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Towards an Industry Standard to Assess Longevity Basis Risk

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Where the project came from

Longevity Basis Risk Working Group

LLMA Life & Longevity Markets Association

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- Formed in Dec 2011
- Focused on market-friendly means of analysing basis risk
- ITT to sponsor in-depth yet practical research in Feb 2013

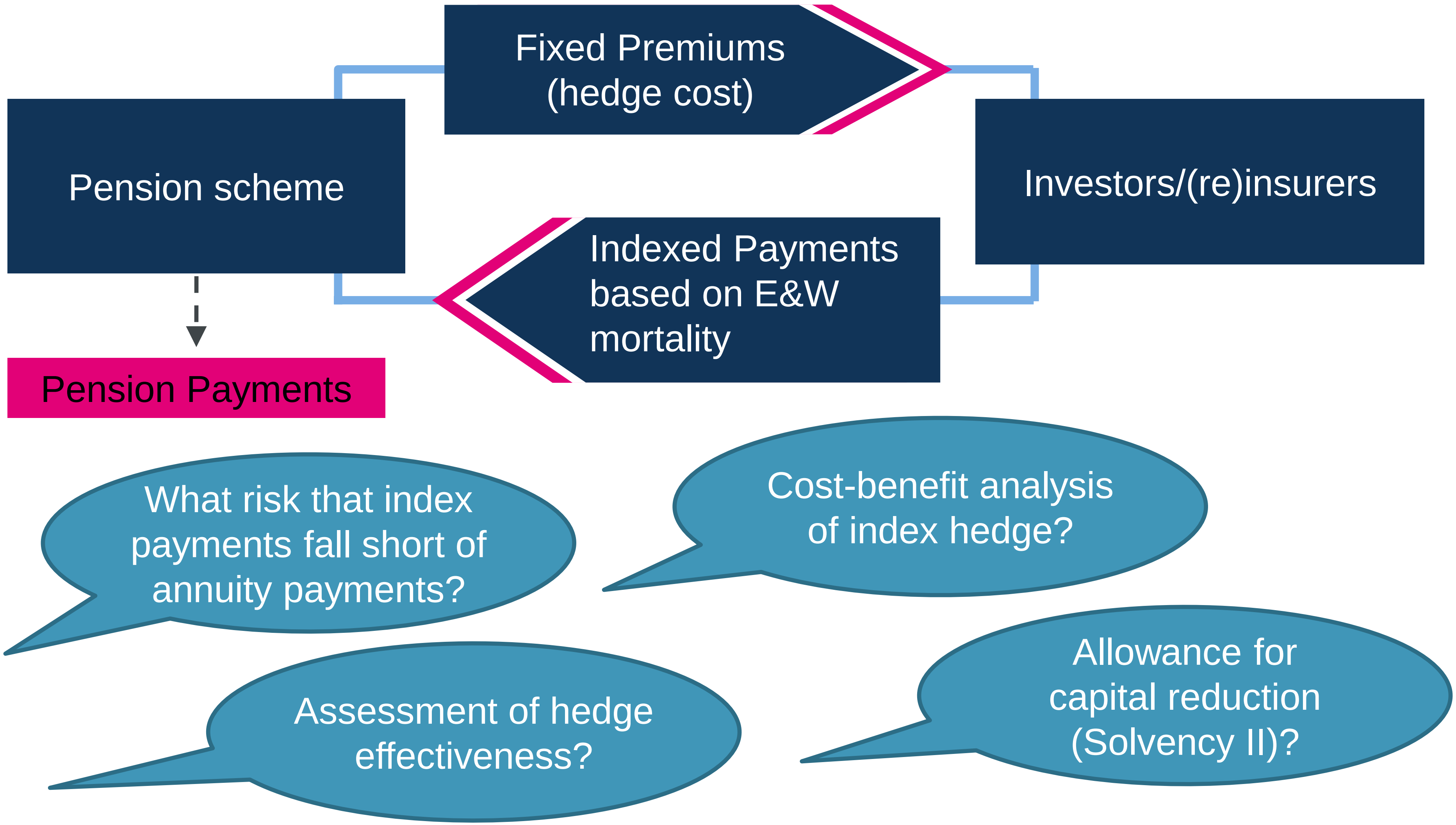
Joint Research Group

Cass Business School CITY UNIVERSITY LONDON

HYMANS ROBERTSON The Spirit of Independence

- Sponsored by LBRWG following selection process
- Formally appointed Oct 2013
- Output will be made publicly available

Introduction:
The Basis Risk Question



Longevity Basis Risk Working Group

What is LBRWG trying to achieve through sponsoring this project?

Life & Longevity Markets Association

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- The biggest challenge is how to model the demographic risk.
- Given the typical inputs for a pension scheme or annuity book:

Pre- Hedge Overlay
Target Population Size – Number of individuals
Target Population Annuity/Pension Amounts
Geographic location
Historical mortality experience information if available

- How do we simulate the two populations? (Hedge and portfolio)
- How are their mortality diffusions related?
- If $\mu_R(x, t)$ is the force of mortality for E&W, we need to generate $\mu_B(x,t)$ (mortality for the pool). What form should $\mu_B(x,t)$ take?

Joint Research Group

How is it tackling the project?

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- Review existing research on different trends (and baseline) for various sub-populations
- Extend trend research (multivariate analysis)

Understand past dynamics
- informs choice / structure of model

- Review existing models
- Criteria for "good model"
- Review models vs criteria

Propose a practical (stochastic) model for multiple populations
- including example parameterisation

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Trends (and baseline) for sub-populations

Wide variations in baseline longevity

Widely known

Geographical[1]	Glasgow City 14.3	Kensington and Chelsea 24.4
Social Class[2]	Routine & Manual 15.8	Higher Managerial 18.8
Deprivation[3]	Most Deprived 15.7	Least Deprived 19.8
Gender[1]	Male 18.0	Female 20.6

Source: [1]Life expectancy at birth and at age 65 by local areas in the United Kingdom, 2004-06 to 2008-10 (ONS, 2011). [2]ONS Longitudinal Study (Johnson, 2011). [3]Inequality in Disability-Free Life Expectancy by Area Deprivation: England, 2003–06 and 2007–10 (ONS,2013)

Variation in life expectancy from age 65 can be as high as 10 years

Annuitant specific (Club Vita)

Lifestyle +4 years

Affluence +4 years

Ret health — +2 years (normal vs. ill health retiree)

Job — <1 year (manual vs. non-manual job)

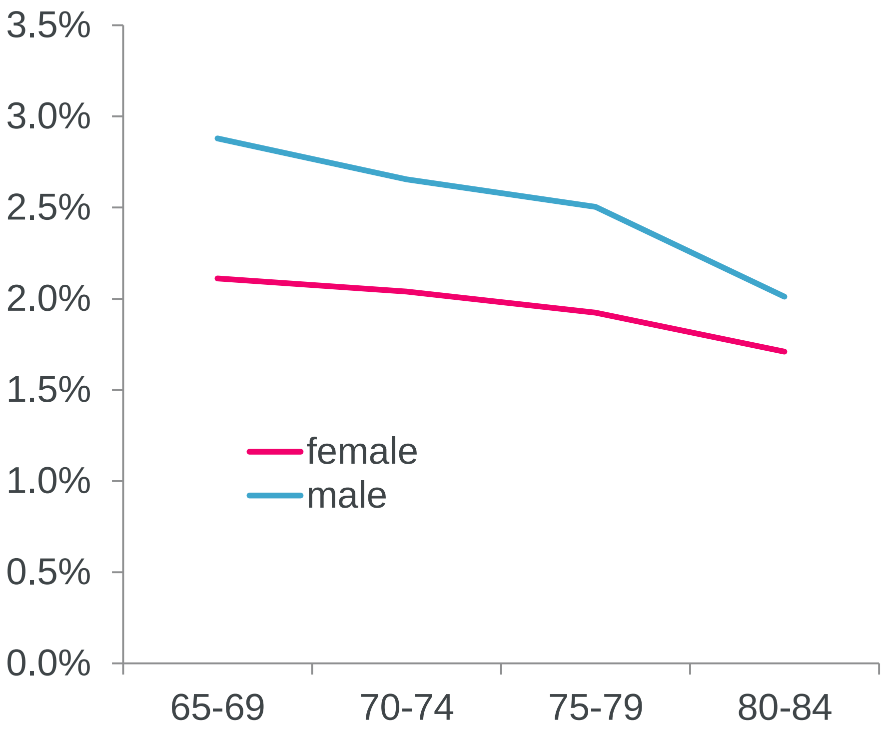
Attribution of 10 years difference in period life expectancy from age 65

Wide variation in life expectancy, but well understood by industry practitioners

Clear differences in improvements (1)

- Gender
- Deprivation
- NS-SEC

Annual rate of improvement in England and Wales by gender (1981-2011)

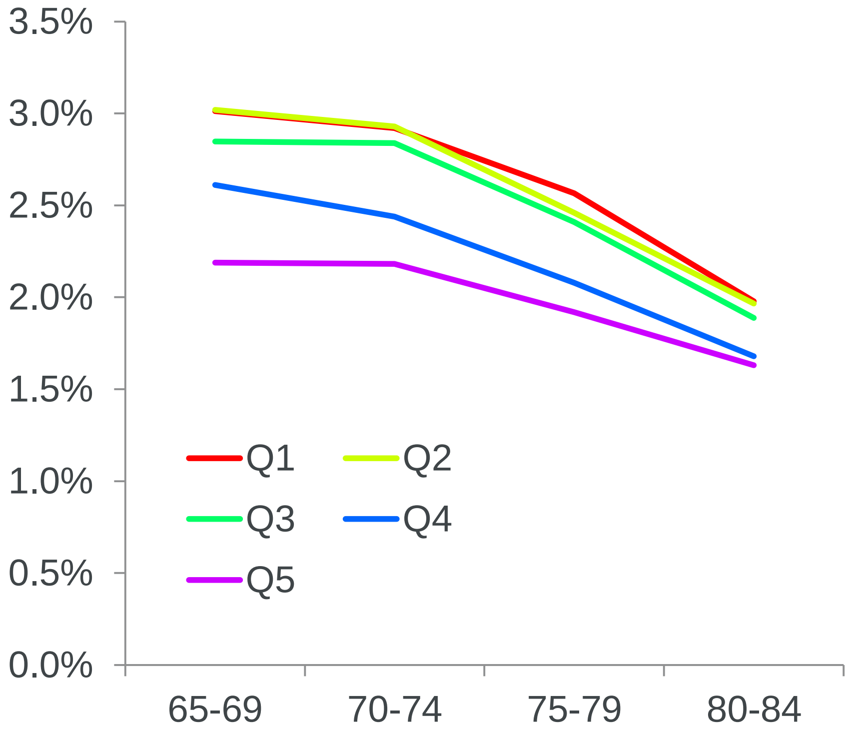


Source: Own calculations based on HMD data

Clear differences in improvements (2)

- Gender
- Deprivation
- NS-SEC

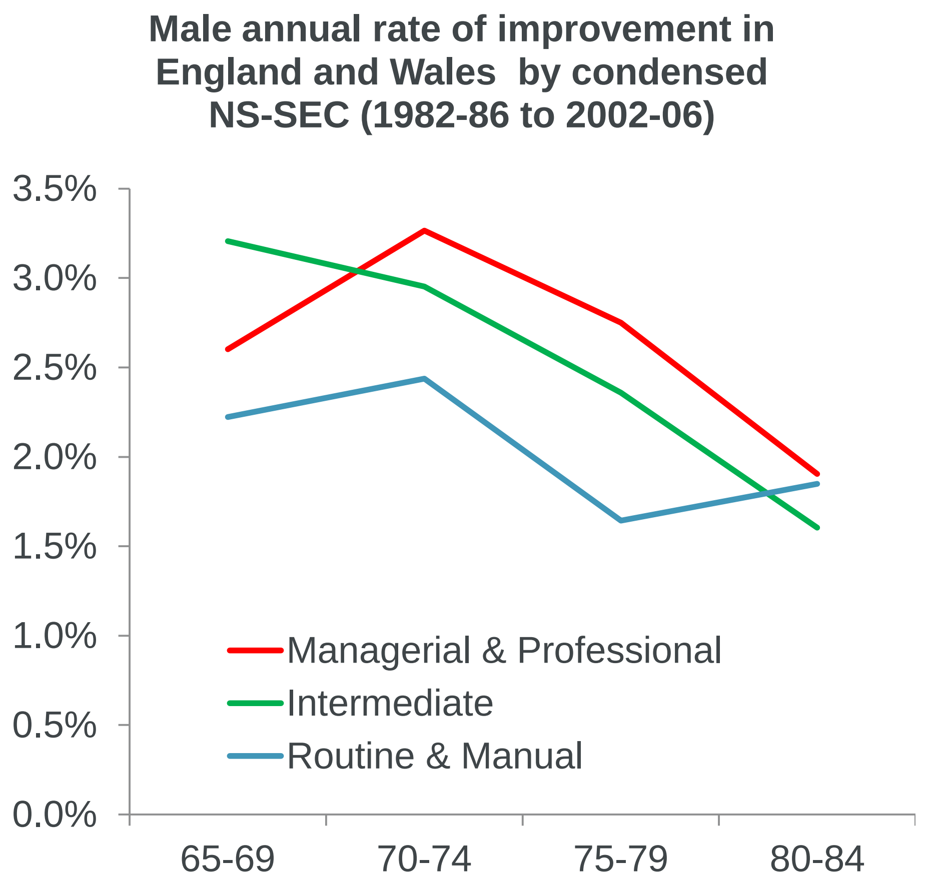
Male annual rate of improvement in England by deprivation quintile (1982 to 2006)



Source: Based on Table 1 in Lu et al (2013)

Clear differences in improvements (3)

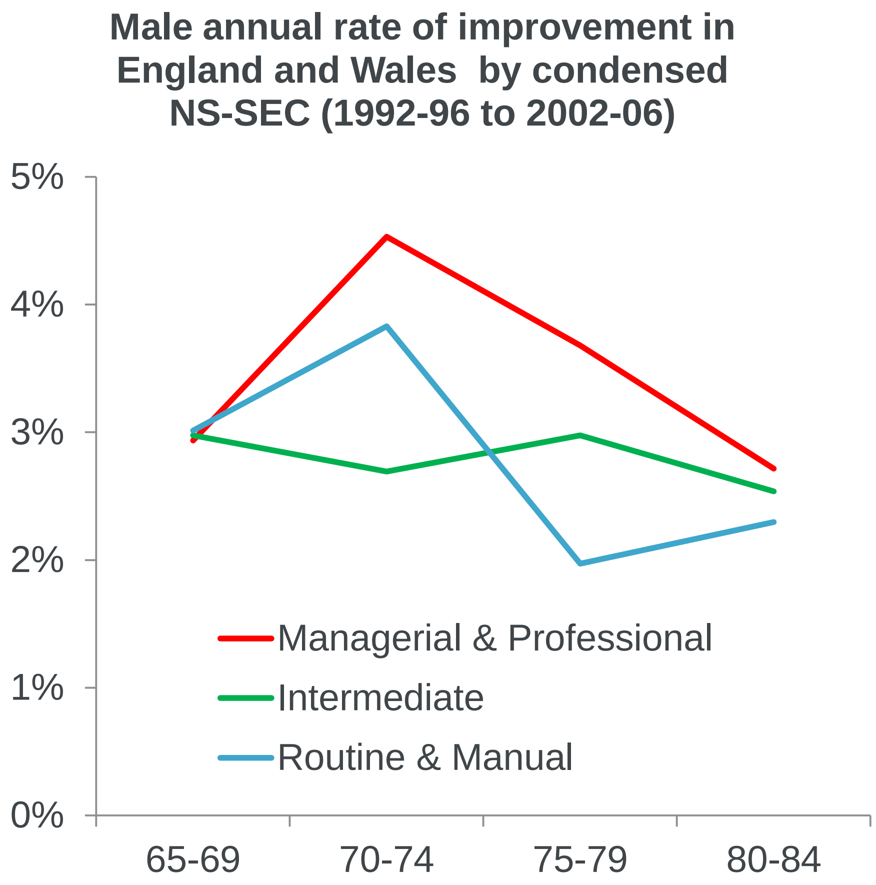
- Gender
- Deprivation
- NS-SEC



Source: ONS Longitudinal Study - Own calculations based on ONS. (2013)

Clear differences in improvements (4)

- Gender
- Deprivation
- NS-SEC



Source: ONS Longitudinal Study - Own calculations based on ONS. (2013)

Key predictors of historic improvements
(GLM analysis of improvements using Club Vita data)

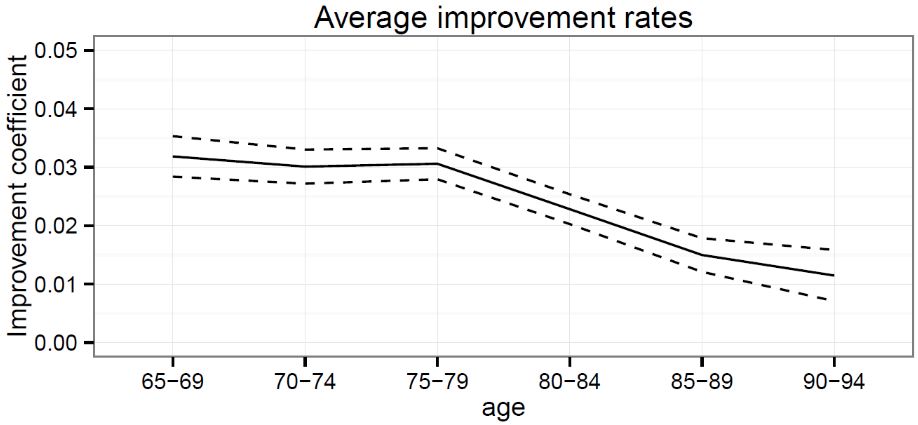
Predictor	Men	Women
Postcode (IMD) + Pension amount	1	1=
Postcode (IMD)	2	1=
Postcode (lifestyle) + Pension amount	4	3
Postcode (lifestyle)	6	4
Pension amount	3	5
"No specific improvements predictor"	5	6

Best balance between fit and simplicity

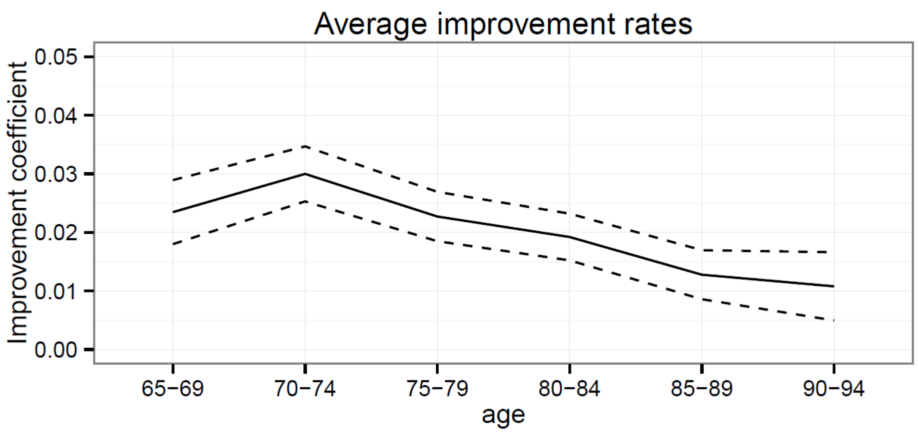
Consistent results for men and women. Postcode (IMD) and pension amount are key predictors of historic improvements.

Key predictors: Age effect

Men



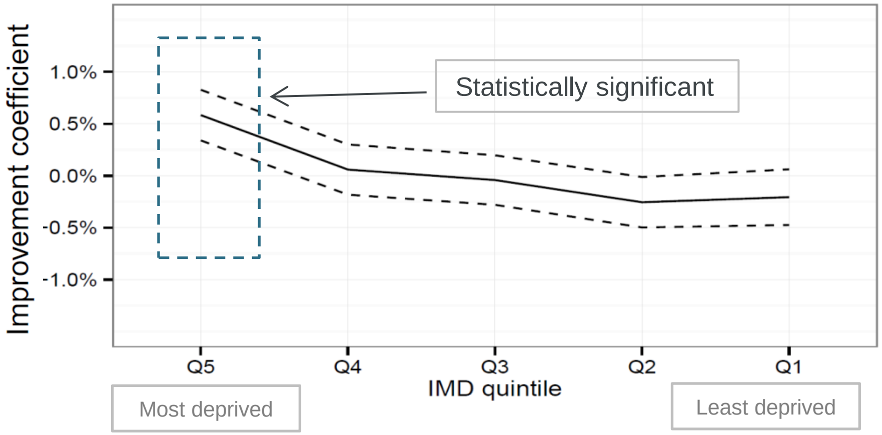
Women



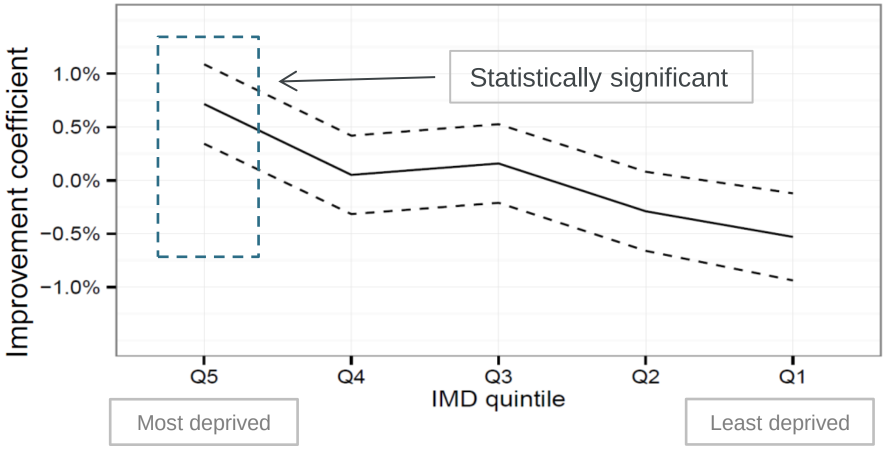
- No surprises – improvements generally decline with age.
- Significant non-zero improvements at the top ages (~1% p.a.).
- Improvements generally lower for women than men.

Key predictors: Postcode (IMD)

Men



Women

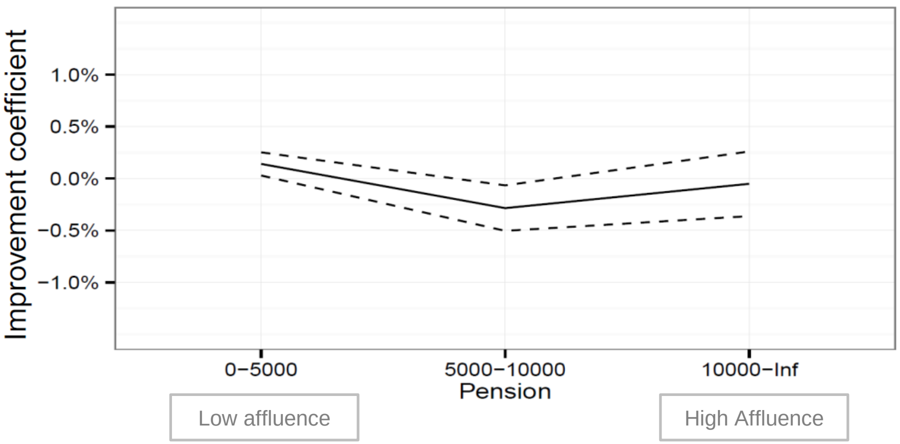


Impact of IMD on improvement rate (controlling for affluence)

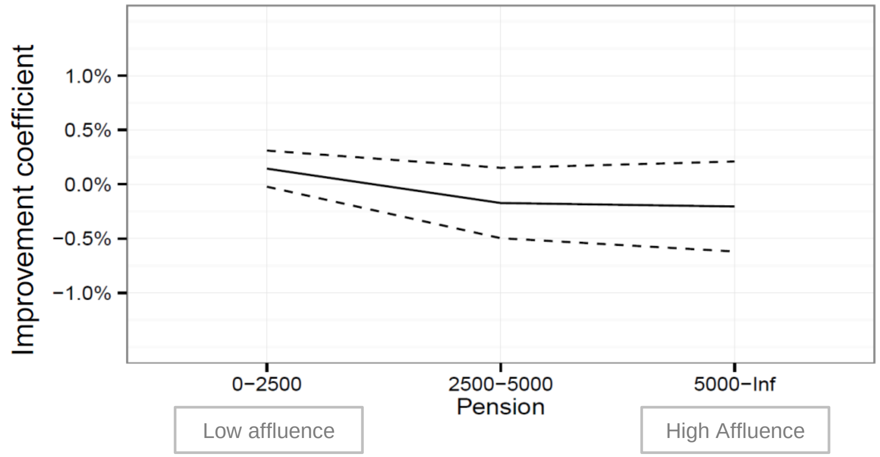
- In general, pensioners living in the most deprived areas have seen significantly faster improvements.
- Pensioners in less deprived areas have very stable and similar improvement rates (especially amongst men).

Key predictors: Affluence (Pension)

Men



Women

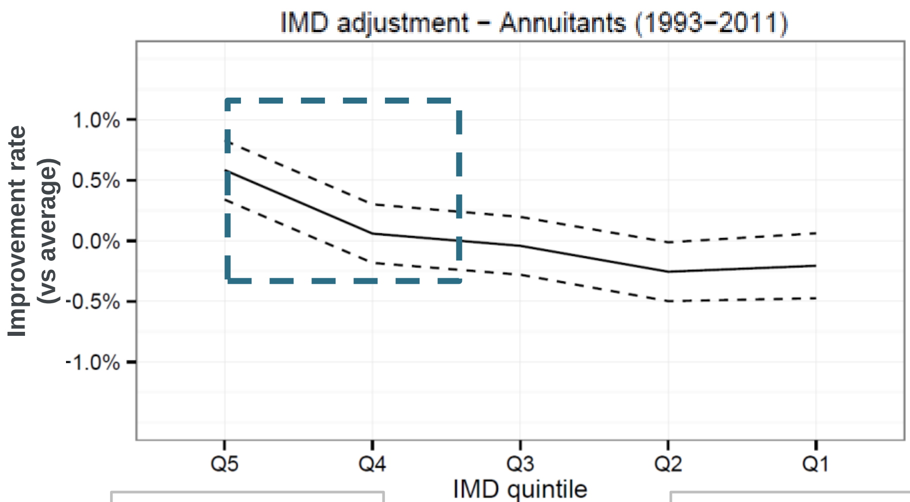


Impact of affluence on improvement rate (controlling for IMD)

- Impact of affluence more modest than deprivation
- Improvements appear to have a ‘smile’ effect
- Weak differences in improvements between different affluence bands (especially for women).

The deprivation paradox

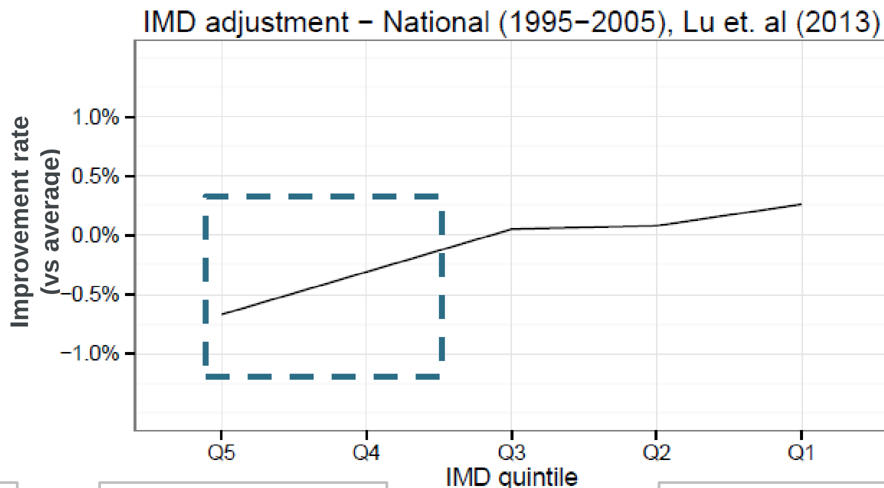
Improvements by IMD (pensioners)



Most deprived

Least deprived

Improvements by IMD (national)



Most deprived

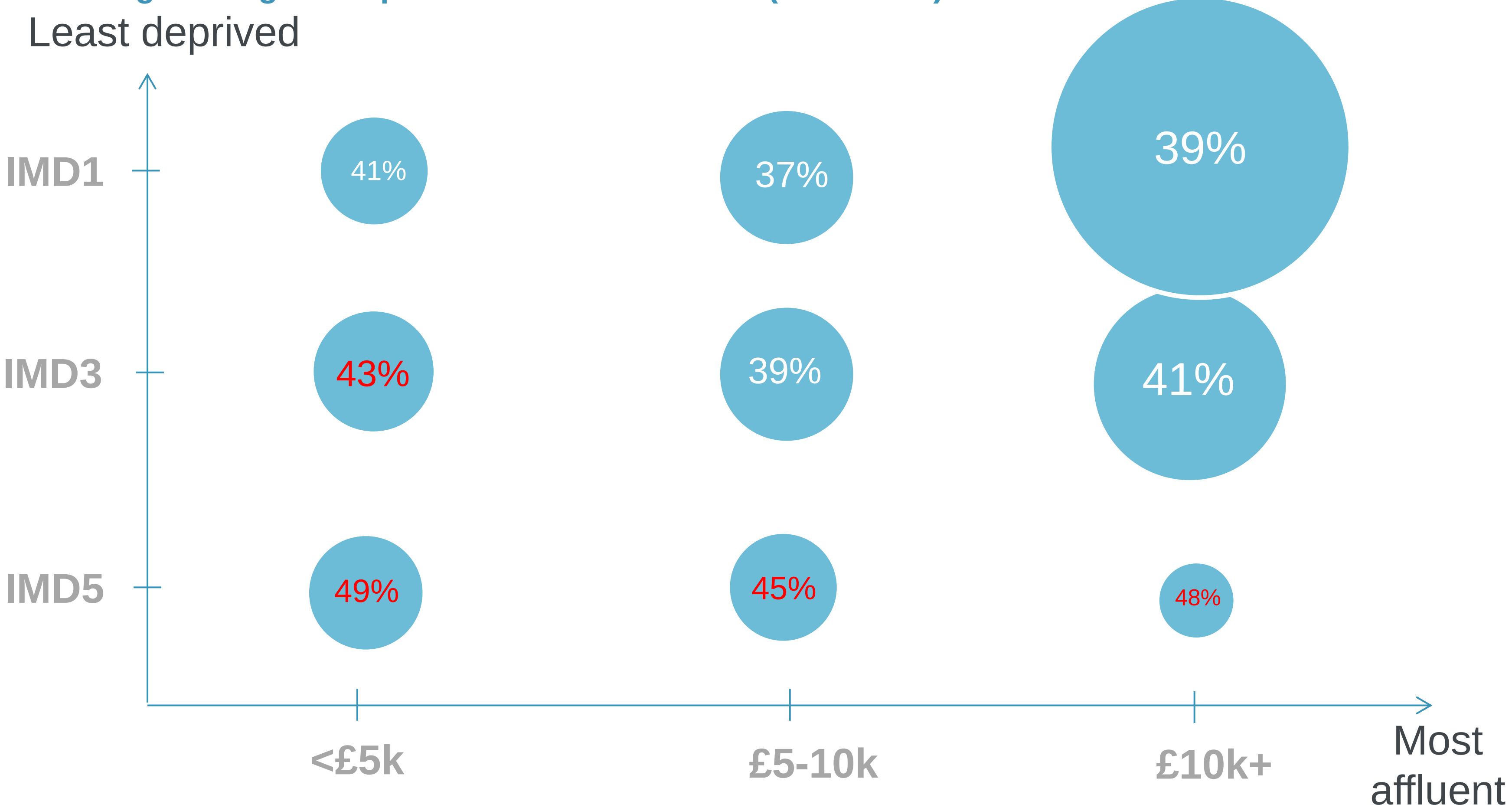
Least deprived

- **Select effect:** Annuitants are likely to differ from the average person in their area, particularly in most deprived quintiles

Models calibrated to national IMD data could be misleading in the management of pension scheme basis risk

Material impact of basis risk

Average change in improvement rates for men (1993-2011)



Over last 18 years 'middle England' saw mortality improve by 42%
The range across pensioners was between 36% and 49%

Different circle sizes refer to relative amount of pension for each socio-economic group.

Key conclusions from research on trends

- Difference by SEC
  - Material differences in trends by SEC – as big as impact of gender
- Key predictors
  - IMD (via postcode) has strongest link to past improvements.
  - Pension has second strongest link.
- Deprivation paradox
  - IMD effect very different to that in whole population (selection effect).
  - Should not parameterise using whole UK IMD data.
- Basis risk matters
  - IMD / pension combination has big impact on improvements
  - 22% to 32% improvement per decade, vs. 26% average UK.

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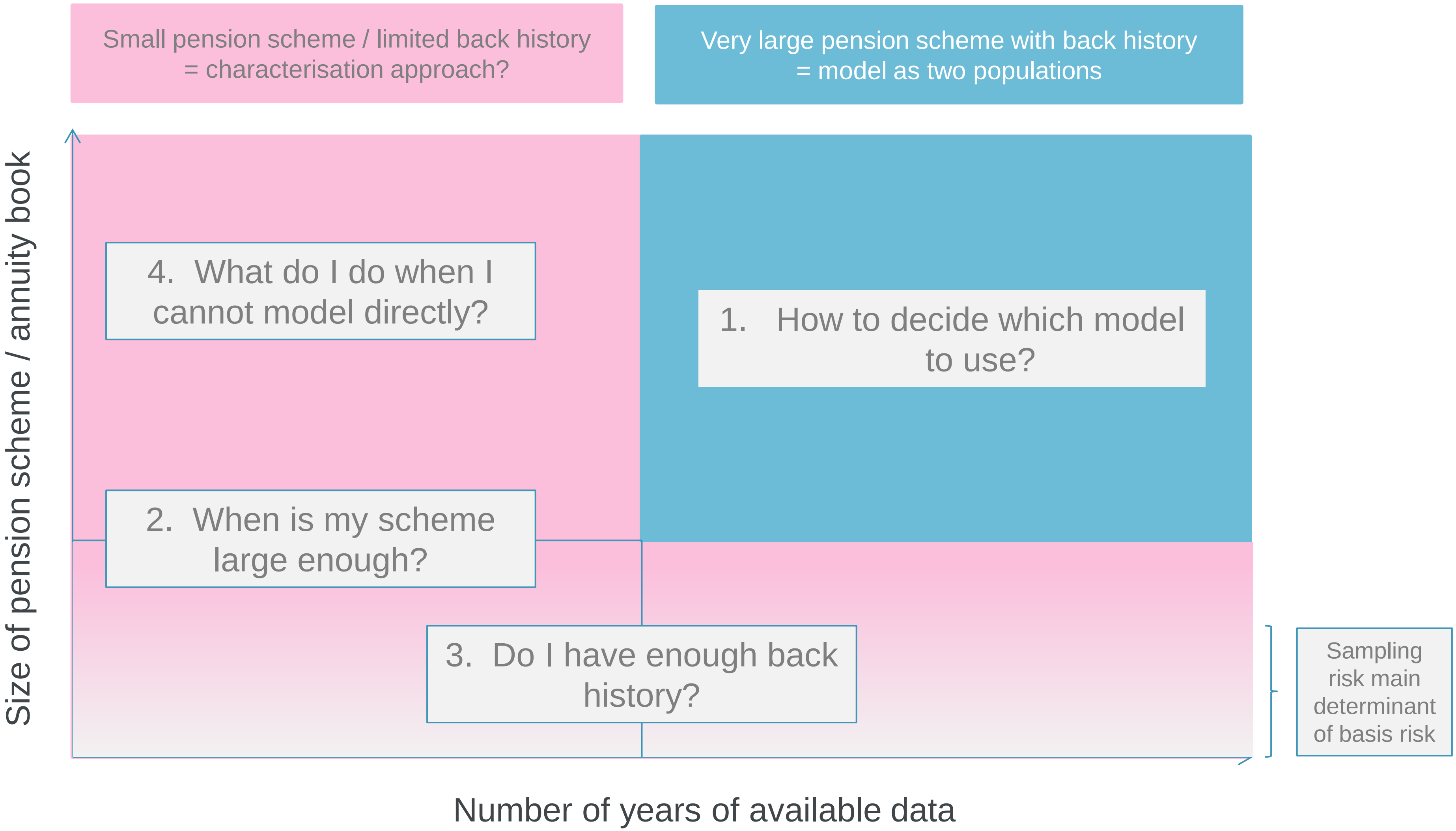
Towards a framework for longevity basis risk

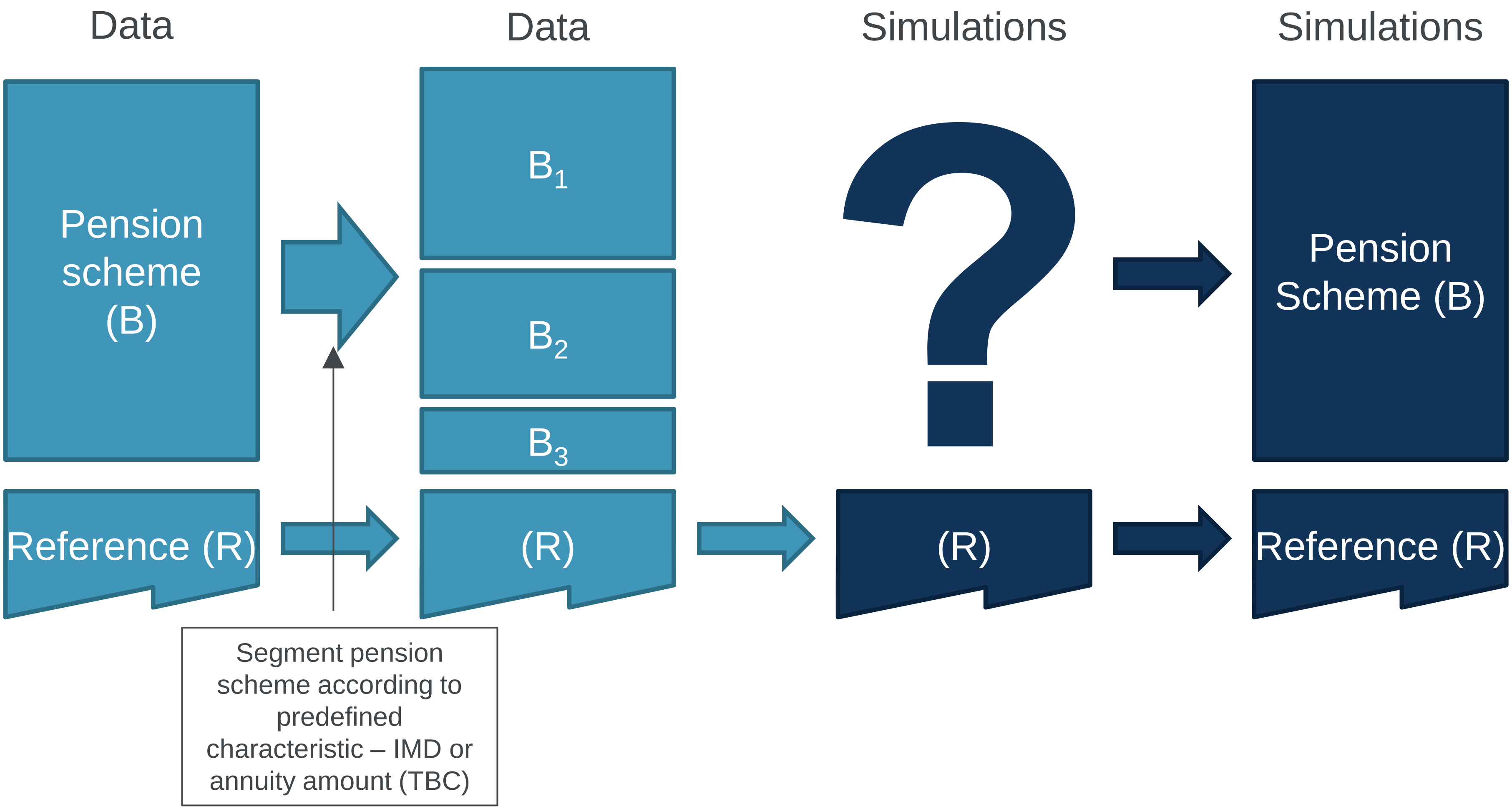
What risk that index payments fall short of pension payments?

Cost-benefit analysis of index hedge vs indemnity hedge?

Developing a practical framework

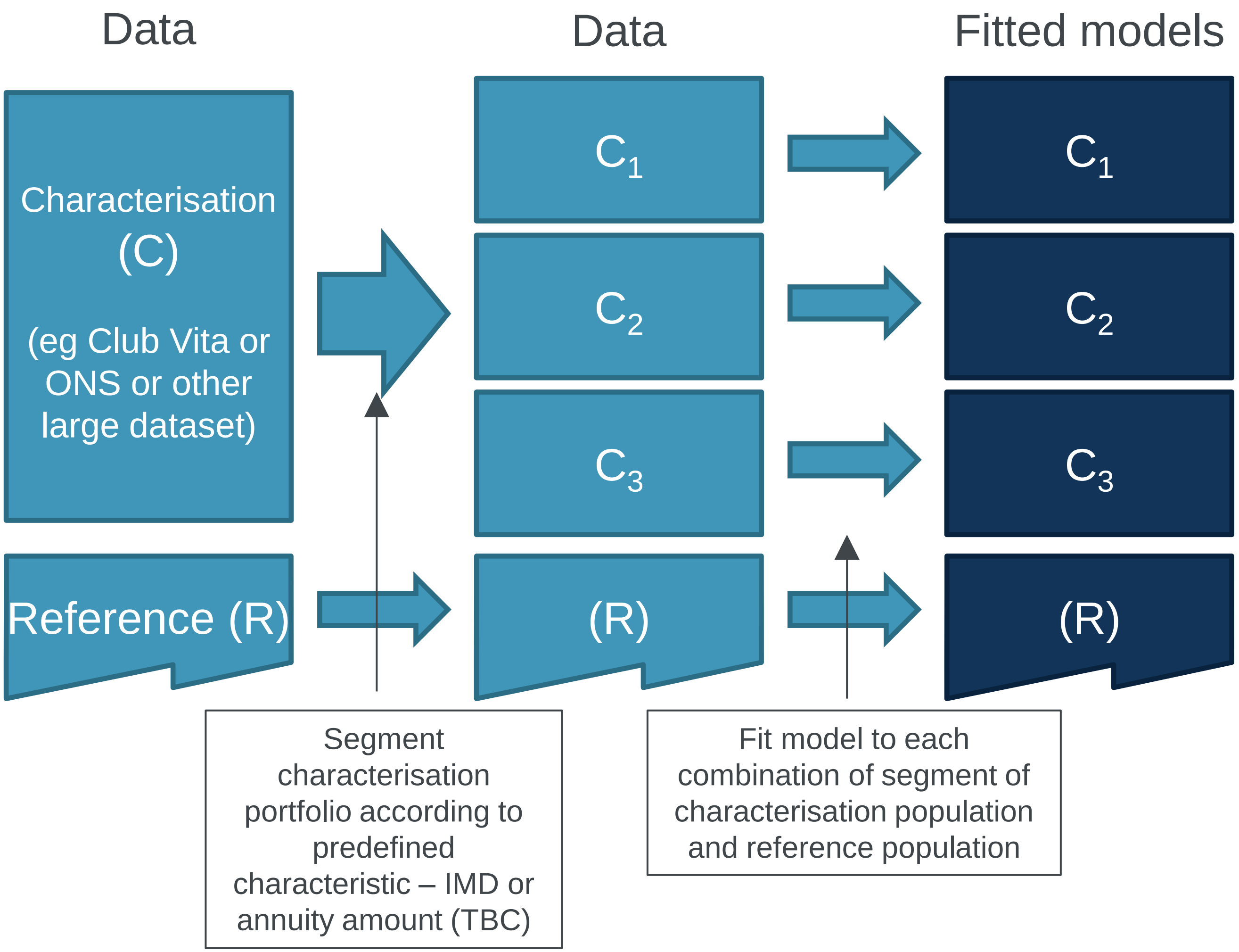


What happens when I cannot model directly? *Characterisation approach*

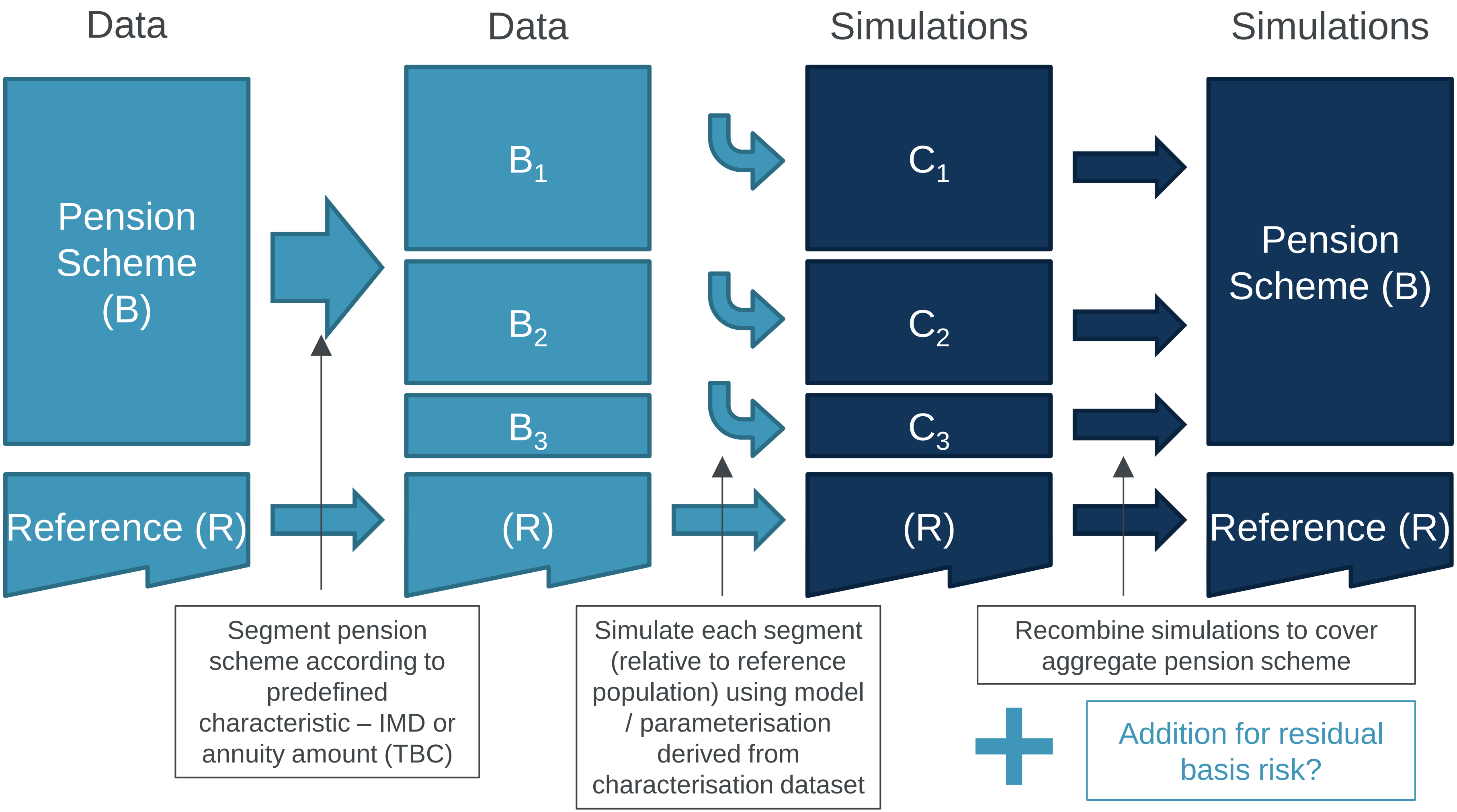


Characterisation approach

Applying direct modelling to the characterising groups



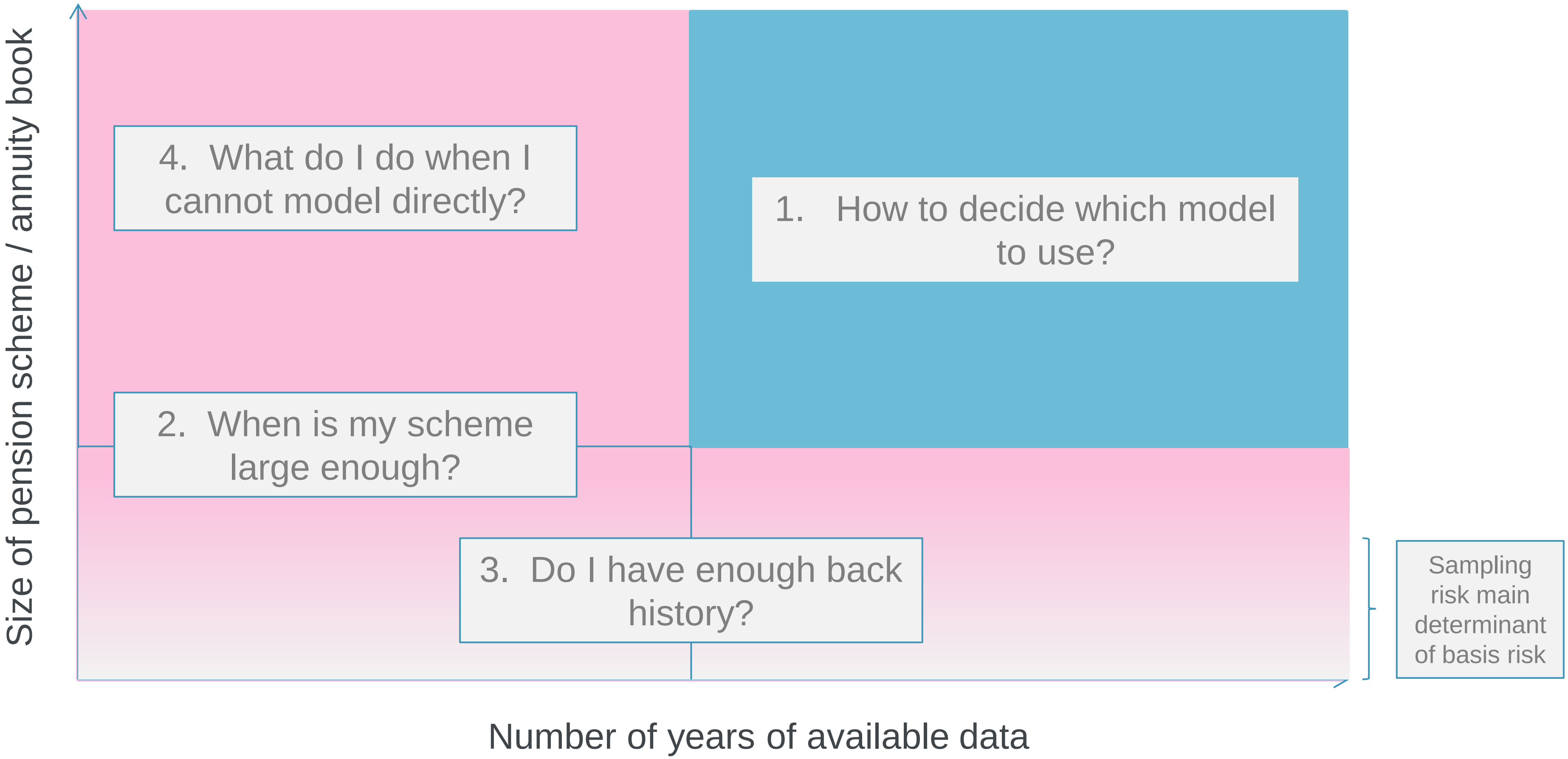
Calculating basis risk for smaller schemes



Developing a practical framework

Small pension scheme / limited back history
= characterisation approach?

Very large pension scheme with back history
= model as two populations



Initial analysis of choice of model

What risk that index payments fall short of annuity payments?

Assessment of hedge effectiveness?

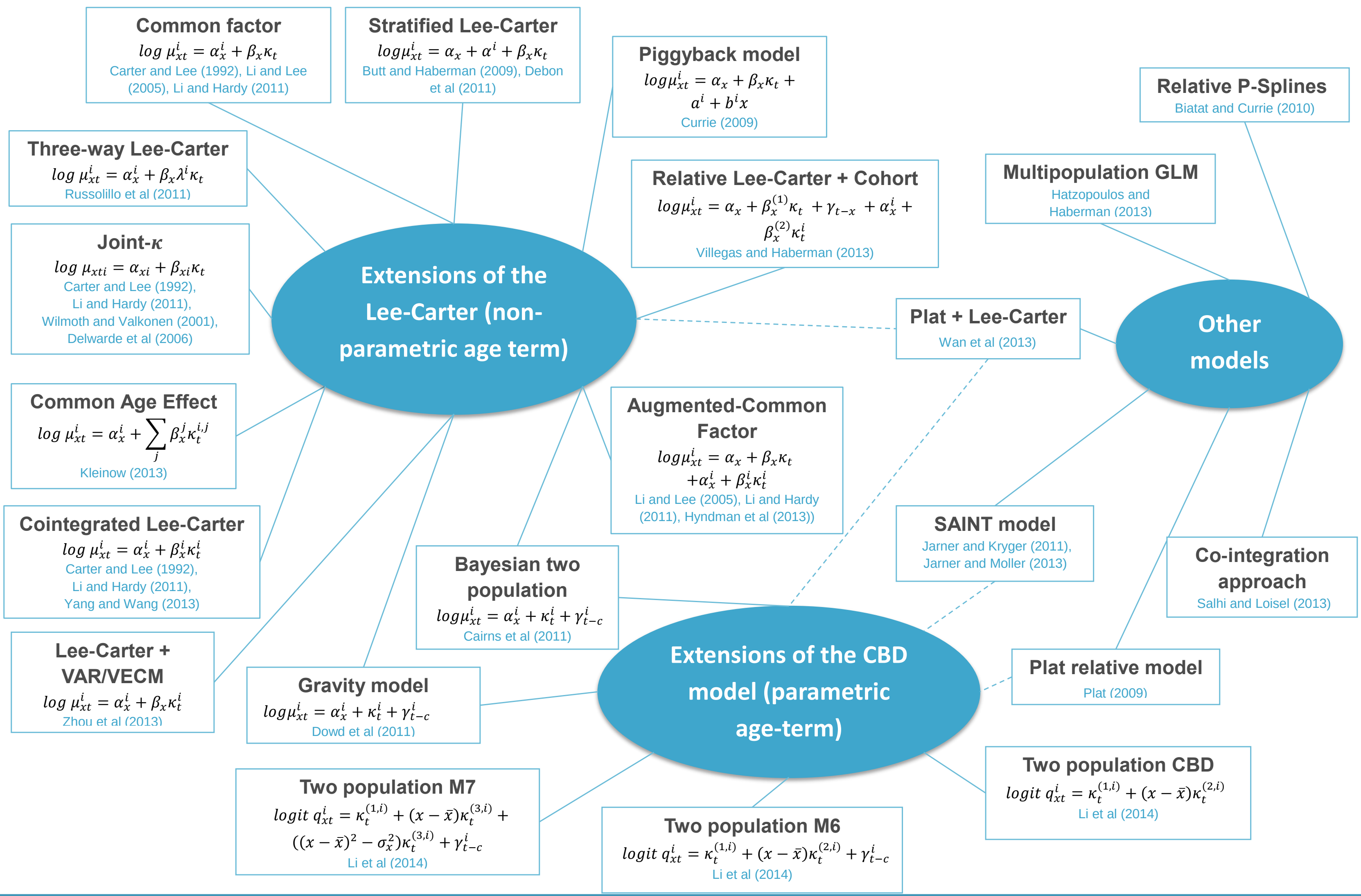
Cost-benefit analysis of index hedge?

Allowance for capital reduction (Solvency II)?

Narrowing down the (long) list of possible models

- To answer key questions requires a model
- Huge selection to choose from
- Define criteria for "good" ***practical*** model
- Review existing models vs. those criteria
  - Work in progress!
  - Example included today

The landscape of two population models



Assessment criteria for single population models

CMI Working paper 25, Cairns et al. (2008, 2009), Haberman and Renshaw (2011)

Practical

- Ease of implementation
- Transparency
- Simple (Parsimony)

Central estimates (Deterministic)

- Cohort effect
- Goodness of fit of rates
- Reasonable central projection of rates

Risk assessment (Stochastic)

- Generate sample paths
- Reasonable forecast levels of uncertainty of rates
- Incorporate parameter uncertainty in simulations
- Non-perfect correlations between year on year changes in mortality at different ages

Assessment criteria for **two population models**

Practical

- Ease of implementation
- Compatible with available data
- Handle portfolio heterogeneity
- Transparency
- Simple (Parsimony)
- Disentangle baseline and improvement differences

Central estimates (Deterministic)

- Cohort effect
- Goodness of fit of rates and rate differences
- Reasonable central projection of rates and rate differences

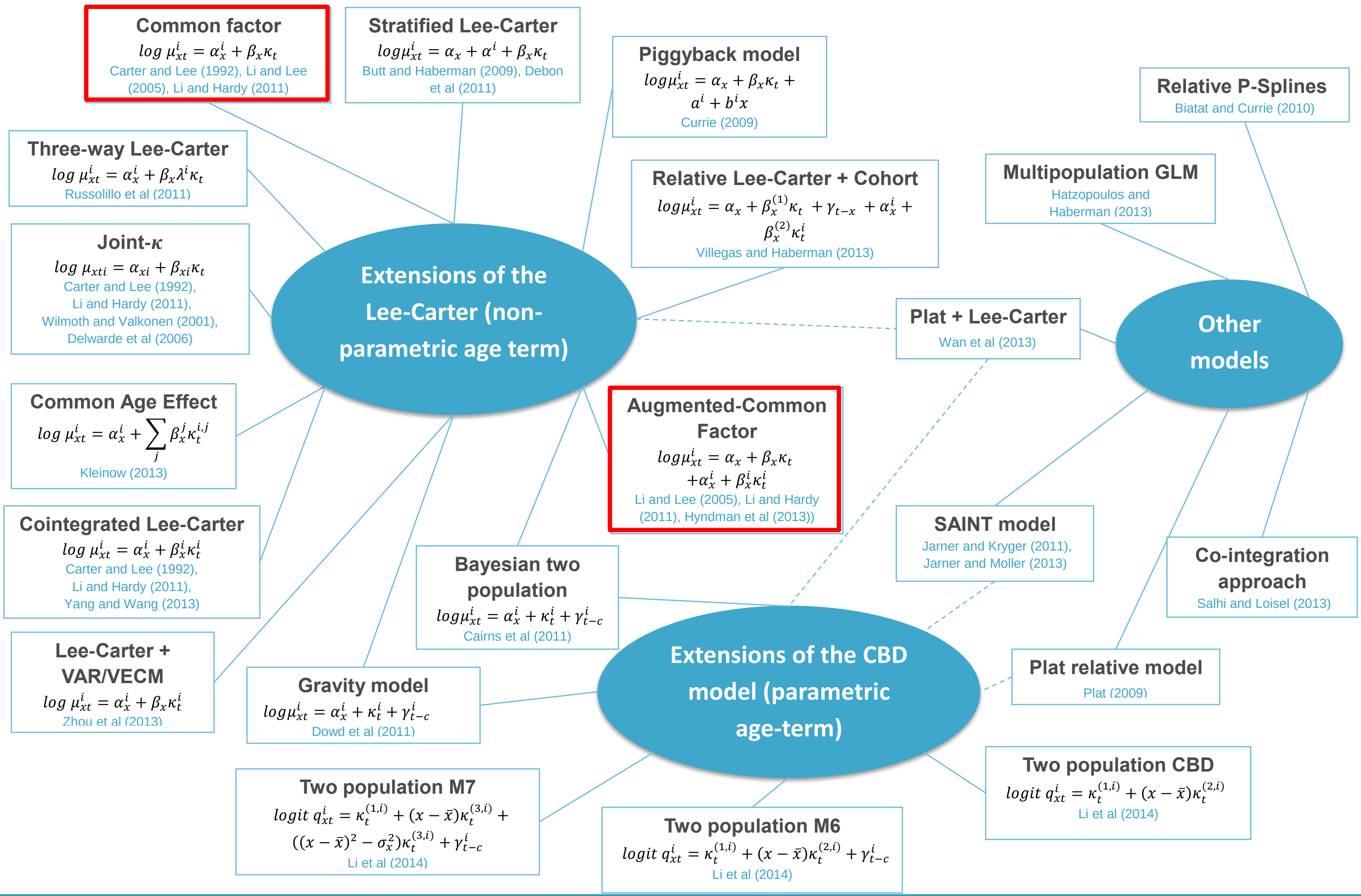
Risk assessment (Stochastic)

- Generate sample paths
- Reasonable forecast levels of uncertainty of rates and rate differences
- Incorporate parameter uncertainty in simulations
- Non-perfect correlations between year on year changes in mortality at different ages
- Non-perfect correlations between mortality rates in the two populations

High level critique of existing models

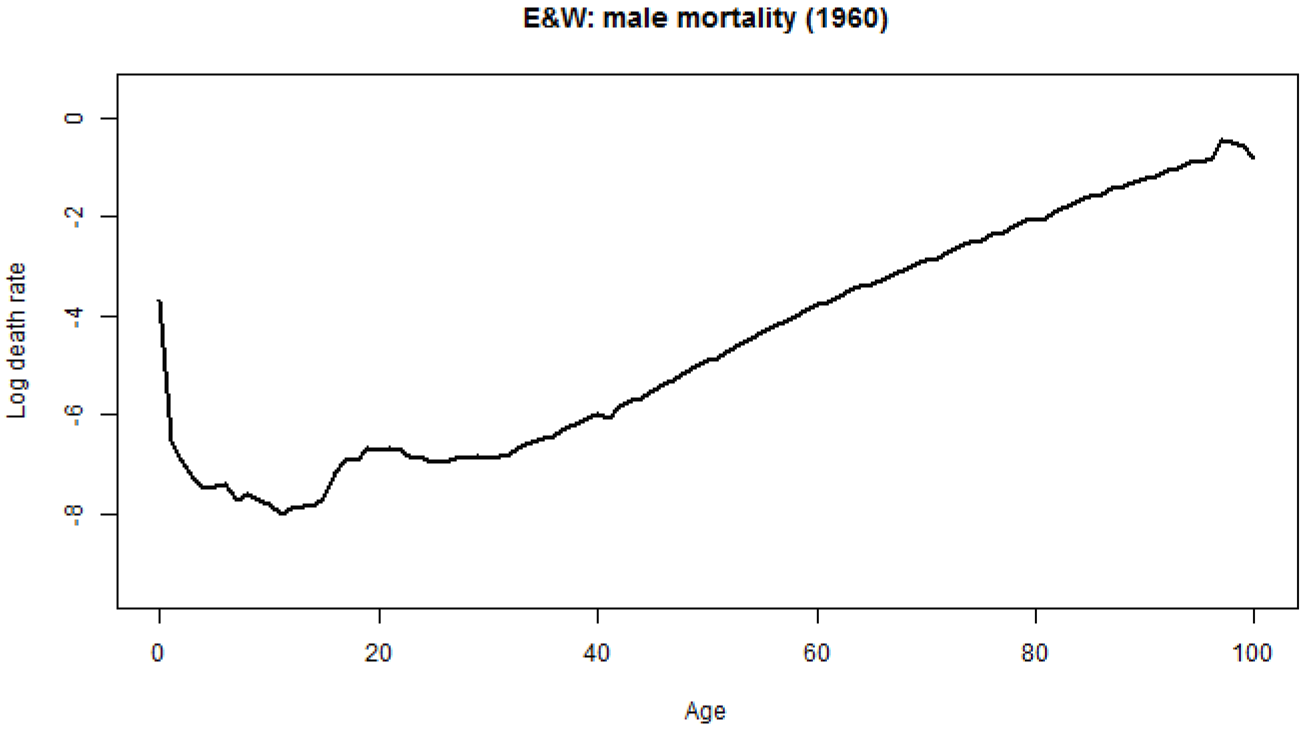
- **Practical vs. Risk Assessment:**
  - Some models imply perfect correlations so will materially underestimate basis risk and so overestimate hedge effectiveness (e.g. common factor, joint-κ)
  - Models with non-perfect correlations tend to require a lot of data or are less transparent / easy to use (e.g. augmented common factor, Bayesian 2 populations)
- **Central Estimates?:**
  - Models often assume convergence in improvement rates which are inconsistent with historic observations (e.g. common factor, joint-κ)
- **Practical?:**
  - Models as originally stated do not allow explicitly for covariates to account for heterogeneity in mortality rates / improvements as implied by analysis of Club Vita data

An example: Common factor vs. Augmented common factor



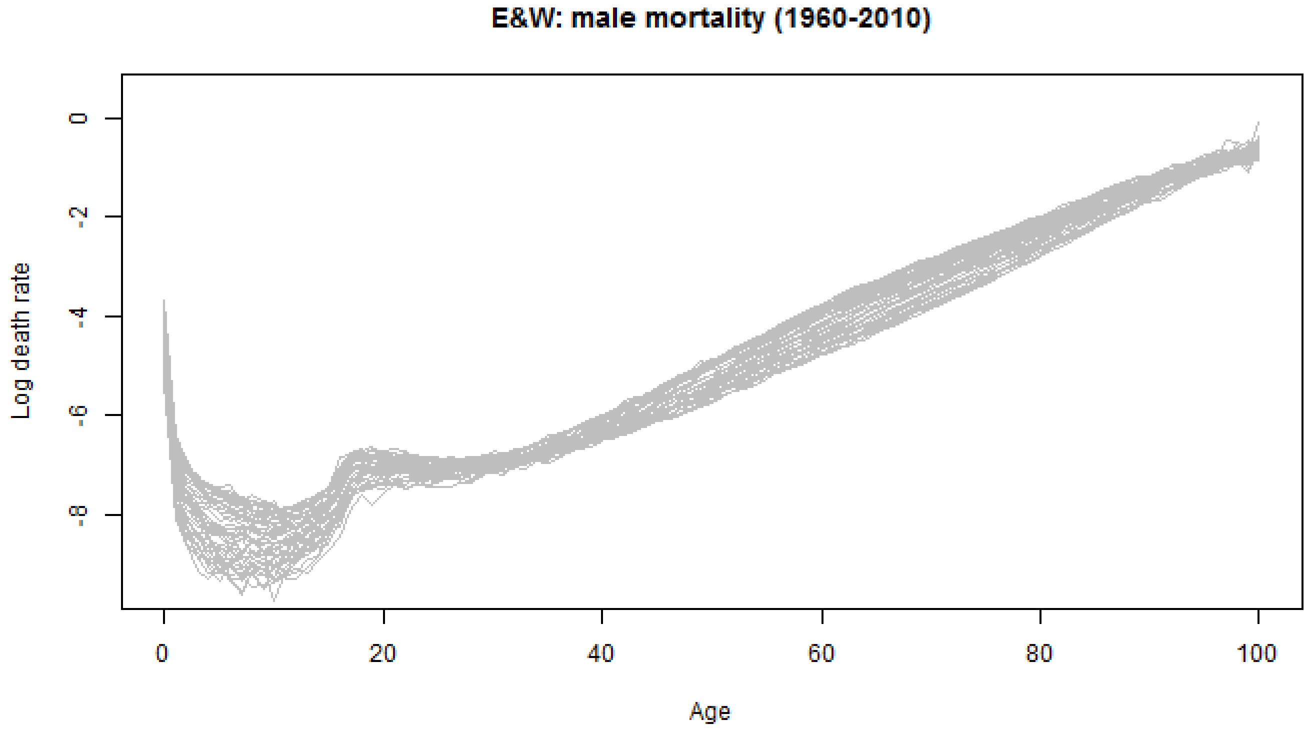
An example

The Lee-Carter Model for one population



An example

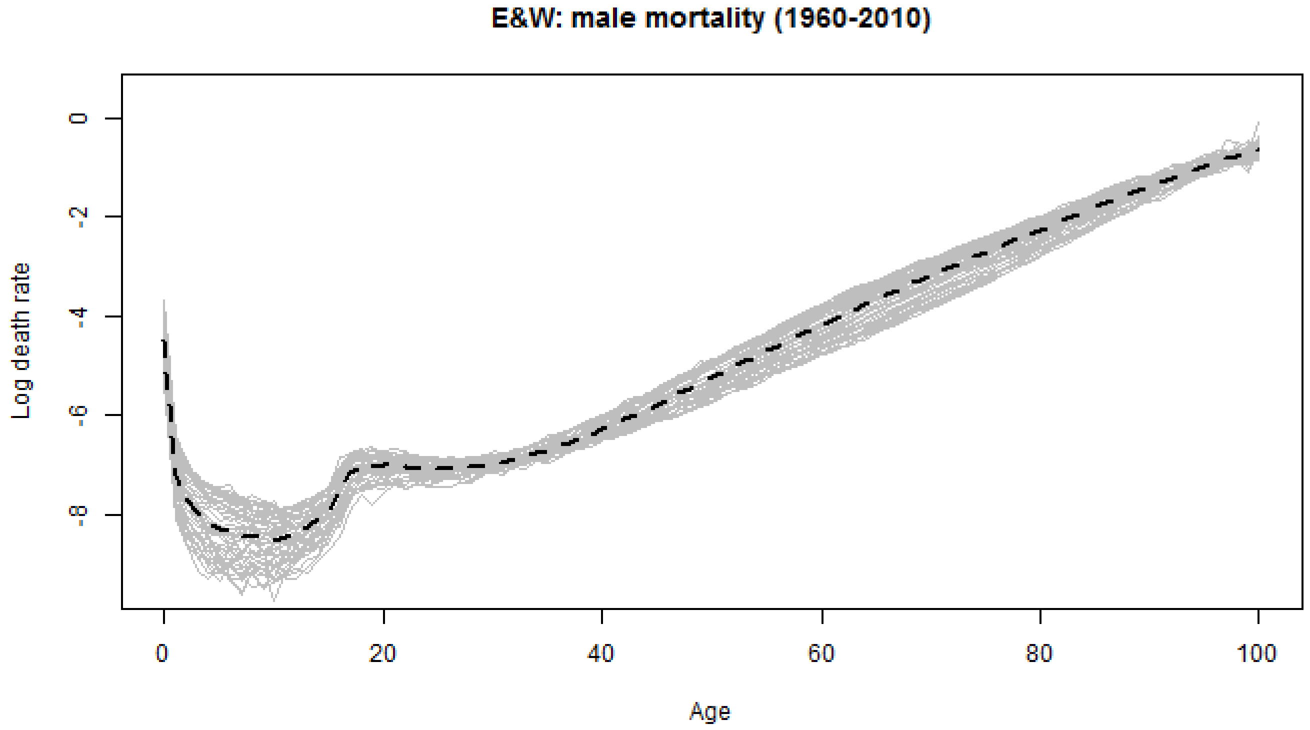
The Lee-Carter Model for one population



$$\log \mu_{xt} \quad =$$

An example

The Lee-Carter Model for one population



$$\log \mu_{xt} = \alpha_x$$

An example

The Lee-Carter Model for one population

$$\log \mu_{xt} \quad = \quad \alpha_x \quad + \quad \kappa_t$$

An example

The Lee-Carter Model for one population

$$\log \mu_{xt} = \alpha_x + \beta_x \kappa_t$$

An example

Common factor vs. Augmented common factor

Reference Population (μ_{xt}^{R})

$$\log(\mu_{xt}^{R}) = \alpha_x^{R} + \beta_x^{R}\kappa_t^{R}$$

$$\kappa_t^{R} = d + \kappa_{t-1}^{R} + \xi_t^{R}$$

Book Population (μ_{xt}^{B})

Common Factor Model

$$\log\left(\frac{\mu_{xt}^{B}}{\mu_{xt}^{R}}\right) = \alpha_x^{B}$$

Baseline differences

Augmented Common Factor Model

$$\log\left(\frac{\mu_{xt}^{B}}{\mu_{xt}^{R}}\right) = \alpha_x^{B} + \beta_x^{B}\kappa_t^{B}$$

Baseline differences

Improvement differences

$$\kappa_t^{B} = \phi_0 + \phi_1\kappa_{t-1}^{B} + \xi_t^{B}$$

An example

England and Wales (Reference) vs. Club Vita (Book)

Common Factor Model

$$log\left(\frac{\mu_{xt}^{B}}{\mu_{xt}^{R}}\right) = \alpha_x^B$$

- **Practical?** Simpler model
- **Practical?** Does not require a long experience, but a relatively big pension scheme
- **Single population central estimates?** Reasonable performance
- **Single population risk assessment?** Reasonable performance

Augmented Common Factor Model

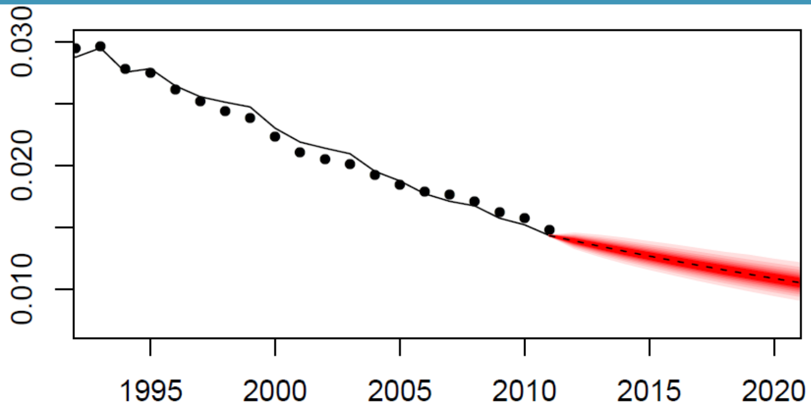
$$log\left(\frac{\mu_{xt}^{B}}{\mu_{xt}^{R}}\right) = \alpha_x^B + \beta_x^B \kappa_t^B$$

- **Practical?** More complicated model
- **Practical?** Requires a longer experience and a bigger pension scheme
- **Single population central estimates?** Reasonable performance
- **Single population risk assessment?** Reasonable performance

Two examples

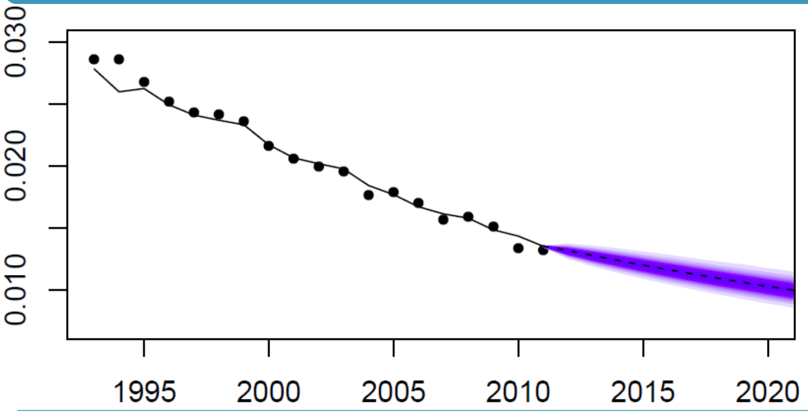
England and Wales (Reference) vs. Club Vita (Book)

Reference Population ($\mu^{R}_{65,t}$)

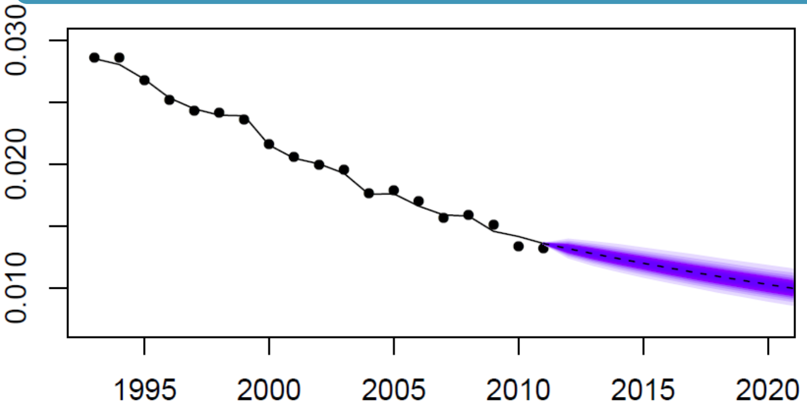


Book Population ($\mu^{B}_{65,t}$)

Common Factor Model



Augmented Common Factor Model



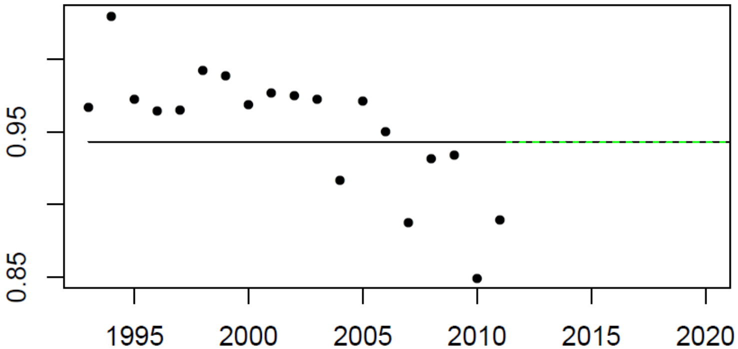
Two examples

England and Wales (Reference) vs. Club Vita (Book)

Common Factor Model

$$log\left(\frac{\mu_{xt}^{B}}{\mu_{xt}^{R}}\right) = \alpha_x^B$$

Book / Reference ($\mu_{65,t}^{B}/\mu_{65,t}^{R}$)

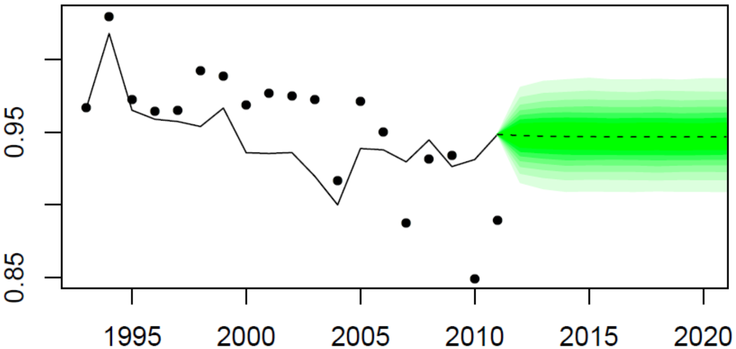


- **Two population central estimates?** Assumes a constant mortality ratio between reference and book
- **Two population risk assessment?:** Assumes a perfect correlation between the populations
- **Conclusion:** Non-adequate for basis risk assessment

Augmented Common Factor Model

$$log\left(\frac{\mu_{xt}^{B}}{\mu_{xt}^{R}}\right) = \alpha_x^B + \beta_x^B \kappa_t^B$$

Book / Reference ($\mu_{65,t}^{B}/\mu_{65,t}^{R}$)



- **Two population central estimates?** Does not assumes a constant mortality ratio between reference and book
- **Two population risk assessment?:** Does not assume a perfect correlation between the populations
- **Conclusion:** Better performance for basis risk assessment but still unsatisfactory

Where we are now – and what's next?

- ✓ Review existing research on different trends (and baseline) for various sub-populations
- ✓ Extend trend research (multivariate analysis)
- ✓ Understand past dynamics - informs choice / structure of model
- ✓ Review landscape of existing models
- ✓ Define criteria for "good model"
- ✓ Review models vs. criteria – shortlist possible models
- ✓ Assess models
  - Stage 1: Core properties
  - Stage 2: Goodness of fit to data and resaonableness
  - Stage 3: Robustness
- Collate strengths & weaknesses / practical considerations of preferred approach(es)
- Develop characterisation approach
- Publish basis risk framework, including example parameterisation

Look out for these results later this year!

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Thank you

Any questions?