The Cairns-Blake-Dowd Model

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Plan for Talk

- Introductory remarks
- The Cairns-Blake-Dowd (CBD) model
  - Pros and cons
- Assessment criteria
- Extension to include a cohort effect
- Backtesting

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Introduction – CBD Model

- Model designed for:
  - Annuities and pensions – *longevity risk*
  - Not for *short-term mortality risk*
- Model for mortality at higher ages
- CBD model:
  - *exploits relative simplicity of mortality curve at higher ages*
  - *Not designed for lower ages*

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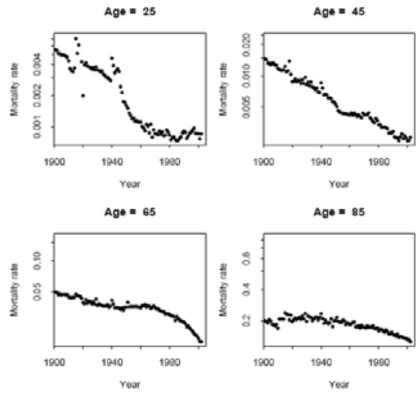
Historical mortality rates (log scale)



Introduction

- Pensions e.g. 30 year old
  - Uncertainty in value of deferred annuity is mostly affected by post-60 mortality
  - Model for mortality below age 60 is relatively unimportant
    - E.g. Prob(Survival to age 60) = 0.96 with St.Dev. 0.005

Background

- Part of wider *LifeMetrics* research programme
  - Comparison of 8 models
  - Within sample fit
  - Out of sample performance/backtesting
  - *Development of new models*
- Focus here on specific models we have developed

Introduction

- Why do we need stochastic mortality models?
- Data => future mortality is uncertain
- Good risk management
- Setting risk reserves
- Annuity contracts with embedded options
  - E.g. guaranteed annuity options
- Pricing and hedging longevity-linked securities
  - E.g. q-forwards
- Many models to choose from:
  - Limited data => model and parameter risk

Measures of mortality

- $q(t,x)$ = underlying mortality rate:
  in year t at age x
- $m(t,x)$ = underlying death rate
- *Poisson model:*

Actual deaths:

$D(t,x)$ ~ *independent* $\text{Poisson}\big(m(t,x)E(t,x)\big)$

$E(t,x)$ = central exposed to risk

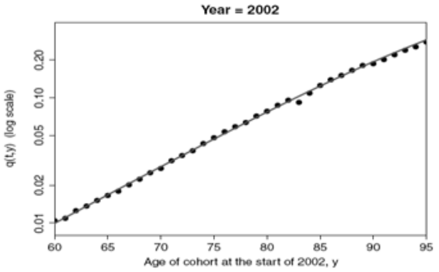
Need good mortality forecasting model

- 'Process-based' models
  - Model process of dying
    - Not used much yet
- 'Explanatory' or 'causal' models
  - Model causes of death
    - e.g. heart disease or socio-economic factors
    - Not used much yet, but post-code modelling more common
- 'Extrapolative' projection models
  - Will only be reliable if the past trends continue:
    - Medical advances can invalidate extrapolative projections by changing the trend

Models

Lee-Carter (1992) LC
$\log m(t,x) = \beta_x^{(1)} + \beta_x^{(2)}\kappa_t^{(2)}$
Cairns-Blake-Dowd (2006) CBD-1
logit$q(t,x) = \kappa_t^{(1)} + (x-\bar{x})\kappa_t^{(2)}$
Cairns et al. (2007) CBD-2
logit$q(t,x) =$ $\kappa_t^{(1)} + (x-\bar{x})\kappa_t^{(2)} + \left((x-\bar{x})^2 - \sigma_x^2\right)\kappa_t^{(3)} + \gamma_{t-x}^{(4)}$

CBD-1 fit at higher ages



Model Notation

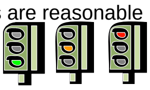
- Beta(x) terms => Age effects
- Kappa(t) terms => Period effects
- Gamma(t-x) terms => Cohort effects

Main extrapolative models: Philosophical differences

- Lee-Carter model:
  - No smoothness across ages or years
- CBD model:
  - Smoothness across ages in same year
- P-splines model:
  - Smoothness across years and ages

How to compare stochastic models

- Quantitative criteria
  - Log-likelihood; BIC
  - Pattern of standardised residuals (i.i.d. ???)
- Qualitative criteria
  - Robust relative to age and period range
  - Biologically reasonable
  - Forecasts are reasonable



  - Suitability for specific applications

Models - LC

Lee-Carter (1992) LC
$\log m(t,x) = \beta_x^{(1)} + \beta_x^{(2)} \kappa_t^{(2)}$
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Lee-Carter Model

- Pros:
  - Robust
  - Simple one-factor model
  - Good fit over wide age ranges
- Cons:
  - Lack of smoothness of age effect (esp. small populations)
  - Cannot cope with improvements at different ages at different times
    - Tendency to use only very recent data
  - Possible underestimation of uncertainty
  - β_x affects both trend and uncertainty at age *x*
    - Cannot decouple
  - One-factor model
    - Perfect correlation across ages
  - No cohort effect

Models – CBD-1

Lee-Carter (1992) LC
$\log m(t,x) = \beta_x^{(1)} + \beta_x^{(2)}\kappa_t^{(2)}$
Cairns-Blake-Dowd (2006) CBD-1
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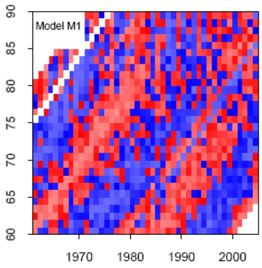
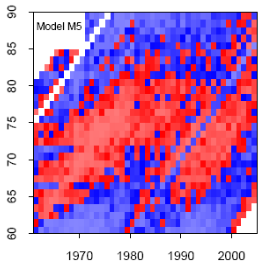
CBD-1 Model

- Our first model independent of LC
- Why?
  - Pensions
  - High ages
  - Simple models
- Pros:
  - Robust
  - Two correlated factors: level and slope
  - Allows different improvements at different ages at different times
  - Simple age effects
  - Easy to incorporate parameter uncertainty
- Cons:
  - No cohort effect
  - Good at big picture but overall fit not as good as LC
  - LC better able to pick up small non-linearities in mortality curve

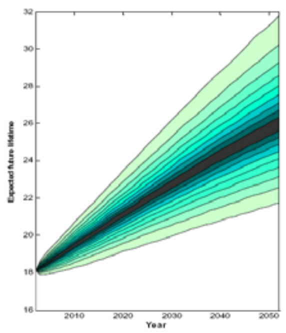
Residuals LC & CBD-1

- Violation of indep. Poisson assumption

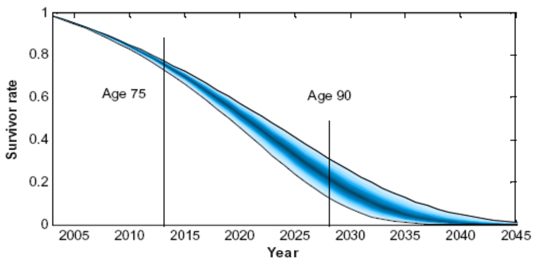




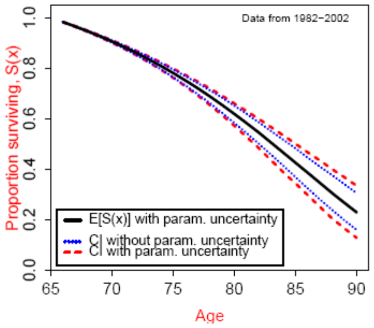
Communicating risk:
(Cohort) Longevity fan chart for 65-year old males



(Cohort) Survivor fan chart for 65-year old males in 2003

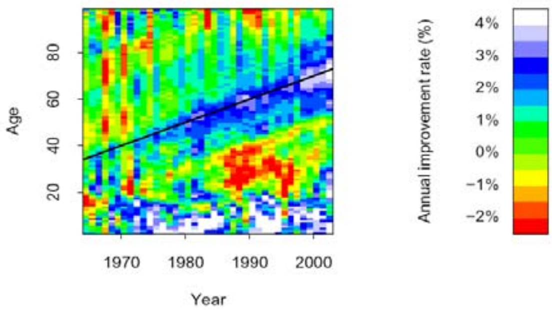


Inclusion of parameter uncertainty



Cohort effect:

Black line: 1930 cohort



Models – CBD-2

Lee-Carter (1992) LC
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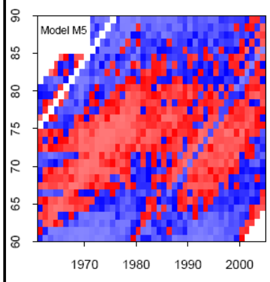
CBD-2 Model

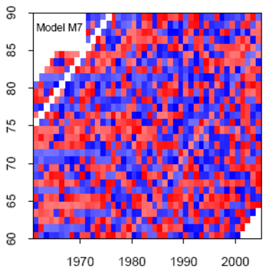
- Developed to address deficiencies of earlier models (LC and CBD-1)
- Builds on pros of earlier models
- Key advance builds on Renshaw-Haberman
- Several cohort extensions investigated: *CBD-2 model was best in terms of balance between goodness of fit, parsimony and robustness*

Are standardised residuals iid?

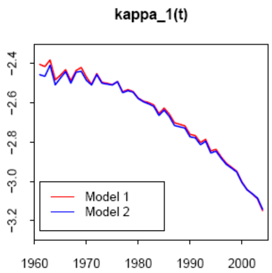
CBD-1

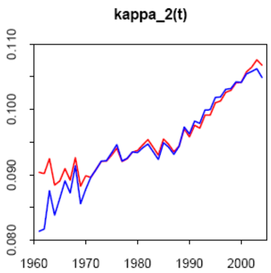


CBD-2

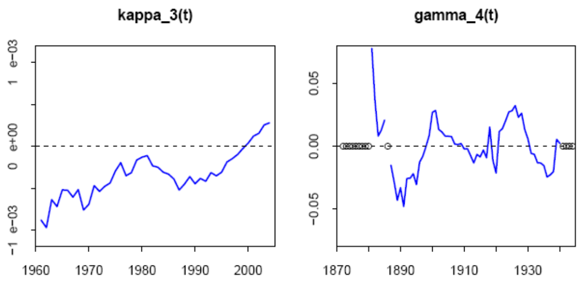


CBD-1 versus CBD-2



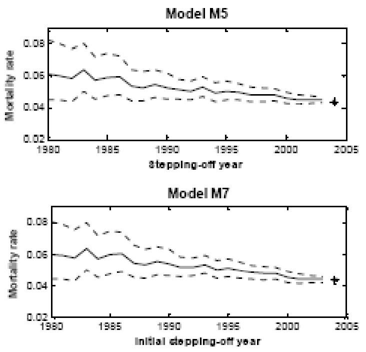


CBD-2 extra terms



Backtesting

- Forecasts of 2004 mortality rates
- Fixed forecast date 2004
- Data: 1960-1980
  - Forecast for 2004
- Data: 1961-1981
  - Forecast for 2004
- …
- Data: 1973-2003
  - Forecast for 2004



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Expanding horizons

- Data from 1960-1980
  - *Forecast for 1985*
- Data from 1961-1981
  - *Forecast for 1986*
- Data from 1962-1982
  - *Forecast for 1987*
- etc.

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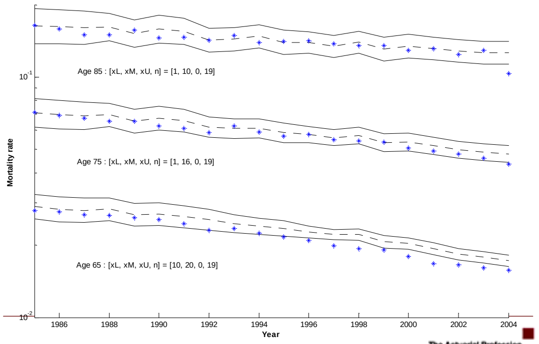
CBD-1: Rolling 5-yr ahead prediction interval
Ages 85, 75, 65



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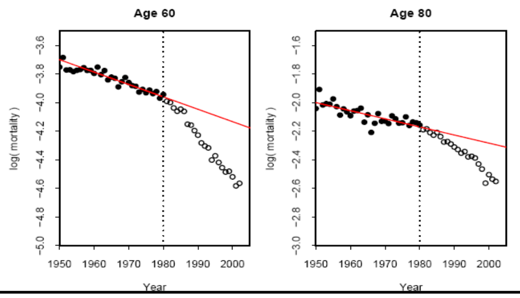
Conclusions

All models had difficulty in capturing the change in trend

Crude projections based on data up to 1980



Conclusions

- Results between models are reasonably consistent
- Backtesting:
  - No model emerges as obviously better
  - Eg general year-on-year noise swamps subtlety of cohort effect
- Revert to other criteria:
  - Quantitative and qualitative
- Recapitulation:
  - CBD-2 is a good, robust model for higher ages
  - CBD-1 good at modelling the bigger picture
  - Alternatives or adaptations needed for lower ages

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References

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  A quantitative comparison of stochastic mortality models using data from England and Wales and the United States. Working paper, Pensions Institute and Heriot-Watt University.
- Cairns, Blake, Dowd, Coughlan, Epstein and Khalaf-Allah (2008)
  Mortality density forecasts: an analysis of six stochastic mortality models. Working paper, Pensions Institute and Heriot-Watt University.

See:
- http://www.ma.hw.ac.uk/~andrewc/papers/
- http://www.pensions-institute.org/papers.html

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