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C2: Are Markets Consistent with the Real World? Insurance Liability Valuation Revisited.

Andrew D Smith

Presentation Outline

The Real World and Option Pricing Theory

- Review of option pricing theory – what are we really assuming?
- The Ultimate Forward Rate paradox
- Challenges in credit modelling
- How option pricing methodology has changed since the 2008 crisis
- Linking market prices to subjective probabilities
- Conclusions

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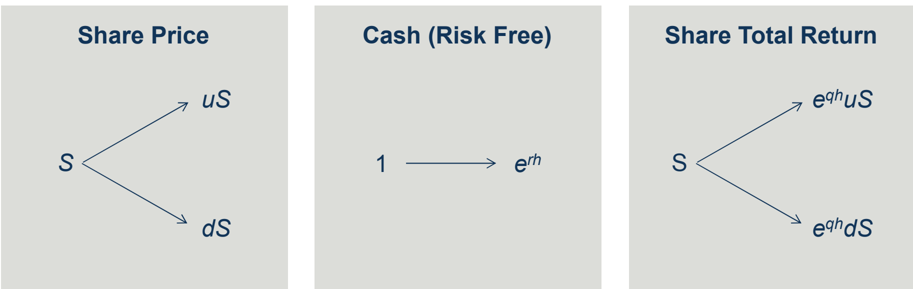
Option Pricing – A Critical Evaluation

Example: Binomial Option Pricing Model

Cox-Ross-Rubinstein - Construction of Base, Upstate and Downstate



Cox, J. C.; Ross, S. A.; Rubinstein, M. (1979). "Option pricing: A simplified approach". Journal of Financial Economics 7 (3): 229

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Binomial Put Option Pricing

Constructing a Risk-Free Portfolio to Find an Initial Option Value P_0

Put Option + Δ Shares

P_0 → P_u, P_d

ΔS_0 → $e^{qh}\Delta u S_0$, $e^{qh}\Delta d S_0$

Risk-Free Portfolio

Equal value in downstate or upstate:

$$P_u + e^{qh}\Delta u S_0 = P_d + e^{qh}\Delta d S_0$$

Hedging Construction:

$$\Delta = \frac{P_d - P_u}{e^{qh}(u-d)S_0}$$

Arbitrage Argument

Risk-free portfolio must earn the risk free rate.

The initial portfolio value is the present value of the terminal value (discount at the risk free rate).

Subtracting the shares, we find the initial option value. This is the "risk neutral" construction.

Rules of Option Pricing Theory

Strict Rules to Enter the Club

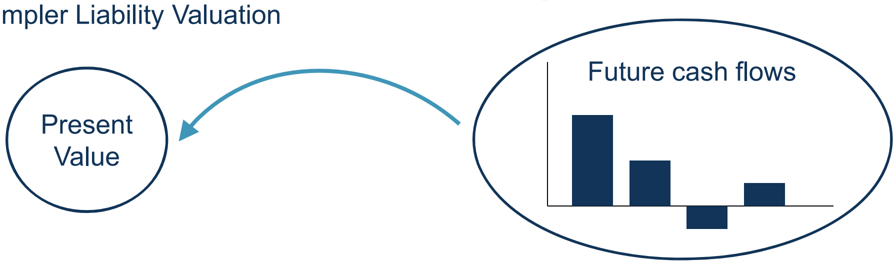
Forbidden

- Bid-ask spreads
- Market impact of trades
- Information asymmetries
- Taxes
- Solvency capital requirements and costs of holding these
- Collateral posting requirements
- Risk of default on derivatives
- Illiquidity premiums or other non-cash-flow valuation effects

Permitted

- Investment and unlimited borrowing at a single risk free rate
- Unlimited and infinitely-divisible supply of underlying assets
- Continuous-time trading (24/7)
- Buying and selling with no impact on the market price
- Consensus on possible price moves in the underlying asset

Benefits of Club Membership
Simpler Liability Valuation



Unique market-consistent present value depends on:
- Future liability cash flows
- Allowing for policyholder and management embedded options

Present value does not depend on:
- Insurers' asset strategy
- Who owns the liability
- Whether the liability is actively traded (liquidity)

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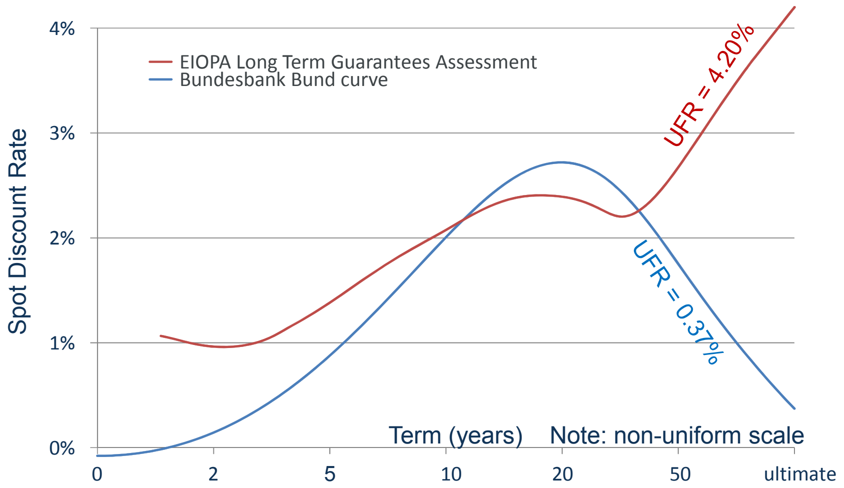
The Ultimate Forward Rate Paradox

Yield Curve Extrapolation (as at end 2011)

Spot (and Forward) Curves Converge to Ultimate Forward Rates

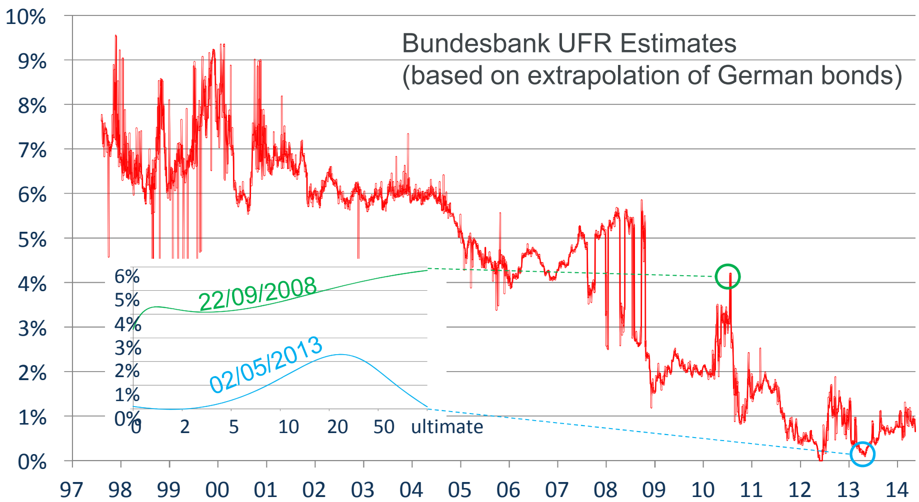


Dybvig-Ingersoll-Ross Paradox

- Many theoretical yield curve models (examples: Vasiček, Cox-Ingersoll-Ross) have the concept of an “ultimate forward rate” (UFR), which is the limit of the forward interest rates for long bonds.
- Under Vasiček, CIR, the UFR is not allowed to change from one valuation date to the next. This has been reflected in the Solvency II use of 4.2% UFR.
- Dybvig, Ingersoll & Ross showed that in general, for an arbitrage-free model, the UFR can never fall.
- Recent attempts to resolve the paradox within an arbitrage-free framework, such as Brody & Hughston (2013), suggest UFR = 0, which is unlikely to be popular with insurance firms.
- Does this mean that financial institutions should not have to test the consequences of a fall in UFR?
- Or does this indicate we rely too heavily on unrealistic option pricing theory?

Dybvig P, Ingersoll P and Ross S (1996). Long forward and zero-coupon rates can never fall. Journal of Business 60, 1-25.

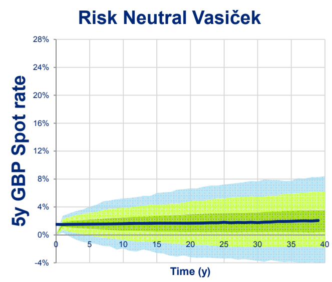
Observed UFR Behaviour

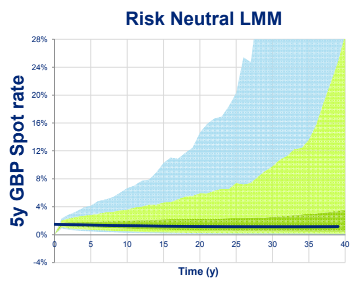


Why Do My Interest Rates Explode?

A Curse of Economic Scenario Generators (ESGs)

- Under Solvency II, discount curves are extrapolated using an ultimate forward rate (UFR) assumption. Theoretical models such as Vasiček, CIR have constant UFR.





- Some insurers instead use a "LIBOR market model". Here the UFR explodes to infinity after time zero. Finite term yields drift up over time and frequently hit thousands of percent for long horizons. At the very least, these create problems with calculation overflow and convergence of mean present values.

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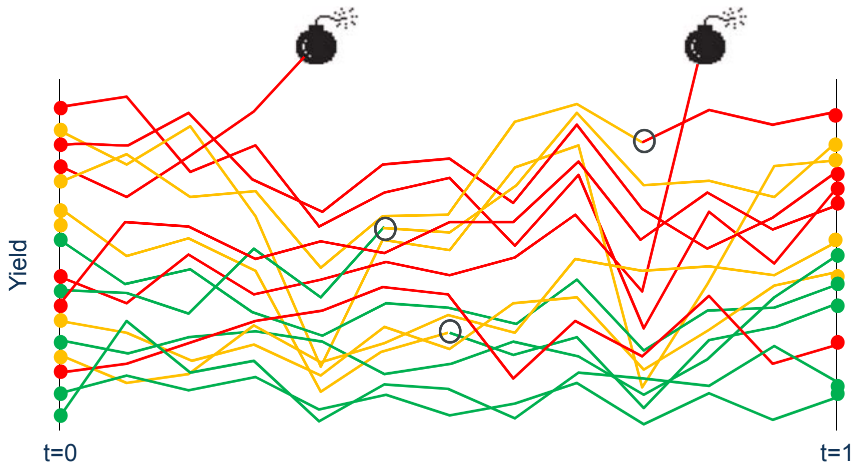
Corporate Bond Paradox

Introducing Credit Grades

The Rating Does Not Uniquely Determine the Yield



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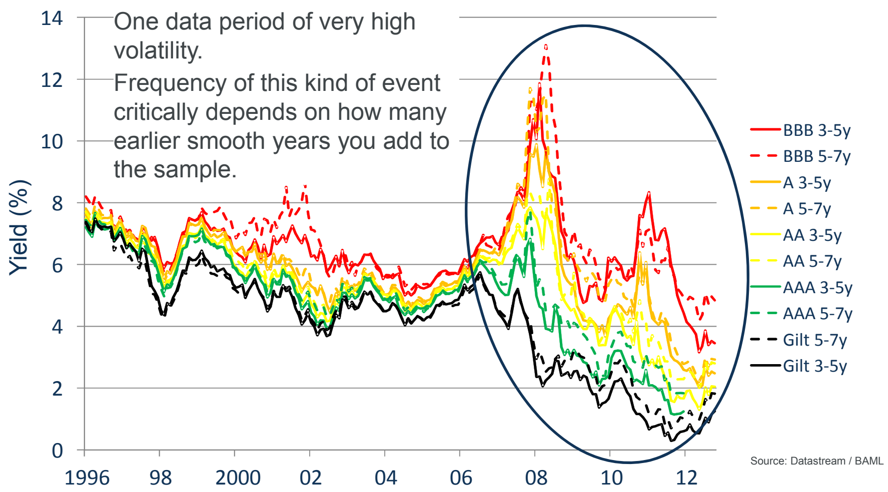
UK Yield Spread History



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Why Can't I Calibrate My Initial Spreads?

Start with a Real World Model and Use Arbitrage Arguments

- For a free-form (real world) model, we have free choice over the distribution of credit spread changes.
- Also consider non-Markov models, e.g. correlated transitions.
- Everything is empirical; little theoretical content.

- The Jarrow-Lando-Turnbull model allows for stochastic migration subject to shocks from a Cox-Ingersoll-Ross process.
- Arbitrage-free theory implies term structure of spreads by grade. CIR makes the maths tractable.
- We can't incorporate jumps / empirical distributions of spread changes as the hedging arguments break down.
- But rigid spreads structure often complicates initial calibration.

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Valuation Adjustments in Option Pricing

Changes in Option Pricing Methodology

Banks Adopt Adjustments to Counter Weaknesses in the Theory

Credit valuation adjustment **CVA**	Allowance for possible default by derivative counterparties
Debit valuation adjustment **DVA**	Reduce stated liabilities with an allowance for own default.
Funding valuation adjustment **FVA**	Allowance for funding of derivative position (borrowing over the risk free rate, stock lending, collateral posting).

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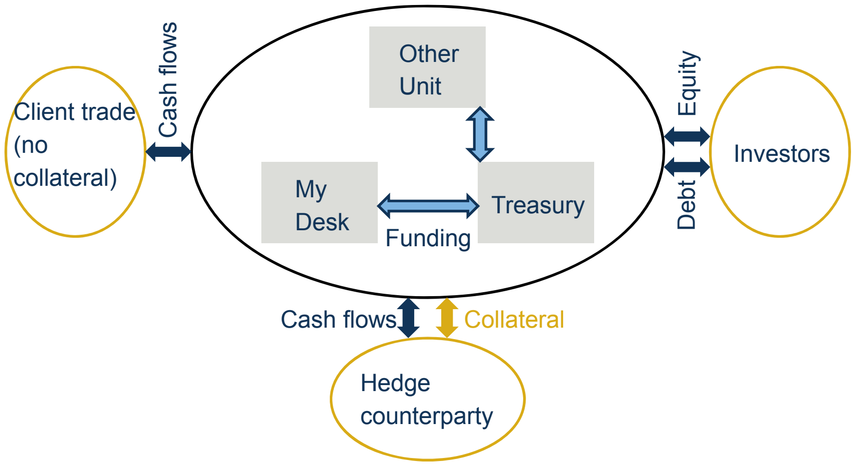
Debit Valuation Adjustment

Is an Insurance Liability Reduced by Insurers' Inability to Pay?

- If someone else owes me money, I allow for the risk that they default
- Suppose I owe someone else money
  - Do I reduce the liability to allow for my risk of default?
  - My default risk makes my promises less valuable to policyholders
  - Should affect the prices that policyholders are willing to pay
  - Implicit in accounting for other forms of corporate debt
- There are no "default free" option prices or interbank instruments
  - So there is no such thing as a market-consistent insurance liability "ignoring" credit risk
  - We can make no adjustment – then implicitly we take credit risk of calibration instruments
- Timing of default matters
  - Suppose I have an asset that pays out in 1% of outcomes but my counterparty has a 1% failure rate, and they're the same events
  - Then the asset is worthless, ignoring default recoveries (do not only add 1% to discount rate)

Funding Derivative Positions

Heated Debate About How to Charge Clients for Funding / Collateral



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Linking Market Prices to Real World Behaviour

How We Use Models in Practice

Market-Consistent Models Are Not What They Seem

- There is an established rationale for market-consistent pricing based on a change of measure (risk neutral probabilities) via a replication / arbitrage argument.
- Market practice seldom follows the theoretical approach. We construct risk neutral laws from market prices, and not from a real world law via a change of measure.
- It might be said that this leads to inconsistency between pricing theory and real world models – however there are many acknowledged respects in which option pricing theory does not fit the real world, so this inconsistency should not disturb us. For example, we readily accept that historic and implied volatilities are not the same thing. Apparent arbitrages are thwarted by market imperfections and model risk.
- We rely implicitly on change of measure arguments when we calibrate assumptions such as volatility (for markets such as property with no significant option market) or correlations between markets.

Insurance Model Validation

Situation	An insurer provides profit-sharing policies with guaranteed minimum benefits. The underlying assets are a diversified portfolio of bonds (government and corporate), equities, property and alternative assets. An ESG (economic scenario generator) is used for valuing the guarantees.
Assumption	• Investment and unlimited borrowing at a single risk free rate. • Unlimited and infinitely-divisible supply of underlying assets. • Continuous-time trading (24/7). • Buying and selling with no impact on the market price. • Consensus on possible price moves in the underlying asset.
Validation	Test trade: Suppose the fund owns a shopping centre. Can I instantly adopt execute an offsetting short position of 1% of that shopping centre, at a price consistent with my valuation of the 100% share, with no costs of dealing or other transaction costs?

Steps from the Real World to Market Prices

More Steps than Are Commonly Acknowledged

My model Her model
Your model His model

Resolve ambiguity and information asymmetry

Funding costs and market price impact of trades

Hedging / arbitrage arguments

Market prices

We lack a comprehensive theory of asset pricing that captures all market features.

Current practice is to calibrate option pricing models to subsets of market prices (as we cannot explain everything at once).

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Conclusions

Observations on Option Pricing Theory

How Has Market Consistency Served Insurers and Policyholders?

The realistic regime has generally served us well, however:

- There have been challenging complications: lack of risk-free rate, liquidity premiums, which sit uneasily with the ESG theoretical framework
- We have in effect fossilised the now discredited pre-crisis bank methodology. Has the complexity of ESG's made it too difficult for us to move forward?
- For more on scenario generators, please go to section F4

Conclusions

The Future of Market Consistent Valuation in Insurance

- Emerging standards, including Solvency II and IFRS, continue to emphasize market consistency, which is generally a good thing
- A robust definition of market consistency has still not been formulated outside the parallel universe of option pricing theory
- All cash flows matter: reverse-engineer explicit replication arguments so that proper allowance can be made for dealing costs, model ambiguity and other difficult effects
- Banking methodology innovations (CVA, DVA, FVA) may hit insurers first via IFRS rather than Solvency II
- This will continue to be a political issue as much as a technical one, with large impact of different methodologies on stated liabilities
- Expect a continuing theoretical mess for some time to come, but of course this is part of what makes insurance professionally rewarding and interesting