Institute and Faculty of Actuaries

Risk and Investment Conference 2013

Brighton, 17 – 19 June

B4: How to blend ‘qualitative’ with ‘quantitative’:
global macro economic themes in strategic asset allocation

18th June 2013

B4: How to blend ‘qualitative’ with ‘quantitative’: global macro economic themes in strategic asset allocation

“As we continue through uncertain times, there will always be a need for qualitative judgement when imposing the high level economic themes that drive the distributions we use to assess risk. In this practical workshop, we will explore the strengths and weaknesses of a simple ‘maximum entropy’ based approach as used to blend these themes into the strategic asset allocation that backs a typical multi-year cashflow liability.”

Alun Marriott
Mark Sinclair-McGarvie

Marriott Sinclair LLP

www.marriottsinclair.com
Tel +44 20 3530 5000

Introduction & Agenda

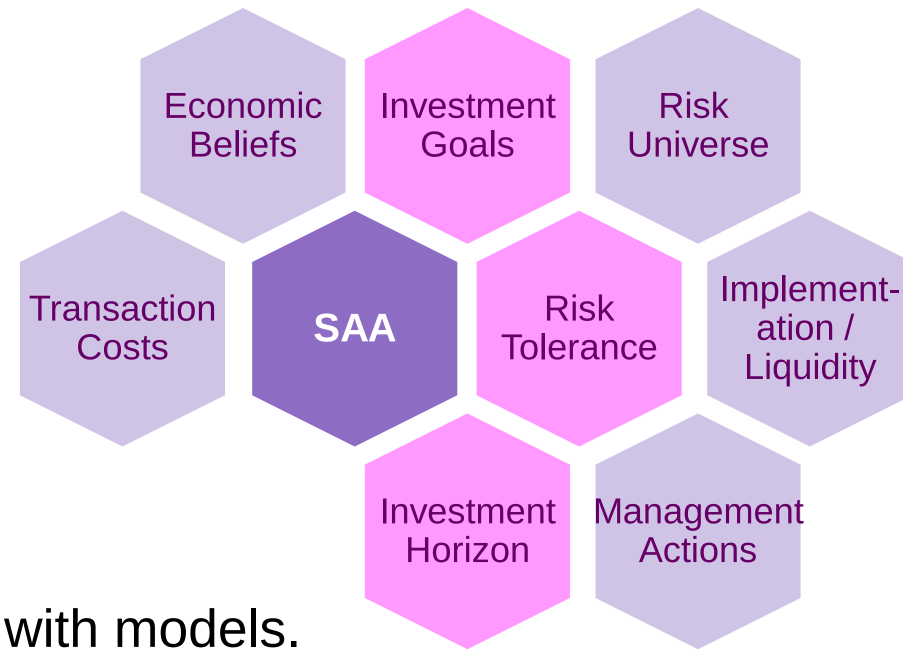
- This workshop follows on from a talk given at GIRO 2010 where we discussed Entropy Maximisation in the context of ESGs (Economic Scenario Generators) and CAT models.
- Our goal today is to consider ways we may take account of the diverse range of economic themes – and how to ensure we reflect our perception of the likelihood of these themes in a Strategic Allocation exercise.
- We are going to focus on two key subjects.
  - Firstly, we will give a brief overview of the Entropy technique – delving into various practical aspects that warrant care
  - Secondly, we discus *how* we can improve the “reliability” of SAA process itself – and in particular the use of numeric optimisation methods
- Questions

Part 1: Background to Thematic Economic Scenarios & Entropy

Why this talk …
Strategic Asset Allocation

INVESTOPEDIA

"An investment strategy that aims to balance risk and reward by apportioning a portfolio's assets according to an individual's **goals**, **risk tolerance** and **investment horizon**."



Many of these can be explored with models.

Hence, we need to ensure these models reflect what we really believe. No point in buying or using a model that we cannot stand behind.

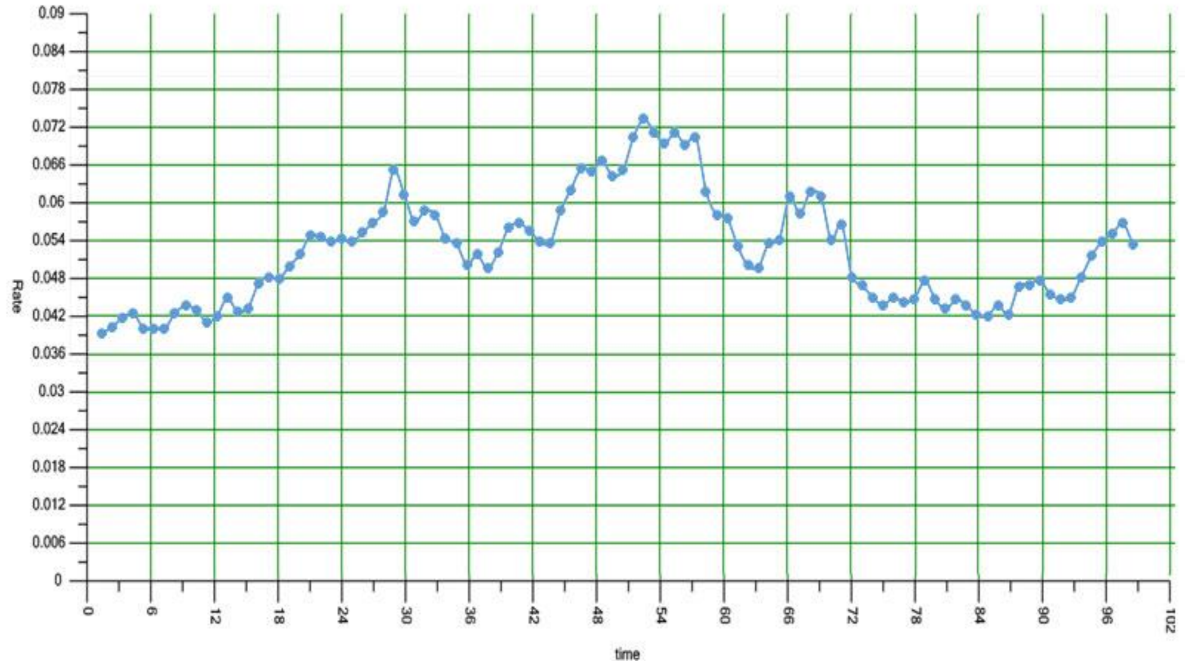
Models – What do we really mean?

Simulations / Projections / Risk – Single Shot, Simulation Based, Thematic etc…

Single Path, Single / Stochastic, Multiple Themes

[illegible]

- Probability plots can show bi-modality of themes.



Economic Beliefs

Economic Themes … (at least a small set of them).

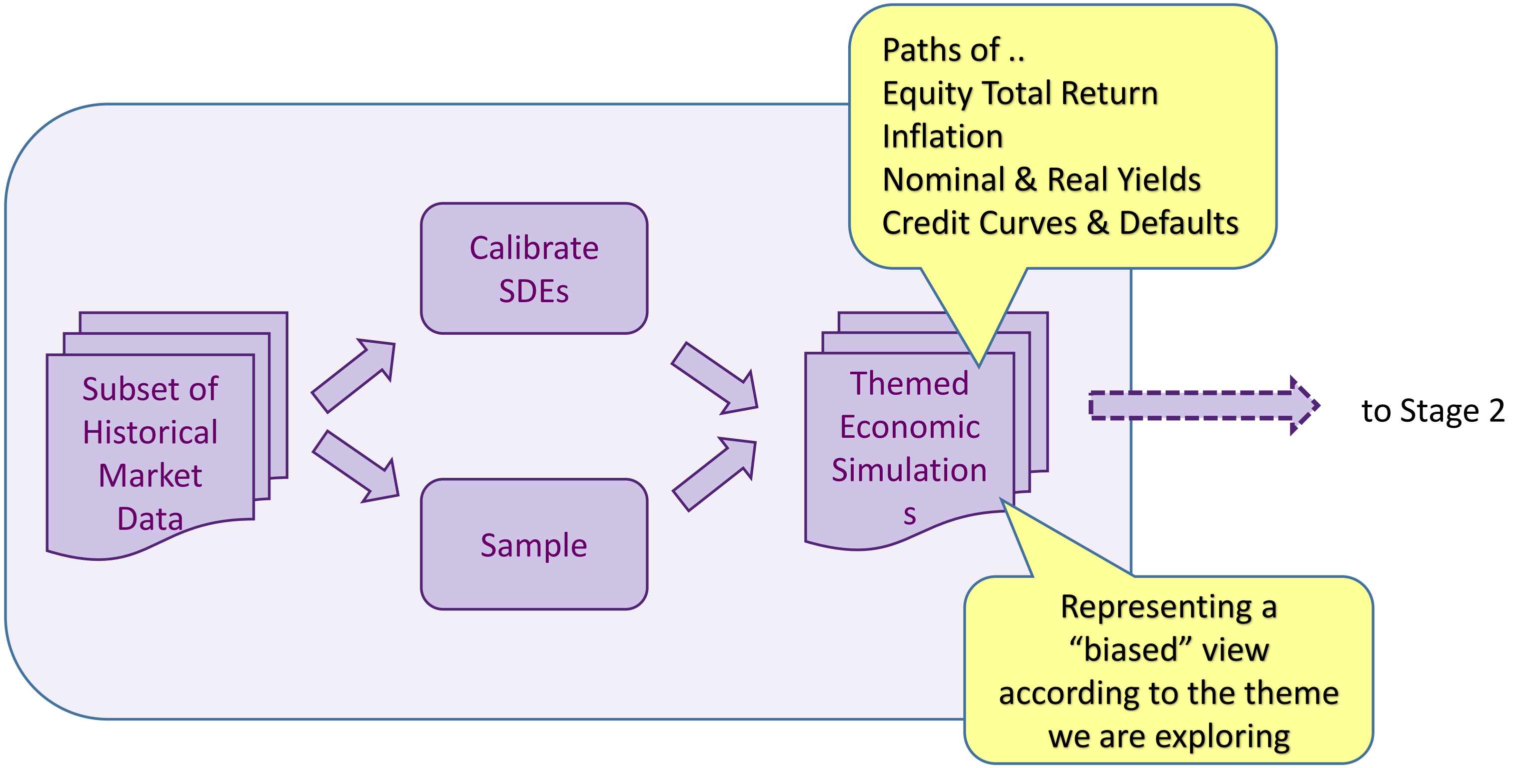
Today
18th June

	Likelihood
China deflation risk	**10%**
UK Continued Low Rate / Medium Inflation	**15%**
UK “imported” inflation spike due to weakness in GBP	**30%**
Bursting bubbles – Stock / Property / Gold …	**40%**
Euro breakup	**5%**

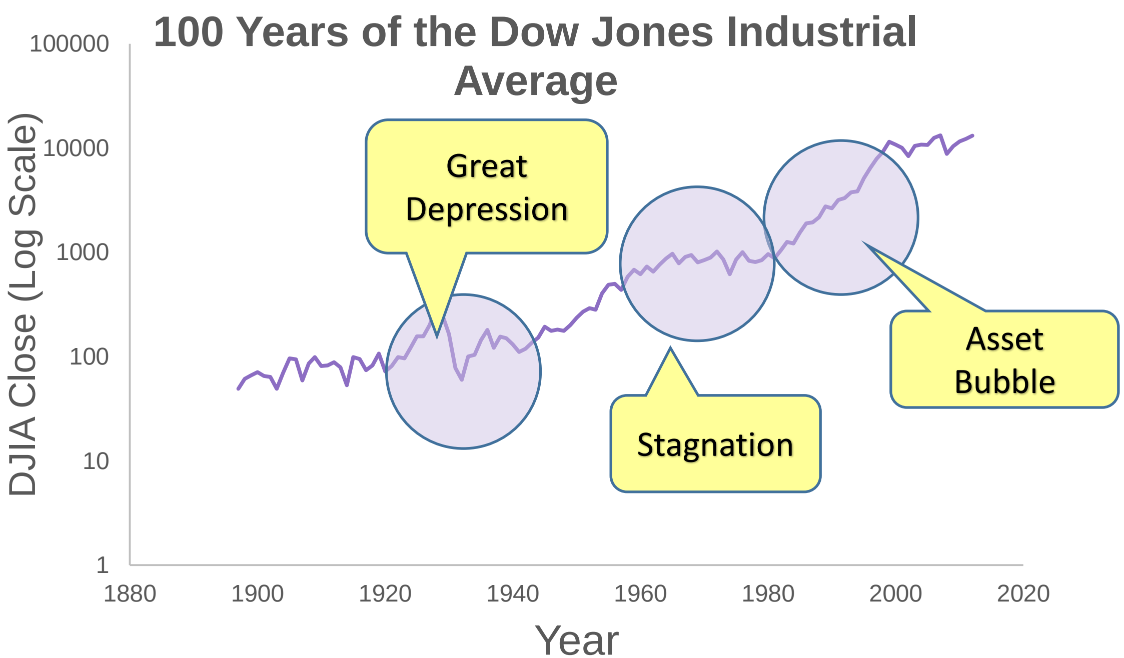
These themes are “Forward Looking” & “Fundamental”
They are mainly derived by economic, political and social analysis.
They may be based on similarities with historical precedent

Creating a "Themed" simulation set

Method A – Calibrate to a specific window of data



Calibrating the themes
Probabilities of Probabilities …

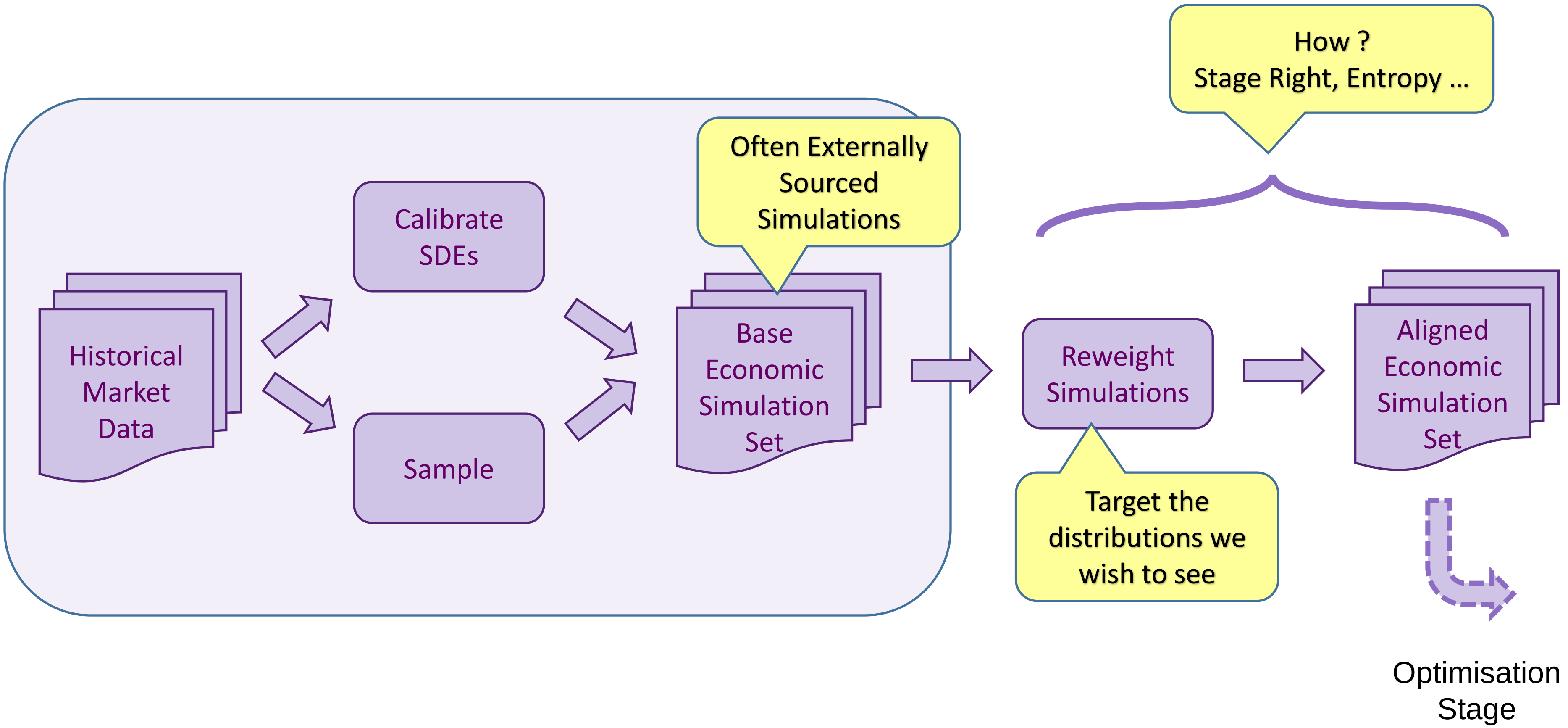


Step 1: For each "theme", we need to calibrate the models, or determine our targets "empirically", based on behaviours we want to generate.

Step 2: We then need to ascribe probabilities to the chance that each "theme" will occur going forward …

We could "assume" …
20% chance of a Great Depression
60% chance of an Asset Bubble
40% chance of Stagnation

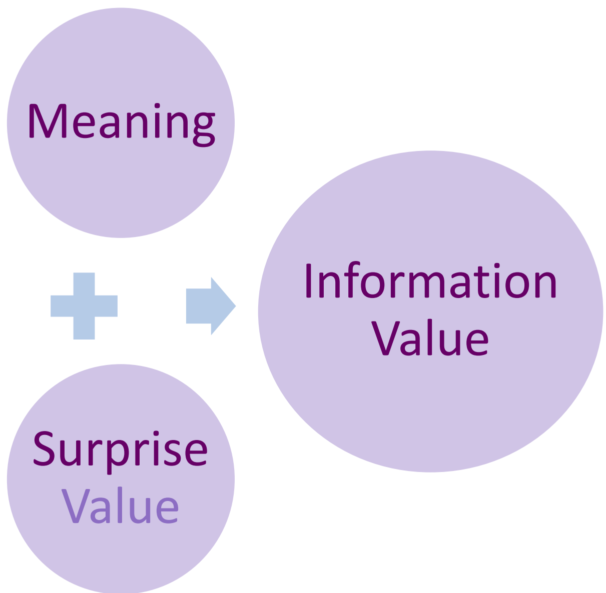
Creating the "Themed" simulations
Method B – Rescale existing simulations

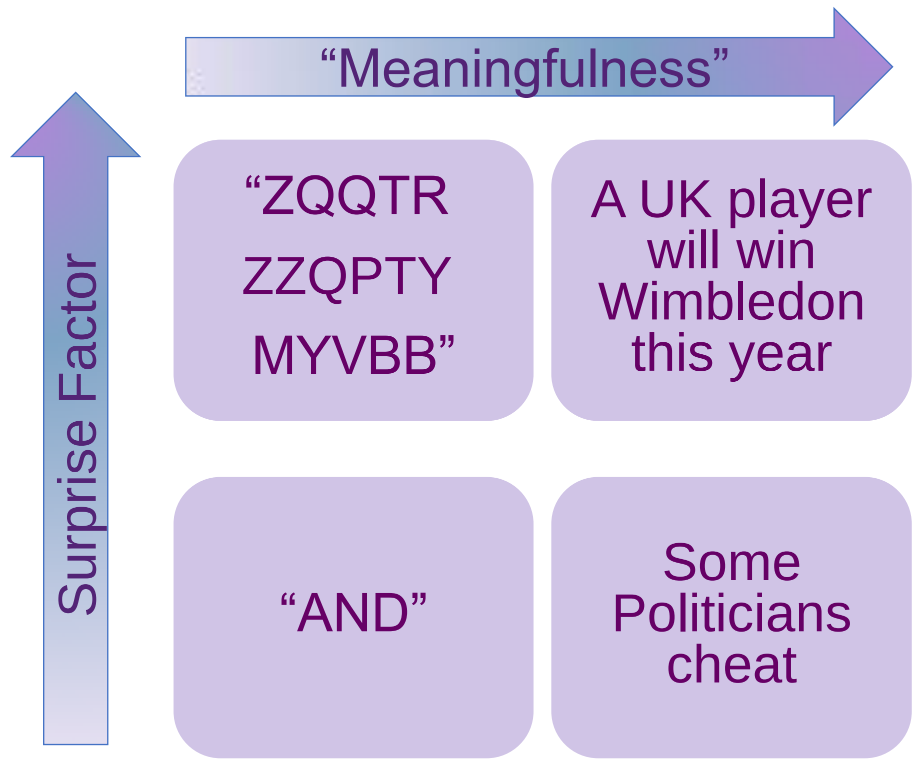


Maximum Entropy
A framework for modifying stochastic data

- Each of the questions posed earlier may be translated into "How do we modify a stochastic distribution"?
- We may be interested in modifying a variety of distributional characteristics
  - Mean
  - Volatility / Variance
  - Percentile
  - Correlation?
- We want to preserve as much of the original distribution as possible – and this is where "Maximum Entropy" comes in.
- "Maximum Entropy" is simply a metric that tells us how much "freedom" we've retained in the original distribution.

Information Value
Quantifying Information





Information Value
Intuitive Properties of Information

Example

$\mathrm{I}[\omega_2]$ is the Information Value of the Event ω_2, picking a King, $\mathrm{P}[\omega_2] = {}^{1}/_{13}$
$\mathrm{I}[\omega_1]$ is the Information Value of the Event ω_1, picking a Heart, $\mathrm{P}[\omega_1] = {}^{1}/_{4}$

Information Value	Description	The Log Function
$\mathrm{I}[\omega_1 \cap \omega_2] \geq \mathrm{I}[\omega_2] \geq \mathrm{I}[\omega_1]$	The lower the probability of the event occurring, the greater the Information Value	"-Log" function is monotonic increasing on (0,1]
$\mathrm{I}[\omega_1 \cap \omega_2] = \mathrm{I}[\omega_2] + \mathrm{I}[\omega_1]$	The Information Value does not depend on how it is received.	-Log(⅓ x ¼) = -Log(⅓) + -Log(¼)
$\mathrm{I}[\omega_1] \geq 0 \; \forall \, i$	Information can never be "lost"	-Log function is strictly positive on (0,1]

Entropy

Information, $\mathbb{I}$(X) to Entropy, $\mathbb{H}$(X)

Definition of Information Content

$$\mathbb{I}[\boldsymbol{X}] = -k \log_a \mathbb{P}[\boldsymbol{X}]$$

Definition of Entropy

Simply the expectation of the Information Value over all events. With n discrete events, this may be written

$$\mathbb{H}[\boldsymbol{X}] = \mathbb{E}\big[\mathbb{I}[\boldsymbol{X}]\big] = -k \sum_{j=1}^{n} p_j \, log_a \, p_j$$

We typically assume k = 1, and use the natural logarithm, i.e. a="e".

- Where $\mathbb{I}[\boldsymbol{X}]$ is a vector of Information Values for each event in **X**
- Where $\mathbb{P}[\boldsymbol{X}]$ is a vector of Probabilities for each event in **X**
- $\mathbb{E}\big[\mathbb{I}[\boldsymbol{X}]\big]$ is the Expected Information Value of each event in **X**
- $\mathbb{H}[\boldsymbol{X}]$ is the Total Entropy of the set of all events in **X**
- N is the number of events in the set of all possible events and j is the iterator over all events
- p_j is the probability of event x_j
- k and a are arbitrary constants

Maximum Entropy
The Maximum Entropy Principle (ME)

Principle of Maximum Entropy

In the absence of any other information, it can be proved that Entropy is maximized when we are most uncertain. i.e. when each simulation is equally likely and we are not biased to any particular outcome.

Why is the Principle of Maximum Entropy helpful ?

The Principle of Maximum Entropy allows us to quantify the effect of introducing information – and hence design a scheme for adjusting probabilities as information is added.

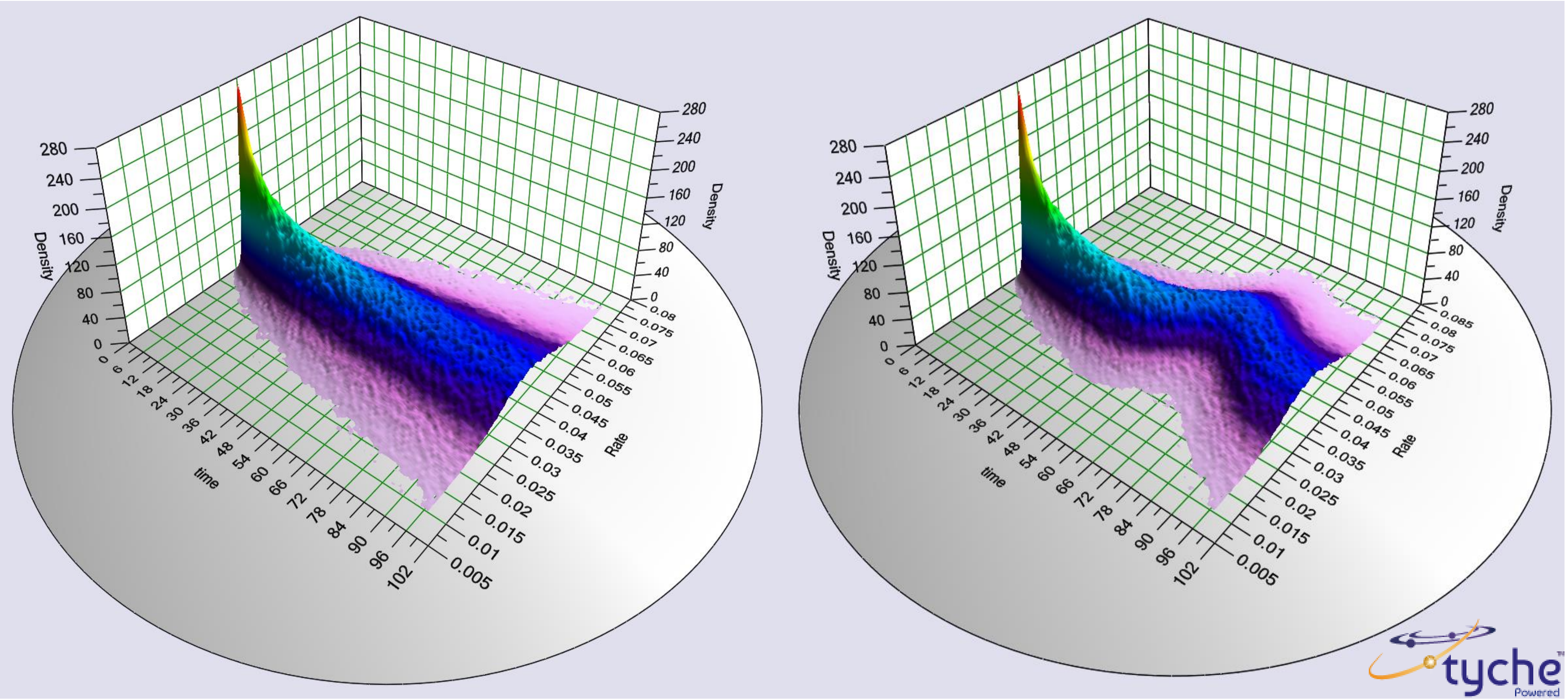
Ensuring the SAA reflects our views

These "reweighted" simulations then feed directly into the SAA process ensuring it too is aligned with our view of the likely high level economic outlook.

Part 2: The Practical Side & Examples

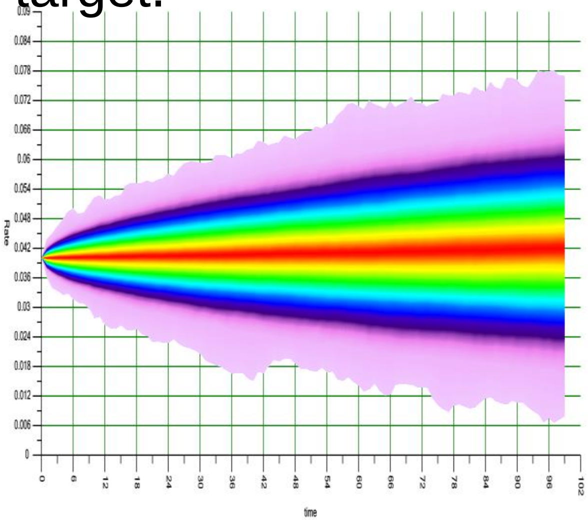
Maximum Entropy

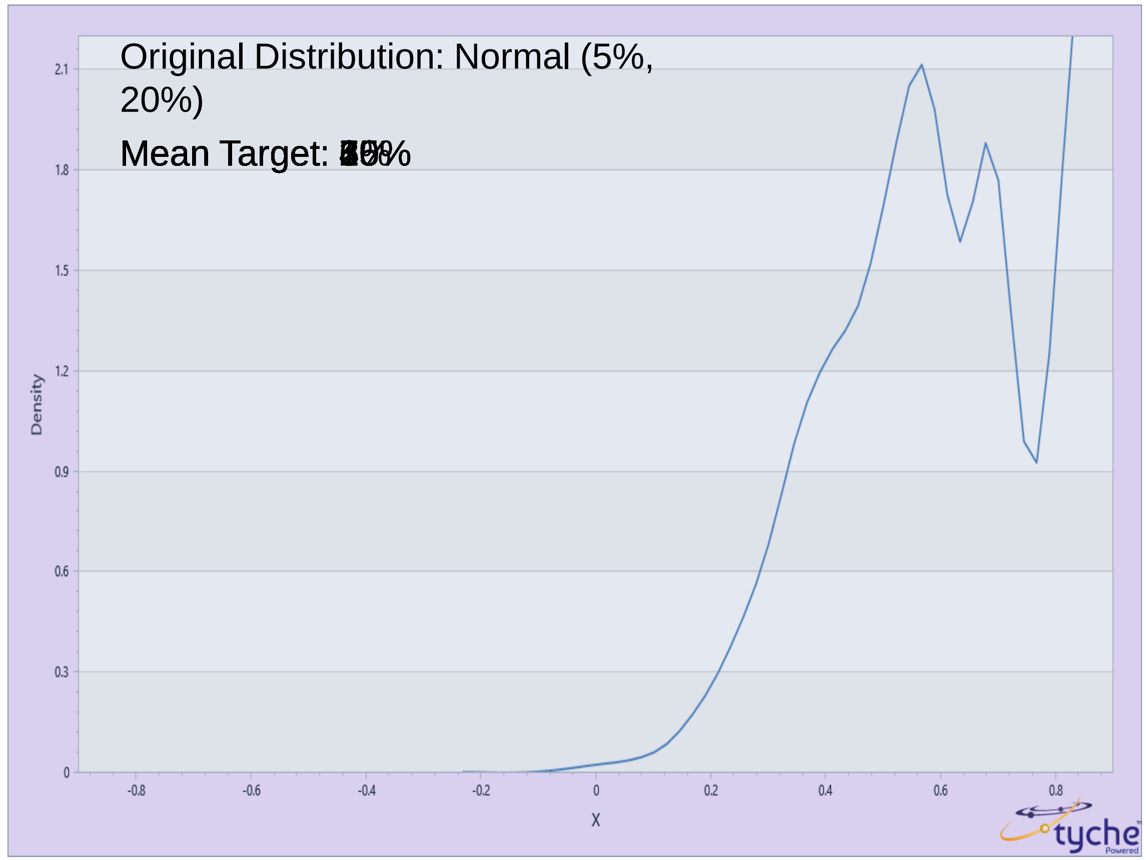
A 4% cash target evolving over ~8 years – Pre and Post



Maximum Entropy
Applying increasingly extreme views.

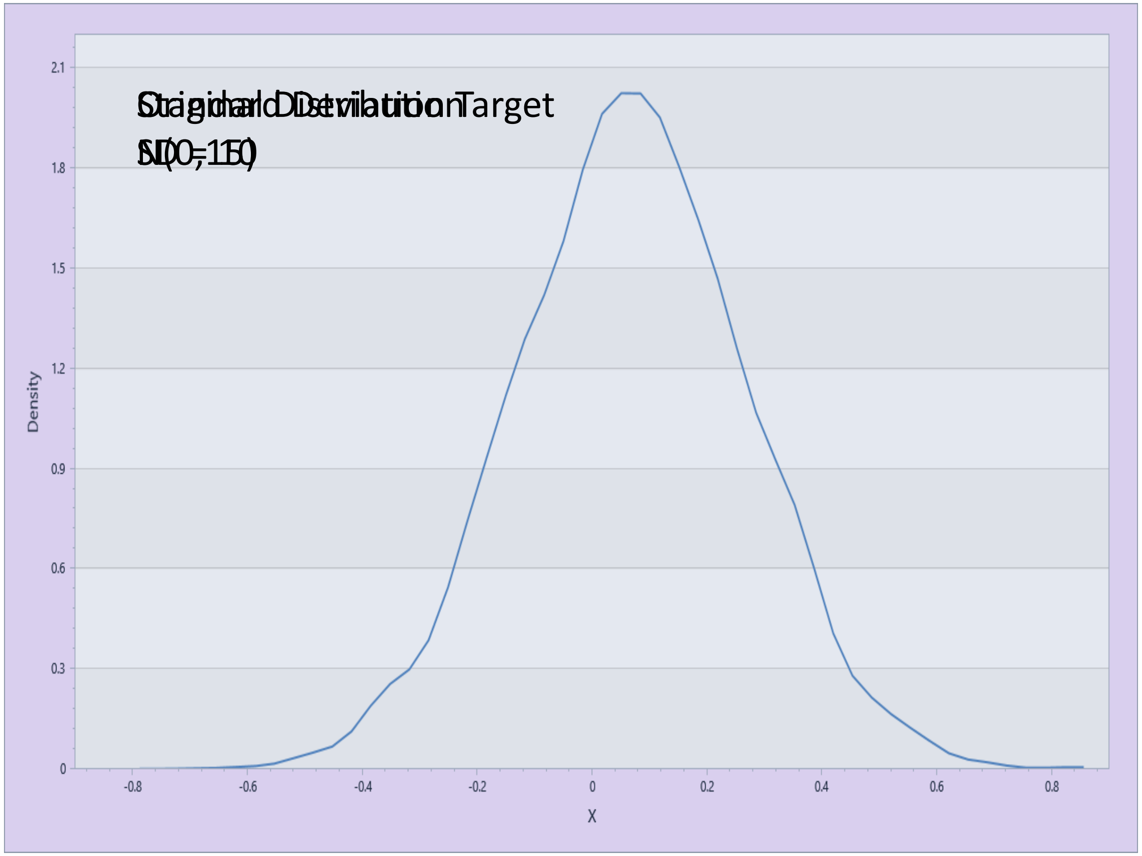
As we our views become increasingly extreme, there are fewer simulations around our target leading to fewer simulations that can be reweighted to achieve our target.



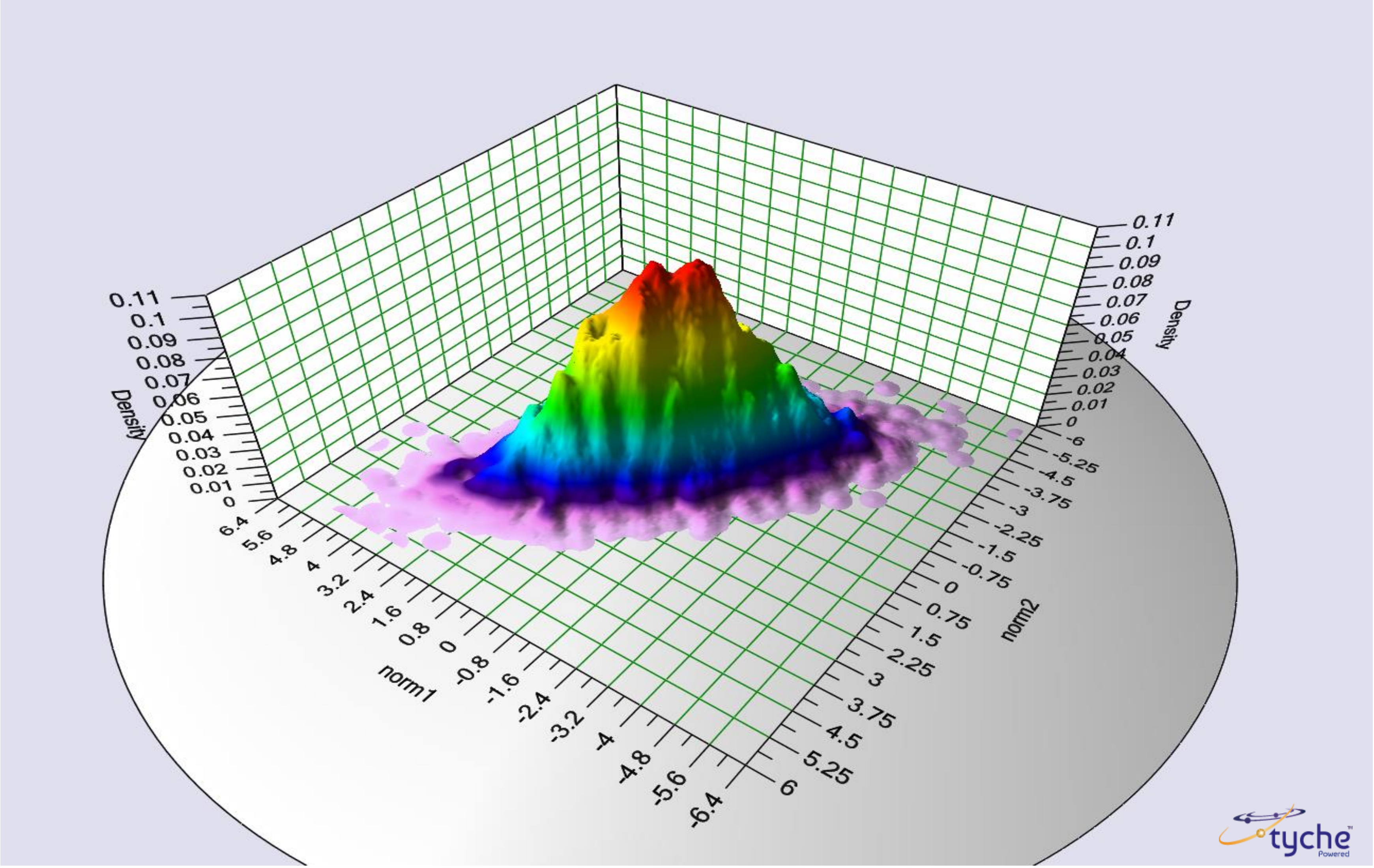


Maximum Entropy
Volatility Targets

In addition to targets on the drift, we can also target other measures including standard deviation and correlation.



Maximum Entropy
Applying correlation targets – This is new!

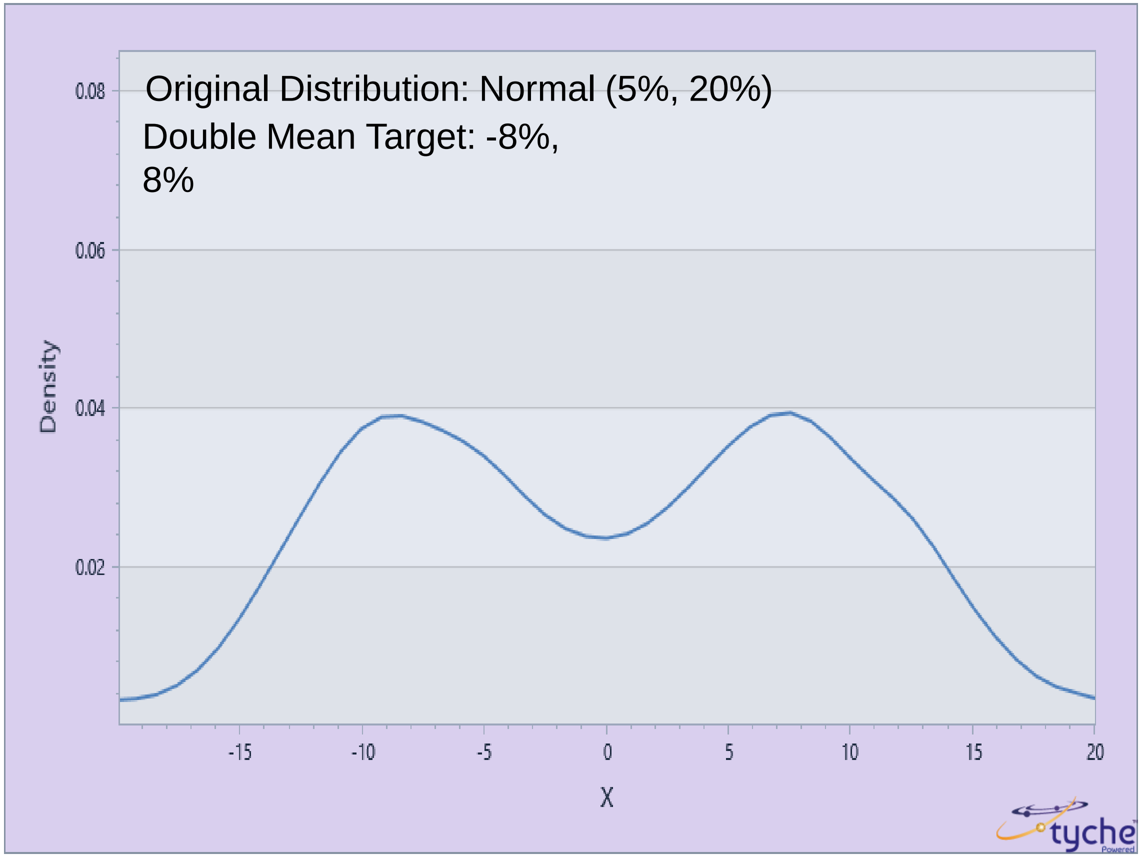


Maximum Entropy
Applying bi-modal targets.

In specific relation to our talk today, “multi-themed” distributions can be achieved in 2 ways ..

Multiple themes may be thought of as being multi-modal – each mode corresponding to the peak of the corresponding theme.

We do also this by creating a simulation set for each theme – then blend these together linearly according to some qualitatively defined weighting factor.



Maximum Entropy

Key Properties of Entropy based alignment….

1 Coherent adjustment of the joint distribution

2 Entropy is a "Minimum Disturbance" Measure

3 Easy to specify – Avoid direct re-parameterisation

When you rescale a simulation – you are rescaling the *entire* simulation – which ensures that the inter-relationships between *all* economic variables are preserved (albeit they do are adjusted accordingly)

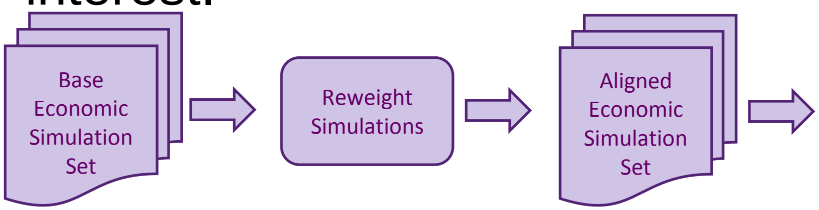
In using an Entropy measure, we are inherently trying to minimize disturbance (as defined by Entropy) …

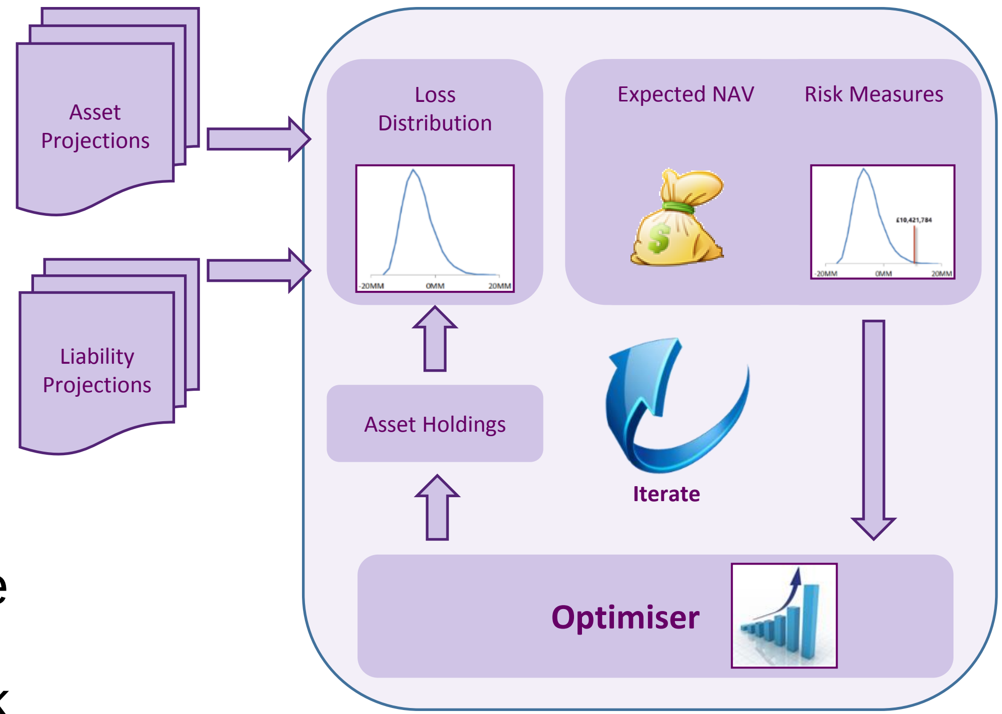
Unlike direct re-parameterisation, Entropy targets the measure we actually observe…

E.g. Inflation in 2013 will be 6%

Optimising SAA

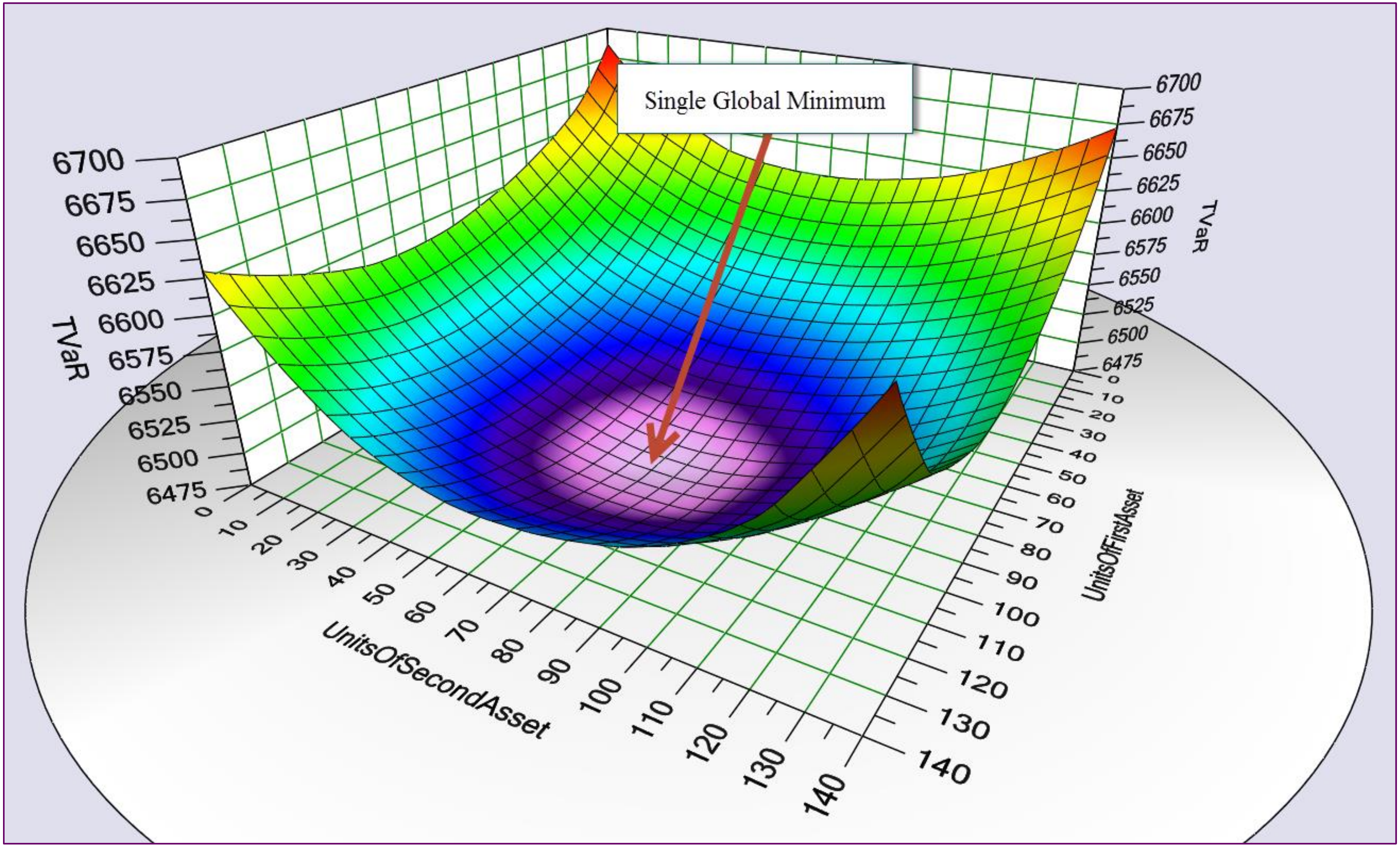
- Economic simulations may be used to perform asset and liability projections.
- Coupled with knowledge of the asset holdings, this implied a **loss distribution** over the horizon of interest.





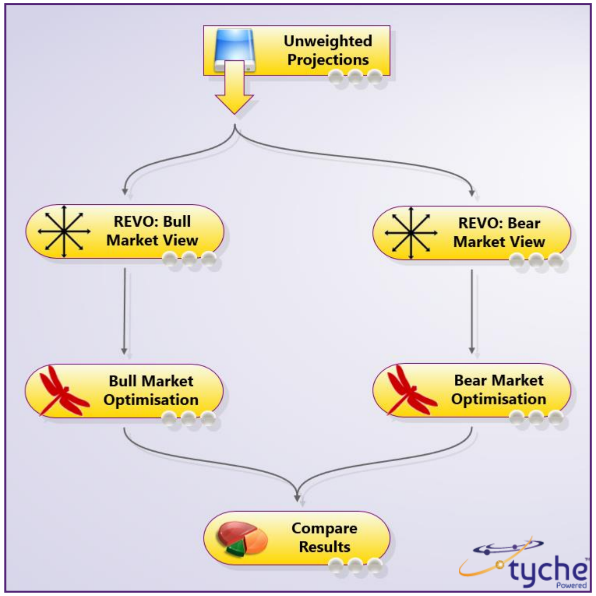
- **Optimisation algorithms** may be used to select the optimal asset holdings, subject to criteria on risk and return.

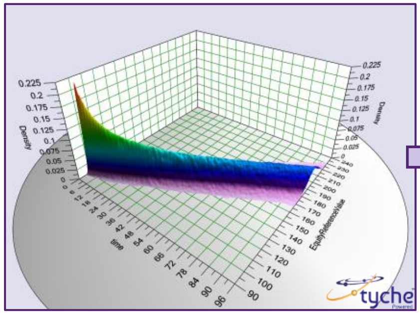
Optimisation Issues

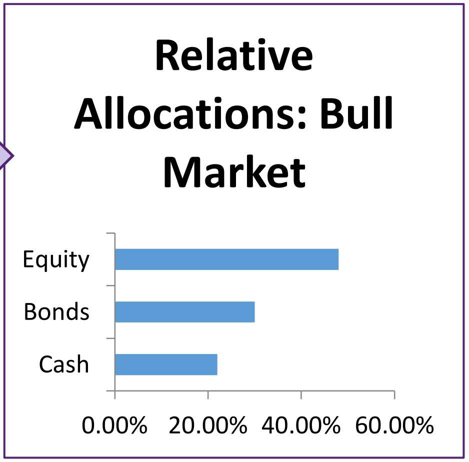


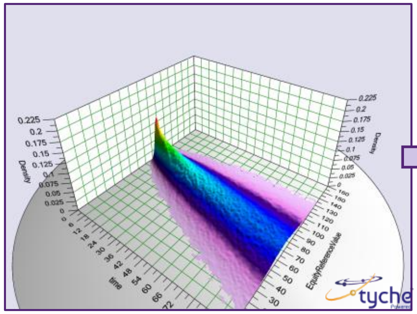
Optimisation & Entropy Maximisation

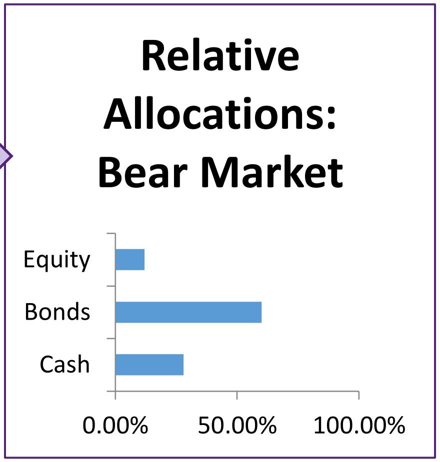
- The results of an SAA optimisation depend on the **underlying economic input**. We may investigate multiple scenarios here using the entropy maximisation approach.
- Consider the following two scenarios:
  - **Bull Market**. positivedrift, and low volatility.
  - **Bear Market**. negative drift and high volatility.
- Given these two scenarios, the asset allocations selected by the optimiser are quite different:





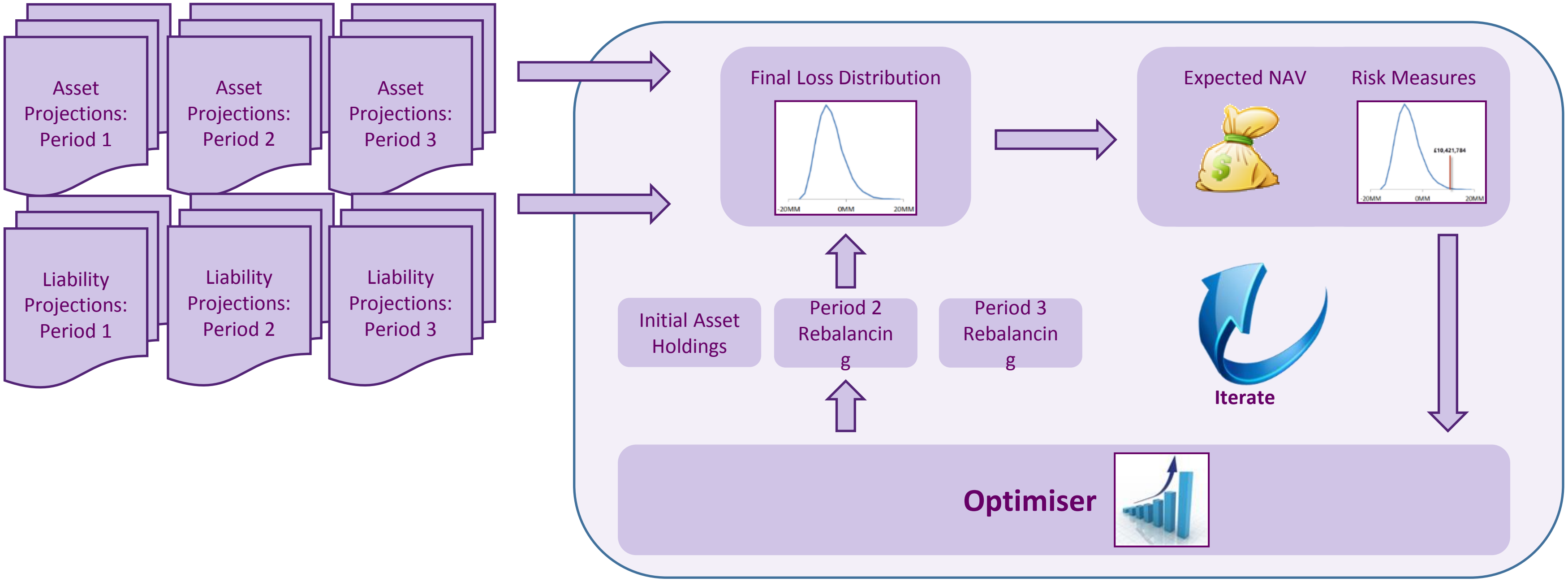






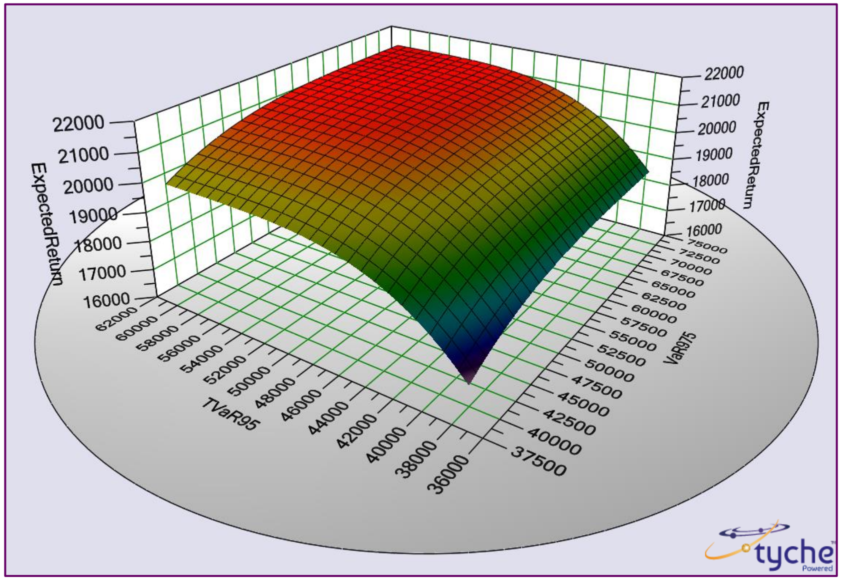
Multi-Period Optimisation

- Optimisation need not be restricted to a single **static** strategy holding over the whole horizon – or a **dynamic** strategy that rebalances at regular intervals with the horizon.
- This increases the problem size significantly: the number of variables is now the initial allocations to be made, as well as the subsequent rebalancings of the portfolio.



Strategy Exploration

- A single optimisation depends upon knowledge of the investor's **risk appetite**.
- However, risk appetite is partly determined by the **reward** available for taking on risk.
- Thus, when determining risk appetite it can be informative to perform many different optimisations for different risk appetites, and inspect the optimal strategy and return for each risk appetite.



- These optimisations may be summarised on an **efficient frontier**. The frontier shown below varies an investor's risk appetite by two measures (VaR @ 97.5%, TVaR @ 95%) and determines the optimal expected return for each appetite:
- Calculation of efficient frontiers can be very expensive: the one above took 10,000 optimisations to generate!

Summary

At an operational level

- We need to consider thematic risks – e.g. explicitly consider the set of high level macro views that affect our business
- Certainly for medium and long term horizons, these high level views (and in particular drifts), and our adjustments to them, may easily dominate investment strategy.
- We must explicitly factor in management action – e.g. rebalancing & reinvestment
- Modern technology can facilitate these more intensive applications

At a practical level

- Maximum Entropy is a great tool to help us make these “high level” adjustments in an easy, coherent way
- “Iterative” optimisation techniques, combined with Maximum Entropy, can lead to more sensible outcomes - in turn improving the “image” of numeric optimisation and it’s relevance to the investment process.

Questions

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