Acknowledging uncertainty in longevity

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&

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PBSS, Edinburgh
26th September 2011

Uncertainty in Mortality Forecasting

- Basis risk
- Model risk
- Parameter risk
- Stochastic risk

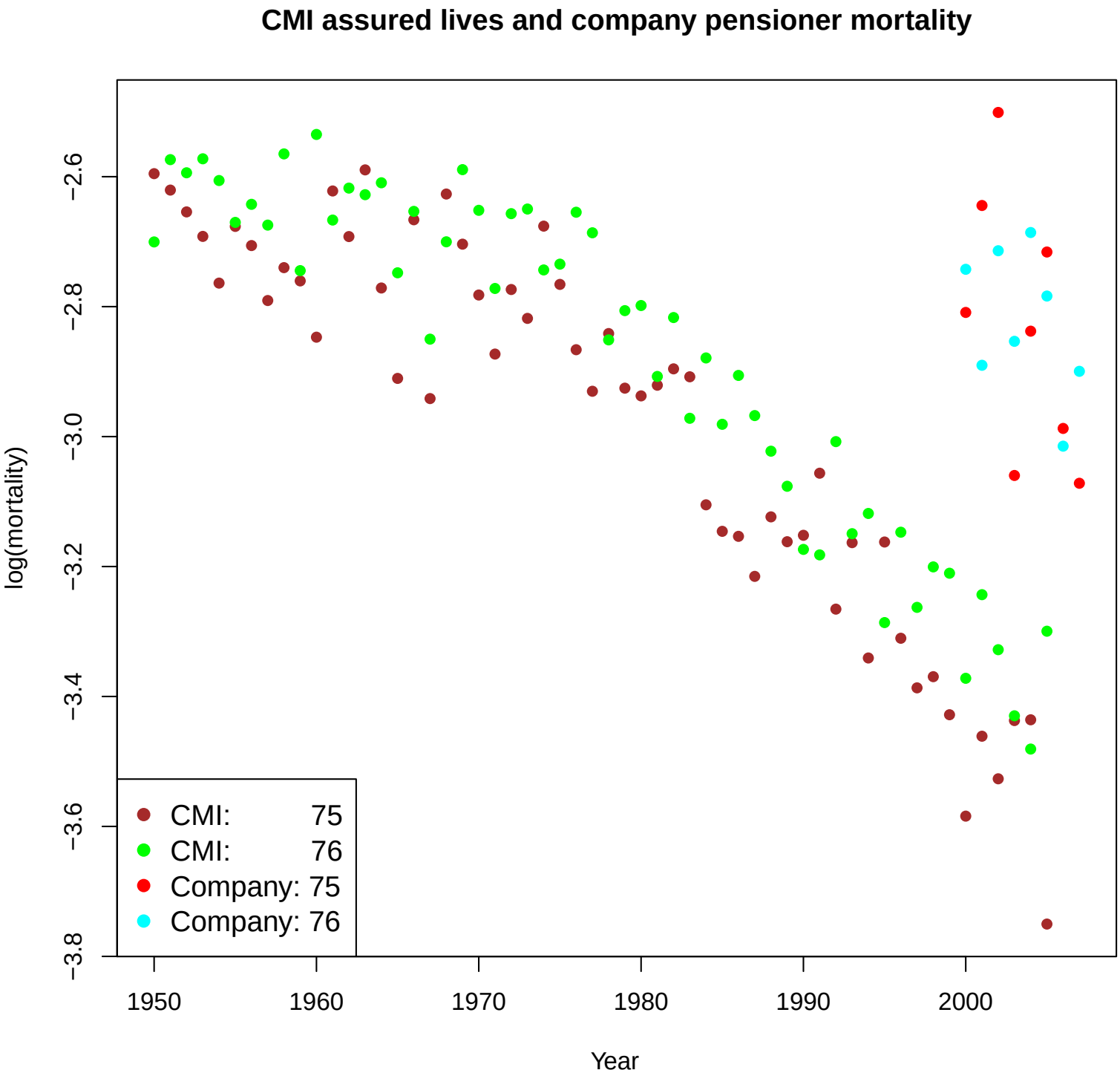
Basis Risk

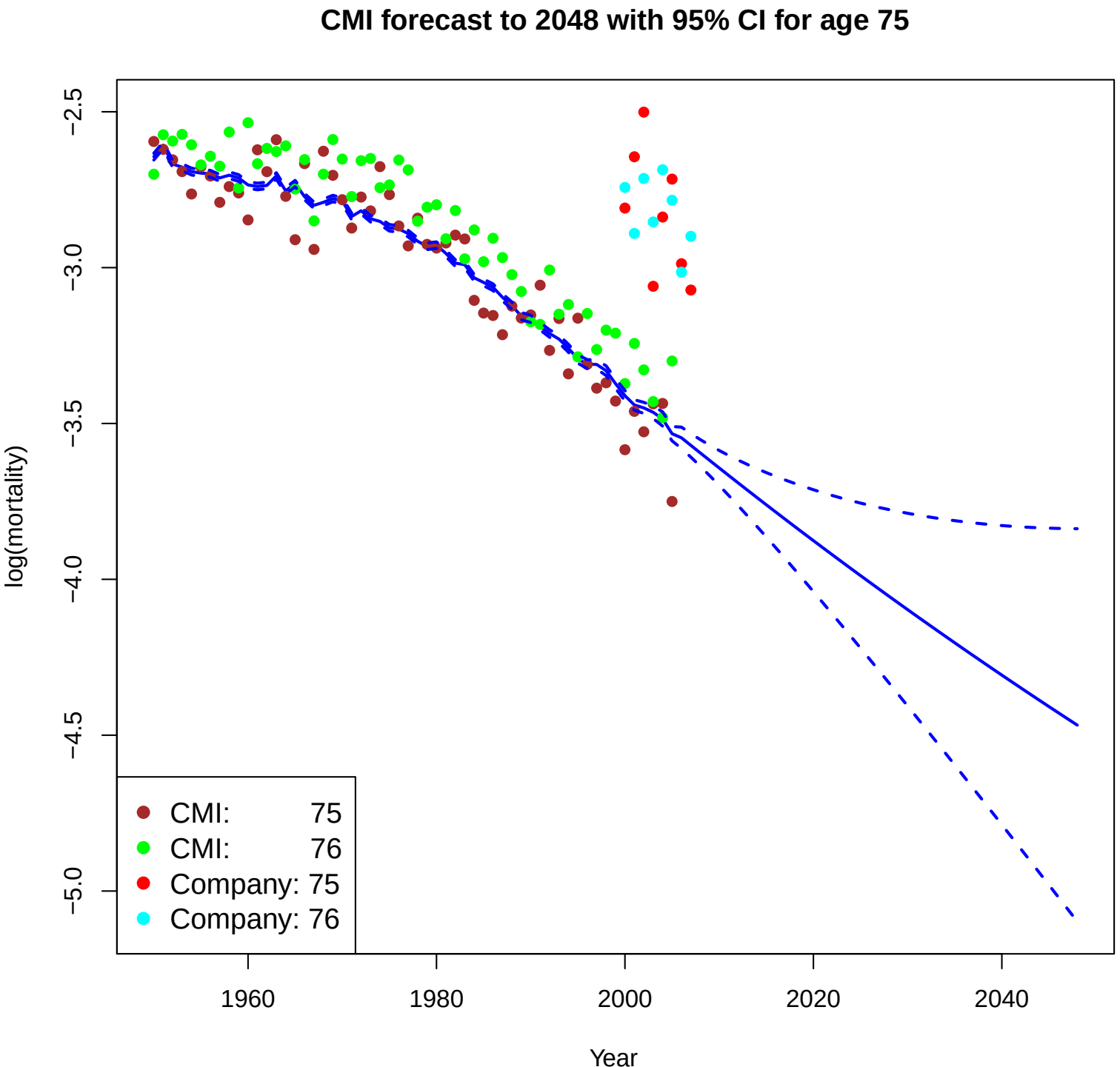
- Portfolio mortality v Reference mortality
- Mortality by Amounts v Mortality by Lives

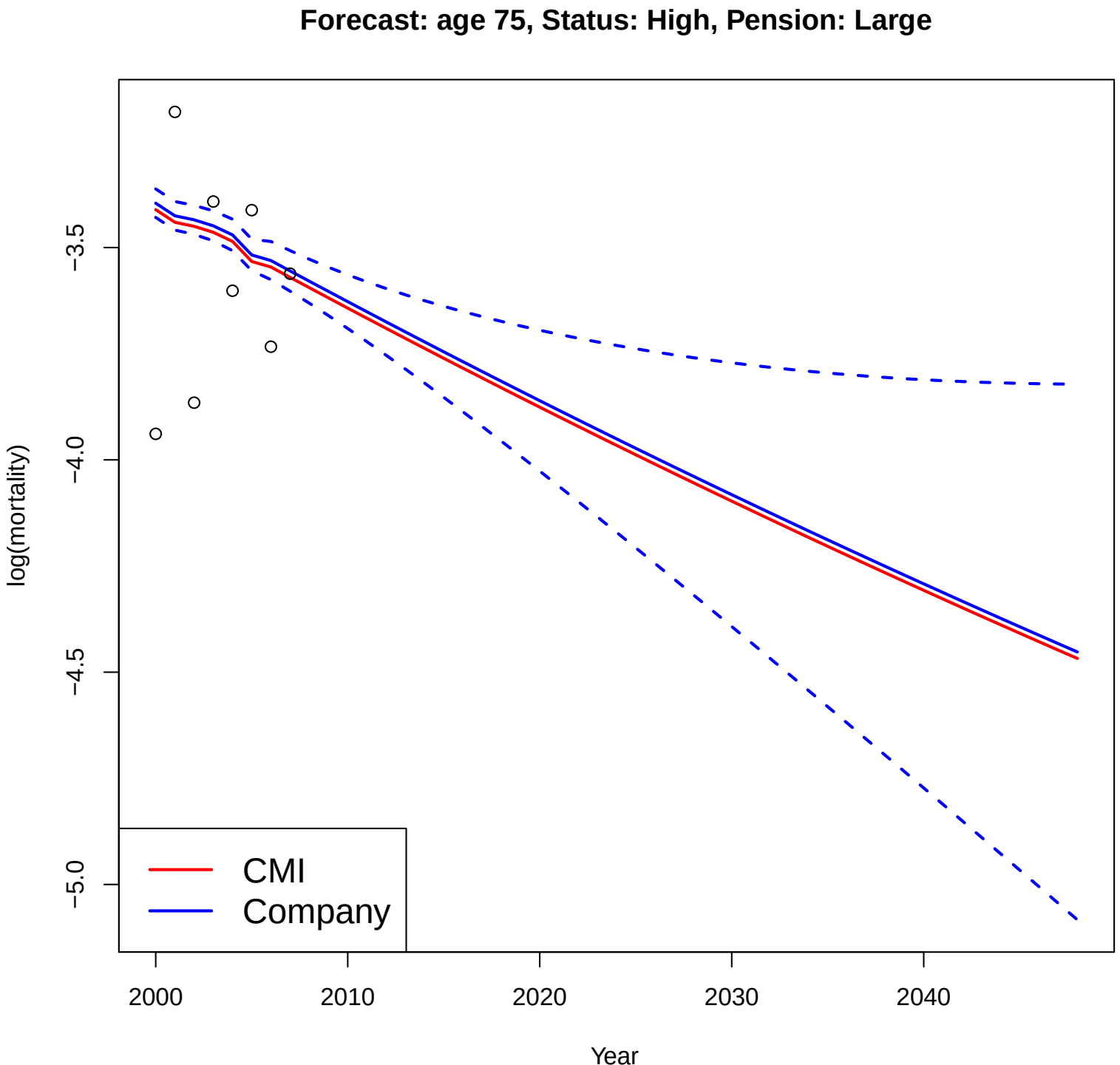
The Piggyback Model adjusts a given forecast wrt relevant data.

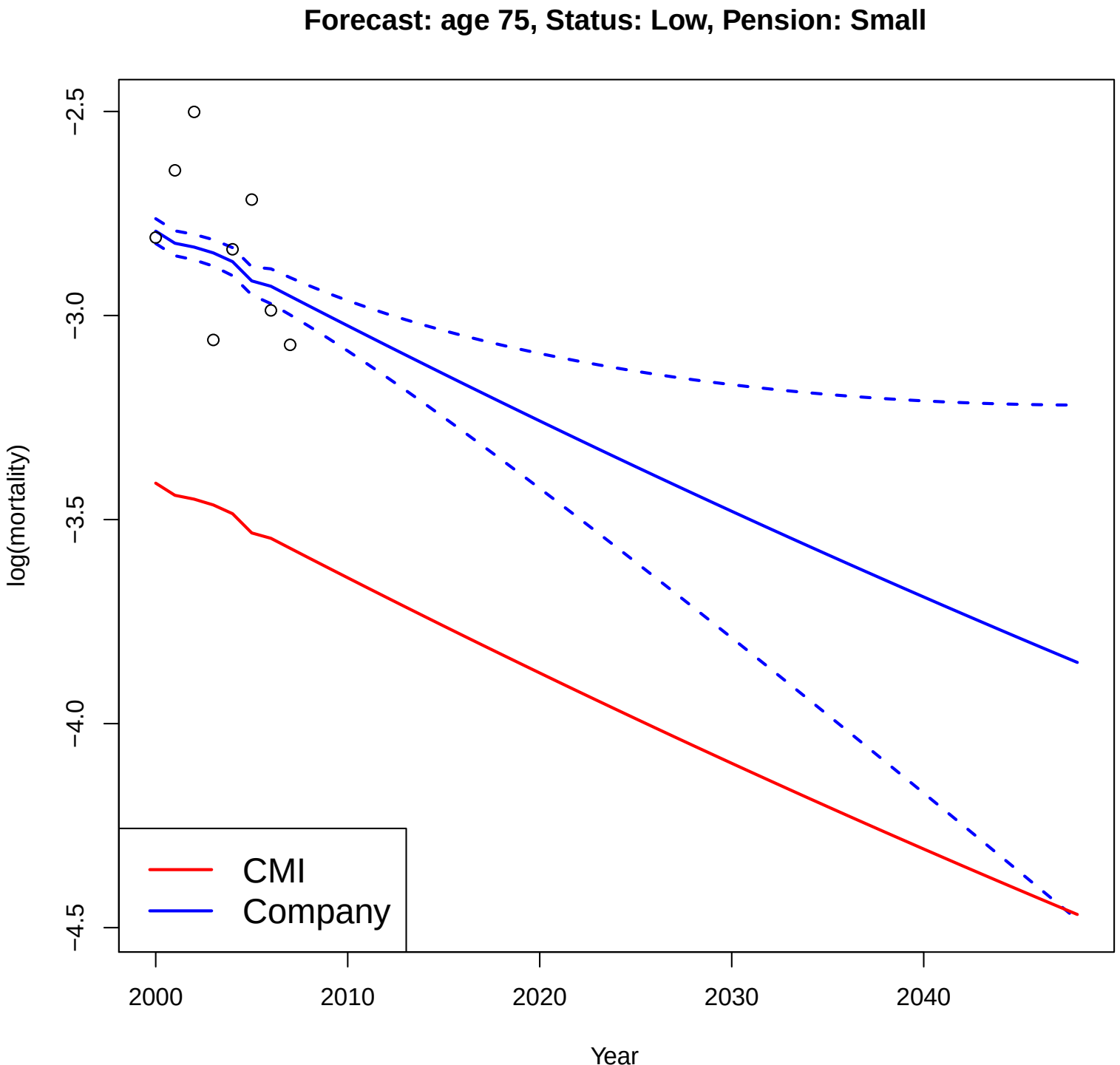
Data splitting

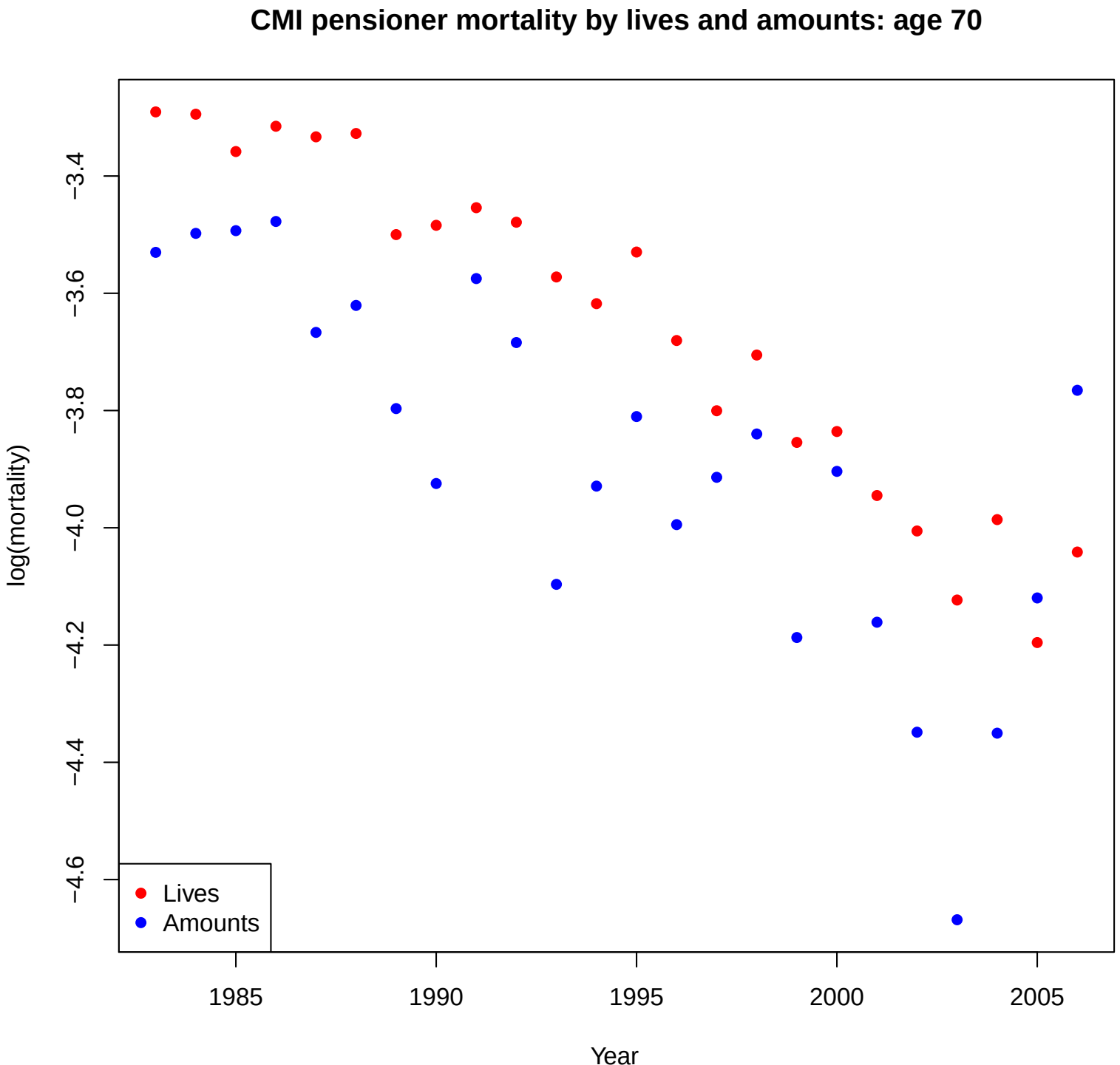
- Social class: two levels
- Pension size: two levels
  - ⋆ Level 00: high status, large pension
  - ⋆ Level 01: high status, small pension
  - ⋆ Level 10: low status, large pension
  - ⋆ Level 11: low status, small pension

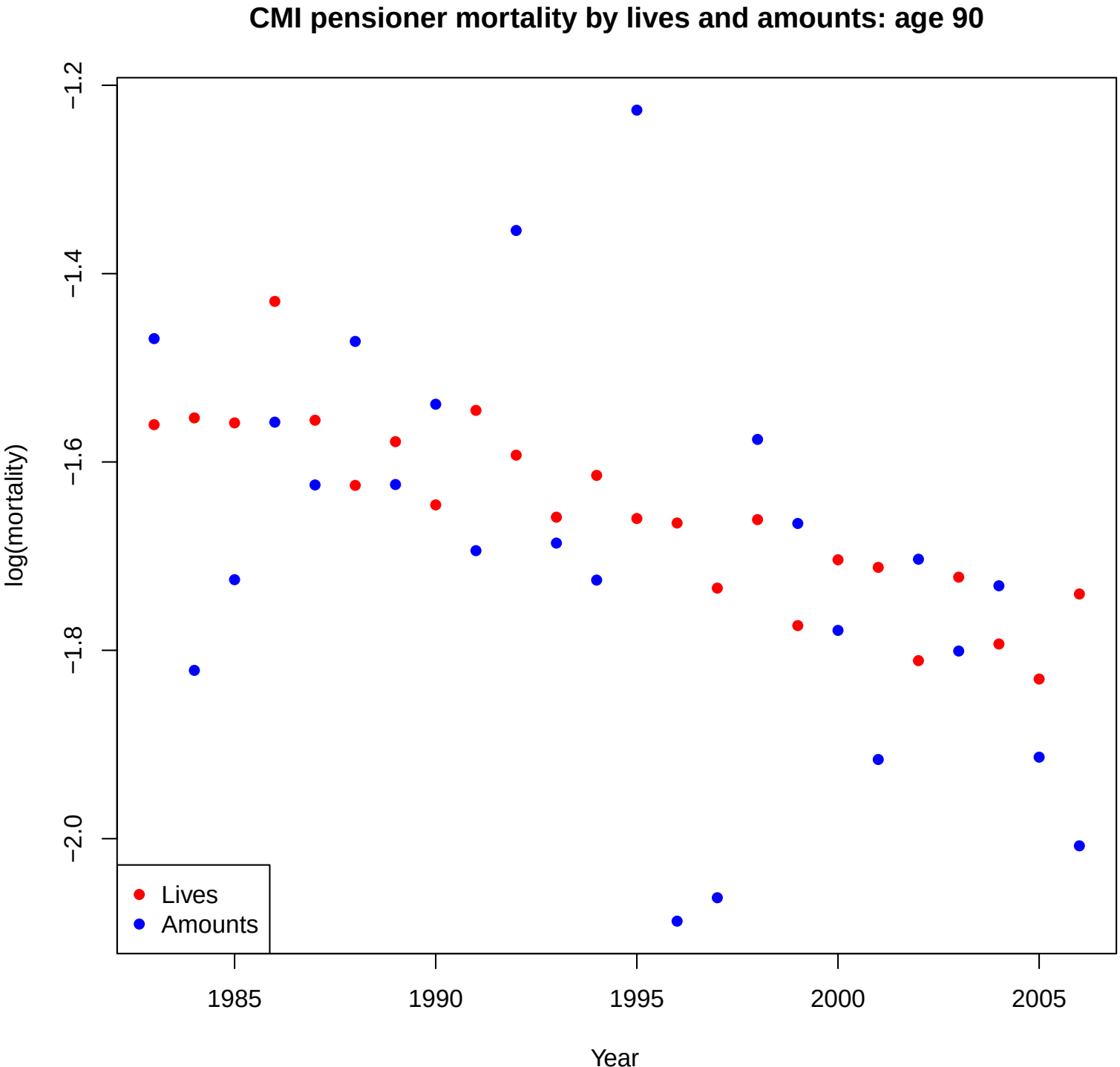

CMI assured lives and company pensioner mortality
log(mortality)
−2.6
−2.8
−3.0
−3.2
−3.4
−3.6
−3.8
1950
1960
1970
1980
1990
2000
Year
CMI: 75
CMI: 76
Company: 75
Company: 76

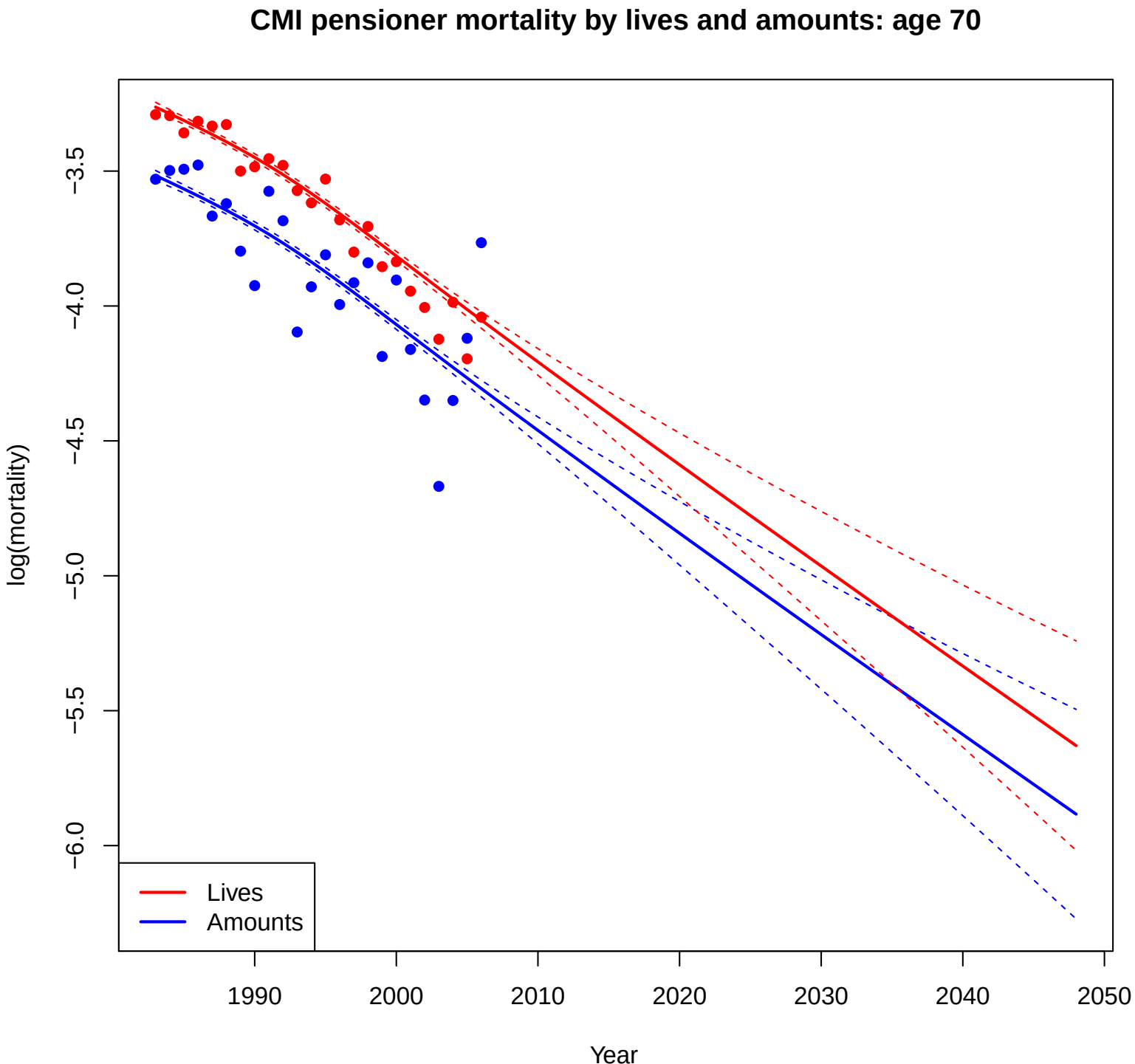

CMI forecast to 2048 with 95% CI for age 75
log(mortality)
−2.5
−3.0
−3.5
−4.0
−4.5
−5.0
1960
1980
2000
2020
2040
Year
CMI: 75
CMI: 76
Company: 75
Company: 76

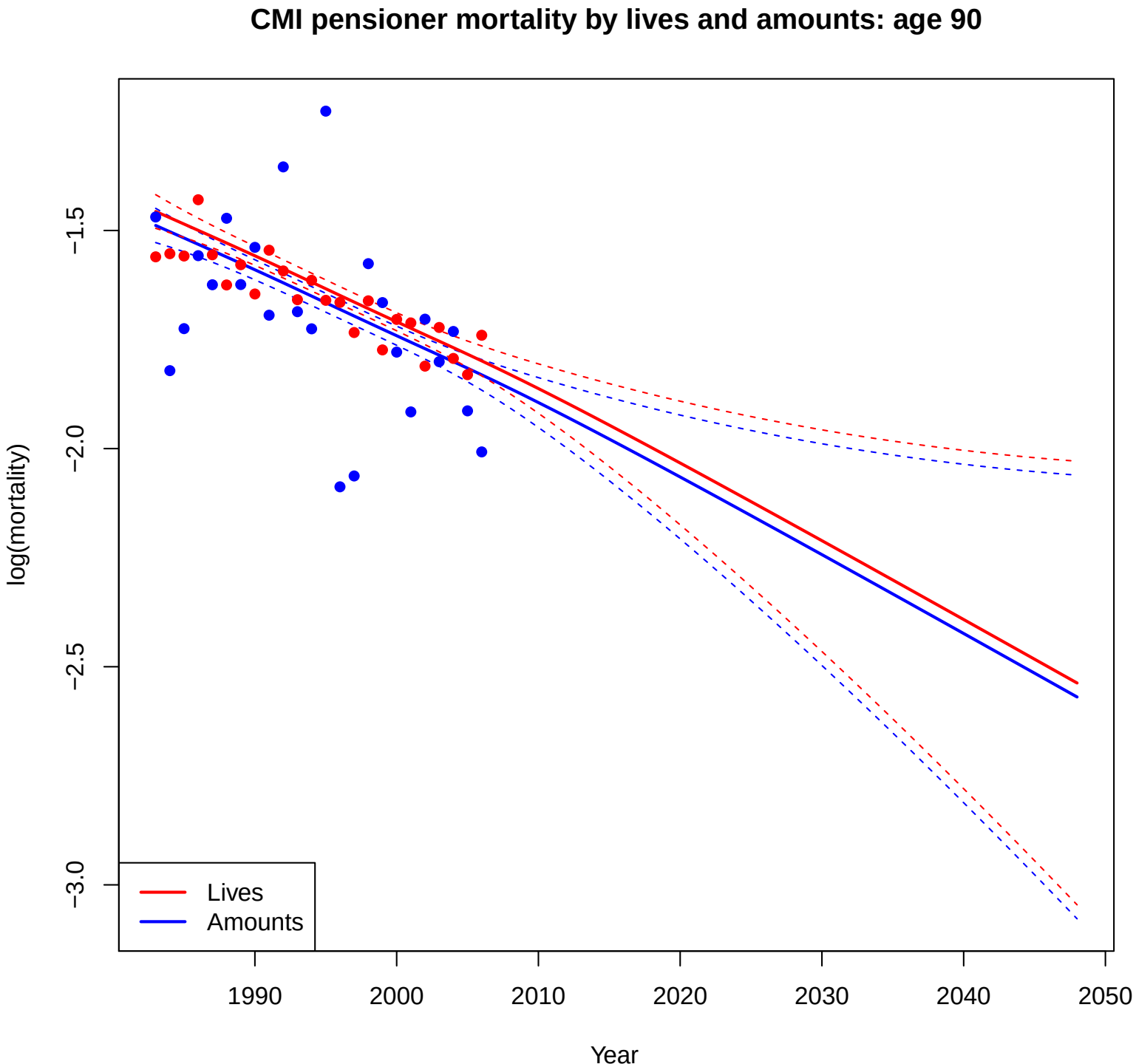

Forecast: age 75, Status: High, Pension: Large
log(mortality)
−3.5
−4.0
−4.5
−5.0
2000
2010
2020
2030
2040
Year
CMI
Company


Forecast: age 75, Status: Low, Pension: Small
log(mortality)
−2.5
−3.0
−3.5
−4.0
−4.5
2000
2010
2020
2030
2040
Year
CMI
Company


CMI pensioner mortality by lives and amounts: age 70
log(mortality)
−3.4
−3.6
−3.8
−4.0
−4.2
−4.4
−4.6
1985
1990
1995
2000
2005
Year
Lives
Amounts


CMI pensioner mortality by lives and amounts: age 90
log(mortality)
−1.2
−1.4
−1.6
−1.8
−2.0
1985
1990
1995
2000
2005
Year
Lives
Amounts

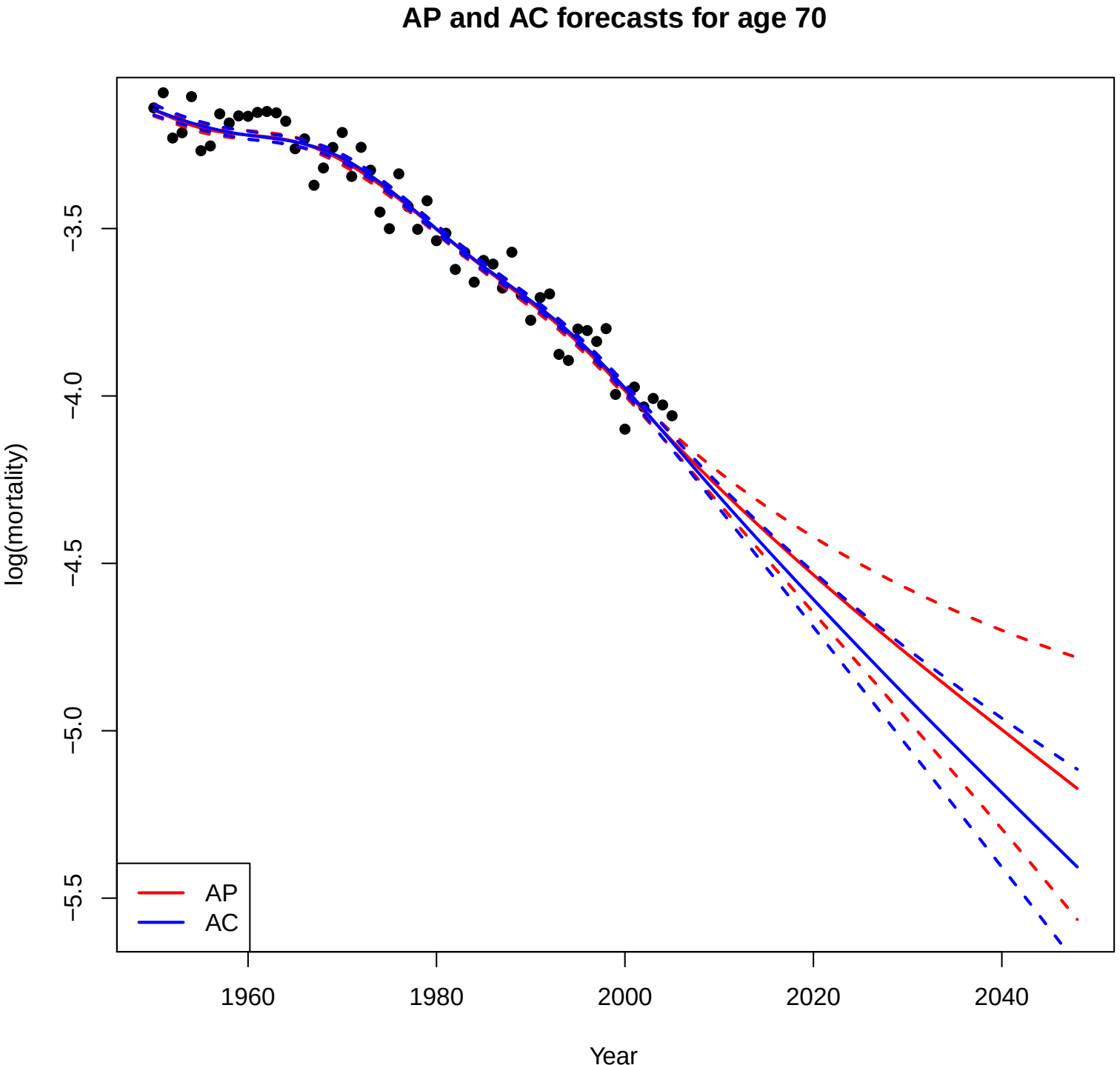

CMI pensioner mortality by lives and amounts: age 70
log(mortality)
−3.5
−4.0
−4.5
−5.0
−5.5
−6.0
1990
2000
2010
2020
2030
2040
2050
Year
Lives
Amounts


CMI pensioner mortality by lives and amounts: age 90
log(mortality)
−1.5
−2.0
−2.5
−3.0
1990
2000
2010
2020
2030
2040
2050
Year
Lives
Amounts

Model Risk

Age-Period v Age-Cohort 2-d P-spline models

- 2-d P-spline model fits a general 2-dimensional surface
- Forecasting is with penalties in
  - Age and year of death (period), or
  - Age and year of birth (cohort)


AP and AC forecasts for age 70
log(mortality)
−3.5
−4.0
−4.5
−5.0
−5.5
1960
1980
2000
2020
2040
Year
AP
AC

Parameter and stochastic error

Lee-Carter model: forecast: random walk with drift

- Parameter error: estimate of the drift parameter
- Stochastic error: estimate of the volatility (of the drift)

Lee-Carter models

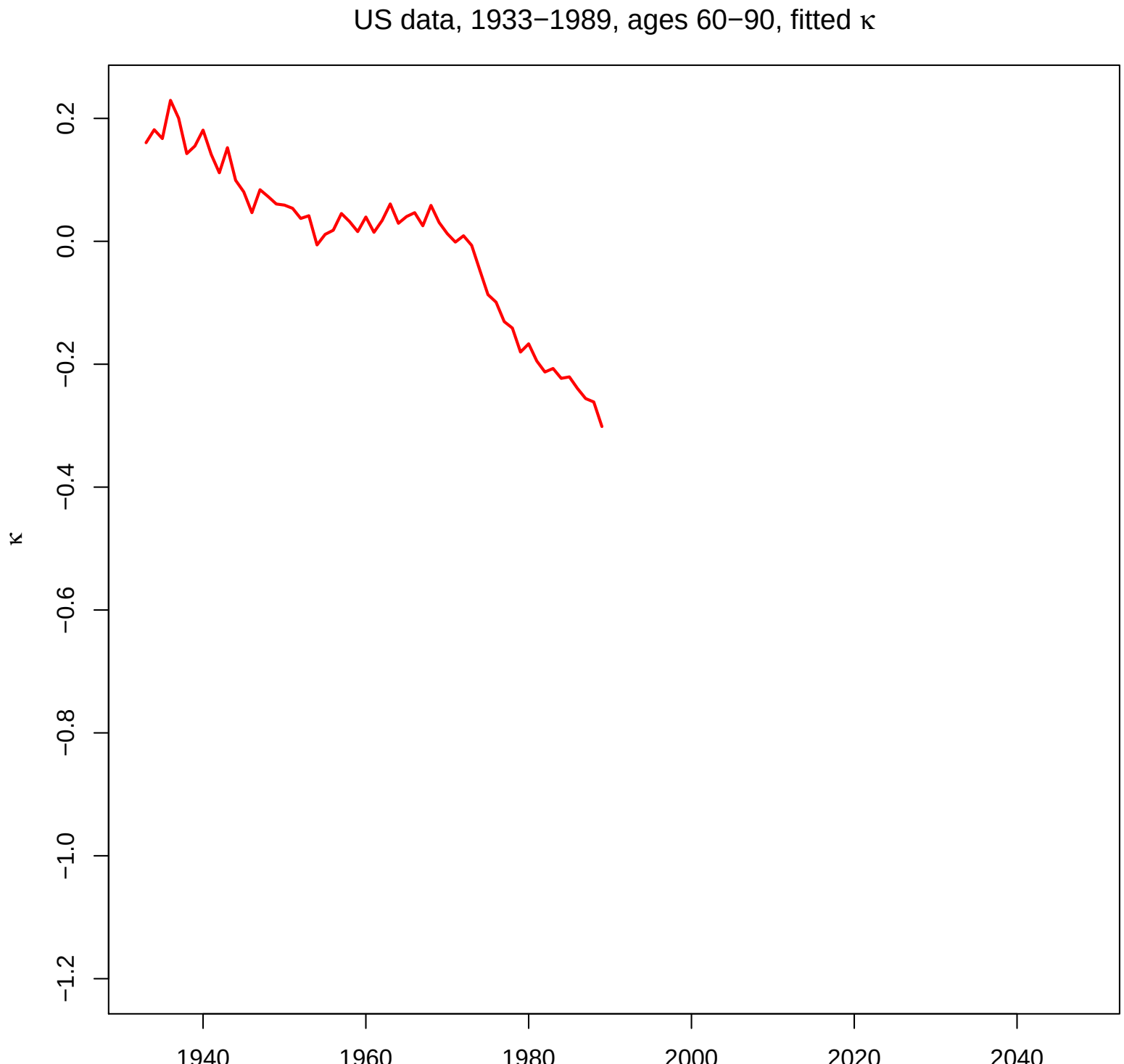
Lee-Carter (1992)

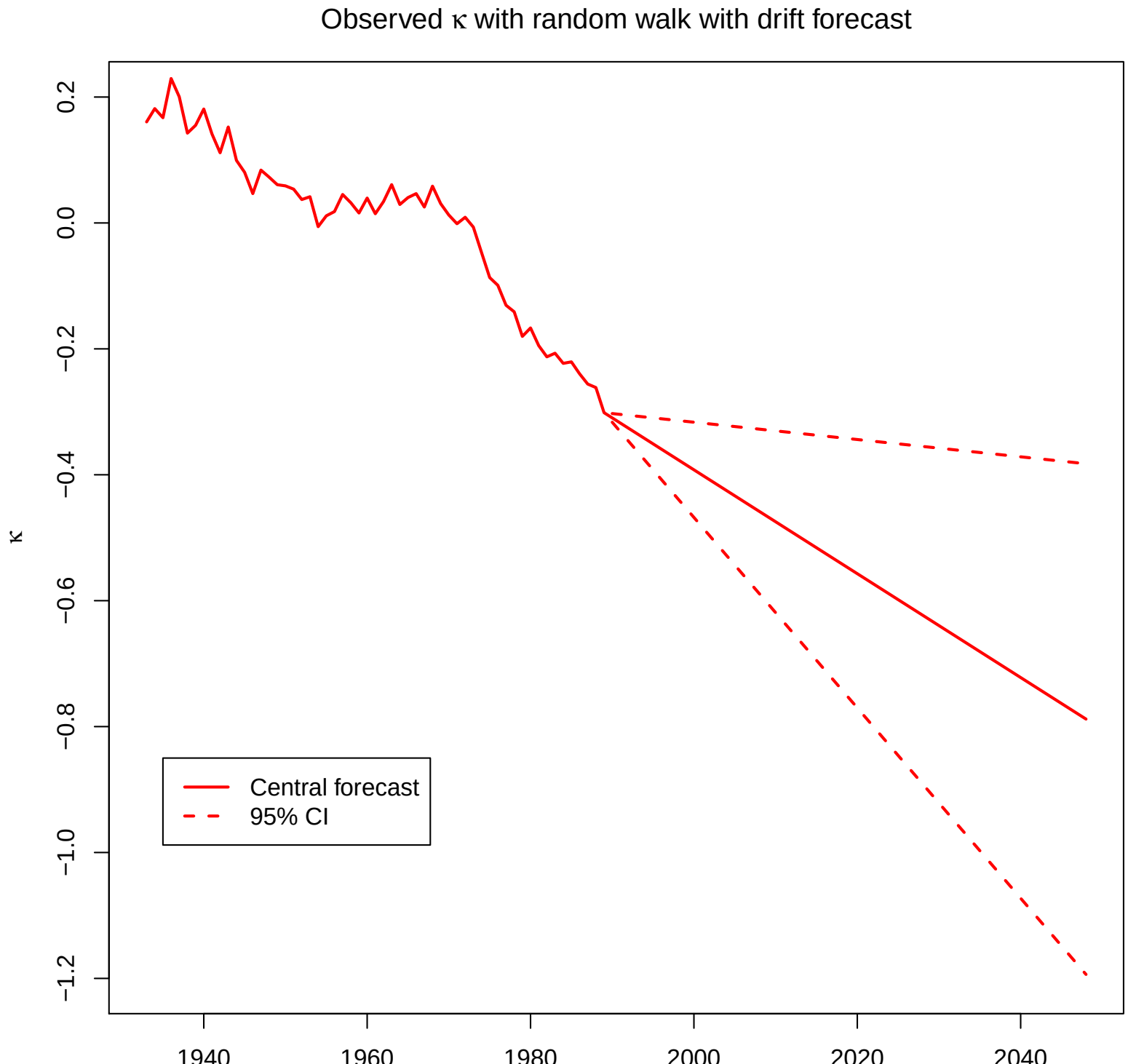
$$\log \mu_{ij} = \alpha_i + \beta_i \kappa_j,$$

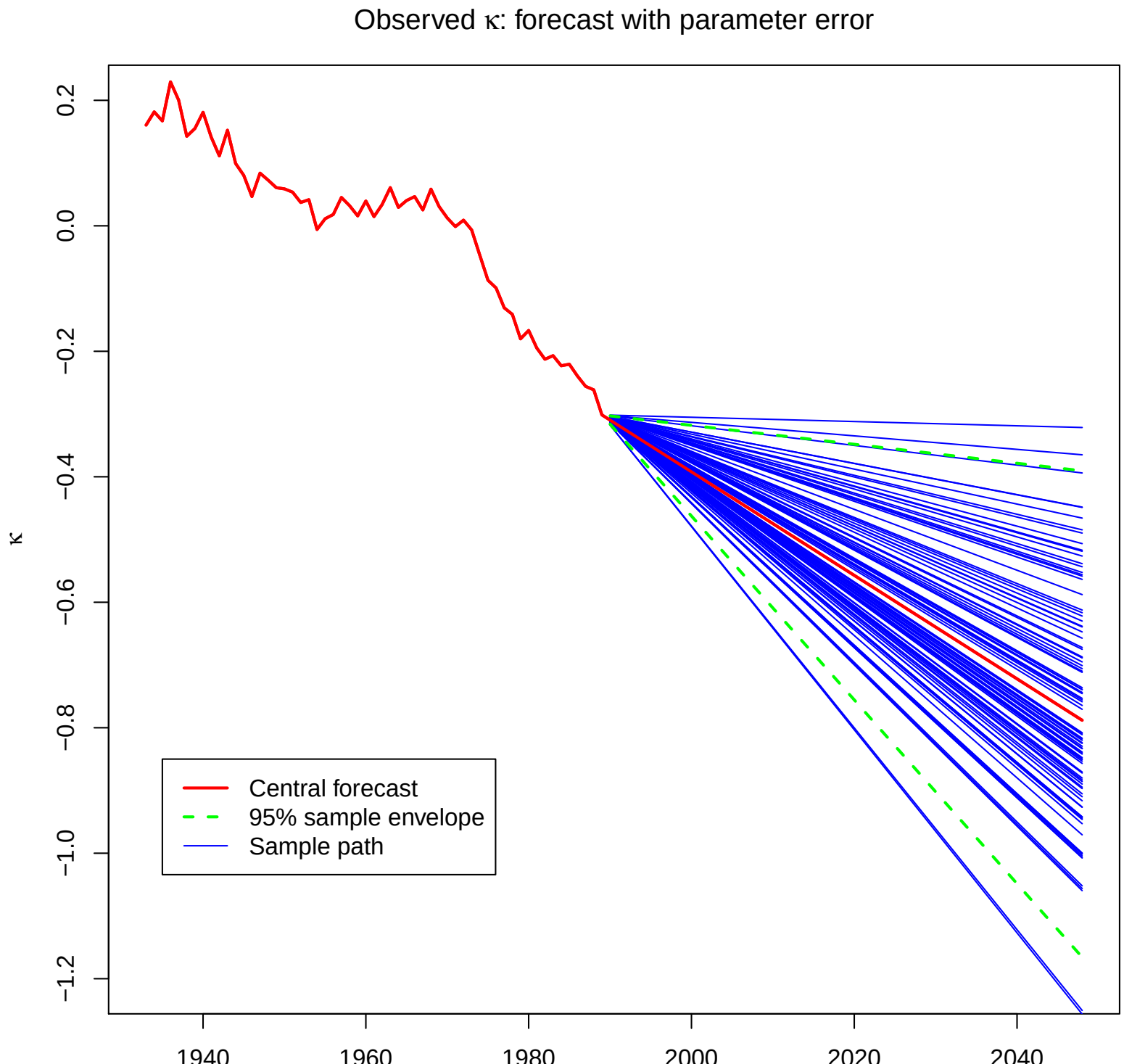
α_i : average mortality by age

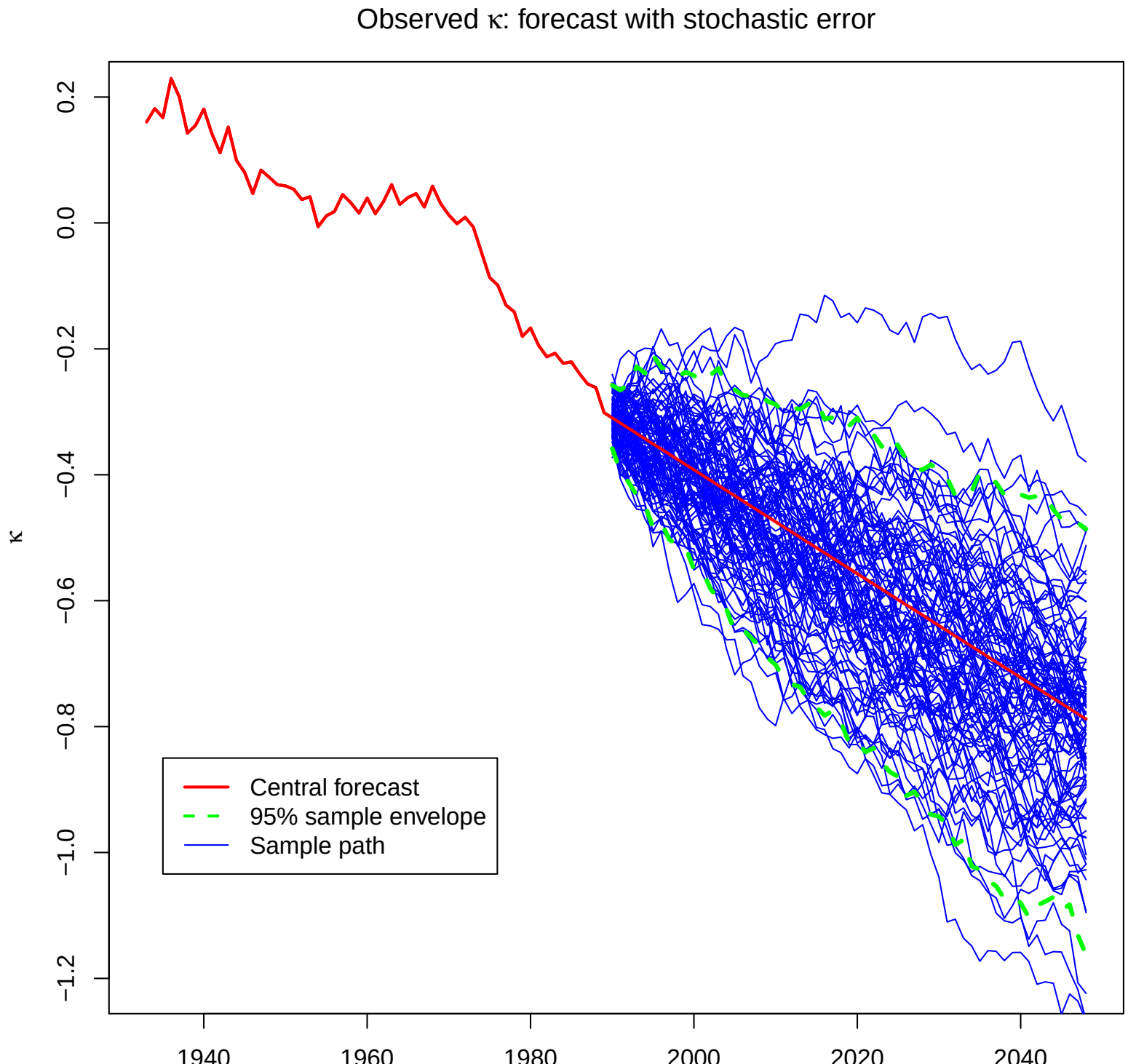
κ_j : time index

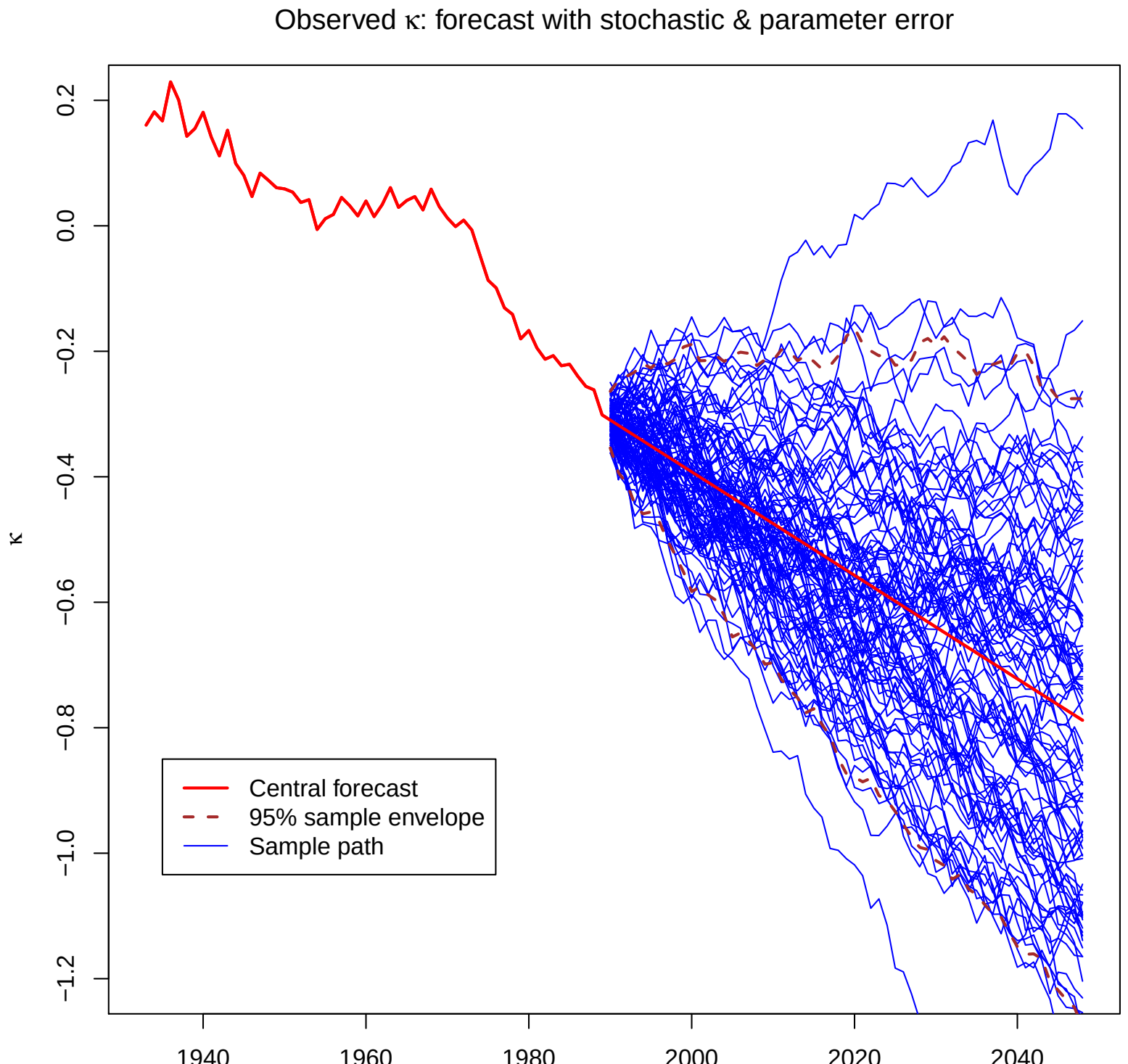
β_i : modulates time index for each age

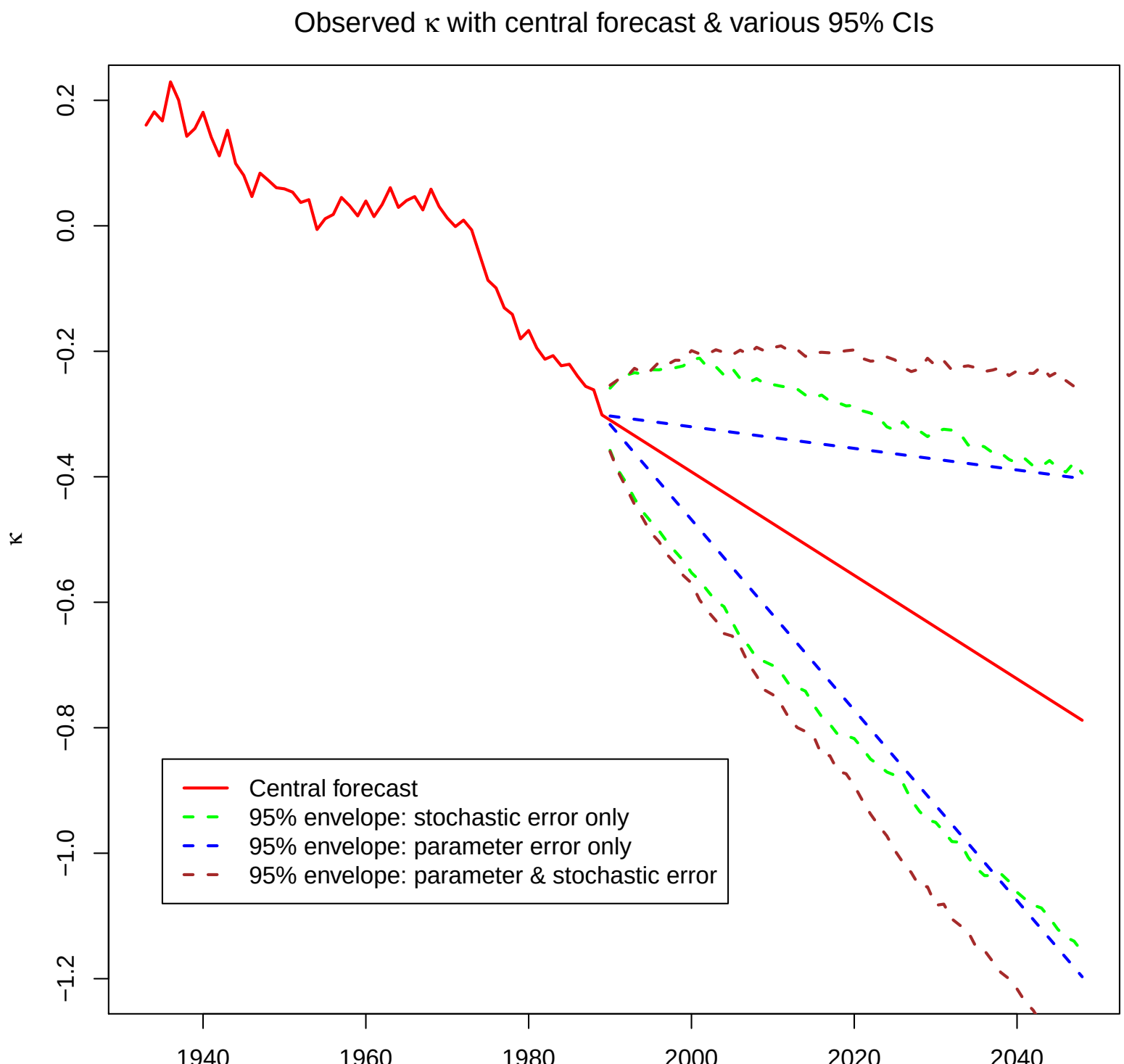

US data, 1933−1989, ages 60−90, fitted κ
κ
0.2
0.0
−0.2
−0.4
−0.6
−0.8
−1.0
−1.2
1940
1960
1980
2000
2020
2040


Observed κ with random walk with drift forecast
κ
0.2
0.0
−0.2
−0.4
−0.6
−0.8
−1.0
−1.2
1940
1960
1980
2000
2020
2040
Central forecast
95% CI


Observed κ: forecast with parameter error
κ
0.2
0.0
−0.2
−0.4
−0.6
−0.8
−1.0
−1.2
1940
1960
1980
2000
2020
2040
Central forecast
95% sample envelope
Sample path


Observed κ: forecast with stochastic error
κ
0.2
0.0
−0.2
−0.4
−0.6
−0.8
−1.0
−1.2
1940
1960
1980
2000
2020
2040
Central forecast
95% sample envelope
Sample path


Observed κ: forecast with stochastic & parameter error
κ
0.2
0.0
−0.2
−0.4
−0.6
−0.8
−1.0
−1.2
1940
1960
1980
2000
2020
2040
Central forecast
95% sample envelope
Sample path


Observed κ with central forecast & various 95% CIs
κ
0.2
0.0
−0.2
−0.4
−0.6
−0.8
−1.0
−1.2
1940
1960
1980
2000
2020
2040
Central forecast
95% envelope: stochastic error only
95% envelope: parameter error only
95% envelope: parameter & stochastic error

Conclusions

Dealing with Longevity Risk

- Basis risk: use piggyback models
- Model risk: use multiple models
- Parameter & stochastic risk: use sample paths

References

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