Edinburgh International Conference Centre, Edinburgh

Portfolio-specific mortality modelling

Stephen J. Richards
11^{th} November 2013

Copyright © Longevitas Ltd. All rights reserved. Electronic versions of related papers and presentations can be found at www.longevitas.co.uk

Contents

1. About the speaker

1. About the speaker

- Consultant on longevity risk since 2005
- Founded longevity-related software businesses in 2006:
- Joint venture with Heriot-Watt in 2009:

2. Data description

2. Data description

Multi-employer pension arrangement in Germany:

— 253,444 pension records.

— 31,842 deaths in 2007–2011.

— 1.03 million life-years lived in 2007–2011.

Source: Richards, Kaufhold and Rosenbusch (2013).

2. Data description

Unequal distribution of liabilities:

— 50% of all pensions are received by just 23.5% of lives.

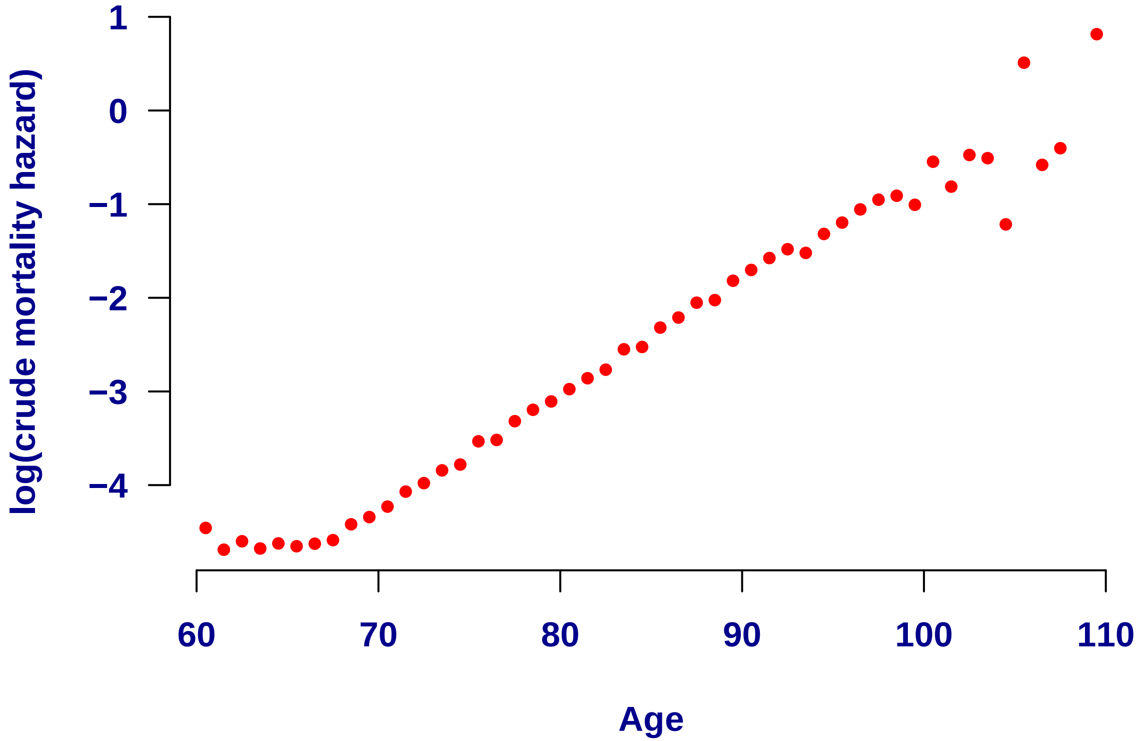
— males are 34.5% of lives, but 59.7% of large-pension cases.

Source: Richards, Kaufhold and Rosenbusch (2013).

3. Exploratory data analysis

3. Exploratory data analysis

$\log_e$(crude mortality hazard) from age 60, males and females combined:



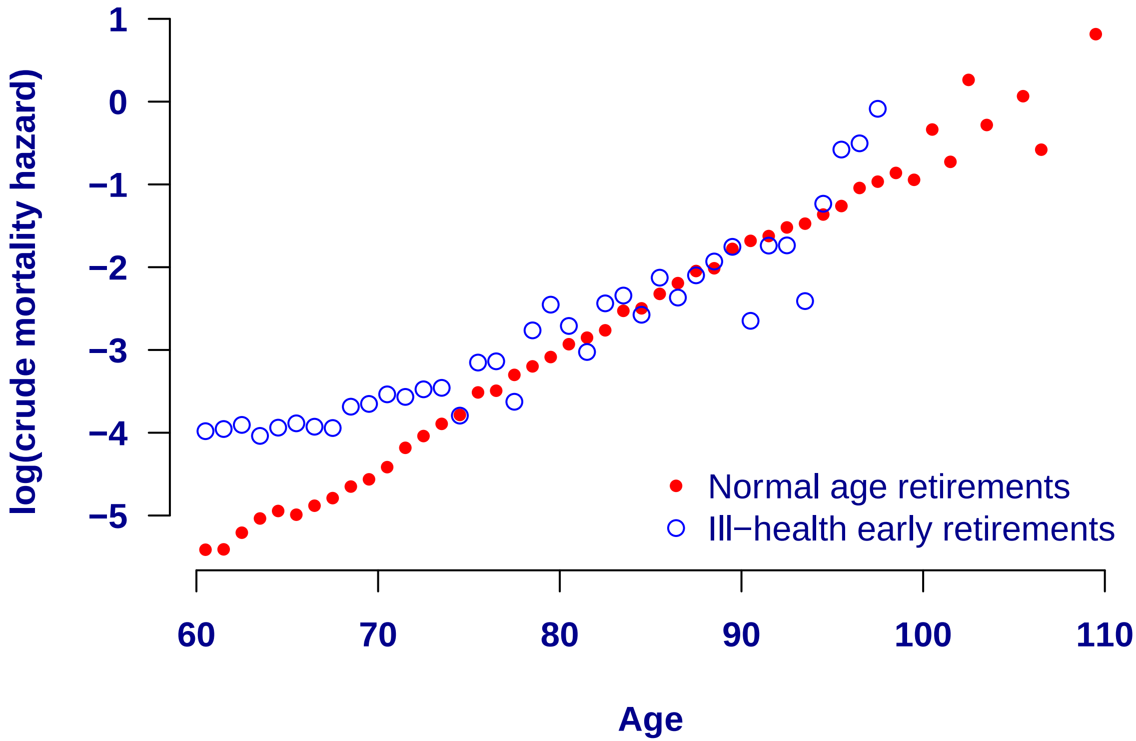
Source: Richards, Kaufhold and Rosenbusch (2013), Figure 1.

3. Exploratory data analysis

- Mortality increases with age.
- Smoothing is needed to iron out random variation.
- Extrapolation is needed for highest ages.

3. Exploratory data analysis

$\log_e$(crude mortality hazard) from age 60 by retirement type:



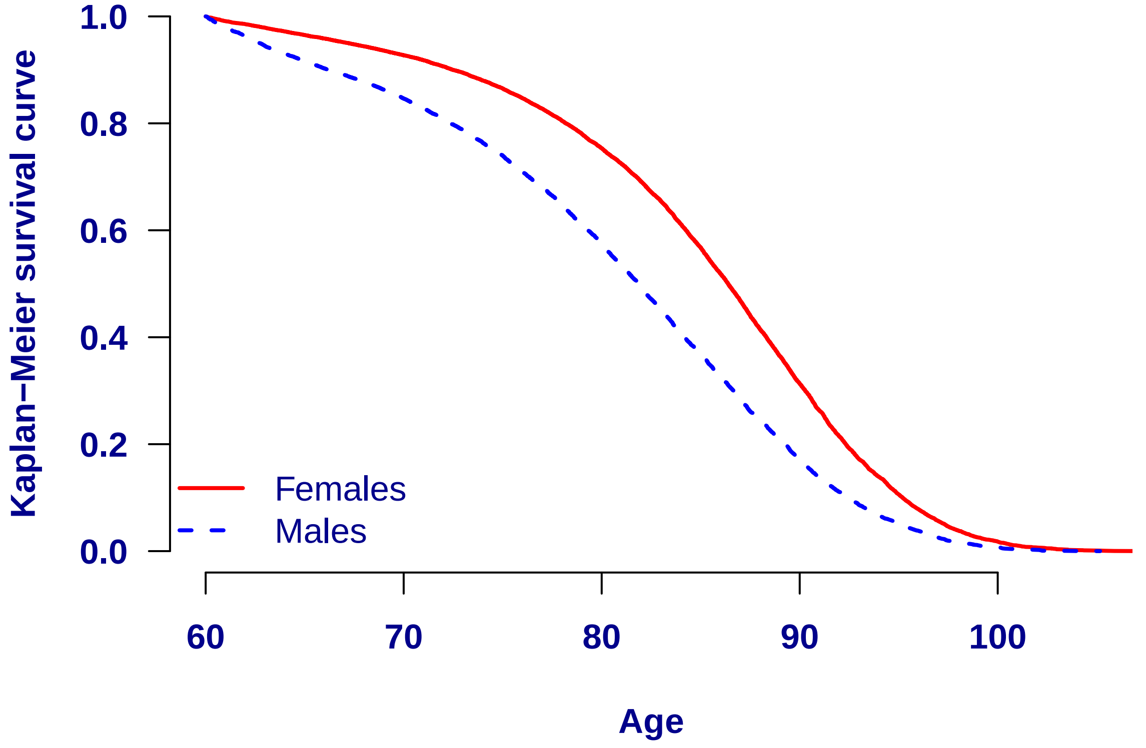
Source: Richards, Kaufhold and Rosenbusch (2013), Figure 4.

3. Exploratory data analysis

- Strong excess mortality for ill-health retirals, but
- Excess ill-health mortality reduces with increasing age.
- This phenomenon is known as *mortality convergence*.

3. Exploratory data analysis

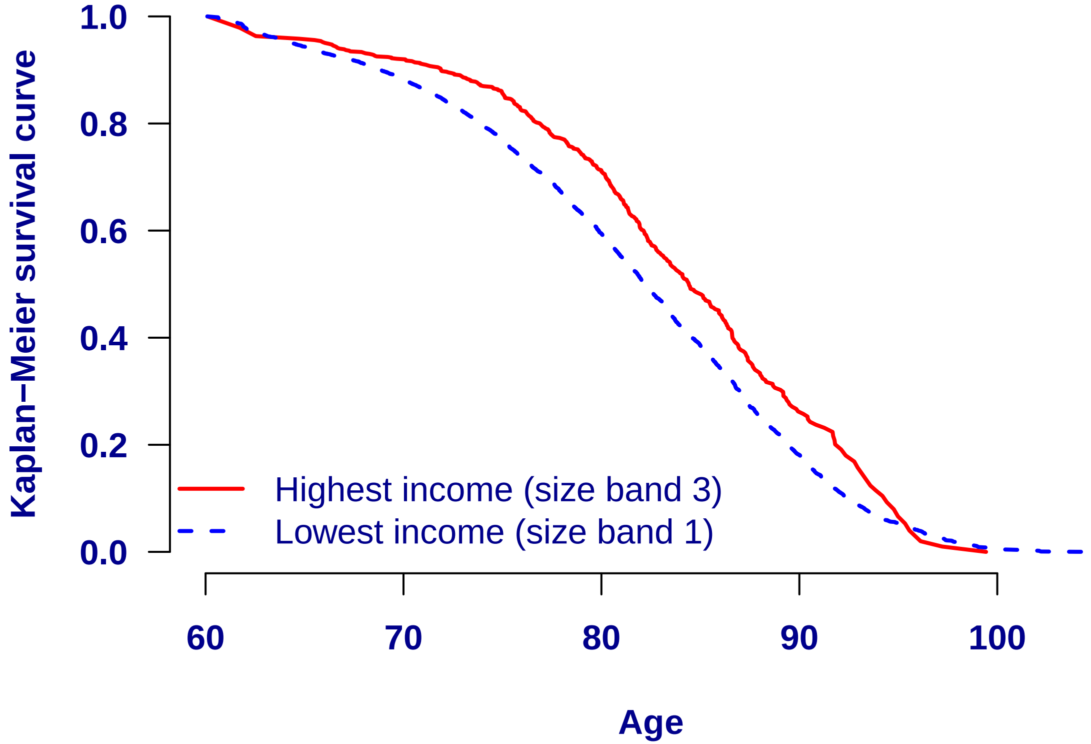
Kaplan-Meier product-limit estimator by gender from age 60:



Source: Richards, Kaufhold and Rosenbusch (2013), Figure 2.

3. Exploratory data analysis

Kaplan-Meier product-limit estimator by income from age 60:



Source: Richards, Kaufhold and Rosenbusch (2013), Figure 3.

3. Exploratory data analysis

The data tell us what the requirements of the model are:

— smooth out random variation,

— extrapolate to higher ages,

— allow for multiple risk factors simultaneously, and

— allow risk factors to vary their impact by age.

4. Model structure and fitting

4. Model structure

- All requirements are fulfilled by a parametric survival model.
- Here we will use the Makeham-Perks law:

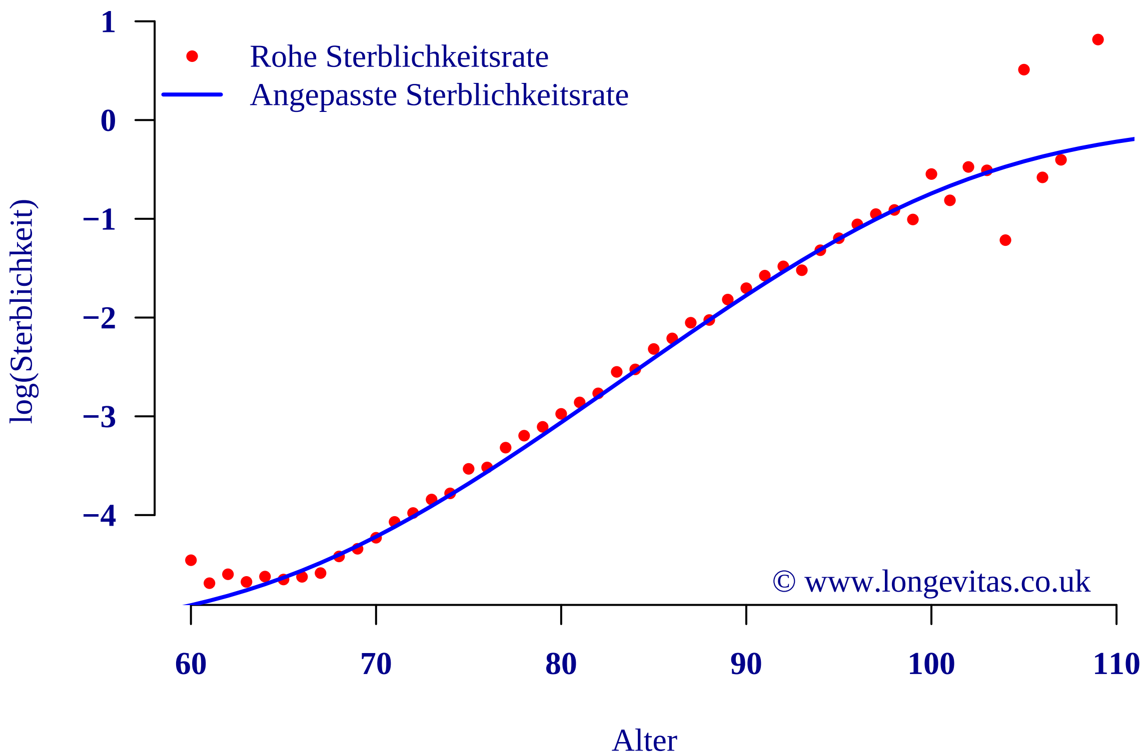
$$\mu_x = \frac{e^{\epsilon} + e^{\alpha+\beta x}}{1 + e^{\alpha+\beta x}}$$

with real-valued age x and real-valued parameters ϵ, α and β.

Source: Richards (2008, 2012).

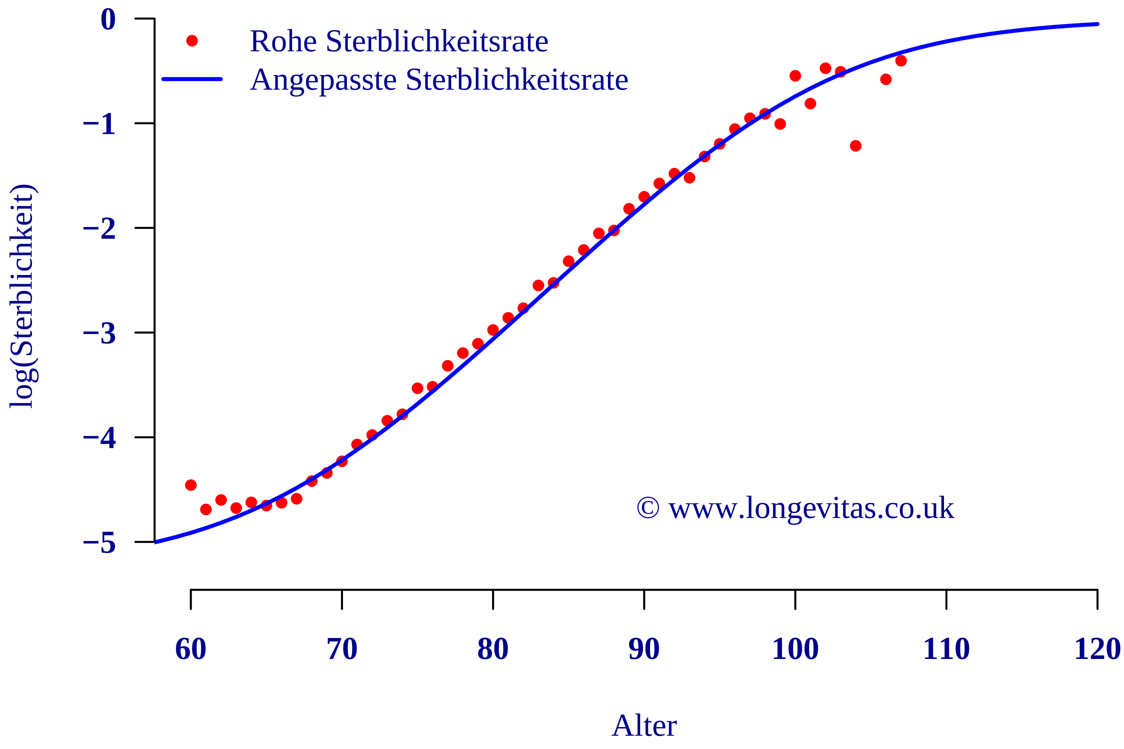
4. Model features

Automatic smoothing of random variation:



4. Model features

Sensible extrapolation to higher ages:



4. Model fitting: method of maximum likelihood

Likelihood function:

$$L = \prod_{i=1}^{n} {}_{t_i}p_{x_i} \mu_{x_i+t_i}^{d_i}$$

where:

- x_i is the entry age for life i of n lives,
- t_i is the time observed, and
- $d_i = 1$ if life i is dead, otherwise $d_i = 0$.

4. Model structure

Simple relationship between μ_x and survival probability ${}_tp_x$:

$$
\begin{aligned}
{}_tp_x &= \exp\left(-\int_0^t \mu_{x+s} ds\right) \\
&= \exp\left(-H_x(t)\right)
\end{aligned}
$$

$H_x(t)$ is the *integrated hazard function*.

4. Model fitting: method of maximum likelihood

Optimisation is often easier with the log-likelihood function:

$$
\begin{aligned}
\ell &= \log L \\
&= \sum_{i=1}^{n} -H_{x_i}(t_i) + \sum_{i=1}^{n} d_i \log \mu_{x_i+t_i}
\end{aligned}
$$

where $H_x(t) = \int_0^t \mu_{x+s} ds$.

Richards (2012) tabulates μ_x and $H_x(t)$ for sixteen models.

4. Model structure

- Assume α should vary by gender:

$$\alpha_i = \alpha_0 + \alpha_M z_i$$

where:

— α_0 is the so-called *baseline*,

— α_M is the effect of being male, and

— $z_i = 1$ if life i is male, otherwise $z_i = 0$ if life i is female.

- α_M measures the mortality difference for being male.
- Alternatively, we could set males as the baseline and estimate α_F.

4. Model structure

- Simple extension to j risk factors:

$$\alpha_i = \alpha_0 + \sum_{j=1}^{m} \alpha_j z_{j,i}$$

where:

— α_j is the effect of risk factor j, and

— $z_{j,i} = 1$ if life i has risk factor j, otherwise $z_{j,i} = 0$.

- $\alpha_j < 0$ when mortality is reduced, $\alpha_j > 0$ when mortality is raised.
- No minimum number of lives for estimating α_j.

5. Results

5. Results for German pensioners

Seven statistically significant risk factors for longevity:

— age,

— gender,

— pension size,

— retirement status: normal, ill-health or widow(er),

— employer type,

— region, and

— time

Source: Richards, Kaufhold and Rosenbusch (2013).

5. Results for German pensioners

Financial impact on annuity factors at age 65:

Risk factor	Change	Annuity factor	Relative change
Base case	-	16.114	
Gender	Female→male	14.529	-9.8%
Retirement health status	Normal→ill-health	12.974	-10.7%
Pension size	Largest→smallest	11.717	-9.7%
Region	B→P	11.025	-5.9%
Employer type	Private→public	10.599	-3.9%
Overall			-34.2%

Source: Richards, Kaufhold and Rosenbusch (2013), Appendix 1.

5. Results — international comparison

- How do these results compare with other data sets?
- Consider annuities with a UK insurer. . .

5. Results for UK annuitants

UK insurer with six available risk factors:

- age,
- gender,
- lifestyle (via postcode),
- duration (time since annuity purchase),
- pension size, and
- region.

Source: Richards and Jones (2004).

5. Results for UK annuitants

Financial impact of mortality rating factors:

Factor	Step change	Reserve	Change
Base case	-	13.39	
Gender	Female→male	12.14	-9.3%
Lifestyle	Top→bottom	10.94	-9.9%
Duration	Short→long	9.88	-9.7%
Pension size	Largest→smallest	9.36	-5.2%
Region	South→North	8.90	-4.9%
Overall			-33.6%

Source: Richards and Jones (2004), page 39.

5. What risk factors should you use?

- Each portfolio is unique.
- Business practice determines available information.
- Fit models to your data using business-relevant risk factors.
- Even small portfolios can have significant characteristics of their own...

5. Impact of scheme-specific mortality

- Return to German pensioner data.
- The largest scheme has approximately 12,000 members.
- Do the seven risk factors explain the mortality variation in this scheme?

5. Impact of scheme-specific mortality

- Mortality around 10% lower for largest scheme.
- Effect exists *even after allowing for all seven other risk factors*.
- Result was highly statistically significant (p-value 0.0001).
- Impact was an extra 2–2$\frac{1}{2}$% on reserves.

6. Conclusions

6. Conclusions

- A parametric survival model simultaneously:
  - — identifies the main risk factors,
  - — identifies any interactions with age,
  - — smoothes (graduates) the rates, and
  - — extrapolates to higher ages.
- Even small portfolios can have significant characteristics of their own.

References

Kaplan, E. L. and Meier, P. **1958** *Nonparametric estimation from incomplete observations*, Journal of the American Statistical Association **53**, 457–481.

Richards, S. J. and Jones, G. L. **2004** *Financial aspects of longevity risk*, Staple Inn Actuarial Society, London.

Richards, S. J. **2012** *A handbook of parametric survival models for actuarial use*, Scandinavian Actuarial Journal, 2012 (**4**), pages 233–257.

Richards, S. J., Kaufhold, K. and Rosenbusch, S. **2013** *Creating portfolio-specific mortality tables: a case study*, European Actuarial Journal, DOI: 10.1007/s13385-013-0076-6.

LONGEVITAS