

The Actuarial Profession
making financial sense of the future

Redefining the deviance objective for generalised linear models

Tony Lovick, Pete Lee



Relieving the pressure from tight fitting models

October 2011

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Agenda

- Motivation for method
- Concepts behind Case Deleted Deviance
- Simple Example
- Concepts behind Noise Reduction Method
- Real Examples
- Other applications



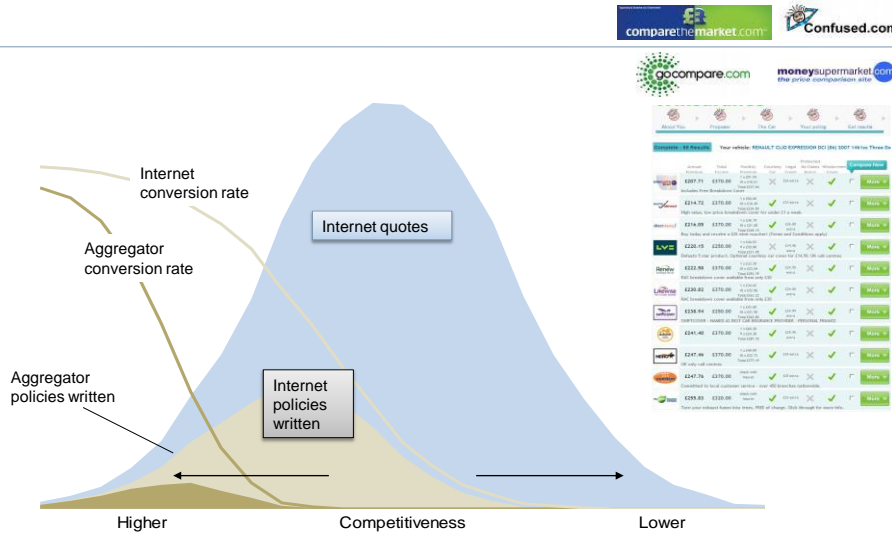
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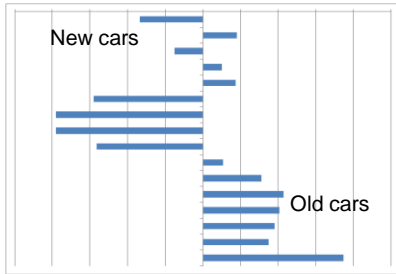
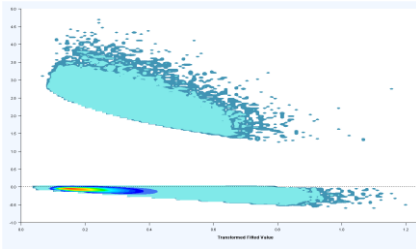
Redefining the deviance objective for generalised linear models

Motivation for method

Motivation for method



Motivation for methods

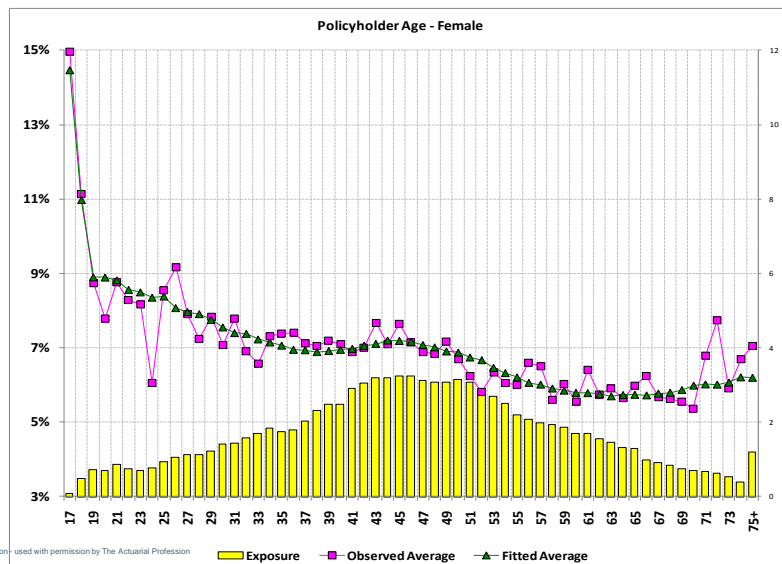


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- 100+ candidate variables
- ~25 factors – more when looking at e.g. external data for postcoding
- Up to ~12 interactions
- Multi-dimension effects via “scores”
- Reduced scope for competitor benchmarking
- Dilemma of over-fitting vs. danger of anti-selection

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Motivation for methods

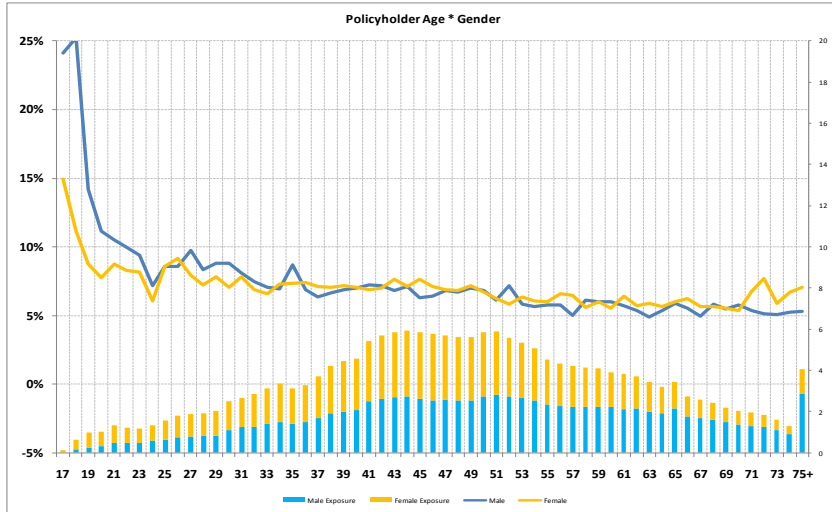


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Exposure Observed Average Fitted Average

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Motivation for methods

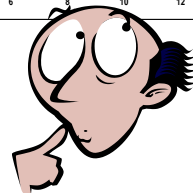
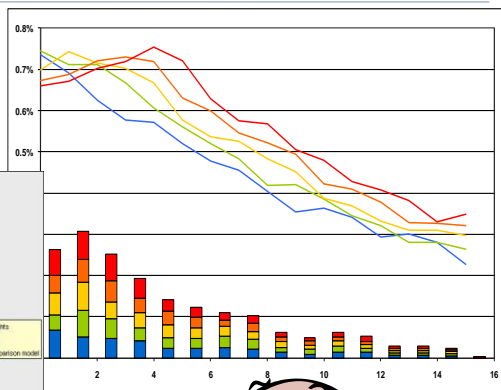
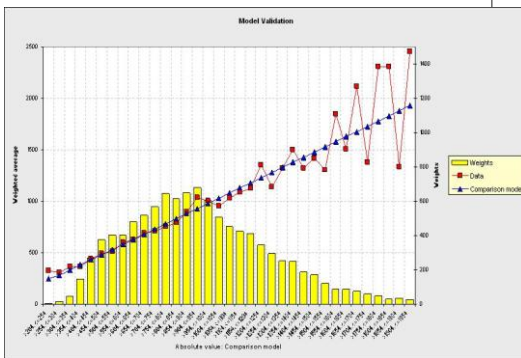


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Motivation for methods

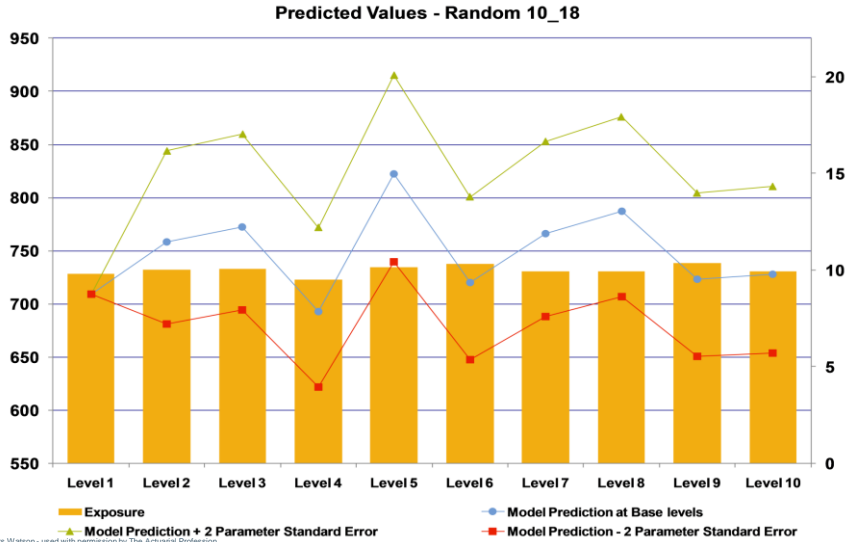
- Chi-squared & F-tests
- Wald p-values
- Akaike information criteria



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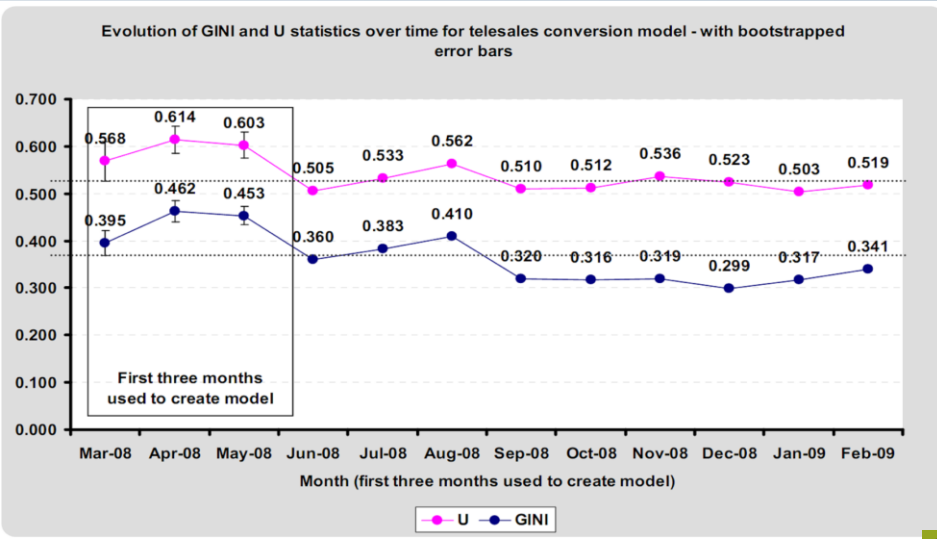
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Motivation for methods



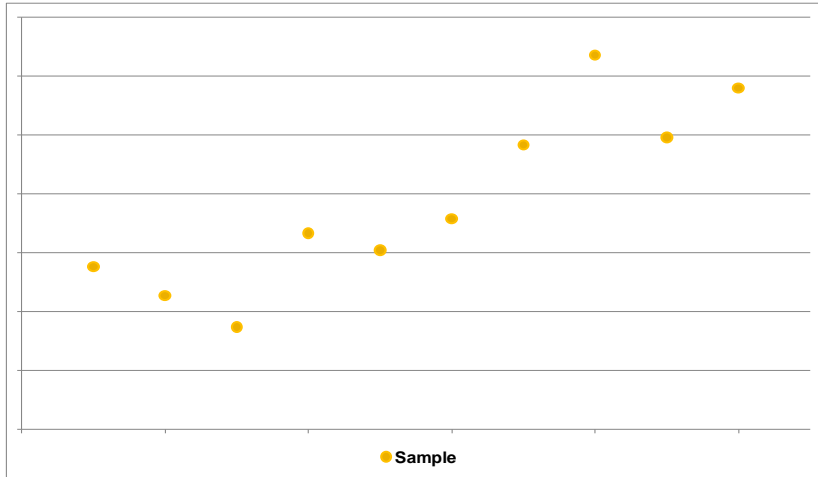
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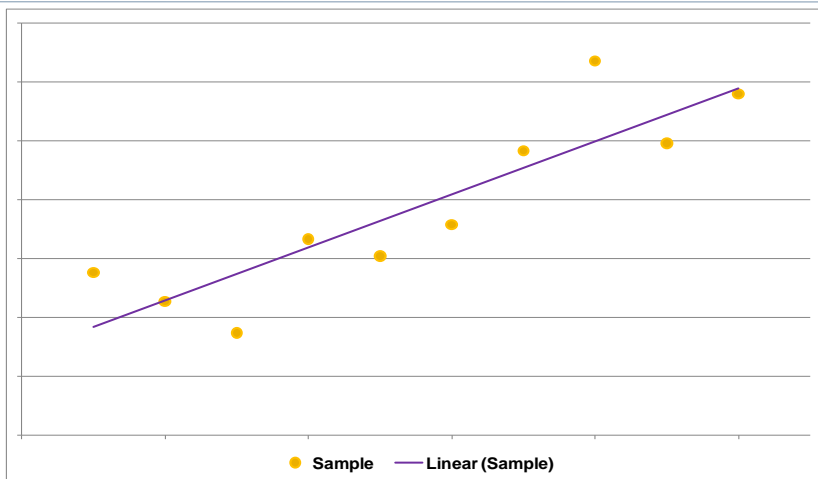
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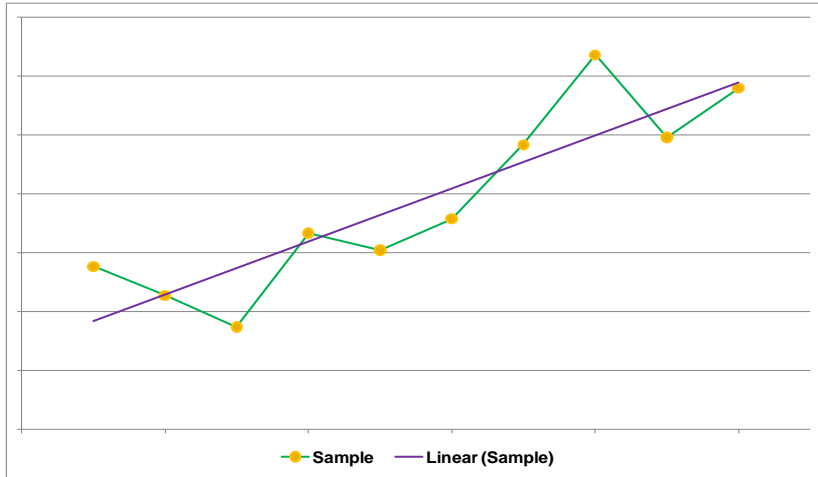
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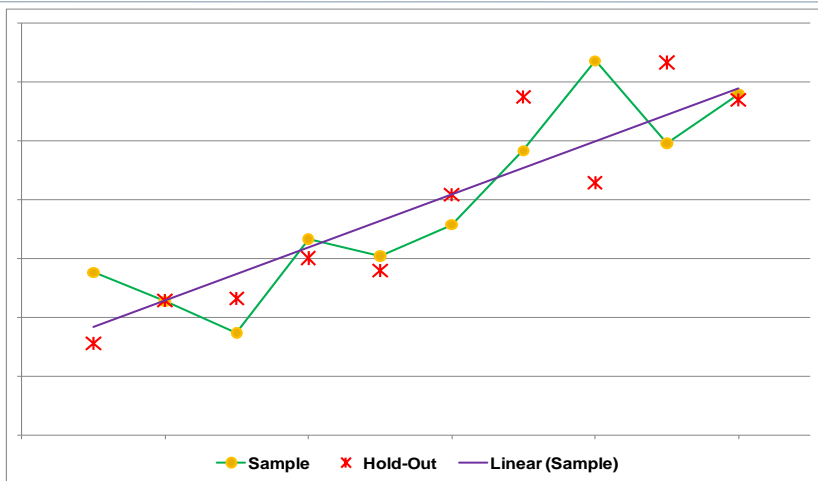
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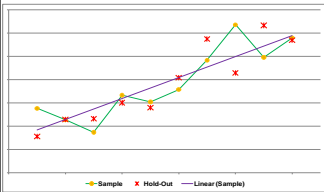
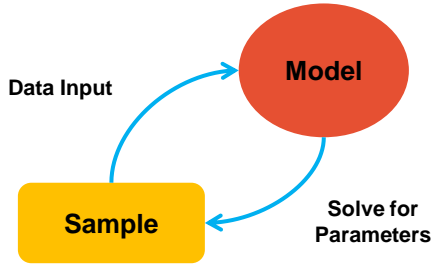
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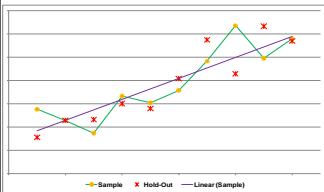
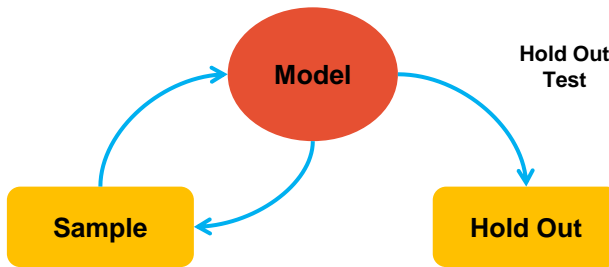
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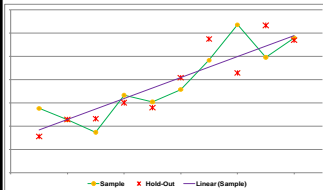
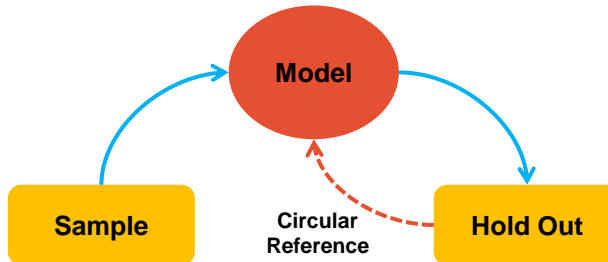
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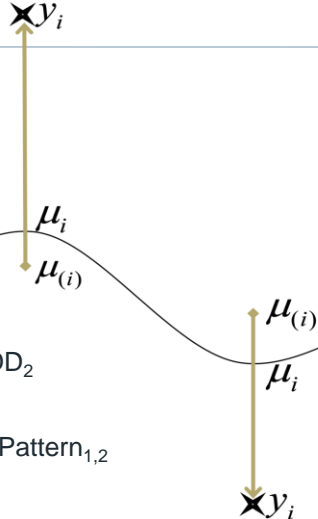
Redefining the deviance objective for generalised linear models

Concepts behind Case Deleted Deviance

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Concepts behind the Case Deleted Deviance

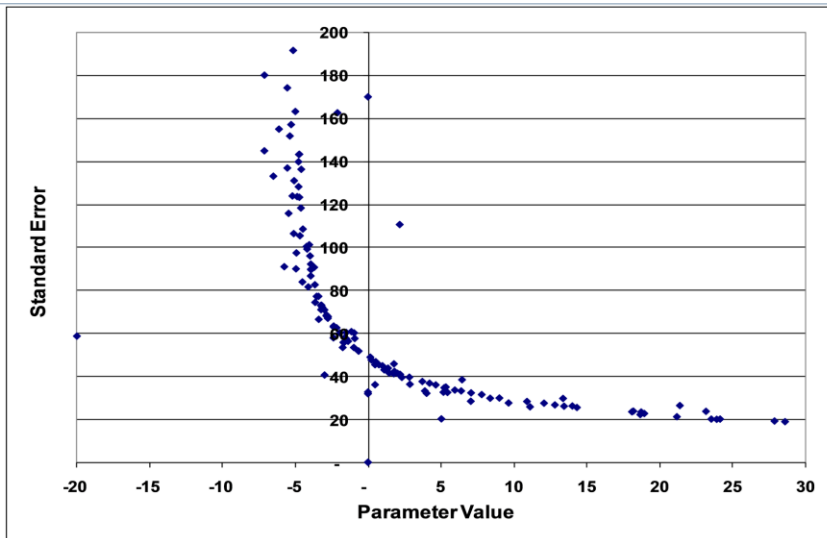
- Standard Deviance
 - $SD(y_i, \mu_i)$
 - “Case Deleted” Deviance
 - $CDD(y_i, \mu_{(i)})$
-
- “Pattern”
 - $Pattern_{1,2} = CDD_1 - CDD_2$
 - “Noise”
 - $Noise_{1,2} = SD_1 - SD_2 - Pattern_{1,2}$
 - “Value”
 - $Value_{1,2} = Pattern_{1,2} - 5 * Noise_{1,2}$



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Concepts behind the Case Deleted Deviance

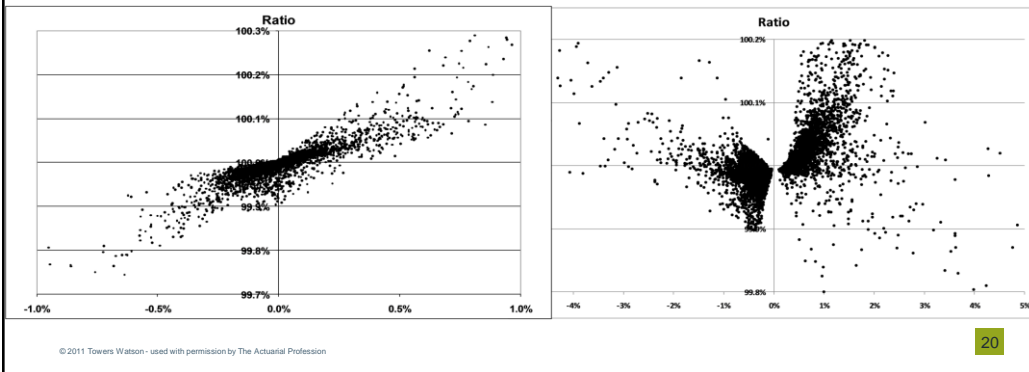


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Concepts behind the Case Deleted Deviance

- The approximate method to calculate μ_i is
 - 99.9% accurate
 - “n” times faster



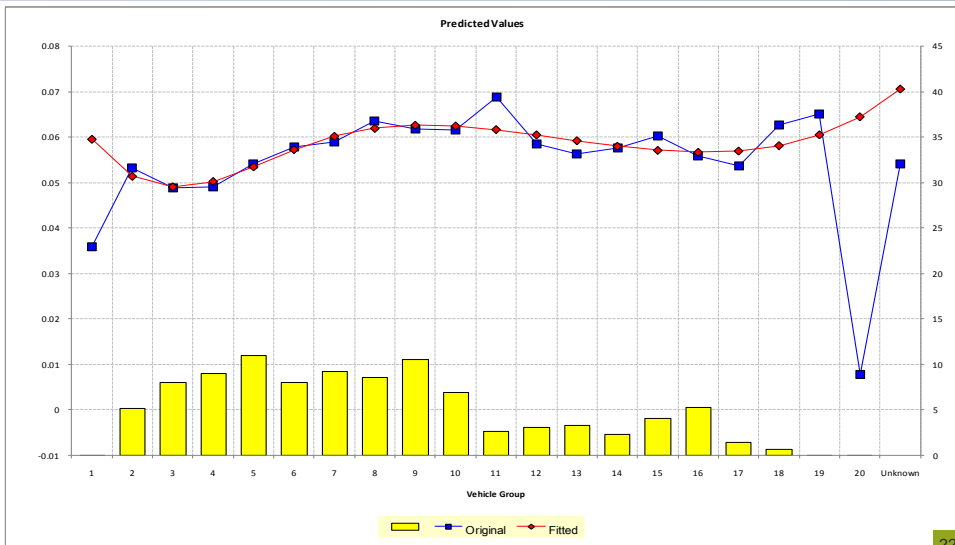
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Simple Example

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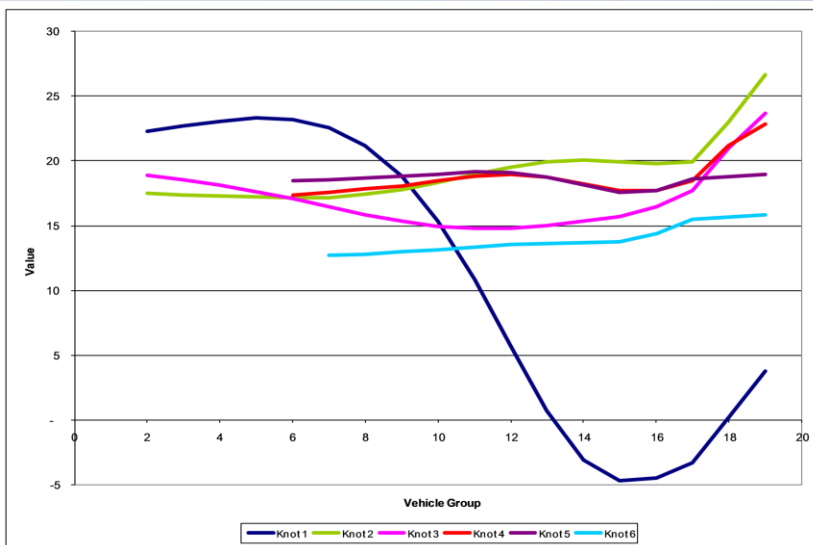
Simple Example



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Simple Example



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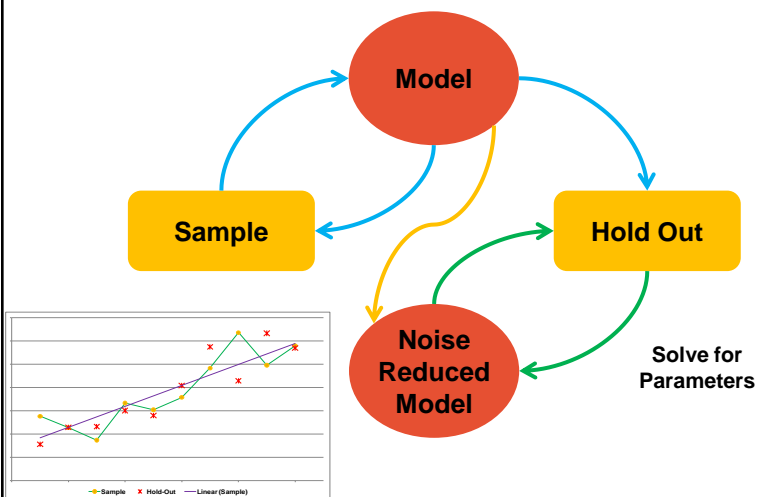
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Redefining the deviance objective for generalised linear models

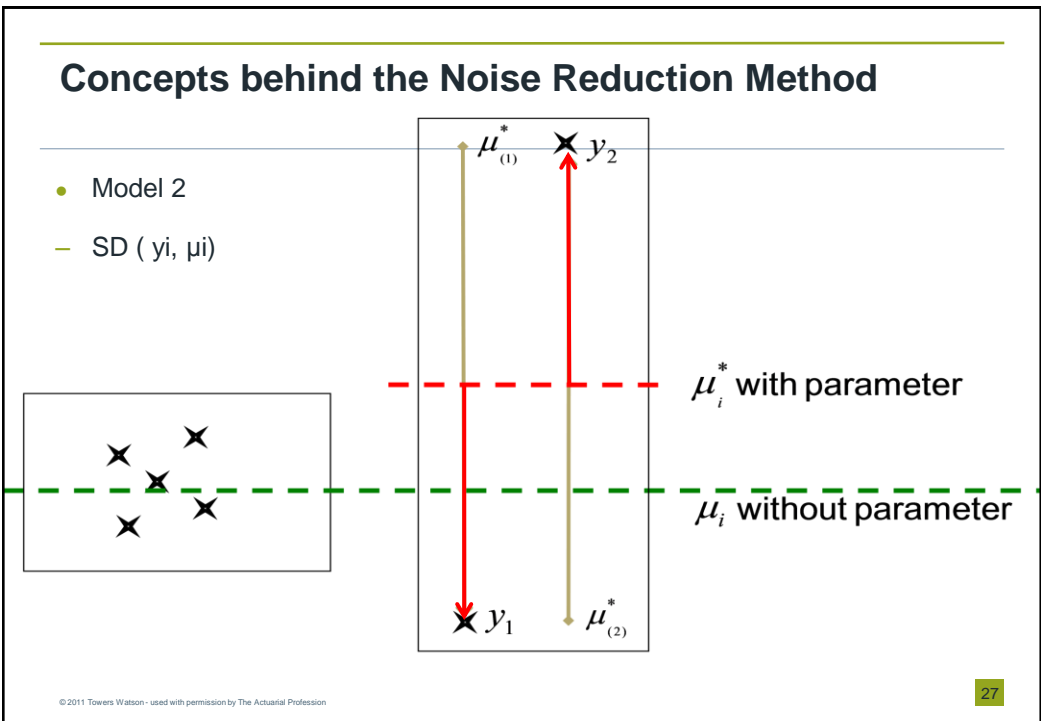
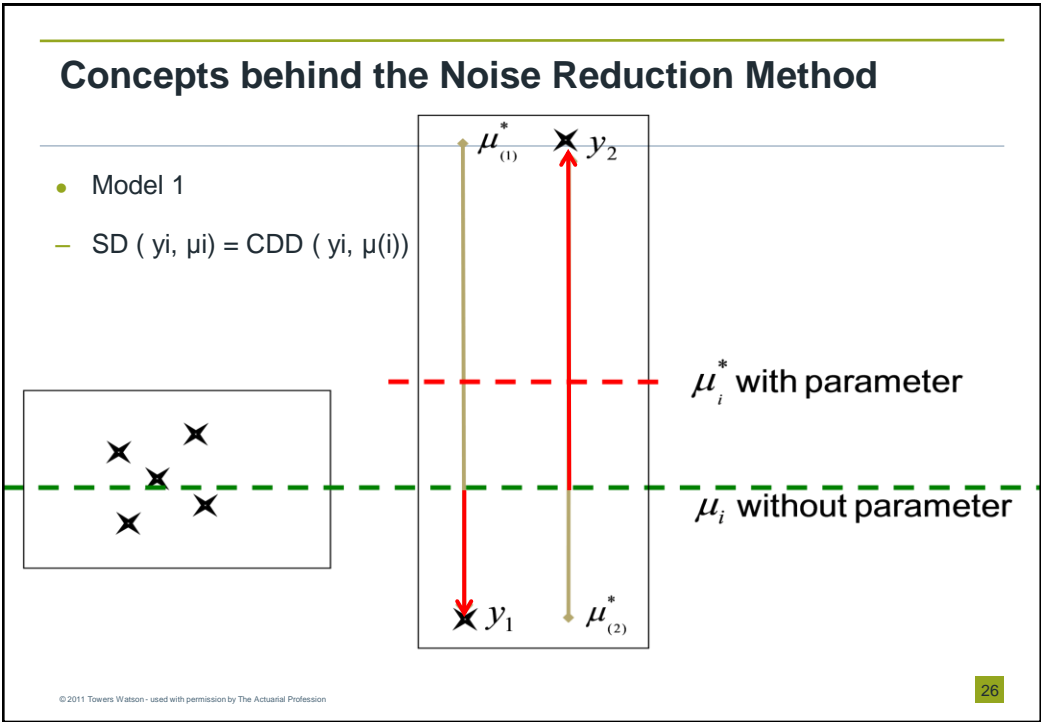
Concepts behind Noise Reduction Method

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Motivation for methods

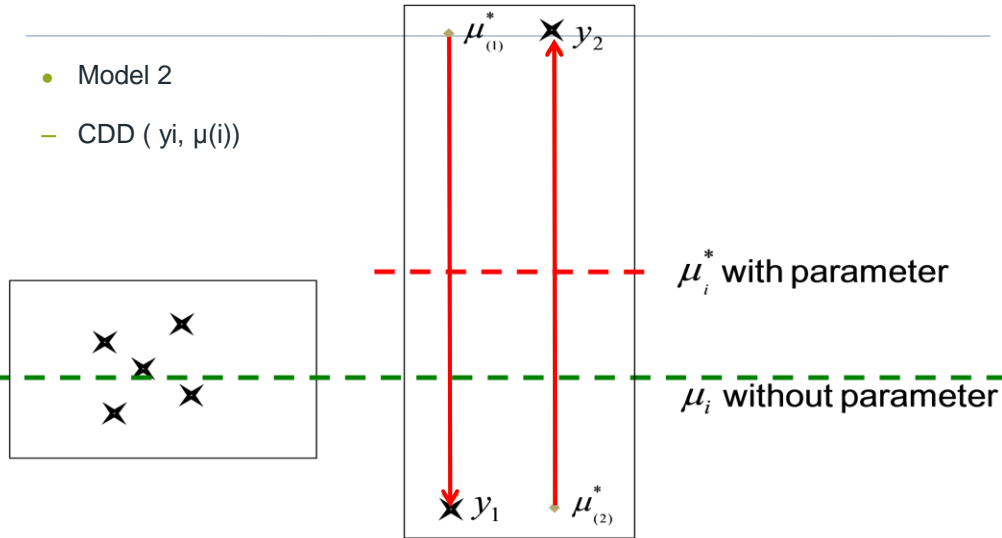


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Concepts behind the Noise Reduction Method

- Model 2
- CDD ($y_i, \mu(i)$)



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Concepts behind the Noise Reduction Method

- Find the “best” scalars in the “Case Deleted Deviance” sense
- Higher Variance parameters get scaled back most
- Take account of parameter correlations

| Standard Error | Standard Error (%) | Alias Indicator (%) | Weight | Weight (%) | Exp(Value) |
|----------------|--------------------|---------------------|---------|------------|------------|
| 0.031 | 1.1 | | 236,207 | 100.0 | 0.064 |
| 0.020 | 34.1 | | 100,320 | 42.5 | 0.942 |
| | | | 135,888 | 57.5 | |
| | | | 200,845 | 85.0 | |
| 0.027 | 23.6 | | 35,362 | 15.0 | 1.120 |
| 0.010 | 8.9 | | 236,207 | 100.0 | 1.124 |

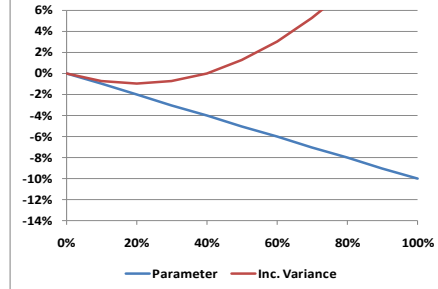
| | | | | | | | |
|----|--------------------------------|--------|-------|-------|---------|-------|-------|
| 14 | VA Curve 1 (OPoly(1)) | -0.265 | 0.016 | 6.0 | 219,928 | 93.1 | 0.768 |
| 15 | VA Curve 1 (OPoly(2)) | -0.076 | 0.017 | 22.5 | 219,928 | 93.1 | 0.926 |
| 16 | PA Curve 1 spline 1 (OPoly(1)) | 0.212 | 0.039 | 18.5 | 8,194 | 3.5 | 1.236 |
| 17 | PA Curve 1 spline 3 (OPoly(1)) | 0.041 | 0.009 | 23.1 | 229,373 | 97.1 | 1.042 |
| 18 | PA Curve 1 spline 4 (OPoly(1)) | -0.064 | 0.009 | 13.9 | 229,373 | 97.1 | 0.938 |
| 19 | YADA Curve 1 (OPoly(1)) | -0.176 | 0.014 | 8.2 | 236,207 | 100.0 | 0.838 |
| 20 | YADA Curve 1 (OPoly(2)) | 0.062 | 0.012 | 19.0 | 236,207 | 100.0 | 1.064 |
| 21 | VG Curve 1 spline 1 (OPoly(1)) | -0.242 | 0.119 | 49.4 | 203,278 | 86.1 | 0.785 |
| 22 | VG Curve 1 spline 2 (OPoly(1)) | -0.116 | 0.070 | 60.0 | 235,863 | 99.9 | 0.890 |
| 23 | VG Curve 1 spline 3 (OPoly(1)) | -0.050 | 0.055 | 109.6 | 235,863 | 99.9 | 0.951 |
| 24 | VG Curve 1 spline 4 (OPoly(1)) | -0.177 | 0.092 | 52.3 | 235,863 | 99.9 | 0.838 |

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Concepts behind the Noise Reduction Method

- Find the “best” scalars in the “Case Deleted Deviance” sense
- Higher Variance parameters get scaled back most
- Take account of parameter correlations



C_{jk} Is the Variance Covariance Matrix

$$C_{jk} = \left(\sum_p X_{pj}^m W_p X_{pk} \right)^{-1}$$

$$C_{jk}^* = \begin{cases} \text{Var}(\lambda_j \beta_j) = \lambda_j^2 \text{Var}(\beta_j), & j = k \\ \text{Cov}(\lambda_j \beta_j, \lambda_k \beta_k) = \lambda_j \lambda_k \text{Cov}(\beta_j, \beta_k), & j \neq k \end{cases}$$

$$h_i = \sum_{jk} X_{ij} C_{jk} X_{ik} W_i$$

$$h_i^* = \sum_{jk} X_{ij} C_{jk}^* X_{ik} W_i$$

$$\eta_{\mathbf{C}} = \eta_i - \left(\frac{h_i}{1-h_i} \right) g'(\eta_i) (\mathbf{y}_i - \mu_i)$$

$$\eta_{\mathbf{C}^*} = \eta_i^* - \left(\frac{h_i^*}{1-h_i^*} \right) g'(\eta_i^*) (\mathbf{y}_i - \mu_i)$$

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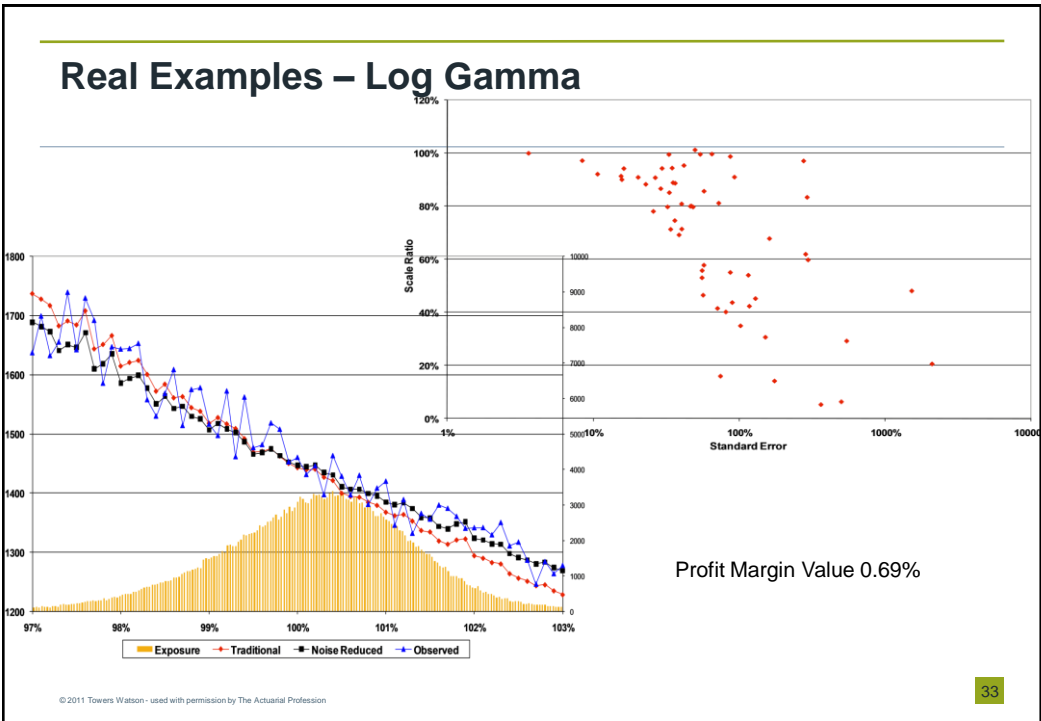
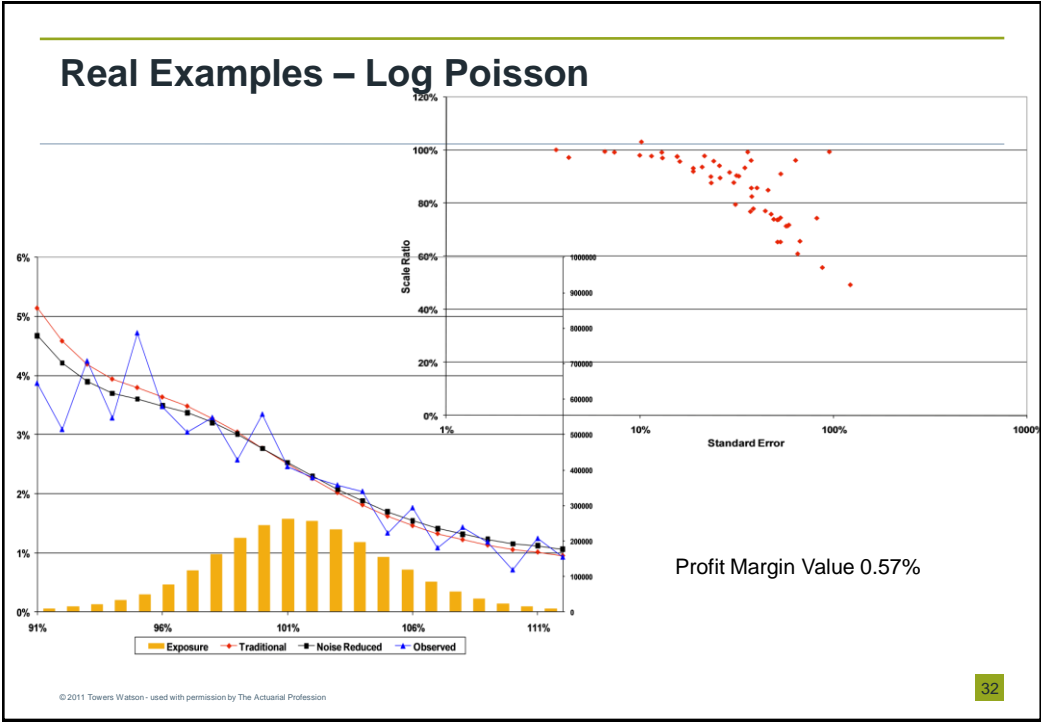
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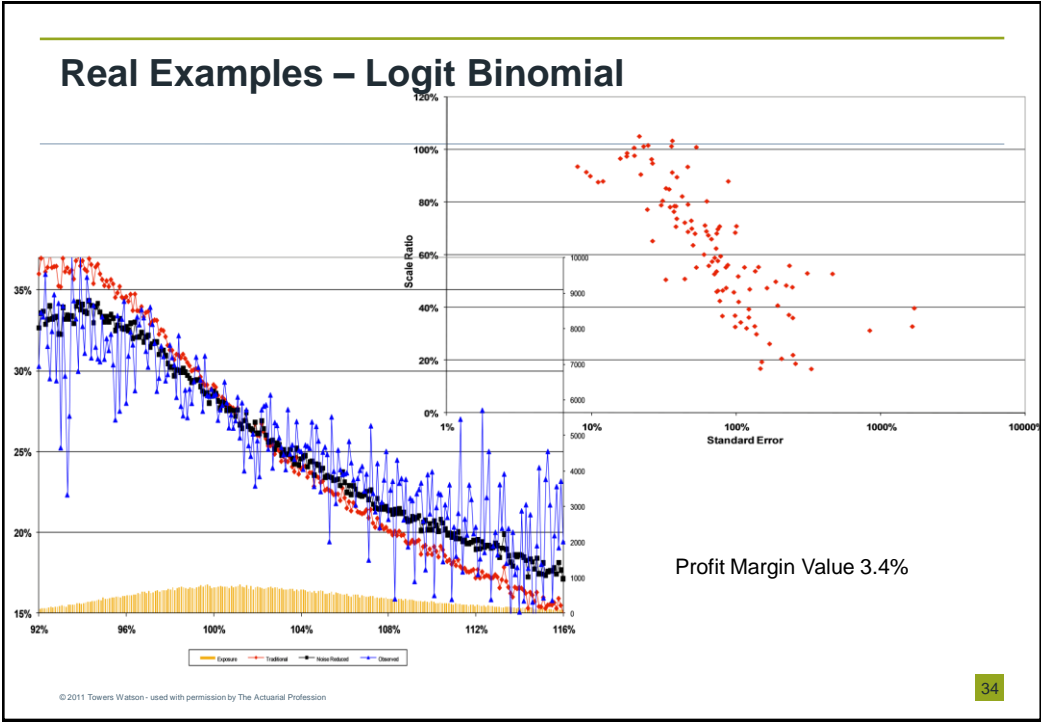
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Real Examples

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Other applications

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Other Applications

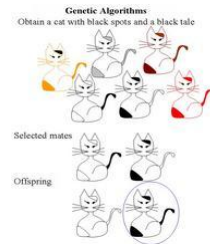
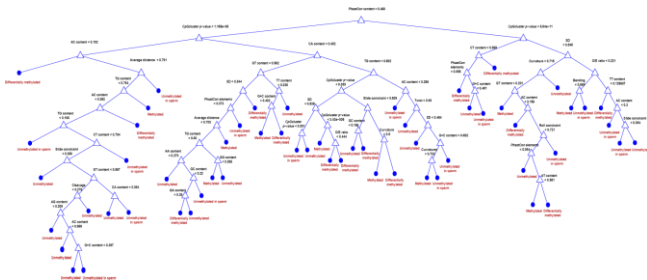
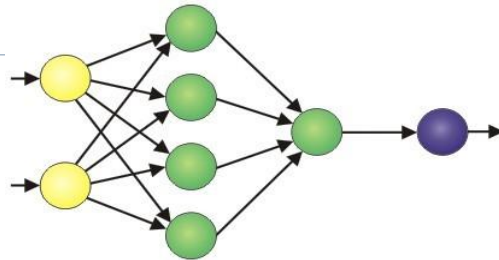
- “Case Deleted” Deviance
 - Completely generic concept, that can be applied to any model type
 - For models with quadratic parameter convergence, a similar approximation should exist
- “Value” Measure
 - An absolute mechanism to compare disparate model options.
- Noise Reduction Method
 - GLMs are convenient in that they generate the Variance Covariance matrix as part of the solution.
 - But could be applied to any model where this can be estimated.
 - Result is a set of scaled parameters which are “most predictive”.

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Other Applications

- Can be applied to
 - Neural Networks,
 - Genetic Algorithms,
 - Decision Trees, etc.



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Questions or comments?

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Tony Lovick, MA FIA
Pricing Actuary

- Tony graduated in Mathematics from Oxford University in 1987, and qualified as a Fellow of the Institute of Actuaries in 1994. He spent twenty one years with Aviva Group, before joining EMB as a Senior Consultant.
- Tony undertook a number of roles within Aviva, most recently as Price Optimisation Actuary, "Pay as you drive" Actuary and Head of Statistics and Development, in the Personal Lines Pricing Division of Norwich Union.*
- Tony is interested in innovative actuarial research and its delivery through pragmatic systems development. As Price Optimisation Actuary he undertook the client side pricing and architecture design, concluding in a successful Motor Renewal pilot.
- As the actuary leading the research for Pay as you drive, he helped inspire the analysis, build of the data warehouse systems**, and launch of the product to market. As part of this project Aviva prepared two patents with Tony listed as the inventor, one of which is now granted***.
- As Head of Statistics he led the implementation of full postcode risk cost models for motor and home insurance, pioneering the introduction of external data to Aviva rating systems.

* <http://www.linkedin.com/in/anthonylovick>

** <http://www.silicon.com/financialservices/0.3800010322.39169285.00.htm>

*** <http://v3.espacenet.com/textdoc?DB=EPODOC&IDX=GB2436880&F=0>

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Peter Lee FIA
Director

- Peter Lee is a Director at Towers Watson and global lead in pricing innovation with over twenty years experience in non-life insurance. Prior to joining EMB Peter worked at Allianz UK as the Personal Lines Actuary.
- Whilst at EMB, Peter worked for a large number of insurers throughout the world in different regulatory regimes, advising over a broad spectrum of areas and products ranging from claims reserving to pricing and the design of management information. Throughout his career Peter has been at the forefront of innovation, being one of the pioneers of the application of statistical modelling to personal lines pricing and then extending these techniques to commercial lines.
- More recently Peter developed EMB's price optimisation solution which has now been implemented in many of the largest general insurers in the world. Much of Peter's work involves embedding technical analysis and demand-based pricing into a wider pricing process, allowing these enhanced capabilities to be more effectively leveraged. Peter is now working with clients to link pricing and marketing to provide an enhanced framework for managing customer value.

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