Compartmental Reserving

a process-based Bayesian reserving approach

Jake Morris — Liberty Specialty Markets

Rob Murray — LCP

22 October 2015

Agenda

- Background
  - Motivations
- Methodology
- Case studies
  - Semi-Bayesian
  - Fully Bayesian
- Conclusions

Overview

Features

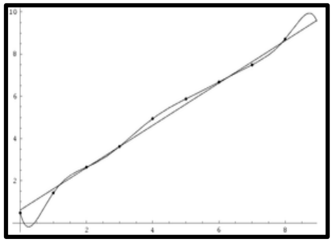
- Intuitive parameters including **case reserve robustness measure**
- Provides coherent measure of **reserve uncertainty**
- Supports **negative development**
- Can capture **calendar effects**
- **Independent** of DFM / BF
- Incorporates **judgement**

Models the claims generation *process*

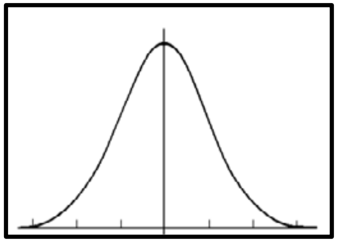
Background

Motivations

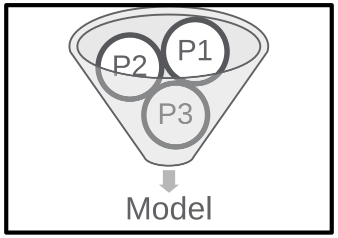
1. **Parsimony**
   – extract signal from noise
   – description of individual cohort vs. average



2. **Quantification of reserve uncertainty**
   – incorporate multiple information sources
   – isolate drivers of uncertainty



3. **Interpretability & Extensibility**
   – meaningful parameters
   – option to capture specific process features



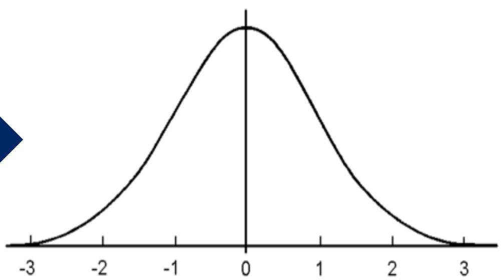
Background

1. Parsimony

Mixed-effects / hierarchical modelling

Cohorts

Parameters a *mixture* of those varying across cohort and those not*



<table>
<tr><th>Cohort</th><th>P_1</th><th>P_2</th><th>P_3</th><th>P_4</th></tr>
<tr><td>1</td><td>$P_{1,1}$</td><td rowspan="4">P_2</td><td>$P_{3,1}$</td><td rowspan="4">P_4</td></tr>
<tr><td>2</td><td>$P_{1,2}$</td><td>$P_{3,2}$</td></tr>
<tr><td>…</td><td>…</td><td>…</td></tr>
<tr><td>N</td><td>$P_{1,N}$</td><td>$P_{3,N}$</td></tr>
</table>

Only estimate mean and s.d. of the variable parameters

*Also known as a mixture of *random effects* and *fixed effects*

Background

2. Reserve uncertainty

Objective:

"***Given*** *any value (estimate of future payments) and* ***our current state of knowledge****, what is the probability that final payments will be no larger than the given value?*"

- Casualty Actuarial Society (2004)
Working Party on Quantifying Variability in Reserve Estimates

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Bayes' theorem:

$$p(\theta \mid y) \propto L(\theta; y)\, p(\theta)$$

Posterior ∝ Likelihood x Prior

$$\boxed{p(ULR \mid incurred) \propto L(ULR; incurred)\, p(ULR)}$$

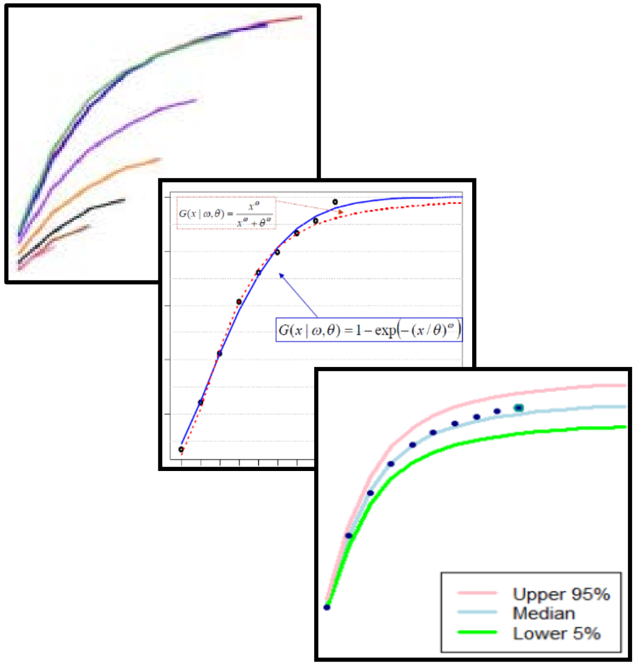
Background

Existing research

- These concepts have been applied to loss reserving:

A Bayesian non-linear model for forecasting insurance loss payments

Yanwei Zhang Vanja Dukic James Guszcza



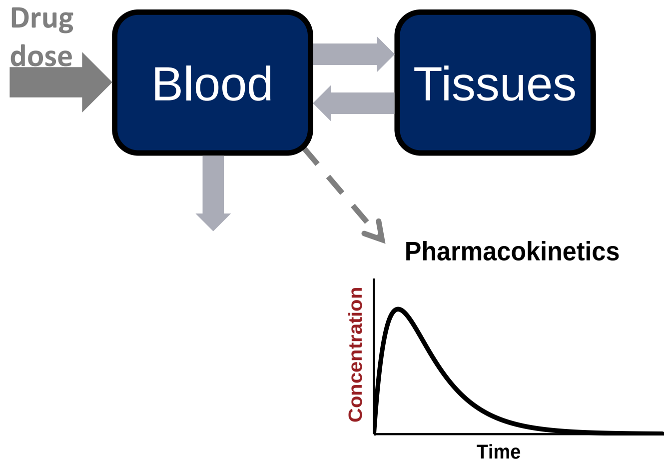
Key idea: fit a nonlinear parametric curve to cumulative paid triangles in a mixed-effects modelling framework

Background

3. Interpretability/Extensibility

- Models provide control over complexity (vs. methods)
- Drug developers use modelling & simulation to predict exposure/response:

“Compartmental” Pharmacokinetic models



Meaningful parameters and extensibility

Reformulate to model claims generation *process*

Methodology

Compartmental loss reserving model

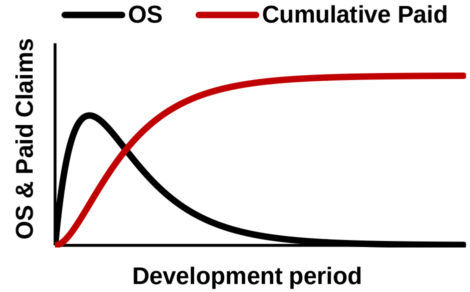
Structural model

Premiums Written → Exposed to risk → Claims reported → Claims paid

- Cash flows between compartments governed by ODEs*

- Fit to paid and outstanding triangles
  - Simultaneously
  - Explicitly estimating tails

Supports negative development



*ODEs: a collection of simultaneous Ordinary Differential Equations

Methodology

Parameters

Parameters have natural interpretations

Reported loss ratio ("**RLR**")

Rate of earning + reporting ("$\mathbf{k_{er}}$")

Reserve robustness factor ("**RRF**")

Rate of payment ("$\mathbf{k_p}$")

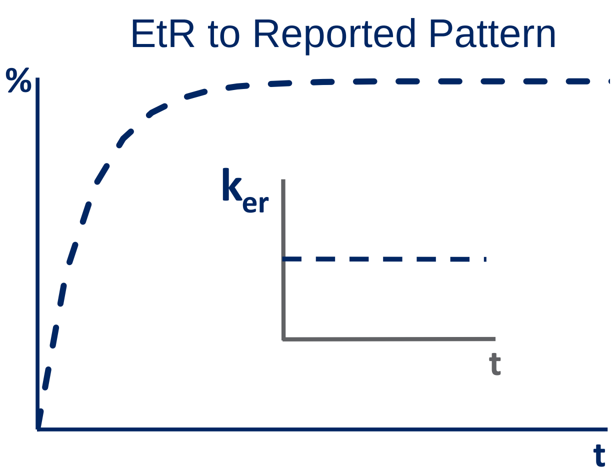
$$\mathbf{ULR} = \mathbf{RLR} * \mathbf{RRF}$$

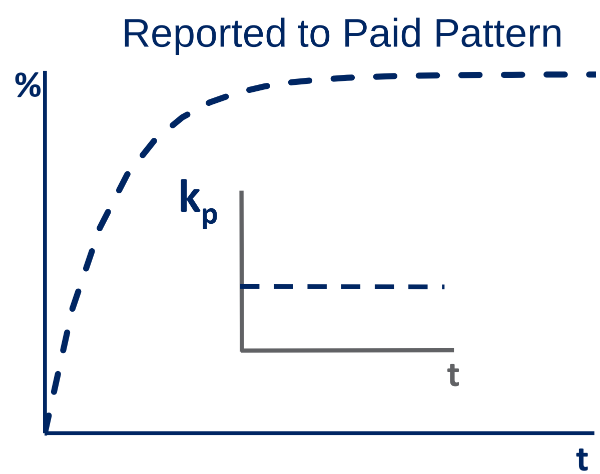
Estimate parameters in a mixed-effects framework

Methodology

Rates → Patterns

Pattern % = 1 – $e^{-rate*t}$



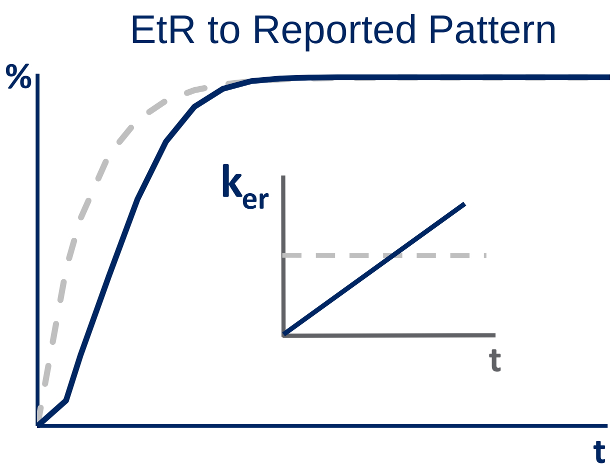


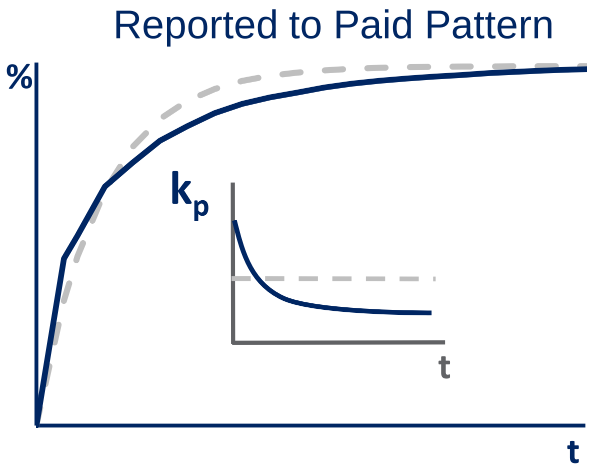
t = development time

Methodology

Rates → Patterns

Pattern % = 1 – $e^{-rate(t)*t}$



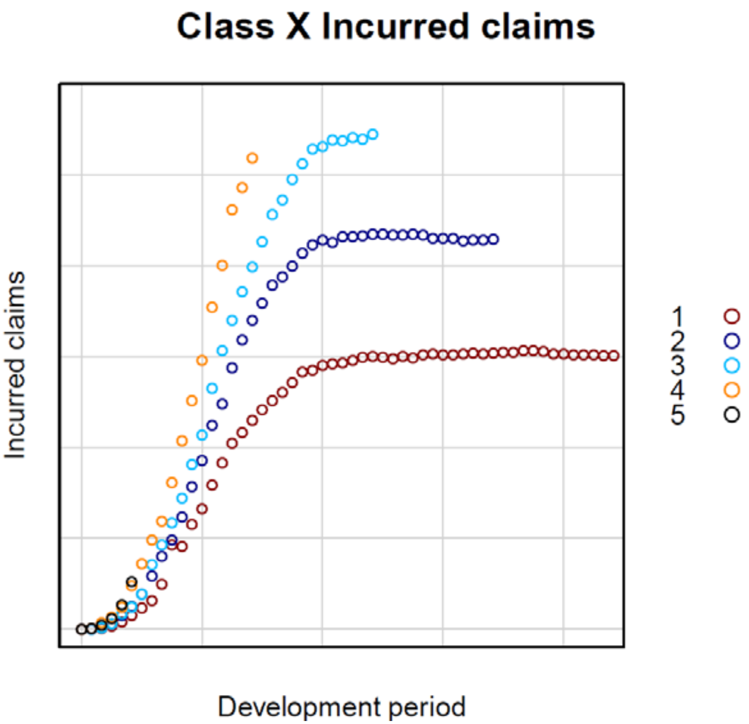


t = development time

Case Study 1

LSM

- **Class X**
  - Underwriting cohorts (1 – 5)
  - Ultimate premiums & writing patterns
  - Paid and incurred claims development



- **Objectives**
  - Fit **semi-Bayesian*** compartmental model
  - Extrapolate fits to ultimate
  - Compare ULRs to LSM

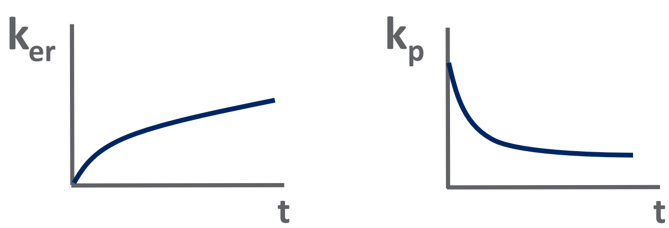
*Specify parameter starting values but not distributions

Case Study 1

Selected model

U/W cohort model:

Variable rates



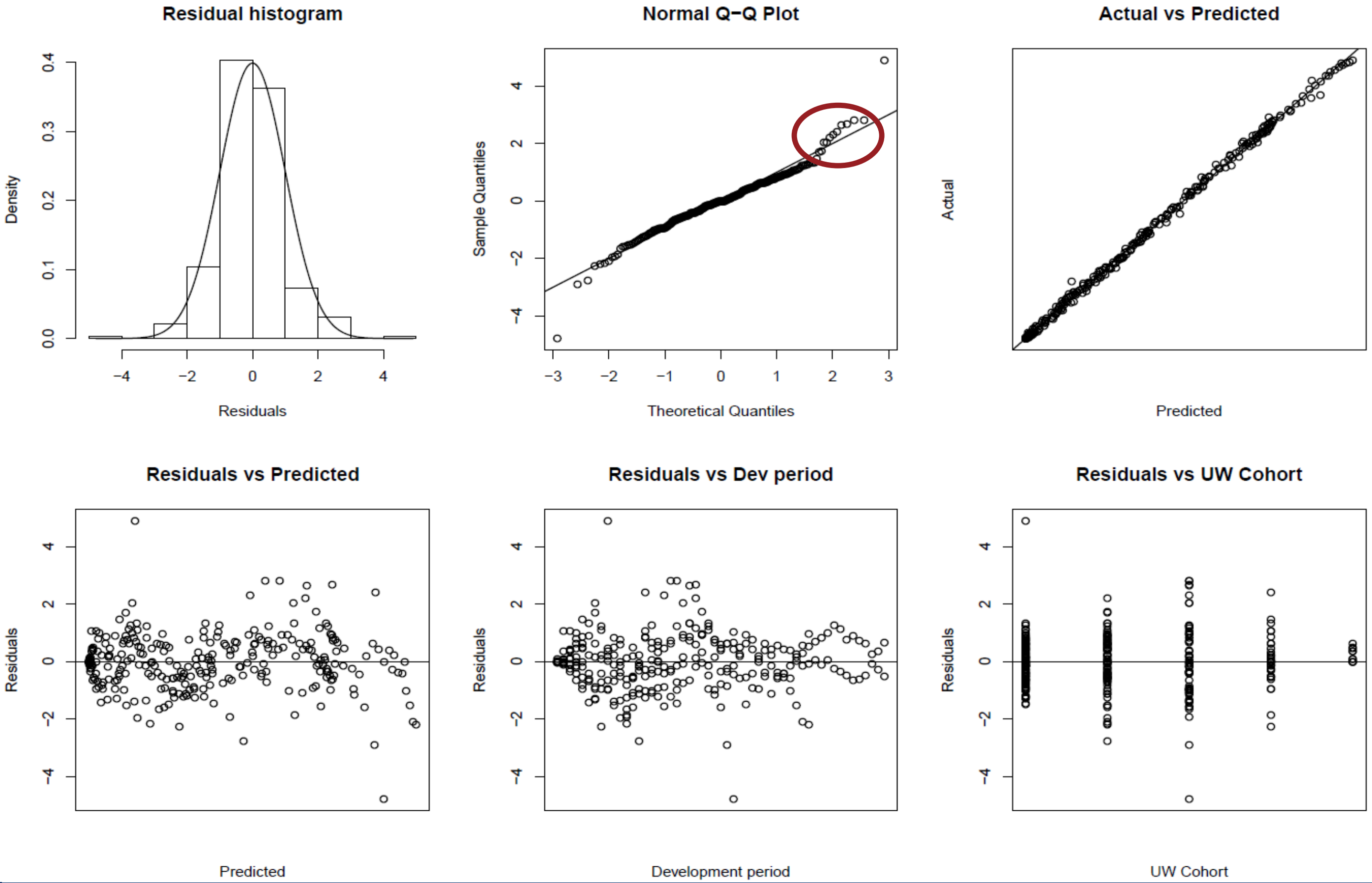
Full random effects structure

Cohort	RLR	ker	RRF	kp
1	RLR_1	ker_1	RRF_1	kp_1
2	RLR_2	ker_2	RRF_2	kp_2
…	…	…	…	…
5	RLR_5	ker_5	RRF_5	kp_5

Fit model and explore diagnostics…

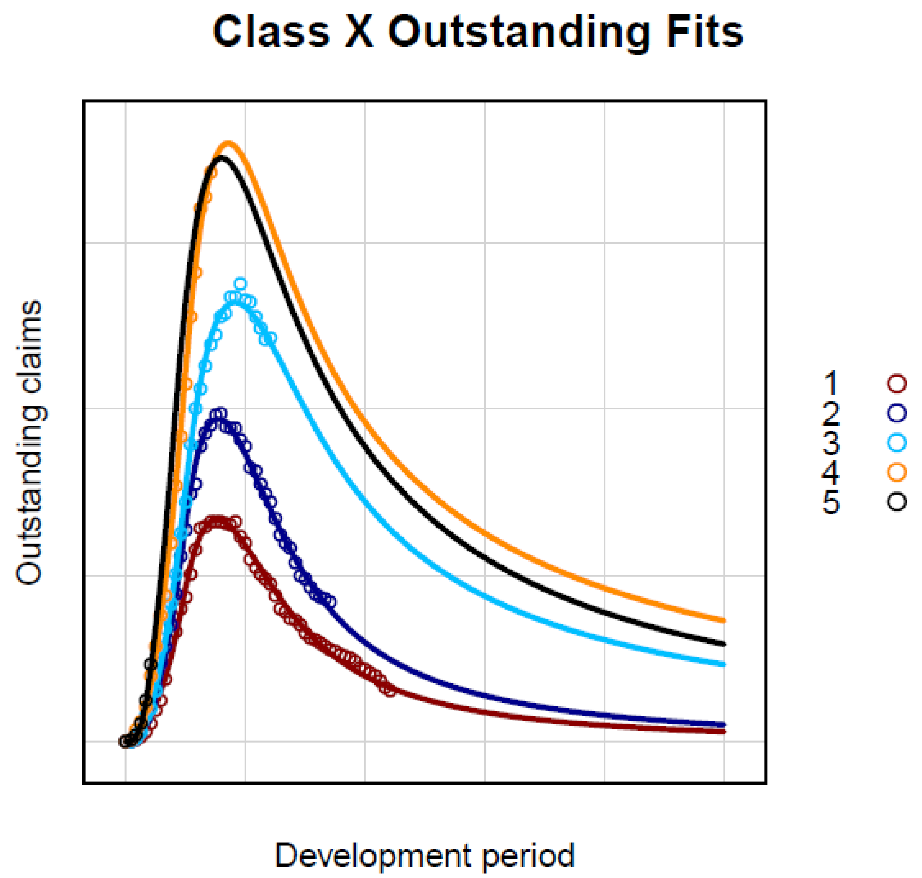
Case Study 1

Model diagnostics



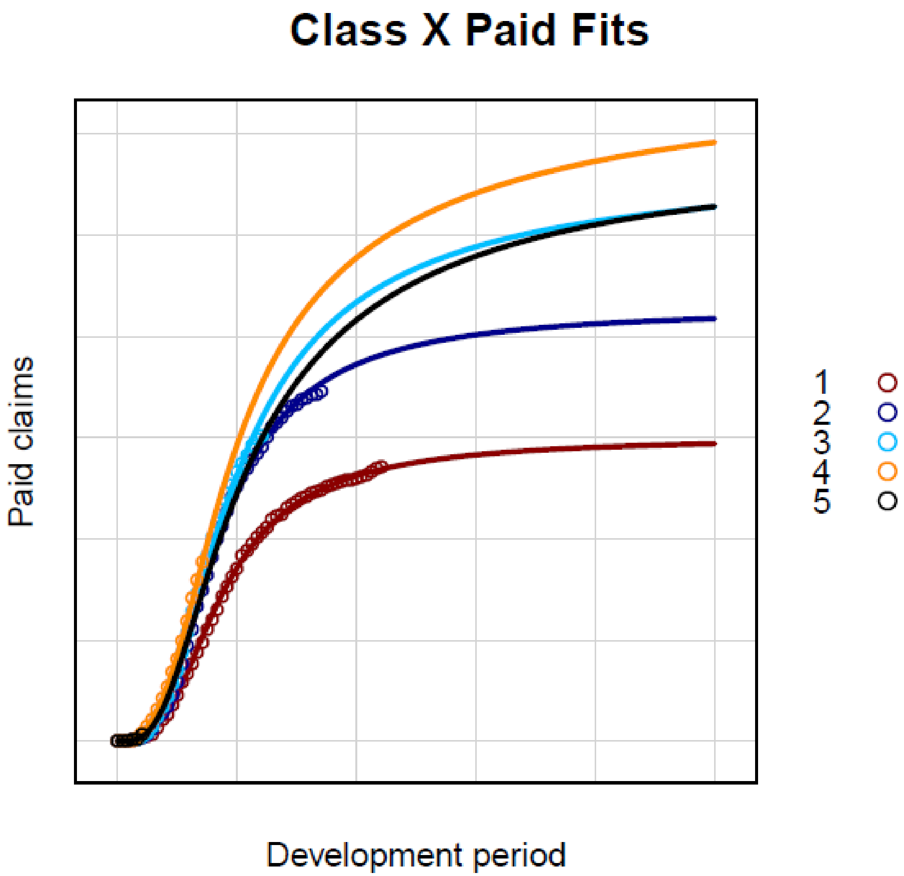
Case Study 1

O/S fits



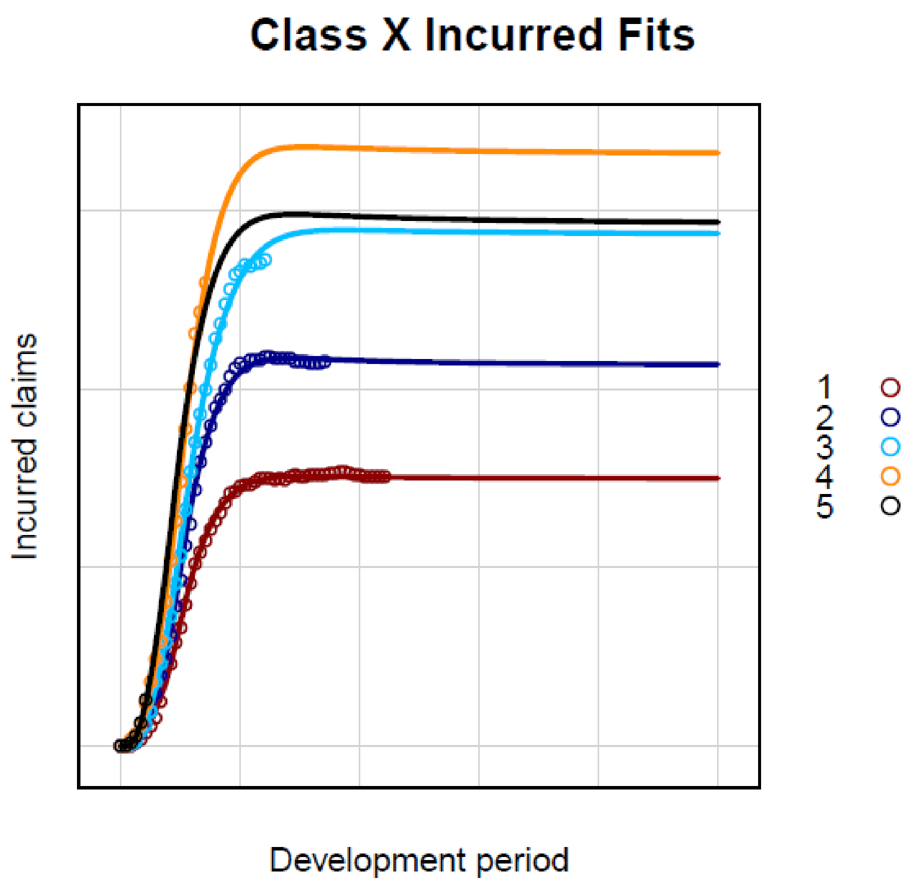
Case Study 1

Paid fits



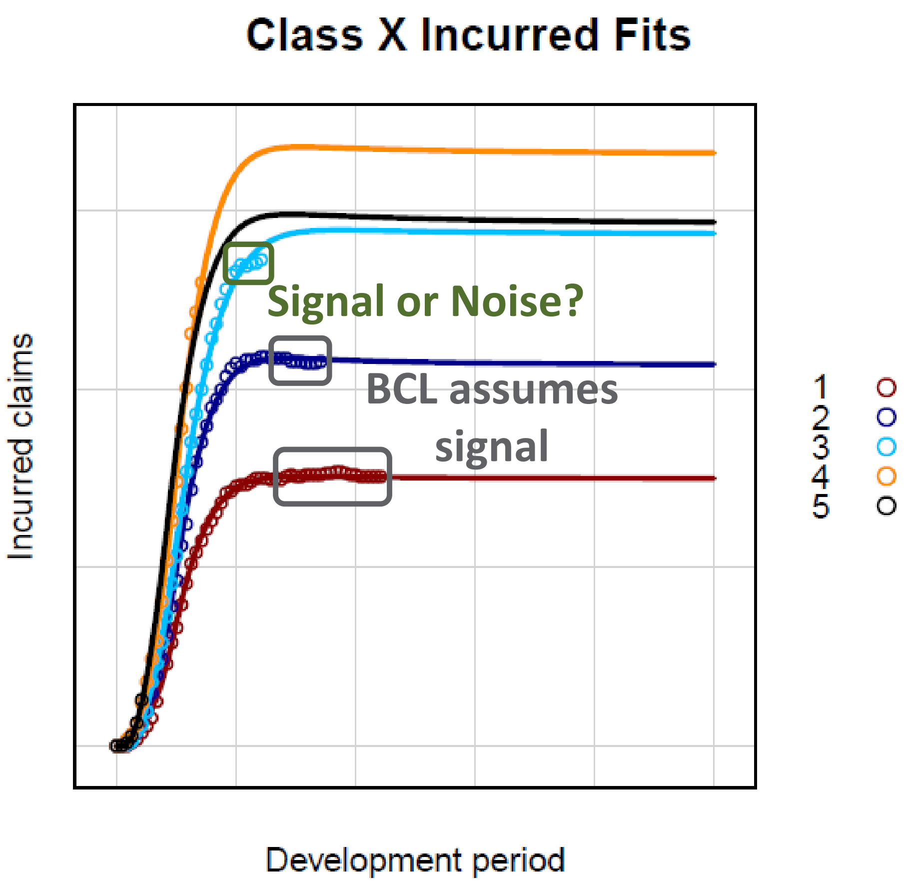
Case Study 1

Incurred fits



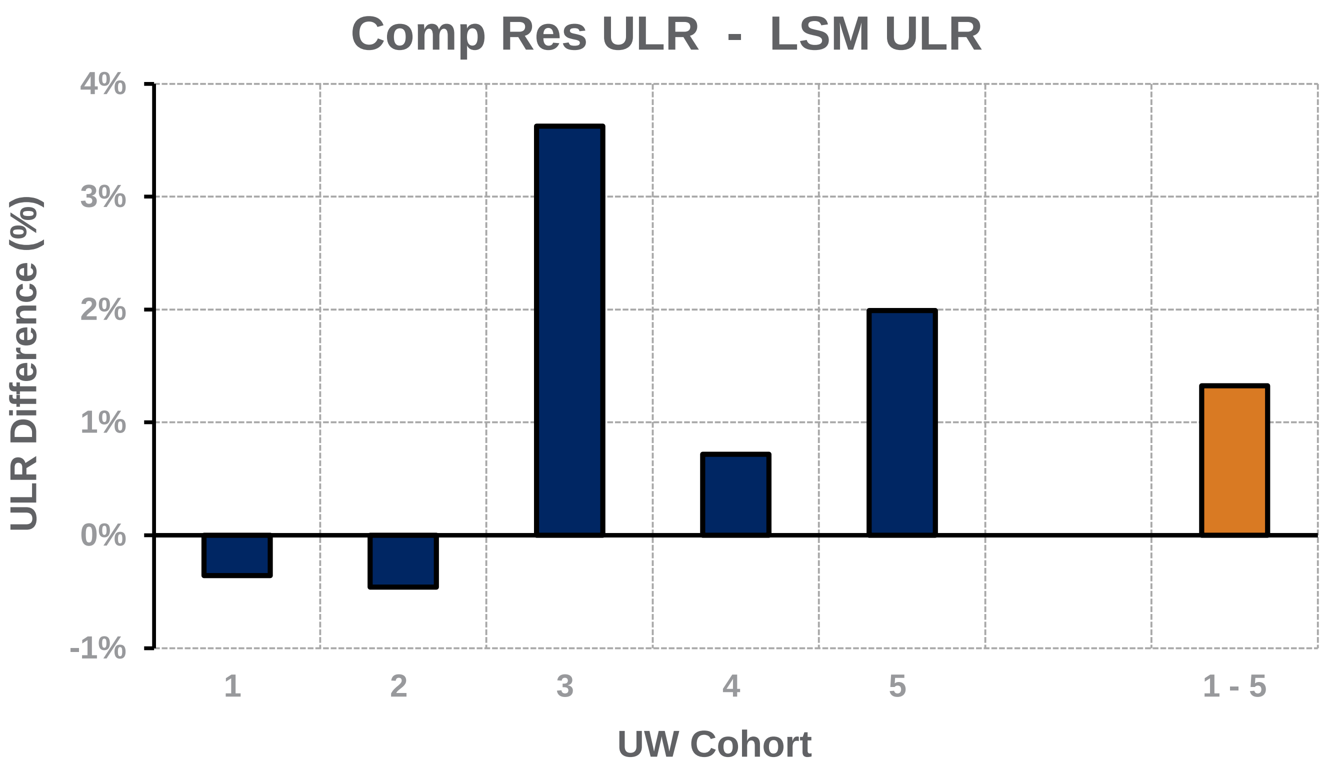
Case Study 1

Incurred fits



Case Study 1

Results Summary



Case Study 2

Why do Bayesian data analysis?

"Modern Bayesian methods provide ***richer information****, with* ***greater flexibility*** *and broader applicability than 20th century methods.*

Bayesian methods are intellectually ***coherent and intuitive*** *…[and] readily computed..."*

- John K. Kruschke

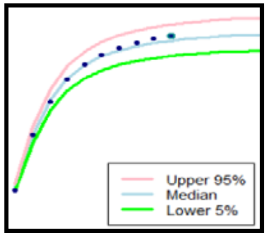
Open Letter extolling the benefits of the Bayesian approach

Case Study 2

Why do Bayesian loss reserving?

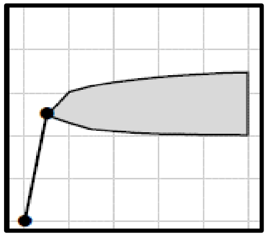
1) Estimate **full probability distributions** of quantities of interest:

"Given our current state of knowledge…"



2) **Incorporate judgement**:

$$ULR_{BF} = f(incurred, exposure, \mathbf{IELR})$$



3) Model structure **flexibility***:

Alternative distributions ***Autocorrelation***

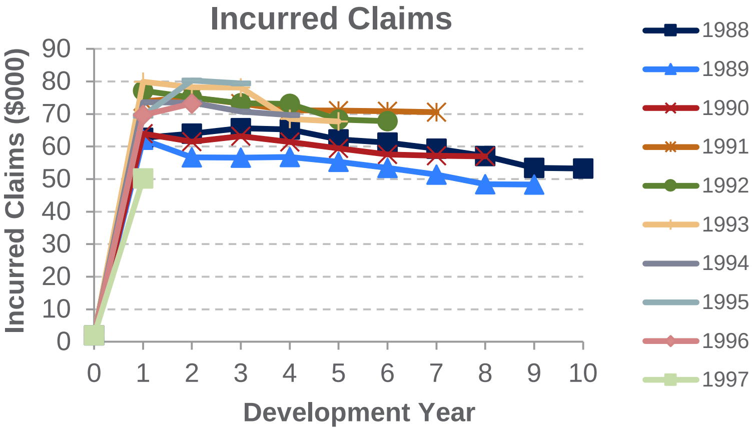
Calendar effects ***External information***

*Provided by Gibbs sampling

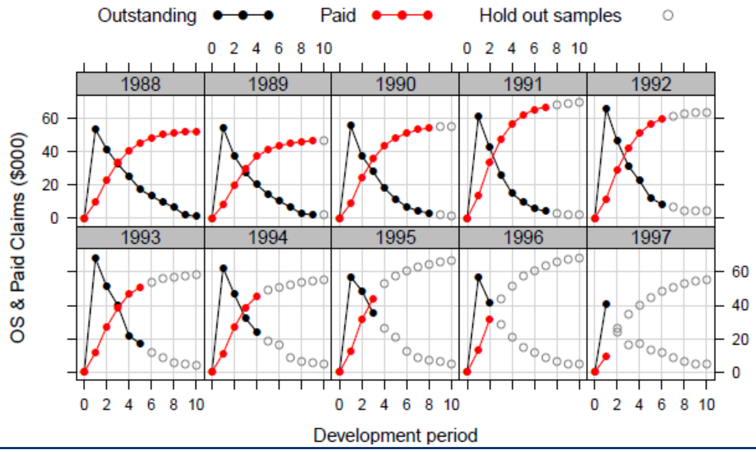
Case Study 2

Data & Objectives

- **Workers' Comp Schedule P data**
  - Accident year cohorts (1988 – 1997)
  - Earned premiums
  - Paid and incurred claims development



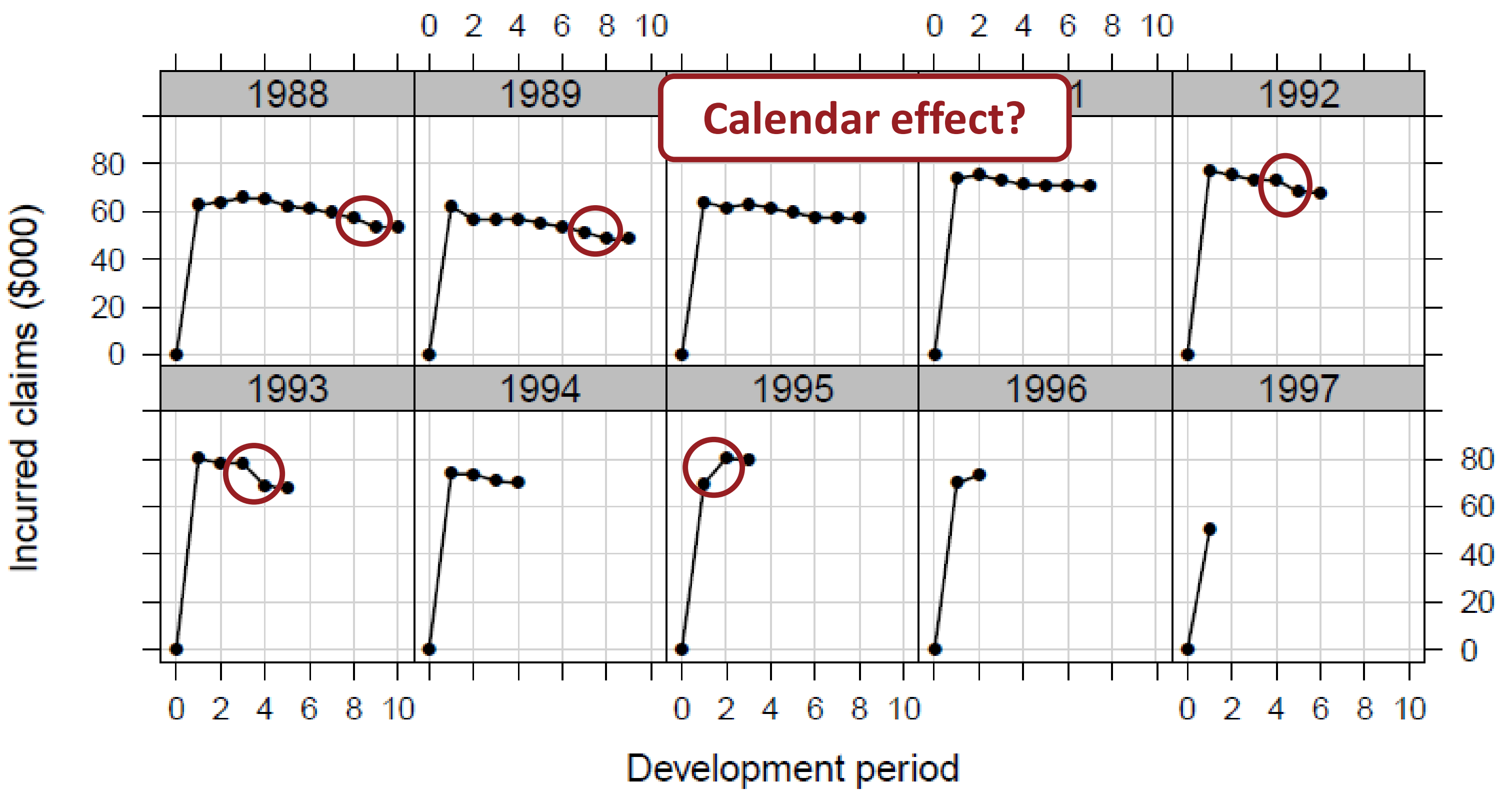
- **Objectives**
  - Fit **Bayesian** compartmental model
  - Extrapolate fits & posterior predictive intervals ("PPIs") to time 10*
  - Compare fits & PPIs to lower triangles



*PPIs account for Parameter and Process uncertainty

Case Study 2

Incurred data visualisation

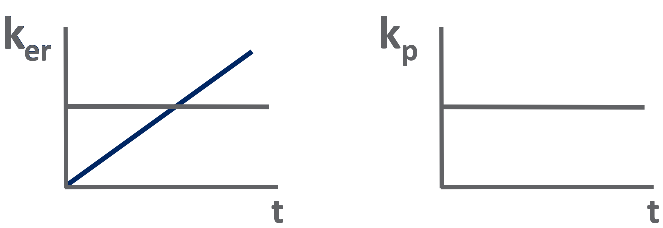


Case Study 2

Model 1

Base model (extended):

~~*Constant rates*~~



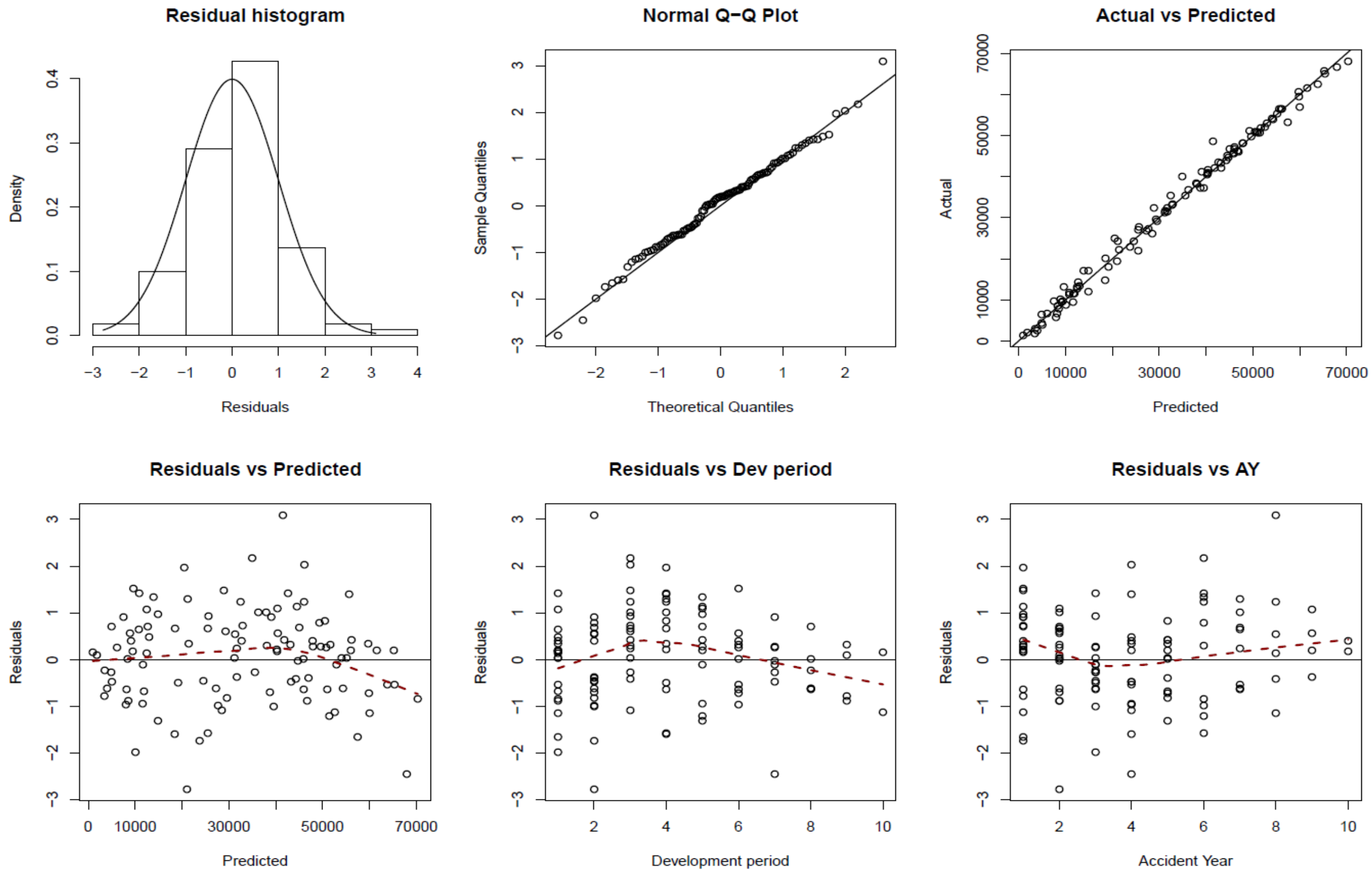
2 random effects

AY	RLR	k_{er}	RRF	k_p
1988	RLR_1	k_{er}	RRF_1	k_p
1989	RLR_2		RRF_2	
…	…		…	
1997	RLR_{10}		RRF_{10}	

Fit model and explore diagnostics…

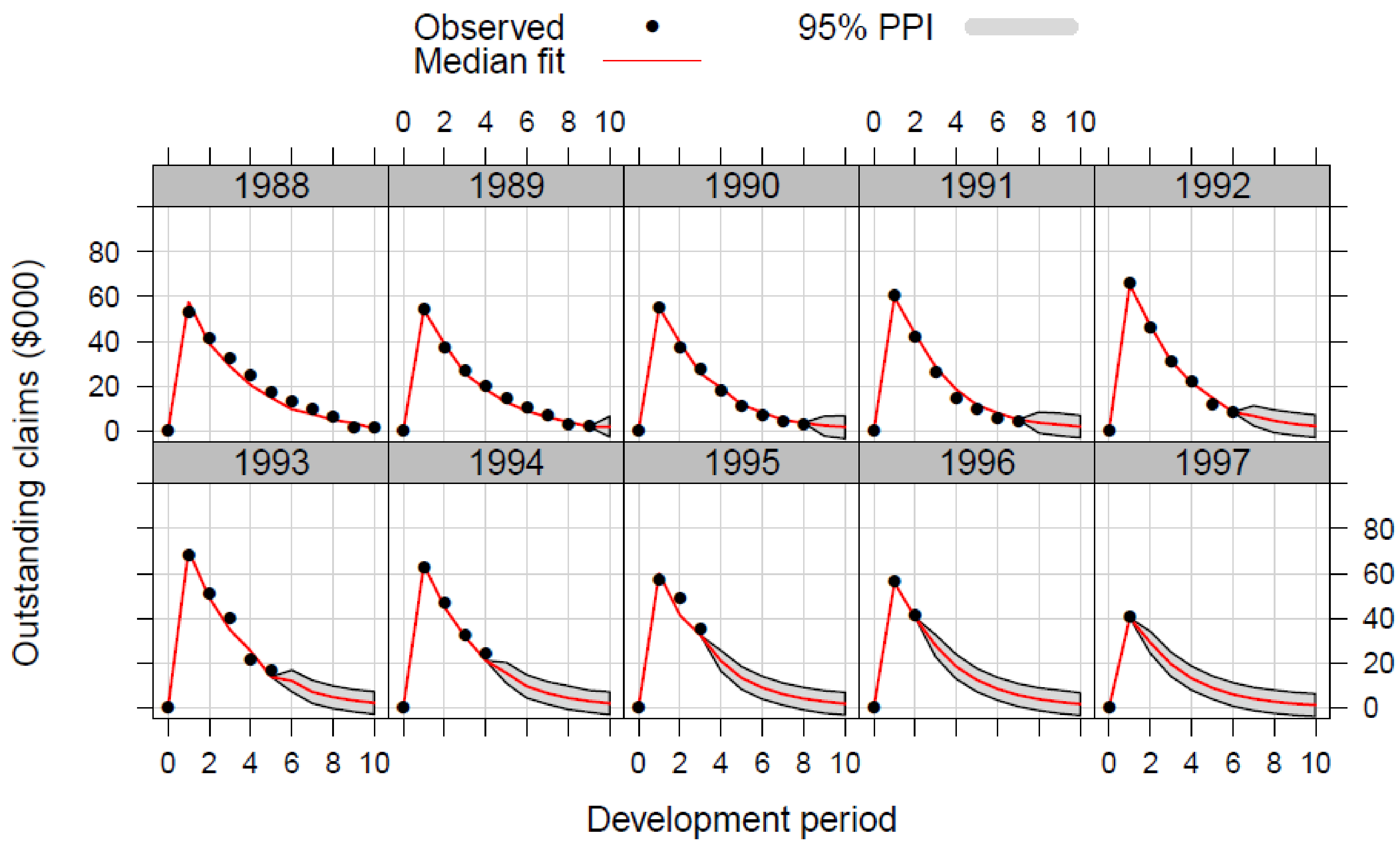
Case Study 2

Model 1 Diagnostics



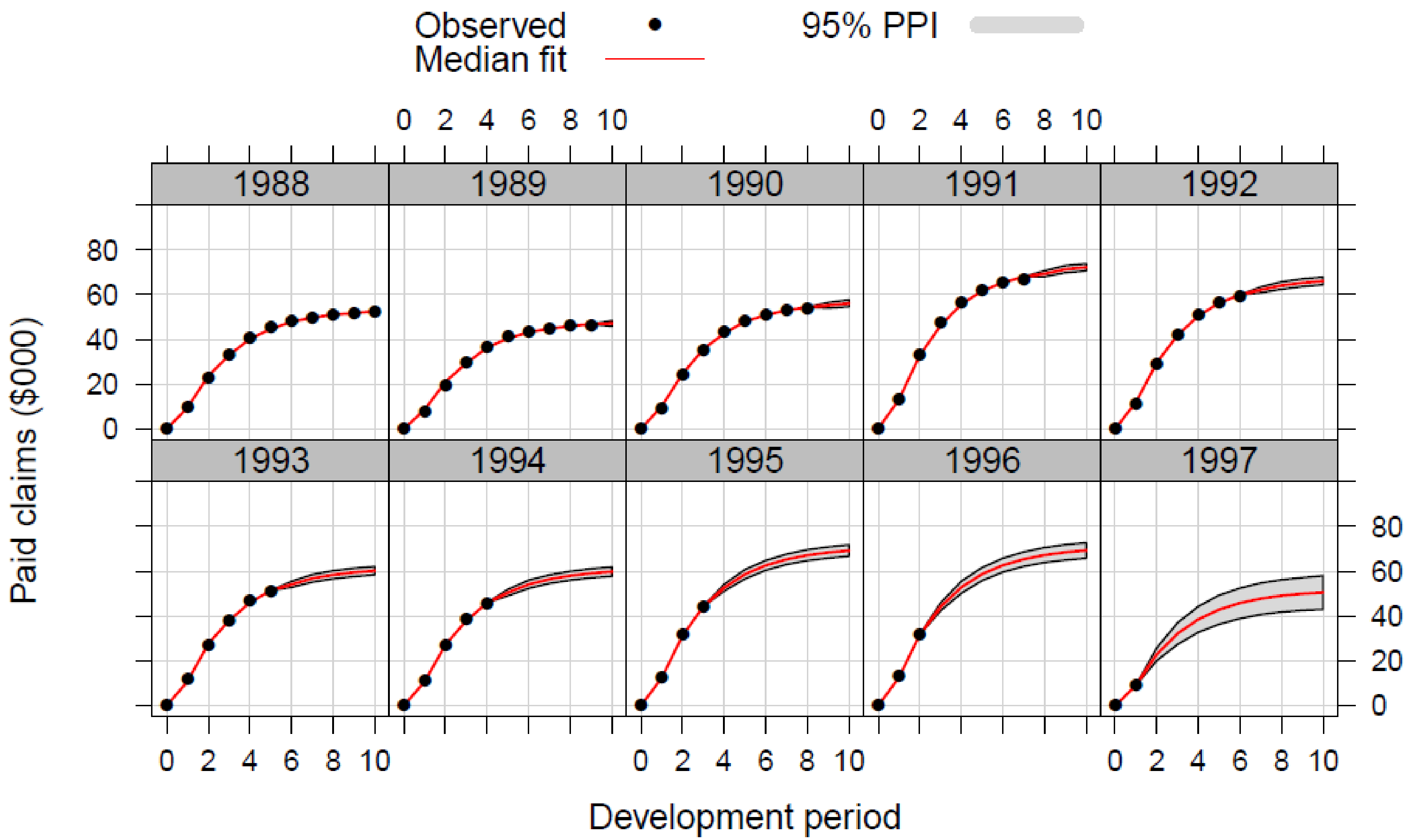
Case Study 2

Model 1 O/S fits



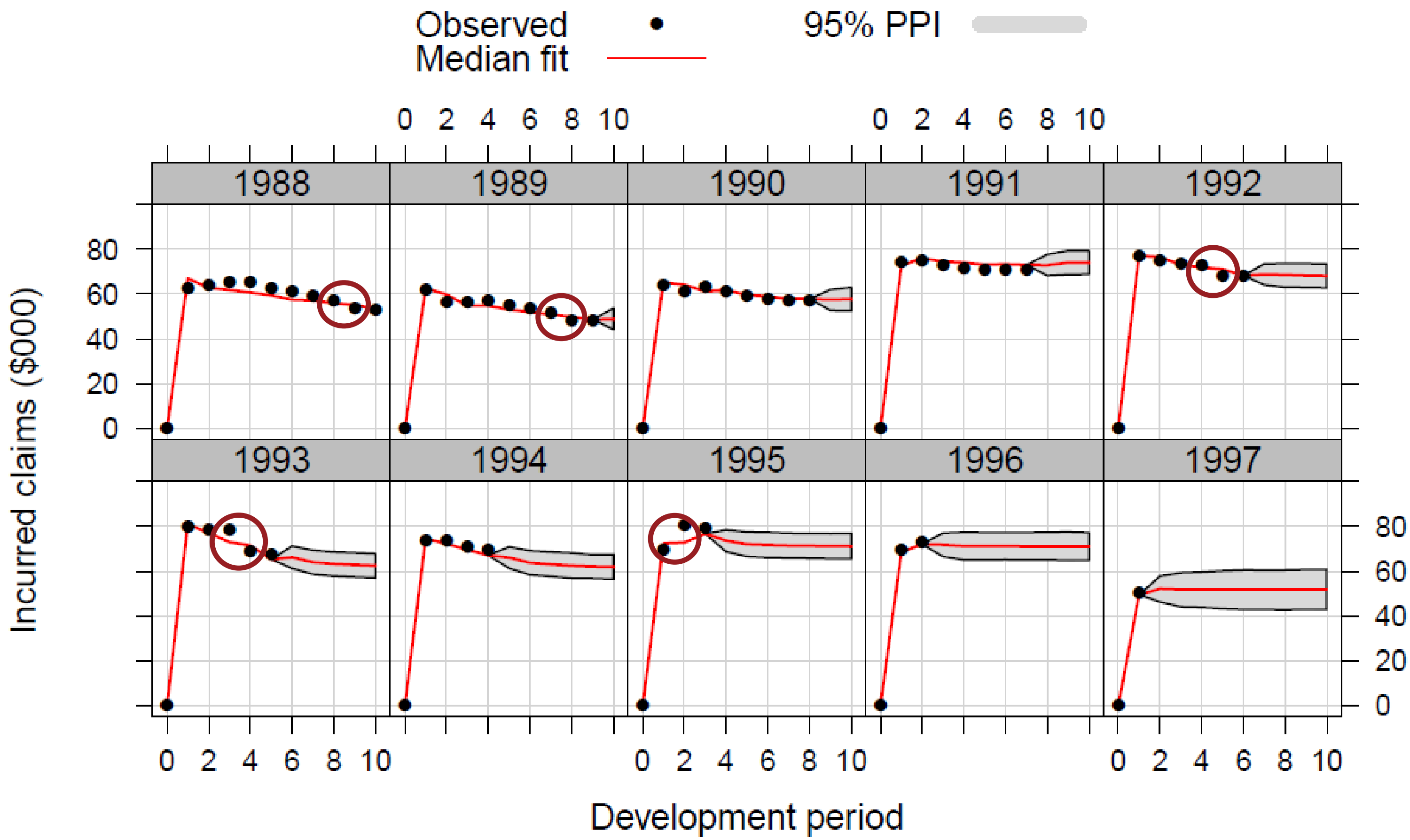
Case Study 2

Model 1 paid fits



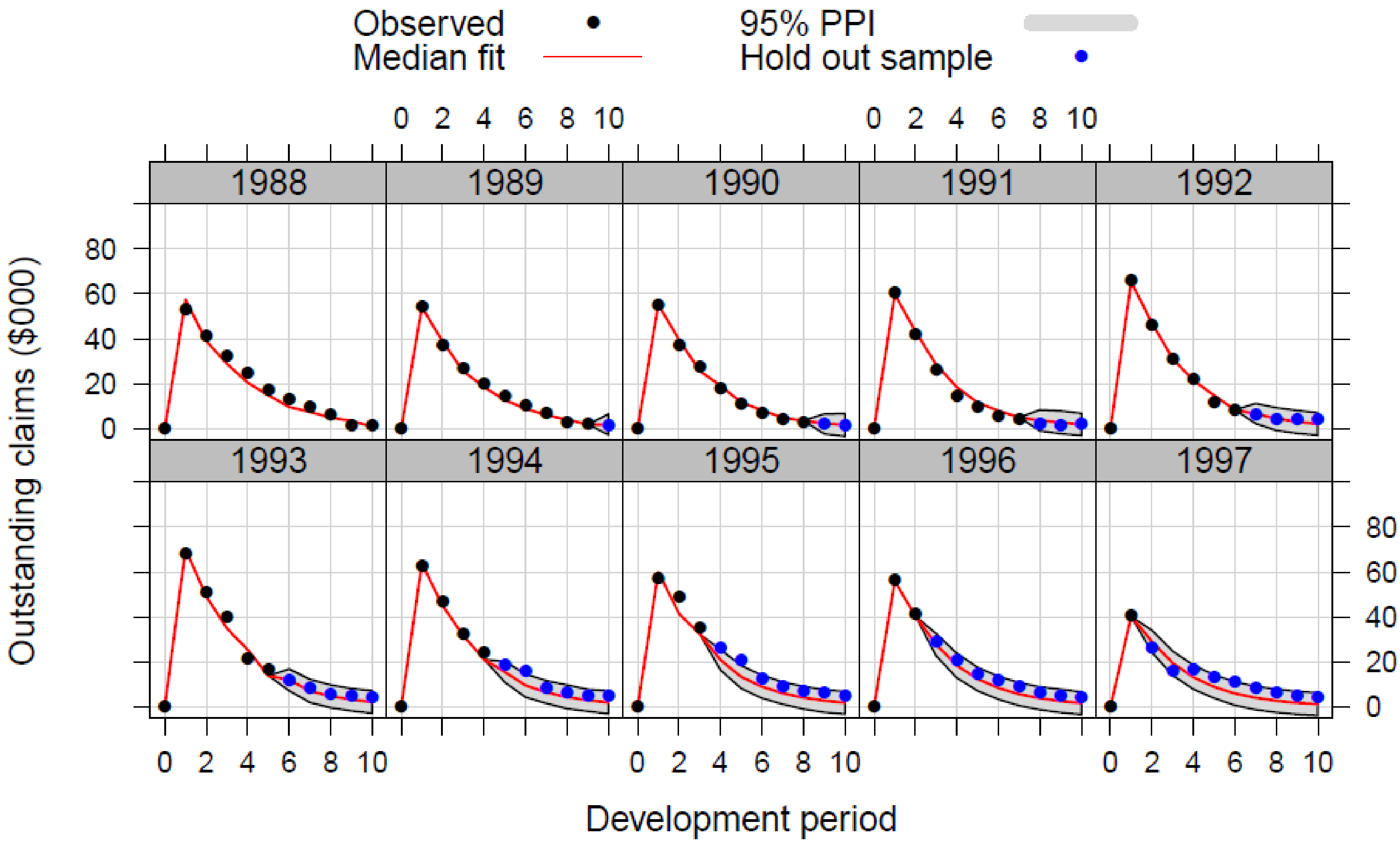
Case Study 2

Model 1 incurred fits



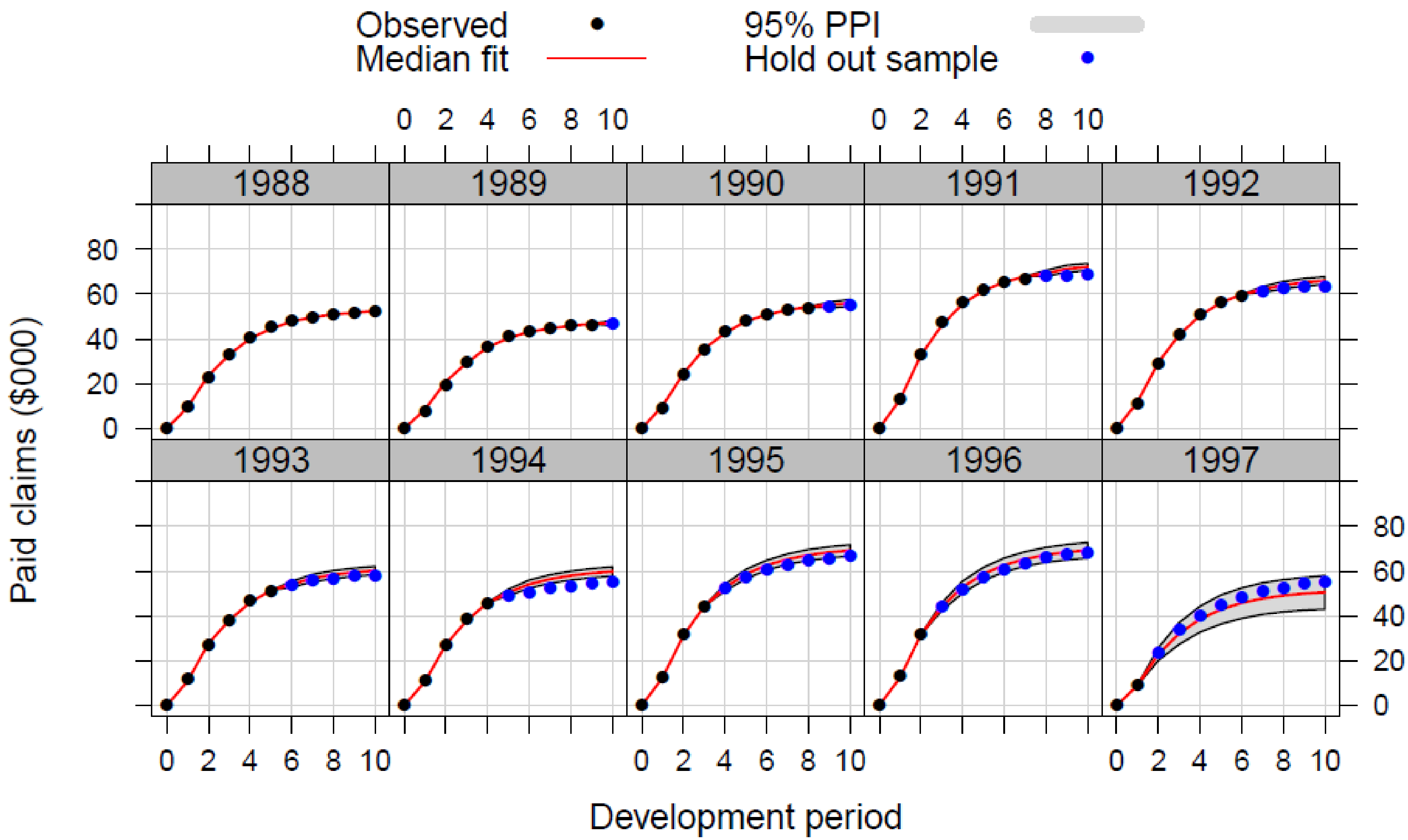
Case Study 2

Model 1 O/S vs hold out sample



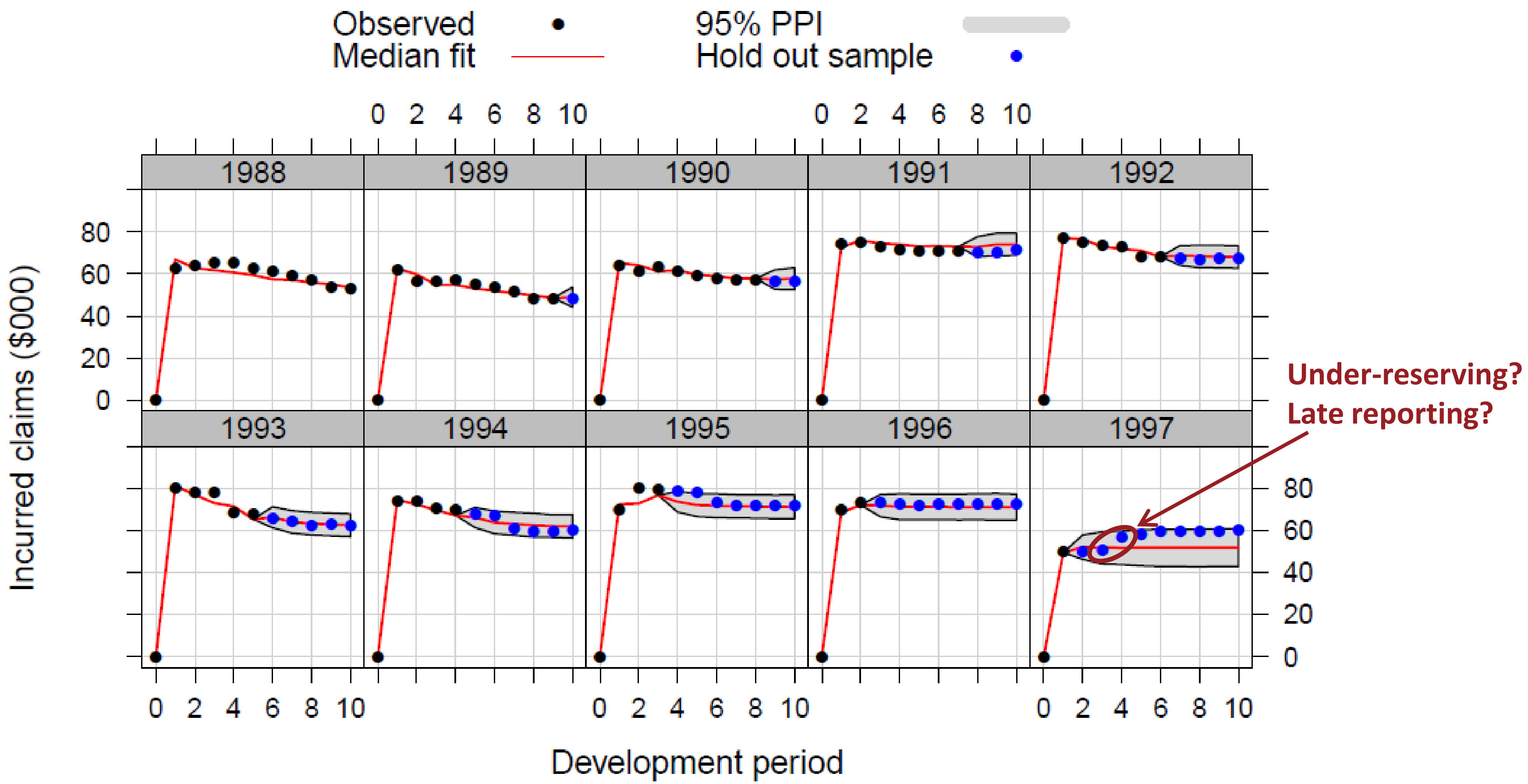
Case Study 2

Model 1 paid vs hold out sample



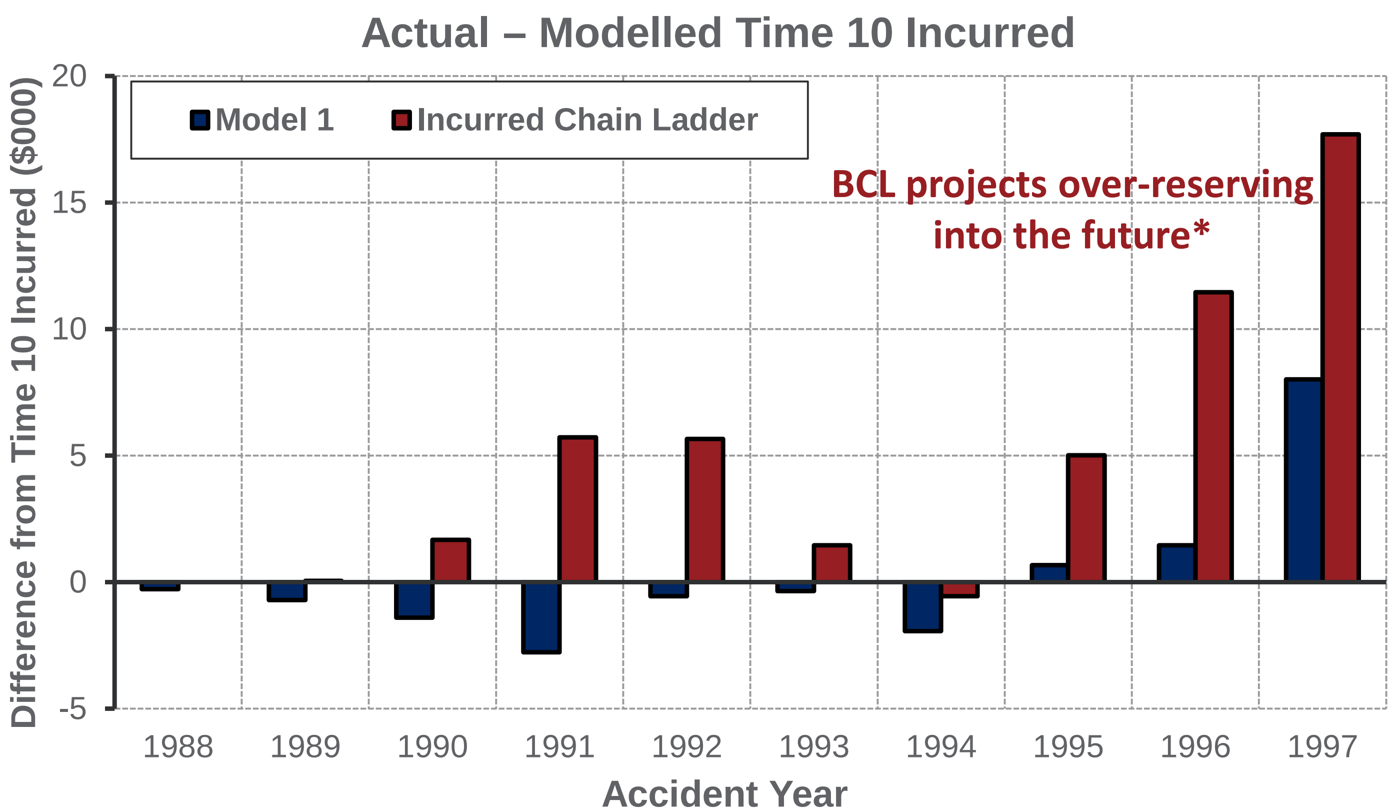
Case Study 2

Model 1 incurred vs. hold out sample



Case Study 2

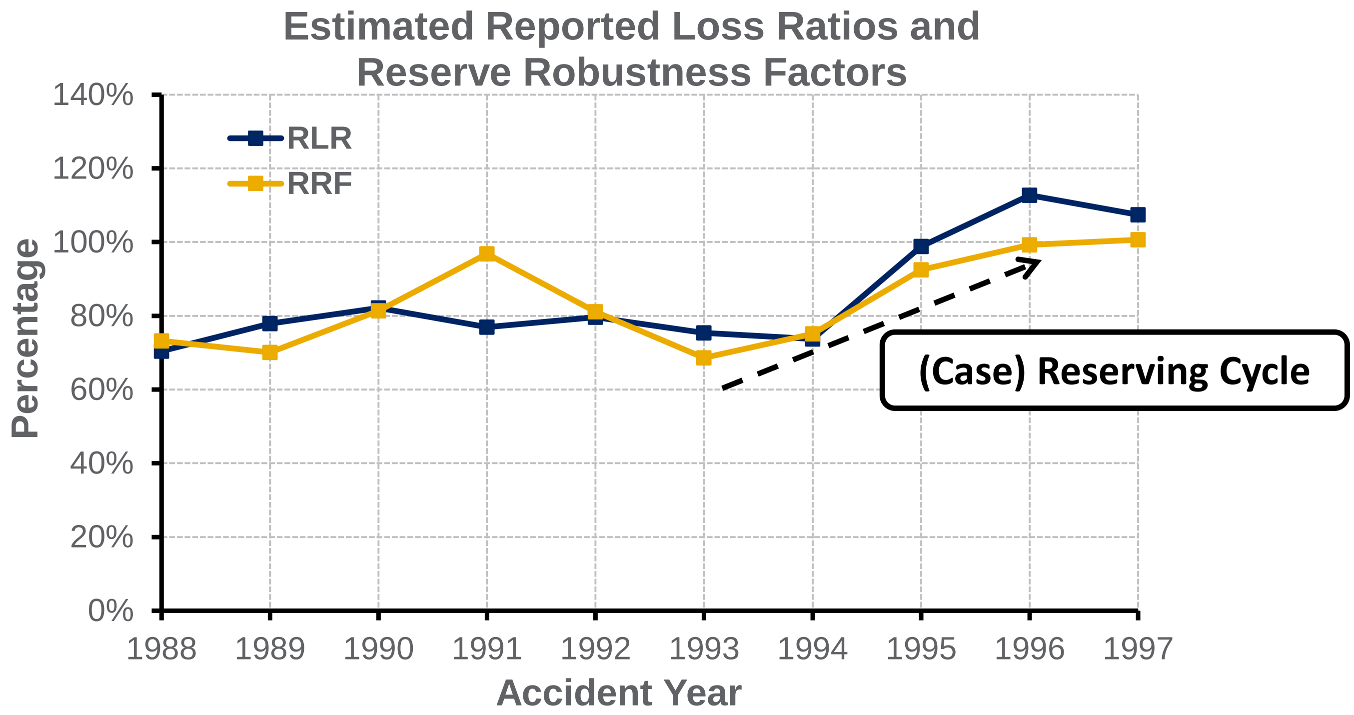
Model 1 Summary (1)



*BF method used in practice

Case Study 2

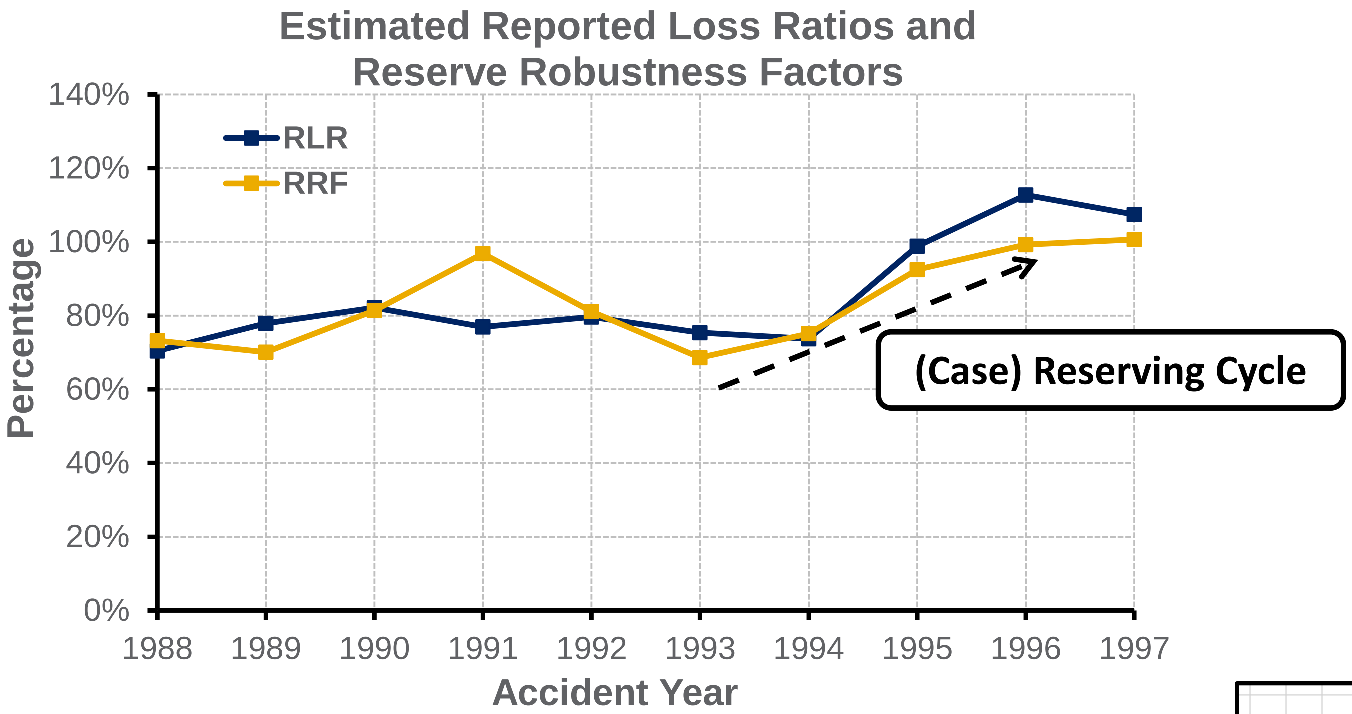
Model 1 Summary (2)

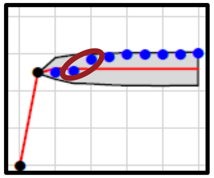


Model estimates less over-reserving over time…

Case Study 2

Model 1 Summary (2)





Model estimates less over-reserving over time…
…but note under-reserving in 1997!*

***In practice:** discuss with case handlers and sensitivity test prediction to changes in RRF

Case Study 2

Model 2

Explicitly model calendar effect:

Premiums Written

Exposed to risk

RLR

k_{er}

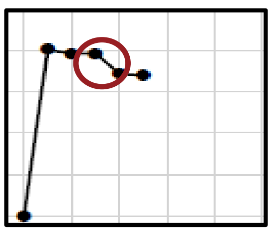
Claims reported

RRF

k_p

Claims paid

C



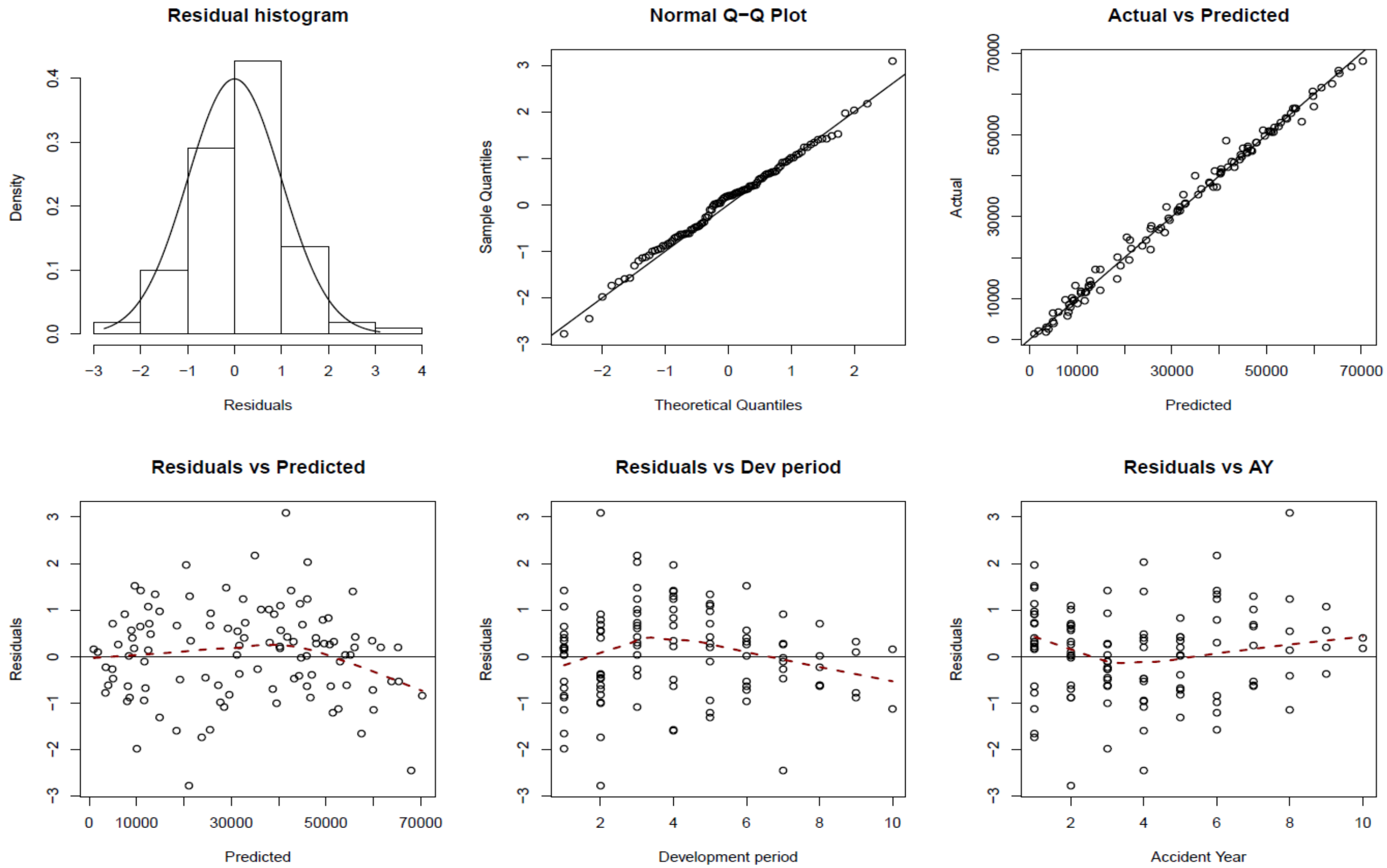
Case reserve review

Full random effects structure

Estimate case reserve % increases/decreases

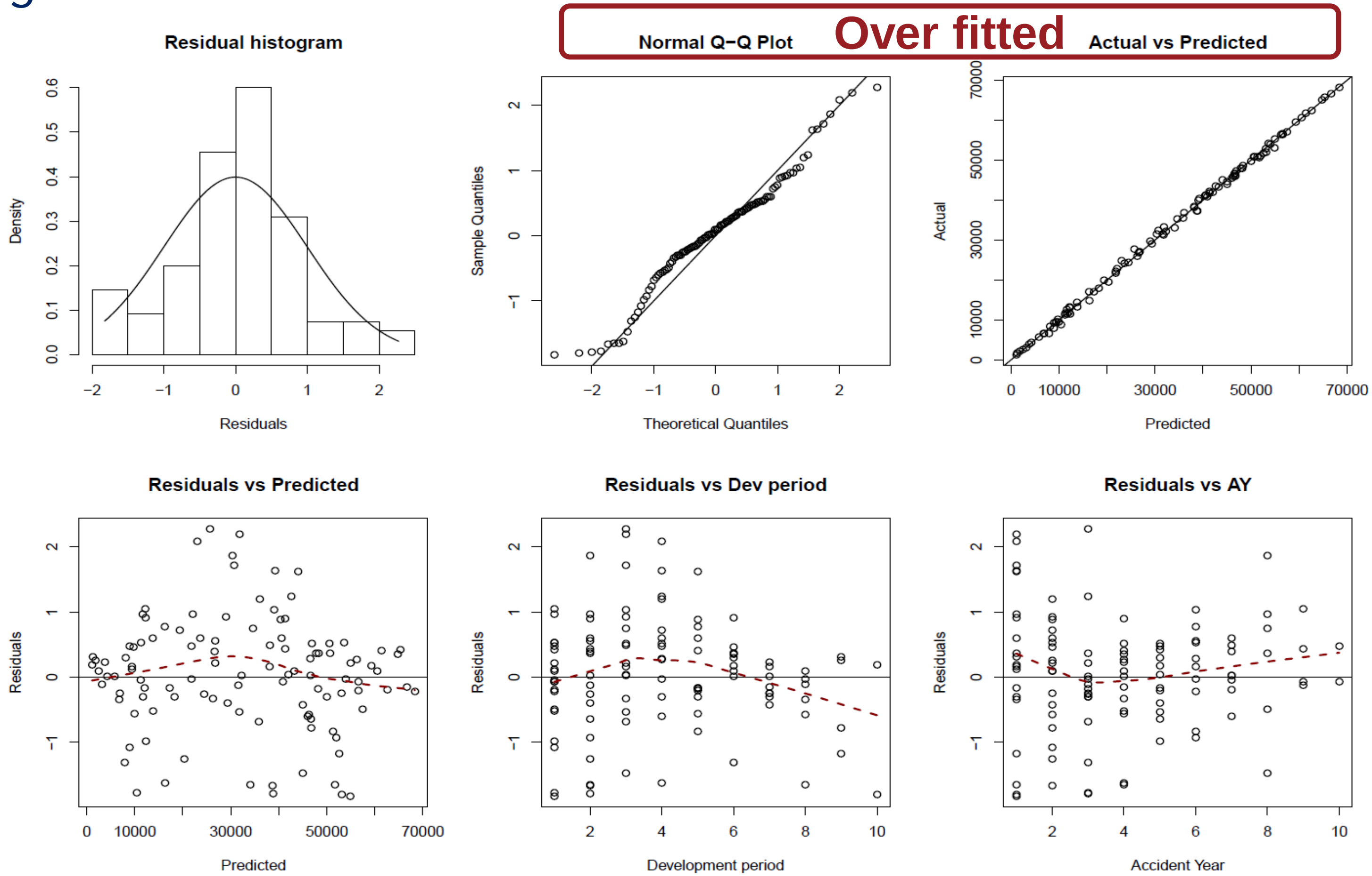
Case Study 2

Diagnostics: Model 1 vs Model 2



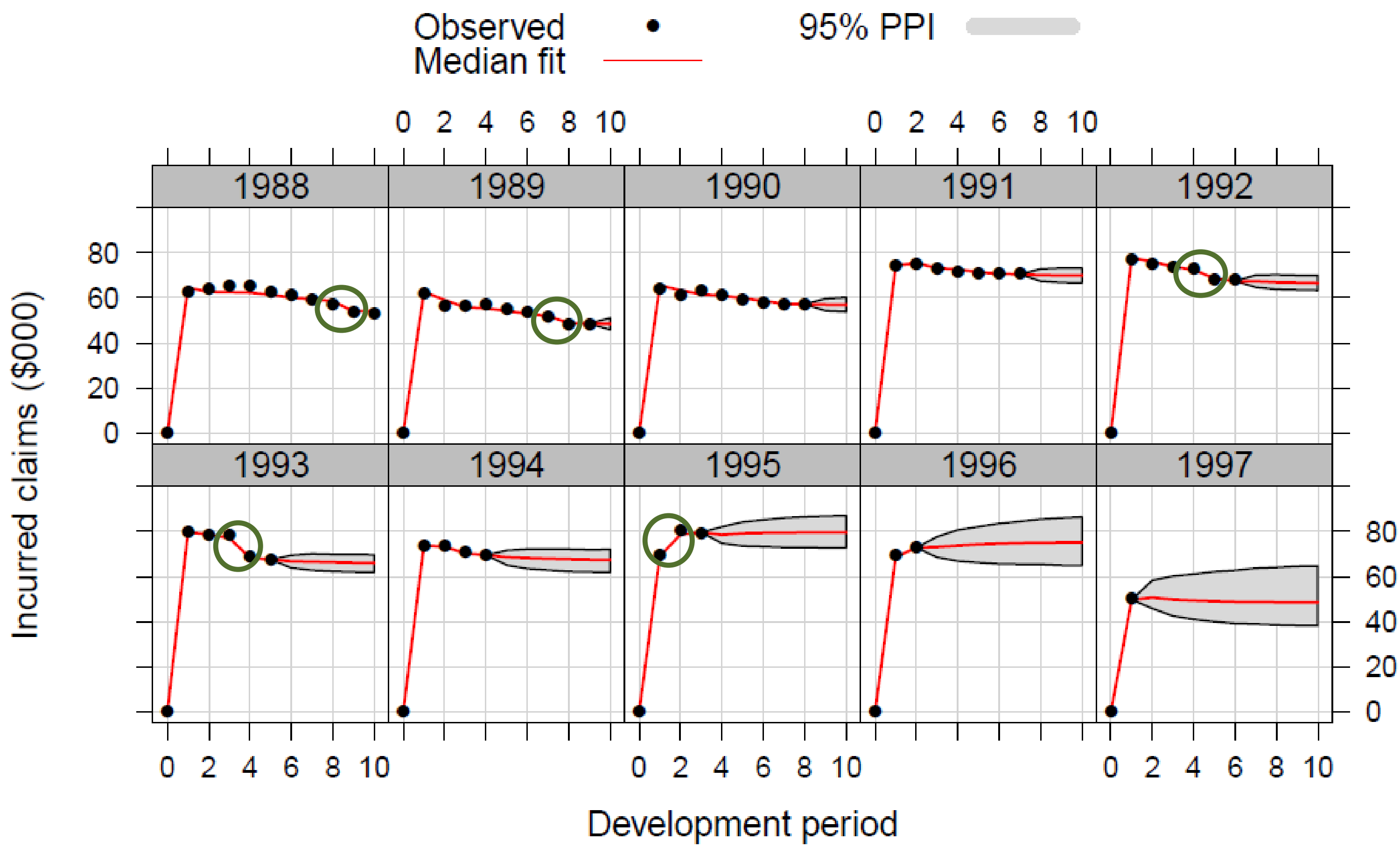
Case Study 2

*Diagnostics: Model 1 vs Model **2***



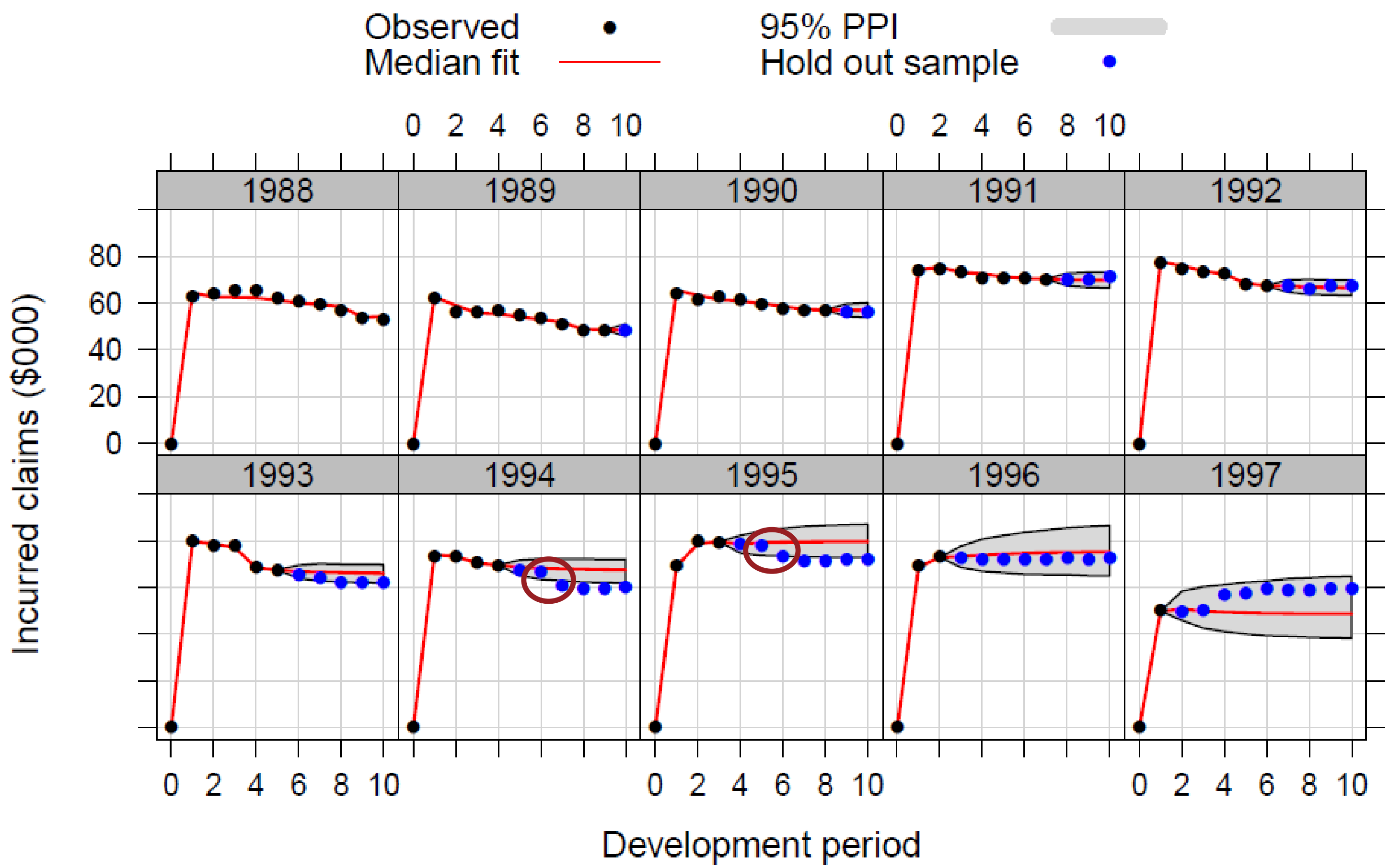
Case Study 2

Model 2 incurred fits



Case Study 2

Model 2 incurred vs. hold out sample



Conclusions (1)

*A modeller's notes**

- **"Fitting nonlinear mixed-effects models can be a tricky (and frustrating) business"**
  - Is the model appropriate?
  - Convergence $\not\Rightarrow$ correctness
  - Different fitting methods $\Rightarrow$ different results

- **"Model diagnostics are (even more) important for these models"**
  - Does the model describe *all* cohorts *reasonably* well?

"There are some ***general*** rules for fitting these models…

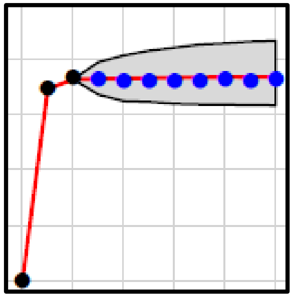
…but experience is the best guide"

*Mike K Smith – Pharmacometrician, Pfizer

Conclusions (2)

Bayesian compartmental reserving

$$p(ULR \mid incurred) \propto L(ULR; incurred)\, p(ULR)$$



Conclusions (2)

Bayesian compartmental reserving

- **Strengths of compartmental reserving:**
  - Independent stochastic method *supports negative incurred development*
  - Meaningful parameters *including measure of reserve robustness*
  - Parsimonious yet extensible *can capture calendar effects*

- **Weaknesses of compartmental reserving:**
  - Model shape constraints with volatile data
  - Sensitivity to starting values / priors *(strength!)*
  - Learning curve

Try it out for yourself!

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Acknowledgements

for sponsoring intermediary development

Including former colleagues:

- **Robert Ruiz**
  - Leading semi-Bayesian mathematical documentation
  - Advocating the method
- **Rob Murray, Charl Cronje, Matthew Pearlman, Richard Holloway, Matt Locke** and **Charlie Stone**
  - Support at various stages
  - Sounding boards

Compartmental Reserving

a process-based Bayesian reserving approach

Jake Morris — Liberty Specialty Markets
Rob Murray — LCP

22 October 2015

Appendix

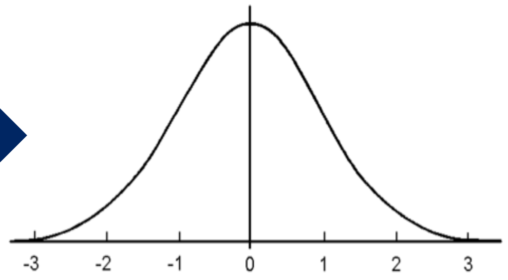
Background

1. Parsimony

Mixed-effects / hierarchical modelling

Cohorts

Parameters a *mixture* of those varying across cohort and those not*



Cohort	P_1	P_2	P_3	P_4
1	$P_{1,1}$	P_2	$P_{3,1}$	P_4
2	$P_{1,2}$		$P_{3,2}$	
…	…		…	
N	$P_{1,N}$		$P_{3,N}$	

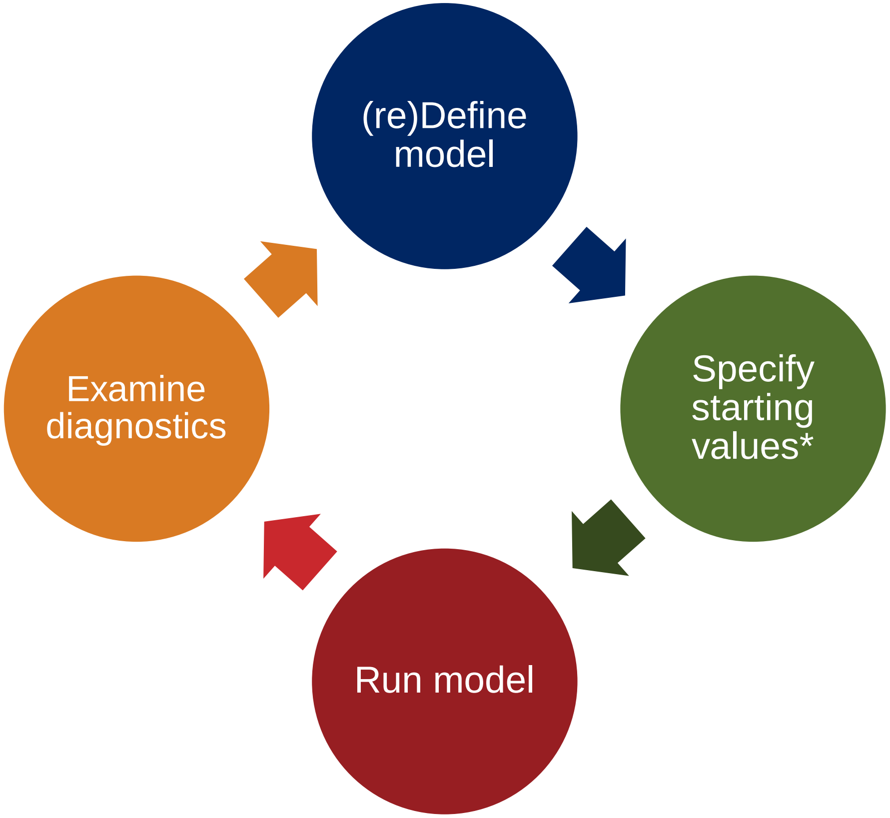
"Borrowing Strength"

Only estimate mean and s.d. of the variable parameters

*Also known as a mixture of *random effects* and *fixed effects*

Implementation

The modelling cycle



*/prior distributions

Liberty Specialty Markets

Implementation

A departure from Excel

- Nonlinear mixed-effects models require **complex solver algorithms**:

Response y {OS,PD}
=
Non-linear function f of
(Parameter vector $\underline{\phi}$ and time t)
+
Noise w

$$L(\underline{\beta},\underline{\eta},\sigma \mid \Im_0^{(\omega_0)}) = \prod_{i\in I}\prod_{c\in C}\int_{\underline{b}^{(i)}\in\Re^{SizeP}} pdf(\underline{y}^{(i,c)}(\omega_0)\mid \underline{b}^{(i)},\underline{\beta},\sigma)\cdot pdf(\underline{b}^{(i)}\mid\underline{\eta},\sigma)\cdot d\underline{b}^{(i)}$$

We don't have to worry about this!

- "f" is derived by solving **ODEs**:

$$dEX/dt = -k_{er} \cdot EX$$
$$dOS/dt = k_{er} \cdot RLR \cdot EX - k_p \cdot OS$$
$$dPD/dt = k_p \cdot RRF \cdot OS$$

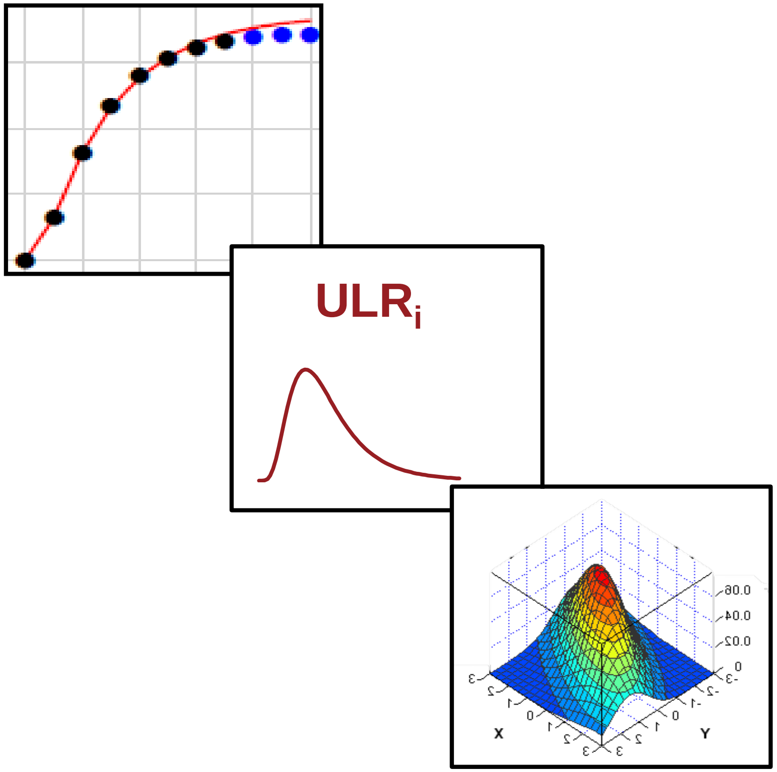
EX	OS	PD
Exposed to risk →	Claims reported →	Claims paid

Semi-Bayesian: **Fully Bayesian:** BUGS

Case Study 2

Bayesian loss reserving in practice

- **Autoregressive sub-models**
  - for consecutive under/over fits
- **Log-normal distributions**
  - for claims process parameters
- **Prior distributions**
  - for *all* other uncertain parameters

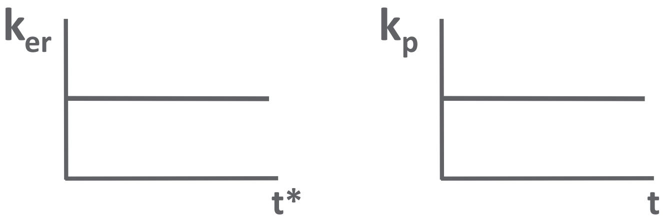


Implementation

Semi-Bayesian

Base model:

Constant rates



2 random effects

Cohort	RLR	k_{er}	RRF	k_p
1	RLR_1	k_{er}	RRF_1	k_p
2	RLR_2		RRF_2	
…	…		…	
N	RLR_N		RRF_N	

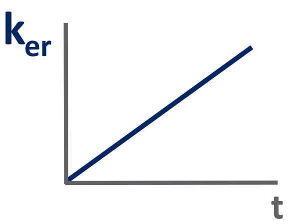
Judgementally select parameter starting values

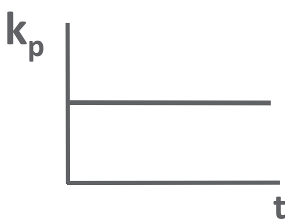
*Development time

Case Study 2

Model 1.5

Base model (extended):





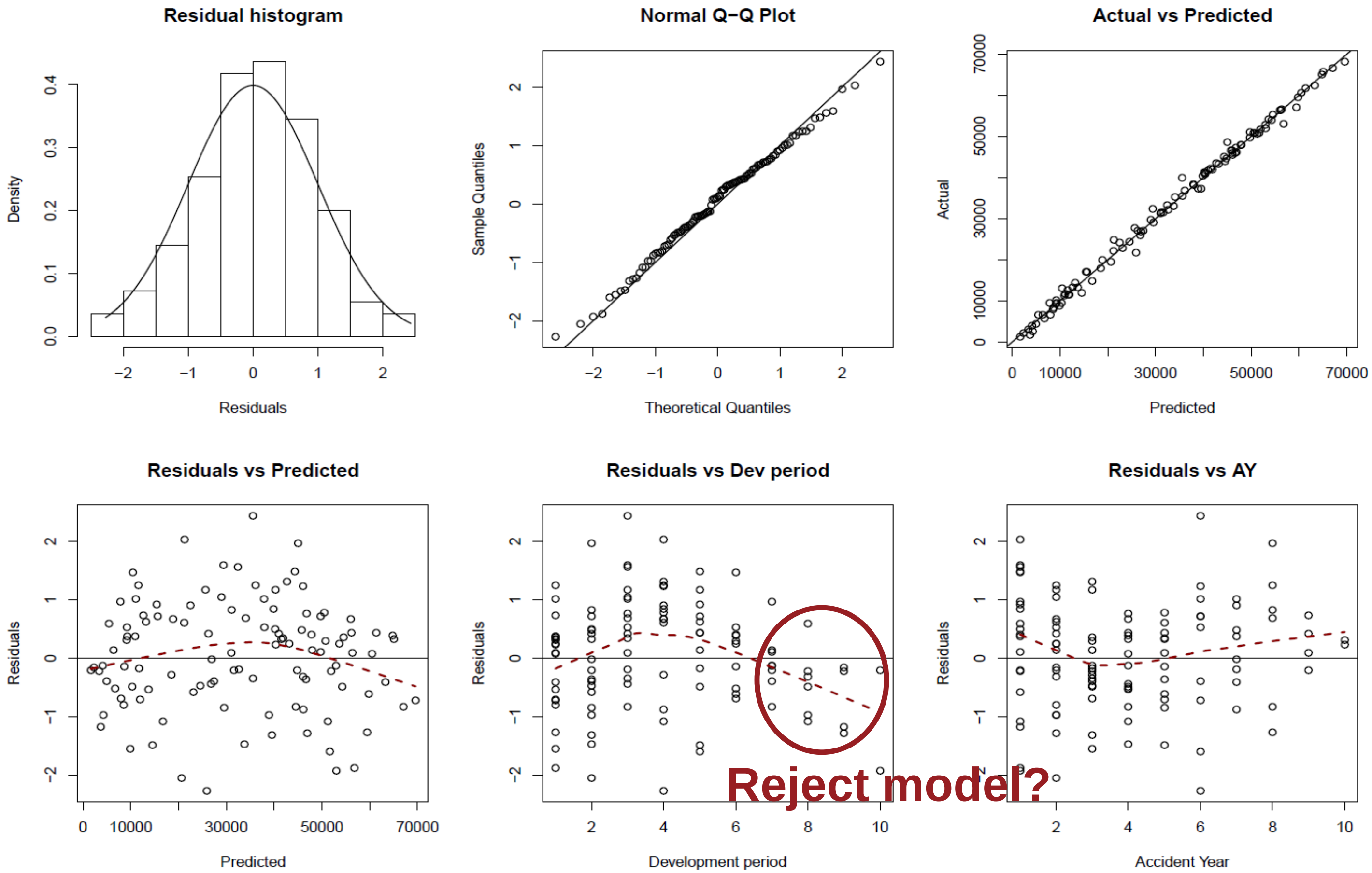
2 random effects

AY	RLR	ker	RRF	kp
1988	RLR_1	ker_1	RRF_1	kp_1
1989	RLR_2	ker_2	RRF_2	kp_2
…	…	…	…	…
1997	RLR_{10}	ker_{10}	RRF_{10}	kp_{10}

Fit new model and explore diagnostics

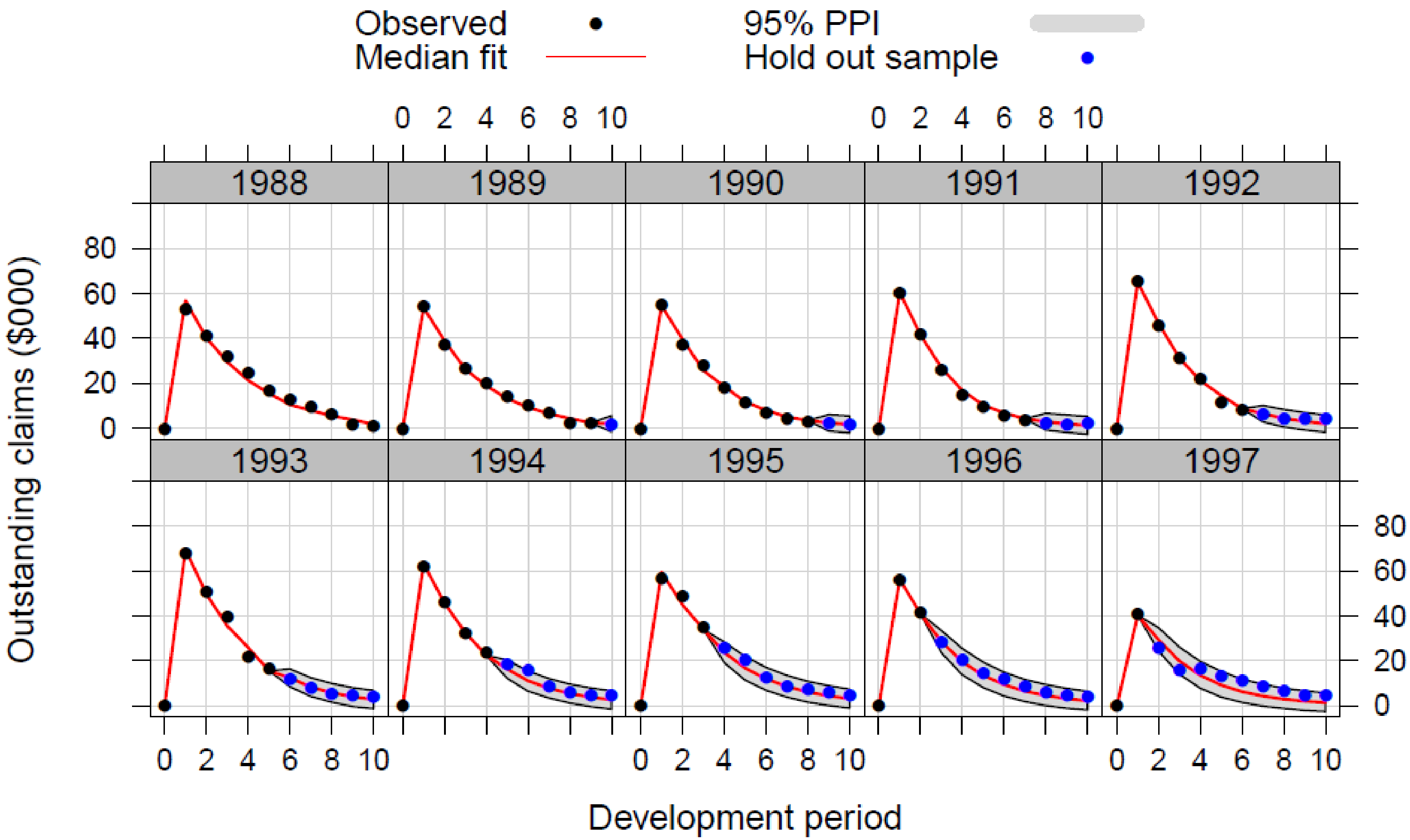
Case Study 2

Model 1.5 Diagnostics



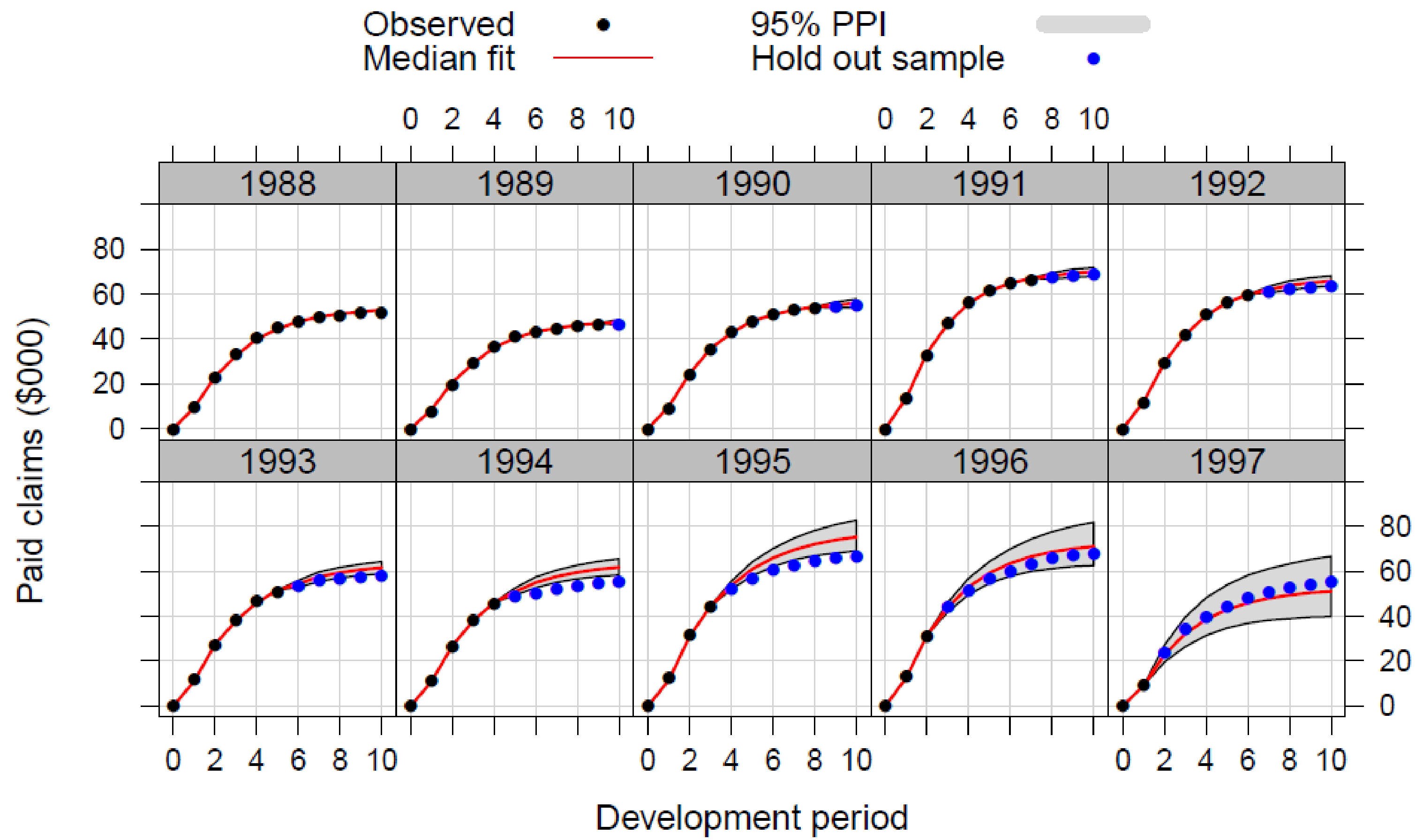
Case Study 2

Model 1.5 O/S vs hold out sample



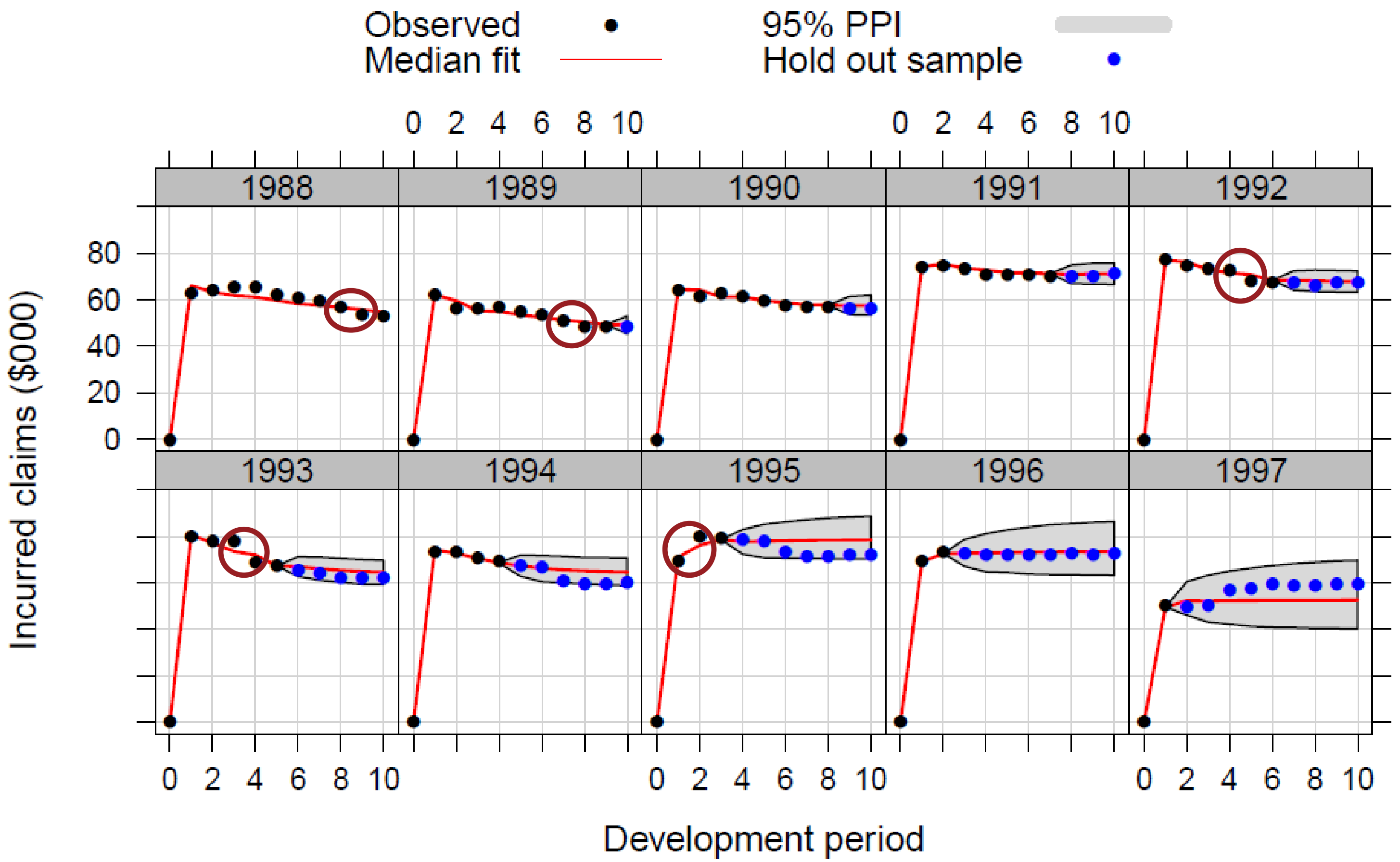
Case Study 2

Model 1.5 paid vs hold out sample



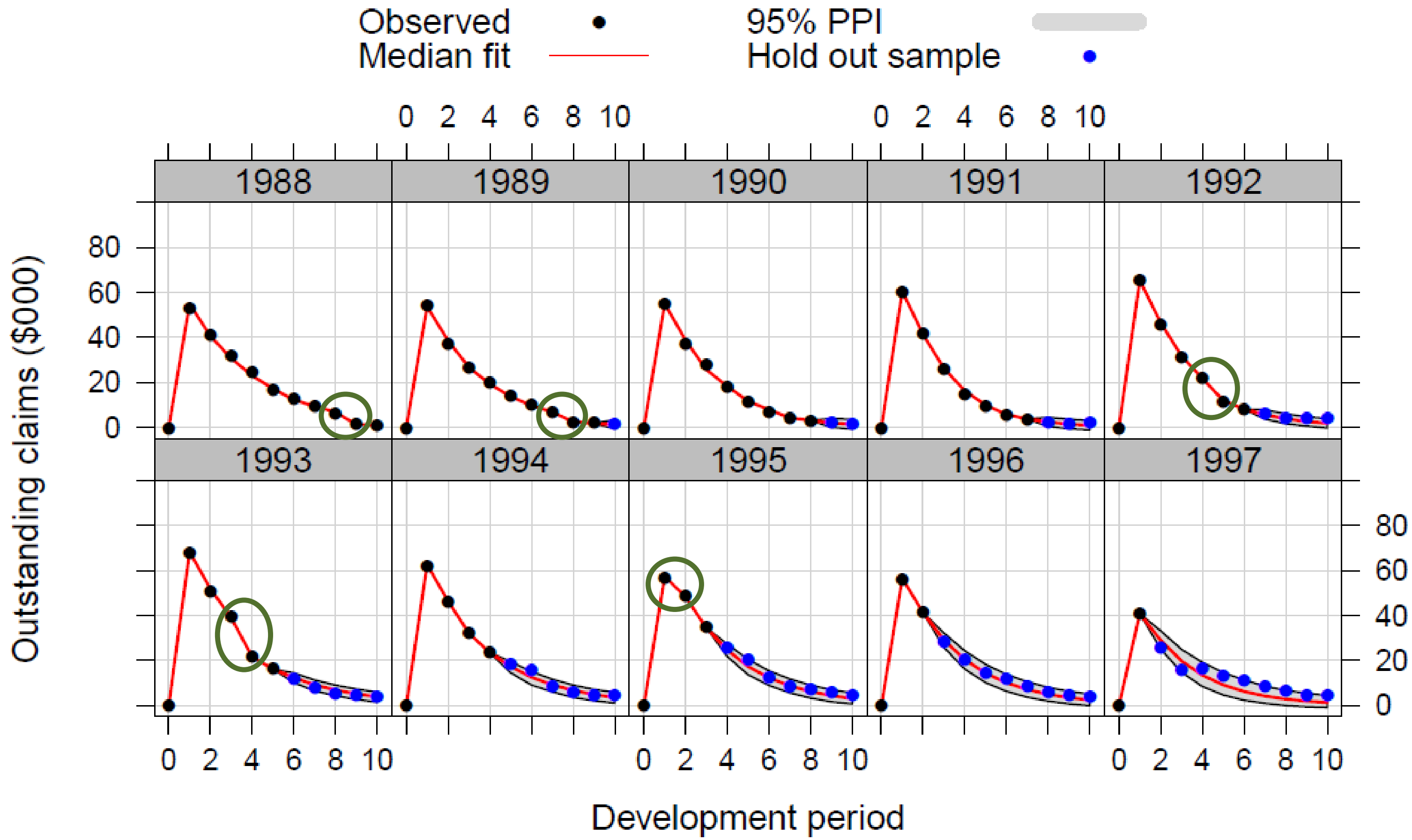
Case Study 2

Model 1.5 incurred vs. hold out sample



Case Study 2

Model 2 O/S vs hold out sample



Case Study 2

Model 2 paid vs hold out sample