Institute and Faculty of Actuaries

ERM in an optimal control framework

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Background and objectives

- Origins of **Enterprise Risk Management ("ERM")**
  - **Dynamic Financial Analysis** from the 1990s
    - **Insurance risk** and **market risk**
  - ERM added all other risks
    - Particularly **operational risk**
- Objectives today
  - Clear mathematical formulation of ERM problem
    - Balance of risk controls against profit objective
    - Comments will apply to a general **risk business**
      - Though we shall usually speak in the context of an insurance company

Definition of ERM

- **COSO (2004)** definition

  (Committee of Sponsoring Organisations of the Treadway Commission)

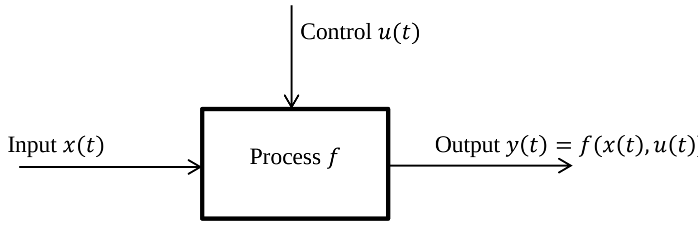
  – "Enterprise risk management is a **process**, effected by an entity's board of directors, management and other personnel, applied in **strategy setting** and **across the enterprise**, designed to identify **potential events** that may affect the entity, and **manage risk** to be **within its risk appetite**, to provide **reasonable assurance** regarding the **achievement of entity objectives**."

ERM – implementation

- COSO **ERM – Integrated Framework**
  – Internal environment
  – Objective setting
  – Event identification
  – Risk assessment
  – Risk response
  – Control activities
  – Information and communication
  – Monitoring

Optimal control theory (1)

- Deterministic control theory



- Objective is to optimise some functional J$[y]$ (**performance measure**) of output y by manipulation of the control $u(t)$

Optimal control theory (2)

- Mathematical formulation
  - Generalise previous system by assuming that $x(t), y(t), u(t)$ are m-, n-, r-vectors
  - Define
    $$X(t) = \{x(s): 0 \leq s \leq t\}$$
    - Similarly $Y(t), U(t)$
  - Assume system output takes the form
    $$y(t) = f(X(t), U(t)) \text{ with } f: \mathcal{F}_t^{m+r} \to \mathfrak{R}^n$$
  - Assume performance measure takes the form
    $$J[y] = \int_0^T L[t, y(t)]\, dt \text{ with } L: \mathfrak{R}^{n+1} \to \mathfrak{R}$$

Loss function

Loss intensity function

Optimal control theory (3)

$$J[y] = \int_0^T L[t, y(t)]\, dt$$

- Optimal control is $u^*(t)$ given by
$$u^*(t) = arg \min_{u(t)} J[y]$$
- A few more details
  - **Terminal value**: extend loss function as follows
$$J[y] = \int_0^T L[t, y(t)]\, dt + \Phi[T, y(T)]$$
Terminal value
  - **Constraints**, e.g.

$y_i(t) \leq c_{(1)}, 0 \leq t \leq T$ **or** $y_j(T) \leq c_{(2)}$ **or** $g\big(y_k(T)\big) \leq c_{(3)}$, etc.

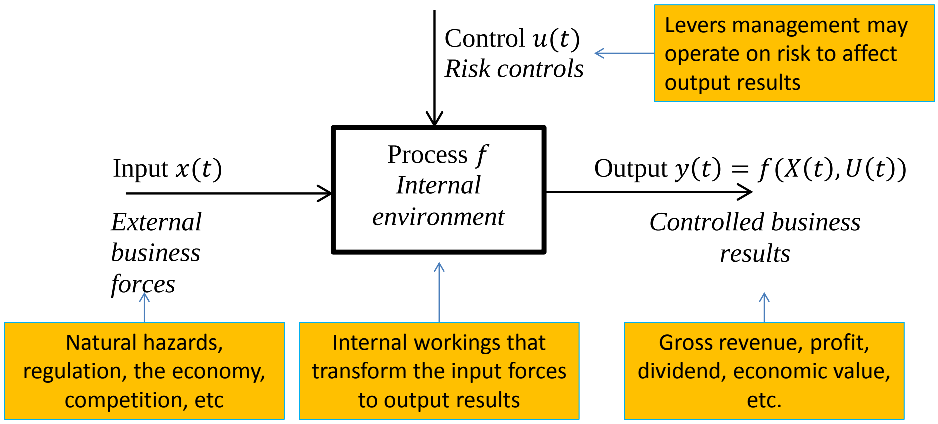
Stochastic control theory

- Suppose inputs are stochastic
  - Then outputs and controls will also be stochastic

Deterministic	Stochastic
Objective function	**Objective function**
$J[y] = \int_0^T L[t, y(t)]\, dt + \Phi[T, y(T)]$	$E[J[y]] = E\left[\int_0^T L[t, y(t)]\, dt + \Phi[T, y(T)]\right]$
Constraints	**Constraints**
$y_i(t) \leq c_{(1)}$, etc	$E[y_i(T)] \leq c_{(1)}$, etc
Control	**Control**
$u^*(t) = arg \min_{u(t)} J[y]$	$u^*(t) = arg \min_{u(t)} E[J[y]]$

ERM formulation (1)

- ERM as a control system



ERM formulation (2)

- Objective function
  - Example: discounted future profit (loss)
  - ⇒ Loss intensity = instantaneous profit (loss) at time t, discounted to time 0
- Constraints
  - Externally sourced, e.g. regulatory restrictions such as minimum capital requirement
  - Internally sourced “business rules”, e.g. “premium volume should not contract by more than 10% in any one year”
- Discussion
  - Computational load is heavy
  - There may be difficulty in parameterising the system
    - Particularly selection of probability distributions

Review of the COSO framework

- Internal environment
- Objective setting
- **Event identification**
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Event identification

- **COSO**: "Internal and external events affecting achievement of an entity's objectives must be identified"
- These are the factors that affect loss intensity $L[t, x(t), u(t, x(t))]$ through inputs $x(t)$

- Usual to consider events by major risk group
  - Credit risk
  - Market risk
  - Liquidity risk
  - Insurance risk
  - Operational risk
  - Group risk
- And sub-groups within these
- Classification of events can be very detailed
  - E.g. British Bankers' Association recognises about 140 operational risks alone

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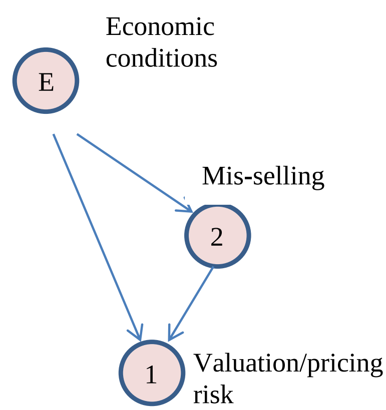
Internal environment (1)

- **COSO:** “The internal environment encompasses the tone of an organization, and sets the basis for how risk is viewed and addressed by an entity’s people, including risk management philosophy and risk appetite, integrity and ethical values, and the environment in which they operate”

Internal environment (2)

- How does an individual risk affect the loss intensity?
- Factors describing individual loss
  - Probability of occurrence
  - Distribution of severity (size of loss)
  - **Effect of given severity on loss intensity**
    - **Determined by internal environment**
- There may be dependency between individual risks

Example of dependency



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Risk assessment - definition

- **COSO:** "Risks are analyzed, considering likelihood and impact, as a basis for determining how they should be managed. Risks are assessed on an inherent and a residual basis"

Risk assessment – likelihood and impact

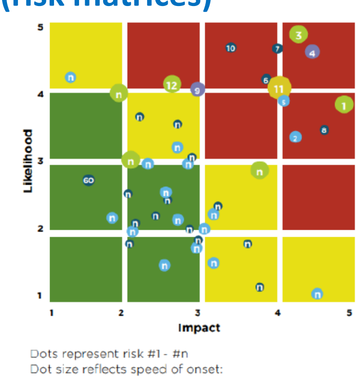
- Input is stochastic vector $x(t) = [x_1(t), \dots, x_m(t)]$
- Characterise by distribution function $F_X(x) = F_X(x_1, \dots, x_m)$
- Analyse by **likelihood (= frequency)** and **impact (= severity)**
  - Cost of r-th risk is

$$X_r = \sum_{i=1}^{N_r} S_{ri}$$

Frequency, d.f. F_{N_r}

Severity, d.f. F_{S_r}

- Leads to familiar **likelihood-impact plots (risk matrices)**



Risk assessment – dependency between risks

- Note that **joint d.f.** $F_X(x_1, \ldots, x_m)$ is required

Copulas

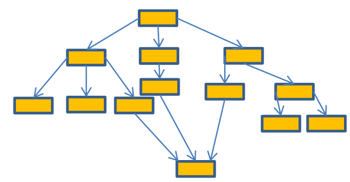
- Mapping $C: [0,1]^m \to [0,1]$ that transforms a set of marginal d.f.'s into a joint d.f., e.g. frequency

$$F_N(n_1, \ldots, n_m) = C(F_{N_1}(n_1), \ldots, F_{N_m}(n_m))$$

- Useful but one still needs to quantify dependencies in some form
  - Correlation or some other parameter

Causal models

- Built on quantitative maps of the dependencies between individual risks



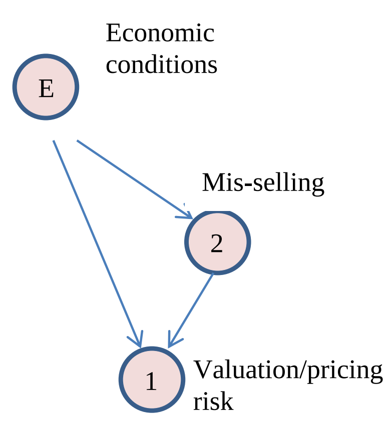
- Example is **Bayesian network**

Risk assessment – Bayesian networks

Technical description

- **Directed graph**, i.e.
  - Collection of nodes
  - Edges connecting pairs of nodes
  - Each edge has a direction
- Graph is **acyclic**, i.e.
  - No path of directed edges leading from a node back to itself
- Graph is **probabilistic**, i.e.
  - Each edge is quantified with the conditional dependency between its nodes

Example (sub-graph only)



Risk assessment – Bayesian networks

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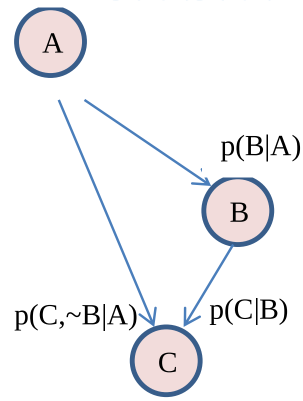
Example (sub-graph only)

$$Prob[C] = Prob[A]\{p(C,\sim B|A) + p(B|A)p(C|B)\}$$



Risk assessment – quantifiability of risks

- Undoubtedly difficult with little or no data for some parts of the model
- Much to be said but just a few points here
  - Decompose graph of Bayesian network into **disconnected sub-graphs**
    - These are stochastically independent
  - If using copulas
    - Must first determine marginals
    - Then dependencies
    - Perhaps commence with marginals of broad collections of risks, e.g. credit risk, market risk, etc. (**vine copulas**?)
- Last resort ("cop out option") is **stress testing**
  - Measure change in objective function for a defined increment in a single risk or group of risks

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Risk response and control activities (1)

- **COSO**
  - **Risk response**

    "Management selects risk responses – avoiding, accepting, reducing, or sharing risk – developing a set of actions to align risks with the entity's risk tolerances and risk appetite"
  - **Risk control**

    "Policies and procedures are established and implemented to help ensure the risk responses are effectively carried out"
- Thus
  - A risk response is a policy
  - A risk control is its implementation

Risk response and control activities (2)

- Two categories of risk control
  - Prevention
    - Seeks to reduce likelihood or reduce severity of occurrence
  - Mitigation
    - Seeks to modify impact on business of event with known severity, e.g. excess-of-loss reinsurance

$$F_{S(net)}(x) = F_S(x), x < D$$
$$= 1, x \geq D$$

- Cost of risk controls
  - **Direct costs**, e.g. physical safety measures
  - Reduction in **expected profit**, e.g. reinsurance
    - Also reduction in the negative tail of the distribution of profit
- Thus compromise between risk control and cost
  - The compromise may be medium or long term
  - e.g. reinsurance may:
    - Generate a short term cost; **BUT**
    - Increase insurer's longevity, and therefore future expected profits

Risk response and control activities (3)

- Each risk control comes with a cost
- If it is a voluntary control, it should improve performance
  - Otherwise, why is imposed?
  - If it is compulsory (regulatory), it may not necessarily improve performance
- The control theoretic optimum seeks
  - to implement only (voluntary) controls that improve performance
  - where the strength of the control may be selected, to implement at that strength where the marginal gain in performance is balanced by the marginal cost

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Objective setting (1)

- **COSO:** "Objectives must exist before management can identify potential events affecting their achievement. Enterprise risk management ensures that management has in place a process to set objectives and that the chosen objectives support and align with the entity's mission and are consistent with its risk appetite"

Objective setting (2)

- Recall basic problem of minimisation of objective function

$$E[J[y]] = E\left[\int_0^T L[t, y(t)]\,dt + \Phi[T, y(T)]\right]$$

- Objective setting amounts to selection of the objective function and its constraints

Example

Objective function

Maximise discounted expected profit over the interval [0,T] where T years is the time horizon; subject to the following **constraints**:

Prob[technical insolvency in [0,T]]<p_1
Prob[revenue account in loss in any one year in [0,T]]<p_2
Prob[net cost of a single natural event>k_1 in [0,T]]<p_3
Prob[reduction in scale of operations>k_1% in any one year in [0,T]]
<p_4

- Objective function, in isolation, encourages risk taking
- Constraints encourage risk mitigation

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- **Monitoring**

Monitoring (1)

- **COSO:** "The entirety of enterprise risk management is monitored and modifications made as necessary. Monitoring is accomplished through ongoing management activities, separate **evaluations**, or both"
- For monitoring of parametric models, see Taylor (2011), based on following principles:
  - Each parameter in the model under test generates one monitoring table
  - Monitoring is stochastic, i.e. each time it compares an observation with a target it gives the probability of a deviation from target as large as observed

Monitoring (2)

- Example (Motor insurance rather than ERM)

Driver age	Claim frequency			
	Observed	Target	Ratio: Observed/Target	Statistical significance
	%	%	%	%
Under 25	17.6	18.4	96	21
25-30	13.0	11.9	109	6
30-40	11.0	10.1	109	2
40-50	11.4	10.8	106	4
50-65	11.0	10.5	105	8
Over 65	14.2	12.3	115	3
Total	**12.0**	**11.2**	**107**	**1**

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Information and communication

- **COSO:** "Relevant information is identified, captured, and communicated in a form and timeframe that enable people to carry out their responsibilities. Effective communication also occurs in a broader sense, flowing down, across, and up the entity"
- Less quantitative subject
  - Not discussed here

Conclusions

- Optimal control formulation integrates in a quantitative manner:
  - Business objectives (profit directed)
  - Risk controls (cost generating)
- Disciplined structure for these opposing forces
- We have **NOT** established an ERM framework
  - **Have not** identified the risks or the risk controls
  - **Have** provided the formal link between these and business performance

References

- COSO (2004). **Enterprise Risk Management -- Integrated Framework**. American Institute of Certified Public Accountants, NYC, USA.
- Taylor G (2011). A statistical basis for claims experience monitoring. **North American Actuarial Journal**, 15(4), 535-552.