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Reducing Model Risk With Goodness-of-fit

Victory Idowu
London School of Economics

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Agenda

I. An overview of Copula Theory
II. Copulas and Model Risk
III. Goodness-of-fit methods for copulas
IV. Presentation of the new method

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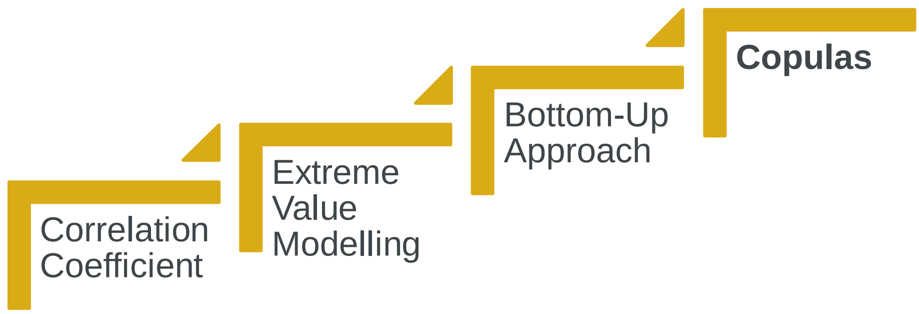
Measuring Dependence



Copula's Definition

- A Mathematical Approach…

"d-dimensional copula is a multivariate distribution function on $[0,1]^d$ with uniform marginals."

- A Conceptual Approach...

"a mixing of distributional functions which allows for flexibility in the dependence structure."

Copulas and Tail Dependence

- Copulas allow for flexibility in their dependence structure; incorporating tail dependence in the model fitting procedure is of upmost importance for risk management professionals
- Internal models: Gaussian and Student-t Copulas
- Other interesting copulas: Empirical, Vine and Archimedean Copulas.

Copula	Lower Tail Dependence, λ_L	Upper Tail Dependence, λ_U
Gumbel	0	≥ 0
Frank	0	0
Clayton	≥ 0	0
Generalised Clayton	≥ 0	≥ 0

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Copulas Gone Wrong

- Recent failures due to erroneous copula usage:

Photo: AP photo/Richard Drew
https://www.wired.com/2009/02/wp-quant/

$$C(u,v) = \phi_2\left(\phi^{-1}(u), \phi^{-1}(v), \rho\right) \text{ for } -1 \leq \rho \leq 1$$

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The Model Risk Problem

"…model risk … is the potential for adverse consequences from decisions based on incorrect or misused model outputs and reports."

Federal Reserve (2011)

Sources of Model Risk:
Incorrect Model Use \\ Expert Judgements \\ Model Changes

- The Model Risk Problem with Copulas is:
  Selecting the wrong copula because of using the wrong selection criteria.

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Limitations of Copula

General Limitations
Data Limitations
Parameter Fitting
Computational Cost
Possibility for Overconfidence

Copula Specific Limitations
Practicality
Use Test
Stability
Communication

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Model Risk ≠ Model Error

Model Risk

Reflects the lack of knowledge in our ability to fully capture all forms of uncertainty in the model.

Assumes the existence of a true model that we can measure our deviances from.

Model Error

For us, there is no such thing as a model error problem.

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Goodness-of-fit and Model Risk

- **Our Objective: to reduce model risk by developing a system that can select a copula and thus reduce uncertainty in the dependency structure between the risks.**

- A definition for Goodness-of-fit

"the degree to which observed data matches the values expected by theory"

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Hypothesis Test

- The hypothesis test under discussion is

$$H_0 : C \in \mathcal{C}_0$$
$$H_1 : C \notin \mathcal{C}_0$$

where the copula family is represented by $\mathcal{C}_0 = \{ C_\theta : \theta \in \Theta\}$ and Θ is the parameter space [Berg, 2009].

Current Goodness-of-fit Approaches

Cramér–von Mises, [Berg, 2009]

- Examines the squared deviances between the suggested copula $C(\boldsymbol{u})$ and the empirical copula $C^*(\boldsymbol{u})$.
- Test Statistic (one sample case)

$$\int_{-\infty}^{\infty} (C^*(\boldsymbol{u}) - C(\boldsymbol{u}))^2 \, dC(\boldsymbol{u})$$

Limitations
Computational Expense \\ Limitations in the Tail of the Distribution

Current Goodness-of-fit Approaches

Anderson–Darling test, [Berg, 2009]

- An extension of the Cramér–von Mises test, and places more weights on the tails of the distribution:

$$n \int_{-\infty}^{\infty} (C^*(\boldsymbol{u}) - C(\boldsymbol{u}))^2 \, w_{AD} \, dC(\boldsymbol{u})$$

where $w_{AD} = \left[C(\boldsymbol{u}) \left(1 - C(\boldsymbol{u})\right)\right]^{-\mathbf{1}}$

Limitations
Computational Expense \\ Requires knowledge of Critical Values

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Current Goodness-of-fit Approaches

Kolmogorov–Smirnov test, [Berg, 2009]

- Quantifies the distance between the suggested copula $C(\boldsymbol{u})$ and the empirical copula $C^*(\boldsymbol{u})$
- Test statistic

$$\sup |C(\boldsymbol{u}) - C^*(\boldsymbol{u})|$$

Limitations
Computational Expense \\ Requires large dataset \\ Distribution must be fully specified

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Current Goodness-of-fit Approaches

Other tests

Ranks

- For any sample $\mathbf{x}_j$, $\frac{R_{j1}}{n+1}, \dots \frac{R_{jd}}{n+1}$ where R_{ji} is the rank of x_{ji} in $\mathbf{x}_j$
- Can be thought of as pseudo-samples from the copula

Rosenblatt's Transform

- Transforms a set of dependent variables into independent uniform variables.
- $\mathcal{V}_i = \mathcal{R}(Z_i)$ where $\mathcal{R}(Z_i) = \mathbb{P}(Z_d \leq x_d \mid Z_1 = z_1, \dots, Z_{d-1} = z_{d-1})$

AIC

- More of a measure of model quality
- Trade-off between goodness-of-fit of a model and its complexity
- $2k - 2\ln L$ where k is the number of parameters and L is the likelihood.

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The New Approach

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Overview: New Approach

- The approach discussed in my paper is a complete reformulation of the goodness-of-fit problem
- By finding a suitable approximation (see paper) to a given copula we can determine the relevant the copula family
- In order to achieve this we need some classical results from the field of uncertainty quantification.

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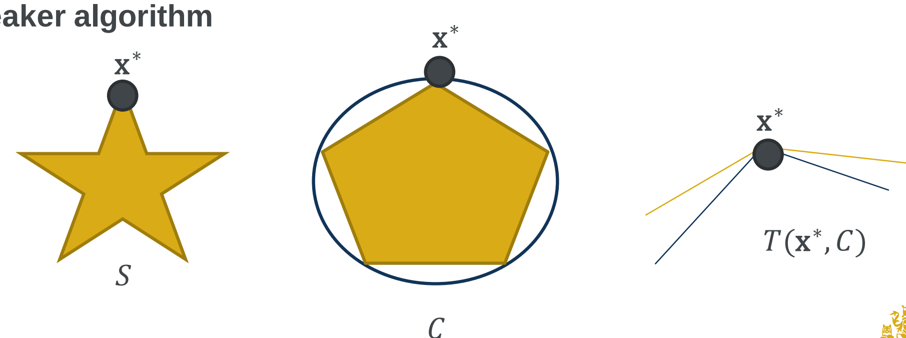
Overview: New Approach

- **Convex Relaxation**
- A trade-off between data usage and numerical computation, we **aim to find a weaker algorithm**



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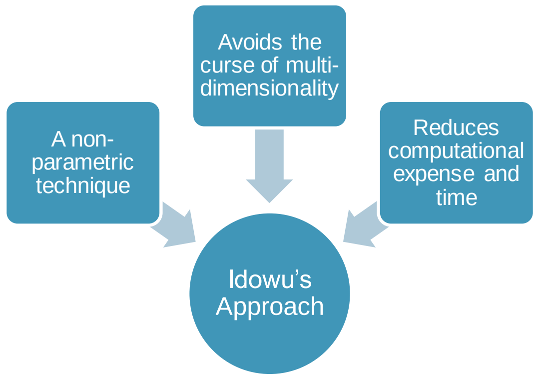
Benefits of the New Model



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Ongoing work

- Great scope for implementation in the financial sector
- Development of a computational package
- For further details of the corresponding mathematics and implementation of the approach see [Idowu, 2017] – Working Paper.

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Further Reading

- Victory Idowu is an academic working on Uncertainty Quantification and Model Risk research with an emphasis in Actuarial science
- Other areas of research include:
  - Structured Expert Judgement
  - Model Validation (see The Model Validator's Manifesto).

https://www.actuaries.digital/2017/05/01/the-model-validators-manifesto/#_ftn1

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Questions **Comments**

Contact Details: V.Idowu@lse.ac.uk

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