

Models of asset pricing: The implications for asset allocation

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Abstract.....	1
Introduction	1
A. Single-factor CAPM methodology.....	2
B. Multi-factor CAPM models in the UK.....	4
C. Multi-factor models and theory	6
D. Multi-factor models and asset allocation	7
Previous orthodoxy	7
Multi-factor extensions of CAPM and asset allocation.....	9
Conclusions	12

Abstract

In the context of asset allocation the asset pricing model plays a central role – allowing investors to optimise returns for a given level of risk. With the single-factor CAPM investing in a simple combination of the market and a riskless asset can do this. However, the single-factor CAPM does not hold empirically. In recent years the Fama-French (1993) model has supplanted the single-factor CAPM both empirically and more recently theoretically as the preferred asset pricing model in many applications. This has implications for asset allocation. The two-dimensional trade-off between risky assets and a riskless asset become a multi-dimensional trade-off between two or more risk factors and the riskless asset. Although the new approach is similar, there are real difficulties. For one, neutral advice needs to be treated far more carefully. In fact the real result may help investment advisors refocus on the real message of the single-factor CAPM – the risk that matters in investment selection is marginal risk. In a multi-dimensional world this requires us to understand the risk preference of investors and their pre-existing risk exposure.

Introduction

The cost of capital is a fundamental concept in financial economics. Calculation of the appropriate measure is essential in portfolio investment and asset allocation. The questions of what model is most appropriate and how it should be implemented is therefore of critical importance to the investment industry.

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It is important to realise that calculating the cost of capital is a developing science and there is still no universally accepted methodology. While the single-factor Capital Asset Pricing Model (CAPM) (see Sharpe (1964) and Lintner (1965)) is arguably one of the better-founded models – having been established over forty years – and provides valuable insights into the analysis and valuation of risk, its ability to explain investor behaviour is widely recognised as severely limited. There are alternative theories, such as the multi-factor Consumption-CAPM or the Arbitrage Pricing Theory (APT), which also address the issue of the measurement of the cost of capital. However, the main difficulty with the APT has been that it does not identify the factors that analysts should include in their estimate of the cost of capital.

There is strong evidence that for some categories of firms the CAPM over- or under-estimates investor return expectations. The evidence of underestimation is most strongly pronounced for firms with high book-to-market ratios such as firms that rely heavily on physical capital and for small firms (see Fama and French (1992)).

As a consequence, one application of a multi-factor model – the Fama-French (1993) model – has been thoroughly tested, is well supported among the available alternatives and has received widespread acceptance.

This note examines the principal models of asset pricing that are currently being employed in finance and points out the far-reaching implications for asset allocation.

A. Single-factor CAPM methodology

In evaluating a specific project's desirability, it is important to consider the project's contribution to both company profitability and risk level. With respect to the single-factor CAPM approach, the contribution of a new project's risk must be evaluated by taking its covariance not only with the firm's own earning but also with other firms' earnings. The logic behind this approach is that investors in the stock market ought to possess a diverse portfolio and thus risk needs to be considered on a marginal basis – not just in terms of its covariance of the return with the company's existing investments, but rather with the covariance of return on the market portfolio as a whole.

The CAPM market beta coefficient essentially measures the component of an asset's risk that cannot be diversified away by holding the market portfolio. It is this "systematic" risk that investors are predicted to demand a return for holding. As the remaining "specific" risk can be diversified away at no extra cost, it is predicted that investors will not demand any additional return for bearing such risk. Apart from reducing the number of parameters that are required to be calculated, the CAPM is also arithmetically simple to implement because the model predicts that the relationship between beta and return should be linear. The expected return on equity shares is:

$$E(r_e) = r_f + \beta_e (E(r_m) - r_f) \quad (1)$$

Where:

r_e is the return on an equity investment

r_f is the risk-free rate

β_e is the beta coefficient

While the appropriate use of CAPM for cost of capital estimation is not without merit, there has been much debate over the best model of expected returns and over the status of the single-factor CAPM versus multi-factor models, especially the Fama-French (1993) model and the APT.

The whole area of asset pricing has been in flux since the late 1970s when significant evidence against the standard form of the CAPM began to emerge. Early discussion focussed on “anomalies”, such as the size effect, but over time it has become clear that many of these so-called anomalies were actually consistent with more general forms of pricing models.

While the early evidence for the single-factor CAPM was generally supportive this was later questioned by more refined empirical approaches and then by more fundamental criticism (see Black, Jensen and Scholes (1972), Fama and MacBeth (1973) and Roll (1977)). Empirical evidence of persistent “anomalies” also began to build up (e.g., Banz, 1981). In addition, a number of competing model were proposed:

- Intertemporal-CAPM, (Merton (1973)) – (I-CAPM);
- Arbitrage Pricing Theory (Ross ((1976)) – (APT); and
- Consumption-CAPM (Breedon (1979)) – (C-CAPM).

In spite of these developments, CAPM continued to retain acceptability and became the standard model for most applications. However, in the 1990s the work of Fama and French first provided evidence that the CAPM market beta had *less* explanatory power than a number of other firm specific variables that had been considered unlikely to reflect systematic risk (Fama and French (1992)); and then proposed (Fama and French (1993)) and refined (Fama and French (1996)) an alternative three-factor model that included the market plus the returns on the portfolios that are known as HML (the value factor) and SMB (the size factor).

However, the model has not been without highly respected critics. Kothari, Shanken and Sloan (1995) questioned the validity of the model on a number of grounds including survivor bias. MacKinlay (1995) questioned whether the results were statistically distinct from CAPM. Furthermore, the Fama-French model has been criticised for being *ad hoc* given that the HML and SMB factors had no known economic meaning.

In spite of these objections, the Fama-French model has been shown empirically to be resilient over time and across markets. As a consequence, it has now become relatively well accepted and appears as a matter of course in academic studies that examine excess returns (e.g., Carhart (1994)). Indeed, multi-factor models of some sort are now employed by 30% of US firms (see Graham and Harvey (2001))².

B. Multi-factor CAPM models in the UK

As stated above, Fama and French (1993) developed a three-factor risk model based on these findings that can be interpreted as an APT model or as a variant of C-CAPM. Their three factors are:

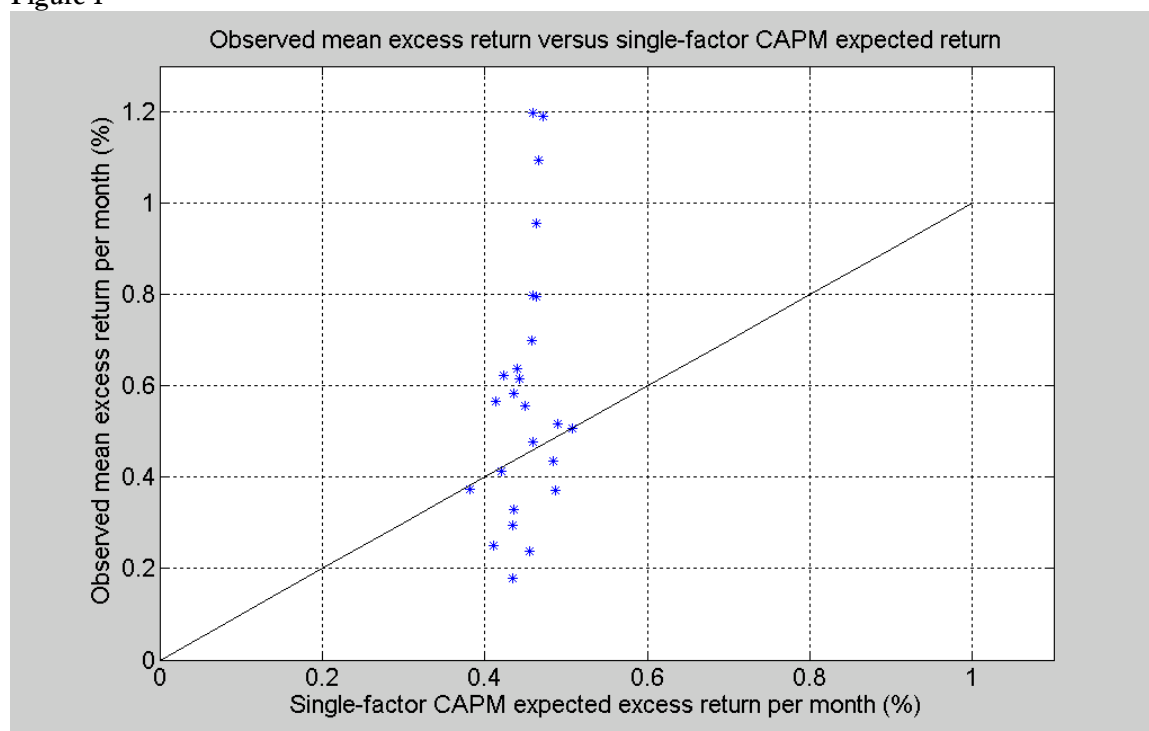
- The market portfolio as for the CAPM;
- The difference in the return between small and large firms (“size” or “SMB”); and
- The differences between the return on firms with large and small book to market value ratios (“value” or “HML”).

We (Giles and Butterworth (2002)) have produced a model based on the work of Fama and French using UK data. One of the intermediary steps involved with this model (see below) is the formation of twenty-five portfolios formed by dividing the entire set of firms (in our case the top 350 UK companies by market value that have accounting data available) using quintiles of market value (size) and book to market value ratio (value).

According to the CAPM, any portfolio formation methodology should still result in returns that are a linear function of the CAPM beta. However, when the twenty-five portfolios are formed using the Fama-French methodology this relationship is not at all evident and the portfolios form an almost vertical pattern (see Figure 1). If the CAPM market beta provided a sufficient explanation of the variation in returns across these portfolios then the measured excess returns should be expected to equate to the estimate derived from the model. In other words, the points on this plot should vary around a line that starts at the origin and continues at forty-five degrees. This is clearly not the case.

² There are a number of helpful reviews of the literature available such as Campbell (2000) and Ferson (2003).

Figure 1



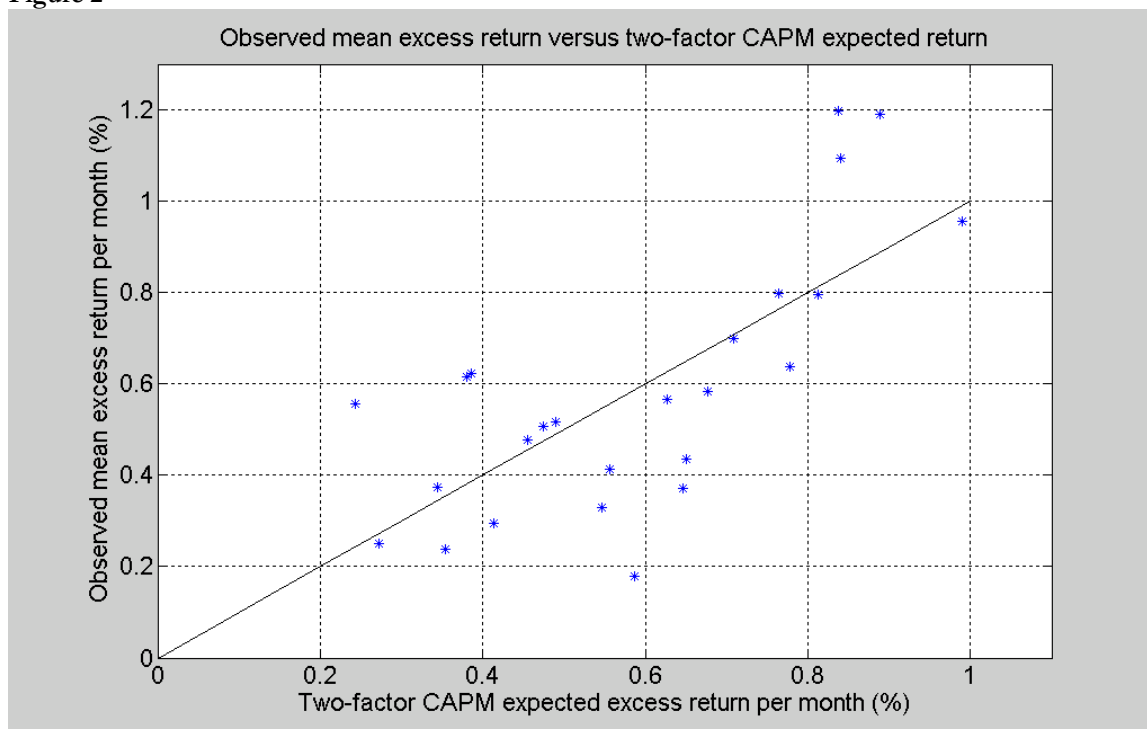
Source: Giles and Butterworth (2002)

In effect, the single-factor CAPM explains almost none of the variation in returns across the twenty-five portfolios. Using the approach adopted by Fama and French we found that the returns are best explained by a combination of the market portfolio and value. That is, the size variable is not found to be an explanatory variable across the 350 firms in this sample³.

In comparison with the CAPM market beta alone, the predictions of our market and value model explains the returns on the twenty-five portfolios to a degree of statistical precision not possible using the CAPM alone (see Figure 2). In this case the expected excess returns are very close to the measured excess returns. Accordingly, points on the plot vary around a line that starts at the origin and increases at forty-five degrees.

³ Although it could still be important across a sample that included smaller firms.

Figure 2



Source: Giles and Butterworth (2002)

Although there may be continuing investigations into the optimal specification and interpretation of these models there is no doubt that even at this stage of development they are of significantly more practical value than the CAPM based on covariance with the market portfolio alone.

C. Multi-factor models and theory

One criticism of the Fama-French model is that the factors are *ad hoc* and lack a theoretical basis. This criticism is misplaced and is now overtaken by recently published evidence. Liew and Vassalou (2000) provide persuasive evidence that the relationship between the Fama-French factors and GDP is significant in most countries tested, including the UK. Subsequent research by Vassalou and Xing (2004) establishes that much of the effect arises from default risk. This research provides a valuable link between the findings of Fama and French and the established consumption focused theoretical framework.

A further objection to the Fama-French model is that the factor premia may represent mis-pricing. However, this argument trivialises the significance of the model. The joint hypotheses problem, which has been accepted for many years, states that it is impossible to distinguish between risk premia and mis-pricing. The real question is whether a model better explains returns over time – and if such explanation can be

repeated in different markets and in periods subsequent to the initial observations. These conditions are true for these models.

D. Multi-factor models and asset allocation

For the actuarial profession the most important consequence is likely to be asset allocation. Where does the traditional two-dimensional trade-off between risk and reward stand if risk itself is multi-dimensional?

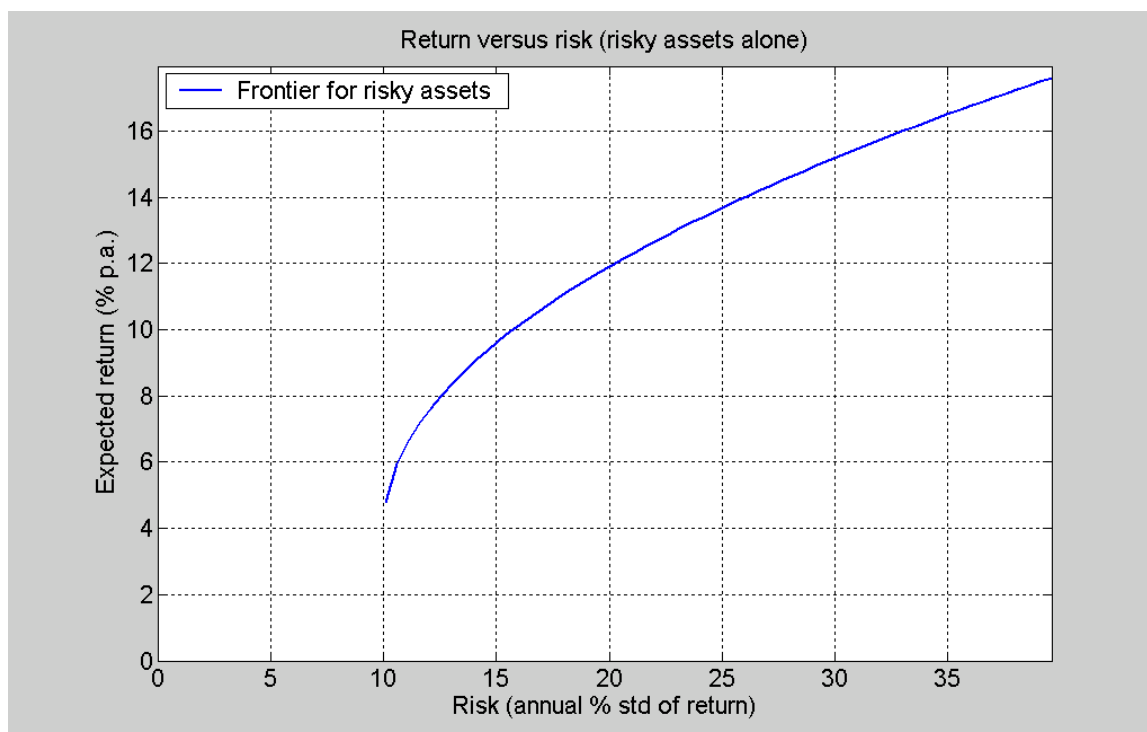
The answer is simple but the implementation is less so. The traditional two-dimensional efficient frontier becomes three- or four-dimensional. Fama (1996) at the University of Chicago provides the theoretical model and it is now for practitioners to find ways that implement it – while maintaining reasonable bounds of error.

But first we review the current approach so that the new approach does not lose its context.

Previous orthodoxy

Up until ten years ago there were many who believed that the major questions in finance had been answered and that “best advice” was clear. An investor should hold the market portfolio and a bond portfolio in a combination that reflects his/her own risk preference. This meant that they should never balance risk through stock selection.

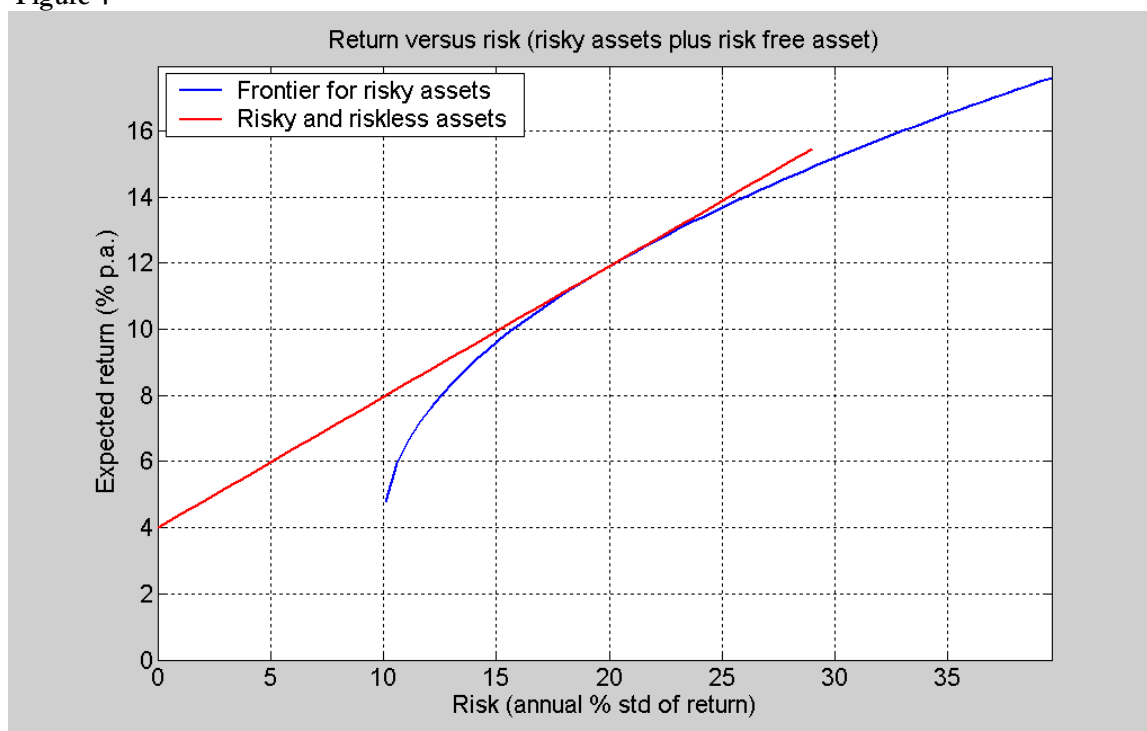
Figure 3



In a world of risky assets the single-factor model of Sharpe-Linter predicted that all investors would select portfolios that fall along the “efficient frontier” depicted in Figure 3. The exact position of their portfolios would depend on their risk preferences. More risk seeking investors would choose portfolios to the right and less risk tolerant to the left.

In a world that includes a riskless asset, the single-factor CAPM predicts that all investors would select the risky portfolio that lies at the intersection of the risky efficient frontier and a tangential line originating at the risk-free return on the y-axis – as depicted in Figure 4. Risk preferences would then be expressed by either borrowing to increase risk above average (moving to the right on the tangential line) or reducing risk by holding a combination of the tangential portfolio and the riskless asset (moving to the left on the tangential line).

Figure 4



Importantly, the tangential portfolio was predicted to be the “market”, i.e., a portfolio that included all assets in the market in proportion to their market values. This clearly led to the idea of benchmarking against the market (and to tracker funds).

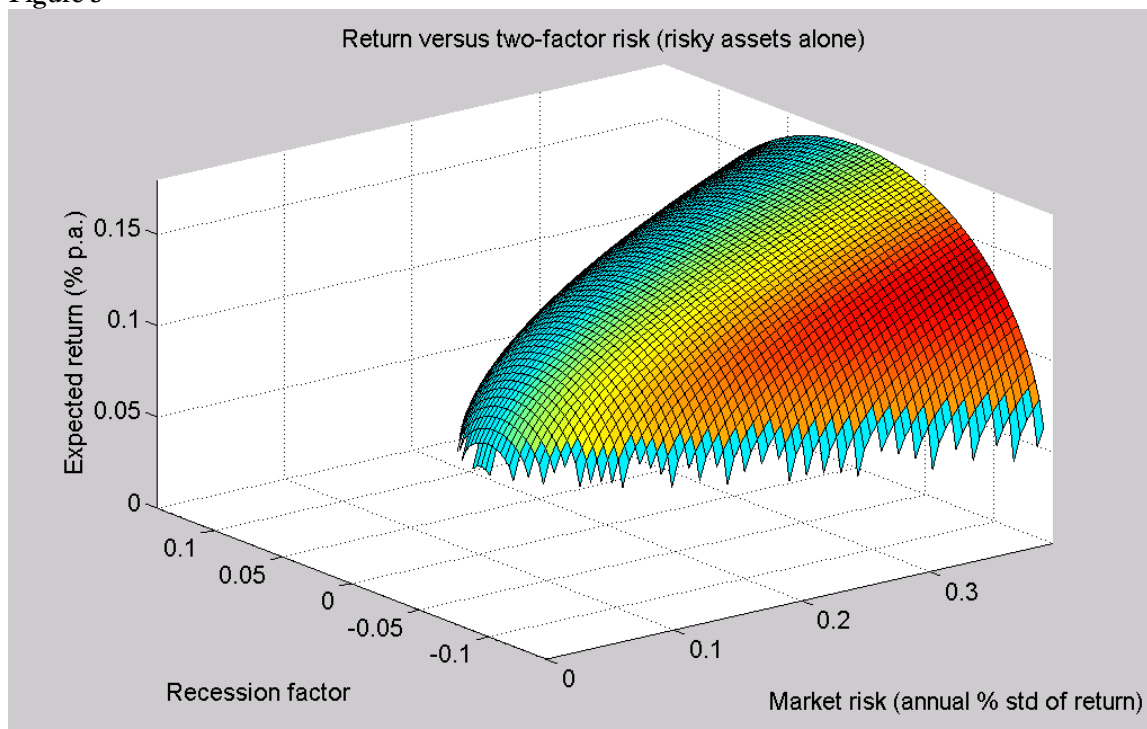
Multi-factor extensions of CAPM and asset allocation

Fortunately, even if the single-factor CAPM does not hold, the approach described above is not completely invalidated. However, it needs to be extended to multiple dimensions as Fama (1996) shows. In this paper we illustrate this using a two-factor model that utilises the market and recession aversion as the two risk factors.

In a world of risky assets, the two-factor model predicts that all investors would select portfolios that fall on the “efficient frontier surface” depicted in Figure 5. The exact position of their portfolios would depend on their risk preferences. Investors who seek more variance would choose portfolios to the right and the less risk tolerant would choose portfolios to the left (on the x-axis).

However, those who are averse to returns that might fall more than the average during recessions would choose portfolios with the negative recession factor (to the right on the y-axis), while those who are neutral to recession risk will choose portfolios with a zero recession factor (i.e., portfolios that lie along the ridge of the surface).

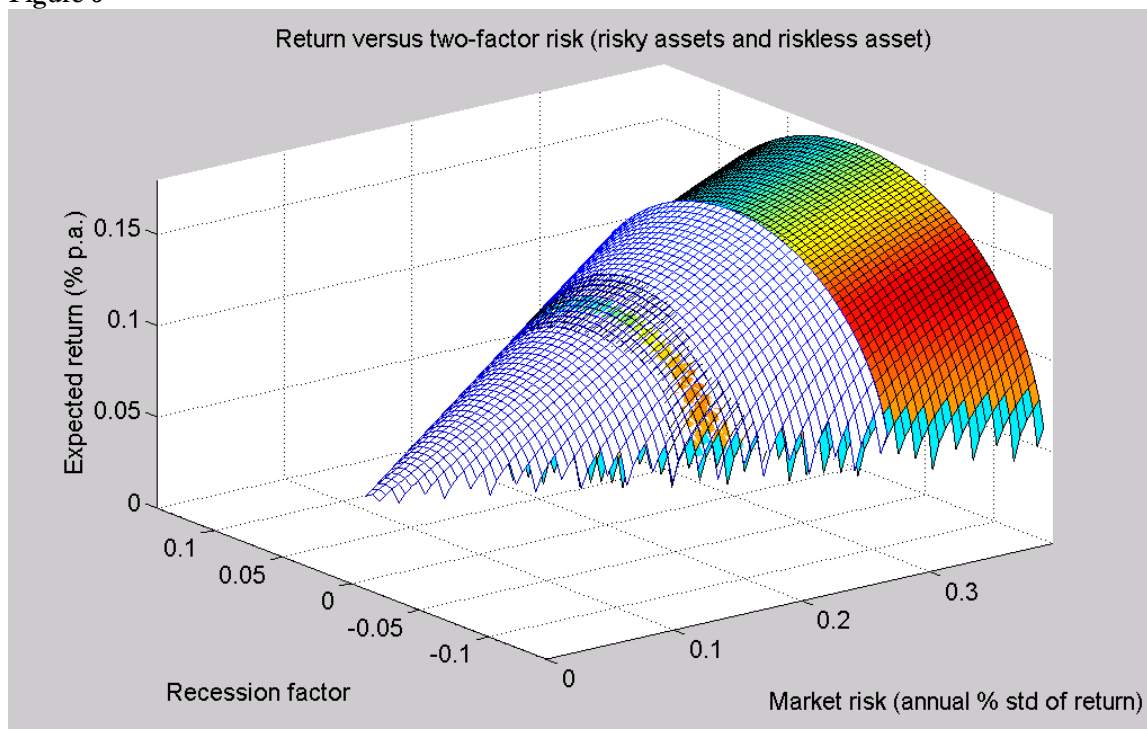
Figure 5



As with the single-factor CAPM, the inclusion of a riskless asset simplifies the problem. However, in this case the tangential line of Figure 4 becomes the tangential cone of Figure 6. Now all investors will choose portfolios on the *curve* where the parabolic risk frontier surface intersects tangentially with a linear cone originating from the point on the z-axis that corresponds to the risk-free return.

If we were to observe this surface side-on, we would have a view that is very similar to Figure 4. In fact, for investors who are neutral in respect to recession risk (i.e., those who only care about mean and variance) this view would fully describe their choices. However, this does not mean that neutral choices remain the same as they did under the single-factor model.

Figure 6

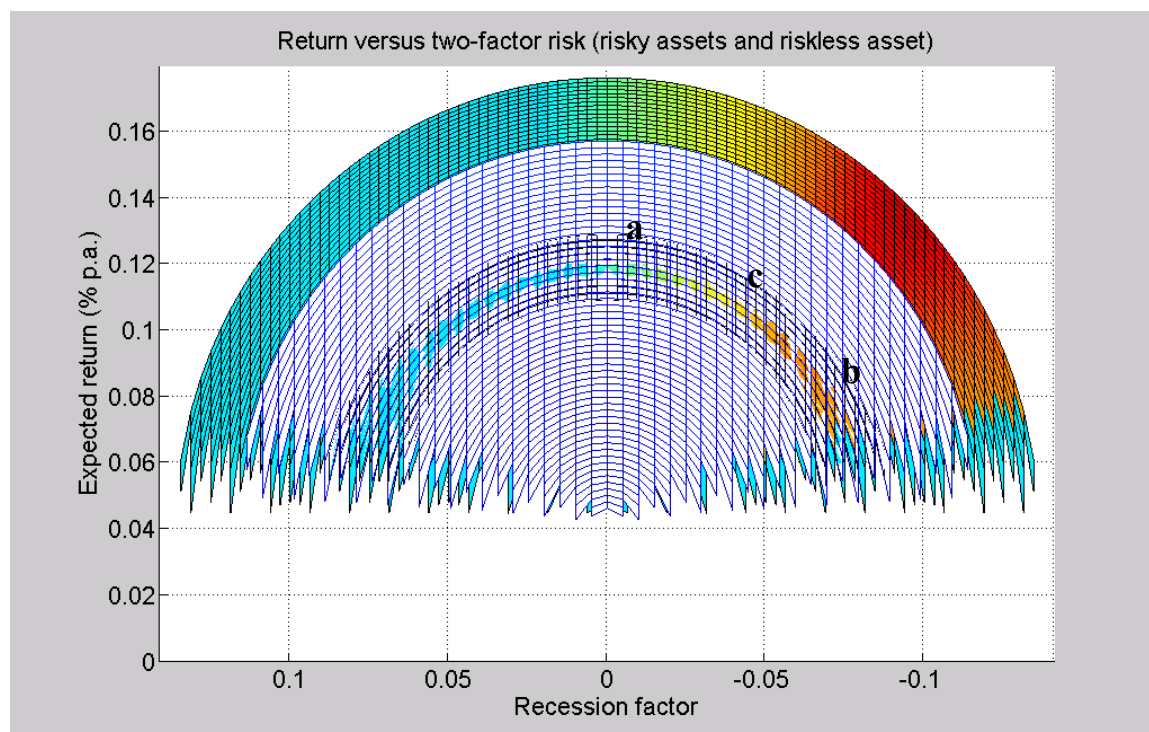


In line with common sense, evidence suggests that the average investor is averse to recession risk. To illustrate the impact of this, we could characterise the population as belonging to one of two groups: those who have greater than average aversion to recession risk and those who are neutral to it.

If we take a front-on view of Figure 6, it looks like Figure 7. The semi-circular arc that passes points 'a', 'b' and 'c' defines the set of efficient portfolios that investors ought to be invested in – given differing tolerance to recession risk. Investors with a greater than average aversion will invest in portfolios in the region of Point 'b'. Investors that are neutral to recession risk will invest at Point 'a'.

One of the interesting corollaries of this is that the average portfolio (the “market”) is at Point 'c'. This means that we have to be very careful if we are trying to make a neutral assumption about an investor's optimal portfolio. If investors are *average* investors they ought to still be invested in a combination of the market and the riskless asset. However, if they are pure mean-variance optimisers then they ought to be invested in a combination of 'a' and the risk-free asset. With a single-factor model these choices were one and the same. With a multi-factor model we need to be more explicit.

Figure 7



Conclusions

The single-factor CAPM is being rapidly supplanted by multi-factor extensions. This has implications both for cost of capital estimation and for asset allocation.

For asset allocation, the two-dimensional trade-off between risky assets and a riskless asset become a multi-dimensional trade-off between two or more risk factors and the riskless asset. Although the new approach is similar there are real difficulties. For one, neutral advice needs to be treated far more carefully. In fact the real result may help investment advisors refocus on the real message of the single-factor CAPM – the risk that matters in investment selection is marginal risk. In a multi-dimensional world this requires us to understand the risk preference of investors and their pre-existing risk exposure.

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