

## ARTHUR MORGAN'S MORTALITY EXPERIENCE

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ARTHUR MORGAN'S mortality experience, which was referred to in the paper by Dr Seal (*J.I.A.* 85, 165) was a pioneer effort in this field and since it has been much misunderstood some notes on the way in which it was compiled may be of interest.

The statistics were contained in a pamphlet whose title-page describes what the statistics were intended to show, the following being a transcript of the title page:

## TABLES

SHOWING

THE TOTAL NUMBER OF PERSONS ASSURED IN

## THE EQUITABLE SOCIETY

FROM ITS COMMENCEMENT IN SEPTEMBER 1762, TO JANUARY 1, 1829,  
DISTINGUISHING THEIR AGES AT THE TIME OF ADMISSION INTO THE SOCIETY,  
AND EXHIBITING

THE NUMBER OF YEARS DURING WHICH THEY HAVE CONTINUED MEMBERS OF IT;

THE PERIODS OF LIFE AT WHICH THEIR ASSURANCES HAVE TERMINATED;

AND

THE AGES WHICH THE SURVIVING MEMBERS HAD ATTAINED ON THE 1st OF JANUARY 1829  
TO WHICH ARE ADDED

## TABLES

OF THE

PROBABILITIES AND EXPECTATIONS OF THE DURATION OF HUMAN LIFE,  
DEDUCED FROM THESE DOCUMENTS;

A STATEMENT OF THE DISORDERS, (AS CERTIFIED TO THE COURT OF DIRECTORS,)

OF WHICH 4095 PERSONS ASSURED HAVE DIED IN THIRTY-TWO YEARS ENDING DECEMBER 31, 1832;

AND A SUPPLEMENT SHOWING THE MORTALITY OF THE SOCIETY

FOR THE YEARS 1829, 1830, 1831 AND 1832

The tabulations were not designed to produce an exposed-to-risk in the way in which a modern mortality investigation would be arranged. They were primarily designed to show the durations of the lives of persons assured with the Equitable Society. Morgan had in mind to show out of the entrants at any given age the number who attained each succeeding year of age, which was the general procedure of Deparcieux, to whom Morgan refers.

On the following page is an extract from Morgan's Table A. As he explains in the Introduction, the table is formed by a continuous process. Under any given age attained, and against the specified age at entry for the row, there are

Extract from Table A showing the duration of *all* the lives assured in the Equitable Society, from September 1762 to 1 January 1829

Age 29				Age 30				Age 31				Age on admission
Attained the above age	Living Jan. 1829, at the above age	Discontinued their Assurances	Died	Attained the above age	Living Jan. 1829, at the above age	Discontinued their Assurances	Died	Attained the above age	Living Jan. 1829, at the above age	Discontinued their Assurances	Died	
5	—	—	—	5	—	—	—	5	1	—	—	7
9	1	1	—	8	—	—	—	8	—	—	—	8
2	—	1	—	1	—	—	—	1	—	—	—	9
6	—	—	—	6	—	—	—	4	—	—	—	10
12	1	—	—	11	1	2	—	10	—	—	—	11
14	—	—	—	14	—	—	—	14	2	1	—	12
15	1	—	1	13	1	—	—	12	1	—	—	13
17	2	1	—	14	2	—	—	12	—	—	—	14
6	—	—	—	6	1	1	—	4	—	—	—	15
22	2	—	—	20	1	3	—	17	1	1	—	16
11	—	—	—	11	—	1	—	10	1	—	—	17
20	1	1	—	18	—	—	—	18	—	1	—	18
26	—	—	—	26	—	6	—	20	2	—	—	19
50	1	3	1	45	2	2	1	40	1	—	—	20
137	3	10	—	124	—	5	—	119	3	9	1	21
104	4	11	2	177	3	13	1	160	1	13	1	22
261	7	19	3	232	4	11	2	215	2	13	—	23
336	1	20	3	314	6	19	3	280	2	19	3	24
462	7	32	2	421	10	29	2	380	5	14	5	25
481	6	22	2	451	7	26	7	411	7	13	4	26
624	10	54	3	557	6	31	5	515	6	29	4	27
726	8	40	2	676	7	70	4	595	3	37	4	28
783	—	—	3	780	1	45	5	729	7	65	4	29
—	—	—	—	702	—	1	2	759	4	30	4	30
—	—	—	—	—	—	—	—	785	—	—	2	31
—	—	—	—	—	—	—	—	—	—	—	—	32
4219	55	214	20	4692	51	265	32	5129	49	251	32	32

stated in successive columns the number of persons who attained the given age, the numbers who were existing at that age on 1 January 1829 and the numbers of discontinuances and deaths at that age. By subtracting the last three items from the first, the number of persons who attain the next succeeding age is found. Morgan explains the procedure by taking as an example the figures tabulated under the age of 30 years and opposite the age of 25 on admission. The table shows that out of the entrants at this age there were 421 persons attaining the age of 30 years. There were 10 persons existing at age 30 on 1 January 1829, 29 persons discontinued their assurances and 2 died. The total deduction was thus 41, which left 380 persons attaining the age of 31 years.

This simple explanation is easy to follow but it is not easy to discover the exact significance of the figures. Some of the ages are undefined and some of the explanations are inconsistent, leading to much confusion. So far as is known, Arthur Morgan never gave any other explanation of the figures than is given in the Introduction and Tables.

T. R. Edmonds, writing to *The Lancet* in 1837, assumed that the experience was of completed policy-years, ending with the policy anniversary in 1829, and that the age on admission was the age next birthday, so being overstated by half a year on the average. He constructed a table which showed, for all ages at entry combined, the rate of mortality for each year of duration. The deaths recorded at duration 0 were assumed to be a full year's deaths and the rate of mortality was about 40% only of the rates at durations 1 to 4.

William Spens, writing in 1861, quotes T. R. Edmonds as saying in a subsequent letter to *The Lancet* (1842) that the age next birthday at entry was assumed to be attained at the end of the year of entry, the mortality in the calendar year of entry (a fraction of duration 0) being ignored. By studying the discontinuances, Spens gives reasons for thinking that the discontinuances and deaths which are tabulated at the same age as the age on admission relate to the fractional period, averaging one-half, of the calendar year of entry.

James Chatham, writing in 1890, thought that the existings, being tabulated at the beginning of the year 1829, should be deducted from the living at the same age so as to exclude them from the exposed to risk. It is true that the tables are headed in a manner which implies that the experience came to an end on 1 January 1829. However, the Introduction specifically states that the mortality of 1829 was included and Morgan gives a numerical example of the calculation of a rate of mortality (in the form  $q^{-1}$ ), which does *not* deduct the existings but treats them as being exposed to risk for that year.

H. W. Manly, about 1908, prepared a survey of the various mortality experiences of the Equitable Life Assurance Society up to the experience of 1863-93, a task which, he said, had occupied his spare time for the past 13 years. The survey was never published but Manly used some of the information in his paper to the Seventh International Congress of Actuaries in 1912. Manly concluded that Arthur Morgan's experience really covered the period from 1 January 1763 to 31 December 1829 and was compiled, in effect, by the calendar year method which was used for the Seventeen Offices' experience and the Institute's H<sup>m</sup> experience. Manly thought that the age next birthday at entry was assumed to be attained at the end of the year of entry and that the period from entry to the end of the calendar year of entry was ignored.

Arthur Morgan's description in the Introduction implies that he worked from exact age  $x$  to exact age  $x + 1$  and specifically states that he used *all* the lives assured. When reviewing the evidence in 1943, Sir William Elderton and

M. E. Ogborn came to the conclusions that the experience should be regarded as a life-year method, as implied by Morgan's own words, and that the deaths at the same age as the age at entry were those occurring before the next birthday, i.e. approximately half a year's deaths, but that this blemish was hidden from Morgan because he calculated the mortality in groups of 5 years (actually  $4\frac{1}{2}$  years on average).

Dr Seal's discovery, that the recorded deaths appear to be not a random distribution but one which has been smoothed or graduated in some way, led us to re-examine carefully the surviving records in the hope of discovering how the experience had been constructed.

So far as can be traced there was no record of the date of birth. The primary fact in the records was the age at entry, usually the age next birthday. However, the age at entry used for the mortality experience was the age last birthday, so that when Morgan referred to the age on admission he meant literally what he said, that being the age already attained.

Records were kept of the various assurances made (though not in the form of a policy register) and separate records of the deaths and discontinuances. The deaths were, of course, in the order of the date of death (actually, the date of proof of claim); the procedure was to go through the records of deaths allocating each to the appropriate age at entry, so that the tabulation of deaths emerged in the form of an analysis primarily by age at entry and, within each age, by date of death. The method facilitated the exclusion of duplicate policies and where a member had more than one policy the earliest would be taken for the purposes of the experience.

The records of deaths, discontinuances and existings would be tabulated under the appropriate ages attained for each age at entry. Starting with the oldest attained age, the respective numbers in the row for (say) age 25 at entry would be added backwards so as to obtain the number of those entrants who were living at each attained age. The process would ultimately produce the number of entrants at the given age of 25 years, which number could be checked directly from the records of entrants. This is the kind of way that the Morgans, father and son, arranged their work so as to be self-checking. An independent check was important because it is unlikely that Arthur Morgan could have had any other satisfactory check on the work. The entries can be verified where the dates of birth, admission and death are known; a number of mistakes have been found, especially in the earliest entries where the original records might well have been unsatisfactory for the purpose of the investigation.

In a strict life-year method, it would be necessary to tabulate the deaths in each year of age. This was impossible because the date of birth was unknown and the age at death had to be found by adding the duration to the age at entry. The duration was computed in such a way as to allocate the deaths, on the average, to the year of age instead of the policy-year.

The analysis of deaths included the month and year of entry and the month and year of death. The duration was found by subtracting the one from the other, so that the duration comprised an integral number of years and a number of months which might range from 0 to 11 months. The analysis also includes the 'age at death' which, though not defined, was calculated from the age at entry by adding the duration taken to the next lower integer where the number of months was 0-6, and taken to the next higher integer where the number of months was 7-11. The assumption seems to have been that if the fractional part of the duration was 0-6 months the member died before the

birthday in that policy-year, whereas if the fractional part of the duration was 7-11 months the member died after the birthday. There is no record of why Morgan adopted this rather strange procedure but the resulting ages at death have been verified in a sufficient number of instances to establish the fact.

As an example where the date of birth is known, there is the case of Charles James Fox who was born on 24 January 1749 and died on 13 September 1806. Assurances on his life were effected in June 1782 at age 34, in December 1791 at age 43 and in September 1792 at age 45, these being the ages next birthday except for the last which was overstated by one year. In the manuscript analysis of deaths this life can be identified amongst those recorded at age 33 at entry, i.e. the age last birthday at the time of the first of the assurances. The duration at death was 24 years 3 months and the age at death 57 years (i.e.  $33 + 24$ ).

The way in which the duration was calculated has a curious effect at duration 0. In respect of those who entered on the first day of the month a full seven months' deaths (i.e. the month of entry and the six following months) were treated as being at duration 0, whereas for those who entered on the last day of the month the mortality at duration 0 included six months' deaths only. Thus the mortality at duration 0 related to  $6\frac{1}{2}$  months on the average. Spens gives figures which show that the percentage of the first year's to the second year's mortality was 72% in an American experience, 74% in the Scottish Amicable and 42% in the Equitable; if the last-named is related to  $6\frac{1}{2}$  months' exposure the percentage becomes 77%, a much more likely figure.

There is no surviving evidence of how the discontinuances were treated but it is reasonable to assume that they were dealt with in the same way as the deaths.

Further evidence about the existings at close of observations has come to light and it now seems possible to complete the picture of what was done.

In Morgan's Table A the existings are simply headed 'Living Jan. 1, 1829, at the above age' and the method of calculating the age is undefined. In the Introduction Morgan says of the columns of existings:

In the second sub-division is the number of persons of each class (i.e. age at entry) who had on 1 January 1829 attained, and were living at, the several ages stated at the head of these columns respectively.

Using the example quoted earlier in this note, Morgan says '10... were existing on 1 January 1829, and would enter upon their 31st year of age in the course of that year', that is to say their thirtieth birthdays were in 1829 and the tabulation of existings was at age next birthday on 1 January 1829. This, however, is incorrect as will appear.

For many years the Society's policies were valued individually, valuations being at decennial intervals, but on 1 January 1829 the assurances were classified by age attained and a continuous classification was started. This classification seems to be the obvious starting point for the mortality investigation though duplicate policies would have to be excluded. Comparison of the assurances in force on 1 January 1829 with the existings tabulated in the mortality investigation confirms this assumption. The comparison was made at the oldest ages where the data were few; the figures agree very well.

The classification of assurances in force gives the age (undefined) at the head of the page with totals of the policies for each year of entry, totals of sums assured, bonus additions and premiums but, of course, neither policy number nor age at entry. It seems reasonable to assume, however, that the age was

the age last birthday at entry plus the duration defined as 1829 *minus* the year of entry, i.e. in effect, that the age next birthday at entry would be attained at the end of the year of entry. The assurances in force at the oldest ages correspond exactly with the existings in the mortality experience when the ages and durations are calculated in this way and allowance is made for what are obviously duplicate assurances on one life; the few discrepancies are too large to be accounted for by some other method of calculating the ages and durations.

The foregoing comparison makes it virtually certain that the existings were tabulated at an age attained which was calculated in the same way as the classic fiction of the assurances. It was, in effect, an approximation to exact age on 1 January 1829, being the age actually attained either in 1828 or 1829 with about an equal distribution in each of these years. It is more important to relate the method of calculating the ages of the existings to the method for the deaths and this is less clear.

Using Morgan's example once again, consider the group who entered in the year 1824 at the age of 25 years. The survivors of this group would be tabulated as existings at the age of 30 years on 1 January 1829. The deaths tabulated at the age of 30 years would be those which occurred at durations 4 years 7 months to 5 years 6 months, as in the following scheme:

Entrants in January 1824	Deaths from August 1828 to July 1829
February 1824	September 1828 to August 1829
	.....
June 1824	January 1829 to December 1829
July 1824	February 1829 to December 1829
	.....
December 1824	July 1829 to December 1829

It will be seen that the later groups were truncated because no deaths that occurred after the close of 1829 were included. Among the July entrants, the deaths related to 11 months; among the August entrants, to 10 months; and so on, the total deficiency on an even distribution being 7/48ths or about one-seventh. The proportion to the total deaths in this group for all years of entry would, of course, be much smaller.

Entrants in January to May 1824 could have died in 1829 at durations 5 years and 7 months or more, and would be treated as having died at age 31, though nothing would be included in the exposed to risk for this group at that age. The proportion of such deaths would be 5/48ths, which taking one age with another would tend to counterbalance the deficiency mentioned in the preceding paragraph.

The difference between 7/48ths and 5/48ths, namely, 2/48ths or one-half of a month, simply reflects the fact that 6½ months', and not 6 months', deaths were included in duration 0. In fact, the entrants in the year 1829 - *t* contributed, on the average, *t* + ½ years of exposure to the experience but 6½ months were included at duration 0 and (taking one age with another) 11½ months at duration *t*, though both fractions were taken by Morgan as being complete years. The mistake was hidden from him by his use of 5-year groups, where it would be less noticeable.

It is clear that the numbers existing at age 30 on 1 January 1829 were assumed to be exposed to risk for the whole year to age 31. It is, however, far from clear how the deaths in 1829 were treated. If they are included in the statistics of Table A, a person who was existing on 1 January 1829 and who died

in 1829 at the same age would be deducted twice or, if the table was constructed backwards as has been suggested, would be counted twice in those attaining the given age. Such an elementary error is unlikely, especially in view of the independent check on the number of entrants. The deaths in 1829 may have been deducted from the existing on 1 January 1829, so that they were not included in both, or the tabular deaths may have been those up to the end of 1828, as stated in the headings, the deaths of 1829 being brought into account in the calculation of the rates of mortality.

The total of the deaths recorded in the published table was 5144. A count of the deaths recorded in the manuscript book of deaths yields a total of 4953 up to 1828, to which should be added the 154 deaths in 1829, i.e. 5107 in all. The deficiency of 37 may be due to the correction of errors or, possibly, to deaths among temporary and survivorship assurances, if the manuscript book is confined to whole life assurances. The figure of 5107 agrees with a note at the beginning of the book recording that number of deaths 'up to 1 January 1829'. This curious phrase may be explained by Morgan's tendency to regard the mortality experience as consisting of blocks of 1 year each, a whole year's experience beginning on 1 January being added each year to the totals of the earlier experience.

In view of what has been said it seems likely that the total of 5144 deaths included the deaths of 1829 and that the existings on 1 January 1829 were adjusted to exclude those who died in 1829. This view is supported by Morgan's statement in reference to his Table B (which excluded discontinuances) that it

contains an account of the duration of the lives of those persons *only*, who, having effected assurances in the Society between September 1762, and January 1, 1829, either lived and continued their assurances to the close of the latter year, or died in the intervening period.

It may also be noted that Morgan referred to the experience as relating to a period of more than 67 years, which implies to the end of 1829.

The deaths appear to have been submitted to some smoothing process though it is impossible now to say exactly what was done. The deaths recorded in the manuscript book for all ages attained from 10 to 46 have been compared with the corresponding deaths in the printed table and the totals agree at each attained age in both records with the exception of ages 21, 22, 23, 39, 40, 44, 46, where there are small counterbalancing differences; we may give Arthur Morgan the benefit of the doubt and assume that the printed figure is the correct one.

An examination of the recorded deaths at ages 10-46 suggests that, within each attained age, the numbers of deaths were adjusted empirically so as to give a smoother distribution by age at entry. The following is a comparison at selected attained ages.

Age at death		Numbers of deaths at successive ages at entry
26	Manuscript	1, 3, 1, 7, 2, 7, 1
	Printed	2, 2, 2, 7, 3, 5, 1
30	Manuscript	0, 0, 1, 1, 4, 1, 7, 3, 5, 8, 2
	Printed	1, 0, 1, 2, 3, 2, 7, 5, 4, 5, 2
31	Manuscript	1, 1, 0, 1, 5, 5, 3, 5, 5, 4, 2
	Printed	1, 1, 0, 3, 5, 4, 4, 4, 4, 2

Though these differences are capable of being explained by some difference in the method of calculating the ages in the two records, it seems much more likely that the printed numbers have been adjusted to give a smoother series.

Comparison of the deaths along the rows of Table A, i.e. at each age at entry, does not give the same type of picture. It is intrinsically unlikely that the deaths would have been adjusted along the rows because that would have meant transferring deaths from one age at death to another. Though the deaths within each attained age have been transferred from one age at entry to another, the transfer was probably made in such a way that the totals of the rows for each age of entry were altered as little as possible.

Where the manuscript gives a large number of deaths at a particular age and duration, the printed table also shows a large, though usually somewhat smaller, number; for example, the deaths at age 39, among entrants at age 37, were recorded as 22 in the manuscript, 18 in the printed table. This would be impossible if the deaths had been averaged in groups of 3 or 5 ages. It may be asked whether the transfer of deaths from one age at entry to another has introduced any bias into the select experience. There does seem (e.g. at attained ages 40, 60 and 80) a marked tendency for the deaths in the manuscript to exceed the deaths in the printed tables at the shorter durations and to fall short of them at the longer durations. Such a bias would tend to accentuate the apparent effect of selection in the printed tables. In view of this tendency, a comparison made by Manly is of considerable interest. It compares the actual deaths at the first seven durations with those expected by the aggregate experience, as graduated by Manly.

Select mortality of 1762-1829 compared with the expected by the graduated aggregate mortality

(All ages combined)

Year of assurance	Actual deaths	Expected deaths
1	103	236.2
2	241	239.1
3	235	229.2
4	212	217.9
5	198	210.5
6	201	204.9
7	212	200.5

It seems difficult to find in such data any evidence of select mortality, save in the first year, though Manly thought he could see the evidence of a continuing effect due to selection. Taking the first year as being  $6\frac{1}{2}$  months only, the expected deaths would be 128 compared with the actual deaths of 103, about 80%.

The transfer within each age at death does not affect the aggregate mortality in any way, but the select mortality would be affected both by the adjustment in the deaths and a consequent alteration (relatively smaller in magnitude) in the exposed to risk. Morgan would have thought of this in much the same light as we should regard a graduation in the mortality, though we should apply the process at a later stage. The deaths at duration 0 are identical in both

records for ages 10-46 and it looks as if the select mortality at duration 0 can be relied upon.

Morgan felt that the data were too few to yield reliable rates of mortality when analysed by age at entry and he remarks:

there is a variance observable in the rates of mortality at different ages when estimated in this manner which renders it scarcely possible to graduate a table from them.

It should be remembered that the concept of a rate of mortality,  $q_x$ , which could be graduated, was then only just emerging. In those days the 'rate of mortality' meant the table of  $l_x$  and  $d_x$ , which was smoothed empirically; John Finlaison was the first to calculate  $q_x$  and to graduate this function, when he was investigating the mortality experiences of the tontines and of Government life annuitants in the 1820s. Thus, Arthur Morgan would be thinking of his table in much the same light as the life table, rather than as the raw material from which a graduated rate of mortality, in the modern sense, could be produced. No doubt he would have regarded it as his duty to produce a table which gave a reasonable picture of the durations of the lives but the adjustment should have been mentioned.

Arthur Morgan calculated two tables of mortality. Table A was produced by the more or less conventional procedure which has been described in this note. Table B was produced by excluding from the exposed to risk the whole of those lives whose assurances were ultimately discontinued within the period of observation.

At first sight, it is difficult to see the purpose of Table B. Morgan explains it as showing the mortality in a population which is continually kept up to the original number by the replacement of those who discontinue their assurances. The explanation does not seem helpful because if the discontinuances were replaced by similar lives it would have no effect; but Morgan, no doubt, had some conception which we find difficult to grasp because our ways of thought are different. As he indicates, Morgan was thinking of the difference between select and ultimate mortality and, in modern terms, might have justified Table B on the following lines.

The conventional method represented by Table A assumes that the same average rate of mortality applies to the whole of the lives exposed to risk, including those who ultimately discontinue their assurances. However, it could be argued that those whose assurances are ultimately discontinued are subject throughout to a lighter rate of mortality, being a select group and possibly one which is continuously select. The actual rate of mortality in this group cannot be known but if these lives be assumed to be subject to zero mortality (which is, in effect, the assumption underlying Table B) the rate of mortality in the remainder of the exposed to risk could be regarded as being an upper limit. Hence, Table A and Table B give lower and upper limits to the true ultimate mortality.

Unfortunately for this argument, some of the assurances were temporary assurances where discontinuance might be merely the expiry of the period of the assurance; but the proportion of such assurances was small. Probably, however, actuaries nowadays would be sceptical of the approach represented by Table B.

Though, no doubt, criticisms could be made of much that Arthur Morgan did, the mortality experience was a worthy pioneer effort in this field.

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