Extending the Asset Share Model

Recognizing the Value of Options in P&C Insurance Rates

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Extending the Asset Share Model

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Background and Motivation

- Well known that renewal business has lower loss costs than new business
- Asset share model calculates premium required to produce desired expected profit over the lifetime of a policy incepting today
  - Company may take a loss during the initial terms, then make it up with profitable renewal terms
- Formula:

$$EPV[P] = \frac{F + EPV[L]}{1 - U - V}$$

  - Extra parameters needed: premium and loss trend, discount rate, renewal probability

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Background and Motivation

- What is the shortcoming of the asset share model? It assumes constant inflation of premium and loss, ignoring risk class transition
- Example 1:
  - 2 risk classes, Low and High
  - Low can have an accident and be reclassified as High
- Premium and loss trend include the effect of Low risks being reclassified and having higher premium and loss
  - E.g. 10% chance of transition * 40% higher premium = 4% trend
  - If we change High risk rate, Low risk trend assumptions no longer valid

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Background and Motivation

- Example 2:
  - Experienced vs Inexperienced Drivers
- Inexperienced drivers charged more, asset share model assumes higher premium will keep trending into the future
  - Premium must drop to experienced rate eventually, leading to less renewal term profit than anticipated
- Is there a way to adapt the asset share model to recognize interactions between the risk classes over time and remedy these problems?

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Background and Motivation

- Accident forgiveness, price lock, rate guarantees are common policy features in personal auto
  - Economics depend on transition between risk classifications
- Common theme is the right but not obligation to purchase an asset (insurance policy) in the future at a price set today
  - That's a call option
- Will our new model be able to price these products?

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The Extended Asset Share Model

- The solution is to use linear algebra to calculate the asset share price for all risks at the same time and explicitly account for risk class transition
- Instead of separate series of parameters for each class used independently, use series of vectors for premium, loss and expense in a unified analysis
  - Each dimension represents one risk class
- Also need a series of transition matrices A(n) which describe how each risk class renews and moves into the others at the n-th renewal

The Extended Asset Share Model

- The Extended Asset Share Model Formula:

$$\sum_{n=0}^{\infty} P(n) * v_n * A_n(R) = \frac{1}{1-U-V}\left[F + \sum_{n=0}^{\infty} v_n * A_n(L_n)\right]$$

- Simple linear equation of form Ax = b
  - Scalars: P, v, U, V; Vectors: R, F, L ; Matrix: A
- LHS is the lifetime present value of premium, RHS is the loaded fixed expense and lifetime present value of loss
  - Transition matrix tracks policies as they renew into different classes or fail to renew

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The Extended Asset Share Model

- We can apply the inverse of the left hand side matrix to get the current premium vector (relativity times base rate) alone:

$$P(0) * R = \frac{1}{1-U-V}\left(\sum_{n=0}^{\infty} \frac{P(n)}{P(0)} * v_n * A_n\right)^{-1}\left(F + \sum_{n=0}^{\infty} v_n * A_n(L_n)\right)$$

- $\frac{P(n)}{P(0)}$ is the rate of overall premium inflation excluding effect of risk class transition, i.e. base rate increases
- Closed form solution for premium and relativities

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Extending the Asset Share Model

Simple Example – Importance of Variables

- Consider the following simple example:
  - Annual policies with two risk classes: Low and High
  - Losses L_{low} = 50 and L_{high} = 70 for the first term
  - Fixed expense 10 at time 0; 20% variable expense every term
  - Low risk renewal probability 90% every term; High 70%
  - Loss trend 1% Low risk, 3% High risk
  - Premium trend 4% per year
  - 5% discounting; 5% profit
  - Every term 10% chance Low risk reclassified as High; High risk not reclassified
  - Low risk class policies renew with 70% probability when reclassified as High

Standard Asset Share Model Data

Extended Model Additional Data

Simple Example – Importance of Variables

- Given that the retention rate interacts with risk class transition and premium trend depends on the resulting rates this would be difficult to do with standard methods

- The extended model can calculate the required premiums with just a few lines of programming code:
  - $P_{low} = 56.22$
  - $P_{high} = 90.92$

- More interesting: what is the impact of the High risk class variables on indicated Low risk class premium, e.g. loss trend?

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Simple Example – Importance of Variables

- Look at derivative of P_{low} with respect to input variables:
  - $\partial ln(P_{low})/\partial ln(L_{low}) = 0.99$
  - $\partial ln(P_{low})/\partial ln(r_0) = 3.91$
  - $\partial ln(P_{low})/\partial ln(r_1) = 1.73$

  - r_0 = Low risk loss trend factor
    - *E.g. 1.01*
  - r_1 = High risk loss trend factor
    - *E.g. 1.03*

- Changing the High risk loss trend by +/- 1% has about 2x the impact on P_{low} as changing L_{low} by +/- 1% (multiplicatively), and 0.5x the impact of changing Low risk trend

- Surprising since transition probability is only 10%; result will vary under different assumptions

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Complex Example – "Accident Forgiveness" and Value of Options

- Consider the following additions to the simple example:
  - Third risk class, Medium
  - L_{med} = 55; loss trend 2%
  - Low risks transition only to Medium risk with 10% probability each renewal
  - Medium risk renewal probability 95%
  - Medium risks transition only to High risk with 25% probability each renewal
  - $P_{med} = P_{low}$, i.e. higher expected loss is "forgiven"
- What should we charge Low/Medium risks? Where's the option and what's it worth?
  - Again, difficult for standard methods to solve but easy using Extended Asset Share model

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Complex Example – "Accident Forgiveness" and Value of Options

- Extended Asset Share model formula gives three equations in three unknowns: P_{low}, P_{med} , P_{high}
- Normally we set each premium equal to lifetime EPV of loss and expense, but with forgiveness we set P_{med} = P_{low} and solve for the other two
- Indicated premiums:
  - P_{low} (no forgiveness) = $54.98
  - P_{med} (no forgiveness) = $64.25
  - P_{low} (with forgiveness) = P_{med} (with forgiveness) = $57.65
  - P_{high} = $90.92

Complex Example – "Accident Forgiveness" and Value of Options

- Low risk policy with forgiveness is given ***the right but not obligation to buy*** renewal policy at the Low risk price, even if reclassified as Medium
- Call option on insurance policy costs Low/Medium risks $2.67 per term
  - Single term value is less for Low risk policies; more for Medium risk policies when option is "in the money"
- Extended Model adds amortized value of the lifetime EPV of option costs to the price

Extending the Asset Share Model

Further Considerations

- Price elasticity of demand
  - Transition matrices will have demand functions as entries
  - Computationally more complex
- More sophisticated premium formulas
  - IRR, RORAC, etc.
- Regulatory constraints
  - Is higher cost of options a valid reason to charge higher rates for otherwise identical risks?