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Report of the Working Party

on

CAPITAL ALLOCATION

Working Party Members

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1   INTRODUCTION

No explicit terms of reference were given for the Working Party but these have been taken as follows:

1. "To consider the reasons for the allocation of capital of an Insurance/Reinsurance company between product lines or portfolios of risks written by that company.

2. To review the effectiveness of the methods available for allocating capital and to identify the key factors that affect the allocation and over time.

Inevitably these lead into many related issues and widen the subject matter considerably, particularly into the realm of solvency levels required to support the writing of each class of business. This is considered outside of the scope of the working party as it falls more naturally into the working party dealing with solvency. Therefore we have considered only relative capital allocation on the assumption that the capital in total will support the business written in the period.

In addition to avoiding the question of the absolute level of solvency, the working party has focused more on capital allocation to support the risk inherent in underwriting activities of the company rather than others such as those arising from investment policy exchange rates and security of reinsurers.

In some circumstances, the risk in these other activities can be of a similar level, if not more than the underwriting activities. We also explored the general framework for considering the risks from whatever source.

2. SUMMARY AND CONCLUSIONS

The Working Party considers the allocation of capital across classes of business or responsibilties an important process in the management of a company. This process should focus attention on the risks for which capital cover is required and whether the associated returns are commensurate with those risks.

The paper attempts to deal with a number of conceptual issues as well as providing a survey of a range of methods. The Working Party has only considered risks arising from the underwriting activities of a company.

In an allocation process we would highlight the following considerations.

(a) It is important to be clear on what constitues capital in particular where this is "locked in" for example, through the use of non-discounted reserves. This capital may or may not be fully absorbed within the insurance operations and therefore the "profit centres" should be clearly identified.

(b) We need to be clear on the underlying criteria used to determine the allocation and the implications on the movement of capital between classes.

(c) The allocation process concentrates on the reward for risk but inevitably leads to the question of solvency and minimum capital levels. This question has not been pursued and warrants further work.

(d) The allocation to business written in the past has to be seen in a different light from that looking at the capital required to support future writings as a result of the greater uncertainties. The evolution of the actual and expected capital requirements by class of business is an important part of the process, in particular the capital deployed to meet the risk in the run-off of funds relative to the current year's exposures.

(e) A variety of methods of allocation have been described which range from simple approaches only requiring regularly available data to those which are more complicated which recognise and quantify variation of results to a greater extent. The latter methods are considered more appropriate, but further work is necessary to improve our understanding of how these would operate for a company writing multiple classes of business.

In additional we have identified the following areas of further work in respect of capital allocation:

(a) Lloyds, Mutuals and Captives.

(b) Assets and exchange risk and the impact of inflation. Are there any links with the work undertaken by AFIR ?

(c) Incorporate non-insurance activities to study the cost of capital between the various areas and the benefits of diversification.

(d) Extend the existing examples in the paper to more complex situations.

On a personal note, I would like to extend my thanks to my colleagues on the Working Party for all their efforts in the preparation of the paper. The subject extends into nearly all facets of the operation of a company and we as actuaries should be well placed to forge the links necessary.

3. **REASONS FOR CAPITAL ALLOCATION**

The insurer's total capital serves a number of purposes, and the extent to which capital is required to fulfil each of these purposes determines the limitations on the insurer's capabilities. Each of these purposes must, therefore, be considered in making these economic decisions, or in making a meaningful allocation of the insurer's capital.

Before embarking on a discussion of the various methods that can be employed, it is important to clarify the objectives of the allocation process. These have been identified as follows:

1. **Planning**

For a company to make efficient use of its capital resource it is important to gauge the effect of alternative strategies on the capital available. This in turn leads to an assessment of the profit required and a basis for management control.

There are also education benefits in this procedure which can lead to a greater understanding of the impact of risks undertaken and how capital is deployed.

2. **Performance Measurement**

Whether it is a particular product line or portfolio of risks the company needs to be able to set a premium pricing formula or profit target which allows for the return on capital. Therefore any assessment of past profitability requires this allocation of capital. Again there are education benefits in highlighting inadequate or above average returns from particular portfolios or products.

The purpose of the allocation is to work out the cost of capital ie the amount of return needed to service the capital allocation. Thus the fundamental objective of making an allocation of capital between portions of an insurers book (be these portions different classes or cohorts) is to allow management to make economic decisions which recognise how each portion of the book restricts the insurers ability to write further risks.

4 **WHAT IS CAPITAL REQUIRED FOR ?**

An insurer's capital resource enables it to provide some guarantee that coverage will still be provided in the event that the total call on the insurer's assets arising from such coverage exceeds the insurer's income. In such a way the capital acts as a catalyst to the insurance process.

In a US context Kneuer (1986) identifies seven purposes of capital, as follows:

(i) Capital must absorb any basic insurance costs (claims or expenses) unable to be met from premiums charged.

In the situation where coverage is not deliberately under-priced then capital is required to meet unexpected inadequacies in unearned premium reserve, expenses incurred in excess of those included in the pricing formula, and any excess cost of incurred claims. Claims cost may be unexpectedly high due to worse than expected loss experience, high claims cost inflation, inadequate claims control or any other of a number of reasons.

(ii) Capital must absorb any <u>deficiencies in claims reserves</u>. Capital is required to fund any deterioration in the run-off of claims reserves. Many insurers, have become acutely aware of this in recent years as they have experienced significant adverse development on reserves for long-tail US Casualty business written many years previously. Where an insurer discounts its outstanding claims reserves then any shortfall in the earned investment return relative to the rate of discount must be met from capital.

(iii) Capital must absorb any <u>declines in asset values</u>. Declines in values of assets taking the shareholders' funds will reduce the amount of capital available. Declines in values of assets backing insurance liabilities (viewed on consistent bases) must be funded from capital in order that these liabilities may be met when they arise.

(iv) Capital must provide protection against all <u>other adverse financial contingencies</u>. Sources of loss to the insurer other than the three outlined above exist. These include:

- mis-matching of assets and liabilities
- default on premium balances held by intermediaries
- failure of reinsurers
- adverse changes in the basis of taxation
- guarantee fund assessments
- casualty losses such as thefts
- foreign exchange losses
- unexpected increases in operating expenses

The insurer may have established a variety of liabilities to reflect the <u>expected</u> losses from these sources, but capital is required to fund losses from these sources in excess of those expected if coverage is to be guaranteed.

In each of the four cases detailed above, capital provides a contingency buffer against unexpected adverse outcomes. A fifth purpose of capital is to fund expected losses when business is knowingly sold at inadequate rates.

(v) Capital is required to fund under-priced business. Management may decide to accept some business at inadequate rates. This may be done for example to maintain market share, to benefit customer relations, obtain other related business which is more profitable, or more generally as a result of the insurance cycle. In any case the adverse outcome is certain, and capital must be used up to fund expected losses. The capital available should be sufficient to fund losses on under-priced business without affecting its ability to act as a contingency buffer for unexpected losses.

(vi) Capital is required to fund dividend payments when income cannot.

An insurer may wish to maintain a stable pattern of dividend payments to its shareholders, even when current income is insufficient to fund a proposed dividend. Dividend payments in excess of those affordable from current income must be funded from an insurer's capital.

(vii) Capital serves to maintain confidence in the insurer amongst consumers and regulators.

Since capital provides for the certain or contingency unfunded payments described above, it serves to uphold confidence in the insurer's ability to provide continuing coverage. An insurer is required by regulators to have capital sufficient to meet statutory minimum solvency requirements, but in order to continue to attract a well balanced portfolio of business, the insurer must maintain capital at a higher level.

The assessment of these risks and the actions pursued inevitably puts capital at risk from poor quality management and the identification of risk and need for capital support can be of value in itself.

5 **WHAT IS CAPITAL?**

Capital can be provided in a number of ways. In the UK and other countries readily realisable assets are typical for normal trading insurance companies. Capital can be provided in other ways e.g by way of guarantees, perhaps backed by deposits such as in Lloyd's. Part-paid shares, preference shares and sub-ordinated loan stock have all been used, but depending on the local regulatory environment.

In addition to considering the balance of assets over liabilities as capital we could also include any items of value "locked" into the balance sheet. Typically, investment earnings on the assets backing undiscounted insurance liabilities might be included. Thus, the definition of capital is unclear, but may cover any guarantees or investments or and any items of value which may not emerge for some time. The ability to raise additional funds is also important. For practical work on limited liability companies with shareholders, the working party would suggest capital may be usefully defined as adjusted net assets where assets are adjusted to full settlement value and liabilities are discounted for future investment income with equalisation and other contingent reserves being released.

6 **PRACTICAL RULES FOR MAKING MEANINGFUL ALLOCATIONS**

By considering the reasons for which an allocation of capital is desirable from Section 4, and the purposes for which capital is required by an insurer, we can arrive at some practical rules to which any meaningful allocation of capital must conform.

The fundamental reason for performing an allocation of capital is to enable informed economic decisions to be made, on matters such as pricing, which recognise how the demands for capital from each portion of the insurers book restricts its ability to write other risks. Each of the purposes for which capital is required contributes to these demands, and so must be considered in any meaningful allocation.

Rule 1: Any allocation must consider each function that capital is performing.

Clearly the sum of the amounts of capital allocated to different sections (classes, cohorts or other "profit centres") of the insurer's portfolio may

not exceed the insurer's total capital. If the sum of the allocated amounts is less than this total then there is a residual amount not being used to enable further business to be written in any section of the book. Thus the ability of the insurer to write more business would be understated for one or more sections of its portfolio, and economic decisions based on the insurer's perception of its own ability to write more business would be distorted. A second practical rule follows:

Rule 2: The sum of the amounts of capital allocated to the various sections of the insurer's portfolio must be exactly equal to the insurer's total capital.

Considering again the purposes that capital performs, capital must absorb any unfunded insurance losses arising from claims costs, or expenses exceeding premiums, for current business, or from deficiencies in claims reserves for expired business. Clearly, making a negative or zero allocation of capital to a section of business which presents exposure to losses of either type does not recognise this exposure to loss, and economic decisions based on such an allocation would be unsound. Another stated purpose of capital is to imbue consumers and regulators with confidence in the insurer's ability to fulfil promises of coverage. A zero or negative allocated amount cannot do this. A third practical rule follows:

Rule 3: The amount of capital allocated to any section of the insurer's book presenting an exposure to loss should be positive.

Sections of the insurer's book giving rise to greater exposure to unfunded losses should be allocated greater amounts of capital under any rational allocation. The capital available to fund unexpected adverse outcomes is separate from that needed to fund expected losses on under-priced business. A fourth practical rule follows:

Rule 4: The amount of capital allocated to a section of the insurers book, excluding that amount which funds under-priced business should increase as that section's exposure to unfunded losses increases.

A corollary here is that the amount of capital allocated to a section of the insurers book should exceed that required to fund under-priced business. The point in time at which losses on under-priced business are recognised will depend on whether the insurer establishes an additional reserve for unexpired

risks over and above the unearned premium reserve. However capital required to fund these losses should be allocated to the under-priced business as soon as it is written, since its existence limits the insurer's ability to write further business.

The nature of the risks underwritten by an insurer within a section of its book will determine the exposure to unfunded losses from that section. A good allocation method should respond to changes in the key components of underlying risk within a section and adjust the allocation to that section accordingly. However, the allocation should not be over-sensitive to minor changes in this risk. It is desirable that the allocation is stable so that it progresses smoothly unless there are major changes in the type or form of the underlying risk.

Rule 5: The allocation should adjust in response to significant changes in the underlying risks.

Other practical considerations suggest the following:

Rule 6: Allocations must be based to some extent on past results, and must be relatively stable with respect to the insurer's results over the short-term.

Rule 7: Any formulae used to make allocations should be explicit, objective and justifiable.

Such a formulae will provide an allocation in situations where management consensus is lacking. Subject to these requirements it is desirable that allocation formulae are as simple as possible.

7   **PRACTICAL CONSIDERATIONS**

To illustrate the practical issues involved in the allocation process, consider the following equation representing the capital structure of a company.

$$U \cdot r = P \cdot L + U \cdot i$$

Where U = initial capital

     P = risk premium

     L = profit loading

     i = investment return on capital

     r = shareholders expected rate of return on capital

Rearranging this becomes

$$U (r - i) = P . L$$

Thus $(r - i)$ is the additional return to be provided by the insurance operations (or possibly other areas of operation) over that earned by the investment of the capital.

The allocation exercise constitutes extending this equation over n classes of business using certain criteria to determine the proportion of the total capital used required by each line (or sphere of operation).

Closer scrutiny of this deceptively simple looking formula raises a number of issues.

(i)   **Shareholders additional return**

    In planning and setting objectives, we need to determine the insurance profit loading L which achieves the rate of return required by shareholders ie

    $$L = \frac{U}{P} . (r - i)$$

    for each class of business.

    The first question is should $(r - i)$ be different for each class of business? This inevitably will relate to the underlying uncertainties in writing each class of business, but for the company in total, will be set by investors expectations. Following from this level, the additional returns by class will be dictated by the levels of risk in a perfect market.

The profit loadings derived may not be obtainable in the period and this should trigger further debate on the underlying risks and establish whether there is scope for adjustment to key variables to revise the level of capital allocated and bring the additional return in balance with the insurance profit anticipated. For example, by a reduction in risk through increased reinsurance.

This debate will not relate solely to the risks underlying the capital allocation, and should be seen as a part of the total management of the company incorporating such issues as marketing, status or perceived security. The aim is to find the optimum deployment of capital by class of business to achieve the company's goals in the wider sense and this will not necessarily be related solely to expected insurance profit.

A balance has to be struck between the return required by shareholders, the levels of capital required by class of business to meet the risks involved and expected insurance profit.

(ii)   **Insurance Profit**

In monitoring performance we require

$$r = \frac{P}{U} \cdot L + i$$

thus the measurement of the expected shareholders return is driven by the allocation of capital relative to premium and the investment return on this allocated capital. As can be seen, the higher the allocation relative to premium the lower the return for a given level of insurance profit loading L. In general, the variability of the profit from the insurance operations will determine this ratio P/U.

The term $P \cdot L$ ie the profit is not clearly defined and if we consider a period of a revenue year and amplify this into its component parts on an accounted basis, we have the following (ignoring tax and dividends in order to simplify analysis.)

$$P \cdot L = (P - CP) + (F_{t-1} - F_t) + I$$

where P = premiums after commission and expenses.

CP = claims paid in the revenue year.

F = technical funds brought forward and carried forward respectively ie at (t - 1) and t.

I = interest on technical funds during the year.

Using c to denote the current year and p to denote prior years, this can be further rearranged as follows.

$$P \cdot L = (P^c - CP^c - F^c_t) + (P^p - CP^p + F^p_{t-1} - F^p_t) + I$$

If D represents the adjustment in funds due to discounting then the equation can be further amplified into the following

$$= (P^c - CP^c - F^c_t + D^c_t)$$

$$+ (P^p - CP^p + F^p_{t-1} - D^p_{t-1} - F^p_t + D^p_t)$$

$$(- D^c_t + D^p_{t-1} - D^p_t)$$

$$+ I$$

As can be seen, with suitable allocation of I, there are three distinct elements to this

(a)   the profit from the current years' business on a discounted basis.

(b)   the savings or losses arising on the discounted funds brought forward.

(c)   The   strain   arising   from   setting   up   non-discounted   (or
       incorporating other margins) in the funds.

From this it can be seen that capital is needed to support the results
of the current years' business which will be estimated to a greater or
less extent and this part requires retaining capital to cover the
run-off risk. The degree to which this is covered by non-discounting is
an important consideration which links reserving with the definition of
capital and hence the allocation process.

This leads into the next aspect.

(iii) <u>Capital</u>

The movement in this discount adjustment, across different classes will
determine the rate of release of this "hidden" requirement from period
to period. It is important that this is recognised as this involves a
certain "overhead" and is similar to an allocation of expenses.

The similarity with expenses can be further drawn upon in that not all
capital will be allocated to the insurance operations since other
"profit centres" may exist. This is a decision which needs to be taken
at the outset and will depend on organisational structure, management
responsibilities and aims of the company.

Whilst this paper does not deal with non-underwriting risks, it can be
seen that these other profit centres would encompass such items as asset
and exchange risks. It should be noted that these items could also be
treated as a risk element in the insurance operation and therefore
attract additional capital to be serviced by the allocation to each
class of business. For example, this can be illustrated in drawing up
an allocation statement in the following form.

Class	Current Year	Prior Years	Total
1			
2			
3			
4			
.			
.			
.			
n			

Asset

Exchange

				Total Capital including discount adjustment

Class	Current Year	Prior Years	Asset Exchange	Total
1				
2				
3				
.				
.				
.				
n				

Total capital
including
discount
adjustment

The statement summarises the deployment of capital across "profit centres" and identifies the major sources of risk. Statements of this type should also highlight where capital is "hidden" and how this is likely to move over time depending on the business strategy. The release (if feasible) of this capital may not coincide with the capital demands of current business and therefore it is important to recognise the time for which capital is required and also how it is held in the books of account.

(iv) **Revenue versus Underwriting Year**

In the main, management decisions will be focused on current and future business, but as can be seen from the above analysis, this can have little impact on the results in the revenue year and hence the return achieved.

This could be dominated by prior years' movements (eg in the case of substantial asbestos or toxic waste involvements, or long tail funds in general) but even if the impact here is minimal it must be appreciated that the premium will contain an element of past years. This can arise either from the influence of unearned premium in the case of a direct writer or booked premium for prior years accounted in the current year for companies on a funded basis. With the complication of exchange rate movements the return on capital on a revenue basis may bear little relationship to whether the current year's business achieves (or will achieve) the profit required to meet its return on capital objective.

(v) **Criteria used in the allocation basis**

Different methods of allocation will use different criteria to determine the capital by class and it is important these are fully understood because of the implications it has for movement of capital between classes.

For example, the method could equate one of the following:

(a)   profit loading.

(b)   variability of claim amount.

(c)   probability of ruin.

Each has different implications, for example, the probability of ruin(c) will increase the capital to a class where the expected insurance profit is reducing whereas (a) would move capital in the opposite direction. Criteria (b) would ignore this.

The conflict is largely between the need to maintain solvency or "security" from period to period for the insurer, thereby requiring more capital as the risk increases as against the investors expectations which would reduce capital if the expected insurance profit reduces.

The suggested approach therefore, is to set capital at a total level which ensures a proper balance between solvency requirements and investors expectations and use criteria for the allocation between classes which reflects the relative risk contributed, such that no one class threatens the security of the company more than any other. To achieve this may well require a number of iterations of the process to arrive at a satisfactory management solution.

(vi)   <u>Appraisal Values</u>

By considering the results of each class of business as above relative to the allocated capital over time provides a picture of the evolution of the business from period to period. If the future business written is incorporated into this, it can be seen that the earnings from

(a)   capital
(b)   past business written
(c)   future business written

can be identified and hence an appraised value calculated. The increase from period to period then constitutes the "shareholders value added" whence the added value from the current years writings

by each class of business can be derived to determine their relative contribution. This provides a valuable management tool in monitoring the economic value of the company.

(vii) Correlations between Classes

It is important to recognise that uncorrelated classes of business are likely to reduce risk. This is a factor that should be considered in formulating a business strategy both in overall capital needs and how any "benefit" is apportioned between classes. Of interest recent market results tend to suggest a degree of positive correlation between classes.

8. **Methods Available**

(i) **'Simpler' Methods**

This section considers a number of methods for allocating capital between sections of the insurers book based on readily available data.

Method A   Allocation in proportion to Net Written Premiums   The capital C, allocated to the ith section of the insurers book is the product of that section's net written premium (NWP_i) and the ratio of the insurers total capital C to its total net written premium.

$$C_i = NWP_i x \frac{C}{NWP}$$

$$\text{where } C = \sum_i C_i, \quad NWP = \sum_i NWP_i$$

The premium figure used to allocate capital at the end of an accounting period is assumed to be the net written premium for that period.   If the volume of business to be written in the section is expected to increase considerably, then an estimate of the premium income following period could be made for all sections and used in the formulae.   However this has the shortcomings that

(a) it is subjective

(b) capital is required to support business other than that taken on in the current accounting period e.g. run-off of claims reserves in respect of old underwriting years, so that even a good estimate of premium income in the following accounting period is not necessarily a better measure of capital requirements.

Comments on Method A

This method of allocation satisfies some of the practical rules laid out in Section 5.   In particular it allocates all of the insurers capital between sections in an explicit and objective manner.   The proportion of capital allocated to each section will progress smoothly from year to year if the premium income for each section grows at a similar rate. However, allocating capital in proportion to net written premiums has the following shortcoming.

(a)   Changing the relative premium rate levels of the different sections changes the allocation.   In particular, decreasing the premium rates _decreases_ the amount of capital allocated to the section.   If rates are decreased for the same cover, the probability of unfunded insurance losses arising increases, so that the capital necessary to fund these losses _increases_.   If rates are decreased to the point where losses are expected then capital is required to fund losses on the under-priced business. So, this allocation method contravenes our allocation Rule 4, in that it does not increase the capital allocated to a section as that section's exposure to unfunded losses increases.

(b)   The allocation method does not take into account the nature of the underlying risks and so contravenes Rule 5.   The uncertainty in losses and potential for unfunded losses is clearly greater from £1 of catastrophe excess of loss premium than £1 of comprehensive motor premium written to the same underwriting ratio, yet this method allocates the same amount of capital to each.

(c)   This method allocates no capital to a line of business which is no longer written since it has no written premiums.   If, however, this line take a number of years to run off and claims reserves still exist, then capital is required to protect against adverse run-off.   Thus this method contravenes Rule 3.   The opposite problem could occur with a rapidly growing line, to which too much capital would be allocated.   These problems arise through the use of one-year flow to allocate year-end capital.
In general, this method does not allow for special characteristics of each section of the insurer's book such as the nature of reinsurance, margins in reserves and growth patterns.

A stated objective of achieving a meaningful allocation of capital is to enable decisions on matters such as pricing which take into account the cost of capital.   However, consider a pricing method that is based on a minimum return on capital employed:

$ROCE_i$ =   the return on capital employed for a section i of the insurers book.

$$ROCE_i \quad = \quad \frac{\text{profit for section i}}{\text{capital allocated to section i}} \quad = \quad \frac{P_i}{C_i}$$

If capital is allocated in proportion to net written premium then

$$C_i = \frac{NWP_i}{NWP} \times C \text{ and so}$$

$$ROCE_i = \frac{Pi}{NWP_i} \times \frac{NWP}{C}$$

but $\quad \dfrac{P_i}{NWP_i} = $ operating margin = profit per unit premium

and $\quad \dfrac{NWP}{C} = $ premium to capital ratio

$= 1/\text{solvency margin}$

So, a pricing method which considers return on capital employed, where the capital has been allocated in proportion to written premium, is merely considering the return on written premium adjusted by a scaling factor.

<u>Method B (Allocation in proportion to accident year incurred claims (including direct claims expenses))</u>. At the end of an accident year accounting period, the period C_i, allocated to the ith section of the insurer's book, is given by

$$C_i = (L_i + E_i) \times \frac{C}{(L + E)}$$

$$C = \sum C_i = \text{the insurer's total capital}$$

$L_i = $ the accident year incurred losses for section 'i'

$E_i = $ the corresponding direct claims expenses for section 'i'

and $\quad L = \sum L_i, \quad E = \sum E_i$

Comments on Method B

This method allocates <u>all</u> of the insurer's capital between sections according to an explicit formula. As with Method A, this method bases the allocation of capital on a variable which might be expected to reflect increases or decreases in exposure. Whilst this method overcomes the problem posed by premium rate changes with Method A, it fails to take account of many factors materially affecting the potential for unfunded losses, for example the nature of the underlying risks and hence the variability of expected claims amounts, and the adequacy of premium rates. Moreover, the allocation is subjective, since it relies on the estimation of accident year incurred losses at the end of the accident year. For long-tailed classes this is obviously difficult to do with any degree of accuracy. In addition, since one purpose of capital is to absorb deficiencies in claims reserves then an underestimation of accident year incurred losses is doubly dangerous. Such an underestimation would result in the establishment of inadequate outstanding claims reserves for a section at the same time as the amount of capital allocated to that section being lower than it might otherwise have been.

<u>Method C (Allocation based on 'imputed equity values'.)</u> Butsic (1985) describes a subjective technique for allocating capital to lines of business in a way that reflects the relative riskiness of lines. The aim of this technique was to enable allowance to be made for the cost of capital when measuring the relative success of profit centres within a company.

The method is as follows:

(a)   Select a "reference" product line, with average perceived risk. (Butsic proposes Commercial Multiple Peril). To this line is assisgned an arbitrary premium to equity ratio close to the long term industry average premium to equity ratio for all lines, say 2.5 to 1.

(b)   Select another line, compare this with the reference line and decide on the premium to equity ratio for this line at which you would be indifferent to writing this line compared to the reference line.

For example, Property Fire, which is short-tailed and has fairly complete pricing data, at a 4 to 1 premium to equity ratio may be considered equally risky as the reference line of business at 2.5 to 1.

(c)   Repeat Stage (b) for all product lines.

These calculations would allocate 'imputed equity' amounts of 1/4 of the Property Fire premiums to that line, 40% of the Commercial Multiple Peril premiums to that line and so on.   In general, Capital allocated to line i = $(NWP)_i$ x (premium to equity ratio)$_i$.

Comments on method C: Like method B, this method is subjective, the allocation being based on a subjective assessment of the relative riskiness of writing business lines.
The method breaks our allocation Rule 2, in that the insurer's total capital is not necessarily allocated. Indeed the sum total of the imputed equity amounts may either fall short of or exceed the capital available to the insurer. A possible adjustment would be to scale allocations so that the total allocated capital equates to the total available capital. In this situation, Method C is a variant of Method A which additionally allows for the relative riskiness of writing £1 of business in each business line.

The premium figures on which an allocation is based may relate to the previous financial planning year, or may be estimates for the coming year, in which case an additional element of subjectivity is introduced. As with other methods of allocation based on written premiums, lines that are growing or shrinking may be misrepresented in the allocation and changes in rate levels can distort the allocation.   However the method does attempt to address the fundamental problem of variability.

Method D Allocation based on unpredictability of losses.

Method C adjusts the allocation subjectively for the perceived relative risk of various lines. An objective technique is difficult since this risk is not stable between periods.

If capital is to be allocated in proportion to the relative
riskiness of various lines, then the unpredictability of losses
may be used as a measure of this riskiness. A model can be
developed for predicting losses, and the error in this model may
be monitored and used to adust the allocation.

As an example, consider a model relating incurred claims to earned
premiums by accident year. Assume that, for each line i of an
insurers business, we have the following data for each of calendar
years 1976 to 1990:

a)   incurred claims $L_{i,t}$ relating to both losses from accidents
        in the calendar year t and changes in reserves for accidents
        in prior years.

b)   earned premiums, $EP_{i,t}$, in calendar year t.

For each line of business consider the set of five year time
periods ending in each of the years 1981 to 1990. For each such
period a regression model may be developed which relates incurred
claims to earned premiums over that period. A model for line i in
the period beginning in calendar year t_o is

$$L_{i,t_o+t} = A_i + B_i \cdot t + C_i \cdot EP_{i,t_o+t} \qquad t = 0,1,2,3,4,\ldots\ldots 9$$

where A_i, B_i and C_i are fitted regression coefficients that vary
by line.

For each line, the total squared error in the above equation may
be calculated for each time period. Chi-square statistics can be
developed for any line to allow comparison between time periods.
Capital may then be allocated in proportion to the square root of
the total squared error in the above equation for each line of
insurance.

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For example, an allocation of capital at 31 December 1990 may be based on the model for the time period 1981 to 1990. For this period, for line i, define

$$U_i = \sum_{t=1981}^{1990} (L_i, t - L_{i'_t})^2$$

where $L_{i,t}$ is the estimate of incurred claims from the model. Then the capital C_i allocated to the ith line is given by

$$C_i = C \times \frac{U_i}{U}$$

where $U = \sum U_i . C = \sum C_i$

Comments on Method D

A worked example would show that an allocation based on this approach would be highly unstable over time. A stochastic approach as described later also attempts to quantify claims variability.

Method E Allocation in proportion to annual marginal profit.

Classical micro-economic theory may be applied to obtain another allocation formula. When a finite, rational company in perfect competition can produce several products, that company maximises its profits when it produces less of the products that yield a smaller marginal return on input, and more of the products that yield a higher marginal return. When this company is in equilibrium, then the marginal expected return on the constraining inputs will be equal for each product that is produced.

In order to apply this theory to allocating the capital of a multi-line insurer, the following four assumptions are needed

a)   The various sections of the book are priced and sold independently.

b)   The insurer is in equilibrium.

c)   Capital is the only constraining factor in production.

d)   Each sections marginal premium-to-capital requirement is equal to the company's average premium-to-capital ratio.

Assumption a) b) and c) are unlikely to hold in practice; but assumption d) is reasonable if the relative mix of writings amongst sections is independent of capital levels. It can be shown that if total profit is a function of product mix and there is one constraining input, then the ratio of marginal profit to the marginal amount of input required to produce the product is equal for all products. For an insurer this means that the marginal profit for each section of the book is in a uniform ratio to the sections marginal premium-to-capital ratio ie

$$\frac{dP_i/dWP_i}{dC_i/dWP_i}$$ has the same value for each section i of the insurer's book

Assumption d) is that each section's marginal premium to capital ratio is equal to the average premium-to-capital ratio ie

$$\frac{dC_i}{dWP_i} = \frac{C_i}{WP_i}$$

For each section i of the insurer's book assume that the marginal profit ratio, r_i, can be derived by adding a proportion t_i, of written premiums (representing fixed expenses) to the reported operating profit P_i, and dividing by written premiums, WP_i

$$r_i = \frac{dP_i}{dWP_i} \qquad \frac{P_i + f_i\,WP_i}{WP_i}$$

So we have that

$$\frac{r_i \cdot WP_i}{C_i}$$

takes the same value for each section i, and this value is equal to

$$\frac{\sum_i (r_i \cdot WP_i)}{C}$$

The capital C_i allocated to each section can then be found from

$$C_i = r_i \cdot WP_i \cdot C / \sum (r_k \cdot WP_k)$$

ie each section's allocated capital is proportional to that section's total marginal profit.

Comments on Method E

Economists would apply this method to expected profits rather than actual, but an explicit calculation of expected marginal profits is generally not available. This method shares practical problems with other simple methods. The use of a one-year alloaction base is arbitrary, and since profits of insurers are extremely volatile, the resulting allocation will be unstable from year-to-year. Where the actual marginal profits of a section are negative then this method allocates negative capital amounts to that section. Where a section is at the bottom of its particular underwriting cycle then its marginal profits will be low at the same time as its potential for unfunded losses is high.

This method also ignores growth patterns. For example, a line that is running off will have no significant expected marginal profits, but will present significant potential to unfunded loss.

Other allocation methods have avoided the instability introduced by basing allocations on year-on-year flows of premiums, losses or profits by considering ratios of capital to claims reserves, total reserves or total liabilities. These methods however reverse the problems of growth patterns and run-off posed by one-year methods. For example, a growing line presents an exposure to unfunded losses on new business which may be out of proportion to its reserves or other liabilities arising in earlier periods. Similarly short-tailed lines presenting potential to catastrophic losses may be under represented in any allocation based on size of reserves.

Examples of these methods are given in Appendix B.

Method F Variability of Loss Ratios

The method discussed in this section is intended to be simple, and has as its aim to serve as a tool to assist management in planning their choice between different possible lines of business. It is more applicable to the collection of different lines of business that might be found in a London Market reinsurance company, in that they show marked differences in:

- typical length of tail to settlement,
- sharp volatility of the possible underwriting outcome (i.e. loss-ratio) from one underwriting year to the next.

The calculations below are not intended to be real life solutions, but are intended to bring out the structure of the method.

To initiate the method, a very simplified model of an underwriting year has been chosen, as follows:

- it is assumed that the correct cost of claims is accurately known by the end of the first development year of any given underwriting year,
- claims reserves are fully discounted.

Once the structure of the capital allocation method has been brought out, then there will be introduced some technical features necessary for a "real life" solution, i.e. :

- the final cost of claims is often not known for some (sometime many) years after the end of the underwriting year, particularly for long-tail classes so that reserve fluctuations on prior years is generally an important component of the underwriting result on a revenue year basis i.e. the best estimate of the loss-ratio for the underwriting year will fluctuate for some years into the future,

- it can often be the practice of the Company not to discount reserves, or else not to discount them at a full prospective rate of interest,

- the practical running of any insurance operation requires the ability to withstand adverse fluctuation perhaps, via an explicit reserve and may increase in importance if reserves are discounted.

Then the bare bones of the model are best understood in five steps, as described in the following paragraphs. Each step has an accompanying table with a numerical example by way of illustration.

Table 1 shows in outline how, given an average investment return of present value equal to 20 points of premium, an average loss-ratio of 102.5% of net premium can give rise to a Return on Equity (RoE) of 25% pre-tax.

Table 1:   From loss-ratio to RoE (Return on Equity)

All classes combined

Definitions :

NWP = Net Written Premium	NWP =	100
C = Claims Cost	C =	102.5
II = Interest earned on Insurance funds	II =	20
M/E = Management Expenses	M/E =	10
RoS = Return on Sales, i.e. the total insurance profit as a percentage of NWP	RoS =	7.5
E = Equity, i.e. capital allocated to the insurance operation	E =	50
i = rate of interest	i =	10%
SI = Interest earned on Shareholders' equity	SI =	5
NWP = Net Written Premium	NWP =	102.5
ROS = Insurance Margin	ROS =	7.5
ROE = Return on Equity	ROE =	25.0%

Table 1 relates to the average of all lines of business. Since different lines of business have different average periods to settlement, they can of course be written at different target loss-ratios to achieve the same RoE. Table 2 gives an example of this for six lines of business. The first two pairs of lines have equal average periods to settlement, so for the purposes of this intermediate stage, where we have assumed equal volumes written and equal capital allocated to each line, they produce indentical target loss-ratios. Later on, when we allocate different amounts of capital in line with the perceived riskiness of each line, then different target loss-ratios will emerge.

Table 2: from loss-ratio to RoE for different lines of business

	Line 1	Line 2	Line 3	Line 4	Line 5	Line 6
NWP =	100	100	100	100	100	100
C =	132.5	132.5	95	95	107.5	147.5
II =	50	50	12.5	12.5	25	65
M/E =	10	10	10	10	10	10
ROS =	7.5	7.5	7.5	7.5	7.5	7.5
E =	50	50	50	50	50	50
i =	10%	10%	10%	10%	10%	10%
SI =	0	0	0	0	0	0
NWP =	100	100	100	100	100	100
ROS =	7.5	7.5	7.5	7.5	7.5	7.5
ROE =	15.0%	15.0%	15.0%	15.0%	15.0%	15.0%

Some lines of business are riskier than others. Therefore they ought on average to be required to produce a higher expected margin to justify the greater risk. The way this higher required margin is assessed is by allocating different amounts of capital per unit of premium to the different lines of business, based upon ad hoc studies of the variability in the loss-ratio.

In reinsurance, sharp volatility of prospective underwriting results, i.e. loss-ratios, must be accepted as part and parcel of the business. One possible strategy is to allocate to each line of business sufficient capital so that a sudden fluctuation that is "reasonably possible" or "quite likely" does not wipe out the solvency margin. The idea is that a feasible result should not prevent the Company from participating in the same line of business in the following underwriting year. Although it would be possible after a very severe adverse fluctuation to ask sharedholders for more capital if the prospective returns warrant, in the course of "normal" fluctuations this should be avoided.

If a line of business existed on its own, it might be necessary to allocate sufficient capital so that, after a "reasonably likely" fluctuation, there still remains a base capital of say 50% of premium left to support the following year's underwriting. So the total capital needed would be 50% of premium plus the likely fluctuation, and target loss-ratios would be examined to see if they produce a sufficient insurance margin to give an adequate return on the capital so allocated.

When a line of business is written in a Company alongside other lines whose fluctuations are "certain" to be independent, it might be possible to reduce the minimum base capital required for each line. For the purposes of the following example, a total "respectability" solvency margin requirement of 75% of NWP has been split into:

- 25% of NWP for each separate line of business to serve as a minimum base capital,
- 25% of total NWP to be allocated between the lines of business to reflect their relative riskiness,
- 25% of total NWP as a buffer against non-underwriting uncertainties, e.g. asset fluctuations.

Suppose that an assessment of the different lines of business suggests the following "reasonably possible" degrees of unpredictability in the loss-ratio:

Line 1	25
Line 2	50
Line 3	80
Line 4	7.5
Line 5	25
Line 6	120

Then Table 3 shows the impact of the allocation method on the required target loss-ratios.

Table 3: from RoE to target loss-ratio for different lines

	Line 1	Line 2	Line 3	Line 4	Line 5	Line 6
Min E =	25	25	25	25	25	25
Fluct E =	25	50	80	7.5	25	120
Total E =	50	75	105	32.5	50	145
i =	10%	10%	10%	10%	10%	10%
SI =	5	7.5	10.5	3.25	5	14.5
NWP =	100	100	100	100	100	100
ROS =	7.5	11.25	15.75	4.875	7.5	21.75
ROE =	25.0%	25.0%	25.0%	25.0%	25.0%	25.0%
NWP =	100	100	100	100	100	100
C =	132.5	128.75	86.75	97.625	107.5	133.25
II =	50	50	12.5	12.5	25	65
M/E =	10	10	10	10	10	10
ROS =	7.5	11.25	15.75	4.875	7.5	21.75

This table already gives some insight into the risk-reward characteristics of the different lines of busines, as follows:

- lines 1 and 2 illustrate similar types of business, e.g. casualty non-proportional treaty excluding U.S. risks, and have similar run-off periods to settlement. The difference is that line 1 contains mainly "working" treaties, whereas line 2 has on average much higher deductibles and is exposed to fluctuations from variation income credits. line 2 has to be written at lower target loss-ratios,

- lines 3 and 4 illustrate similar types of business, e.g. commercial property type risks excluding the US., except that line 3 is written on a facultative basis whereas line 4 is written as proportional treaties. The facultative business in line 3 is much less "balanced" than the same business written under treaties in line 4, and so must be written to a target loss-ratio of over 10 points better,

- although line 6 has 15 points more investment income that in line 1, it has to be written to a similar loss-ratio to reflect the greater risk.

When planning the approach on how much volume to write in each line of business, the target loss-ratio needs to be compared with the prospective actual loss-ratio. A useful starting point is a study of the past loss-ratios. Table 4 shows loss-ratios for the past 10 underwriting years, with the achieved average and a "reasonably likely" degree of fluctuation. From these inputs, a prospective Return on Equity is calculated.

The fluctuation has been assessed on a cautious ad hoc basis. There are not really sufficient sample points to estimate a distribution about a mean. The underwriter will be more concerned to estimate how the likely prospective loss-ratio might differ from the historic average, taking into account recent trends in claim levels, exposures and premium rates, and whether the fluctuation margin is representative of the future account.

Table 4: From prospective loss-ratios to "risk-adjusted" RoE

4(a) : History of past loss-ratios

U/w Year	Line 1	Line 2	Line 3	Line 4	Line 5	Line 6
1981	131.7	89.8	63.4	93.9	101.0	102.8
1982	142.1	146.9	54.8	98.3	106.2	72.5
1983	125.6	129.8	31.6	98.1	99.3	57.8
1984	129.8	97.5	47.2	98.8	112.4	86.6
1985	102.9	117.6	108.9	93.7	85.8	69.2
1986	113.4	151.7	141.0	99.8	48.8	62.8
1987	85.9	85.3	53.3	90.4	74.9	202.8
1988	87.1	100.0	34.5	91.5	109.4	98.5
1989	124.7	122.6	70.8	95.8	131.6	85.0
1990	115.0	120.7	111.0	104.8	109.2	85.0
Average	115.8	116.2	71.6	96.5	97.9	92.3
Prospective	115	115	75	97.5	105	85
Fluctuation	25	50	80	7.5	25	120

4(b): Prospective loss-ratios and "risk-adjusted" RoE

	Line 1	Line 2	Line 3	Line 4	Line 5	Line 6
NWP =	100	100	100	100	100	100
C =	115	115	75	97.5	105	85
II =	50	50	10	12.5	25	37.5
M/E =	10	10	10	10	10	10
ROS =	25	25	25	5	10	42.5
Min E =	25	25	25	25	25	25
Fluct E =	25	50	80	7.5	25	120
Total E =	50	75	105	32.5	50	145
i =	10%	10%	10%	10%	10%	10%
SI =	5	7.5	10.5	3.25	5	14.5
NWP =	100	100	100	100	100	100
ROS =	25	25	25	5	10	42.5
ROE =	60.0%	43.3%	33.8%	25.4%	30.0%	39.3%

In principle, it is then possible to start planning the choice between the relative volume to write in each line of business by seeking a mix of business that maximises the total RoE subject to the constraint that the total fluctuation margin is equal to 50% of the total NWP. The RoE's shown above look high, but it should be remembered that there are still some features to be adjusted for, described in paragraph 3.7.

The balance chosen would also have regard to the normal business criteria such as:

- access to the relevant business volumes, and market penetration,
- ability and availability of underwriting staff,
- relationship with clients,
- cost and availability of reinsurance etc

The model shown above has been kept deliberately simple in order to focus on the "riskiness" versus capital allocation component. In practice, three technical features deserve immediate attention, as mentioned earlier.

(a) **Uncertainties in the cost of claims/loss-ratio**

The loss-ratio for an underwriting year is sometimes not known for sometime after the end of the year. For example, with casualty reinsurance, even excluding US exposures, the loss-ratio can be estimated by 36 months, but still be quite uncertain until 60 months. For higher level casualty, where frequency is more important to the result than severity, the uncertainty is less than the lower layers. On the latter, when a new influence on claims appears, it can often hit several underwriting years in the same way. It is assumed that line 1 requires a fluctuation margin to cover the possible short-term fluctuation of 10 points in loss-ratio on each of 3 underwriting years, i.e. 30% of NWP in all. It is also assumed that line 2 requires an additional 10% of NWP, and that the other lines can be reasonably reserved at 12 months.

(b) <u>Undiscounted reserves</u>

When reserves are not discounted, a working capital requirement arises because reserves are put up in excess of the net cash flow plus interest received. A sophisticated "profit-testing" model, tracking the early years of new business strain followed by the ultimate release of accounting profits, all discounted at a risk rate of return, is the proper way to value the cost of the capital used this way. For current purposes, an ad hoc approach is used, as follows:

Supposed on line 1 undiscounted reserves of 125% are required, and that premium is received 80% in year 1, less expenses of 10%, and the final 20% in year 2. Then the amounts of capital borrowed are approximately 55%, 25%, 15%, 5% in years 1,2,3,4. Then if the level of premium stays roughly constant in real terms over many years, this line has a permanent loan from the shreholders of 100% NWP across all underwriting years. This loan is tied up in accounting requirements and is not available to meet fluctuations or support new business. Similarly, lines 2 and 5 are required to have a "new business strain capital allocation" of 100% and 40%.

(c) <u>Fluctuation reserves</u>

If fluctuation reserves were maintained, e.g. for line 6, then they would replace the need for the fluctuation margin allocation of capital. In this example, these have been ignored.

Taking into account the above features gives a more realistic model of the prospective return on equity as follows:

Table 5:   Prospective loss-ratios and "risk-adjusted" RoE

	Line 1	Line 2	Line 3	Line 4	Line 5	Line 6
NWP =	100	100	100	100	100	100
C =	115	115	75	97.5	105	85
II =	50	50	10	12.5	25	37.5
M/E =	10	10	10	10	10	10
ROS =	25	25	25	5	10	42.5
Min E =	25	25	25	25	25	25
Fluct E =	25	50	80	7.5	25	120
Reserve uncerty =	30	10	-	-	-	-
New bus strain =	100	100	-	-	30	-
Total E =	180	185	105	32.5	80	145
i =	10%	10%	10%	10%	10%	10%
SI =	18	18.5	10.5	3.25	8	14.5
NWP =	100	100	100	100	100	100
ROS =	25	25	25	5	10	42.5
ROE =	23.9%	23.5%	33.8%	25.4%	22.5%	39.3%

The calculations shown above are designed purely to demonstrate a method and nothing should be read too deeply into the actual absolute figures used. However, they do bring out immediately a feature which we believe to be an enduring feature of the underlying economics of the longer tail casualty classes, namely that the uncertainty in the old year reserves and the need to put up undiscounted reserves impose the requirement for a much higher average insurance margin than is perhaps generally understood. It should be noted that the new business strain has not been "allocated" to the line of business by choice, but that it is an element of equity that must be serviced when considering prospective returns on this account. It should also be realised that lines 3 and 6 are "niche" areas, and that it is not possible to increase sharply the Company's presence.

(ii) **Risk theory**

Risk theory in its purest and original form looks only at the uncertainty in the claims distribution and focuses on the ruin probability.

The normal approximation gives the following formula for the required reserve U, for a ruin probability of e (with y the normal deviate at (1-e)), with a profit loading of L and pure premium of P.

(a) $U = yan^{\frac{1}{2}} - LP$

> where a   = 2nd moment of individual claim amount distribution
>   m   = 1st moment of individual claim amount distribution
> and   n   = expected number of claims
> [hence P = mn]

> [with n Poisson distributed, and large for Normal approximation to be valid]

> This expression applies over a single time frame.   However the expression over an infinite time frame, whilst appearing very different, produces similar results.

(b) $e = \exp(-RU)$

> and in the case of a poisson process this approximates to

> $R = 2Lm / a^2$
> $e = \exp(-2LPU/a^2 n)$

> i.e. $U = P * a^2 n/P^2 / (L * -2/\log(e))$

(c) $= P *$ fluctuation / loading * risk willingness

> [as described by Straub]

It can be seen that the expression for ruin probability (equation (b)) is independent of the size of the insurance company. This odd result is a consequence of the assumed continuous monitoring of the solvency of the company. In practice this assumption results in a conservative estimate for the true ruin probability.

By introducing excess of loss reinsurance the insurer may limit his claim distribution to size M. Be defining a^2 ≈ KMm, and noting that in practice K is approx. ≈ 0.6 we have

M ≑ 0.64 LP - finite time frame
M ≑ 0.43 LP - infinite time frame

for e = 0.01

Sanders 1991 has gone further in introducing the concept of a charge on the capital into the infinite time frame equation (b) above.

Using the terminology above one can substitute for R the following

$R = 2(Lm-iU)/a^2$

where i = charge on free reserves (might represent required shareholders rate of return)

This can then be used to determine the profit loading required for a given level of U. Capital may be defined as being used "most effectively" if the capital is chosen to minimise the profit loading per policy.

This gives formulae for U and L.

$U = a [n . \log(1/e) / (2 . i)]^{\frac{1}{2}}$
$L = a [n . \log(1/e) . 2 . i / m^2]^{\frac{1}{2}}$

The implications of these are:

- more capital is allocated to riskier classes (as defined by a)

- less capital is allocated to classes where i is increased

- the profit loading increases with riskier classes

- the profit loading increases with increased i

These results are not disimilar to those of Meyers in which he considers the amount of capital subscribed to an insurer by shareholders. The premise is that only expected rates of return - and not risk of ruin - enter into the amount of capital shareholders are willing to subscribe. It is the regulators who are interested in minimising the ruin probability, using the mechanism of a minimum capital level. These may come into conflict in say a downturn in the underwriting cycle when investors will look to reduce capital to achieve their required return, whilst regulators will be looking to ensuring that levels of free reserves are maintained. This conflict can only eventually be resolved by pushing up the loading to draw in sufficient extra capital to restore equilibrium.

To consider the implications for capital allocation consider the simple single time frame normal approximation model.

Suppose there are two classes of business being transacted and capital is to be allocated between them to equate their ruin probability.

If total capital is U then

$$U = y \, (a_1^2 . n_1 + a_2^2 . n_2)^{\frac{1}{2}} - (L_1 . P_1 + L_2 . P_2)$$

if split of capital is $\beta : 1-\beta$

then $(\beta U + L_1 . P_1) \, / \, (a_1^2 . n_1)^{\frac{1}{2}} = ((1-\beta)U + L_2 . P_2) \, / \, (a_2^2 . n_2)$

This cannot be solved readily except in special cases

(a) if $L_1 = L_2 = 0$

ß : 1-ß = $(a_1^2 . n_1)^{\frac{1}{2}}$: $(a_2^2 . n_2)$

i.e. allocated in proportion to the standard deviation of the claim distribution

(b) if $P_1 = P_2 = P$, $n_1 = n_2 = n$, and $a_1 / m_1 = a_2 / m_2$

then $ß = 0.5 + (L_2 - L_1) . P / U$

In general the LP term is small compared to the capital and the first expression for allocation may be good approximation.

An example of the approach with possible solutions to particular problems are given in Appendix C.

Comments on Risk Theory Approach

Risk theory concentrates solely on the claims distribution. Allocation of capital will reflect the uncertainty of the claim distribution and the expected profitability of each line of business, but will not reflect the extent to which these elements are themselves correlated with other financial elements of the insurance operation. In particular inflation is likely to increase the uncertainty in the claims and impact the value of the assets, thereby partly immunising the value of the insurance operation.

The normal approximation used in risk theory may in practice suffice to measure the risk arising from claim uncertainty. Further refinement is normally unwarranted in view of the uncertainty surrounding the other elements of the risk measure, namely the profit loading, the parameters to represent the claims distribution, and the choice of the ruin probability.

In its pure form the approach looks at the risk during the forthcoming period of exposure, and considers the claims to be independent, identically distributed with the number of claims poisson distributed. In practice these restrictive assumptions can be loosened to provide a greater degree of realism at the cost of some loss of simplicity and a degree of crude approximation. Past periods of exposure still to be run-off will still contain some uncertainty primarily in the claim distribution as the extent of incurred but not reported claims will be slight compared to the uncertainty in numbers pertaining at the commencement of the year of exposure. Correlations in the claim amounts may be modelled by allowing for an element of covariance.

Finally, the number of claims may be better modelled by allowing the underlying poisson parameter to vary over time in a cyclical fashion (to represent seasonal variation for example).

Reinsurance can be incorporated into the claims process to indicate the reduction in capital needed as a result of the reductions in claim uncertainty.

The profit loading to use can be viewed on a short or long term basis. The long term approach is favoured as in practice insurance returns will need to be looked at over the insurance cycle and capital will need to be allocated for this purpose. (The allocation will also be more robust from year to year, and provide a stable backdrop against which to measure performance.)

Risk theory could be used to determine the long run required return as once the capital is allocated to equate risk the relative levels of profit loading are uniquely determined. If the ruin probability is specified then the actual required profit loading for each class can be determined.

However it should be stressed that the limitations of risk theory (as regards the extent to which undue concentration on claim uncertainty ignores other risks to the business) should militate against reading too much into the actual level of risk given by this approach. Rather the risk theory approach is seen as a way of indicating the relative uncertainty of the claims process and how capital should be allocated accordingly.

(iii) **Stochastic Methods**

    (a) <u>Introduction</u>

This section of the paper investigates the levels of capital that are required to underwrite insurance business. The method used for the investigation is stochastically based. A model of the claim process is adopted and simulations performed in order to estimate the distribution of claims arising from a cohort of business.

Capital could be considered necessary to meet the variations in future experience, which might be quantifiable from consideration of the distribution of payments. One important concept that arises from this is the allocation of capital during the lifetime of the policy rather than the year of underwriting. Some of the implications that this has for the return on capital of the long tail and short tail lines of business are considered.

The stochastic approach allows a clear evaluation of the variability inherent in the business, both in terms of the timing and amounts of payments. The appropriate levels of capital required to support this variability are discussed briefly. This work is very much exploratory, rather than a definitive answer to the question of capital allocation.

    (b) <u>Conclusions</u>

The allocation of capital to an insurance policy for the period during which there is potential for reserve deficiency to materialise is intuitively correct and yields some interesting results. This can be investigated deterministically. However, the stochastic approach allows the consideration of the full distribution rather than one or two of the moments of that distribution and may highlight areas that would otherwise be overlooked.

Stochastic investigation offers the possibility of determining how much capital should be allocated to different products in order to attain   equal   levels   of   risk.   Different

products will require varying amounts of capital to achieve equal risk. The pricing of these products will allow for the provision of equal return, but on different amounts of capital per unit. The prices actually available can then be compared to those required (which must be done given that a perfect market does not exist).

It would appear that the levels of capital currently allocated by way of crude solvency in the exposure period, and via undiscounted reserves during the run-off imply higher premiums than capital allocation via the stochastic approach. Alternatively, these levels of capital are sufficient to meet stochastic fluctuations with very high confidence levels. This is particularly true of the long tail class.

We have investigated superficially the impact of unanticipated future inflation on the distribution of claims costs (refer to Section (e)). We did not examine the capital requirements implied by this additional variability. Stochastic models for inflation could be included in future work. This is obviously an extremely important aspect, and one which will proportionately impact longtail classes to a greater extent.

Section (d) highlights two problems that need to be resolved. First, the revision of the expected claims distribution as experience emerges and the implications of this to the release of capital. Second there is the problem that arises from the use of total claims costs to determine the confidence limits.

Initial indications suggest that further work will yield very interesting results in the areas of capital allocation, product pricing and appraised values. Investigation of appropriate claim models including inflation, attempts to derive and verify realistic parameters for different classes of business, and completely rigorous simulations should form part of further work. At this stage, we have also omitted to investigate the effects that writing differing volumes of business has on claim distributions.

(c)   Reasons for Requiring Capital

It can be useful to divide the capital requirements into those during the exposure period, usually one year, and then the requirements to run-off the claims.   This is because the ending of the exposure period is a crucial point, particularly for short tail business.   The simulated distributions are of claims from the start of the exposure period.   Once this first year has elapsed, the conditional distribution of claims given the current position is likely to be very different from the original distribution.

Considering the short tail class, by the end of the exposure period, it is known whether any catastrophes have occurred. These events give rise to a great percentage of the variability of the assumed claims model.   Thus the claims distribution given one year's experience will be much less variable.   Alternatively the amount of capital required during the exposure period is very high compared to the capital required to run-off the claims.

For the long tail class, the expiry of the exposure period is less crucial, first because there is no "catastrophe" exposure. Second the information available after one year will not alter the expected future claims distribution as much as in the short tail case because it represents a much lower percentage of total claims.   Third, events may have occurred that are not currently defined as claims, but will be at some point in the future.

Thus the capital requirements of the long tail business will be more evenly spread over the period of exposure and run-off.

The variability after the end of year one can be split into two elements.   First is the variation in the reserve requirement, arising from the number and average severity of claims assuming that inflation is known.   The second source of variation is inflation itself.

The variation in the reserve requirement, arising from claim severity and frequency can be divided further, into two elements: failure to set the reserve at the desired confidence level (estimation error); random fluctuation of the reserve about the desired confidence level. For example, one might decide to set reserves equal to the mean of future payments. Errors in the reserve may then arise either because the mean is incorrectly estimated or because future payments are not equal to the mean.

The second type of error, the problem of varying inflation, can be somewhat mitigated, under certain circumstances, by investing the technical reserves in assets whose yields will move with inflation. This should be investigated together with the introduction of stochastic inflation models.

We do not believe different amounts of capital are required, depending on the position of the insurance cycle. Provided that the characteristics of a product remain unchanged,then the variability of the claims distribution will be constant over time. The position on the cycle determines the amount of capital provided by the premium and the amount, if any, that must be allocated from shareholders funds.

(d) **Stochastic Variation in Claim Costs**

In order to investigate the capital required, we have considered a very simple profit test approach to pricing. Expenses are taken as 30% of office premiums and investment income on the premium is assumed to be received at the start of the exposure period i.e. there are no delays in the receipt of premiums.

We suggest an approach whereby all classes should yield the same return on capital and that the capital allocated should be such that the risks of different classes are equal. We assume that risk may be reflected by selecting equal confidence limits for determining the capital. However, ultimately this must be a question of the owners utility.

Capital is allocated on the basis of confidence limits of the claims distribution. Claims themselves are assumed to follow the mean amounts. Thus, if the 70% confidence limit is selected, then the capital allocated is the amount required to maintain reserves equal to the amount required were claims at that level, rather than at the mean level. Premiums are charged to meet the claims, expenses and provide a return on capital. Investment income is assumed to be earned on all invested funds. Taxation is ignored.

Initially, we ignore accounting requirements when calculating premiums. Therefore, all claim reserves are discounted to allow for future investment income. We have assumed income will be earned at 5% and arbitrarily, that the shareholders require a return of 15% on capital. The return on capital is available immediately. We have not considered when it might be released at this stage.

Assuming a constant return at all levels of risk is artificial since the return required should increase with the level of risk, resulting in the same premium for all pairings of risk and return. There is only one level of confidence at which the 15% return is appropriate and the premium at this level should equate to that charged in a perfect market.

The resulting claim ratios for a cohort are shown in Tables 1 and 2 below, for the long and short tail classes respectively (note that details of the expected claims costs are given in Appendix A). The claim ratios are based on the mean claims costs. We show the implied operating ratios as well.

Table 1 - Claim Ratios for the Long Tail Class

Confidence Limit	Claims Ratio	Operating Ratio
70.0%	105.3%	135.3%
75.0%	102.9%	132.9%
80.0%	100.2%	130.2%
85.0%	96.7%	126.7%
90.0%	92.8%	122.8%
95.0%	86.8%	116.8%
99.0%	71.7%	101.7%

Table 1 demonstrates the very long tail nature of this class of business. The significant decreases in the claim ratio for 95% and 99% confidence limits demonstrates the high additional capital that is required at these levels.

It is interesting to compare the long tail class with the results under a more traditional analysis where a solvency margin of 20% of premium is required in the year of exposure and undiscounted claims reserves are established. The capital "locked in" by the accounting standards' requirement to set undiscounted reserves in considerable. This feeds through to the premium calculations given the additional amounts of capital that must be serviced. The premium required to provide a 15% return on capital implies a 93.75% claim ratio. Thus this is roughly equivalent to capital allocated on a 90% confidence level basis, although the phasing of capital releases between the two are different.

Table 2 - Claim Ratios for the Short Tail Class

Confidence Limit	Claims Ratio	Operating Ratio
70.0%	71.0%	101.0%
75.0%	70.7%	100.7%
80.0%	70.6%	100.6%
85.0%	70.2%	100.2%
90.0%	70.1%	100.1%
95.0%	69.7%	99.7%
99.0%	67.1%	97.1%

As with the long tail classes, it is only the extreme confidence levels that demand significant additional capital allocations and hence, lower claim ratios.

If there were perfect markets, then the "fair" premium that a company could charge would exactly compensate the shareholder for the risk and would be the market rate. We have assumed arbitrarily that a shareholder requires a 15% return at the 90% confidence level, and from this have estimated the "fair" premium.

We are then able to calculate the risk at different confidence levels by solving for the IRR.   The return available at 100% confidence should be the risk free return.   These returns are shown in Table 3 below:

Table 3 - Returns Available on Short Tail from Fair Premium

Confidence Level	Return on Capital Available
65.0%	67.0%
70.0%	47.2%
75.0%	26.5%
80.0%	23.1%
85.0%	19.1%
90.0%	15.0%
95.0%	14.0%
99.0%	8.0%

There is a problematic area that we have not yet overcome. Suppose a product is priced with capital allocated to meet claims at a 75% confidence level, for example. The 75% confidence limit relates to the total claims cost.   Our first problem is how to release capital throughout the life time of a policy as the actual claims experience provides "prior information" that alters the distribution of future claims.

Second, because the confidence limits relate to overall claims costs, there can be situations where the payments during a particular period, implicit in the overall mean claims cost can exceed those implicit in the 75% confidence limit.   Given our method of capital allocation, this leads to negative capital requirements, which is not a sensible concept with our definition of capital.   This problem is particularly noticeable in the short tail class which experiences catastrophes.

An alternative approach might be to actually create a distribution for claims in each individual payment period, however there are then problems convoluting these distributions to estimate aggregate payments.

(e) Capital Required to Guard Against Unexpected Inflation

Graph A in Exhibit 1 shows the distribution of ultimate claims cost from a long tail and short tail cohort of policies on the assumption that inflation follows that assumed in the pricing basis, and with inflation a constant 1% above the assumed level. The difference between the two sets of distributions under the two assumptions is shown in Graph B.

At the point of sale an assumption is being made as to the rate of future inflation expected. As we would expect, the margin, measured as a percentage of the reserve, required to cover an adverse 1% deviation in inflation is much higher for the long tail class versus the short tail class due to the higher mean term of the liabilities.

The relatively constant level of the increase of the short tail claims cost, for all but the extremes of the distribution, indicates that there is very little variation of the mean term of the payments. A full treatment of the effects of inflation could probably be derived from a model for inflation from the point of sale to the mean term of payments.

For the long tail class, the amount of variation increases considerably with the distribution percentile. This indicates that a longer mean term is implicit in the more severe claim cost realisations - as we would expect given the model used. The full treatment of inflation effects in this instance would require a model for future inflation over a period of time. The more severe claim cost realisations have longer mean terms, and would, therefore, be subject to proportionately greater inflationary variations.

Comparison of Short and Long Tail Business

Distribution Functions with 1% Inflation Increase

Claims Cost
Thousands

Short Tail

Long Tail

Percentile
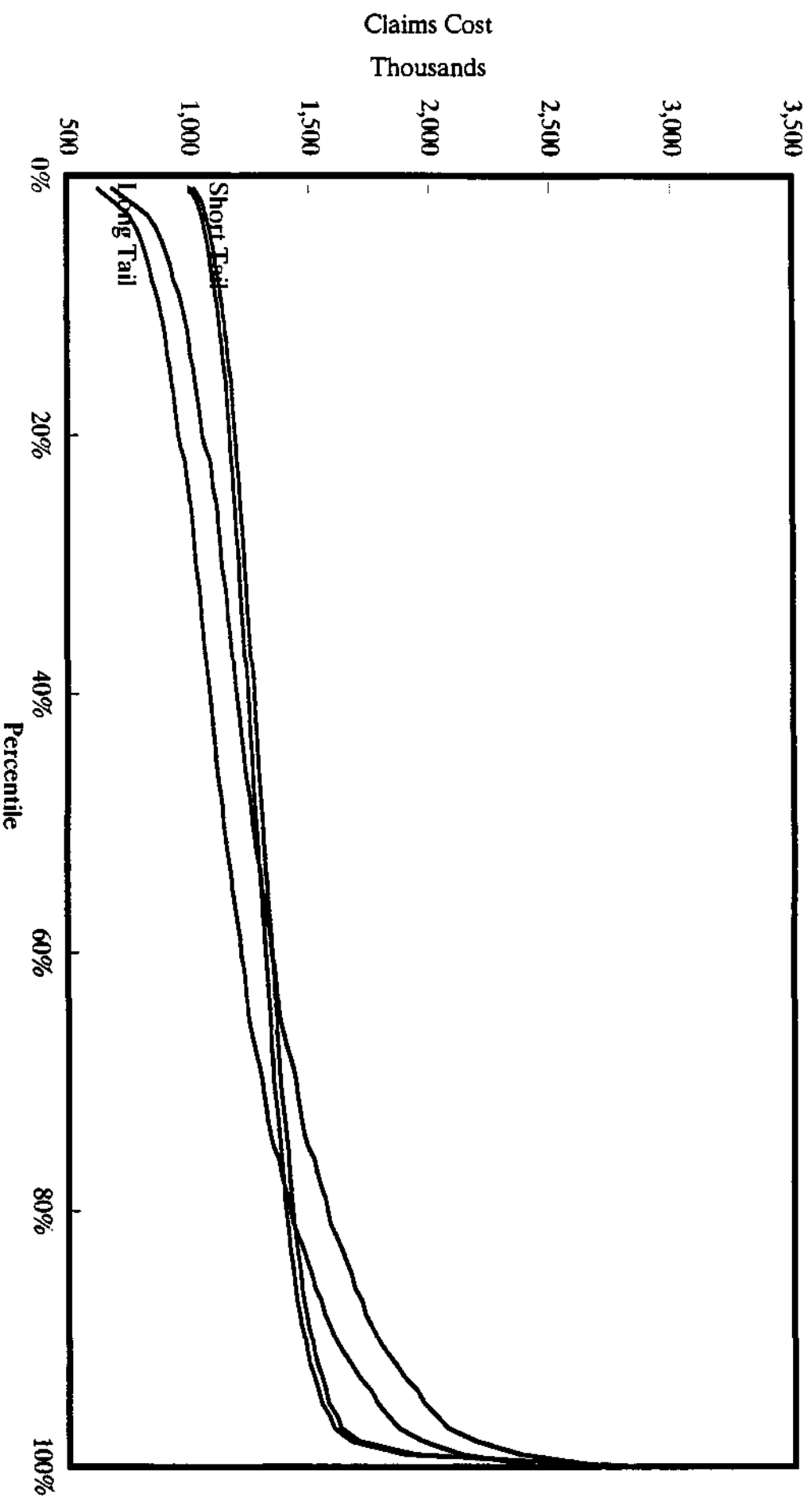
Comparison of Short and Long Tail Business

Change in Claims Distribution for a 1% Inflation Increase

Increase in Claims Cost

Thousands

Long Tail

Short Tail

Percentile
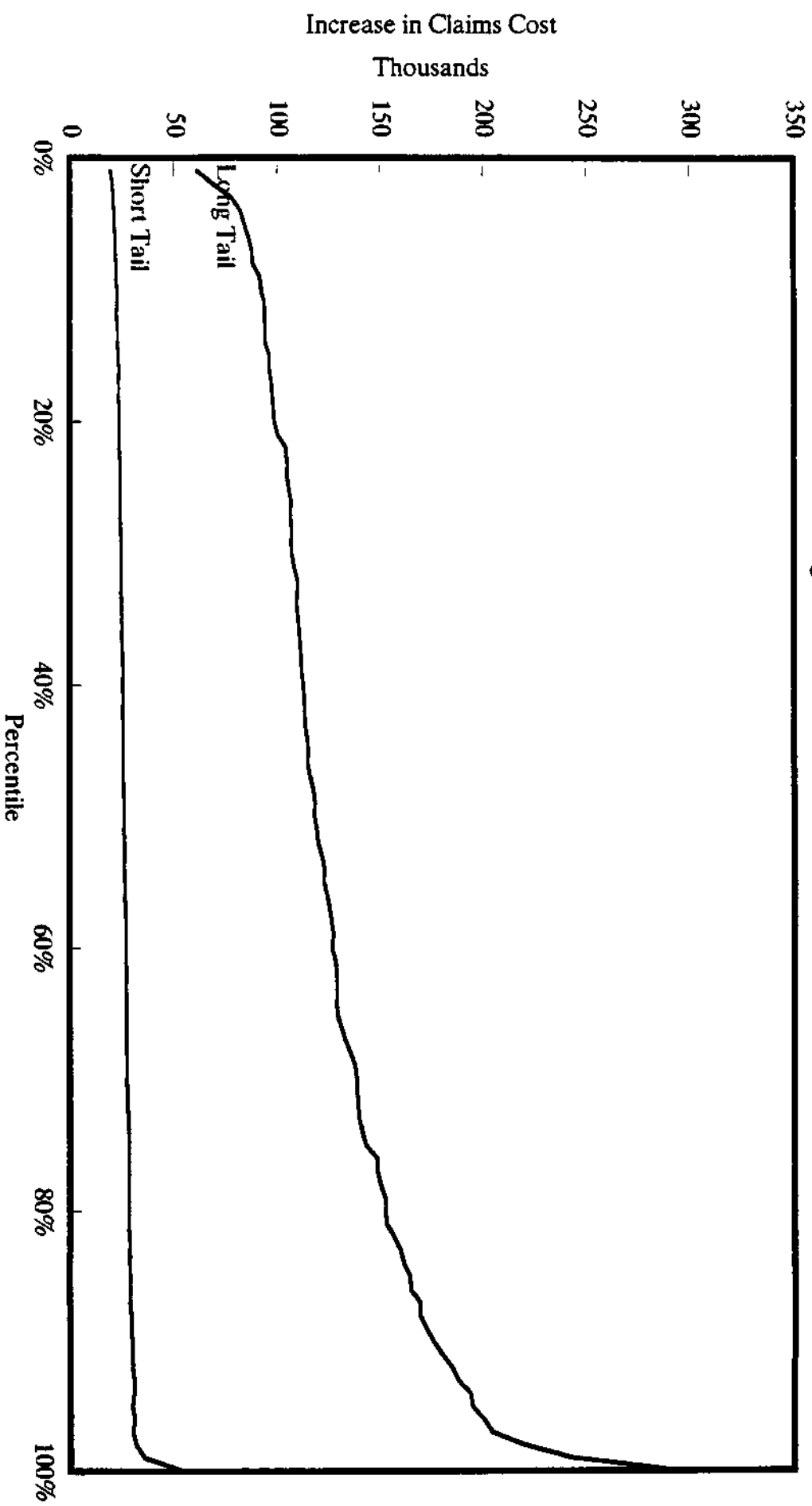
f) The Claim Process Model

The claims process model assumed is based upon that described by T. Wright in a recent paper in the JIA Vol 117 Part III "A Stochastic Model for Claims Reserving in General Insurance". In summary, the model assumes that the delay to payment has a Gamma distribution, that the number of payments is Poisson, that the mean payment at time D is proportional to D^x for some constant x, and that the coefficient of variation of the severity is independent of the time to payment.

Within this framework, payments have been assumed to be lognormally distributed. In addition, for the short tail line of business a catastrophe element is introduced. The number of catastrophes is modelled as a Poisson variable, the severity (measured as a percentage of the expected non-catastrophe claims cost) a Pareto variable, the point of occurrence during the exposure period a uniform variable, and the monthly payment pattern from the date of occurrence is assumed to be fixed for all catastrophes. The number of individual claims related to each catastrophe is not considered.

The model implicitly assumes a constant force of future inflation, which can be varied by altering the value of D. If future inflation is not assumed to be constant, then this must be introduced after the initial simulation. Further details of the models, inlcuding the parameter assumptions, are given in Appendix A.

Considerable further development of the claims process model is required. This model assumes one payment per claim, rather than a variable number. All types of insurance are subject to dependent claims arising from events. These claims need to be recognised since they may account for a considerable percentage of overall variability. Before the model could be used in practice, extensive validation against historic data would be required.

9. Comparison of Result of Methods

To be presented at the Giro conference.

10 **Lloyd's**

The Working Party did not spend much of its time on Lloyd's but there are areas worthy of future study.

The structure and operation of Lloyd's syndicates give rise to special issues differing from those facing companies. Names supporting a syndicate essentially provide a guarantee of unlimited liability. Also Premium capacity is limited by a formula calculation linked to deposited assets. In this sense, allocation of capital at the syndicate level is explicit. There is no mechanism, however, of passing capital from one year's Names to the next year's Names. The Reinsurance to Close is a premium but does not include margins for uncertainty nor credit for potential future investment income.

Syndicates often specialise in the type of business written following the underwriters special skills and knowledge. We are not aware of capital allocation for, say, profit monitoring being undertaken by underwriters on segments of a syndicates business. Underwriters, however, are keenly aware of the potential returns to Names as their future year's capacity depends on retaining and obtaining new Names.

The organisational structure of Managing Agents, Managing Syndicates and of Members Agents looking after the Names is important. Typically, Members Agents will spread a Name's capital amongst several syndicates. The selection of syndicates and the long term support of Names clearly represents a mechanism for allocating capital, or in the Lloyd's terminology, "Capacity". Agents will address a number of issues with potential Names and the likely volatility of results from a syndicate is important. We would welcome views as to how important this factor is in practice.

11 **The Capital Asset Pricing, Option Pricing and Other Frameworks**

The paper has discussed in some detail capital allocation in relation to shareholders' required returns on capital and other concepts from Financial Economic theory. There are other approaches being explored for pricing insurance products or monitoring insurance profitability in competitive markets. These approaches were taken as outside the scope of our work for this year's Working Group. The approaches are, in many cases, at an embryonic stage anyway. However, we would note that the approaches at the

moment essentially aim at determining the relationship between capital allocated and return required amongst other variables. The methods do not aim to provide a solution to the problem of how much capital is required. Having not explored this area further the Working Party would welcome views from others.

Short Tail Business

The time until payment was assumed to be a Gamma distribution with mean 1. If the parameters of the Gamma are b and c, then the mean is given by b/c, and the variance by b/c^2. In this instance b=c=2. A sample of 10,000 such gammas were generated using Statgraphics. These were then re-sampled, as required, during the main simulation.

Given the timing of a payment, the severity of the payment is assumed to be lognormally distributed with mean kD^x where D is the realisation of the gamma distribution. x is taken as 2, and k as 1,000 which, with the expected value of D^2 being 3/2, gives an expected cost per payment of 1,500. The constant coefficient of variation of the log-normal is taken as 1. The log-normal was simulated as the exponential of the underlying Normal distribution, which was generated using the Box-Muller transformation of a spreadsheet uniform variate.

Cohorts have an expected number of 800 payments assumed to follow a Poisson distribution. The expected amount of the total payments on a cohort is, therefore, 1,200,000. This amount is derived as the Poisson mean multiplied by K multiplied by the expected value of the $Gamma^2$. This last value is given by $c(c+1)/b^2$. The Poisson simulations, the re-sampling of the gamma sample and the log-normal simulations are all performed using a spreadsheet uniform random variable.

It is also reasonable to assume that a short tail class of business will be exposed to catastrophe type losses. Therefore, in addition to the normal claims simulated above, a catastrophe element is included. The event frequency is taken to be Poisson with mean 1. The event is assumed to occur with uniform likelihood at any point during the first year (i.e. the period of exposure). The monthly payment pattern from occurrence of the event is taken to be constant. The pattern assumed was 10%, 30%, 40%, 10%, 10%.

The severity of the event is taken as a Pareto with parameter 1.5. The actual severity of claims is expressed as a percentage of the expected "normal" claims cost. The minimum catastrophe cost was taken as 2% of normal claims cost. An upper limit to the cost of one catastrophe was set at 100% of normal claims cost. The number of individual claims involved in the catastrophe is not considered. If the Pareto distribution were uncapped, then the mean catastrophe cost would be 6% of "normal" claims, giving an overall mean cost of 1,272,000.

Once the simulation of one cohort is complete, then the claim payments are grouped into quarterly amounts. A total of one thousand cohorts are simulated and then bootstrapped where necessary to provide a larger sample.

Long Tail Business

The time until payment was assumed to be a Gamma distribution with mean 5. In this instance b=2 and c=0.4. A sample of 10,000 such gammas were generated using Statgraphics. These were then re-sampled, as required, during the main simulation.

Given the timing of a payment, the severity of the payment is assumed to be lognormally distributed with mean kD^x where D is the realistion of the gamma distribution. x is taken as 2, and k as 80 which, with the expected value of D^2 being 37.5, gives an expected cost per payment of 3,000. The constant coefficient of variation of the log-normal is taken as 3.

Cohorts with an expected number of 400 payments are simulated. The precise number of payments is assumed to follow a Poisson distribution. The expected amount of the total payments on a cohort is, therefore, 1,200,000. This amount is derived as the Poisson mean multiplied by K multiplied by the expected value of the Gamma^2. This last value is given by $c(c+1)/b^2$. The Poisson variables, the re-sampling of the gamma sample and the log-normal variates are all simulated using a spreadsheet uniform random variable.

Once the simulation of one cohort is complete, then the claim payments are grouped into quarterly amounts. A total of one thousand cohorts are simulated and then bootstrapped where necessary to provide a larger sample.

If we compare the long tail and short tail simulation models the following differences are evident:

1. The coefficient of variation for the long tail class is take as 3 compared to the 1 for the short tail class. Both these parameters are selected to be reasonably realistic.

2.   The average cost of short tail claims is 1,500 compared to 3,000 for long
     tail.   Again these are reasonably representative of a large UK book of
     business.

3.   The mean time to payment of a short tail claim is one year, versus five years
     for the long tail class.   The short tail assumption is realistic, however,
     the assumption for the long tail class might be considered to be on the high
     side.

4.   The variance of the time until payment of the long tail class is much higher
     than for the short tail class.

Comments on the Simulation

The quarterly payments in the simulated cohorts do not demonstrate smooth first
differences in the tail, or in the case of the long tail class for much of the
development.   This is undoubtedly the result of basing the main simulation on
samples of only 10,000 gammas.   Further work in this area should include a fuller
simulation.

Simple Allocation Methods

	DTI Net Written Premiums	DTI Net Earned Premiums	DTI Net Incurred Claims	DTI Operating Result	Assumed Fixed Expenses	Assumed Actual Marginal Profit	Assumed Premium to Capital Ratio	Imputed Capital Amounts
Year To 31st December 1989								
One Year Business								
Accounting Class								
Accident & Health	2,927	2,705	1,873	(128)	591	463	2.00	1,464
Motor Vehicle	31,598	30,979	25,274	(3,992)	6,959	2,967	2.00	15,799
Property Damage	68,215	65,621	34,267	9,935	14,633	24,568	1.67	40,847
General Liability	5,882	5,411	3,396	819	1,826	2,645	1.20	4,902
Pecuniary Loss	2,171	2,241	1,031	266	901	1,167	1.33	1,632
Total	110,793	106,957	65,841	6,900	24,910	31,810	1.71	64,644

	Method A	Method B	Method C	Method D	Method E
End 1989 Allocation					
One Year Business					
Accounting Class					
Accident & Health	0.026	0.028	0.023	0.012	0.015
Motor Vehicle	0.285	0.384	0.244	0.214	0.093
Property Damage	0.616	0.520	0.632	0.510	0.772
General Liability	0.053	0.052	0.076	0.227	0.083
Pecuniary Loss	0.020	0.016	0.025	0.037	0.037
Total	1.000	1.000	1.000	1.000	1.000

	————————Related to Method A allocation————————				
	Method A	Method B	Method C	Method D	Method E
Accounting Class					
Accident & Health	1.000	1.077	0.857	0.460	0.551
Motor Vehicle	1.000	1.346	0.857	0.751	0.327
Property Damage	1.000	0.845	1.026	0.828	1.254
General Liability	1.000	0.972	1.428	4.272	1.566
Pecuniary Loss	1.000	0.799	1.289	1.882	1.872

Note

Method A : Allocation in proportion to net written premiums.
Method B : Allocation in proportion to net incurred claims.
Method C : Allocation in proportion to 'imputed capital amounts'.
Method D : Allocation based on unpredictability of losses.
Method E : Allocation in proportion to annual marginal profit.

Capital Allocation Working Party
GISC Conference : October 1991

Simple Allocation Methods : Method D - Allocation based on unpredictability of losses

One Year Business
Accounting Class

Incurred Claims

	1981	1982	1983	1984	1985	1986	1987	1988	1989
Accident & Health	731	901	802	1,090	1,278	1,309	1,472	1,627	1,873
Motor Vehicle	12,651	9,672	9,875	11,784	15,143	14,854	16,792	20,886	25,274
Property Damage	18,435	24,001	25,705	28,315	33,466	34,640	36,142	28,236	34,267
General Liability	2,948	2,047	2,095	5,981	1,926	2,908	3,448	4,825	3,396
Pec Loss	441	476	578	743	1,568	1,531	1,514	1,456	1,031
Total	35,206	37,097	39,055	47,913	53,381	55,242	59,368	57,030	65,841

Earned Premiums

	1981	1982	1983	1984	1985	1986	1987	1988	1989
Accident & Health	1,328	1,422	1,528	1,655	1,722	1,873	2,055	2,374	2,705
Motor Vehicle	15,701	15,386	14,459	15,569	16,762	18,223	21,660	27,722	30,979
Property Damage	31,604	36,719	39,883	43,512	47,353	51,082	54,944	60,323	65,621
General Liability	3,615	3,535	3,488	3,508	3,778	3,760	4,152	4,757	5,411
Pec Loss	1,137	1,119	1,264	1,596	1,860	2,152	2,507	2,361	2,241
Total	53,465	58,181	60,622	65,840	71,475	77,090	85,318	97,537	106,957

	Deviance	Root Deviance	Allocation
Accident & Health	37,491	193.6	0.012
Motor Vehicle	11,639,478	3411.7	0.214
Property Damage	66,034,756	8126.2	0.510
General Liability	13,057,846	3613.6	0.227
Pec Loss	345,361	587.7	0.037
Total		15932.7	1.000

A regression model of the form $IC(t) = A + B.t + C.EP(t)$ has been fitted using GLIM to the above incurred claims and earned premiums data.
Deviance is the residual sum of squares.

Risk Theory Example

Particular problems arise from

(i)   liabilities take >1yr to run off

(ii)   covariance of claim amounts (legal precedents setting higher levels of award etc.)

(iii)   claim numbers vary seasonally/secularly

Solutions to these could be:

(i)   allow for sum of diminishing variances over many years

(ii)   build in covariance

(iii)   use compound poisson

For (i) we could say that the variance is only for claims amount, as the numbers should be relatively certain fairly quickly.   Also it is only required for outstanding claims (though these are likely to be for higher claim amounts).

Consider a single class of business:

Class A - characteristics

(a) medium/long tail

(b) large average claim size

(c) low frequency

(d) profit margin (inc investment income)

e.g.

no. of policies		400,000
no. of claims in year	1	3.0%
(poisson with known no.)	2	3.0%
	3	2.0%
	4	2.0%
Total number of claims		40000

X lognormal with parameters (u, sigma^2): = (5, 1.6)

    with $E(X)$ =   534

    $\text{Std}(X)$ =   1,844

    $E(X.X)$ =   3,685,807

let $\text{Cov}(X_i, X_j)$ =   0.002% of $\text{Var}(X)$

let negative binomial parameter h = 400

Assume this is year 1 distribution

If we assume each year's development alters u as given below the risk premium is as follows:

Year	u change	risk prem
1	0	6,405,464
2	0.2	7,823,651
3	0.4	6,370,553
4	0.6	7,781,011
Total		28,380,679

The variance of the claim distribution is given by

$$\text{Var}(s) = na^2 + u^2(\text{Var}(N)-n) + (n^2-n+\text{Var}(N)).\text{Cov}(X_i, X_j)$$

case 1: poisson - $\text{Var}(N) = n$: $\text{Var}(S) = na^2$

case 2: Neg Bin - $\text{Var}(N) = n + n^2/h$: $\text{Var}(S) = n.a^2 + u^2.n^2/h$

if n <u>known</u>

$$\text{Var}(S) = n.a^2 - n.u^2 + (n^2-n).\text{Cov}(X_i, X_j)$$

Variance is <u>viewed at start of each year</u>

As an approximation we can regard the number of claims outstanding at each development year following the initial year of exposure as known, with a lognormal claim amount distribution.

The variances of the claim distribution is given by the following for each separate year of origin (the column heading refers to the approximation method employed).

Year of Origin	poisson	negative binomial	poisson + covariance	negative binomial + covariance
0	1.47E+11	1.29E+12	2.56E+11	1.40E+12
-1	1.42E+11	1.42E+11	2.22E+11	2.22E+11
-2	1.21E+11	1.21E+11	1.60E+11	1.60E+11
-3	9.03E+10	9.03E+10	1.05E+11	1.05E+11
Total	5.01E+11	1.64E+12	7.43E+11	1.88E+12
Std Dev	707,758	1,280,876	861,685	1,372,040

Suppose we have a second class of business.

Class B - characteristics

(a) shorter tail
(b) smaller average claim size
(c) high frequency
(d) profit margin in premium   10%

no. of policies   400,000

no. of claims in year   1   20.0%
(poisson with known no.)   2   5.0%
                                  3   0.5%
                                  4   0.0%

Total number of claims   102000

X lognormal with parameters $(u, sigma^2)$: =   (5, 1.1)
with $E(X)$ =   272
     $Std(X)$ =   417
     $E(X.X)$ =   247,707
let $Cov(X_i, X_j)$ =   0.001% of $Var(X)$
let negative binomial parameter h = 1000

If we assume each year's development alters u as given below the risk premium is as follows:

Year	u change	risk prem
1	0	21,742,554
2	0.2	6,639,104
3	0.4	810,901
4	0.6	0
Total		29,192,560

then the variances are:

Year of Origin	poisson	negative binomial	poisson + covariance	negative binomial + covariance
0	2.53E+10	7.94E+11	4.34E+10	8.12E+11
-1	5.71E+09	5.71E+09	6.96E+09	6.96E+09
-2	7.74E+08	7.74E+08	7.89E+08	7.89E+08
-3	0.00E+00	0.00E+00	0.00E+00	0.00E+00
Total	3.17E+10	8.00E+11	5.11E+10	8.20E+11
Std Dev	178,172	894,562	226,058	905,327

If we assume the negative binomial describes the frequency distribution and that there is a covariance term in the claim amount distribution then the risk element for each class as given by their ruin probabilities is given by:

If class A has a proportion of total capital ß then the ruin probability for a class A is given by

$$y(e) = \frac{ßU + L_1 P_1}{Std(S_1)}$$

Similarly for class B

$$y(e) = \frac{(1-ß)U + L_2 P_2}{Std(S_2)}$$

i.e.

CLASS A $\frac{\beta U+(.10/.90) \times 28,380,679}{1,372,040}$

CLASS B $\frac{(1-\beta)U+(.05/.95) \times 29,192,560}{905,327}$

Suppose solvency margin U to P = 50%

then

gross premium = 63,970,266
free capital   = 31,985,133

Equating the two implies:

β =   59.2%

giving solvency margins for A and B as follows

A: 60%
B: 40%

These results are very sensitive to the choices of the parameters particularly the negative binomial and covariance parameters.

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