5 | 3 | 2 | 1 | | |
| 1977 | 1 | Actual | 120 | 113 | 70 | 43 | 30 | 18 | 6 | 1 | 1 |
| | | (i) | 120 | 120 | 87 | 53 | 34 | 12 | 3 | 0 | 0 |
| | | (ii) | 120 | 107 | 85 | 50 | 31 | 11 | 3 | 0 | 0 |
| | | (iii) | 120 | 113 | 80 | 51 | 34 | 15 | 4 | 1 | 0 |

Under each of the three models used the quality of the fit of the means is about the same. When comparing the actual claim distribution with the modelled claim distribution it appears that for each model the number of modelled claims is too high at the low claim sizes and too low at the high claim sizes. However the fit of model (iii) is noticeably better than that of the other two. In addition to model (i) various other model of a corresponding type were fitted in the same way but similar results were produced. The problem is that the very small number of claims in the account being considered produces great variability so that it is very difficult to establish any clear pattern. Despite the encouraging results obtained under model (iii) it may be necessary to obtain meaningful results to combine several small accounts of similar nature into an overall experience if this is possible. It should be emphasised that the interpolation method is no worse than the original method.

(3) With a large portfolio there are likely to be so many claims that the effect of a single large claim not being "properly" represented in the projection into the future for reserving purposes should not be serious. However for a small portfolio this may not be true as even at fairly low levels of claim size the upper end of the estimated distribution of outstanding claims will contain very few claims. Case estimates could possibly be used to provide some fine tuning to the reserves produced by D.H.R.'s method as follows. At the date the reserves are calculated one can produce both from D.H.R.'s method and from the case estimates

Cont'd....

a cumulative distribution (for all origin years combined) of the size of the future settlements under each method. A comparison of the upper end of the two cumulative distributions might indicate whether the case estimates suggest there are any unusual features which require the model reserves to be altered, e.g. if there are more "large" claims than the model is projecting. Of course this depends on how much faith one has in one's case estimates.

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Although a small portfolio will always produce problems purely because of the variability inherent in its size it would seem possible to overcome these by using some or all of the techniques discussed above. As regards the second approach discussed, namely fitting a M surface from another portfolio, for large offices this should present no problem as they could use a M surface from a not too dissimilar account. However for the smaller offices this approach might well not be possible unless standard surfaces were to be published.

The account was remodelled on the interpolation approach using data from the account itself. The base year was 1973 (the same as the base year for the Private Car Comprehensive model) and the data arrays used were:

Non-Zero Claims

| Claim Size | Time 0 | .5 | 1 | 2 | 3 |
|---|---|---|---|---|---|
| 1 | 498 | 398 | 240 | 38 | 11 |
| 25 | 371 | 312 | 202 | 31 | 10 |
| 100 | 183 | 163 | 119 | 22 | 7 |
| 200 | 111 | 102 | 78 | 19 | 7 |
| 300 | 89 | 82 | 62 | 18 | 7 |
| 500 | 60 | 56 | 43 | 13 | 6 |
| 1,000 | 30 | 28 | 24 | 9 | 2 |
| 2,000 | 6 | 5 | 5 | 3 | 2 |
| 5,000 | 1 | 1 | 1 | 1 | 1 |

Zero Claims

| 0 | .5 | 1 | 2 |
|---|---|---|---|
| 255 | 230 | 131 | 15 |

All the above claims were settled within 3 years. Values of the M function at times later than 3 (2 for zero claims) were obtained by simple extrapolation.

Results

| | $r_{jk}$ | | | | $p_j$ |
|---|---|---|---|---|---|
| 1975 | 1.16 | 2.06 | 3.50 | - | .290 |
| 1976 | 1.10 | 2.32 | 3.02 | | .310 |
| 1977 | 1.06 | 2.16 | | | .294 |
| 1978 | 1.11 | | | | .270 |

## Numbers of Settlements

### Non-Zero Claims

| | Actual | | | | Modelled | | | |
|---|---|---|---|---|---|---|---|---|
| 1975 | 220 | 85 | 16 | 0 | 209 | 95 | 19 | 0 |
| 1976 | 180 | 82 | 9 | | 116 | 98 | 8 | |
| 1977 | 220 | 120 | | | 205 | 135 | | |
| 1978 | 259 | | | | 259 | | | |

### Zero Claims

| | Actual | | | | Modelled | | | |
|---|---|---|---|---|---|---|---|---|
| 1975 | 73 | 52 | 9 | 0 | 85 | 42 | 7 | 0 |
| 1976 | 60 | 62 | 2 | | 73 | 48 | 3 | |
| 1977 | 70 | 75 | | | 83 | 62 | | |
| 1978 | 94 | | | | 94 | | | |

### $b_{jk}$

### Development Year

| | 1 | 2 |
|---|---|---|
| 1975 | 1.04 | 1.22 |
| 1976 | 1.66 | 1.67 |
| 1977 | 1.70 | |

## Comparison of Means (Actual on Top)

| | Development Year 1 | Development Year 2 |
|---|---|---|
| 1975 | 305 | 613 |
| | 283 | 651 |
| 1976 | 396 | 728 |
| | 462 | 890 |
| 1977 | 427 | |
| | 454 | |

## Comparison of Distributions

| Notification Year | Development Year | | Claim Size 1 | 25 | 100 | 300 | 500 | 1000 | 2000 | 5000 | 10000 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 1975 | 1 | Actual | 85 | 74 | 47 | 21 | 13 | 7 | 1 | | |
| | | Model | 85 | 72 | 42 | 21 | 14 | 8 | 1 | | |
| | 2 | Actual | 16 | 14 | 9 | 5 | 4 | 4 | 2 | | |
| | | Model | 16 | 13 | 9 | 7 | 5 | 4 | 1 | | |
| 1976 | 1 | Actual | 82 | 76 | 40 | 21 | 17 | 8 | 5 | | |
| | | Model | 82 | 71 | 49 | 27 | 19 | 11 | 5 | | |
| | 2 | Actual | 9 | 9 | 7 | 5 | 5 | 2 | 1 | | |
| | | Model | 9 | 7 | 6 | 4 | 4 | 3 | 2 | | |
| 1977 | 1 | Actual | 120 | 113 | 70 | 43 | 30 | 18 | 6 | 1 | 1 |
| | | Model | 120 | 104 | 72 | 39 | 28 | 16 | 7 | 0 | 0 |

## THE APPLICATION OF THE MODEL TO A LARGE PORTFOLIO

A. DHR's model was applied to a fairly large class where there was over 20,000 claims notified per year. The class was private car comprehensive and the data used was for origin years 1972-76. This data was from another company than that from which the model was derived.

The model used was derived from private car comprehensive experience for base year 1973 and covered six years of development and claims up to £20,000. This allowed margins at the boundaries of the model in the later phases 2 and 3 when the model was applied to only five years of development and claims up to £10,000.

The terms and coefficients of the model are:-

| | Term | Coefficient | |
|---|---|---|---|
| Non-Zero | $x$ | -.63500 | 00 |
| | $x^2$ | .13240 | 01 |
| | $x^4$ | -.37301 | -01 |
| | $t$ | .10917 | 01 |
| | $xt$ | .43541 | 00 |
| | $x^2t$ | -.60653 | 00 |
| | $x^4t$ | .26142 | -01 |
| | $xt^2$ | -.55224 | -01 |
| | $x^2t^2$ | .71649 | -01 |
| | $x^4t2$ | -.33646 | -02 |
| | $t^2e^{-t}$ | -.26825 | 01 |
| | $t^{2.3}e^{-t}$ | .42790 | 01 |
| | $te^{-6t}$ | .13357 | 01 |
| | $x^4e^{-4x}e^{-2t}$ | .17927 | 02 |
| | $x^2e^{-4x}e^{-2t}$ | .14153 | 02 |
| | $x^{10.5}e^{-5x}t^6e^{-9t}$ | .59583 | 05 |
| | $x^{12}e^{-8x}t^6 3^{-9t}$ | -.17857 | 07 |
| | $x^{12}e^{-7x}t^4e^{-t}$ | .12887 | 02 |
| | $x^9e^{-4x}t^8e^{-3t}$ | .18687 | 01 |
| | $x^9e^{-3x}t^5e^{-3t}$ | .54616 | 00 |
| Zero | $t$ | .20851 | 01 |
| | $t^2e^{-3t}$ | -.10275 | 02 |

Continued ...

Phase 2

For years of notification 1972-76 the $r_{jk}$ and $p_j$ values which were obtained are:-

| | $r_{jk}$ | | | | | $p_j$ |
|---|---|---|---|---|---|---|
| 1972 | 1.08 | 2.47 | 3.58 | 4.76 | 5.47 | .188 |
| 1973 | 1.07 | 2.49 | 3.54 | 4.44 | | .187 |
| 1974 | 1.06 | 2.49 | 3.50 | | | .182 |
| 1975 | 1.08 | 2.36 | | | | .172 |
| 1976 | 1.08 | | | | | .160 |

The above rates differ from the company the model is derived from where the $r_{jk}$ are close to integer values.

A comparison of the actual numbers of claims settled in each development year with the corresponding numbers produced by the model gives the following tables:-

Numbers Settled

| | Actual | | | | | Modelled | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | Non Zero Settlements | | | | | | | | | |
| 1972 | 16160 | 6682 | 354 | 92 | 24 | 16044 | 6775 | 365 | 94 | 29 |
| 1973 | 20366 | 9173 | 429 | 96 | | 20400 | 9104 | 448 | 105 | |
| 1974 | 18635 | 8105 | 402 | | | 18446 | 8295 | 400 | | |
| 1975 | 16490 | 6890 | | | | 16427 | 6951 | | | |
| 1976 | 13914 | | | | | 13914 | | | | |
| | Zero Settlements | | | | | | | | | |
| 1972 | 3232 | 2081 | 69 | 6 | 3 | 3320 | 2004 | 62 | 6 | 0 |
| 1973 | 4197 | 2632 | 92 | 12 | | 4163 | 2687 | 75 | 7 | |
| 1974 | 3483 | 2523 | 60 | | | 3633 | 2367 | 65 | | |
| 1975 | 2991 | 1904 | | | | 3041 | 1854 | | | |
| 1976 | 2385 | | | | | 2385 | | | | |

Continued ...

The corresponding $b_{jk}$ values are:-

| | Development Year | | | |
|---|---|---|---|---|
| | 1 | 2 | 3 | 4 |
| 1972 | .91 | .94 | 1.06 | .87 |
| 1973 | 1.08 | .91 | 1.17 | |
| 1974 | 1.20 | 1.17 | | |
| 1975 | 1.42 | | | |

and the comparison of actual means with 80% estimated limits is:-

| | Development Year | | | |
|---|---|---|---|---|
| | 1 | 2 | 3 | 4 |
| 1972 | 176<br>174-184 | 516<br>478-619 | 1318<br>978-1519 | 1841<br>844-2162 |
| 1973 | 210<br>205-214 | 527<br>472-598 | 1242<br>986-1510 | |
| 1974 | 226<br>227-238 | 544<br>518-729 | | |
| 1975 | 271<br>264-277 | | | |

Another way of considering the fit of the model to the data is to compare the actual and modelled distributions by size of claims settled in each development year. This is done in the following table for development years 1 to 4 for 1972. The numbers of modelled claims have been grossed up or down so that the total number of claims is equal to the corresponding actual number.

| Development Year | | Numbers of claims settled at an amount greater than:- £ | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | 10 | 300 | 500 | 1000 | 2000 | 3000 | 5000 | 7000 | 10000 |
| 1 | Actual | 6682 | 1034 | 441 | 104 | 30 | 11 | 3 | 3 | 2 |
| | Model | 6682 | 1033 | 441 | 115 | 26 | 11 | 5 | 2 | 1 |
| 2 | Actual | 354 | 122 | 95 | 53 | 24 | 13 | 5 | 5 | 3 |
| | Model | 354 | 129 | 94 | 53 | 24 | 13 | 5 | 2 | 1 |
| 3 | Actual | 92 | 51 | 46 | 34 | 25 | 18 | 11 | 6 | 4 |
| | Model | 92 | 47 | 40 | 32 | 23 | 17 | 11 | 8 | 5 |
| 4 | Actual | 24 | 16 | 15 | 15 | 11 | 7 | 5 | 5 | 4 |
| | Model | 24 | 13 | 11 | 10 | 8 | 7 | 5 | 4 | 3 |

Continued ...

Phase 3

The B's calculated by combining the b's so as to correspond to equivalent groups rather than diverse calendar years are:-

$B_{jk}$

Group

| | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| 1972 | .90 | .92 | .98 | 1.02 |
| 1973 | 1.07 | 1.02 | 1.01 | |
| 1974 | 1.18 | 1.19 | | |
| 1975 | 1.38 | | | |

By looking at the past increase in the claims cost from year to year, the suggestion is that a rate around 17% could be applied for the future.

It was decided to use rates of inflation of (a) 15% and (b) 20% per annum when projecting the model forward to calculate reserves to show the effect of different rates of inflation.

The resulting mean reserves calculated as being required as at 31.12.76 were:-

| Year | (1) Mean reserves including payments made on account (a) at 15% £m | (1) (b) at 20% £m | (2) Less payments made on account £m | (3) Outstanding reserves (1) - (2) (a) at 15% £m | (3) (b) at 20% £m |
|---|---|---|---|---|---|
| 1973 | .51 | .51 | .12 | .39 | .39 |
| 1974 | .83 | .87 | .15 | .68 | .72 |
| 1975 | 1.36 | 1.48 | .24 | 1.12 | 1.24 |
| 1976 | 3.11 | 3.37 | .89 | 2.22 | 2.48 |
| Total | 5.81 | 6.23 | 1.40 | 4.41 | 4.83 |

Probability Limits

| | | | | | |
|---|---|---|---|---|---|
| 10% | 5.36 - 6.28 | 5.75 - 6.75 | | 3.96 - 4.88 | 4.35 - 5.35 |
| 5% | 5.25 - 6.43 | 5.63 - 6.91 | | 3.85 - 5.03 | 4.23 - 5.51 |

B. The interpolation method was used on the same data with the following results:-

| | $r_{jk}$ | | | | | $p_j$ |
|---|---|---|---|---|---|---|
| 1972 | 1.08 | 2.53 | 3.61 | 4.77 | 5.48 | .188 |
| 1973 | 1.06 | 2.55 | 3.59 | 4.44 | | .187 |
| 1974 | 1.06 | 2.55 | 3.52 | | | .182 |
| 1975 | 1.07 | 2.42 | | | | .171 |
| 1976 | 1.08 | | | | | .160 |

Numbers of Settlements

Non Zero Settlements

| | Actual | | | | | Modelled | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| 1972 | 16160 | 6682 | 354 | 92 | 24 | 16010 | 6787 | 385 | 97 | 28 |
| 1973 | 20366 | 9173 | 429 | 96 | | 20381 | 9102 | 472 | 106 | |
| 1974 | 18635 | 8105 | 402 | | | 18409 | 8312 | 419 | | |
| 1975 | 16490 | 6890 | | | | 16400 | 6974 | | | |
| 1976 | 13914 | | | | | 13915 | | | | |

Zero Settlements

| | Actual | | | | | Modelled | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| 1972 | 3232 | 2081 | 69 | 6 | 3 | 3336 | 1999 | 50 | 6 | 1 |
| 1973 | 4197 | 2632 | 92 | 12 | | 4180 | 2682 | 60 | 7 | |
| 1974 | 3483 | 2523 | 60 | | | 3640 | 2371 | 52 | | |
| 1975 | 2991 | 1904 | | | | 3047 | 1849 | | | |
| 1976 | 2385 | | | | | 2387 | | | | |

The corresponding $b_{jk}$ values are:-

| | Development Year | | | |
|---|---|---|---|---|
| | 1 | 2 | 3 | 4 |
| 1972 | .90 | .94 | 1.30 | .88 |
| 1973 | 1.08 | 1.00 | 1.23 | |
| 1974 | 1.22 | 1.27 | | |
| 1975 | 1.44 | | | |

Continued ...

The comparison of actual averages with 80% limits from the model are:-

| | Development Year | | | |
|---|---|---|---|---|
| | 1 | 2 | 3 | 4 |
| 1972 | 176<br>170-180 | 516<br>445-584 | 1318<br>1045-1594 | 1841<br>775-1851 |
| 1973 | 210<br>202-212 | 527<br>481-613 | 1242<br>957-1467 | |
| 1974 | 226<br>226-238 | 544<br>512-726 | | |
| 1975 | 271<br>265-279 | | | |

The comparison of distributions for 1972:-

| Development Year | Claim Size | 10 | 300 | 500 | 1000 | 2000 | 3000 | 5000 | 7000 | 10000 |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | Actual | 6682 | 1034 | 441 | 104 | 30 | 11 | 3 | 3 | 2 |
| | Model | 6682 | 1023 | 455 | 116 | 29 | 14 | 5 | 3 | 1 |
| 2 | Actual | 354 | 122 | 95 | 53 | 24 | 13 | 5 | 5 | 3 |
| | Model | 354 | 122 | 87 | 51 | 28 | 13 | 6 | 3 | 2 |
| 3 | Actual | 92 | 51 | 46 | 34 | 25 | 18 | 11 | 6 | 4 |
| | Model | 92 | 51 | 42 | 33 | 25 | 19 | 11 | 8 | 5- |
| 4 | Actual | 24 | 16 | 15 | 15 | 11 | 7 | 5 | 5 | 4 |
| | Model | 24 | 14 | 12 | 10 | 8 | 6 | 4 | 3 | 3 |

The B's calculated by combining the b's so as to correspond to equivalent groups rather than diverse calendar years are:-

$B_{jk}$

| | Group | | | |
|---|---|---|---|---|
| | 1 | 2 | 3 | 4 |
| 1972 | .92 | .91 | 1.05 | 1.22 |
| 1973 | 1.09 | 1.05 | 1.08 | |
| 1974 | 1.23 | 1.23 | | |
| 1975 | 1.45 | | | |

Continued ...

These reserves were calculated assuming (a) 15% and (b) 20% per annum future inflation.

The resulting mean reserves calculated as being required as at 31.12.76 were:-

| Year | Mean reserves including payments made on account (a) at 15% | Mean reserves including payments made on account (b) at 20% | Less payments made on account | Outstanding reserves (1) - (2) (a) at 15% | Outstanding reserves (1) - (2) (b) at 20% |
|---|---|---|---|---|---|
| | £m | £m | £m | £m | £m |
| 1973 | .55 | .55 | .12 | .43 | .43 |
| 1974 | .83 | .87 | .15 | .68 | .72 |
| 1975 | 1.39 | 1.51 | .24 | 1.15 | 1.27 |
| 1976 | 3.21 | 3.49 | .89 | 2.32 | 2.60 |
| Total | 5.98 | 6.42 | 1.40 | 4.58 | 5.02 |

Probability Limits

| | | | | | |
|---|---|---|---|---|---|
| 10% | 5.53 - 6.45 | 5.93 - 6.93 | | 4.13 - 5.05 | 4.53 - 5.53 |
| 5% | 5.41 - 6.60 | 5.80 - 7.09 | | 4.01 - 5.20 | 4.40 - 5.69 |

C. Outstanding reserves using the chain ladder adjusted and unadjusted for inflation using (1) settlements only and (2) all payments to date were performed on the same data.

The inflation assumptions used were:-

| | 73/72 | 74/73 | 75/74 | 76/75 | 77/76 & later |
|---|---|---|---|---|---|
| Earnings Inflation | 14.0 | 17.3 | 27.4 | 15.8 | (a) 15% (b) 20% |

(1) Considering the table of cumulative payments on settled claims only.

| Year | Development Year | | | | | incl. all payments o/s case estimates |
|---|---|---|---|---|---|---|
| | 0 | 1 | 2 | 3 | 4 | 5 |
| 1972 | 1681422 | 2895410 | 3141072 | 3348846 | 3455441 | 3900626 |
| 1973 | 2462920 | 4495506 | 4721604 | 4898365 | | |
| 1974 | 2496906 | 4329427 | 4629014 | | | |
| 1975 | 2470023 | 4317118 | | | | |
| 1976 | 2494697 | | | | | |

| Ratio | | | | | |
|---|---|---|---|---|---|
| Unadjusted | 1.7602 | 1.0658 | 1.0489 | 1.0318 | 1.1288 |
| Adjusted (a) at 15% | 1.6421 | 1.0500 | 1.0314 | 1.0177 | 1.0632 |
| Adjusted (b) at 20% | 1.6421 | 1.0500 | 1.0314 | 1.0177 | 1.0605 |

This results in the following:

| | (1) Outstanding including payments on account | | | (2) Less payments on account | (3) Outstanding reserves (1) - (2) | | |
|---|---|---|---|---|---|---|---|
| | Unadjusted | Adjusted (a) | Adjusted (b) | | Unadjusted | Adjusted (a) | Adjusted (b) |
| | £m | £m | £m | £m | £m | £m | £m |
| 1973 | .81 | .81 | .85 | .12 | .69 | .69 | .73 |
| 1974 | 1.03 | .98 | 1.06 | .15 | .88 | .83 | .91 |
| 1975 | 1.30 | 1.19 | 1.31 | .24 | 1.06 | .95 | 1.07 |
| 1976 | 3.22 | 3.03 | 3.29 | .89 | 2.33 | 2.14 | 2.40 |
| Total | 6.36 | 6.01 | 6.51 | 1.40 | 4.96 | 4.61 | 5.11 |

(2) Considering the table of cumulative payments on all claims.

| Year | Development Year 0 | 1 | 2 | 3 | 4 | incl. o/s case estimates 5 |
|---|---|---|---|---|---|---|
| 1972 | 1951972 | 2872040 | 3218530 | 3519066 | 3718731 | 3900626 |
| 1973 | 2963018 | 4456826 | 4753559 | 5022866 | | |
| 1974 | 2995346 | 4404323 | 4781972 | | | |
| 1975 | 3281331 | 4553366 | | | | |
| 1976 | 3383843 | | | | | |

| | | | | | |
|---|---|---|---|---|---|
| Ratios Unadjusted | 1.4552 | 1.0870 | 1.0715 | 1.0567 | 1.0489 |
| Adjusted (a) at 15% | 1.3896 | 1.0655 | 1.0453 | 1.0317 | 1.0243 |
| Adjusted (b) at 20% | 1.3896 | 1.0655 | 1.0453 | 1.0317 | 1.0233 |

This results in the following:

| Year | Outstanding Claims Reserves Unadjusted £m | Adjusted (a) £m | Adjusted (b) £m |
|---|---|---|---|
| 1973 | .45 | .55 | .58 |
| 1974 | .82 | .87 | .94 |
| 1975 | 1.22 | 1.23 | 1.34 |
| 1976 | 2.84 | 2.82 | 3.07 |
| Total | 5.33 | 5.47 | 5.93 |

D. Summary of Results

(1) At 15% inflation

| | D.H.R.'s Method | | Chain Ladder | | | | Outstanding as at 31.12.76 bas on payments o/s to 31.12.78 |
|---|---|---|---|---|---|---|---|
| | | | Based on Settlements | | Based on all payments | | |
| | Formula | Interpola-tion | Unadjusted | Adjusted | Unadjusted | Adjusted | |
| | £m | £m | £m | £m | £m | £m | £m |
| 1972 | | | | | | | .17 |
| 1973 | .39 | .43 | .69 | .69 | .45 | .55 | .33 |
| 1974 | .68 | .68 | .88 | .83 | .82 | .87 | .83 |
| 1975 | 1.12 | 1.15 | 1.06 | .95 | 1.22 | 1.23 | .87 |
| 1976 | 2.22 | 2.32 | 2.33 | 2.14 | 2.84 | 2.82 | 2.22 |
| Total | 4.41 | 4.58 | 4.96 | 4.61 | 5.33 | 5.47 | 4.25 |

Probability Limits

10% 3.96-4.88 4.13-5.05
5% 3.85-5.03 4.01-5.20

(a) At 20% inflation

| | D.H.R.'s Method | | Chain Ladder | | | | Outstanding as at 31.12.76 bas on payments o/s to 31.12.78 |
|---|---|---|---|---|---|---|---|
| | | | Based on Settlements | | Based on all payments | | |
| | Formula | Interpola-tion | Unadjusted | Adjusted | Unadjusted | Adjusted | |
| | £m | £m | £m | £m | £m | £m | £m |
| 1972 | | | | | | | .17 |
| 1973 | .39 | .43 | .69 | .73 | .45 | .58 | .33 |
| 1974 | .72 | .72 | .88 | .91 | .82 | .94 | .83 |
| 1975 | 1.24 | 1.27 | 1.06 | 1.07 | 1.22 | 1.34 | .87 |
| 1976 | 2.48 | 2.60 | 2.33 | 2.40 | 2.84 | 3.08 | 2.22 |
| Total | 4.83 | 5.02 | 4.96 | 5.11 | 5.33 | 5.93 | 4.25 |

Probability Limits

10% 4.35-5.35 4.53-5.53
5% 4.23-5.51 4.40-5.69

Commentary

The above results are not meant to prove one method better than another which could not be done by one example anyway. It is intended to show the different estimates obtainable using the same data. The inadequacy of the data may affect the results to a greater or lesser degree depending on the method used.

D.H.R.'s method requires far more detailed information than the chain ladder method. The information provided was not adequate in that the base year 1972 did not have a complete run-off. The large claims data for the original company was different from the company the formula was applied to so that the proportion of large claims and the parameter of the pareto density were changed to be more representative of the data being worked on.

The above shows that a good fit can be obtained by applying a formula for one company's data using D.H.R.'s method to the same class using another company's data with only the R's and B's changing to allow for different settlement rates and claims inflation. Taking this a stage further, if a formula for a class of business with a large volume of data was devised then it could be applied to this class of business for any company with only the R's and B's changing.