The Actuarial Profession
making financial sense of the future

Variable annuities: bridging the divide

Alexis Zervoglos, Commerzbank; Paul Coleman, Barclays Capital

B2: Hedge instruments – from vanilla to exotic

17 September 2010

Agenda

Introduction to variable annuity benefits

Greeks and how to hedge them

Variable annuity underlyings

Variable annuity guarantees

Agenda

Introduction to variable annuity benefits

- How do we value the benefits?
- What are the risks?
- How do we approach hedging?

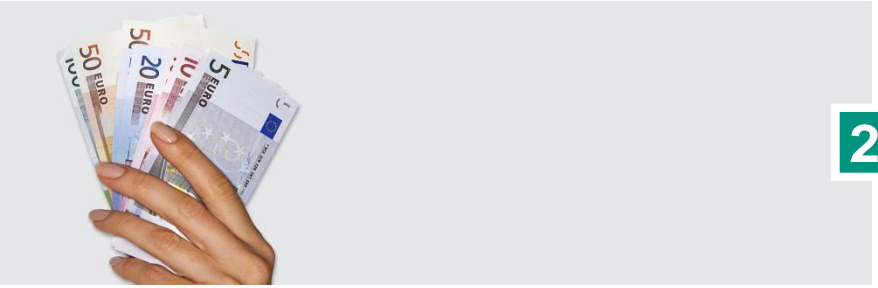
Greeks and how to hedge them

Variable annuity underlyings

Variable annuity guarantees

What determines the value of a Variable Annuity?

1. Performance of the investment portfolio



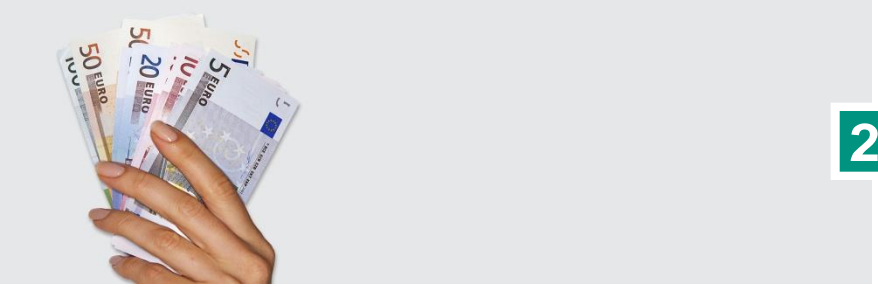
2. Benefits guaranteed to the policyholder

3. Biometric assumptions (mortality, behaviour)

How can we determine the value of each component?

1. Performance of the investment portfolio

Risk neutral, market consistent



2. Benefits guaranteed to the policyholder

Policy terms and conditions

3. Biometric assumptions (mortality, behaviour)

Mortality tables, lapse model

Overview of risks in VAs

Risk 1

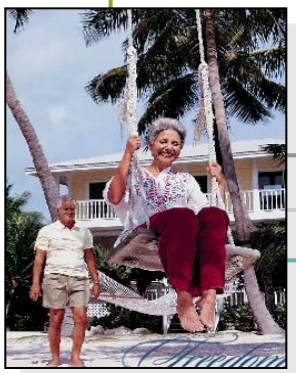
Underlying Funds

- Tracking error / basis
- Volatility
- Correlation
- Model risk

Risk 2

Interest rates

- Present value of cash flows
- Level of guaranteed yield
- Correlation with bond funds (hybrid)



Risk 3

Mortality / Longevity

- Cost of DB hedging
- IB and WB-for-life longevity risk
- Mortality linked to market movements

Risk 4

Marketing and customer behaviour

- Lapsing / surrender
- Cessation of contributions
- Maintenance of guarantee fee

Variable annuity benefits

GMDB (death)

- Investment performance at the date of death

GMAB (accumulation)

- Investment performance at the maturity date

GMWB (withdrawal)

- Investment performance at each withdrawal date

GMIB (income)

- Investment performance relative to income cost at the maturity date
- Composite interest rate and asset risk

Variable annuities: hedging approach

What model is to be assumed for assets and liabilities?

- What accounting and regulatory standards apply?
- Fully assess the drawbacks of the chosen approach

What risk factors should be hedged?

- What hedging instruments are available and in what circumstances should each of these be used?
- How should value be assessed?
  - Price and liquidity of hedges
  - Capital cost of unhedged risks
  - Short dated v. long dated hedges

What is the process for rebalancing hedges?

Agenda

Introduction to variable annuity benefits

Greeks and how to hedge them

- Black-Scholes-Merton
- The Greeks (rho, delta, gamma, vega)

Variable annuity underlyings

Variable annuity guarantees

Black-Scholes-Merton pricing model

The standard option-pricing methodology

An extension of the capital asset pricing model (CAPM)

Based on a number of ideal assumptions known not to fully represent reality:

- Prices move continuously without jumps
- Markets are deep so that trading does not affect prices
- Zero transaction costs

Hedging all exposures shown by any model will not eliminate all risks

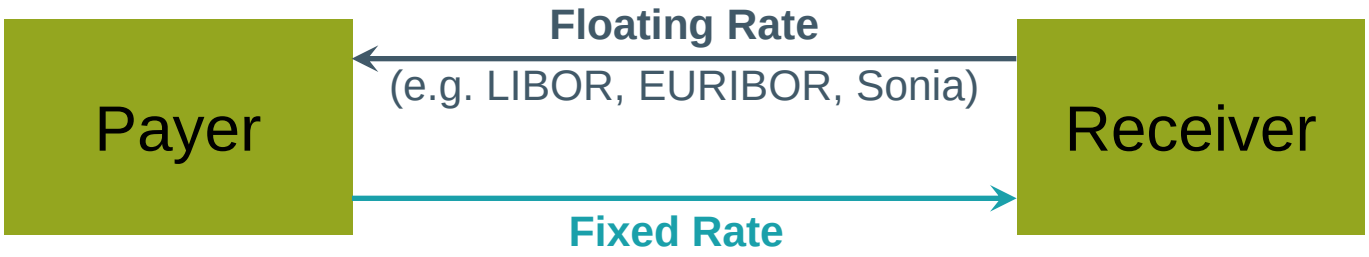
Option Greeks

Black-Scholes-Merton (and similar) leads to components of risk

- **Rho:** exposure to interest rates
- **Delta:** exposure to the underlying asset value(s)
  - first derivative of the option value
- **Gamma:** exposure to the change in delta
  - second derivative of the option value
  - affects the effectiveness and cost of delta hedging
- **Vega:** exposure to the volatility used to value the option
- **Theta:** exposure to the passing of time
  - time decay

Hedging the Greeks: Rho

Interest rate swaps



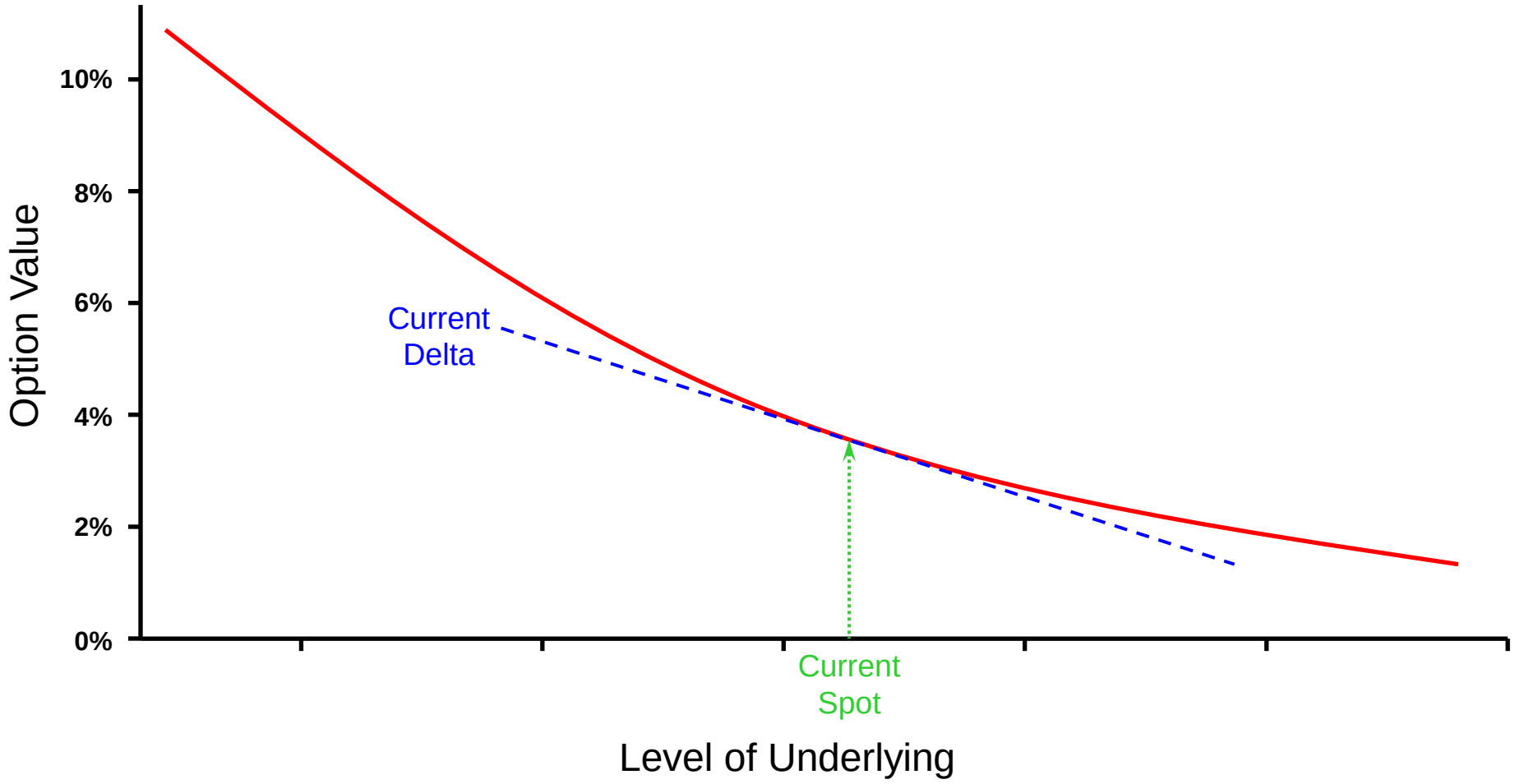
Hedging the Greeks: Rho

Interest rate swaps

- Available in multiple currencies from many counterparties
- Available to long term (50 years)
- Can usually be traded with relatively small bid:offer spreads and in large size with little market impact

	-----YEN-----	----------USD----------		----------CAD----------
	Act365/Act360	Act360/Act360		Act365/Act365
1 Yr	0.4625-0.4025		0.472-0.442	
2 Yrs	0.4650-0.4050	18.50/15.50	0.709-0.679	011.3/007.3 1.566-1.526
3 Yrs	0.4900-0.4300	24.50/21.50	1.031-1.001	018.5/014.5 1.916-1.876
4 Yrs	0.5300-0.4700	20.75/17.75	1.379-1.349	021.5/017.5 2.222-2.182
5 Yrs	0.5950-0.5350	16.75/13.75	1.730-1.700	018.5/014.5 2.469-2.429
6 Yrs	0.6750-0.6150	14.00/11.00	2.051-2.021	023.5/019.5 2.684-2.644
7 Yrs	0.7750-0.7150	5.00/2.00	2.320-2.290	026.3/022.3 2.877-2.837
8 Yrs	0.8850-0.8250	6.25/3.25	2.536-2.506	028.0/024.0 3.059-3.019
9 Yrs	1.0000-0.9400	3.00/	2.714-2.684	028.0/024.0 3.225-3.185
10Yrs	1.1100-1.0500	-2.75/-5.75	2.865-2.835	027.3/023.3 3.382-3.342
12Yrs	1.3075-1.2275	21.25/18.25	3.105-3.075	048.8/042.8 3.655-3.595
15Yrs	1.5225-1.4425	18.50/15.50	3.355-3.325	067.3/061.3 3.926-3.866
20Yrs	1.7425-1.6625	10.75/7.75	3.558-3.528	074.0/068.0 4.138-4.078
25Yrs	1.8175-1.7375	-7.50/-10.5	3.655-3.625	053.8/047.8 4.080-4.020
30Yrs	1.8475-1.7675	-29.7/-32.7	3.715-3.685	031.8/025.8 4.004-3.944

Hedging the Greeks: Delta



Hedging the Greeks: Delta

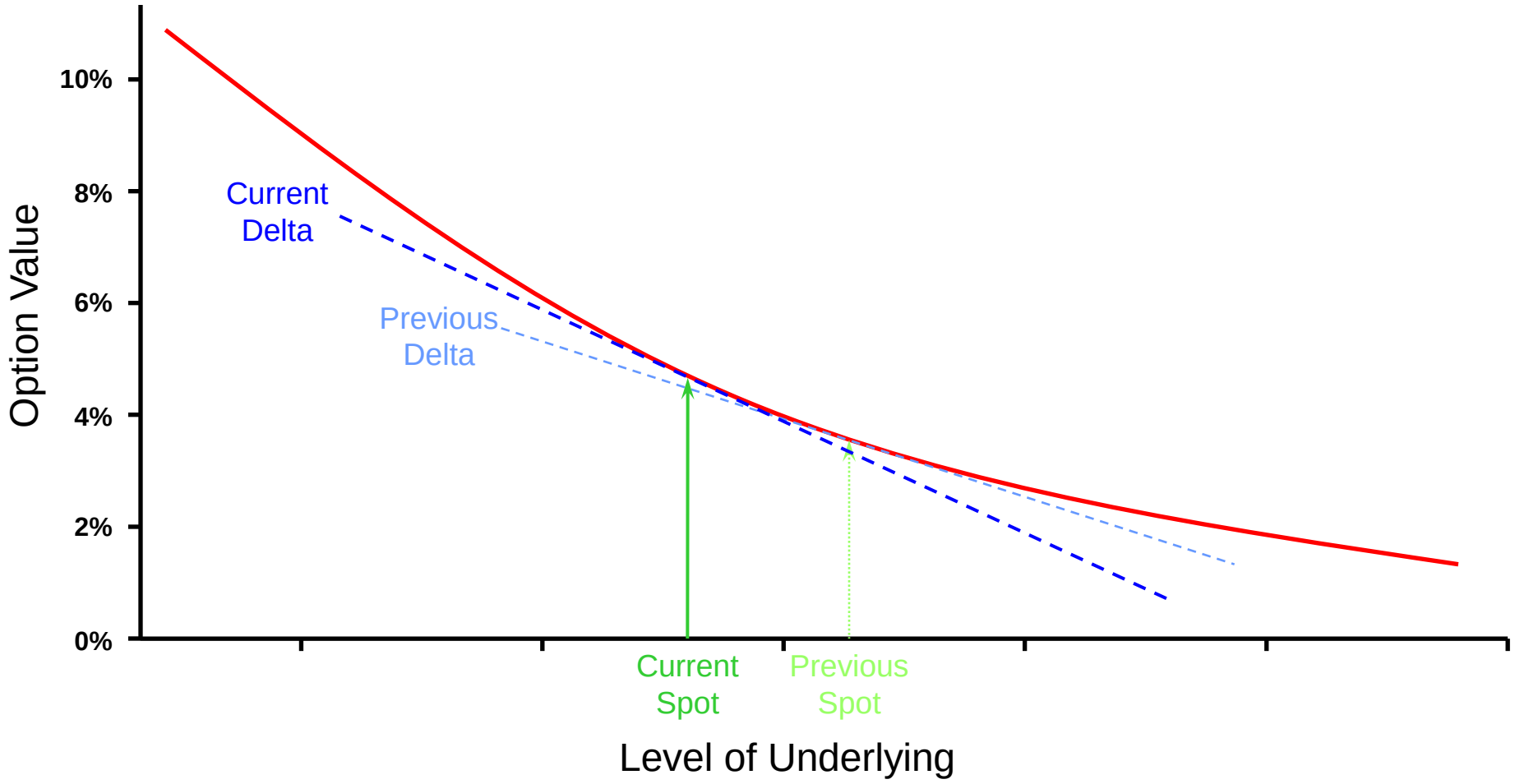
Exchange traded futures contracts

- When available can often be traded with small bid:offer spreads and in fairly large size with little market impact
- Available on a restricted number of underlyings
- Little or no availability beyond a 12 month term

OTC forwards and total return swaps

- Wider availability than futures but still restricted underlyings
- Often available longer than 1 year
  - particularly beyond 5 years, liquidity may be poor and expensive if available at all

Hedging the Greeks: Gamma



Hedging the Greeks: Gamma

Delta hedging a short option position

- Time decay generates a profit for the option writer (theta)
- Each hedge rebalance causes a loss to the option writer

Hedge Change	Asset Value	Option Value	Current Delta	Hedge Value	P&L	Total P&L
0	100%	-4.48%	-39%	4.48%		
1	98%	-5.36%	-48%	5.27%	-0.09%	-0.09%
2	100%	-4.48%	-39%	4.30%	-0.09%	-0.18%
3	103%	-3.47%	-29%	3.12%	-0.17%	-0.35%
4	100%	-4.48%	-39%	3.99%	-0.15%	-0.50%

Hedging the Greeks: Vega

Vega is the sensitivity of an option's value to the change in the price of the underlying asset

A vega hedge involves buying or selling an instrument that is sensitive to implied volatility

- Another option
- A variance swap

A vega hedge may give gamma and delta exposure

- Greeks must be hedged together

Hedging the Greeks: Vega

Variance swaps

- A swap of the realized (actual) variance (volatility squared) against a fixed rate

  Realized Variance $(\sigma^2) = S \times {}^{t=N}\sum_{t=1} \{ \ln [\text{Price}(t) / \text{Price}(t-1)] \}^2$

  where S is a fixed amount allowing for annualisation and scaling

- The payoff of a variance swap is convex in volatility
  - A long variance swap position gains more for an increase in volatility than it loses for a similar decrease in volatility
  - The cost of convexity is reflected in the fixed rate payable
- A pure play on realized volatility
  - Vega and gamma but no delta
  - Combination with delta hedging may be preferred to using options

Agenda

Introduction to variable annuity benefits

Greeks and how to hedge them

Variable annuity underlyings

- The challenges
- Actively managed funds
- Index baskets

Variable annuity guarantees

Variable annuity underlyings: issues

Equity index tracking funds

- But may be tracking a non-hedgeable index

Actively managed equity funds

- What is the closest hedgeable index?

Bond funds

- Yield curve and credit exposure

Multi-asset funds

- How stable is the allocation between asset classes?

Alternative assets

- Is there any proxy that could be used for hedging?

Hedging possibilities:
OTC options on actively managed funds

Call options are available on a limited number of funds

- Large single priced funds are most accessible
- Put options are rarely available due to a lack of shorting opportunities for potential providers

Options on funds are likely to be relatively expensive

- Poor transparency and high rebalancing costs

Large size is usually precluded

- Liquidity or market impact concerns when trading fund units
- Risk limits for the provider (vega basis risk)

Termination risk if fund shrinks or undergoes material change

Eliminates tracking error on proxy indices but limited availability

Hedging possibilities:
Options on baskets

Baskets of equity indices are widely available

- S&P500, DJ EuroStoxx 50, Nikkei 225, FTSE 100, etc

Potential to include non-equity underlyings

- Commodities and commodity indices
- Bond indices
- Swaps (for yield curve exposure within bond portfolios)
- Credit indices (for credit exposure within bond portfolios)

Potential to manage related risks

- Exposure to exchange rates

Potential where multi-asset exposures exist and are reasonably stable

Agenda

Introduction to variable annuity benefits

Greeks and how to hedge them

Variable annuity underlyings

Variable annuity guarantees

- Overview of guarantees
- Timing risks
- Additional risks
- Tailored hedging solutions

Variable annuity guarantees

GMDB (death)

- Investment performance at the date of death

GMAB (accumulation)

- Investment performance at the maturity date

GMWB (withdrawal)

- Investment performance at each withdrawal date

GMIB (income)

- Investment performance relative to income cost at the maturity date
- Composite interest rate and asset risk

Hedging possibilities:
Timing risk of staggered withdrawal (GMDB, GMWB)

Average rate (Asian) options

- Put Return = Units x Max { 0, Strike – $\sum_t$ $Weight_t$ x $Price_t$ }
- Can mitigate risk better than European options
  - Still residual timing risk where withdrawal varies relative to the averaging profile specified (the $Weight_t$ assumptions)
- Path dependency but not too complex to model, value or understand

Hedging possibilities:
Timing risk of staggered withdrawal (GMDB, GMWB)

Putable structures

- The holder has the right, but not the obligation, to sell the specified underlying at a fixed price on defined dates
  - Expensive if multiple dates are allowed
- Path dependency (probability of exercise before expiry)
  - Complex to model, value and understand
- Could be a useful tool to supplement a vanilla hedge
  - Predictable policyholder behaviour reduces the need for expensive financial options

Hedging possibilities:
Mitigating additional risks

Exotic OTC derivatives

- Relative performance options
  - Equities relative to bonds or swaps (annuity proxy) for GMIB hedging
- Lookback options
  - Put options with ratcheting strike rates to cover escalating product guarantees
- Inflation hybrids
  - Put options with inflation linked strike rates to cover real money product guarantees

Hedging possibilities:
Tailored asset packages

Hedges designed to closely match the entire product offering

- Accumulation period linked to a defined investment strategy
  - Equity and other asset classes
  - Capital protection on defined dates (putability)
  - With or without inflation protection
- A option for the insurer to choose an annuity at maturity on terms defined at outset
  - Annuity payable for a fixed period
  - With or without inflation protection

Hedging possibilities:
Tailored asset packages II

Hedges designed to precisely match the entire product offering

- Seriatim hedging of all capital market risk on a per policy basis

Tailored Asset Packages II:
The difference between insurance and banking

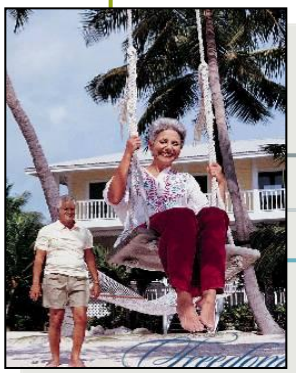
Banking

- Capital markets focus – “risk neutral”
- Live market risk management systems
- Direct links to stock exchanges, brokerages, money markets
- Back office functions geared to capital market trade settlement, documentation, risk reporting

Insurance

- Actuarial risk focus – “risk aware”
- Sophisticated risk prediction based on statistical data and events
- Back office functions geared towards insurance/reinsurance contract settlement, documentation and Asset/Liability risk reporting

Tailored Asset Packages II:
Overview of risks in VAs



Risk 1

Underlying Funds

- Tracking error / basis
- Volatility
- Correlation
- Model risk

Risk 2

Interest rates

- Present value of cash flows
- Level of guaranteed yield
- Correlation with bond funds (hybrid)

Risk 3

Mortality / Longevity

- Cost of DB hedging
- IB and WB-for-life longevity risk
- Mortality linked to market movements

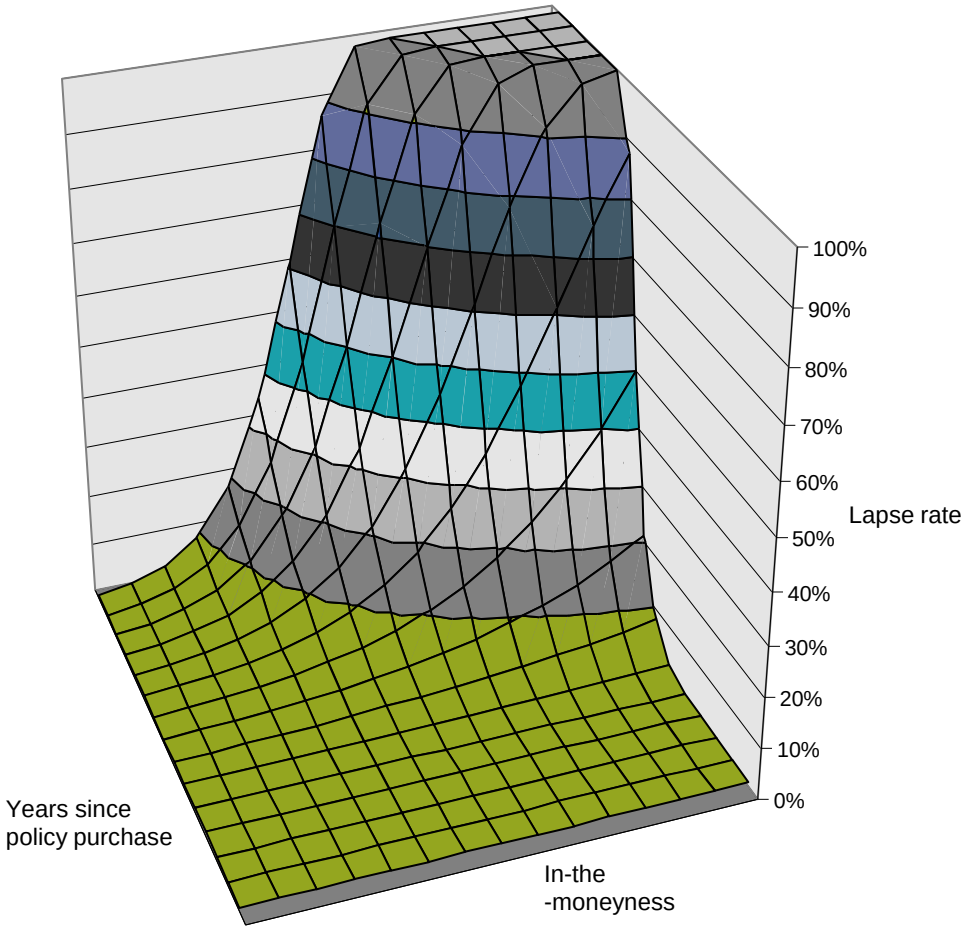
Risk 4

Marketing and customer behaviour

- Lapsing / surrender
- Cessation of contributions
- Maintenance of guarantee fee

Tailored Asset Packages II:
Lapse models must show all behavioural influences

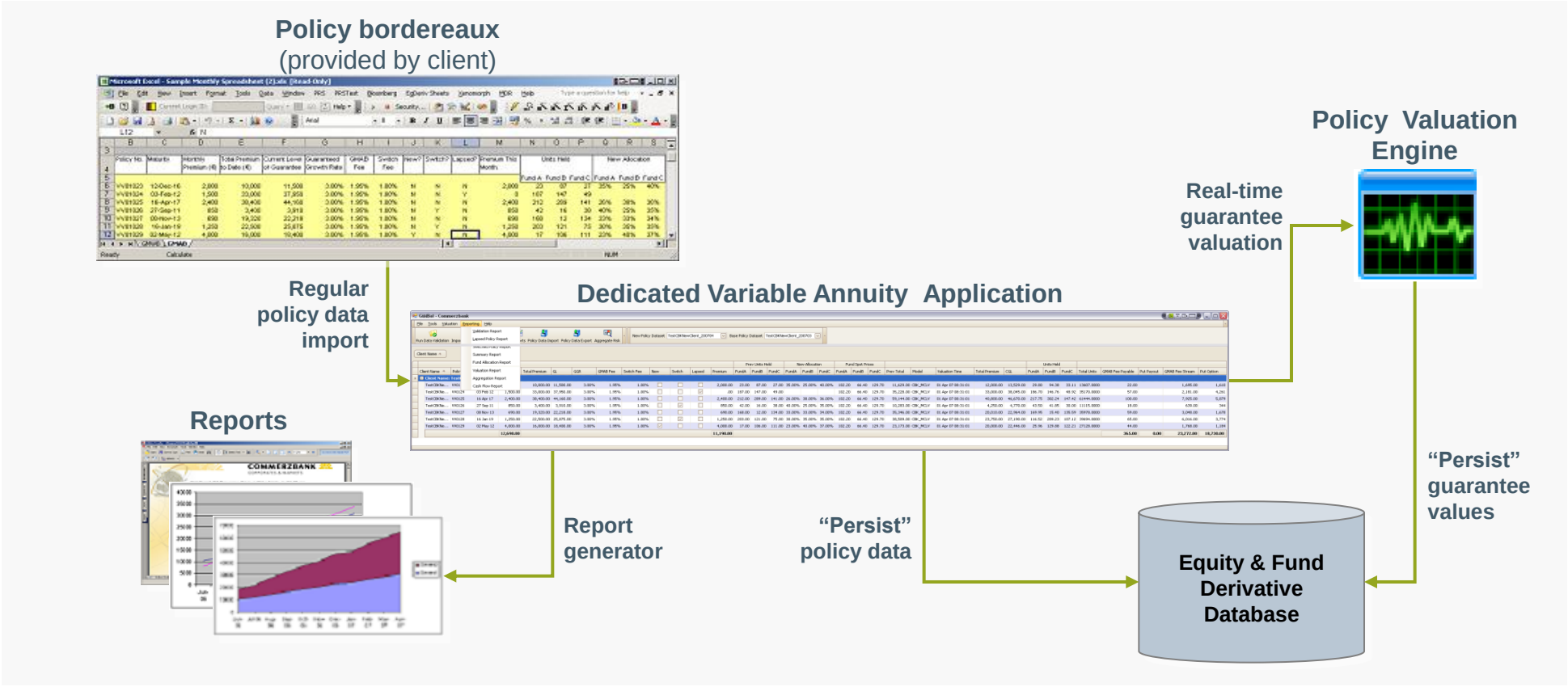
Sample lapse model



Lapsing is dynamic

- Responds to changes in:
  - **Portfolio value**
  - **Time**
  - **Policyholder age**
- Lapses have “greeks” which must be calculated on a risk-neutral basis
  - **Delta**
  - **Gamma**
  - **Vega**
  - **Rho**

Tailored Asset Packages II:
Seriatim hedging of capital market risks



- Precise hedging of all market risks
- Actuarial risk hedged or unhedged
- Derivative or reinsurance delivery wrapper

Conclusions

Hedging VAs requires a multi-faceted approach:

- Good design is the best hedge
- Sustainability of product and pricing: hedgeability and capital efficiency
- The purpose of risk neutral modelling is to determine the cost of hedging, not to estimate the portfolio path
- All costs need to be taken into account
- Behavioural assumptions have market components that need hedging

Questions or comments?

Expressions of individual views by members of The Actuarial Profession and its staff are encouraged.

The views expressed in this presentation are those of the presenters.