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THE CALCULATION OF PREMIUM RATES FOR DECREASING TEMPORARY ASSURANCES

by

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INTRODUCTION

In their text book, *The Practice of Life Assurance*, published in 1952, Coe and Ogborn devoted less than four pages to the methods of calculating premiums for varying temporary assurances, including family income policies. At that time decreasing temporary assurances were relatively uncommon except for a block of single-premium policies transacted mainly by a small number of offices which specialized in them.

Since that date, however, the transaction of this type of policy on an annual-premium basis has assumed very substantial proportions, particularly in connexion with mortgages, and the cover provided varies considerably among different offices. Some are prepared to go to considerable lengths to provide exact cover, whereas others appear reluctant to go into any great refinements. This is understandable as the calculation of premium rates is quite a formidable task whichever of the known methods is used. The not infrequent changes in building societies' interest rates add considerably to the difficulties.

The annual-premium net rates themselves are quite small over a wide range of ages and terms, and the expense loading is a very substantial part of the total office premium. Under such conditions great refinement can hardly be justified, but, nevertheless, the methods used must be reasonably accurate. Approximations are almost inevitable (at least until computers become standard instruments for premium calculations) and existing methods are quite numerous, some being tucked away in the files of individual offices.

The purpose of this note is:

(*a*) to summarize the known methods,

(*b*) to indicate the extent to which they are currently used, and

(*c*) to show how the work may be further reduced.

METHODS

Coe and Ogborn indicated three general processes that could be used to arrive at decreasing temporary assurance premiums, viz.

(1) Detailed calculation by the formula

$$\sum_t S_t C_{[x]+t-1}/(N_{[x]}-N_{[x]+k}),$$

where k is the term over which premiums are payable. This method is accurate but very laborious, particularly if premium rates have to be changed frequently because of changes in interest rates. The net premiums are calculated and a loading for expenses added.

(2) Equally detailed calculation applying the office premiums for level temporary assurances to the differences in sums assured from year to year, i.e. $\sum_t (S_t - S_{t+1}) P'^{\,1}_{[x]\overline{t}|}\,\ddot{a}_{[x]\overline{t}|}/\ddot{a}_{[x]\overline{k}|}$.

Since office premiums are used, expense loadings are already included, but the resulting loading in the emerging premium is not necessarily suitable for the decreasing temporary assurance. This formula was best suited for use for equal decrements of sum assured. (Asbury [6] described this method, but using net premiums.)

(3) A rather lighter task by the use of a formula of the form

$$\frac{a_{\overline{n}|} - a_{[x]\overline{n}|}}{\ddot{a}_{[x]\overline{k}|}} \times R \times f(n),$$

where R is the level annual repayment for the term n at the mortgage rate of interest j. McAlpin [1] showed that $f(n)$ could be taken with reasonable accuracy as

$$\frac{(1+j)\sum_{t=1}^{n} a^{\,j}_{\overline{n-t+1}|}}{(1+i)\sum_{t=1}^{n} a^{\,i}_{\overline{n-t+1}|}},$$

which is independent of age.

USE OF METHODS IN PRACTICE

In investigating methods currently used for premium calculation, reference was made to 37 offices known to be actively competing for this class of business. The questions asked were:

(*a*) Which approach do you use in arriving at your net premium (Coe and Ogborn mention three in particular)?
(*b*) Do you allow for decrements other than yearly?
(*c*) Do you allow for different rates of interest, and if so by what process—e.g. separate calculation, or approximate adjustment to a basic rate?
(*d*) Do you also provide for level repayments of capital such as arises in some private mortgages (i.e. in effect interest at 0%)? If so, is your method for that variety of cover any different?

In the light of replies to the second question a supplementary question was later asked of some offices:

> Was adherence to yearly decrements due to (i) the nature of the calculation and the feeling that unnecessary complications are to be avoided, or (ii) some other reasons?

No reply was asked for unless (ii) applied, and the replies received did not disclose anything of outstanding importance.

Table 1 shows a summary of the replies received to the first four questions.

Of the eight offices classified under (*a*) as using 'other methods', four employed Knapman's method [4] or a slight modification of it. Under this method the net single premium is expressed as

$$\frac{(1+j)}{j a^{j}_{\overline{n}|}}\left(A^{1}_{[x]\overline{n}|}{}^{i} - v_j^{n+1} A^{1}_{[x]\overline{n}|}{}^{I}\right),$$

where $$1+I = (1+i)/(1+j).$$

Two others use a method described as equating area and first moment. The process used in this method is to express the premium in the form

$$S \times \pi^{1}_{[x]\overline{r}|},$$

where, if ϕ_t is the sum assured in year t,

$$rS = \phi_1+\phi_2+\ldots+\phi_n,$$

and $$\tfrac{1}{2}r(r+1)S = \phi_1+2\phi_2+\ldots+n\phi_n.$$

It is easily shown, as follows, that the method is accurate if C_{x+t} is linear.

If $C_{x+t} = C_x(1+kt)$ the value of the benefits is

$$\frac{C_x}{D_x}[\phi_1+\phi_2(1+k)+\phi_3(1+2k)+\ldots+\phi_n(1+\overline{n-1}k)],$$

i.e. $$\frac{SC_x}{D_x}[r+\tfrac{1}{2}kr(r-1)],$$

i.e. $$\frac{SC_x}{D_x}[1+(1+k)+(1+2k)+\ldots+(1+\overline{r-1}k)],$$

i.e. $$\frac{S}{D_x}[C_x+C_{x+1}+\ldots+C_{x+r-1}].$$

For a level decrease of sum assured,

$$S = \text{Initial Sum Assured}\times\frac{n+1}{2r},$$

r being $\tfrac{1}{3}(2n+1)$ taken to the nearer integer, whilst for normal mortgage protection cover with a mortgage rate of j,

$$S = \text{Initial Sum Assured}\times\frac{n-a^j_{\overline{n}|}}{jra^j_{\overline{n}|}},$$

where $$r = \frac{n^2+a^j_{\overline{n}|}}{n-a^j_{\overline{n}|}}-\frac{2}{j},$$

to which $\cdot 8(n-1)$ has been found to be a reasonable approximation.

The seventh office found it worth while to construct commutation columns of C'_x where $C'_x = C_x\times(1+j)^x$, where j is the mortgage rate of interest.

Finally, one office was content to use the $R(a^j_{\overline{n}|}-a^j_{[x]\overline{n}|})$ method at the mortgage rate of interest without adjustment, presumably relying on its loadings to produce satisfactory office premiums.

Table 1

(a)

Method	Number
$S \times C$	16
$SA \times A$	3
McAlpin type of formula	9
All three above methods	1
Other methods	8

(b)

Reply	Number
No	22
Not normally	6
Yes	9

(c)

Reply	Number
Yes (recalculate)	5
Yes (approx. adjustment)	19
No	13

(d)

Reply	Number
Yes (same formula as for (a))	13
Yes (theoretical formula)	15
Factor adjustment to (a) or (c)	5
Respread of term premiums	1
Not answered	3

Notes

(a) Three offices use the $S \times C$ method, replace the S by an average sum assured over a period and apply an $(M-M)$ factor instead of C, thus reducing the work.

(b) Most of those saying 'Yes' stated that monthly decrements were normal.

A MODIFIED FORM OF McALPIN'S METHOD

McAlpin [1] was concerned with the valuation of decreasing temporary assurances and not with the calculation of premiums. He assumed that the level annual repayment R would be tabulated on the valuation schedules, and expressed the value of the sum assured, assuming payment at the end of the year of death of $(1+j)$ times the amount outstanding at the beginning of the year, as

$$R(a^i_{\overline{n}|} - a^i_{x\overline{n}|}) \times \frac{(1+j) \sum_{t=0}^{n-1} a^j_{\overline{n-t}|} C^i_{x+t}}{(1+i) \sum_{t=0}^{n-1} a^i_{\overline{n-t}|} C^i_{x+t}}.$$

This he replaced by the approximation

$$R(a^i_{\overline{n}|} - a^i_{x\overline{n}|}) \times f(n),$$

where

$$f(n) = \frac{(1+j) \sum_{t=0}^{n-1} a^j_{\overline{n-t}|}}{(1+i) \sum_{t=0}^{n-1} a^i_{\overline{n-t}|}}.$$

For the calculation of premiums we need to replace R by $1/a^{i}_{\overline{n}|}$. The single premium corresponding to a loan of 1, again assuming payment at the end of the year of death of $(1+j)$ times the amount outstanding at the beginning of the year, is

$$\frac{a^{i}_{\overline{n}|} - a^{i}_{[x]\overline{n}|}}{a^{i}_{\overline{n}|}} \times f'(n).$$

Here
$$f'(n) = \frac{a^{i}_{\overline{n}|}}{a^{j}_{\overline{n}|}} \times f(n)$$
$$= \phi(n,j)/\phi(n,i),$$

where
$$\phi(n,i) = \frac{(1+i)(n-a_{\overline{n}|})}{1-v^n},$$

or
$$\frac{n-a_{\overline{n}|}}{da_{\overline{n}|}}.$$

The values of ϕ can be calculated for the premium rate of interest i and for any mortgage rate j, and $f'(n)$ obtained by division.

Carosone [2] used an adaptation of McAlpin's method on the above lines.

Table 2 compares $f'(n)$ with the accurate values using $C_{[x]+t}$ on the basis of A 1949–52 Select, with $i = \cdot 03$ and $j = \cdot 04$ and $\cdot 06$.

Table 2

$j = \cdot 04$

		Accurate values			
		x			
n	$f'(n)$	20	30	40	50
10	1·024		1·025	1·028	1·027
20	1·037	1·037	1·044	1·047	1·045
30	1·049	1·053	1·066	1·066	

$j = \cdot 06$

		Accurate values			
		x			
n	$f'(n)$	20	30	40	50
10	1·071		1·077	1·084	1·083
20	1·111	1·111	1·132	1·143	1·135
30	1·144	1·157	1·197	1·197	

It has been found that $f'(n)$ is very nearly linear for variations in both term and rate of interest j. It can be represented accurately enough for practical purposes by

$$1+\frac{n+10}{8}(j-i),$$

and in this form it is considerably easier to calculate. This approximation applies equally well whether j is greater or less than i, so that level decrements—corresponding to a mortgage rate of 0 %—can be covered.

Using this modification of McAlpin's development the general formula for the single premium for any mortgage cover on this pattern for each £100 initial sum assured becomes

$$\frac{100}{a^i_{\overline{n}|}}(a^i_{\overline{n}|}-a^i_{[x]\overline{n}|})\left\{1+\frac{n+10}{8}(j-i)\right\},$$

and if monthly decrements of sum assured are to be provided for it is merely necessary to substitute

$$(a^{(12)i}_{\overline{n}|}-a^{(12)i}_{[x]\overline{n}|}) \quad \text{for} \quad (a^i_{\overline{n}|}-a^i_{[x]\overline{n}|})$$

with similar adjustments for half-yearly or quarterly if they arise. The process may be further simplified for monthly decrements with little loss of accuracy if the more easily calculated $(\bar{a}^i_{\overline{n}|}-\bar{a}^i_{[x]\overline{n}|})$ is used. It is already fairly common practice to use the expression $(\bar{a}^i_{\overline{n}|}-\bar{a}^i_{[x]\overline{n}|})$ in the calculation of premiums for family income benefits, and the use of the same expression for mortgage cover enables one set of basic calculations to be employed for both purposes.

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